Chapter 6

Vector Algebra

Contents

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The word vector is derived from Latin word meaning "to carry". The subject vector analysis was developed in the later part of the 19th century by the American Physicist and mathematician Josiah Willard Gibbs (1839-1903 A.D.) and the English enginner Oliver Heaviside (1850-1925 A.D.) workign independently. Many of the ideas came earlier, however, especially from Irish mathematician William Rowen Hamilton (1805-1865 A.D.) Scottish physicist James Clerk Maxwell (1831-1879 A.D) and H.G. Grassmann (1809-1877 A.D.). It was Hamiliton who introduced scalar and vector terms In 1844, Grassman published his work in lineale ausdehnungslehre and in 1883 appeared Hamilton's Lectures on Quatermions. Hamilton's method of quaternions was a solution to the problem of multiplying vectors in three dimensional space. Maxwell used some of Hamilton's ideas in his study of electro-magnetic theory.

Vector Algebra

6.1 Introduction

Vectors represent one of the most important mathematical systems, which is used to handle certain types of problems in Geometry, Mechanics and other branches of Applied Mathematics, Physics and Engineering.

Scalar and vector quantities : Physical quantities are divided into two categories – scalar quantities and vector quantities. Those quantities which have only magnitude and which are not related to any fixed direction in space are called *scalar quantities*, or briefly scalars. Examples of scalars are mass, volume, density, work, temperature etc.

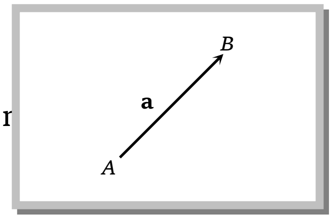
A scalar quantity is represented by a real number along with a suitable unit.

Second kind of quantities are those which have both magnitude and direction. Such quantities are called vectors. Displacement, velocity, acceleration, momentum, weight, force etc. are examples of vector quantities.

6.2 Representation of Vectors

Geometrically a vector is represented by a line segment. For example, $\mathbf{a} = \overrightarrow{AB}$. Here A is called the initial point and B, the terminal point or tip.

Magnitude or modulus of $\mathbf{a}$ is expressed as $|\mathbf{a}| = |\overrightarrow{AB}| = AB$.



Note : ❒ The magnitude of a vector is always a non-negative real r

❒ Every vector $\overrightarrow{AB}$ has the following three characteristics:

Length : The length of $\overrightarrow{AB}$ will be denoted by $|\overrightarrow{AB}|$ or AB.

Support : The line of unlimited length of which AB is a segment is called the support of the vector $\overrightarrow{AB}$.

Sense : The sense of $\overrightarrow{AB}$ is from A to B and that of $\overrightarrow{BA}$ is from B to A. Thus, the sense of a directed line segment is from its initial point to the terminal point.

6.3 Types of Vector

(1) **Zero or null vector :** A vector whose magnitude is zero is called zero or null vector and it is represented by $\vec{O}$.

The initial and terminal points of the directed line segment representing zero vector are coincident and its direction is arbitrary.

(2) **Unit vector :** A vector whose modulus is unity, is called a unit vector. The unit vector in the direction of a vector $\mathbf{a}$ is denoted by $\hat{\mathbf{a}}$, read as "*a cap*". Thus, $|\hat{\mathbf{a}}| = 1$.

$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{\text{Vector } a}{\text{Magnitude of } a}$$

Note : ❐ Unit vectors parallel to x-axis, y-axis and z-axis are denoted by $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ respectively.

❐ Two unit vectors may not be equal unless they have the same direction.

(3) **Like and unlike vectors :** Vectors are said to be like when they have the same sense of direction and unlike when they have opposite directions.

(4) **Collinear or parallel vectors :** Vectors having the same or parallel supports are called collinear vectors.

(5) **Co-initial vectors :** Vectors having the same initial point are called *co-initial vectors*.

(6) **Co-planar vectors :** A system of vectors is said to be coplanar, if their supports are parallel to the same plane.

Note : ❐ Two vectors having the same initial point are always coplanar but such three or more vectors may or may not be coplanar.

(7) **Coterminous vectors :** Vectors having the same terminal point are called *coterminous vectors*.

(8) **Negative of a vector :** The vector which has the same magnitude as the vector $\mathbf{a}$ but opposite direction, is called the negative of $\mathbf{a}$ and is denoted by $-\mathbf{a}$. Thus, if $\overrightarrow{PQ}=\mathbf{a}$, then $\overrightarrow{QP}=-\mathbf{a}$.

(9) **Reciprocal of a vector :** A vector having the same direction as that of a given vector $\mathbf{a}$ but magnitude equal to the reciprocal of the given vector is known as the reciprocal of $\mathbf{a}$ and is denoted by $\mathbf{a}^{-1}$. Thus, if $|\mathbf{a}|=a, |\mathbf{a}^{-1}|=\frac{1}{a}$

Note : ❐ A unit vector is self reciprocal.

(10) **Localized and free vectors :** A vector which is drawn parallel to a given vector through a specified point in space is called a localized vector. For example, a force acting on a rigid body is a localized vector as its effect depends on the line of action of the force. If the value of a vector depends only on its length and direction and is independent of its position in the space, it is called a free vector.

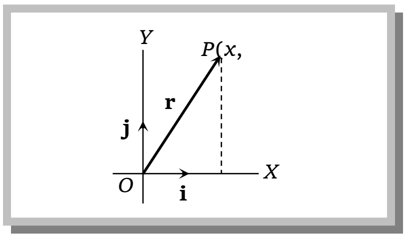
(11) **Position vectors :** The vector $\overrightarrow{OA}$ which represents the position of the point A with respect to a fixed point O (called origin) is called position vector of the point A. If (x, y, z) are co-ordinates of the point A, then $\overrightarrow{OA}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$.

(12) **Equality of vectors :** Two vectors $\mathbf{a}$ and $\mathbf{b}$ are said to be equal, if

(i) $|\mathbf{a}|=|\mathbf{b}|$ (ii) They have the same or parallel support and (iii) The same sense.

6.4 Rectangular resolution of a Vector in Two and Three dimensional systems

(1) Any vector $\mathbf{r}$ can be expressed as a linear combination of two unit vectors $\mathbf{i}$ and $\mathbf{j}$ at right angle *i.e.,* $\mathbf{r}=x\mathbf{i}+y\mathbf{j}$



The vector $x\mathbf{i}$ and $y\mathbf{j}$ are called the perpendicular component vectors of $\mathbf{r}$. The scalars x and y are called the components or resolved parts of $\mathbf{r}$ in the directions of x-axis and y-axis respectively and the ordered pair (x, y) is known as co-ordinates of point whose position vector is $\mathbf{r}$.

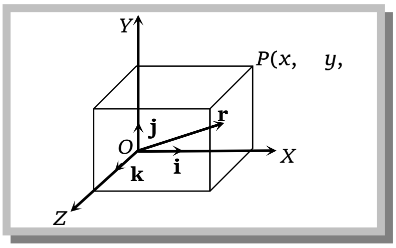
Also the magnitude of $\mathbf{r} = \sqrt{x^2 + y^2}$ and if θ be the inclination of $\mathbf{r}$ with the x-axis, then $\theta = \tan^{-1}(y/x)$

(2) If the coordinates of P are (x, y, z) then the position vector of $\mathbf{r}$ can be written as $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

The vectors $x\mathbf{i}, y\mathbf{j}$ and $z\mathbf{k}$ are called the right angled components of $\mathbf{r}$.

The scalars x, y, z are called the components or resolved parts of $\mathbf{r}$ in the directions of x-axis, y-axis and z-axis respectively and ordered triplet (x, y, z) is known as coordinates of P whose position vector is $\mathbf{r}$.

Also the magnitude or modulus of $\mathbf{r} = |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$



Direction cosines of $\mathbf{r}$ are the cosines of angles that the vector $\mathbf{r}$ makes with the positive direction of x, y and z-axes.

$\cos\alpha = l = \dfrac{x}{\sqrt{x^2 + y^2 + z^2}} = \dfrac{x}{|\mathbf{r}|}$, $\cos\beta = m = \dfrac{y}{\sqrt{x^2 + y^2 + z^2}} = \dfrac{y}{|\mathbf{r}|}$ and

$\cos\gamma = n = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}} = \dfrac{z}{|\mathbf{r}|}$

Clearly, $l^2 + m^2 + n^2 = 1$. Here $\alpha = \angle POX$, $\beta = \angle POY$ $\gamma = \angle POZ$ and $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the unit vectors along OX, OY, OZ respectively.

Example: 1 If $\mathbf{a}$ is a non-zero vector of modulus a and m is a non-zero scalar, then $m\mathbf{a}$ is a unit vector if

(a) $m = \pm 1$ (b) $m = |\mathbf{a}|$ (c) $m = \dfrac{1}{|\mathbf{a}|}$ (d) $m = \pm 2$

Solution: (c) As $m\mathbf{a}$ is a unit vector, $|m\mathbf{a}| = 1 \Rightarrow |m||\mathbf{a}| = 1 \Rightarrow |m| = \dfrac{1}{|\mathbf{a}|} \Rightarrow m = \pm\dfrac{1}{|\mathbf{a}|}$

Example: 2 For a non-zero vector $\mathbf{a}$, the set of real numbers, satisfying $|(5-x)\mathbf{a}| < |2\mathbf{a}|$ consists of all x such that

(a) $0 < x < 3$ (b) $3 < x < 7$ (c) $-7 < x < -3$ (d) $-7 < x < 3$

Solution: (b) We have, $|(5-x)\mathbf{a}| < |2\mathbf{a}|$

$\Rightarrow |5-x||\mathbf{a}| < 2|\mathbf{a}| \Rightarrow |5-x| < 2 \Rightarrow -2 < 5-x < 2 \Rightarrow 3 < x < 7$.

Example: 3 The direction cosines of the vector $3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ are

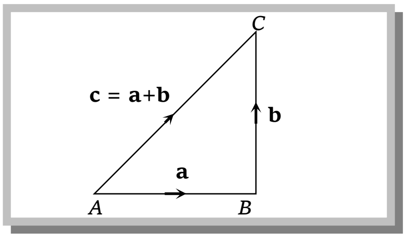
(a) $\dfrac{3}{5}, \dfrac{-4}{5}, \dfrac{1}{5}$ (b) $\dfrac{3}{5\sqrt{2}}, \dfrac{-4}{5\sqrt{2}}, \dfrac{1}{\sqrt{2}}$ (c) $\dfrac{3}{\sqrt{2}}, \dfrac{-4}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}$ (d) $\dfrac{3}{5\sqrt{2}}, \dfrac{4}{5\sqrt{2}}, \dfrac{1}{\sqrt{2}}$

Solution: (b) $\mathbf{r} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$; $|\mathbf{r}| = \sqrt{3^2 + (-4)^2 + 5^2} = 5\sqrt{2}$

Hence, direction cosines are $\dfrac{3}{5\sqrt{2}}, \dfrac{-4}{5\sqrt{2}}, \dfrac{5}{5\sqrt{2}}$ *i.e.,* $\dfrac{3}{5\sqrt{2}}, \dfrac{-4}{5\sqrt{2}}, \dfrac{1}{\sqrt{2}}$.

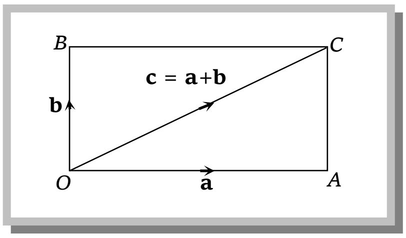
6.5 Properties of Vectors

(1) **Addition of vectors**



(i) **Triangle law of addition :** If two vectors are represented by two consecutive sides of a triangle then their sum is represented by the third side of the triangle, but in opposite direction. This is known as the triangle law of addition of vectors. Thus, if $\overrightarrow{AB} = \mathbf{a}, \overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$ then $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ *i.e.,* $\mathbf{a} + \mathbf{b} = \mathbf{c}$.

(ii) **Parallelogram law of addition :** If two vectors are represented by two adjacent sides of a parallelogram, then their sum is represented by the diagonal of the parallelogram whose initial point is the same as the initial point of the given vectors. This is known as parallelogram law of addition of vectors.



Thus, if $\overrightarrow{OA} = \mathbf{a}, \overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$

Then $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$ *i.e.,* $\mathbf{a} + \mathbf{b} = \mathbf{c}$, where OC is a diagonal of the parallelogram $OABC$.

(iii) **Addition in component form :** If the vectors are defined in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$, *i.e.,* if $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then their sum is defined as $\mathbf{a} + \mathbf{b} = (a_1 + b_1)\mathbf{i} + (a_2 + b_2)\mathbf{j} + (a_3 + b_3)\mathbf{k}$.

Properties of vector addition : Vector addition has the following properties.

(a) **Binary operation :** The sum of two vectors is always a vector.

(b) **Commutativity :** For any two vectors $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$

(c) **Associativity :** For any three vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$, $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$

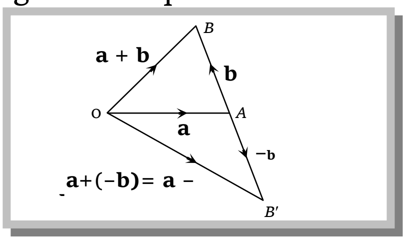
(d) **Identity :** Zero vector is the identity for addition. For any vector $\mathbf{a}$, $\mathbf{0} + \mathbf{a} = \mathbf{a} = \mathbf{a} + \mathbf{0}$

(e) **Additive inverse :** For every vector $\mathbf{a}$ its negative vector $-\mathbf{a}$ exists such that $\mathbf{a} + (-\mathbf{a}) = (-\mathbf{a}) + \mathbf{a} = \mathbf{0}$ *i.e.,* $(-\mathbf{a})$ is the additive inverse of the vector **a**.

(2) **Subtraction of vectors** : If $\mathbf{a}$ and $\mathbf{b}$ are two vectors, then their subtraction $\mathbf{a} - \mathbf{b}$ is defined as $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$ where $-\mathbf{b}$ is the negative of $\mathbf{b}$ having magnitude equal to that of $\mathbf{b}$ and direction opposite to $\mathbf{b}$.

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$

Then $\mathbf{a} - \mathbf{b} = (a_1 - b_1)\mathbf{i} + (a_2 - b_2)\mathbf{j} + (a_3 - b_3)\mathbf{k}$.



Properties of vector subtraction

(i) $\mathbf{a} - \mathbf{b} \neq \mathbf{b} - \mathbf{a}$

(ii) $(\mathbf{a} - \mathbf{b}) - \mathbf{c} \neq \mathbf{a} - (\mathbf{b} - \mathbf{c})$

(iii) Since any one side of a triangle is less than the sum and greater than the difference of the other two sides, so for any two vectors a and b, we have

(a) $|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$

(b) $|\mathbf{a} + \mathbf{b}| \geq |\mathbf{a}| - |\mathbf{b}|$

(c) $|\mathbf{a} - \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$

(d) $|\mathbf{a} - \mathbf{b}| \geq |\mathbf{a}| - |\mathbf{b}|$

(3) **Multiplication of a vector by a scalar :** If $\mathbf{a}$ is a vector and m is a scalar (*i.e.,* a real number) then $m\mathbf{a}$ is a vector whose magnitude is m times that of $\mathbf{a}$ and whose direction is the same as that of $\mathbf{a}$, if m is positive and opposite to that of $\mathbf{a}$, if m is negative.

$\therefore$ Magnitude of $m\mathbf{a} = |m\mathbf{a}| \Rightarrow m$ (magnitude of $\mathbf{a}$) = $m|\mathbf{a}|$

Again if $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ then $m\mathbf{a} = (ma_1)\mathbf{i} + (ma_2)\mathbf{j} + (ma_3)\mathbf{k}$

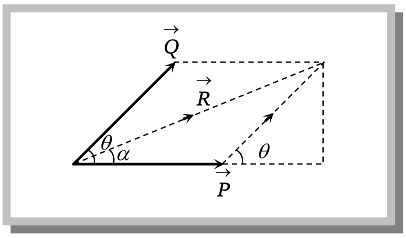
Properties of Multiplication of vectors by a scalar : The following are properties of multiplication of vectors by scalars, for vectors $\mathbf{a}, \mathbf{b}$ and scalars m, n

(i) $m(-\mathbf{a}) = (-m)\mathbf{a} = -(m\mathbf{a})$ (ii) $(-m)(-\mathbf{a}) = m\mathbf{a}$

(iii) $m(n\mathbf{a}) = (mn)\mathbf{a} = n(m\mathbf{a})$ (iv) $(m+n)\mathbf{a} = m\mathbf{a} + n\mathbf{a}$

(v) $m(\mathbf{a}+\mathbf{b}) = m\mathbf{a} + m\mathbf{b}$

(4) **Resultant of two forces**



$\vec{R} = \vec{P} + \vec{Q}$

$|\vec{R}| = R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

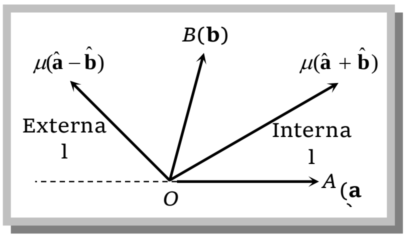
where $|\vec{P}| = P$, $|\vec{Q}| = Q$, $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

Deduction : When $|\vec{P}| = |\vec{Q}|$, *i.e.*, $P = Q$, $\tan\alpha = \dfrac{P\sin\theta}{P + P\cos\theta} = \dfrac{\sin\theta}{1+\cos\theta} = \tan\dfrac{\theta}{2}$; $\therefore \alpha = \dfrac{\theta}{2}$

Hence, the angular bisector of two unit vectors $\mathbf{a}$ and $\mathbf{b}$ is along the vector sum $\mathbf{a}+\mathbf{b}$.

