Probability

Multiple Choice Questions (MCQs)

1. When a pair of dice is rolled (one is yellow and the other is green), then

(a) total number of elements in sample space is 6×6.
(b) sample space is {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)}.
(c) both (a) and (b).
(d) total number of elements in sample space is $6 + 6$.

2. One die of red colour(R), one of white colour(W) and one of blue colour (B) are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Then, the sample space is

(a) $S = \{R1, R2, R3, R4, R5, R6, W1, W2, W3, W4, W5, W6, B1, B2, B3, B4, B5, B6\}$
(b) $S = \{R1, R2, R3, R4, R5, R6, W1, W2, W4, W5, B1, B3, B4, B5, B6\}$
(c) Both (a) and (b) (d) None of these

3. A die is thrown repeatedly until a six comes up. Then, the sample space for this experiment is

(a) $S = \{6, (1, 6), (2, 6), (3, 6)\}$
(b) $S = \{(6, 1), (6, 2), (6, 4), (6, 6)\}$
(c) infinite number of possibilities occur
(d) None of these

4. A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 yellow and 3 red balls. If it shows head, we throw a die. Then, the sample space for this experiment is

(a) $S = \{TY_1, TY_2, TR_1, TR_2, TR_3, H1, H2, H3, H4, H5, H6\}$
(b) $S = \{TY_1, TY_2, TR_1, H1, H4\}$
(c) $S = \{Y_1, Y_2\}$ (d) None of these

5. The empty set ϕ is called _______ event.

(a) sure (b) impossible
(c) simple (d) possible

6. The whole sample space S is called _______ event.

(a) sure (b) impossible
(c) distinct (d) negative

7. Event can be classified into various types on the basis of the ______ they have.

(a) experiment (b) sample space
(c) elements (d) None of these

8. A event which has only X sample point(s) of a sample space, is called simple event. Here, X refers to ______.

(a) two (b) three (c) one (d) zero

9. If an event has more than one sample point, then it is called a/an ______.

(a) simple event (b) elementary event
(c) compound event (d) None of these

10. For every event A, there corresponds another event A' called the ______ of A.

(a) complementary event
(b) simple event
(c) not complementary event
(d) None of these

11. If A and B are mutually exclusive events and $P(B) = \frac{1}{3}$, $P(A \cup B) = \frac{13}{21}$, then $P(A)$ is equal to

(a) $\frac{1}{7}$ (b) $\frac{4}{7}$ (c) $\frac{2}{7}$ (d) $\frac{5}{7}$

12. A bag contains 9 discs of which 4 are red, 3 are blue and 2 are yellow. The discs are similar in shape and size. A disc is drawn at random from the bag. Find the probability that it is either red or blue.

(a) $\frac{2}{9}$ (b) $\frac{7}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4}{9}$

13. The probability that a patient visiting a dentist will have a tooth extracted is 0.06, the probability that he will have a cavity filled is 0.2, and the probability that he will have a tooth extracted or a cavity filled is 0.23. What is the probability that he will have a tooth extracted as well as a cavity filled?

(a) 0.03 (b) 0.04 (c) 0.05 (d) 0.06

14. A coin is tossed once, then the sample space is

(a) $\{H\}$ (b) $\{T\}$
(c) $\{H, T\}$ (d) None of these

15. The total number of elementary events associated to the random experiment of throwing three die together is

(a) 210 (b) 216 (c) 215 (d) 220

16. A box contains 1 red and 3 identical blue balls. Two balls are drawn at random in succession without replacement. Then, the sample space for this experiment is

(a) $\{RB, BR, BB\}$ (b) $\{R, B, B\}$
(c) $\{RB\}$ (d) $\{RB, BR\}$

17. A coin is tossed twice. If the second throw result in a tail, a die is thrown. Then, the total number of possible outcomes of this experiment is

(a) 11 (b) 13 (c) 14 (d) 16

18. A coin is tossed repeatedly until a tail comes up for the first time. Then, the sample space for this experiment is

(a) $\{T, HT, HTT\}$
(b) $\{TT, TTT, HTT, THH\}$
(c) $\{T, HT, HHT, HHHT, HHHHT, ...\}$
(d) None of these

19. A pair of dice is rolled. If the outcome is a doublet, a coin is tossed. Then, the total number of outcomes for this experiment is

(a) 40 (b) 42 (c) 41 (d) 43

20. A bag contains 4 identical red balls and 3 identical black balls. The experiment consists of drawing one ball, then putting it into the bag and again drawing a ball. Then, the possible outcomes of this experiment is

(a) $\{RR, BB\}$ (b) $\{RR, B,B, RR\}$
(c) $\{BB, R\}$ (d) $\{RR, RB, BR, BB\}$

21. Two coins (a ₹ 2 coin and a ₹ 5 coin) are tossed once. The total number of elements in sample space is

(a) 2 (b) 4 (c) 3 (d) 11

22. When the sets A and B are two events associated with a sample space. Then, event $A \cup B$ denotes

(a) A and B (b) only A
(c) A or B (d) only B

23. If A and B are two events, then the set $A \cap B$ denotes the event

(a) A or B (b) A and B
(c) only A (d) only B

24. The set $A - B$ denotes the event

(a) A and B (b) A or B
(c) only A (d) A but not B

25. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows an even number". Then, E and F are

(a) mutually exclusive events
(b) exhaustive events
(c) mutually exclusive and exhaustive events
(d) None of these

26. Two dice are thrown and the sum of the numbers which come up on the dice is noted. Let us consider the following events associated with this experiment.

A : "the sum is even".
B : "the sum is a multiple of 3".
C : "the sum is less than 4".
D : "the sum is greater than 11".

Which pair of these events is mutually exclusive?

(a) A and B (b) B and C
(c) C and D (d) A and C

27. An experiment involves rolling a pair of dice. The following events are recorded.

P : The sum is greater than 9.
Q : 1 occurs on either die.
R : The sum is atleast 8 and a multiple of 3.

Which pair of these events is/are mutually exclusive?

(a) P and Q (b) Q and R
(c) Both (a) and (b) (d) None of these

28. A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$.

Find (i) $P(A' \cap B')$ (ii) $P(A \cap B')$

	(i)	(ii)
(a)	0.12	0.19
(b)	0.19	0.12
(c)	0.13	0.20
(d)	0.19	0.18

29. One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, then the probability that the card drawn is not a diamond.

(a) $\frac{1}{4}$ (b) $\frac{1}{7}$ (c) $\frac{3}{4}$ (d) $\frac{1}{5}$

30. Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that only one of them will qualify the examination.