Important Tips

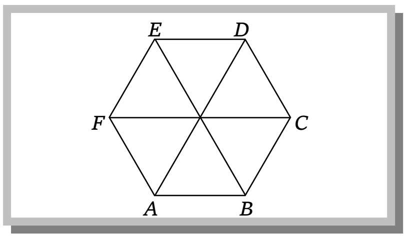
- *The internal bisector of the angle between any two vectors is along the vector sum of the corresponding unit vectors.*
- *The external bisector of the angle between two vectors is along the vector difference of the corresponding unit vectors.*



Example: 4 If $ABCDEF$ is a regular hexagon, then $\overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC} =$ **[Karnataka CET 2002]**

(a) $\vec{O}$ (b) $2\overrightarrow{AB}$ (c) $3\overrightarrow{AB}$ (d) $4\overrightarrow{AB}$

Solution: (d) We have $\overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC}$



$= (\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}) + (\overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB}) + \overrightarrow{FC}$

$= \overrightarrow{AB} + (\overrightarrow{BC} + \overrightarrow{CB}) + (\overrightarrow{CD} + \overrightarrow{DC}) + \overrightarrow{ED} + \overrightarrow{FC}$

$= \overrightarrow{AB} + \vec{O} + \vec{O} + \overrightarrow{AB} + 2\overrightarrow{AB} = 4\overrightarrow{AB}$ $[\overrightarrow{ED} = \overrightarrow{AB}, \overrightarrow{FC} = 2\overrightarrow{AB}]$

Example: 5 The unit vector parallel to the resultant vector of $2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ and $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ is

(a) $\frac{1}{7}(3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})$ (b) $\dfrac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$ (c) $\dfrac{\mathbf{i}+\mathbf{j}+2\mathbf{k}}{\sqrt{6}}$ (d) $\dfrac{1}{\sqrt{69}}(-\mathbf{i} - \mathbf{j} + 8\mathbf{k})$

Solution: (a) Resultant vector $\mathbf{r} = (2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}) + (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$

Unit vector parallel to $\mathbf{r}$ $= \dfrac{1}{|\mathbf{r}|}\mathbf{r} = \dfrac{1}{\sqrt{3^2 + 6^2 + (-2)^2}}(3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}) = \dfrac{1}{7}(3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})$

Example: 6 If the sum of two vectors is a unit vector, then the magnitude of their difference is

[Kurukshetra CEE 1996; Rajasthan PET 1996]

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) 1

Solution: (b) Let $|\mathbf{a}|=1$, $|\mathbf{b}|=1$ and $|\mathbf{a}+\mathbf{b}|=1$ $\Rightarrow$ $|\mathbf{a}+\mathbf{b}|^2=1$ $\Rightarrow$ $1+1+2\cos\theta=1$ $\Rightarrow \cos\theta=-\frac{1}{2}$ $\Rightarrow \theta=120^o$

$\therefore$ $|\mathbf{a}-\mathbf{b}|^2=1+1-2\cos\theta=3 \Rightarrow |\mathbf{a}-\mathbf{b}|=\sqrt{3}$.

Example: 7 The length of longer diagonal of the parallelogram constructed on 5**a** + 2**b** and **a** – 3**b**, it is given that $|\mathbf{a}|=2\sqrt{2}, |\mathbf{b}|=3$ and angle between **a** and **b** is $\frac{\pi}{4}$, is

(a) 15 (b) $\sqrt{113}$ (c) $\sqrt{593}$ (d) $\sqrt{369}$

Solution: (c) Length of the two diagonals will be $d_1=|(5\mathbf{a}+2\mathbf{b})+(\mathbf{a}-3\mathbf{b})|$ and $d_2=|(5\mathbf{a}+2\mathbf{b})-(\mathbf{a}-3\mathbf{b})|$

$\Rightarrow d_1=|6\mathbf{a}-\mathbf{b}|$, $d_2=|4\mathbf{a}+5\mathbf{b}|$

Thus, $d_1=\sqrt{|6\mathbf{a}|^2+|-\mathbf{b}|^2+2|6\mathbf{a}||-\mathbf{b}|\cos(\pi-\pi/4)}$ = $\sqrt{36(2\sqrt{2})^2+9+12.2\sqrt{2}.3.\left(-\frac{1}{\sqrt{2}}\right)}$ = 15.

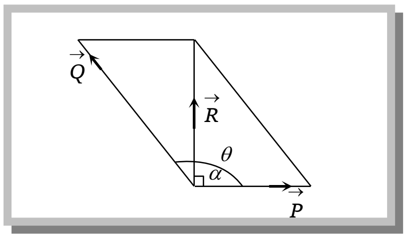
$$d_2=\sqrt{|4\mathbf{a}|^2+|5\mathbf{b}|^2+2|4\mathbf{a}||5\mathbf{b}|\cos\frac{\pi}{4}} = \sqrt{16\times 8+25\times 9+40\times 2\sqrt{2}\times 3\times\frac{1}{\sqrt{2}}} = \sqrt{593}.$$

$\therefore$ Length of the longer diagonal = $\sqrt{593}$

Example: 8 The sum of two forces is $18\ N$ and resultant whose direction is at right angles to the smaller force is $12N$. The magnitude of the two forces are

(a) 13, 5 (b) 12, 6 (c) 14, 4 (d) 11, 7

Solution: (a) We have, $|\vec{P}|+|\vec{Q}|=18N$; $|\vec{R}|=|\vec{P}+\vec{Q}|=12N$



$\alpha=90^o \Rightarrow P+Q\cos\theta=0 \Rightarrow Q\cos\theta=-P$

Now, $R^2=P^2+Q^2+2PQ\cos\theta \Rightarrow R^2=P^2+Q^2+2P(-P)=Q^2-P^2$

$\Rightarrow 12^2=(P+Q)(Q-P)=18(Q-P)$

$\Rightarrow Q-P=8$ and $Q+P=18 \Rightarrow Q=13$, $P=5$

$\therefore$ Magnitude of two forces are $5N$, $13N$.

Example: 9 The vector $\mathbf{c}$, directed along the internal bisector of the angle between the vectors $\mathbf{a}=7\mathbf{i}-4\mathbf{j}-4\mathbf{k}$ and $\mathbf{b}=-2\mathbf{i}-\mathbf{j}+2\mathbf{k}$ with $|\mathbf{c}|=5\sqrt{6}$, is

(a) $\frac{5}{3}(\mathbf{i}-7\mathbf{j}+2\mathbf{k})$ (b) $\frac{5}{3}(5\mathbf{i}+5\mathbf{j}+2\mathbf{k})$ (c) $\frac{5}{3}(\mathbf{i}+7\mathbf{j}+2\mathbf{k})$ (d) $\frac{5}{3}(-5\mathbf{i}+5\mathbf{j}+2\mathbf{k})$

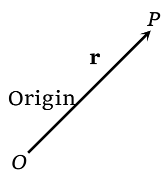
Solution: (a) Let $\mathbf{a}=7\mathbf{i}-4\mathbf{j}-4\mathbf{k}$ and $\mathbf{b}=-2\mathbf{i}-\mathbf{j}+2\mathbf{k}$

Now required vector $\mathbf{c}=\lambda\left(\frac{\mathbf{a}}{|\mathbf{a}|}+\frac{\mathbf{b}}{|\mathbf{b}|}\right)=\lambda\left(\frac{7\mathbf{i}-4\mathbf{j}-4\mathbf{k}}{9}+\frac{-2\mathbf{i}-\mathbf{j}+2\mathbf{k}}{3}\right)=\frac{\lambda}{9}(\mathbf{i}-7\mathbf{j}+2\mathbf{k})$

$|\mathbf{c}|^2=\frac{\lambda^2}{81}\times 54=150 \Rightarrow \lambda=\pm 15 \Rightarrow \mathbf{c}=\pm\frac{5}{3}(\mathbf{i}-7\mathbf{j}+2\mathbf{k})$

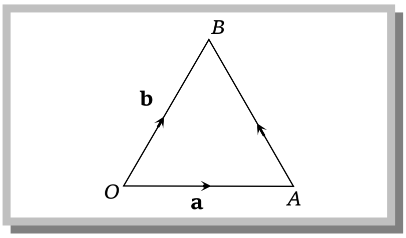
6.6 Position Vector

If a point O is fixed as the origin in space (or plane) and P is any point, then $\overrightarrow{OP}$ is called the position vector of P with respect to O.



If we say that P is the point $\mathbf{r}$, then we mean that the position vector of P is $\mathbf{r}$ with respect to some origin O.

(1) $\overrightarrow{AB}$ **in terms of the position vectors of points A and B :** If $\mathbf{a}$ and $\mathbf{b}$ are position vectors of points A and B respectively. Then, $\overrightarrow{OA} = \mathbf{a}, \overrightarrow{OB} = \mathbf{b}$



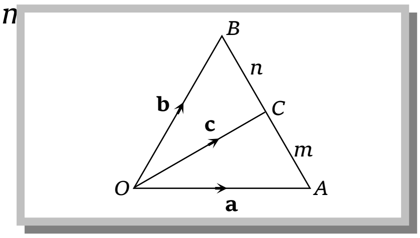
In ΔOAB, we have $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB} \Rightarrow \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$

$\Rightarrow \quad \overrightarrow{AB}$ = (Position vector of B) – (Position vector of A)

$\Rightarrow \quad \overrightarrow{AB}$ = (Position vector of head) – (Position vector of tail)

(2) **Position vector of a dividing point**

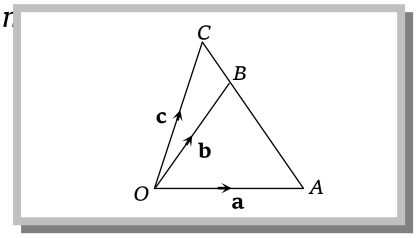
(i) **Internal division :** Let A and B be two points with position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, and let C be a point dividing AB internally in the ratio m



Then the position vector of C is given by

$$\overrightarrow{OC} = \frac{m\mathbf{b} + n\mathbf{a}}{m+n}$$

(ii) **External division :** Let A and B be two points with position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively and let C be a point dividing AB externally in the ratio m



Then the position vector of C is given by

$$\overrightarrow{OC} = \frac{m\mathbf{b} - n\mathbf{a}}{m-n}$$

Important Tips

☞ *Position vector of the mid point of AB is* $\frac{\mathbf{a}+\mathbf{b}}{2}$

☞ *If* $\mathbf{a}, \mathbf{b}, \mathbf{c}$ *are position vectors of vertices of a triangle, then position vector of its centroid is* $\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$

☞ *If* $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ *are position vectors of vertices of a tetrahedron, then position vector of its centroid is* $\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}}{4}$.

Example: 10 If position vector of a point A is $\mathbf{a}+2\mathbf{b}$ and $\mathbf{a}$ divides AB in the ratio 2 : 3, then the position vector of B is **[MP PET 2002]**

(a) $2\mathbf{a}-\mathbf{b}$ (b) $\mathbf{b}-2\mathbf{a}$ (c) $\mathbf{a}-3\mathbf{b}$ (d) $\mathbf{b}$

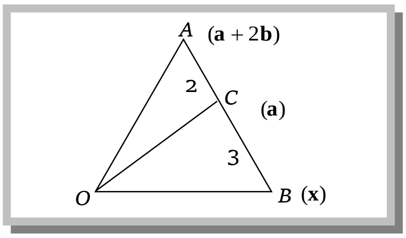
Solution: (c) Let position vector of B is $\mathbf{x}$.

The point $C(\mathbf{a})$ divides AB in 2 : 3.

$\therefore \quad \mathbf{a} = \frac{2\mathbf{x}+3(\mathbf{a}+2\mathbf{b})}{2+3}$

$\Rightarrow 5\mathbf{a} = 2\mathbf{x}+3\mathbf{a}+6\mathbf{b}$

$\therefore \quad \mathbf{x} = \mathbf{a}-3\mathbf{b}$



Example: 11 Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\mathbf{i}+\beta\mathbf{j}+\gamma\mathbf{k}$, $\beta\mathbf{i}+\gamma\mathbf{j}+\alpha\mathbf{k}$, $\gamma\mathbf{i}+\alpha\mathbf{j}+\beta\mathbf{k}$

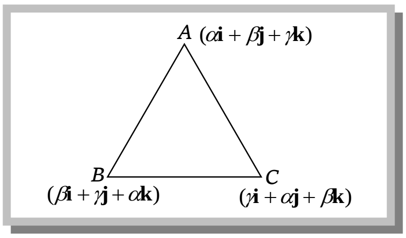
[IIT Screening 1994]

(a) Are collinear

(b) Form an equilateral triangle

(c) Form a scalene triangle

(d) Form a right angled triangle



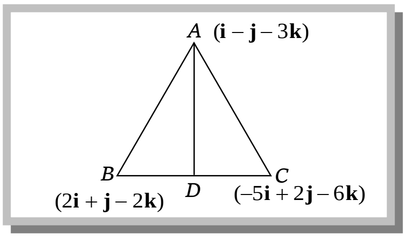
Solution: (b) $AB = \sqrt{(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2} = BC = CA$

$\therefore$ ABC is an equilateral triangle.

Example: 12 The position vectors of the vertices *A, B, C* of a triangle are $\mathbf{i}-\mathbf{j}-3\mathbf{k}$, $2\mathbf{i}+\mathbf{j}-2\mathbf{k}$ and $-5\mathbf{i}+2\mathbf{j}-6\mathbf{k}$ respectively. The length of the bisector *AD* of the angle *BAC* where *D* is on the segment *BC*, is

(a) $\frac{3}{4}\sqrt{10}$ (b) $\frac{1}{4}$ (c) $\frac{11}{2}$ (d) None of these

Solution: (a) $|\overrightarrow{AB}| = |(2\mathbf{i}+\mathbf{j}-2\mathbf{k})-(\mathbf{i}-\mathbf{j}-3\mathbf{k})| = |\mathbf{i}+2\mathbf{j}+\mathbf{k}| = \sqrt{1^2+2^2+1^2} = \sqrt{6}$



$|\overrightarrow{AC}| = |(-5\mathbf{i}+2\mathbf{j}-6\mathbf{k})-(\mathbf{i}-\mathbf{j}-3\mathbf{k})| = |-6\hat{\mathbf{i}}+3\hat{\mathbf{j}}-3\hat{\mathbf{k}}| = \sqrt{(-6)^2+3^2+(-3)^2}$

$= \sqrt{54} = 3\sqrt{6}$.

$BD : DC = AB : AC = \frac{\sqrt{6}}{3\sqrt{6}} = \frac{1}{3}$.

$\therefore$ Position vector of D $= \frac{1.(-5\mathbf{i}+2\mathbf{j}-6\mathbf{k})+3(2\mathbf{i}+\mathbf{j}-2\mathbf{k})}{1+3} = \frac{1}{4}(\mathbf{i}+5\mathbf{j}-12\mathbf{k})$

$\therefore$ $\overrightarrow{AD} =$ position vector of D – Position vector of A $= \frac{1}{4}(\mathbf{i}+5\mathbf{j}-12\mathbf{k})-(\mathbf{i}-\mathbf{j}-3\mathbf{k}) = \frac{1}{4}(-3\mathbf{i}+9\mathbf{j}) = \frac{3}{4}(-\mathbf{i}+3\mathbf{j})$

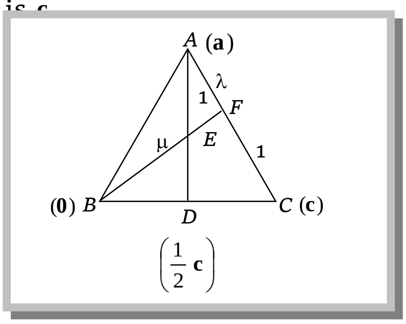
$|\overrightarrow{AD}| = \frac{3}{4}\sqrt{(-1)^2+3^2} = \frac{3}{4}\sqrt{10}$.

Example: 13 The median *AD* of the triangle *ABC* is bisected at *E*, *BE* meets *AC* in *F*. Then *AF* : *AC* =

(a) 3/4 (b) 1/3 (c) 1/2 (d) 1/4

Solution: (b) Let position vector of *A* with respect to *B* is $\mathbf{a}$ and that of *C w.r.t. B* is $\mathbf{c}$

Position vector of *D w.r.t. B* = $\frac{\mathbf{0}+\mathbf{c}}{2} = \frac{\mathbf{c}}{2}$



Position vector of E = $\frac{\mathbf{a}+\frac{\mathbf{c}}{2}}{2} = \frac{\mathbf{a}}{2}+\frac{\mathbf{c}}{4}$ .....(i)

Let $AF : FC = \lambda : 1$ and $BE : EF = \mu : 1$

Position vector of F = $\frac{\lambda\mathbf{c}+\mathbf{a}}{1+\lambda}$

Now, position vector of E = $\frac{\mu\left(\frac{\lambda\mathbf{c}+\mathbf{a}}{1+\lambda}\right)+1.\mathbf{0}}{\mu+1}$ ....(ii). From (i) and (ii) ,

$$\frac{\mathbf{a}}{2}+\frac{\mathbf{c}}{4} = \frac{\mu}{(1+\lambda)(1+\mu)}\mathbf{a}+\frac{\lambda\mu}{(1+\lambda)(1+\mu)}\mathbf{c}$$

$$\Rightarrow \frac{1}{2} = \frac{\mu}{(1+\lambda)(1+\mu)} \text{ and } \frac{1}{4} = \frac{\lambda\mu}{(1+\lambda)(1+\mu)} \Rightarrow \lambda = \frac{1}{2}, \therefore \frac{AF}{AC} = \frac{AF}{AF+FC} = \frac{\lambda}{1+\lambda} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}.$$

6.7 Linear Combination of Vectors

A vector $\mathbf{r}$ is said to be a linear combination of vectors $\mathbf{a},\mathbf{b},\mathbf{c}$..... etc, if there exist scalars *x*, *y*, *z* etc., such that $\mathbf{r} = x\mathbf{a}+y\mathbf{b}+z\mathbf{c}+....$

Examples : Vectors $\mathbf{r}_1 = 2\mathbf{a}+\mathbf{b}+3\mathbf{c}, \mathbf{r}_2 = \mathbf{a}+3\mathbf{b}+\sqrt{2}\mathbf{c}$ are linear combinations of the vectors $\mathbf{a},\mathbf{b},\mathbf{c}$.

(1) **Collinear and Non-collinear vectors :** Let $\mathbf{a}$ and $\mathbf{b}$ be two collinear vectors and let $\mathbf{x}$ be the unit vector in the direction of $\mathbf{a}$. Then the unit vector in the direction of $\mathbf{b}$ is $\mathbf{x}$ or $-\mathbf{x}$ according as $\mathbf{a}$ and $\mathbf{b}$ are like or unlike parallel vectors. Now, $\mathbf{a} = |\mathbf{a}|\,\hat{\mathbf{x}}$ and $\mathbf{b} = \pm|\mathbf{b}|\,\hat{\mathbf{x}}$.

$\therefore \quad \mathbf{a} = \left(\frac{|\mathbf{a}|}{|\mathbf{b}|}\right)|\mathbf{b}|\,\hat{\mathbf{x}} \Rightarrow \mathbf{a} = \left(\pm\frac{|\mathbf{a}|}{|\mathbf{b}|}\right)\mathbf{b} \Rightarrow \mathbf{a} = \lambda\mathbf{b}$, where $\lambda = \pm\frac{|\mathbf{a}|}{|\mathbf{b}|}$.

Thus, if $\mathbf{a}, \mathbf{b}$ are collinear vectors, then $\mathbf{a} = \lambda\mathbf{b}$ or $\mathbf{b} = \lambda\mathbf{a}$ for some scalar λ.

(2) **Relation between two parallel vectors**

(i) If $\mathbf{a}$ and $\mathbf{b}$ be two parallel vectors, then there exists a scalar k such that $\mathbf{a} = k\,\mathbf{b}$.

i.e., there exist two non-zero scalar quantities x and y so that $x\,\mathbf{a} + y\,\mathbf{b} = \mathbf{0}$.

If $\mathbf{a}$ and $\mathbf{b}$ be two non-zero, non-parallel vectors then $x\mathbf{a} + y\mathbf{b} = \mathbf{0} \Rightarrow x = 0$ and $y = 0$.

Obviously $x\mathbf{a} + y\mathbf{b} = \mathbf{0} \Rightarrow \begin{cases} \mathbf{a} = \mathbf{0}, \mathbf{b} = \mathbf{0} \\ \text{or} \\ x = 0, y = 0 \\ \text{or} \\ \mathbf{a} \parallel \mathbf{b} \end{cases}$

(ii) If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then from the property of parallel vectors, we have

$$\mathbf{a} \parallel \mathbf{b} \Rightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

(3) **Test of collinearity of three points :** Three points with position vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are collinear iff there exist scalars x, y, z not all zero such that $x\mathbf{a} + y\mathbf{b} + z\mathbf{c} = \mathbf{0}$, where $x + y + z = 0$. If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j}$, then the points with position vector $\mathbf{a}, \mathbf{b}, \mathbf{c}$ will be collinear iff $\begin{vmatrix} a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \\ c_1 & c_2 & 1 \end{vmatrix} = 0$.

(4) **Test of coplanarity of three vectors :** Let $\mathbf{a}$ and $\mathbf{b}$ two given non-zero non-collinear vectors. Then any vectors $\mathbf{r}$ coplanar with $\mathbf{a}$ and $\mathbf{b}$ can be uniquely expressed as $\mathbf{r} = x\mathbf{a} + y\mathbf{b}$ for some scalars x and y.

(5) **Test of coplanarity of Four points :** Four points with position vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ are coplanar iff there exist scalars x, y, z, u not all zero such that $x\mathbf{a} + y\mathbf{b} + z\mathbf{c} + u\mathbf{d} = \mathbf{0}$, where $x + y + z + u = 0$.

Four points with position vectors

$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$, $\mathbf{d} = d_1\mathbf{i} + d_2\mathbf{j} + d_3\mathbf{k}$

will be coplanar, *iff* $\begin{vmatrix} a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \\ d_1 & d_2 & d_3 & 1 \end{vmatrix} = 0$

6.8 Linear Independence and Dependence of Vectors

(1) **Linearly independent vectors :** A set of non-zero vectors $\mathbf{a}_1, \mathbf{a}_2, \ldots\ldots \mathbf{a}_n$ is said to be linearly independent, if $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \ldots\ldots + x_n\mathbf{a}_n = \mathbf{0} \Rightarrow x_1 = x_2 = \ldots\ldots = x_n = 0$.

(2) **Linearly dependent vectors :** A set of vectors $\mathbf{a}_1, \mathbf{a}_2, ...\mathbf{a}_n$ is said to be linearly dependent if there exist scalars $x_1, x_2, ..., x_n$ not all zero such that $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + ... + x_n\mathbf{a}_n = \mathbf{0}$

Three vectors $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$ will be linearly dependent vectors *iff* $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$.

Properties of linearly independent and dependent vectors

(i) Two non-zero, non-collinear vectors are linearly independent.

(ii) Any two collinear vectors are linearly dependent.

(iii) Any three non-coplanar vectors are linearly independent.

(iv) Any three coplanar vectors are linearly dependent.

(v) Any four vectors in 3-dimensional space are linearly dependent.

Example: 14 The points with position vectors $60\mathbf{i} + 3\mathbf{j}$, $40\mathbf{i} - 8\mathbf{j}, a\mathbf{i} - 52\mathbf{j}$ are collinear, if a =

[Rajasthan PET 1991; IIT 1983; MP PET 2002]

(a) – 40 (b) 40 (c) 20 (d) None of these

Solution: (a) As the three points are collinear, $x(60\mathbf{i} + 3\mathbf{j}) + y(40\mathbf{i} - 8\mathbf{j}) + z(a\mathbf{i} - 52\mathbf{j}) = 0$

such that x, y, z are not all zero and $x + y + z = 0$.

$\Rightarrow$ $(60x + 40y + az)\mathbf{i} + (3x - 8y - 52z)\mathbf{j} = 0$ and $x + y + z = 0$

$\Rightarrow$ $60x + 40y + az = 0$, $3x - 8y - 52z = 0$ and $x + y + z = 0$

For non-trivial solution, $\begin{vmatrix} 60 & 40 & a \\ 3 & -8 & -52 \\ 1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow a = -40$

Trick : If A, B, C are given points, then $\overrightarrow{AB} = k(\overrightarrow{BC}) \Rightarrow -20\mathbf{i} - 11\mathbf{j} = k[(a-40)\mathbf{i} - 44\mathbf{j}]$

On comparing, $-11 = -44k \Rightarrow k = \frac{1}{4}$ and $-20 = \frac{1}{4}(a - 40) \Rightarrow a = -40$.

Example: 15 If the position vectors of A, B, C, D are $2\mathbf{i} + \mathbf{j}, \mathbf{i} - 3\mathbf{j}, 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{i} + \lambda\mathbf{j}$ respectively and $\overrightarrow{AB} \parallel \overrightarrow{CD}$, then λ will be

[Rajasthan PET 1988]

(a) – 8 (b) – 6 (c) 8 (d) 6

Solution: (b) $\overrightarrow{AB} = (\mathbf{i} - 3\mathbf{j}) - (2\mathbf{i} + \mathbf{j}) = -\mathbf{i} - 4\mathbf{j}$; $\overrightarrow{CD} = (\mathbf{i} + \lambda\mathbf{j}) - (3\mathbf{i} + 2\mathbf{j}) = -2\mathbf{i} + (\lambda - 2)\mathbf{j}$; $\overrightarrow{AB} \parallel \overrightarrow{CD} \Rightarrow \overrightarrow{AB} = x\,\overrightarrow{CD}$

$-\mathbf{i} - 4\mathbf{j} = x\{-2\mathbf{i} + (\lambda - 2)\mathbf{j}\} \Rightarrow -1 = -2x, -4 = (\lambda - 2)x \Rightarrow x = \frac{1}{2}, \lambda = -6$.

Example: 16 Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\mathbf{a} + 2\mathbf{b}$ is collinear with $\mathbf{c}$ and $\mathbf{b} + 3\mathbf{c}$ is collinear with $\mathbf{a}$ (λ being some non-zero scalar) then $\mathbf{a} + 2\mathbf{b} + 6\mathbf{c}$ equals **[AIEEE 2004]**

(a) **0** (b) $\lambda\mathbf{b}$ (c) $\lambda\mathbf{c}$ (d) $\lambda\mathbf{a}$

Solution: (a) As $\mathbf{a} + 2\mathbf{b}$ and $\mathbf{c}$ are collinear $\mathbf{a} + 2\mathbf{b} = \lambda\mathbf{c}$ ......(i)

Again $\mathbf{b} + 3\mathbf{c}$ is collinear with $\mathbf{a}$

$\therefore$ $\mathbf{b} + 3\mathbf{c} = \mu\mathbf{a}$ .....(ii)

Now, $\mathbf{a} + 2\mathbf{b} + 6\mathbf{c} = (\mathbf{a} + 2\mathbf{b}) + 6\mathbf{c} = \lambda\mathbf{c} + 6\mathbf{c} = (\lambda + 6)\mathbf{c}$ .....(iii)

Also, $\mathbf{a} + 2\mathbf{b} + 6\mathbf{c} = \mathbf{a} + 2(\mathbf{b} + 3\mathbf{c}) = \mathbf{a} + 2\mu\mathbf{a} = (2\mu + 1)\mathbf{a}$ ......(iv)

From (iii) and (iv), $(\lambda+6)\mathbf{c}=(2\mu+1)\mathbf{a}$

But $\mathbf{a}$ and $\mathbf{c}$ are non-zero , non-collinear vectors,

$\therefore \quad \lambda+6=0=2\mu+1$. Hence, $\mathbf{a}+2\mathbf{b}+6\mathbf{c}=\mathbf{0}$.

Example: 17 If the vectors $4\mathbf{i}+11\mathbf{j}+m\mathbf{k}, 7\mathbf{i}+2\mathbf{j}+6\mathbf{k}$ and $\mathbf{i}+5\mathbf{j}+4\mathbf{k}$ are coplanar, then m is **[Karnataka CET 2003]**

(a) 38 (b) 0 (c) 10 (d) – 10

Solution: (c) As the three vectors are coplanar, one will be a linear combination of the other two.

$\therefore \quad 4\mathbf{i}+11\mathbf{j}+m\mathbf{k}=x(7\mathbf{i}+2\mathbf{j}+6\mathbf{k})+y(\mathbf{i}+5\mathbf{j}+4\mathbf{k}) \Rightarrow \quad 4=7x+y$ .....(i)

$11=2x+5y$ .....(ii)

$m=6x+4y$ .....(iii)

From (i) and (ii), $x=\frac{3}{11}, y=\frac{23}{11}$; From (iii), $m=6\times\frac{3}{11}+4\times\frac{23}{11}=10$.

Trick : $\because$ Vectors $4\mathbf{i}+11\mathbf{j}+m\mathbf{k}$, $7\mathbf{i}+2\mathbf{j}+6\mathbf{k}$ and $\mathbf{i}+5\mathbf{j}+4\mathbf{k}$ are coplanar.

$$\therefore \begin{vmatrix} 4 & 11 & m \\ 7 & 2 & 6 \\ 1 & 5 & 4 \end{vmatrix}=0$$

$\Rightarrow \quad 4(8-30)-11(28-6)+m(35-2)=0 \Rightarrow -88-11\times 22+33m=0 \Rightarrow -8-22+3m=0 \Rightarrow 3m=30 \Rightarrow m=10$.

Example: 18 The value of λ for which the four points $2\mathbf{i}+3\mathbf{j}-\mathbf{k}, \mathbf{i}+2\mathbf{j}+3\mathbf{k}, 3\mathbf{i}+4\mathbf{j}-2\mathbf{k}, \mathbf{i}-\lambda\mathbf{j}+6\mathbf{k}$ are coplanar **[MP PET 2004]**

(a) 8 (b) 0 (c) – 2 (d) 6

Solution: (c) The given four points are coplanar

$\therefore \; x(2\mathbf{i}+3\mathbf{j}-\mathbf{k})+y(\mathbf{i}+2\mathbf{j}+3\mathbf{k})+z(3\mathbf{i}+4\mathbf{j}-2\mathbf{k})+w(\mathbf{i}-\lambda\mathbf{j}+6\mathbf{k})=\mathbf{0}$ and $x+y+z+w=0$,

where x, y, z, w are not all zero.

$\Rightarrow \quad (2x+y+3z+w)\mathbf{i}+(3x+2y+4z-\lambda w)\mathbf{j} + (-x+3y-2z+6w)\mathbf{k}=0$ and $x+y+z+w=0$

$\Rightarrow \quad 2x+y+3z+w=0$, $3x+2y+4z-\lambda w=0$, $-x+3y-2z+6w=0$ and $x+y+z+w=0$

For non-trivial solution, $\begin{vmatrix} 2 & 1 & 3 & 1 \\ 3 & 2 & 4 & -\lambda \\ -1 & 3 & -2 & 6 \\ 1 & 1 & 1 & 1 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 2 & 1 & 3 & 1 \\ 0 & 0 & 0 & -(\lambda+2) \\ -1 & 3 & -2 & 6 \\ 1 & 1 & 1 & 1 \end{vmatrix}=0$, Operating $[R_2 \to R_2-R_1-R_4]$

$$\Rightarrow -(\lambda+2)\begin{vmatrix} 2 & 1 & 3 \\ -1 & 3 & -2 \\ 1 & 1 & 1 \end{vmatrix}=0 \Rightarrow \lambda=-2.$$

Example: 19 If $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{b}=4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ and $\mathbf{c}=\mathbf{i}+\alpha\mathbf{j}+\beta\mathbf{k}$ are linearly dependent vectors and $|\mathbf{c}|=\sqrt{3}$, then **[IIT 1998]**

(a) $\alpha=1, \beta=-1$ (b) $\alpha=1, \beta=\pm 1$ (c) $\alpha=-1, \beta=\pm 1$ (d) $\alpha=\pm 1, \beta=1$

Solution: (d) The given vectors are linearly dependent hence, there exist scalars x, y, z not all zero, such that $x\mathbf{a}+y\mathbf{b}+z\mathbf{c}=\mathbf{0}$

i.e., $x(\mathbf{i}+\mathbf{j}+\mathbf{k})+y(4\mathbf{i}+3\mathbf{j}+4\mathbf{k})+z(\mathbf{i}+\alpha\mathbf{j}+\beta\mathbf{k})=\mathbf{0}$,

i.e., $(x+4y+z)\mathbf{i}+(x+3y+\alpha z)\mathbf{j}+(x+4y+\beta z)\mathbf{k}=\mathbf{0}$

$\Rightarrow \quad x+4y+z=0$, $x+3y+\alpha z=0$, $x+4y+\beta z=0$

For non-trivial solution, $\begin{vmatrix} 1 & 4 & 1 \\ 1 & 3 & \alpha \\ 1 & 4 & \beta \end{vmatrix}=0 \Rightarrow \quad \beta=1$

$|c|^2=3 \Rightarrow \; 1+\alpha^2+\beta^2=3 \Rightarrow \alpha^2=2-\beta^2=2-1=1$; $\therefore \; \alpha=\pm 1$

Trick : $|\mathbf{c}|=\sqrt{1+\alpha^2+\beta^2}=\sqrt{3} \Rightarrow \alpha^2+\beta^2=2$

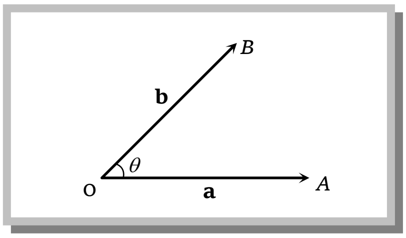
$\because \quad \mathbf{a}, \mathbf{b}, \mathbf{c}$ are linearly dependent, hence $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix}=0 \Rightarrow \beta=1$.

$\therefore \; \alpha^2=1 \Rightarrow \alpha=\pm 1$.

6.9 Product of Two Vectors

Product of two vectors is processed by two methods. When the product of two vectors results is a scalar quantity, then it is called scalar product. It is also known as dot product because we are putting a dot (.) between two vectors.

When the product of two vectors results is a vector quantity then this product is called vector product. It is also known as cross product because we are putting a cross (×) between two vectors.

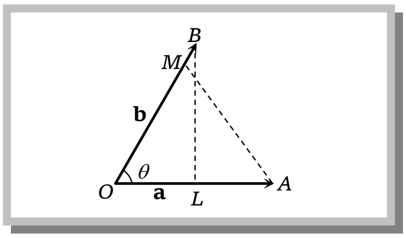


(1) **Scalar or Dot product of two vectors :** If $\mathbf{a}$ and $\mathbf{b}$ are two non-zero vectors and θ be the angle between them, then their scalar product (or dot product) is denoted by $\mathbf{a}.\mathbf{b}$ and is defined as the scalar $|\mathbf{a}||\mathbf{b}|\cos\theta$, where $|\mathbf{a}|$ and $|\mathbf{b}|$ are modulii of $\mathbf{a}$ and $\mathbf{b}$ respectively and $0 \le \theta \le \pi$.

Important Tips

- $\mathbf{a}.\mathbf{b} \in R$
- $\mathbf{a}.\mathbf{b} \le |\mathbf{a}||\mathbf{b}|$
- $\mathbf{a}.\mathbf{b} > 0 \Rightarrow$ *angle between* $\mathbf{a}$ *and* $\mathbf{b}$ *is acute.*
- $\mathbf{a}.\mathbf{b} < 0 \Rightarrow$ *angle between* $\mathbf{a}$ *and* $\mathbf{b}$ *is obtuse.*
- *The dot product of a zero and non-zero vector is a scalar zero.*

(i) **Geometrical Interpretation of scalar product :** Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ respectively. Let θ be the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Draw $BL \perp OA$ and $AM \perp OB$.



From Δs OBL and OAM, we have $OL = OB\cos\theta$ and $OM = OA\cos\theta$. Here OL and OM are known as projection of $\mathbf{b}$ on $\mathbf{a}$ and $\mathbf{a}$ on $\mathbf{b}$ respectively.

Now $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta = |\mathbf{a}|(OB\cos\theta) = |\mathbf{a}|(OL)$

$= (\text{Magnitude of } \mathbf{a})(\text{Projection of } \mathbf{b} \text{ on } \mathbf{a})$ .....(i)

Again, $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta = |\mathbf{b}|(|\mathbf{a}|\cos\theta) = |\mathbf{b}|(OA\cos\theta) = |\mathbf{b}|(OM)$

$\mathbf{a}.\mathbf{b}$ = (Magnitude of $\mathbf{b}$) (Projection of $\mathbf{a}$ on $\mathbf{b}$) .....(ii)

Thus geometrically interpreted, the scalar product of two vectors is the product of modulus of either vector and the projection of the other in its direction.

(ii) **Angle between two vectors :** If $\mathbf{a}, \mathbf{b}$ be two vectors inclined at an angle θ, then, $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

$$\Rightarrow \cos\theta = \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \Rightarrow \theta = \cos^{-1}\left(\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right)$$

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$; $\theta = \cos^{-1}\left(\frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}\right)$

(2) **Properties of scalar product**

(i) **Commutativity :** The scalar product of two vector is commutative *i.e.,* $\mathbf{a}.\mathbf{b} = \mathbf{b}.\mathbf{a}$.