(a) 0.11 (b) 0.10 (c) 0.12 (d) 0.13

31. If A and B are any two events having $P(A \cup B) = \frac{1}{2}$ and $P(\bar{A}) = \frac{2}{3}$, then the probability of $\bar{A} \cap B$ is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

32. If A and B be two events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$, find $P(\overline{A} \cap B)$.

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

33. If A and B be two events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$, find $P(A \cap \overline{B})$.

(a) 0.3 (b) 0.1 (c) 0.2 (d) 1.1

34. The probability of an event A occurring is 0.5 and B occurring is 0.3. If A and B are mutually exclusive, then the probability of neither A nor B occurring is

(a) 0.9 (b) 0.7 (c) 0.1 (d) 0.2

35. If A and B are any two events, then $P(A \cap B') =$

(a) $P(A) + P(B')$

(b) $P(A) + P(A \cap B)$

(c) $P(B) - P(A \cap B)$

(d) $P(A) - P(A \cap B)$

Case Based MCQs

Case I : Read the following passage and answer the questions from 36 to 40.

In Richa's home, She, Suhani and Reetika who are best friends are playing with colourful discs which are similar in shape and size. Richa has 4 red discs, Suhani has 3 blue discs and Reetika has 2 yellow discs. They put all the 9 discs in a bag. Suhani ask Richi's mother to drawn any discs from the bag randomly.

36. Find the probability that the drawn disc to be red.

(a) $\frac{1}{3}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{13}$

37. Find the probability that the drawn disc is yellow.

(a) $\frac{2}{9}$ (b) $\frac{4}{9}$ (c) $\frac{3}{9}$ (d) $\frac{2}{13}$

38. Probability that the drawn disc is not blue, is

(a) $\frac{1}{3}$ (b) $\frac{2}{9}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$

39. The probability that the drawn disc is either red or blue, is

(a) $\frac{7}{9}$ (b) $\frac{1}{3}$ (c) $\frac{3}{22}$ (d) $\frac{7}{13}$

40. Which disc has more probability to be drawn?

(a) yellow (b) orange

(c) blue (d) red

Case II : Read the following passage and answer the questions from 41 to 45.

Nishant has an electricity shop. From an agent he got three bulbs which are manufactured by a new company. Nishant wants to know if any of three bulb is defective and classified as Good 'non-defective' and bad 'defective'.

41. What is the sample space of three bulb to be good or bad?

(a) $\{BBB, BBG, BGB, GBB, GBG, BGG, GGG\}$

(b) $\{BBB, BGG, GBG, GGB, GGG\}$

(c) $\{BBB, BBG, BGG, GGG\}$

(d) $\{BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG\}$

42. Find the probability that there are no defective bulb?

(a) $\frac{1}{8}$ (b) $\frac{2}{8}$ (c) $\frac{3}{8}$ (d) $\frac{5}{8}$

43. What is the probability that there is exactly one bad bulb?

(a) $\frac{7}{8}$ (b) $\frac{5}{8}$ (c) $\frac{3}{8}$ (d) $\frac{6}{8}$

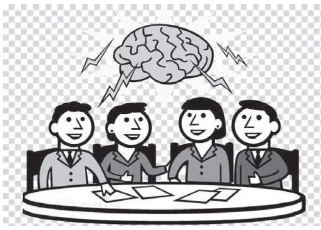
44. The probability that there are atleast two defective bulb is

(a) $\frac{9}{32}$ (b) $\frac{1}{2}$ (c) $\frac{5}{9}$ (d) $\frac{7}{13}$

45. The probability that all the bulbs are good is

(a) $\frac{1}{8}$ (b) $\frac{5}{8}$ (c) $\frac{2}{8}$ (d) $\frac{3}{8}$

Case III : Read the following passage and answer the questions from 46 to 50.



A management committee of a residential colony decided to award two members which will be selected from two men and two women for honesty, helping others and for supervising the workers to keep the colony neat and clean.

46. The total number of ways in which any of two members will be selected is

(a) 4 (b) 5 (c) 6 (d) 7

47. The probability that the committee will select no man is

(a) $\frac{2}{6}$ (b) $\frac{1}{6}$ (c) $\frac{4}{6}$ (d) $\frac{5}{6}$

48. The probability that the committee will select 1 man is

(a) $\frac{9}{10}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

49. The probability that the committee will select at most 1 man is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$

50. The probability that the committee will select exactly 2 men is

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{3}{8}$ (d) $\frac{7}{9}$

Assertion & Reasoning Based MCQs

Directions (Q.-51 to 55) : In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.
(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.
(c) Assertion is correct statement but Reason is wrong statement.
(d) Assertion is wrong statement but Reason is correct statement..

51. Assertion : A coin is tossed. If it shows head, we draw a ball from a bag consisting of 3 brown and 4 red balls; if it **shows** tail we throw a die, then the sample space of this experiment is $S = \{HB_1, HB_2, HB_3, HR_1, HR_2, HR_3, HR_4, T1, T2, T3, T4, T5, T6\}$

Reason : Consider the experiment in which a coin is tossed repeatedly until a head comes up, then the sample space is

$S = \{H, TH, TTH, TTTH, ...\}$

52. Assertion : A coin is tossed and then a die is rolled only in case a head is shown on the coin. The sample space for the experiment is

$S = \{H1, H2, H3, H4, H5, H6, T\}$

Reason : 2 boys and 2 girls are in room X, and 1 boy and 3 girls are in room Y. Then, the sample space for the experiment in which a room is selected and then a person, is

$S = \{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$ where B_i, denote the boys and G_j, denote the girls.

53. Consider the experiment of rolling a die. Then, sample space is $S = \{ 1, 2, 3, 4, 5, 6\}$

Assertion : The event E : "the number appears on the die is a multiple of 7", is an impossible event.

Reason : The event F : "the number turns up is odd or even", is a sure event.

54. Assertion : If sample space of an experiment is $S = \{1, 2, 3, 4, 5, 6\}$ and the events A and B are defined as

A : "a number less than or equal to 3 appears"

B : "a number greater than or equal to 3 appears", then A and B are exhaustive events.

Reason : Events are exhaustive if atleast one of them necessarily occur whenever the experiment is performed.

55. Assertion : The probability of drawing either an ace or a king from a pack of cards in a single draw is 2/13.

Reason : For two events A and B which are not mutually exclusive,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. An experiment consists of tossing a coin once and then throwing it second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once. Find the sample space.

2. A coin is tossed once. If it shows head, we draw a ball from a bag consisting of 3 blue and 4 white balls. If it shows tail, we throw a die twice. Describe sample space.

3. Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non-defective (N). Write the sample space of this experiment.

4. The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment.

5. If A and B are mutually exclusive events of a random experiment and $P(A \cup B) = 0.75$, $P(\overline{A}) = 0.6$, find $P(B)$.

6. Given two mutually exclusive events A and B such that $P(A) = \dfrac{1}{2}$ and $P(B) = \dfrac{1}{3}$, find $P\,(A \cup B)'$.

7. The probability that at least one of the events E_1 and E_2 occurs is 0.6. If the probability of the simultaneous occurrence of E_1 and E_2 is 0.2, find $P(\overline{E}_1) + P(\overline{E}_2)$.