(ii) **Distributivity of scalar product over vector addition:** The scalar product of vectors is distributive over vector addition *i.e.,*

(a) $\mathbf{a}.(\mathbf{b}+\mathbf{c})=\mathbf{a}.\mathbf{b}+\mathbf{a}.\mathbf{c}$ (Left distributivity) (b) $(\mathbf{b}+\mathbf{c}).\mathbf{a}=\mathbf{b}.\mathbf{a}+\mathbf{c}.\mathbf{a}$ (Right distributivity)

(iii) Let $\mathbf{a}$ and $\mathbf{b}$ be two non-zero vectors $\mathbf{a}.\mathbf{b}=0 \Leftrightarrow \mathbf{a}\perp\mathbf{b}$.

As $\mathbf{i},\mathbf{j},\mathbf{k}$ are mutually perpendicular unit vectors along the co-ordinate axes, therefore $\mathbf{i}.\mathbf{j}=\mathbf{j}.\mathbf{i}=0$; $\mathbf{j}.\mathbf{k}=\mathbf{k}.\mathbf{j}=0$; $\mathbf{k}.\mathbf{i}=\mathbf{i}.\mathbf{k}=0$.

(iv) For any vector $\mathbf{a}, \mathbf{a}.\mathbf{a}=|\mathbf{a}|^2$.

As $\mathbf{i},\mathbf{j},\mathbf{k}$ are unit vectors along the co-ordinate axes, therefore $\mathbf{i}.\mathbf{i}=|\mathbf{i}|^2=1$, $\mathbf{j}.\mathbf{j}=|\mathbf{j}|^2=1$ and $\mathbf{k}.\mathbf{k}=|\mathbf{k}|^2=1$

(v) If m is a scalar and $\mathbf{a},\mathbf{b}$ be any two vectors, then $(m\mathbf{a}).\mathbf{b}=m(\mathbf{a}.\mathbf{b})=\mathbf{a}.(m\mathbf{b})$

(vi) If m, n are scalars and $\mathbf{a},\mathbf{b}$ be two vectors, then $m\mathbf{a}.n\mathbf{b}=mn(\mathbf{a}.\mathbf{b})=(mn\,\mathbf{a}).\mathbf{b}=\mathbf{a}.(mn\,\mathbf{b})$

(vii) For any vectors $\mathbf{a}$ and $\mathbf{b}$, we have (a) $\mathbf{a}.(-\mathbf{b})=-(\mathbf{a}.\mathbf{b})=(-\mathbf{a}).\mathbf{b}$ (b) $(-\mathbf{a}).(-\mathbf{b})=\mathbf{a}.\mathbf{b}$

(viii) For any two vectors $\mathbf{a}$ and $\mathbf{b}$, we have

(a) $|\mathbf{a}+\mathbf{b}|^2=|\mathbf{a}|^2+|\mathbf{b}|^2+2\mathbf{a}.\mathbf{b}$ (b) $|\mathbf{a}-\mathbf{b}|^2=|\mathbf{a}|^2+|\mathbf{b}|^2-2\mathbf{a}.\mathbf{b}$

(c) $(\mathbf{a}+\mathbf{b}).(\mathbf{a}-\mathbf{b})=|\mathbf{a}|^2-|\mathbf{b}|^2$ (d) $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}|+|\mathbf{b}| \Rightarrow \mathbf{a}\parallel\mathbf{b}$

(e) $|\mathbf{a}+\mathbf{b}|^2=|\mathbf{a}|^2+|\mathbf{b}|^2 \Rightarrow \mathbf{a}\perp\mathbf{b}$ (f) $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}-\mathbf{b}| \Rightarrow \mathbf{a}\perp\mathbf{b}$

(3) **Scalar product in terms of components :** If $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$, then, $\mathbf{a}.\mathbf{b}=a_1b_1+a_2b_2+a_3b_3$. Thus, scalar product of two vectors is equal to the sum of the products of their corresponding components. In particular, $\mathbf{a}.\mathbf{a}=|\mathbf{a}|^2=a_1^2+a_2^2+a_3^2$.

Example: 20 $(\mathbf{a}.\mathbf{i})\mathbf{i}+(\mathbf{a}.\mathbf{j})\mathbf{j}+(\mathbf{a}.\mathbf{k})\mathbf{k}=$ **[Karnataka CET 2004]**

(a) $\mathbf{a}$ (b) $2\mathbf{a}$ (c) $3\mathbf{a}$ (d) $\mathbf{0}$

Solution: (a) Let $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$

$\therefore$ $\mathbf{a}.\mathbf{i}=(a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}).\mathbf{i}=a_1$, $\mathbf{a}.\mathbf{j}=a_2, \mathbf{a}.\mathbf{k}=a_3$

$\therefore$ $(\mathbf{a}.\mathbf{i})\mathbf{i}+(\mathbf{a}.\mathbf{j})\mathbf{j}+(\mathbf{a}.\mathbf{k})\mathbf{k}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}=\mathbf{a}$.

Example: 21 If $|\mathbf{a}|=3, |\mathbf{b}|=4$ then a value of λ for which $\mathbf{a}+\lambda\mathbf{b}$ is perpendicular to $\mathbf{a}-\lambda\mathbf{b}$ is

(a) 9/16 (b) 3/4 (c) 3/2 (d) 4/3

Solution: (b) $\mathbf{a}+\lambda\mathbf{b}$ is perpendicular to $\mathbf{a}-\lambda\mathbf{b}$

$\therefore$ $(\mathbf{a}+\lambda\mathbf{b}).(\mathbf{a}-\lambda\mathbf{b})=0 \Rightarrow |\mathbf{a}|^2-\lambda(\mathbf{a}.\mathbf{b})+\lambda(\mathbf{b}.\mathbf{a})-\lambda^2|\mathbf{b}|^2=0 \Rightarrow |\mathbf{a}|^2-\lambda^2|\mathbf{b}|^2=0 \Rightarrow \lambda=\pm\frac{|\mathbf{a}|}{|\mathbf{b}|}=\pm\frac{3}{4}$

Example: 22 A unit vector in the plane of the vectors $2\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{i}-\mathbf{j}+\mathbf{k}$ and orthogonal to $5\mathbf{i}+2\mathbf{j}+6\mathbf{k}$ is

(a) $\frac{6\mathbf{i}-5\mathbf{k}}{\sqrt{61}}$ (b) $\frac{3\mathbf{j}-\mathbf{k}}{\sqrt{10}}$ (c) $\frac{2\mathbf{i}-5\mathbf{j}}{\sqrt{29}}$ (d) $\frac{2\mathbf{i}+\mathbf{j}-2\mathbf{k}}{3}$

Solution: (b) Let a unit vector in the plane of $2\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{i}-\mathbf{j}+\mathbf{k}$ be

$\hat{\mathbf{a}}=\alpha(2\mathbf{i}+\mathbf{j}+\mathbf{k})+\beta(\mathbf{i}-\mathbf{j}+\mathbf{k})=(2\alpha+\beta)\mathbf{i}+(\alpha-\beta)\mathbf{j}+(\alpha+\beta)\mathbf{k}$

As $\hat{\mathbf{a}}$ is unit vector, we have

$=(2\alpha+\beta)^2+(\alpha-\beta)^2+(\alpha+\beta)^2=1$

$\Rightarrow$ $6\alpha^2+4\alpha\beta+3\beta^2=1$ .....(i)

As $\hat{\mathbf{a}}$ is orthogonal to $5\mathbf{i}+2\mathbf{j}+6\mathbf{k}$, we get $5(2\alpha+\beta)+2(\alpha-\beta)+6(\alpha+\beta)=0 \Rightarrow 18\alpha+9\beta=0 \Rightarrow \beta=-2\alpha$

From (i), we get $6\alpha^2 - 8\alpha^2 + 12\alpha^2 = 1 \Rightarrow \alpha = \pm\frac{1}{\sqrt{10}} \Rightarrow \beta = \mp\frac{2}{\sqrt{10}}$. Thus $\hat{\mathbf{a}} = \pm\left(\frac{3}{\sqrt{10}}\mathbf{j} - \frac{1}{\sqrt{10}}\mathbf{k}\right)$

Example: 23 If θ be the angle between the vectors $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, then

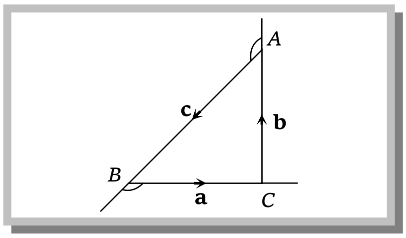
(a) $\cos\theta = \frac{4}{21}$ (b) $\cos\theta = \frac{3}{19}$ (c) $\cos\theta = \frac{2}{19}$ (d) $\cos\theta = \frac{5}{21}$

Solution: (a) Angle between $\mathbf{a}$ and $\mathbf{b}$ is given by, $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{(2\mathbf{i}+2\mathbf{j}-\mathbf{k}).(6\mathbf{i}-3\mathbf{j}+2\mathbf{k})}{\sqrt{2^2+2^2+(-1)^2}.\sqrt{6^2+(-3)^2+2^2}} = \frac{12-6-2}{3.7} = \frac{4}{21}$

Example: 24 Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be vectors with magnitudes 3, 4 and 5 respectively and $\mathbf{a}+\mathbf{b}+\mathbf{c} = 0$, then the values of $\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}$ is **[DCE 2001; AIEEE 2002; UPSEAT 2002]**

(a) 47 (b) 25 (c) 50 (d) – 25

Solution: (d) We observe, $|\mathbf{a}|^2 + |\mathbf{b}|^2 = 3^2 + 4^2 = 5^2 = |\mathbf{c}|^2$



$\therefore \quad \mathbf{a}.\mathbf{b} = 0$

$\mathbf{b}.\mathbf{c} = |\mathbf{b}||\mathbf{c}|.\cos\left(\pi - \cos^{-1}\frac{4}{5}\right) = 4\times5\left\{-\cos\left(\cos^{-1}\frac{4}{5}\right)\right\} = 4\times5\times\left(-\frac{4}{5}\right) = -16$

$\mathbf{c}.\mathbf{a} = |\mathbf{c}||\mathbf{a}|.\cos\left(\pi - \cos^{-1}\frac{3}{5}\right) = 5.3.\left\{-\cos\left(\cos^{-1}\frac{3}{5}\right)\right\} = 5.3.\left(-\frac{3}{5}\right) = -9$

$\therefore \quad \mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a} = 0 - 16 - 9 = -25$

Trick: $\because \quad \mathbf{a}+\mathbf{b}+\mathbf{c} = \mathbf{0}$

Squaring both the sides $|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2 = 0$

$\Rightarrow \quad |\mathbf{a}^2| + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}) = 0 \Rightarrow 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}) = -(9+16+25) \Rightarrow \mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a} = -25$.

Example: 25 The vectors $\mathbf{a} = 2\lambda^2\mathbf{i} + 4\lambda\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 7\mathbf{i} - 2\mathbf{j} + \lambda\mathbf{k}$ make an obtuse angle whereas the angle between $\mathbf{b}$ and $\mathbf{k}$ is acute and less than $\pi/6$, then domain of λ is

(a) $0 < \lambda < \frac{1}{2}$ (b) $\lambda > \sqrt{159}$ (c) $-\frac{1}{2} < \lambda < 0$ (d) Null set

Solution: (d) As angle between $\mathbf{a}$ and $\mathbf{b}$ is obtuse, $\mathbf{a}.\mathbf{b} < 0$

$\Rightarrow \quad (2\lambda^2\mathbf{i} + 4\lambda\mathbf{j} + \mathbf{k}).(7\mathbf{i} - 2\mathbf{j} + \lambda\mathbf{k}) < 0 \Rightarrow 14\lambda^2 - 8\lambda + \lambda < 0 \Rightarrow \lambda(2\lambda - 1) < 0 \Rightarrow 0 < \lambda < \frac{1}{2}$ ......(i)

Angle between $\mathbf{b}$ and $\mathbf{k}$ is acute and less than $\frac{\pi}{6}$.

$\mathbf{b}.\mathbf{k} = |\mathbf{b}|.|\mathbf{k}|\cos\theta \Rightarrow \lambda = \sqrt{53+\lambda^2}.1.\cos\theta \Rightarrow \cos\theta = \frac{\lambda}{\sqrt{53+\lambda^2}}$

$\theta < \frac{\pi}{6} \Rightarrow \cos\theta > \cos\frac{\pi}{6} \Rightarrow \cos\theta > \frac{\sqrt{3}}{2} \Rightarrow \frac{\lambda}{\sqrt{53+\lambda^2}} > \frac{\sqrt{3}}{2} \Rightarrow 4\lambda^2 - 3(53+\lambda^2) > 0 \Rightarrow \lambda^2 > 159 \Rightarrow \lambda < -\sqrt{159}$

or $\quad \lambda > \sqrt{159}$ ......(ii)

From (i) and (ii), $\lambda = \phi$. $\therefore$ Domain of λ is null set.

Example: 26 In cartesian co-ordinates the point A is (x_1, y_1) where $x_1 = 1$ on the curve $y = x^2 + x + 10$. The tangent at A cuts the x-axis at B. The value of the dot product $\overrightarrow{OA}.\overrightarrow{AB}$ is

(a) $-\frac{520}{3}$ (b) -148 (c) 140 (d) 12

Solution: (b) Given curve is $y = x^2 + x + 10$ ......(i)

when $x = 1$, $y = 1^2 + 1 + 10 = 12$

$\therefore \quad A \equiv (1, 12)$; $\therefore \quad \overrightarrow{OA} = \mathbf{i} + 12\mathbf{j}$

From (i), $\frac{dy}{dx} = 2x + 1$

Equation of tangent at A is $y - 12 = \left(\frac{dy}{dx}\right)_{(1,12)}(x-1) \Rightarrow \quad y - 12 = (2\times 1 + 1)(x-1) \Rightarrow y - 12 = 3x - 3$

$\therefore \quad y = 3(x+3)$

This tangent cuts x-axis (*i.e.*, $y = 0$) at $(-3,0)$

$\therefore \quad B \equiv (-3, 0)$

$\overrightarrow{OB} = -3\mathbf{i} + 0.\,\mathbf{j} = -3\mathbf{i}$; $\overrightarrow{OA}.\,\overrightarrow{AB} = \overrightarrow{OA}\,.(\overrightarrow{OB} - \overrightarrow{OA}) = (\mathbf{i} + 12\mathbf{j}).(-3\mathbf{i} - \mathbf{i} - 12\mathbf{j}) = (\mathbf{i} + 12\mathbf{j}).(-4\mathbf{i} - 12\mathbf{j}) = -4 - 144 = -148$.

Example: 27 If three non-zero vectors are $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$. If $\mathbf{c}$ is the unit vector perpendicular to the vectors $\mathbf{a}$ and $\mathbf{b}$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to **[IIT 1986]**

(a) 0 (b) $\frac{3(\Sigma a_1^2)(\Sigma b_1^2)(\Sigma c_1^2)}{4}$ (c) 1 (d) $\frac{(\Sigma a_1^2)(\Sigma b_1^2)}{4}$

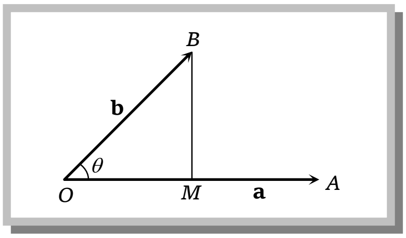
Solution: (d) As $\mathbf{c}$ is the unit vector perpendicular to $\mathbf{a}$ and $\mathbf{b}$, we have $|\mathbf{c}| = 1, \mathbf{a}.\mathbf{c} = 0 = \mathbf{b}.\mathbf{c}$

Now, $$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1^2 + a_2^2 + a_3^2 & a_1b_1 + a_2b_2 + a_3b_3 & a_1c_1 + a_2c_2 + a_3c_3 \\ a_1b_1 + a_2b_2 + a_3b_3 & b_1^2 + b_2^2 + b_3^2 & b_1c_1 + b_2c_2 + b_3c_3 \\ a_1c_1 + a_2c_2 + a_3c_3 & b_1c_1 + b_2c_2 + b_3c_3 & c_1^2 + c_2^2 + c_3^2 \end{vmatrix}$$

$$= \begin{vmatrix} |\mathbf{a}|^2 & \mathbf{a}.\mathbf{b} & \mathbf{a}.\mathbf{c} \\ \mathbf{a}.\mathbf{b} & |\mathbf{b}|^2 & \mathbf{b}.\mathbf{c} \\ \mathbf{a}.\mathbf{c} & \mathbf{b}.\mathbf{c} & |\mathbf{c}|^2 \end{vmatrix} = \begin{vmatrix} |\mathbf{a}|^2 & \mathbf{a}.\mathbf{b} & 0 \\ \mathbf{a}.\mathbf{b} & |\mathbf{b}|^2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a}.\mathbf{b})^2$$

$$= |\mathbf{a}|^2|\mathbf{b}|^2 - \left(|\mathbf{a}||\mathbf{b}|\cos\frac{\pi}{6}\right)^2 = |\mathbf{a}|^2|\mathbf{b}|^2\left(1 - \frac{3}{4}\right) = \frac{1}{4}|\mathbf{a}|^2|\mathbf{b}|^2 = \frac{1}{4}(\Sigma a_1^2)(\Sigma b_1^2)$$

(4) **Components of a vector along and perpendicular to another vector :** If $\mathbf{a}$ and $\mathbf{b}$ be two vectors represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. Draw $BM \perp OA$. In ΔOBM, we have $\overrightarrow{OB} = \overrightarrow{OM} + \overrightarrow{MB} \Rightarrow \mathbf{b} = \overrightarrow{OM} + \overrightarrow{MB}$



Thus, $\overrightarrow{OM}$ and $\overrightarrow{MB}$ are components of $\mathbf{b}$ along $\mathbf{a}$ and perpendicular to $\mathbf{a}$ respectively.

Now, $\overrightarrow{OM} = (OM)\,\hat{\mathbf{a}} = (OB\cos\theta)\,\hat{\mathbf{a}} = (|\mathbf{b}|\cos\theta)\hat{\mathbf{a}} = \left(|\mathbf{b}|\frac{(\mathbf{a}.\mathbf{b})}{|\mathbf{a}||\mathbf{b}|}\right)\hat{\mathbf{a}} = \left(\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|}\right)\hat{\mathbf{a}} = \left(\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|} = \left(\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$

$\therefore \ \mathbf{b} = \overrightarrow{OM} + \overrightarrow{MB} \Rightarrow \overrightarrow{MB} = \mathbf{b} - \overrightarrow{OM} = \mathbf{b} - \left(\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$

Thus, the components of $\mathbf{b}$ along and perpendicular to $\mathbf{a}$ are $\left(\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$ and $\mathbf{b} - \left(\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$ respectively.

Example: 28 The projection of $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ on $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ is

(a) $\frac{1}{\sqrt{14}}$ (b) $\frac{2}{\sqrt{14}}$ (c) $\sqrt{14}$ (d) $\frac{-2}{\sqrt{14}}$

Solution: (b) Projection of $\mathbf{a}$ on $\mathbf{b}$ = $\mathbf{a}.\hat{\mathbf{b}} = \mathbf{a}.\frac{\mathbf{b}}{|\mathbf{b}|}$ = $\frac{(2\mathbf{i}+3\mathbf{j}-2\mathbf{k}).(\mathbf{i}+2\mathbf{j}+3\mathbf{k})}{|\mathbf{i}+2\mathbf{j}+3\mathbf{k}|}$ = $\frac{2+6-6}{\sqrt{14}} = \frac{2}{\sqrt{14}}$

Example: 29 Let $\mathbf{u},\mathbf{v},\mathbf{w}$ be such that $|\mathbf{u}|=1, |\mathbf{v}|=2, |\mathbf{w}|=3$. If the projection $\mathbf{v}$ along $\mathbf{u}$ is equal to that of $\mathbf{w}$ along $\mathbf{u}$ and $\mathbf{v}, \mathbf{w}$ are perpendicular to each other then $|\mathbf{u}-\mathbf{v}+\mathbf{w}|$ equals

(a) 14 (b) $\sqrt{7}$ (c) $\sqrt{14}$ (d) 2

Solution: (c) Without loss of generality, we can assume $\mathbf{v}=2\mathbf{i}$ and $\mathbf{w}=3\mathbf{j}$. Let $\mathbf{u}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, $|\mathbf{u}|=1 \Rightarrow$ $x^2+y^2+z^2=1$ .....(i)

Projection of $\mathbf{v}$ along $\mathbf{u}$ = Projection of $\mathbf{w}$ along $\mathbf{u}$

$\Rightarrow \mathbf{v}.\mathbf{u}=\mathbf{w}.\mathbf{u} \Rightarrow 2\mathbf{i}.(x\mathbf{i}+y\mathbf{j}+2\mathbf{k})=3\mathbf{j}.(x\mathbf{i}+y\mathbf{j}+z\mathbf{k}) \Rightarrow 2x=3y \Rightarrow 3y-2x=0$

Now, $|\mathbf{u}-\mathbf{v}-\mathbf{w}| = |x\mathbf{i}+y\mathbf{j}+z\mathbf{k}-2\mathbf{i}+3\mathbf{j}|$ = $|(x-2)\mathbf{i}+(y+3)\mathbf{j}+z\mathbf{k}|$ = $\sqrt{(x-2)^2+(y-3)^2+z^2}$

$$= \sqrt{(x^2+y^2+z^2)+2(3y-2x)+13} = \sqrt{1+2\times 0+13} = \sqrt{14}\,.$$

Example: 30 Let $\mathbf{b}=3\mathbf{j}+4\mathbf{k}$, $\mathbf{a}=\mathbf{i}+\mathbf{j}$ and let $\mathbf{b}_1$ and $\mathbf{b}_2$ be component vectors of $\mathbf{b}$ parallel and perpendicular to $\mathbf{a}$. If $\mathbf{b}_1=\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}$, then $\mathbf{b}_2=$

(a) $\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}+4\mathbf{k}$ (b) $-\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}+4\mathbf{k}$ (c) $-\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}$ (d) None of these

Solution: (b) $\mathbf{b}=\mathbf{b}_1+\mathbf{b}_2$

$\therefore \mathbf{b}_2=\mathbf{b}-\mathbf{b}_1$ = $(3\mathbf{j}+4\mathbf{k})-\left(\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}\right)$ = $-\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}+4\mathbf{k}$

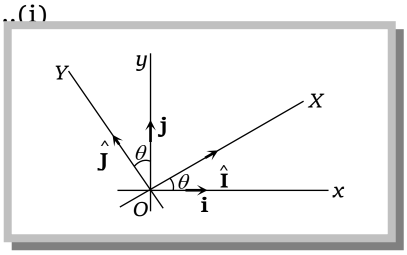
Clearly, $\mathbf{b}_1=\frac{3}{2}(\mathbf{i}+\mathbf{j})=\frac{3}{2}\mathbf{a}$ *i.e.,* $\mathbf{b}_1$ is parallel to $\mathbf{a}$

$\mathbf{b}_2.\mathbf{a}=\left(-\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}+4\mathbf{k}\right).(\mathbf{i}+\mathbf{j})=0$; $\therefore \mathbf{b}_2$ is $\perp r$ to $\mathbf{a}$.

Example: 31 A vector $\mathbf{a}$ has components $2p$ and 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the anti-clockwise sense. If $\mathbf{a}$ has components $p+1$ and 1 with respect to the new system, then

(a) $p=0$ (b) $p=1$ or $-\frac{1}{3}$ (c) $p=-1$ or $\frac{1}{3}$ (d) $p=1$ or -1

Solution: (b) Without loss of generality, we can write $\mathbf{a}=2p\mathbf{i}+\mathbf{j}=(p+1)\hat{\mathbf{I}}+\hat{\mathbf{J}}$ .....(i)



Now, $\hat{\mathbf{I}}=\cos\theta\,\mathbf{i}+\sin\theta\,\mathbf{j}$

$\hat{\mathbf{J}}=-\sin\theta\,\mathbf{i}+\cos\theta\,\mathbf{j}$

$\therefore$ From (i), $2p\mathbf{i}+\mathbf{j}=(p+1)(\cos\theta\,\mathbf{i}+\sin\theta\,\mathbf{j})+(-\sin\theta\,\mathbf{i}+\cos\theta\,\mathbf{j})$

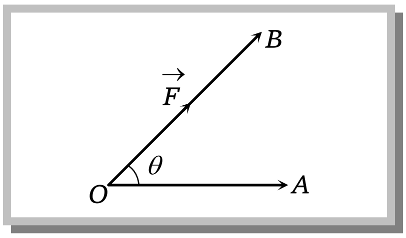
$\Rightarrow 2p\mathbf{i}+\mathbf{j}=\{(p+1)\cos\theta-\sin\theta\}\,\mathbf{i}+\{(p+1)\sin\theta+\cos\theta\}\,\mathbf{j}$

$\Rightarrow 2p=(p+1)\cos\theta-\sin\theta$ ....(ii) and $1=(p+1)\sin\theta+\cos\theta$ ....(iii)

Squaring and adding, $4p^2+1=(p+1)^2+1$

$\Rightarrow (p+1)^2=4p^2 \Rightarrow p+1=\pm 2p \Rightarrow p=1, -\frac{1}{3}$.

(5) **Work done by a force :** A force acting on a particle is said to do work if the particle is displaced in a direction which is not perpendicular to the force.



The work done by a force is a scalar quantity and its measure is equal to the product of the magnitude of the force and the resolved part of the displacement in the direction of the force.

If a particle be placed at O and a force $\vec{F}$ represented by $\overrightarrow{OB}$ be acting on the particle at O. Due to the application of force $\vec{F}$ the particle is displaced in the direction of $\overrightarrow{OA}$. Let $\overrightarrow{OA}$ be the displacement. Then the component of $\overrightarrow{OA}$ in the direction of the force $\vec{F}$ is $|\overrightarrow{OA}|\cos\theta$.

$\therefore$ Work done $= |\vec{F}||\overrightarrow{OA}|\cos\theta = \vec{F}.\overrightarrow{OA} = \vec{F}.\mathbf{d}$, where $\mathbf{d} = \overrightarrow{OA}$ Or Work done = (Force) . (Displacement)

If a number of forces are acting on a particle, then the sum of the works done by the separate forces is equal to the work done by the resultant force.

Example: 32 A particle is acted upon by constant forces $4\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ which displace it from a point $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ to the point $5\mathbf{i}+4\mathbf{j}+\mathbf{k}$. The work done in standard units by the force is given by

(a) 15 (b) 30 (c) 25 (d) 40

Solution: (d) Total force $\vec{F} = (4\mathbf{i}+\mathbf{j}-3\mathbf{k})+(3\mathbf{i}+\mathbf{j}-\mathbf{k}) = 7\mathbf{i}+2\mathbf{j}-4\mathbf{k}$

Displacement $\mathbf{d} = (5\mathbf{i}+4\mathbf{j}+\mathbf{k})-(\mathbf{i}+2\mathbf{j}+3\mathbf{k}) = 4\mathbf{i}+2\mathbf{j}-2\mathbf{k}$

Work done $= \vec{F}.\mathbf{d} = (7\mathbf{i}+2\mathbf{j}-4\mathbf{k}).(4\mathbf{i}+2\mathbf{j}-2\mathbf{k}) = 28+4+8=40$.

Example: 33 A groove is in the form of a broken line ABC and the position vectors of the three points are respectively $2\mathbf{i}-3\mathbf{j}+2\mathbf{k}$, $3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $\mathbf{i}+\mathbf{j}+\mathbf{k}$. A force of magnitude $24\sqrt{3}$ acts on a particle of unit mass kept at the point A and moves it along the groove to the point C. If the line of action of the force is parallel to the vector $\mathbf{i}+2\mathbf{j}+\mathbf{k}$ all along, the number of units of work done by the force is

(a) $144\sqrt{2}$ (b) $144\sqrt{3}$ (c) $72\sqrt{2}$ (d) $72\sqrt{3}$

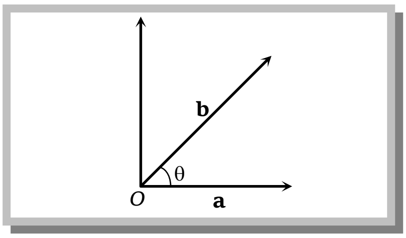
Solution: (c) $\vec{F}=(24\sqrt{3})\dfrac{\mathbf{i}+2\mathbf{j}+\mathbf{k}}{|\mathbf{i}+2\mathbf{j}+\mathbf{k}|} = \dfrac{24\sqrt{3}}{\sqrt{6}}(\mathbf{i}+2\mathbf{j}+\mathbf{k}) = 12\sqrt{2}(\mathbf{i}+2\mathbf{j}+\mathbf{k})$

Displacement $\mathbf{r}=$ position vector of C – Position vector of A = $(\mathbf{i}+\mathbf{j}+\mathbf{k})-(2\mathbf{i}-3\mathbf{j}+2\mathbf{k}) = (-\mathbf{i}+4\mathbf{j}-\mathbf{k})$

Work done by the force $W=\mathbf{r}.\vec{F} = (-\mathbf{i}+4\mathbf{j}-\mathbf{k}).12\sqrt{2}(\mathbf{i}+2\mathbf{j}+\mathbf{k}) = 12\sqrt{2}(-1+8-1)=72\sqrt{2}$.

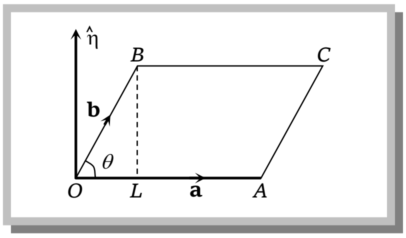
6.10 Vector or Cross product of Two Vectors

Let $\mathbf{a},\mathbf{b}$ be two non-zero, non-parallel vectors. Then the vector product $\mathbf{a}\times\mathbf{b}$, in that order, is defined as a vector whose magnitude is $|\mathbf{a}||\mathbf{b}|\sin\theta$ where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ whose direction is perpendicular to the plane of $\mathbf{a}$ and $\mathbf{b}$ in such a way that $\mathbf{a},\mathbf{b}$ and this direction constitute a right handed system.



In other words, $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\boldsymbol{\eta}}$ where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, $\hat{\boldsymbol{\eta}}$ is a unit vector perpendicular to the plane of $\mathbf{a}$ and $\mathbf{b}$ such that $\mathbf{a},\mathbf{b},\hat{\boldsymbol{\eta}}$ form a right handed system.

(1) **Geometrical interpretation of vector product :** If $\mathbf{a},\mathbf{b}$ be two non-zero, non-parallel vectors represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ respectively and let θ be the angle between them. Complete the parallelogram $OACB$. Draw $BL\perp OA$.



In ΔOBL, $\sin\theta = \frac{BL}{OB} \Rightarrow BL = OB\sin\theta = |\mathbf{b}|\sin\theta$ ......(i)

Now, $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\boldsymbol{\eta}} = (OA)(BL)\hat{\boldsymbol{\eta}}$

$= (\text{Base} \times \text{Height})\,\hat{\boldsymbol{\eta}} = (\text{area of parallelogram } OACB)\,\hat{\boldsymbol{\eta}}$

= Vector area of the parallelogram *OACB*

Thus, $\mathbf{a}\times\mathbf{b}$ is a vector whose magnitude is equal to the area of the parallelogram having $\mathbf{a}$ and $\mathbf{b}$ as its adjacent sides and whose direction $\hat{\boldsymbol{\eta}}$ is perpendicular to the plane of $\mathbf{a}$ and $\mathbf{b}$ such that $\mathbf{a},\mathbf{b},\hat{\boldsymbol{\eta}}$ form a right handed system. Hence $\mathbf{a}\times\mathbf{b}$ represents the vector area of the parallelogram having adjacent sides along $\mathbf{a}$ and $\mathbf{b}$.

Thus, area of parallelogram *OACB* = $|\mathbf{a}\times\mathbf{b}|$.

Also, area of $\Delta OAB = \frac{1}{2}$ area of parallelogram *OACB* = $\frac{1}{2}|\mathbf{a}\times\mathbf{b}| = \frac{1}{2}|\overrightarrow{OA}\times\overrightarrow{OB}|$

(2) **Properties of vector product**

(i) Vector product is not commutative *i.e.,* if $\mathbf{a}$ and $\mathbf{b}$ are any two vectors, then $\mathbf{a}\times\mathbf{b} \neq \mathbf{b}\times\mathbf{a}$, however, $\mathbf{a}\times\mathbf{b} = -(\mathbf{b}\times\mathbf{a})$

(ii) If $\mathbf{a},\mathbf{b}$ are two vectors and *m* is a scalar, then $m\mathbf{a}\times\mathbf{b} = m(\mathbf{a}\times\mathbf{b}) = \mathbf{a}\times m\mathbf{b}$

(iii) If $\mathbf{a},\mathbf{b}$ are two vectors and *m, n* are scalars, then $m\mathbf{a}\times n\mathbf{b} = mn(\mathbf{a}\times\mathbf{b}) = m(\mathbf{a}\times n\mathbf{b}) = n(m\mathbf{a}\times\mathbf{b})$

(iv) Distributivity of vector product over vector addition.

Let $\mathbf{a},\mathbf{b},\mathbf{c}$ be any three vectors. Then

(a) $\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = \mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$ (Left distributivity)

(b) $(\mathbf{b}+\mathbf{c})\times\mathbf{a} = \mathbf{b}\times\mathbf{a}+\mathbf{c}\times\mathbf{a}$ (Right distributivity)

(v) For any three vectors $\mathbf{a},\mathbf{b},\mathbf{c}$ we have $\mathbf{a}\times(\mathbf{b}-\mathbf{c}) = \mathbf{a}\times\mathbf{b}-\mathbf{a}\times\mathbf{c}$

(vi) The vector product of two non-zero vectors is zero vector *iff* they are parallel (Collinear) *i.e.,* $\mathbf{a}\times\mathbf{b} = \mathbf{0} \Leftrightarrow \mathbf{a}\,||\,\mathbf{b}, \mathbf{a},\mathbf{b}$ are non-zero vectors.

It follows from the above property that $\mathbf{a}\times\mathbf{a} = \mathbf{0}$ for every non-zero vector $\mathbf{a}$, which in turn implies that $\mathbf{i}\times\mathbf{i} = \mathbf{j}\times\mathbf{j} = \mathbf{k}\times\mathbf{k} = \mathbf{0}$

(vii) Vector product of orthonormal triad of unit vectors **i, j, k** using the definition of the vector product, we obtain $\mathbf{i}\times\mathbf{j} = \mathbf{k}, \mathbf{j}\times\mathbf{k} = \mathbf{i}, \mathbf{k}\times\mathbf{i} = \mathbf{j}$, $\mathbf{j}\times\mathbf{i} = -\mathbf{k}, \mathbf{k}\times\mathbf{j} = -\mathbf{i}, \mathbf{i}\times\mathbf{k} = -\mathbf{j}$

(viii) Lagrange's identity: If $\mathbf{a}$, $\mathbf{b}$ are any two vector then $|\mathbf{a}\times\mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a}.\mathbf{b})^2$ or $|\mathbf{a}\times\mathbf{b}|^2 + (\mathbf{a}.\mathbf{b})^2 = |\mathbf{a}|^2|\mathbf{b}|^2$

(3) **Vector product in terms of components :** If $\mathbf{a} = a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$.

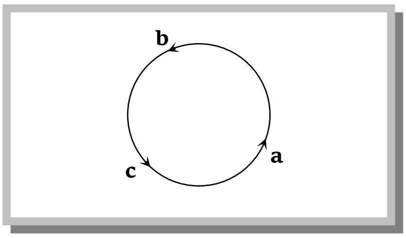
Then, $\mathbf{a}\times\mathbf{b} = (a_2b_3-a_3b_2)\mathbf{i}-(a_1b_3-a_3b_1)\mathbf{j}+(a_1b_2-a_2b_1)\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$.