8. If $P\,(E' \cap F') = 0.87$, then find $P(E \cup F)$.

9. If E and F are events such that $P(E) = \dfrac{1}{4}$, $P(F) = \dfrac{1}{2}$ and $P(E \text{ and } F) = \dfrac{1}{8}$, find $P(E \text{ or } F)$.

10. If $P(A) = \dfrac{3}{5}$ and $P(B) = \dfrac{1}{5}$, find $P(A \text{ or } B)$, if A and B are mutually exclusive events.

Short Answer Type Questions (SA-I)

11. A coin is tossed. If the outcome is a head, a die is thrown. If the die shows up an even number, the die is thrown again. What is the sample space for the experiment?

12. Two dice are rolled. Let A, B, C be the events of getting a sum of 2, a sum of 3 and a sum of 4 respectively. Then, show that

(i) A is a simple event

(ii) B and C are compound events

(iii) A and B are mutually exclusive events.

13. A die is rolled. Let 'E' be the event "die shows prime number" and 'F' be the event "die shows even number". Are E and F mutually exclusive?

14. Probability that a truck stopped at a roadblock will have faulty brakes or badly worn tires are 0.23 and 0.24 respectively. Also, the probability is 0.38 that a truck stopped at the roadblock will have faulty brakes or badly working tires. What is the probability that a truck stopped at this roadblock will have faulty brakes as well as badly worn tires?

15. The probability of two events A and B are 0.25 and 0.50 respectively. The probability of their simultaneous occurrence is 0.14. Find the probability that neither A nor B occurs.

16. If a card is drawn from a deck of 52 cards, then find the probability of getting a king or a heart or a red card.

17. In an essay competition, the odds in favour of competitors P, Q, R, S are 1 : 2, 1 : 3, 1 : 4 and 1 : 5 respectively. Find the probability that one of them wins the competition.

18. Consider the event $A = \{2, 4, 6, 8\}$ associated with the experiment of drawing a card from a deck of ten cards numbered from 1 to 10. Clearly, the sample space is $S = \{1, 2, 3, \ldots\ldots, 10\}$. Then, which of the following is/are true?

(i) $P(A') = 1 - P(A)$ (ii) $P(A') = \dfrac{3}{5}$

(iii) $P(A) + P(A') = 1$

19. If $P(A) = 0.59$, $P(B) = 0.30$ and $P(A \cap B) = 0.21$, then find $P(A' \cap B')$.

20. A coin whose faces are marked by 3, 4 is tossed 5 times. Find the probability of getting a total of 24.

Short Answer Type Questions (SA-II)

21. From a group of 2 boys and 3 girls, two children are selected at random. Consider the following events:

(i) A : Event that both the selected children are girls

(ii) B : Event that the selected group consists of one boy and one girl

(iii) C : Event that at least one boy is selected

Which pairs of events are mutually exclusive?

22. A coin is tossed three times, consider the following events :

A : 'No head appears'

B : 'Exactly one head appears'

C : 'At least two heads appear'

Do they form a set of mutually exclusive and exhaustive events?

23. A drawer has 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one item is chosen at random, what is the probability that it is rusted or a bolt?

24. (i) Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither divisible by 3 nor by 4?

(ii) What is the probability that the sum of the numbers on the two faces is divisible by 3 or 4?

25. For a post, three persons A, B and C appear in the interview. The probability of A being selected is twice that of B and the probability of B being selected is thrice that of C. What are the individual probabilities of A, B, C being selected?

26. P and Q are two candidates seeking admission in I.I.T. The probability that P is selected is 0.5 and the probability that both P and Q are selected is atmost 0.3. Prove that the probability of Q being selected is atmost 0.8.

27. Two dice are thrown simultaneously. Let E_1 denote getting a doublet, E_2 denote getting sum of the numbers appearing on the dice to be at least 10.

(i) Find $P(E_1 \text{ or } E_2)$.

(ii) Are E_1 and E_2 mutually exclusive?

28. If E and F are two events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$, find

(i) $P(E \text{ but not } F)$ (ii) $P(F \text{ but not } E)$

29. If E and F are events such that

$$P(E) = \frac{1}{4},\ P(F) = \frac{1}{2} \text{ and } P(E \text{ and } F) = \frac{1}{8},$$

(i) $P(E \text{ or } F)$ (ii) $P(\text{not } E \text{ and not } F)$.

30. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination? Explain briefly the importance of examination.

31. What is the probability that all L's come together in the word PARALLEL?

32. What is the probability that in a group of

(i) 2 people, both will have the same birthday?

(ii) 3 people, at least two will have the same birthday?

(Assuming that there are 365 days in a year.)

33. A five digit number is formed by the digits 1, 2, 3, 4, 5 without repetition. Find the probability that the number is divisible by 4.

34. Four boys and two girls sit in a row at random. Find the probability that two girls do not sit together.

35. In a lot of 12 Microwave ovens, there are 3 defective units. A person has ordered 4 of these units and since each is identically packed, the selection will be random. What is the probability that (i) all 4 units are good (ii) exactly 3 units are good (iii) at least 2 units are good?

Long Answer Type Questions (LA)

36. 20 cards are numbered from 1 to 20. One card is then drawn at random. What is the probability that the number on the card drawn is

(i) a prime number?

(ii) an odd number?

(iii) a multiple of 5?

(iv) not divisible by 3?

37. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events :

A : the sum is greater than 8

B : 2 occurs on either die

C : the sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

38. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. If the probability of getting at least one contract is 4/5, what is the probability that he will get both?

39. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(a) the student opted for NCC or NSS.

(b) the student has opted neither NCC nor NSS.

(c) the student has opted NSS but not NCC.

40. In a single throw of two dice, determine the probability of obtaining a total of 7 or 9.

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. (a) : The sample space is given by $S = \{(x, y) : x$ is the number on the yellow die and y is the number on the green die$\}$. The number of elements in this sample space is $6 \times 6 = 36$ and the sample space is given below:

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

2. (a) : Let R, W and B denote the red, white and blue die respectively. When a die will be selected, then there will be possibilities for colour R, B or W. Also, there will be 6 possibilities for 6 numbers (1 to 6) with each colour. So, the sample space is

$S = \{R1, R2, R3, R4, R5, R6, W1, W2\ W3, W4, W5, W6, B1, B2, B3, B4, B5, B6\}$

3. (c) : The sample space is

$S = \{6, (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (1, 1, 6), (1, 2, 6), (1, 3, 6), ...\}$

Hence, infinite number of possibilities occur.

4. (a) : Let H and T represent a head and a tail of a coin, respectively.

Also, let red balls are represented by R_1, R_2 and R_3 and yellow balls are represented by Y_1 and Y_2. Then, the sample space,

$S = \{TR_1, TR_2, TR_3, TY_1, TY_2, H1, H2, H3, H4, H5, H6\}$

5. (b) **6.** (a)

7. (c) : Events can be classified into various types on the basis of the elements they have.