(4) **Angle between two vectors :** If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, then $\sin\theta = \frac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}||\mathbf{b}|}$

Expression for $\sin\theta$ **:** If $\mathbf{a} = a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$, $\mathbf{b} = b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$ and θ be angle between $\mathbf{a}$ and $\mathbf{b}$, then

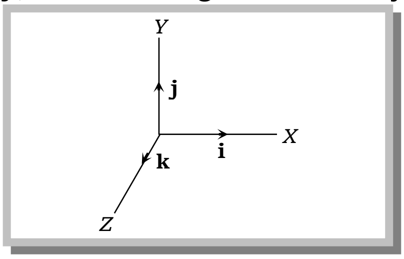
$$\sin^2\theta = \frac{(a_2b_3 - a_3b_2)^2 + (a_1b_3 - a_3b_1)^2 + (a_1b_2 - a_2b_1)^2}{(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)}$$

(5) (i) **Right handed system of vectors :** Three mutually perpendicular vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ form a right handed system of vector *iff* $\mathbf{a}\times\mathbf{b} = \mathbf{c}$, $\mathbf{b}\times\mathbf{c} = \mathbf{a}$, $\mathbf{c}\times\mathbf{a} = \mathbf{b}$



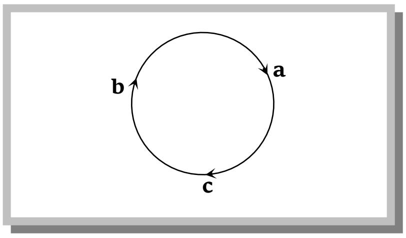
Example: The unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ form a right-handed system,

$\mathbf{i}\times\mathbf{j} = \mathbf{k}, \mathbf{j}\times\mathbf{k} = \mathbf{i}, \mathbf{k}\times\mathbf{i} = \mathbf{j}$



(ii) **Left handed system of vectors :** The vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, mutually perpendicular to one another form a left handed system of vector *iff*

$\mathbf{c}\times\mathbf{b} = \mathbf{a}, \mathbf{a}\times\mathbf{c} = \mathbf{b}, \mathbf{b}\times\mathbf{a} = \mathbf{c}$



(6) **Vector normal to the plane of two given vectors :** If $\mathbf{a}, \mathbf{b}$ be two non-zero, nonparallel vectors and let θ be the angle between them. $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\boldsymbol{\eta}}$ where $\hat{\boldsymbol{\eta}}$ is a unit vector $\perp$ to the plane of $\mathbf{a}$ and $\mathbf{b}$ such that $\mathbf{a}, \mathbf{b}, \boldsymbol{\eta}$ from a right-handed system.

$$\Rightarrow \quad (\mathbf{a}\times\mathbf{b}) = |\mathbf{a}\times\mathbf{b}|\hat{\boldsymbol{\eta}} \Rightarrow \hat{\boldsymbol{\eta}} = \frac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}\times\mathbf{b}|}$$

Thus, $\dfrac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}\times\mathbf{b}|}$ is a unit vector $\perp$ to the plane of $\mathbf{a}$ and $\mathbf{b}$. Note that $-\dfrac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}\times\mathbf{b}|}$ is also a unit vector $\perp$ to the plane of $\mathbf{a}$ and $\mathbf{b}$. Vectors of magnitude 'λ' normal to the plane of $\mathbf{a}$ and $\mathbf{b}$ are given by $\pm\dfrac{\lambda(\mathbf{a}\times\mathbf{b})}{|\mathbf{a}\times\mathbf{b}|}$.

Example: 34 If $\mathbf{a}$ is any vector, then $(\mathbf{a}\times\mathbf{i})^2 + (\mathbf{a}\times\mathbf{j})^2 + (\mathbf{a}\times\mathbf{k})^2$ is equal to

[EAMCET 1988; Rajasthan PET 2000; Orissa JEE 2003; MP PET 1984, 2004]

(a) $|\mathbf{a}|^2$ (b) 0 (c) $3|\mathbf{a}|^2$ (d) $2|\mathbf{a}|^2$

Solution: (d) Let $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$

$\therefore \quad \mathbf{a}\times\mathbf{i} = (a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k})\times\mathbf{i} = -a_2\mathbf{k} + a_3\mathbf{j}$

$(\mathbf{a}\times\mathbf{i})^2 = (\mathbf{a}\times\mathbf{i}).(\mathbf{a}\times\mathbf{i}) = (-a_2\mathbf{k} + a_3\mathbf{j}).(-a_2\mathbf{k} + a_3\mathbf{j}) = a_2^2 + a_3^2$

Similarly $(\mathbf{a}\times\mathbf{j})^2 = a_3^2 + a_1^2$ and $(\mathbf{a}\times\mathbf{k})^2 = a_1^2 + a_2^2$

$\therefore \quad (\mathbf{a}\times\mathbf{i})^2 + (\mathbf{a}\times\mathbf{j})^2 + (\mathbf{a}\times\mathbf{k})^2 = 2(a_1^2 + a_2^2 + a_3^2) = 2|\mathbf{a}|^2$.

Example: 35 $(\mathbf{a}\times\mathbf{b})^2 + (\mathbf{a}.\mathbf{b})^2$ is equal to

(a) $\mathbf{a}^2+\mathbf{b}^2$ (b) $\mathbf{a}^2\mathbf{b}^2$ (c) $2\mathbf{a}.\mathbf{b}$ (d) 1

Solution: (b) $(\mathbf{a}\times\mathbf{b})^2+(\mathbf{a}.\mathbf{b})^2 = (|\mathbf{a}||\mathbf{b}|\sin\theta\hat{\boldsymbol{\eta}})^2+(|\mathbf{a}||\mathbf{b}|\cos\theta)^2$

$= |\mathbf{a}|^2|\mathbf{b}|^2(\sin^2\theta(\hat{\boldsymbol{\eta}}.\hat{\boldsymbol{\eta}})+\cos^2\theta) = (\mathbf{a}.\mathbf{a})(\mathbf{b}.\mathbf{b})(\sin^2\theta+\cos^2\theta) = \mathbf{a}^2\mathbf{b}^2.1=\mathbf{a}^2\mathbf{b}^2$.

Example: 36 The unit vector perpendicular to the vectors $6\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $3\mathbf{i}-6\mathbf{j}-2\mathbf{k}$, is **[IIT 1989; Rajasthan PET 1996]**

(a) $\frac{2\mathbf{i}-3\mathbf{j}+6\mathbf{k}}{7}$ (b) $\frac{2\mathbf{i}-3\mathbf{j}-6\mathbf{k}}{7}$ (c) $\frac{2\mathbf{i}+3\mathbf{j}-6\mathbf{k}}{7}$ (d) $\frac{2\mathbf{i}+3\mathbf{j}+6\mathbf{k}}{7}$

Solution: (c) Let $\mathbf{a}=6\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $\mathbf{b}=3\mathbf{i}-6\mathbf{j}-2\mathbf{k}$

$$\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 6 & 2 & 3\\ 3 & -6 & -2\end{vmatrix}=14\mathbf{i}+21\mathbf{j}-42\mathbf{k}=7(2\mathbf{i}+3\mathbf{j}-6\mathbf{k})\,;\quad |\mathbf{a}\times\mathbf{b}|=7|2\mathbf{i}+3\mathbf{j}-6\mathbf{k}|=7.7$$

$\therefore\ \frac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}\times\mathbf{b}|}=\frac{1}{7}(2\mathbf{i}+3\mathbf{j}-6\mathbf{k})$, which is a unit vector perpendicular to $\mathbf{a}$ and $\mathbf{b}$.

Example: 37 The sine of the angle between the vectors $\mathbf{a}=3\mathbf{i}+\mathbf{j}+\mathbf{k}, \mathbf{b}=2\mathbf{i}-2\mathbf{j}+\mathbf{k}$ is **[Pb. CET 1988]**

(a) $\sqrt{\frac{74}{99}}$ (b) $\sqrt{\frac{25}{99}}$ (c) $\sqrt{\frac{37}{99}}$ (d) $\frac{5}{\sqrt{41}}$

Solution: (a) $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & 1 & 1\\ 2 & -2 & 1\end{vmatrix}=3\mathbf{i}-\mathbf{j}-8\mathbf{k}$; $\sin\theta=\frac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}||\mathbf{b}|}=\frac{\sqrt{74}}{\sqrt{11}.\sqrt{9}}=\sqrt{\frac{74}{99}}$

Example: 38 The vectors $\mathbf{c}$, $\mathbf{a}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ and $\mathbf{b}=\mathbf{j}$ are such that **a, c, b** form a right handed system, then **c** is **[DCE 1999]**

(a) $z\mathbf{i}-x\mathbf{k}$ (b) 0 (c) $y\mathbf{j}$ (d) $-z\mathbf{i}+x\mathbf{k}$

Solution: (a) $\mathbf{a},\mathbf{c},\mathbf{b}$ form a right handed system. Hence, $\mathbf{b}\times\mathbf{a}=\mathbf{c}\Rightarrow$ $\mathbf{c}$ = $\mathbf{j}\times(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})=-x\mathbf{k}+z\mathbf{i}=z\mathbf{i}-x\mathbf{k}$

(7) Area of parallelogram and Triangle

(i) The area of a parallelogram with adjacent sides $\mathbf{a}$ and $\mathbf{b}$ is $|\mathbf{a}\times\mathbf{b}|$.

(ii) The area of a parallelogram with diagonals $\mathbf{a}$ and $\mathbf{b}$ is $\frac{1}{2}|\mathbf{a}\times\mathbf{b}|$.

(iii) The area of a plane quadrilateral $ABCD$ is $\frac{1}{2}|\overrightarrow{AC}\times\overrightarrow{BD}|$, where AC and BD are its diagonals.

(iv) The area of a triangle with adjacent sides $\mathbf{a}$ and $\mathbf{b}$ is $\frac{1}{2}|\mathbf{a}\times\mathbf{b}|$

(v) The area of a triangle ABC is $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$ or $\frac{1}{2}|\overrightarrow{BC}\times\overrightarrow{BA}|$ or $\frac{1}{2}|\overrightarrow{CB}\times\overrightarrow{CA}|$

(vi) If $\mathbf{a},\mathbf{b},\mathbf{c}$ are position vectors of vertices of a ΔABC, then its area = $\frac{1}{2}|(\mathbf{a}\times\mathbf{b})+(\mathbf{b}\times\mathbf{c})+(\mathbf{c}\times\mathbf{a})|$

Note : ❑ Three points with position vectors $\mathbf{a},\mathbf{b},\mathbf{c}$ are collinear if $(\mathbf{a}\times\mathbf{b})+(\mathbf{b}\times\mathbf{c})+(\mathbf{c}\times\mathbf{a})=\mathbf{0}$

Example: 39 The area of a triangle whose vertices are $A(1,-1,2)$, $B(2,1,-1)$ and $C(3,-1,2)$ is

(a) 13 (b) $\sqrt{13}$ (c) 6 (d) $\sqrt{6}$

Solution: (b) $\overrightarrow{AB}=(2\mathbf{i}+\mathbf{j}-\mathbf{k})-(\mathbf{i}-\mathbf{j}+2\mathbf{k})=\mathbf{i}+2\mathbf{j}-3\mathbf{k}$, $\overrightarrow{AC}=(3\mathbf{i}-\mathbf{j}+2\mathbf{k})-(\mathbf{i}-\mathbf{j}+2\mathbf{k})=2\mathbf{i}$

Area of triangle ABC = $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$ = $\frac{1}{2}|(\mathbf{i}+2\mathbf{j}-3\mathbf{k})\times 2\mathbf{i}|$ = $\frac{1}{2}|-4\mathbf{k}-6\mathbf{j}|=|-3\mathbf{j}-2\mathbf{k}|=\sqrt{13}$

Example: 40 If $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}, \mathbf{b}=\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ and $\mathbf{c}=7\mathbf{i}+9\mathbf{j}+11\mathbf{k}$, then the area of the parallelogram having diagonals $\mathbf{a}+\mathbf{b}$ and $\mathbf{b}+\mathbf{c}$ is **[Haryana CET 2002]**

(a) $4\sqrt{6}$ (b) $\frac{1}{2}\sqrt{21}$ (c) $\frac{\sqrt{6}}{2}$ (d) $\sqrt{6}$

Solution: (a) Area of the parallelogram with diagonals $\mathbf{a}+\mathbf{b}$ and $\mathbf{b}+\mathbf{c}$ = $\frac{1}{2}|(\mathbf{a}+\mathbf{b})\times(\mathbf{b}+\mathbf{c})|$

$= \frac{1}{2}|\{(\mathbf{i}+\mathbf{j}+\mathbf{k})+(\mathbf{i}+3\mathbf{j}+5\mathbf{k})\}\times\{(\mathbf{i}+3\mathbf{j}+5\mathbf{k})+(7\mathbf{i}+9\mathbf{j}+11\mathbf{k})\}| = \frac{1}{2}|\{(2\mathbf{i}+4\mathbf{j}+6\mathbf{k})\times(8\mathbf{i}+12\mathbf{j}+16\mathbf{k})\}|$

$$= 4|(\mathbf{i}+2\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}+3\mathbf{j}+4\mathbf{k})| = 4\left|\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 2 & 3\\ 2 & 3 & 4\end{vmatrix}\right| = 4|-\mathbf{i}+2\mathbf{j}-\mathbf{k}| = 4\sqrt{6}$$

Example: 41 The position vectors of the vertices of a quadrilateral $ABCD$ are **a, b, c** and **d** respectively. Area of the quadrilateral formed by joining the middle points of its sides is

(a) $\frac{1}{4}|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{d}+\mathbf{d}\times\mathbf{a}|$

(b) $\frac{1}{4}|\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{d}+\mathbf{a}\times\mathbf{d}+\mathbf{b}\times\mathbf{a}|$

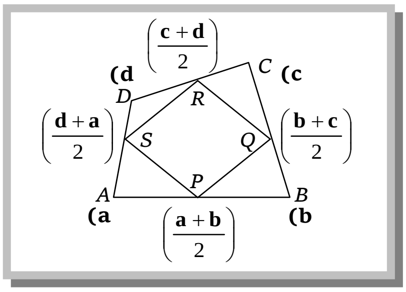
(c) $\frac{1}{4}|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{d}+\mathbf{d}\times\mathbf{a}|$

(d) $\frac{1}{4}|\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{d}+\mathbf{d}\times\mathbf{b}|$

Solution: (c) Let P, Q, R, S be the middle points of the sides of the quadrilateral $ABCD$.

Position vector of P = $\frac{\mathbf{a}+\mathbf{b}}{2}$, that of $Q=\frac{\mathbf{b}+\mathbf{c}}{2}$, that of R = $\frac{\mathbf{c}+\mathbf{d}}{2}$ and that of S = $\frac{\mathbf{d}+\mathbf{a}}{2}$

Mid point of diagonal $SQ \equiv \left(\frac{\mathbf{d}+\mathbf{a}}{2}+\frac{\mathbf{b}+\mathbf{c}}{2}\right)\frac{1}{2}=\frac{1}{4}(\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d})$



Similarly mid point of $PR \equiv \frac{1}{4}(\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d})$

As the diagonals bisect each other, $PQRS$ is a parallelogram.

$\overrightarrow{SP}=\frac{\mathbf{a}+\mathbf{b}}{2}-\frac{\mathbf{d}+\mathbf{a}}{2}=\frac{\mathbf{b}-\mathbf{d}}{2}$; $\overrightarrow{SR}=\frac{\mathbf{c}+\mathbf{d}}{2}-\frac{\mathbf{d}+\mathbf{a}}{2}=\frac{\mathbf{c}-\mathbf{a}}{2}$

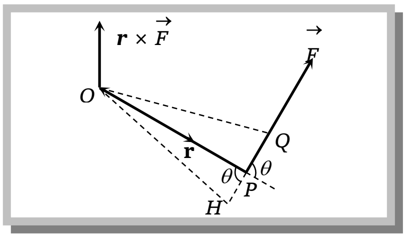
Area of parallelogram $PQRS$ = $|\overrightarrow{SP}\times\overrightarrow{SR}|=\left|\left(\frac{\mathbf{b}-\mathbf{d}}{2}\right)\times\left(\frac{\mathbf{c}-\mathbf{a}}{2}\right)\right|$

$$=\frac{1}{4}|\mathbf{b}\times\mathbf{c}-\mathbf{b}\times\mathbf{a}-\mathbf{d}\times\mathbf{c}+\mathbf{d}\times\mathbf{a}|=\frac{1}{4}|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{d}+\mathbf{d}\times\mathbf{a}|.$$

6.11 Moment of a Force and Couple

(1) **Moment of a force**

(i) **About a point :** Let a force $\vec{F}$ be applied at a point P of a rigid body. Then the moment of $\vec{F}$ about a point O measures the tendency of $\vec{F}$ to turn the body about point O. If this tendency of rotation about O is in anticlockwise direction, the moment is positive, otherwise it is negative.



Let $\mathbf{r}$ be the position vector of P relative to O. Then the moment or torque of $\vec{F}$ about the point O is defined as the vector $\vec{M} = \mathbf{r} \times \vec{F}$.

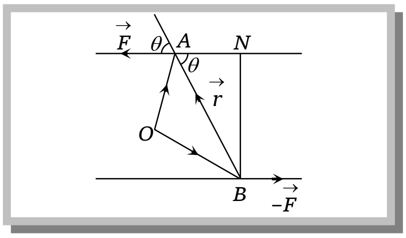
If several forces are acting through the same point P, then the vector sum of the moment of the separate forces about O is equal to the moment of their resultant force about O.

(ii) **About a line:** The moment of a force $\vec{F}$ acting at a point P about a line L is a scalar given by $(\mathbf{r} \times \vec{F}).\hat{\mathbf{a}}$ where $\hat{\mathbf{a}}$ is a unit vector in the direction of the line, and $\overrightarrow{OP} = \mathbf{r}$, where O is any point on the line.

Thus, the moment of a force $\vec{F}$ about a line is the resolved part (component) along this line, of the moment of $\vec{F}$ about any point on the line.

Note : ❑ The moment of a force about a point is a vector while the moment about a straight line is a scalar quantity.

(2) **Moment of a couple :** A system consisting of a pair of equal unlike parallel forces is called a couple. The vector sum of two forces of a couple is always zero vector.



The moment of a couple is a vector perpendicular to the plane of couple and its magnitude is the product of the magnitude of either force with the perpendicular distance between the lines of the forces.

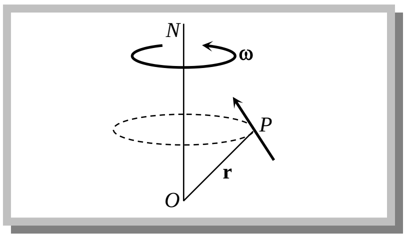
$\vec{M} = \mathbf{r} \times \vec{F}$, where $\mathbf{r} = \overrightarrow{BA}$

$|\vec{M}| = |\overrightarrow{BA} \times \vec{F}| = |\vec{F}||\overrightarrow{BA}|\sin\theta$, where θ is the angle between $\overrightarrow{BA}$ and $\vec{F}$

$= |\vec{F}|(BN) = |\vec{F}|a$

where $a = BN$ is the arm of the couple and $+ve$ or $-ve$ sign is to be taken according as the forces indicate a counter-clockwise rotation or clockwise rotation.