8. (c) : If an event has only one sample point of a sample space, then it is called a simple (or elementary) event.

9. (c) : If an event has more than one sample point, then it is called a compound event.

10. (a) : For every event A, there corresponds another event A' called the complementary event of A.

11. (c) : For mutually exclusive events

$P(A \cup B) = P(A) + P(B)$

$\Rightarrow \quad \frac{13}{21} = P(A) + \frac{1}{3}$

$\Rightarrow \quad P(A) = \frac{13}{21} - \frac{1}{3} = \frac{13-7}{21} = \frac{6}{21} = \frac{2}{7}$

12. (b) : There are 9 discs in all so the total number of possible outcomes is 9.

Let the events A, B, C be defined as

A : 'the disc drawn is red'

B : 'the disc drawn is yellow'

C : 'the disc drawn is blue'

The number of red discs = 4, so $n(A) = 4$.

$\therefore \quad P(A) = \frac{4}{9}$

The number of blue discs = 3, so $n(C) = 3$.

$\therefore \quad P(C) = \frac{3}{9} = \frac{1}{3}$

The event 'either red or blue' may be described by the set 'A or C'.

Since, A and C are mutually exclusive events, we have

$P(A \text{ or } C) = P(A \cup C) = P(A) + P(C) = \frac{4}{9} + \frac{1}{3} = \frac{7}{9}$

13. (a) : Let E_1 be the event that patient visiting a dentist will have a tooth extracted and E_2 be the event that the patient will have a cavity filled. Then,

$P(E_1) = 0.06$, $P(E_2) = 0.2$ and $P(E_1 \cup E_2) = 0.23$

Now, $P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1 \cup E_2)$
$= 0.06 + 0.2 - 0.23 = 0.03$

14. (c) : A coin is tossed once, then the sample space is $S = \{H, T\}$.

15. (b) : When three dice are tossed together, then the total number of outcomes $= 6^3 = 6 \times 6 \times 6 = 216$

16. (a) : Let red ball is denoted by R and blue ball is denoted by B. Now, two balls drawn at random in succession without replacement.

Then, the sample space is $S = \{RB, BR, BB\}$

17. (c) : The sample space S is

$S = \{HH, TH, (HT, 1), (HT, 2), (HT, 3), (HT, 4), (HT, 5), (HT, 6), (TT, 1), (TT, 2), (TT, 3), (TT, 4), (TT, 5), (TT, 6)\}$

Hence, total number of outcomes = 14

18. (c) : The sample space is

$S = \{T, HT, HHT, HHHT, HHHHT, \ldots\ldots.\}$

19. (b) : The sample space associated with the given random experiment is

$S = \{(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (1, 1, H), (1, 1, T), (2, 2, H), (2, 2, T), (3, 3, H), (3, 3, T), (4, 4, H), (4, 4, T), (5, 5, H), (5, 5, T), (6, 6, H), (6, 6, T)\}$

Hence, total number of sample points = 42

20. (d) : The sample space for this experiment is $S = \{RR, RB, BR, BB\}$, where R denotes the red ball and B denotes the black ball.

21. (b) : The sample space is $S = \{HH, HT, TH, TT\}$

22. (c) **23. (b)** **24. (d)**

25. (d) : The sample space is S = {1, 2, 3, 4, 5, 6}.

E = The event "die shows 4" = {4}

F = The event "die shows an even number"
= {2, 4, 6}

$\because \quad E \cap F = \{4\} \neq \phi$

$\therefore$ E and F are not mutually exclusive.

26. (c) : There are 36 elements in the sample space, given by $S = \{(x, y) : x, y \in \{1, 2, 3, 4, 5, 6\}\}$.

Then,

$A = \{(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)\}$

$B = \{(1, 2), (2, 1), (1, 5), (5, 1), (3, 3), (2, 4), (4, 2), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)\}$

$C = \{(1, 1), (2, 1), (1, 2)\}$ and $D = \{(6, 6)\}$

We find that

$A \cap B = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 6)\} \neq \phi$

Therefore, A and B are not mutually exclusive events.

Similarly, $A \cap C \neq \phi$ and $B \cap C \neq \phi$.

Thus, the pairs, (A, B), (A, C), (B, C) are not mutually exclusive events.

But, $C \cap D = \phi$, C and D are mutually exclusive events.

27. (c) : Clearly, $P = \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$.

$Q = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)\}$.

and $R = \{(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)\}$.

Here, $P \cap Q = \phi$ and $Q \cap R = \phi$

$\therefore$ Both P, Q and Q, R are mutually exclusive.

28. (a) : Given, $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$

(i) $P(A' \cap B') = P(\overline{A \cup B}) = 1 - P(A \cup B)$
$= 1 - [P(A) + P(B) - P(A \cap B)]$
$= 1 - [0.54 + 0.69 - 0.35]$
$= 1 - 0.88 = 0.12$

(ii) $P(A \cap B') = P(A) - P(A \cap B) = 0.54 - 0.35 = 0.19$

29. (c) : When a card is drawn from a well shuffled deck of 52 cards, the number of possible outcomes is 52.

Let A be the event that 'card drawn is a diamond'

Clearly, the number of elements in set A is 13.

Therefore, $P(A) = \frac{13}{52} = \frac{1}{4}$

i.e., Probability of a diamond card $= \frac{1}{4}$

Since, A is the event 'card drawn is a diamond', so the event 'card drawn is not a diamond' is denoted by A' or 'not A'

Now, $P(\text{not } A) = 1 - P(A) = 1 - \frac{1}{4} = \frac{3}{4}$

30. (a) : Let E and F denote the events that Anil and Ashima will qualify the examination, respectively.

Given that $P(E) = 0.05$, $P(F) = 0.10$ and $P(E \cap F) = 0.02$.

P(only one of them will qualify)

$= P((E \cap F') \text{ or } (E' \cap F)) = P(E \cap F') + P(E' \cap F)$
$= P(E) - P(E \cap F) + P(F) - P(E \cap F)$
$= 0.05 - 0.02 + 0.10 - 0.02 = 0.11$

31. **(c) :** We have $P(A \cup B) = \frac{1}{2}$

$\Rightarrow$ $P(A \cup (B-A)) = \frac{1}{2}$ $\Rightarrow$ $P(A) + P(B-A) = \frac{1}{2}$

(Since A and $B - A$ are mutually exclusive)

$\Rightarrow$ $1 - P(\bar{A}) + P(B-A) = \frac{1}{2}$ $\Rightarrow$ $1 - \frac{2}{3} + P(B-A) = \frac{1}{2}$

$\Rightarrow$ $P(B-A) = \frac{1}{6}$

$\Rightarrow$ $P(\bar{A} \cap B) = \frac{1}{6}$ (Since $\bar{A} \cap B = B - A$)

32. **(a) :** $P(\overline{A} \cap B) = P(B) - P(A \cap B) = 0.2 - 0.1 = 0.1$

33. **(c)**

34. **(d) :** Here, $P(A) = 0.5$ and $P(B) = 0.3$

Now, $P(A \cup B) = P(A) + P(B)$ (Since $A \cap B = \phi$)

$= 0.5 + 0.3 = 0.8$

$\therefore$ $P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

35. **(d) :** Since $P(A \cap B') = P(A) - P(A \cap B)$

36 **(b) :** Let the event A : 'the disc drawn is red'.

The number of red disc = 4 *i.e.* $n(A) = 4$

$\Rightarrow$ $P(A) = \frac{4}{9}$

37. **(a) :** Let the event B : 'the disc drawn is yellow'?

$\Rightarrow$ $P(B) = \frac{2}{9}$

38. **(c) :** Let the event C : 'the disc drawn is blue'

The number of blue discs = 3 *i.e.*, $n(C) = 3$

$\Rightarrow$ $P(C) = \frac{3}{9} = \frac{1}{3}$

Thus $P(\overline{C}) = 1 - \frac{1}{3} = \frac{2}{3}$.

39. **(a) :** $P(A \text{ or } C) = P(A \cup C) = P(A) + P(C)$

$= \frac{4}{9} + \frac{1}{3} = \frac{7}{9}$

40. **(d) :** Since $P(A) = \frac{4}{9}$, $P(B) = \frac{2}{9}$, $P(C) = \frac{1}{3}$

and $\frac{4}{9} > \frac{3}{9} > \frac{2}{9}$

Thus, the probability of disc drawn of red colour is more.

41. **(d) :** A sample space associated with this is {BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG}

where B stands for defective or bad bulb and G for non-defective or good bulb.

42. **(a) :** Let event A : there are no defective bulb.

$\Rightarrow$ $A = \{GGG\}$

$\therefore$ $P(A) = \frac{1}{8}$

43. **(c) :** Let event B : there is exactly one bad bulb.

$\Rightarrow$ $B = \{BGG, GBG, GGB\}$

$\therefore$ $P(B) = \frac{3}{8}$

44. **(b) :** Let event C : there are at least two defective bulbs.

$\Rightarrow$ $C = \{BBG, BGB, GBB, BBB\}$

$\therefore$ $P(C) = \frac{4}{8} = \frac{1}{2}$

45. **(a) :** Let event D : there are all bulbs good.

$\Rightarrow$ $D = \{GGG\}$

$\therefore$ $P(B) = \frac{1}{8}$

46. **(c) :** Total number of persons = 2 + 2 = 4

Out of these four persons, 2 can be selected in $^4C_2$ ways *i.e.*, 6 ways.

47. **(b) :** No men will be selected means they will select two women.

Two women can be selected in $^2C_2$ ways *i.e.*, 1 way

$\therefore$ Required probability = $\frac{1}{6}$

48. **(c) :** One man will be selected means that they will select 1 man and one woman. One man out of 2 can be selected in $^2C_1$ ways and one woman out of 2 can be selected in $^2C_1$ ways.

Together they can be selected in

$^2C_1 \times {}^2C_1 = 2 \times 2 = 4$ ways.

$\therefore$ Required probability $= \frac{4}{6} = \frac{2}{3}$.

49. **(d) :** Committee will select at most 1 man means there can be no man or 1 man.

$\therefore$ Required probability $= \frac{1}{6} + \frac{2}{3} = \frac{5}{6}$.

50. **(a) :** Two men can be selected in $^2C_2$ ways *i.e.*, 1 way.

$\therefore$ Required probability $= \frac{1}{6}$.

51. **(b) : Assertion** : Let us denote brown balls by B_1, B_2, B_3 and the red balls by R_1, R_2, R_3, R_4. Then, a sample space of the experiment is

$S = \{HB_1, HB_2, HB_3, HR_1, HR_2, HR_3, HR_4, T1, T2, T3, T4, T5, T6\}$.

Reason : In the experiment, head may come up on the 1st toss, or the 2nd toss, or the 3rd toss and so on till head is obtained. Hence, the desired sample space is

$S = \{H, TH, TTH, TTTH, TTTTH, ...\}$

52. (b) : Assertion : The sample space is

$S = \{H1, H2, H3, H4, H5, H6, T\}$

where, H and T represents head and tail respectively of a coin.

Reason :

X	Y
B_1, B_2 G_1, G_2	B_3 G_3, G_4, G_5

When room X is selected, then there are four possibilities for selection of a person which are B_1, B_2, G_1, G_2. Similarly, there will be four possibilities for room Y.

So, the sample space is

$S = \{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$

53. (b) : Given, E be the event "the number appears on the die is a multiple of 7". It is impossible to have a multiple of 7 on the upper face of the die. Thus, the event $E = \phi$ is an impossible event.

The another event F is "the number turns up is odd or even". Clearly, $F = \{1, 2, 3, 4, 5, 6\} = S$, *i.e.*, all possible outcomes of the experiment ensure the occurrence of the event F. Thus, the event F is a sure event.

54 (a) : We have $A = \{1, 2, 3\}$ and $B = \{3, 4, 5, 6\}$

Since $A \cup B = S$, so, A and B are exhaustive events.

55. (b) : $P(\text{Drawing either an ace or a king}) = \dfrac{4}{52} + \dfrac{4}{52}$

$= \dfrac{2}{13}$

($\because$ Both events are mutually exclusive)

Clearly, both Assertion and Reason are correct but Reason is not the correct explanation for Assertion.

SUBJECTIVE TYPE QUESTIONS

1. The sample space S for given experiment is :

$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}$

2. When a coin is tossed and it shows head, then sample space, $S_1 = \{HB_1, HB_2, HB_3, HW_1, HW_2, HW_3, HW_4\}$

When the coin shows tail (T) then a die is thrown twice.

$S_2 = \{T(1, 1), T(1, 2), \ldots, T(1, 6), T(2, 1), T(2, 2), \ldots, T(2, 6), T(3, 1), \ldots, T(3, 6), \ldots, T(6, 1), \ldots, T(6, 6)\}$

Therefore, $S = \{HB_1, HB_2, HB_3, HW_1, HW_2, HW_3, HW_4, T(1,1), T(1,2), \ldots, T(1,6), T(2,1), \ldots, T(2,6), T(3,1), \ldots, T(3,6), T(4,1), \ldots, T(4,6), T(5,1), \ldots, T(5,6), T(6,1), \ldots, T(6,6)\}$

3. The sample space S for selecting three bulbs at random from a lot is given by

$S = \{DDD, DDN, DND, DNN, NDD, NDN, NND, NNN\}$

where D indicates a defective bulb and N a non-defective bulb.

4. The sample space S for the given experiment is

$S = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$.