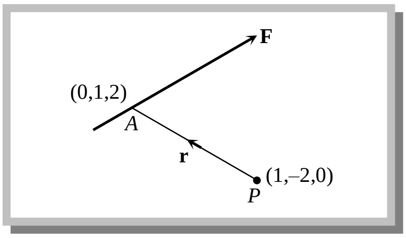
(3) **Rotation about an axis :** When a rigid body rotates about a fixed axis ON with an angular velocity ω, then the velocity $\mathbf{v}$ of a particle P is given by $\mathbf{v} = \omega \times \mathbf{r}$, where $\mathbf{r} = \overrightarrow{OP}$ and $\omega = |\omega|$ (unit vector along ON)



Example: 42 Three forces $\mathbf{i}+2\mathbf{j}-3\mathbf{k}, 2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ and $\mathbf{i}-\mathbf{j}+\mathbf{k}$ are acting on a particle at the point (0, 1, 2). The magnitude of the moment of the forces about the point (1, – 2, 0) is **[MNR 1983]**

(a) $2\sqrt{35}$ (b) $6\sqrt{10}$ (c) $4\sqrt{17}$ (d) None of these

Solution: (b) Total force $\vec{F} = (\mathbf{i}+2\mathbf{j}-3\mathbf{k})+(2\mathbf{i}+3\mathbf{j}+4\mathbf{k})+(\mathbf{i}-\mathbf{j}+\mathbf{k}) = 4\mathbf{i}+4\mathbf{j}+2\mathbf{k}$



Moment of the forces about $P = \mathbf{r}\times\vec{F} = \overrightarrow{PA}\times\vec{F}$

$\overrightarrow{PA} = (0-1)\mathbf{i}+(1+2)\mathbf{j}+(2-0)\mathbf{k} = -\mathbf{i}+3\mathbf{j}+2\mathbf{k}$

$\therefore$ Moment about $P = (-\mathbf{i}+3\mathbf{j}+2\mathbf{k})\times(4\mathbf{i}+4\mathbf{j}+2\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 2 \\ 4 & 4 & 2 \end{vmatrix} = -2\mathbf{i}+10\mathbf{j}-16\mathbf{k}$

Magnitude of the moment $= |-2\mathbf{i}+10\mathbf{j}-16\mathbf{k}| = 2\sqrt{1^2+5^2+8^2} = 2\sqrt{90} = 6\sqrt{10}$

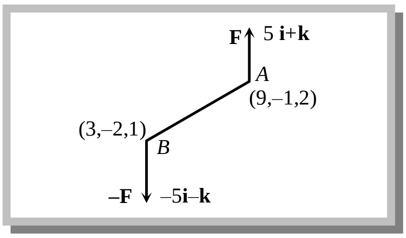
Example: 43 The moment of the couple formed by the forces $5\mathbf{i}+\mathbf{k}$ and $-5\mathbf{i}-\mathbf{k}$ acting at the points $(9,-1,2)$ and $(3,-2,1)$ respectively is **[AMU 1998]**

(a) $-\mathbf{i}+\mathbf{j}+5\mathbf{k}$ (b) $\mathbf{i}-\mathbf{j}-5\mathbf{k}$ (c) $2\mathbf{i}-2\mathbf{j}-10\mathbf{k}$ (d) $-2\mathbf{i}+2\mathbf{j}+10\mathbf{k}$

Solution: (b) Moment of the couple,

$= \overrightarrow{BA}\times\vec{F} = \{(9-3)\mathbf{i}+(-1+2)\mathbf{j}+(2-1)\mathbf{k}\}\times(5\mathbf{i}+\mathbf{k})$

$= (6\mathbf{i}+\mathbf{j}+\mathbf{k})\times(5\mathbf{i}+\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 1 & 1 \\ 5 & 0 & 1 \end{vmatrix} = \mathbf{i}-\mathbf{j}-5\mathbf{k}$



6.12 Scalar Triple Product

If $\mathbf{a},\mathbf{b},\mathbf{c}$ are three vectors, then their scalar triple product is defined as the dot product of two vectors $\mathbf{a}$ and $\mathbf{b}\times\mathbf{c}$. It is generally denoted by $\mathbf{a}.(\mathbf{b}\times\mathbf{c})$ or $[\mathbf{abc}]$. It is read as box product of $\mathbf{a},\mathbf{b},\mathbf{c}$. Similarly other scalar triple products can be defined as $(\mathbf{b}\times\mathbf{c}).\mathbf{a}, (\mathbf{c}\times\mathbf{a}).\mathbf{b}$. By the property of scalar product of two vectors we can say, $\mathbf{a}.(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b}).\mathbf{c}$

(1) **Geometrical interpretation of scalar triple product :** The scalar triple product of three vectors is equal to the volume of the parallelopiped whose three coterminous edges are represented by the given vectors. $\mathbf{a},\mathbf{b},\mathbf{c}$ form a right handed system of vectors. Therefore $(\mathbf{a}\times\mathbf{b}).\mathbf{c} = [\mathbf{abc}]$ = volume of the parallelopiped, whose coterminous edges are **a**, **b** and **c**.

(2) **Properties of scalar triple product**

(i) If $\mathbf{a},\mathbf{b},\mathbf{c}$ are cyclically permuted, the value of scalar triple product remains the same. *i.e.,* $(\mathbf{a}\times\mathbf{b}).\mathbf{c} = (\mathbf{b}\times\mathbf{c}).\mathbf{a} = (\mathbf{c}\times\mathbf{a}).\mathbf{b}$ or $[\mathbf{a\,b\,c}] = [\mathbf{b\,c\,a}] = [\mathbf{c\,a\,b}]$

(ii) The change of cyclic order of vectors in scalar triple product changes the sign of the scalar triple product but not the magnitude *i.e.,* $[\mathbf{a\,b\,c}] = -[\mathbf{b\,a\,c}] = -[\mathbf{c\,b\,a}] = -[\mathbf{a\,c\,b}]$

(iii) In scalar triple product the positions of dot and cross can be interchanged provided that the cyclic order of the vectors remains same *i.e.,* $(\mathbf{a}\times\mathbf{b}).\mathbf{c} = \mathbf{a}.(\mathbf{b}\times\mathbf{c})$

(iv) The scalar triple product of three vectors is zero if any two of them are equal.

(v) For any three vectors $\mathbf{a},\mathbf{b},\mathbf{c}$ and scalar λ, $[\lambda\,\mathbf{a}\;\mathbf{b}\;\mathbf{c}]=\lambda[\mathbf{a}\;\mathbf{b}\;\mathbf{c}]$

(vi) The scalar triple product of three vectors is zero if any two of them are parallel or collinear.

(vii) If $\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}$ are four vectors, then $[(\mathbf{a}+\mathbf{b})\;\mathbf{c}\;\mathbf{d}]=[\mathbf{a}\;\mathbf{c}\;\mathbf{d}]+[\mathbf{b}\mathbf{c}\mathbf{d}]$

(viii) The necessary and sufficient condition for three non-zero non-collinear vectors $\mathbf{a},\mathbf{b},\mathbf{c}$ to be coplanar is that $[\mathbf{a}\;\mathbf{b}\;\mathbf{c}]=0$ *i.e.,* $\mathbf{a},\mathbf{b},\mathbf{c}$ are coplanar $\Leftrightarrow[\mathbf{a}\;\mathbf{b}\mathbf{c}]=0$.

(ix) Four points with position vectors **a**, **b**, **c** and **d** will be coplanar, if $[\mathbf{a}\;\;\mathbf{b}\;\;\mathbf{c}]+[\mathbf{d}\;\;\mathbf{c}\;\;\mathbf{a}]+[\mathbf{d}\;\;\mathbf{a}\;\;\mathbf{b}]\;=[\mathbf{a}\;\;\mathbf{b}\;\;\mathbf{c}]$.

(3) **Scalar triple product in terms of components**

(i) If $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$, $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$ and $\mathbf{c}=c_1\mathbf{i}+c_2\mathbf{j}+c_3\mathbf{k}$ be three vectors.

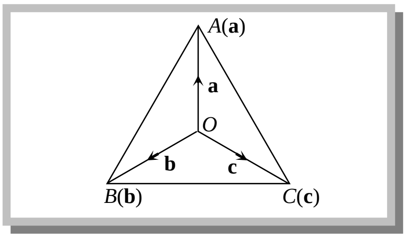
Then, $[\mathbf{a}\;\mathbf{b}\;\mathbf{c}]=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

(ii) If $\mathbf{a}=a_1\mathbf{l}+a_2\mathbf{m}+a_3\mathbf{n},\mathbf{b}=b_1\mathbf{l}+b_2\mathbf{m}+b_3\mathbf{n}$ and $\mathbf{c}=c_1\mathbf{l}+c_2\mathbf{m}+c_3\mathbf{n}$, then $[\mathbf{a}\;\mathbf{b}\;\mathbf{c}]=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}[\mathbf{l}\;\mathbf{m}\;\mathbf{n}]$

(iii) For any three vectors $\mathbf{a},\mathbf{b}$ and $\mathbf{c}$

(a) $[\mathbf{a}+\mathbf{b}\;\;\mathbf{b}+\mathbf{c}\;\;\mathbf{c}+\mathbf{a}]=2[\mathbf{a}\;\;\mathbf{b}\;\;\mathbf{c}]$ (b) $[\mathbf{a}-\mathbf{b}\;\;\mathbf{b}-\mathbf{c}\;\;\mathbf{c}-\mathbf{a}]=0$ (c) $[\mathbf{a}\times\mathbf{b}\;\;\mathbf{b}\times\mathbf{c}\;\;\mathbf{c}\times\mathbf{a}]=[\mathbf{a}\;\;\mathbf{b}\;\;\mathbf{c}]^2$

(4) **Tetrahedron :** A tetrahedron is a three-dimensional figure formed by four triangle *OABC* is a tetrahedron with ΔABC as the base. OA,OB,OC,AB,BC and CA are known as edges of the tetrahedron. $OA,BC;OB,CA$ and OC,AB are known as the pairs of opposite edges. A tetrahedron in which all edges are equal, is called a regular tetrahedron.



Properties of tetrahedron

(i) If two pairs of opposite edges of a tetrahedron are perpendicular, then the opposite edges of the third pair are also perpendicular to each other.

(ii) In a tetrahedron, the sum of the squares of two opposite edges is the same for each pair.

(iii) Any two opposite edges in a regular tetrahedron are perpendicular.

Volume of a tetrahedron

(i) The volume of a tetrahedron = $\dfrac{1}{3}$(area of the base) (corresponding altitude)

$=\dfrac{1}{3}.\dfrac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}||\overrightarrow{ED}|\;=\dfrac{1}{6}|\overrightarrow{AB}\times\overrightarrow{AC}||\overrightarrow{ED}|\cos 0^o$ for $\overrightarrow{AB}\times\overrightarrow{AC}\,||\;\overrightarrow{ED}$

$=\dfrac{1}{6}(\overrightarrow{AB}\times\overrightarrow{AC}).\overrightarrow{ED}=\dfrac{1}{6}[\overrightarrow{AB}\;\overrightarrow{AC}\;\overrightarrow{EA}+\overrightarrow{AD}]=\dfrac{1}{6}[\overrightarrow{AB}\;\overrightarrow{AC}\;\overrightarrow{AD}]$.

Because $\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{EA}$ are coplanar, so $[\overrightarrow{AB}\;\overrightarrow{AC}\;\overrightarrow{EA}]=0$

(ii) If $\mathbf{a},\mathbf{b},\mathbf{c}$ are position vectors of vertices *A*, *B* and *C* with respect to *O*, then volume of tetrahedron

$OABC=\dfrac{1}{6}[\mathbf{a}\;\;\mathbf{b}\;\;\mathbf{c}]$

(iii) If $\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}$ are position vectors of vertices *A*, *B*, *C*, *D* of a tetrahedron *ABCD*, then

its volume = $\dfrac{1}{6}[\mathbf{b}-\mathbf{a}\;\mathbf{c}-\mathbf{a}\;\mathbf{d}-\mathbf{a}]$

(5) **Reciprocal system of vectors :** Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three non-coplanar vectors, and let $\mathbf{a}' = \dfrac{\mathbf{b}\times\mathbf{c}}{[\mathbf{abc}]}$, $\mathbf{b}' = \dfrac{\mathbf{c}\times\mathbf{a}}{[\mathbf{abc}]}$, $\mathbf{c}' = \dfrac{\mathbf{a}\times\mathbf{b}}{[\mathbf{abc}]}$.

$\mathbf{a}', \mathbf{b}', \mathbf{c}'$ are said to form a reciprocal system of vectors for the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$.

If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{a}', \mathbf{b}', \mathbf{c}'$ form a reciprocal system of vectors, then

(i) $\mathbf{a}.\mathbf{a}' = \mathbf{b}.\mathbf{b}' = \mathbf{c}.\mathbf{c}' = 1$ (ii) $\mathbf{a}.\mathbf{b}' = \mathbf{a}.\mathbf{c}' = 0; \mathbf{b}.\mathbf{c}' = \mathbf{b}.\mathbf{a}' = 0; \mathbf{c}.\mathbf{a}' = \mathbf{c}.\mathbf{b}' = 0$

(iii) $[\mathbf{a}'\mathbf{b}'\mathbf{c}'] = \dfrac{1}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$ (v) $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar *iff* so are $\mathbf{a}', \mathbf{b}', \mathbf{c}'$.

Example: 44 If **u, v** and **w** are three non-coplanar vectors, then $(\mathbf{u}+\mathbf{v}-\mathbf{w}).[(\mathbf{u}-\mathbf{v})\times(\mathbf{v}-\mathbf{w})]$ equals **[AIEEE 2003]**

(a) 0 (b) $\mathbf{u}.(\mathbf{v}\times\mathbf{w})$ (c) $\mathbf{u}.(\mathbf{w}\times\mathbf{v})$ (d) $3\mathbf{u}.(\mathbf{v}\times\mathbf{w})$

Solution: (b) $(\mathbf{u}+\mathbf{v}-\mathbf{w}).[\mathbf{u}-\mathbf{v}\times(\mathbf{v}-\mathbf{w})] = (\mathbf{u}+\mathbf{v}-\mathbf{w}).[(\mathbf{u}\times\mathbf{v})-(\mathbf{u}\times\mathbf{w})-0+(\mathbf{v}\times\mathbf{w})]$

$= [\mathbf{u}\,\mathbf{u}\,\mathbf{v}]+[\mathbf{v}\,\mathbf{u}\,\mathbf{v}]-[\mathbf{w}\,\mathbf{u}\,\mathbf{v}]-[\mathbf{u}\,\mathbf{u}\,\mathbf{w}] - [\mathbf{v}\,\mathbf{u}\,\mathbf{w}]+[\mathbf{w}\,\mathbf{u}\,\mathbf{w}]+[\mathbf{u}\,\mathbf{v}\,\mathbf{w}]+[\mathbf{v}\,\mathbf{v}\,\mathbf{w}]-[\mathbf{w}\,\mathbf{v}\,\mathbf{w}]$

$= 0+0-[\mathbf{u}\,\mathbf{v}\,\mathbf{w}]-0+[\mathbf{u}\,\mathbf{v}\,\mathbf{w}]+0+[\mathbf{u}\,\mathbf{v}\,\mathbf{w}]+0-0 = [\mathbf{u}\,\mathbf{v}\,\mathbf{w}] = \mathbf{u}.(\mathbf{v}\times\mathbf{w})$.

Example: 45 The value of 'a' so that the volume of parallelopiped formed by $\mathbf{i}+a\mathbf{j}+\mathbf{k}$; $\mathbf{j}+a\mathbf{k}$ and $a\mathbf{i}+\mathbf{k}$ becomes minimum is **[IIT Screening 2003]**

(a) -3 (b) 3 (c) $1/\sqrt{3}$ (d) $\sqrt{3}$

Solution: (c) Volume of the parallelepiped

V = $[\mathbf{i}+a\mathbf{j}+\mathbf{k}\quad \mathbf{j}+a\mathbf{k}\quad a\mathbf{i}+\mathbf{k}] = (\mathbf{i}+a\mathbf{j}+\mathbf{k}).\{(\mathbf{j}+a\mathbf{k})\times(a\mathbf{i}+\mathbf{k})\} = (\mathbf{i}+a\mathbf{j}+\mathbf{k}).\{\mathbf{i}+a^2\mathbf{j}-a\mathbf{k}\} = 1+a^3-a$

$\dfrac{dV}{da} = 3a^2-1$; $\dfrac{d^2V}{da^2} = 6a$; $\dfrac{dV}{da} = 0 \Rightarrow 3a^2-1=0 \Rightarrow a = \pm\dfrac{1}{\sqrt{3}}$

At $a = \dfrac{1}{\sqrt{3}}, \dfrac{d^2V}{da^2} = \dfrac{6}{\sqrt{3}} > 0$

$\therefore$ V is minimum at $a = \dfrac{1}{\sqrt{3}}$

Example: 46 If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be any three non-zero non-coplanar vectors, then any vector $\mathbf{r}$ is equal to

(a) $z\mathbf{a}+x\mathbf{b}+y\mathbf{c}$ (b) $x\mathbf{a}+y\mathbf{b}+z\mathbf{c}$ (c) $y\mathbf{a}+z\mathbf{b}+x\mathbf{c}$ (d) None of these

Where $x = \dfrac{[\mathbf{r}\,\mathbf{b}\,\mathbf{c}]}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$, $y = \dfrac{[\mathbf{r}\,\mathbf{c}\,\mathbf{a}]}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$, $z = \dfrac{[\mathbf{r}\,\mathbf{a}\,\mathbf{b}]}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$

Solution: (b) As **a, b, c** are three non-coplanar vectors, we may assume $\mathbf{r} = \alpha\mathbf{a}+\beta\mathbf{b}+\gamma\mathbf{c}$

$[\mathbf{r}\,\mathbf{b}\,\mathbf{c}] = (\alpha\mathbf{a}+\beta\mathbf{b}+\gamma\mathbf{c}).(\mathbf{b}\times\mathbf{c}) = \alpha\{\mathbf{a}.(\mathbf{b}\times\mathbf{c})\} = \alpha[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] \Rightarrow \alpha = \dfrac{[\mathbf{r}\,\mathbf{b}\,\mathbf{c}]}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$

But $x = \dfrac{[\mathbf{r}\,\mathbf{b}\,\mathbf{c}]}{[\mathbf{abc}]}$; $\therefore \alpha = x$

Similarly $\beta = y, \gamma = z$; $\therefore$ $\mathbf{r} = x\mathbf{a}+y\mathbf{b}+z\mathbf{c}$.