5. Given, $P(A \cup B) = 0.75$ and $P(\bar{A}) = 0.6$

Now, $P(\bar{A}) = 1 - P(A) \Rightarrow 0.6 = 1 - P(A)$

$\Rightarrow P(A) = 0.4$

Also, $P(A \cup B) = P(A) + P(B) \Rightarrow 0.75 = 0.4 + P(B)$

($\because A$ and B are mutually exclusive events)

$\Rightarrow P(B) = 0.75 - 0.4$

$\therefore P(B) = 0.35$

6. Given, $P(A) = \dfrac{1}{2}, P(B) = \dfrac{1}{3}$

$\therefore P(A \cup B) = P(A) + P(B) = \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{3+2}{6} = \dfrac{5}{6}$

$P(A \cup B)' = 1 - P(A \cup B) = 1 - \dfrac{5}{6} = \dfrac{1}{6}$

7. Given, $P(E_1 \cup E_2) = 0.6$ and $P(E_1 \cap E_2) = 0.2$

$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow P(E_1) + P(E_2) = P(E_1 \cup E_2) + P(E_1 \cap E_2)$

$= (0.6 + 0.2) = 0.8$

$\Rightarrow P(E_1) + P(E_2) = 0.8$

$\Rightarrow \{1 - P(\bar{E}_1)\} + \{1 - P(\bar{E}_2)\} = 0.8$

$\Rightarrow P(\bar{E}_1) + P(\bar{E}_2) = (2 - 0.8) = 1.2$

Hence, $P(\bar{E}_1) + P(\bar{E}_2) = 1.2$

8. Given, $P(E' \cap F') = 0.87$

$\Rightarrow P(E' \cap F') = 1 - P(E \cup F) \Rightarrow 0.87 = 1 - P(E \cup F)$

$\Rightarrow P(E \cup F) = 1 - 0.87 = 0.13$

9. Given, $P(E) = \dfrac{1}{4}$, $P(F) = \dfrac{1}{2}$, $P(E \cap F) = \dfrac{1}{8}$

Now, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\Rightarrow P(E \cup F) = \dfrac{1}{4} + \dfrac{1}{2} - \dfrac{1}{8} = \dfrac{2+4-1}{8}$

$\therefore P(E \cup F) = \dfrac{5}{8}$

10. Given, $P(A) = \dfrac{3}{5}, P(B) = \dfrac{1}{5}$

Now, $P(A \cup B) = P(A) + P(B) = \dfrac{3}{5} + \dfrac{1}{5}$

[$\because A$ and B are mutually exclusive events]

$\therefore P(A \text{ or } B) = P(A \cup B) = \dfrac{4}{5}$

11. The sample space S for the experiment is given by $S = \{T, H1, H3, H5, H21, H22, H23, H24, H25, H26, H41, H42, H43, H44, H45, H46, H61, H62, H63, H64, H65, H66\}$.

12. We have, $A = \{(1, 1)\}$, $B = \{(1, 2), (2, 1)\}$ and $C = \{(1, 3), (3, 1), (2, 2)\}$

(i) Since A consists of a single sample point, it is a simple event.

(ii) Since both B and C contain more than one sample point, hence each one of them is a compound event.

(iii) Since $A \cap B = \phi$,

$\therefore$ A and B are mutually exclusive events.

13. $E = \{2, 3, 5\}$ and $F = \{2, 4, 6\}$

$\therefore$ $E \cap F = \{2\} \neq \phi$

$\therefore$ E and F are not mutually exclusive.

14. Let B be the event that a truck stopped at the roadblock will have faulty brakes and T be the event that it will have badly worn tires.

Given, $P(B) = 0.23$, $P(T) = 0.24$ and $P(B \cup T) = 0.38$.

We have to find $P(B \cap T)$.

Now, $P(B \cup T) = P(B) + P(T) - P(B \cap T)$

[By addition theorem]

$\Rightarrow$ $P(B \cap T) = P(B) + P(T) - P(B \cup T)$

$= 0.23 + 0.24 - 0.38 = 0.09$

15. We have, $P(A) = 0.25$, $P(B) = 0.50$ and $P(A \cap B) = 0.14$

$\therefore$ Required probability $= P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - (0.25 + 0.50 - 0.14) = 0.39$

16. Let S be the sample space $\therefore$ $n(S) = 52$

Let A, B, C be the events of getting a king, a heart and a red card respectively.

$\therefore$ $P(A) = \frac{4}{52} = \frac{1}{13}, P(B) = \frac{13}{52} = \frac{1}{4}, P(C) = \frac{26}{52} = \frac{1}{2}$

$P(A \cap B) = \frac{1}{52}, P(A \cap C) = \frac{2}{52} = \frac{1}{26},$

$P(B \cap C) = \frac{13}{52} = \frac{1}{4}$ and $P(A \cap B \cap C) = \frac{1}{52}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{1}{13} + \frac{1}{4} + \frac{1}{2} - \frac{1}{52} - \frac{1}{4} - \frac{1}{26} + \frac{1}{52} = \frac{7}{13}$

$\therefore$ Probability of getting a king or a heart or a red card $= \frac{7}{13}$.

17. Let A, B, C, D be the events that the competitors P, Q, R and S respectively win the competition. Then,

$P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P(C) = \frac{1}{5}$ and $P(D) = \frac{1}{6}$

Since only one competitor can win the competition. Therefore, A, B, C, D are mutually exclusive events.

$\therefore$ Required probability $= P(A \cup B \cup C \cup D)$

$= P(A) + P(B) + P(C) + P(D)$ [By addition theorem]

$= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{19}{20}$

18. If all the outcomes 1, 2, ..., 10 are considered to be equally likely, then the probability of each outcome is $\frac{1}{10}$.

Now, $P(A) = P(2) + P(4) + P(6) + P(8)$

$= \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$

Also event (not A) $= A' = \{1, 3, 5, 7, 9, 10\}$

$\therefore$ $P(A') = P(1) + P(3) + P(5) + P(7) + P(9) + P(10)$

$= \frac{6}{10} = \frac{3}{5}$

Also, we know that A' and A are mutually exclusive and exhaustive events *i.e.*,

$A \cap A' = \phi$ and $A \cup A' = S$ or $P(A \cup A') = P(S)$

$\Rightarrow$ $P(A) + P(A') = 1$

$\Rightarrow$ $P(A') = P(\text{not } A) = 1 - P(A)$

19. $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - P(A) - P(B) + P(A \cap B) = 1 - 0.59 - 0.30 + 0.21 = 0.32$

20. When we toss a coin five times and denote the sum by S, then $15 \leq S \leq 20$ which means we never get the sum 24

$\therefore$ Required probability is 0.

21. Let us name the boys as B_1 and B_2, and the girls as G_1, G_2 and G_3.

Then, $S = \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, G_1G_2, G_1G_3, G_2G_3\}$.

We have, (i) $A = \{G_1G_2, G_1G_3, G_2G_3\}$

(ii) $B = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$

(iii) $C = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, B_1B_2\}$

Clearly, $A \cap B = \phi$ and $A \cap C = \phi$.

Hence, (A, B) and (A, C) are mutually exclusive events.

22. If S be the sample space of tossing a coin three times then $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Now, A (Event when no head appears) $= \{TTT\}$

B (Event when exactly one head appears)

$= \{HTT, THT, TTH\}$

C (Event when atleast two heads appear)

$= \{HHT, HHH, HTH, THH\}$

Clearly, $A \cap B \cap C = \phi$

So, A, B and C are mutually exclusive events

Clearly, $A \cup B \cup C = S$

So, A, B and C are exhaustive events.

23. Let A be the event that the item chosen is rusted and B be the event that the item chosen is a bolt.

Total number of possible outcomes = 200

As, half of nuts and bolts are rusted

$\therefore$ Number of rusted items = 25 + 75 = 100

Hence, $P(A) = \frac{100}{200}$, $P(B) = \frac{50}{200}$

and $P(A \cap B) = \frac{25}{200}$

$\therefore$ Required probability = $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{100}{200} + \frac{50}{200} - \frac{25}{200} = \frac{125}{200} = \frac{5}{8}$.

24. Let S be the sample space associated with the experiment of throwing a pair of dice.

Then, $n(S) = 36$.

$\therefore$ Total number of possible outcomes = 36

Consider the following events.

A : The sum of the numbers on two faces is divisible by 3

B : The sum of the numbers on two faces is divisible by 4.

Then, $A = \{(1, 2), (2, 1), (1, 5), (5, 1), (3, 3), (2, 4), (4, 2), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)\}$

$B = \{(2, 2), (1, 3), (3, 1), (2, 6), (6, 2), (4, 4), (3, 5), (5, 3), (6, 6)\}$ and $A \cap B = \{(6, 6)\}$

$\therefore$ $P(A) = \frac{12}{36} = \frac{1}{3}$, $P(B) = \frac{9}{36} = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{36}$

(i) Required probability = $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B) = 1 - \{P(A) + P(B) - P(A \cap B)\}$

$= 1 - \left\{\frac{1}{3} + \frac{1}{4} - \frac{1}{36}\right\} = \frac{4}{9}$

(ii) Required probability = $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{36} = \frac{5}{9}$.

25. Let A_1, A_2 and A_3 be three events as defined below:

A_1 = Person A is selected, A_2 = Person B is selected, A_3 = Persons C is selected.

We have, $P(A_1) = 2P(A_2)$ and $P(A_2) = 3P(A_3)$

$\Rightarrow$ $P(A_1) = 6P(A_3)$ and $P(A_2) = 3P(A_3)$.

Since A_1, A_2, A_3 are mutually exclusive and exhaustive events.

$\therefore$ $A_1 \cup A_2 \cup A_3 = S$

$\Rightarrow$ $P(A_1) + P(A_2) + P(A_3) = 1$

$\Rightarrow$ $6P(A_3) + 3P(A_3) + P(A_3) = 1$

$\Rightarrow$ $10P(A_3) = 1$

$\Rightarrow$ $P(A_3) = \frac{1}{10}$

$\therefore$ $P(A_1) = \frac{6}{10}$ and $P(A_2) = \frac{3}{10}$

26. Let A_1 and A_2 be two events defined as:

$A_1 = P$ is selected, $A_2 = Q$ is selected.

We have, $P(A_1) = 0.5$ and $P(A_1 \cap A_2) \le 0.3$

Now, $P(A_1 \cup A_2) \le 1$

$\Rightarrow$ $P(A_1) + P(A_2) - P(A_1 \cap A_2) \le 1$

$\Rightarrow$ $0.5 + P(A_2) - P(A_1 \cap A_2) \le 1$

$\Rightarrow$ $P(A_2) \le 0.5 + P(A_1 \cap A_2)$

$\Rightarrow$ $P(A_2) \le 0.5 + 0.3 \Rightarrow P(A_2) \le 0.8$

27. Total number of possible outcomes = 36

$E_1 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$E_2 = \{(6, 4), (4, 6), (5, 5), (6, 5), (5, 6), (6, 6)\}$

$\therefore$ $P(E_1) = \frac{6}{36}$, $P(E_2) = \frac{6}{36}$

and $P(E_1 \cap E_2) = \frac{2}{36}$.

(i) $P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{6}{36} + \frac{6}{36} - \frac{2}{36} = \frac{6+6-2}{36} = \frac{5}{18}$

(ii) $\because$ $E_1 \cap E_2 = \{(5, 5), (6, 6)\} \neq \phi$

$\therefore$ E_1 and E_2 are not mutually exclusive.

28. Given, $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and

$P(E \text{ and } F) = \frac{1}{8} \Rightarrow P(E \cap F) = \frac{1}{8}$

(i) $P(E \text{ but not } F) = P(E \cap \bar{F}) = P(E) - P(E \cap F)$

$= \frac{1}{4} - \frac{1}{8} = \frac{2-1}{8} = \frac{1}{8}$.

(ii) $P(F \text{ but not } E) = P(F \cap \bar{E}) = P(F) - P(F \cap E)$

$= \frac{1}{2} - \frac{1}{8} = \frac{4-1}{8} = \frac{3}{8}$.

29. Given, $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \cap F) = \frac{1}{8}$

(i) $P(E \text{ or } F) = P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$

(ii) $P(\text{not } E \text{ and not } F) = P(E' \cap F') = P(E \cup F)'$

$= 1 - P(E \cup F) = 1 - \frac{5}{8} = \frac{3}{8}$

30. Let E represents the event that student passes the English examination and H represents the event that student passes the Hindi examination.

Given, $P(E \cap H) = 0.5$, $P(\bar{E} \cap \bar{H}) = 0.1$, $P(E) = 0.75$

Now, $P(\bar{E} \cap \bar{H}) = 1 - P(E \cup H)$

$\Rightarrow$ $0.1 = 1 - [P(E) + P(H) - P(E \cap H)]$

$\Rightarrow$ $0.1 = 1 - [0.75 + P(H) - 0.5]$

$\Rightarrow$ $P(H) = 1 - 0.75 + 0.5 - 0.1 = 1.5 - 0.85 = 0.65$

Therefore, the probability of passing the Hindi examination is 0.65.

31. Total number of possible outcomes $= \dfrac{8!}{3!\,2!} = 3360$

Total number of favourable outcomes (3L's as one letter)

$= \dfrac{6!}{2!} = 360$

Hence, required probability $= \dfrac{360}{3360} = \dfrac{3}{28}$

32. (i) First person may have any one of the 365 days of the year as a birthday.

Similarly, second person may have any one of 365 days of the year as a birthday.

So, the total number of ways in which two persons may have their birthdays = 365 × 365 = $(365)^2$

The number of ways in which two persons have the same birthday = 365.

Hence, required probability $= \dfrac{365}{365^2} = \dfrac{1}{365}$

(ii) Let A be the event "At least two people have the same birthday". Then, $\bar{A}$ = No two or more people have the same birthday = All the three persons have distinct birthdays.

$\therefore \quad P(\bar{A}) = \dfrac{365 \times 364 \times 363}{365^3} = \dfrac{364 \times 363}{365^2}$

Hence, required probability $= 1 - P(\bar{A})$

$$= 1 - \frac{364 \times 363}{365^2}$$

33. Total number of five digit numbers formed by the digits 1, 2, 3, 4, 5 is 5!

$\therefore$ Total number of possible outcomes = 5! = 120.

We know that a number is divisible by 4 if the number formed by last two digits is divisible by 4. Therefore last two digits can be 12, 24, 32, 52 that is, last two digits can be filled in 4 ways. But corresponding to each of these ways there are 3! = 6 ways of filling the remaining three places.

Therefore the total number of five digit numbers formed by the digits 1, 2, 3, 4, 5 and divisible by 4 = 4 × 6 = 24

$\therefore$ Favourable number of elementary events = 24

So, required probability $= \dfrac{24}{120} = \dfrac{1}{5}$

34. Consider 2 girls as a single unit. Then possible arrangements such that 2 girls sit together = 5! = 120

The number of required ways in which 2 girls are never sit together = 6! – (5! × 2)

= 720 – 240 = 480

$\therefore$ Required probability $= \dfrac{480}{6!} = \dfrac{480}{720} = \dfrac{2}{3}$

35. Total number of possible outcomes = ${}^{12}C_4$

Number of defective units = 3

Number of units that are good = 9

(i) Number of ways of selecting all 4 good units = ${}^{9}C_4$

So, required probability $= \dfrac{{}^{9}C_4}{{}^{12}C_4} = \dfrac{126}{495} = \dfrac{14}{55}$.

(ii) Number of ways of selecting exactly 3 good units $= {}^{9}C_3 \times {}^{3}C_1$

$\therefore$ Required probability $= \dfrac{{}^{9}C_3 \times {}^{3}C_1}{{}^{12}C_4} = \dfrac{84 \times 3}{495} = \dfrac{28}{55}$

(iii) Number of favourable events in which atleast 2 units are good = ${}^{9}C_2 \times {}^{3}C_2 + {}^{9}C_3 \times {}^{3}C_1 + {}^{9}C_4$

Required probability $= \dfrac{{}^{9}C_2 \cdot {}^{3}C_2 + {}^{9}C_3 \cdot {}^{3}C_1 + {}^{9}C_4}{{}^{12}C_4}$

$= \dfrac{108 + 252 + 126}{495} = \dfrac{486}{495} = \dfrac{54}{55}$

36. Clearly, the sample space is given by

$S = \{1, 2, 3, 4, 5, \ldots, 19, 20\}$ $\quad \therefore n(S) = 20$.

(i) Let E_1 denotes the event of getting a prime number.

Then, $E_1 = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$\therefore \quad n(E_1) = 8$.

$\therefore \quad P(\text{getting a prime number}) = \dfrac{n(E_1)}{n(S)} = \dfrac{8}{20} = \dfrac{2}{5}$

(ii) Let E_2 denotes the event of getting an odd number.

Then, $E_2 = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

$\therefore \quad n(E_2) = 10$.

$\therefore \quad P(\text{getting an odd number}) = \dfrac{n(E_2)}{n(S)} = \dfrac{10}{20} = \dfrac{1}{2}$.

(iii) Let E_3 be the event of getting a multiple of 5. Then,

$E_3 = \{5, 10, 15, 20\}$ $\quad \therefore n(E_3) = 4$.

$\therefore \quad P(\text{getting a multiple of 5}) = \dfrac{n(E_3)}{n(S)} = \dfrac{4}{20} = \dfrac{1}{5}$.

(iv) Let E_4 be the event of getting a number which is not divisible by 3.

Then, $E_4 = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\}$

$\therefore \quad n(E_4) = 14$.

$\therefore \quad P(\text{getting a number which is not divisible by 3})$

$$= \frac{n(E_4)}{n(S)} = \frac{14}{20} = \frac{7}{10}.$$

37. When two dice are thrown, there are 6 × 6 = 36 possible outcomes.

$\therefore \quad S$ = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore \quad A$ = The sum is greater than 8

$= \{(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)\}$

B = 2 occurs on either die

$= \{(1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (4, 2), (5, 2), (6, 2)\}$

C = The sum is at least 7 and a multiple of 3

$= \{(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)\}$

Also, $A \cap B = \phi$, $B \cap C = \phi$

and $A \cap C = \{(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)\}$

(i) Since $A \cap B = \phi$. So A and B are mutually exclusive.

(ii) Since $B \cap C = \phi$. So B and C are mutually exclusive.

(iii) Since $A \cap C \neq \phi$. So A and C are not mutually exclusive.

38. Let A be the event that a contractor will get a plumbing contract and B be the event that a contractor will get an electric contract.

$$P(A) = \frac{2}{3}, P(\overline{B}) = \frac{5}{9}, P(A \cup B) = \frac{4}{5}$$

$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad \frac{4}{5} = \frac{2}{3} + [1 - P(\overline{B})] - P(A \cap B)$$

$$\Rightarrow \quad \frac{4}{5} = \frac{2}{3} + 1 - \frac{5}{9} - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = \frac{2}{3} + 1 - \frac{5}{9} - \frac{4}{5}$$

$$= \frac{30 + 45 - 25 - 36}{45} = \frac{14}{45}$$

39. Let A represents the event that student opted for NCC and B represents the event that student opted for NSS.

Given, $n(A) = 30$, $n(B) = 32$, $n(A \cap B) = 24$

(a) P(Student opted for NCC or NSS) $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{30}{60} + \frac{32}{60} - \frac{24}{60} = \frac{30 + 32 - 24}{60} = \frac{38}{60} = \frac{19}{30}$$

(b) Probability for students that opted neither NCC nor NSS $= P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$$= 1 - \frac{19}{30} = \frac{11}{30}$$

Probability that students opted for NSS but not NCC

$$= P(\overline{A} \cap B) = P(B) - P(A \cap B)$$

$$= \frac{32}{60} - \frac{24}{60} = \frac{8}{60} = \frac{2}{15}$$

40. Total number of events = 36

Let A be the event of getting sum of 7

$= \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

Let B be the event of getting sum of 9

$= \{(3, 6), (4, 5), (5, 4), (6, 3)\}$

$$P(A) = \frac{6}{36} = \frac{1}{6}, P(B) = \frac{4}{36} = \frac{1}{9}, P(A \cap B) = \phi$$

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{6} + \frac{1}{9} = \frac{3 + 2}{18} = \frac{5}{18}$$