Example: 47 If **a, b, c** are non-coplanar vectors and λ is a real number, then the vectors $\mathbf{a}+2\mathbf{b}+3\mathbf{c}, \lambda\mathbf{b}+4\mathbf{c}$ and $(2\lambda-1)\mathbf{c}$ are non-coplanar for **[AIEEE 2004]**

(a) No value of λ (b) All except one value of λ

(c) All except two values of λ (d) All values of λ

Solution: (c) As $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar vectors. $\therefore$ $[\mathbf{abc}] \neq 0$

Now, $\mathbf{a}+2\mathbf{b}+3\mathbf{c}, \lambda\mathbf{b}+4\mathbf{c}$ and $(2\lambda-1)\mathbf{c}$ will be non-coplanar *iff* $(\mathbf{a}+2\mathbf{b}+3\mathbf{c}).\{(\lambda\mathbf{b}+4\mathbf{c})\times(2\lambda-1)\mathbf{c}\} \neq 0$

i.e., $(\mathbf{a}+2\mathbf{b}+3\mathbf{c}).\{\lambda(2\lambda-1)(\mathbf{b}\times\mathbf{c})\} \neq 0$ *i.e.,* $\lambda(2\lambda-1)[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] \neq 0$

$\therefore$ $\lambda \neq 0, \dfrac{1}{2}$

Thus, given vectors will be non-coplanar for all values of λ except two values: $\lambda = 0$ and $\dfrac{1}{2}$.

Example: 48 x, y, z are distinct scalars such that $[x\mathbf{a}+y\mathbf{b}+z\mathbf{c}, x\mathbf{b}+y\mathbf{c}+z\mathbf{a}, x\mathbf{c}+y\mathbf{a}+z\mathbf{b}]=0$ where **a**, **b**, **c** are non-coplanar vectors then

(a) $x+y+z=0$ (b) $xy+yz+zx=0$ (c) $x^3+y^3+z^3=0$ (d) $x^2+y^2+z^2=0$

Solution: (a) **a, b, c** are non-coplanar

$\therefore$ $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]\neq 0$

Now, $[x\mathbf{a}+y\mathbf{b}+z\mathbf{c}, x\mathbf{b}+y\mathbf{c}+z\mathbf{a}, x\mathbf{c}+y\mathbf{a}+z\mathbf{b}]=0$

$\Rightarrow (x\mathbf{a}+y\mathbf{b}+z\mathbf{c}).\{(x\mathbf{b}+y\mathbf{c}+z\mathbf{a})\times(x\mathbf{c}+y\mathbf{a}+z\mathbf{b})\}=0 \Rightarrow (x\mathbf{a}+y\mathbf{b}+z\mathbf{c}).\{(x^2-yz)(\mathbf{b}\times\mathbf{c})+(z^2-xy)(\mathbf{a}\times\mathbf{b})+(y^2-zx)(\mathbf{c}\times\mathbf{a})\}=0$

$\Rightarrow x(x^2-yz)[\mathbf{abc}]+y(y^2-zx)[\mathbf{b}\ \mathbf{c}\ \mathbf{a}]+z(z^2-xy)[\mathbf{c}\ \mathbf{a}\ \mathbf{b}]=0 \Rightarrow \quad (x^3-xyz)[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]+(y^3-xyz)[\mathbf{abc}]+(z^3-xyz)[\mathbf{abc}]=0$

As $[\mathbf{abc}]\neq 0$, $x^3+y^3+z^3-3xyz=0 \Rightarrow (x+y+z)(x^2+y^2+z^2-xy-yz-zx)=0$

$\Rightarrow \quad \frac{1}{2}(x+y+z)\{(x-y)^2+(y-z)^2+(z-x)^2\}=0 \Rightarrow x+y+z=0$ or $x=y=z$

But x, y, z are distinct. $\therefore \quad x+y+z=0$.

6.13 Vector Triple Product

Let $\mathbf{a},\mathbf{b},\mathbf{c}$ be any three vectors, then the vectors $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ and $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ are called vector triple product of **a, b, c** .

Thus, $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=(\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{a}.\mathbf{b})\mathbf{c}$

(1) **Properties of vector triple product**

(i) The vector triple product $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is a linear combination of those two vectors which are within brackets.

(ii) The vector $\mathbf{r}=\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is perpendicular to $\mathbf{a}$ and lies in the plane of $\mathbf{b}$ and $\mathbf{c}$.

(iii) The formula $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=(\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{a}.\mathbf{b})\mathbf{c}$ is true only when the vector outside the bracket is on the left most side. If it is not, we first shift on left by using the properties of cross product and then apply the same formula.

Thus, $(\mathbf{b}\times\mathbf{c})\times\mathbf{a}=-\{\mathbf{a}\times(\mathbf{b}\times\mathbf{c})\} = -\{(\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{a}.\mathbf{b})\mathbf{c}\} = (\mathbf{a}.\mathbf{b})\mathbf{c}-(\mathbf{a}.\mathbf{c})\mathbf{b}$

(iv) If $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$, $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$ and $\mathbf{c}=c_1\mathbf{i}+c_2\mathbf{j}+c_3\mathbf{k}$

then $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & a_3\\ b_2c_3-b_3c_2 & b_3c_1-b_1c_3 & b_1c_2-b_2c_1\end{vmatrix}$

Note : ❑ Vector triple product is a vector quantity.

❑ $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})\neq(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$

Example: 49 Let $\mathbf{a},\mathbf{b}$ and $\mathbf{c}$ be non-zero vectors such that $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}=\frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$. If θ is the acute angle between the vectors $\mathbf{b}$ and $\mathbf{c}$, then $\sin\theta$ equals **[AIEEE 2004]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

Solution: (a) $\because \quad (\mathbf{a}\times\mathbf{b})\times\mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a} \Rightarrow (\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{b}.\mathbf{c})\mathbf{a}=\frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$

$\Rightarrow \quad (\mathbf{a}.\mathbf{c})\mathbf{b}=\{(\mathbf{b}.\mathbf{c})+\frac{1}{3}|\mathbf{b}||\mathbf{c}|\}\mathbf{a} \Rightarrow (\mathbf{a}.\mathbf{c})\mathbf{b}=|\mathbf{b}||\mathbf{c}|\left\{\cos\theta+\frac{1}{3}\right\}\mathbf{a}$

As $\mathbf{a}$ and $\mathbf{b}$ are not parallel, $\mathbf{a}.\mathbf{c}=0$ and $\cos\theta+\frac{1}{3}=0$

$\Rightarrow \quad \cos\theta=-\frac{1}{3} \Rightarrow \sin\theta=\frac{2\sqrt{2}}{3}$

Example: 50 If $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}, \mathbf{b}=\mathbf{i}+\mathbf{j}, \mathbf{c}=\mathbf{i}$ and $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}=\lambda\mathbf{a}+\mu\mathbf{b}$, then $\lambda+\mu=$ **[EAMCT 2003]**

(a) 0 (b) 1 (c) 2 (d) 3

Solution: (a) $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}=\lambda\mathbf{a}+\mu\mathbf{b} \Rightarrow (\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{b}.\mathbf{c})\mathbf{a}=\lambda\mathbf{a}+\mu\mathbf{b} \Rightarrow \lambda=-\mathbf{b}.\mathbf{c}$, $\mu=\mathbf{a}.\mathbf{c}$

$\therefore\ \lambda+\mu=\mathbf{a}.\mathbf{c}-\mathbf{b}.\mathbf{c} = (\mathbf{a}-\mathbf{b}).\mathbf{c} = \{(\mathbf{i}+\mathbf{j}+\mathbf{k})-(\mathbf{i}+\mathbf{j})\}.\mathbf{i} = \mathbf{k}.\mathbf{i}=0$.

Example: 51 If $\mathbf{a},\mathbf{b},\mathbf{c}$ and $\mathbf{p},\mathbf{q},\mathbf{r}$ are reciprocal system of vectors, then $\mathbf{a}\times\mathbf{p}+\mathbf{b}\times\mathbf{q}+\mathbf{c}\times\mathbf{r}$ equals

(a) $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$ (b) $(\mathbf{p}+\mathbf{q}+\mathbf{r})$ (c) $\mathbf{0}$ (d) $\mathbf{a}+\mathbf{b}+\mathbf{c}$

Solution: (c) $\mathbf{p}=\dfrac{\mathbf{b}\times\mathbf{c}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$, $\mathbf{q}=\dfrac{\mathbf{c}\times\mathbf{a}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$, $\mathbf{r}=\dfrac{\mathbf{a}\times\mathbf{b}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$

$$\mathbf{a}\times\mathbf{p}=\mathbf{a}\times\frac{(\mathbf{b}\times\mathbf{c})}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}=\frac{(\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{a}.\mathbf{b})\mathbf{c}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$$

Similarly $\mathbf{b}\times\mathbf{q}=\dfrac{(\mathbf{a}.\mathbf{b})\mathbf{c}-(\mathbf{b}.\mathbf{c})\mathbf{a}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$ and $\mathbf{c}\times\mathbf{r}=\dfrac{(\mathbf{b}.\mathbf{c})\mathbf{a}-(\mathbf{a}.\mathbf{c})\mathbf{b}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$

$$\therefore\quad \mathbf{a}\times\mathbf{p}+\mathbf{b}\times\mathbf{q}+\mathbf{c}\times\mathbf{r} = \frac{1}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}\{(\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{a}.\mathbf{b})\mathbf{c}+(\mathbf{a}.\mathbf{b})\mathbf{c}-(\mathbf{b}.\mathbf{c})\mathbf{a}+(\mathbf{b}.\mathbf{c})\mathbf{a}-(\mathbf{a}.\mathbf{c})\mathbf{b}\} = \frac{1}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}\times\mathbf{0}=\mathbf{0}$$

6.14 Scalar product of Four Vectors

$(\mathbf{a}\times\mathbf{b}).(\mathbf{c}\times\mathbf{d})$ is a scalar product of four vectors. It is the dot product of the vectors $\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}\times\mathbf{d}$.

It is a scalar triple product of the vectors $\mathbf{a},\mathbf{b}$ and $\mathbf{c}\times\mathbf{d}$ as well as scalar triple product of the vectors $\mathbf{a}\times\mathbf{b},\mathbf{c}$ and $\mathbf{d}$.

$$(\mathbf{a}\times\mathbf{b}).(\mathbf{c}\times\mathbf{d})=\begin{vmatrix}\mathbf{a}.\mathbf{c} & \mathbf{a}.\mathbf{d}\\ \mathbf{b}.\mathbf{c} & \mathbf{b}.\mathbf{d}\end{vmatrix}$$

6.15 Vector product of Four Vectors

(1) $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})$ is a vector product of four vectors.

It is the cross product of the vectors $\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}\times\mathbf{d}$.

(2) $\mathbf{a}\times\{\mathbf{b}\times(\mathbf{c}\times\mathbf{d})\},\{(\mathbf{a}\times\mathbf{b})\times\mathbf{c}\}\times\mathbf{d}$ are also different vector products of four vectors $\mathbf{a},\mathbf{b},\mathbf{c}$ and $\mathbf{d}$.

Example: 52 $\mathbf{a}\times[\mathbf{a}\times(\mathbf{a}\times\mathbf{b})]$ is equal to **[AMU 2001]**

(a) $(\mathbf{a}\times\mathbf{a}).(\mathbf{b}\times\mathbf{a})$ (b) $\mathbf{a}.(\mathbf{b}\times\mathbf{a})-\mathbf{b}.(\mathbf{a}\times\mathbf{b})$ (c) $[\mathbf{a}.(\mathbf{a}\times\mathbf{b})]\mathbf{a}$ (d) $(\mathbf{a}.\mathbf{a})(\mathbf{b}\times\mathbf{a})$

Solution: (d) $\mathbf{a}\times[\mathbf{a}\times(\mathbf{a}\times\mathbf{b})] = \mathbf{a}\times[(\mathbf{a}.\mathbf{b})\mathbf{a}-(\mathbf{a}.\mathbf{a})\mathbf{b}]=(\mathbf{a}.\mathbf{b})(\mathbf{a}\times\mathbf{a})-(\mathbf{a}.\mathbf{a})(\mathbf{a}\times\mathbf{b}) = (\mathbf{a}.\mathbf{b})\mathbf{0}+(\mathbf{a}.\mathbf{a})(\mathbf{b}\times\mathbf{a})= (\mathbf{a}.\mathbf{a})(\mathbf{b}\times\mathbf{a})$

Example: 53 $[\mathbf{b}\times\mathbf{c}\ \mathbf{c}\times\mathbf{a}\ \mathbf{a}\times\mathbf{b}]$ is equal to **[MP PET 2004]**

(a) $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ (b) $2[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$ (c) $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]^2$ (d) $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$

Solution: (c) $[(\mathbf{b}\times\mathbf{c})(\mathbf{c}\times\mathbf{a}),(\mathbf{a}\times\mathbf{b})] = (\mathbf{b}\times\mathbf{c}).\{[\mathbf{c}\times\mathbf{a})\times(\mathbf{a}\times\mathbf{b})\} = (\mathbf{b}\times\mathbf{c}).\{[\mathbf{c}\,\mathbf{a}\,\mathbf{b}]\mathbf{a}-[\mathbf{a}\,\mathbf{a}\,\mathbf{b}]\mathbf{c}\}$

$= (\mathbf{b}\times\mathbf{c}).\{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]\mathbf{a}-0\} = [\mathbf{b}\,\mathbf{c}\,\mathbf{a}][\mathbf{abc}] = [\mathbf{abc}]^2$

Example: 54 Let the vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ be such that $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})=\mathbf{0}$. Let P_1 and P_2 be planes determined by pair of vectors $\mathbf{a},\mathbf{b}$ and $\mathbf{c},\mathbf{d}$ respectively. Then the angle between P_1 and P_2 is **[IIT Screening 2000]**

(a) 0^o (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

Solution: (a) $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})=\mathbf{0} \Rightarrow (\mathbf{a}\times\mathbf{b})$ is parallel to $(\mathbf{c}\times\mathbf{d})$

Hence plane P_1, determined by vectors $\mathbf{a},\mathbf{b}$ is parallel to the plane P_2 determined by $\mathbf{c},\mathbf{d}$

$\therefore$ Angle between P_1 and P_2 $=0$ (As the planes P_1 and P_2 are parallel).

6.16 Vector Equations

Solving a vector equation means determining an unknown vector or a number of vectors satisfying the given conditions. Generally, to solve a vector equation, we express the unknown vector as a linear combination of three known non-coplanar vectors and then we determine the coefficients from the given conditions.

If $\mathbf{a},\mathbf{b}$ are two known non-collinear vectors, then $\mathbf{a},\mathbf{b},\mathbf{a}\times\mathbf{b}$ are three non-coplanar vectors.

Thus, any vector $\mathbf{r}=x\mathbf{a}+y\mathbf{b}+z(\mathbf{a}\times\mathbf{b})$ where x,y,z are unknown scalars.

Example: 55 If $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k},\ \mathbf{a}.\mathbf{b}=1$ and $\mathbf{a}\times\mathbf{b}=\mathbf{j}-\mathbf{k}$, then $\mathbf{b}=$ **[IIT Screening 2004]**

(a) $\mathbf{i}$ (b) $\mathbf{i}-\mathbf{j}+\mathbf{k}$ (c) $2\mathbf{j}-\mathbf{k}$ (d) $2\mathbf{i}$

Solution: (a) Let $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$

Now, $\mathbf{j}-\mathbf{k}=\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 1 & 1\\ b_1 & b_2 & b_3\end{vmatrix}\Rightarrow b_3-b_2=0, b_1-b_3=1, b_2-b_1=-1\Rightarrow b_3=b_2, b_1=b_2+1$

Now, $\mathbf{a}.\mathbf{b}=1\Rightarrow b_1+b_2+b_3=1\Rightarrow 3b_2+1=1\Rightarrow b_2=0\Rightarrow b_1=1, b_3=0$. Thus $\mathbf{b}=\mathbf{i}$

Example: 56 The point of intersection of $\mathbf{r}\times\mathbf{a}=\mathbf{b}\times\mathbf{a}$ and $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$ where $\mathbf{a}=\mathbf{i}+\mathbf{j}$ and $\mathbf{b}=2\mathbf{i}-\mathbf{k}$ is **[Orissa JEE 2004]**

(a) $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ (b) $3\mathbf{i}-\mathbf{k}$ (c) $3\mathbf{i}+2\mathbf{j}+\mathbf{k}$ (d) None of these

Solution: (a) We have $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$ and $\mathbf{r}\times\mathbf{a}=\mathbf{b}\times\mathbf{a}$

Adding, $\mathbf{r}\times(\mathbf{a}+\mathbf{b})=(\mathbf{a}\times\mathbf{b})+(\mathbf{b}\times\mathbf{a})$

$\Rightarrow\ \mathbf{r}\times(\mathbf{a}+\mathbf{b})=\mathbf{0}\ \Rightarrow \mathbf{r}$ is parallel to $\mathbf{a}+\mathbf{b}$

$\therefore\ \mathbf{r}=\lambda(\mathbf{a}+\mathbf{b})=\lambda\{(\mathbf{i}+\mathbf{j})+(2\mathbf{i}-\mathbf{k})\}=\lambda\{3\mathbf{i}+\mathbf{j}-\mathbf{k}\}$

For $\lambda=1$, $\mathbf{r}=3\mathbf{i}+\mathbf{j}-\mathbf{k}$

Example: 57 Let $\mathbf{a}=\mathbf{i}-\mathbf{j}, \mathbf{b}=\mathbf{j}-\mathbf{k}, \mathbf{c}=\mathbf{k}-\mathbf{i}$. If $\hat{\mathbf{d}}$ is a unit vector such that $\mathbf{a}.\mathbf{d}=0=[\mathbf{b}\,\mathbf{c}\,\mathbf{d}]$, then $\hat{\mathbf{d}}$ is equal to **[IIT 1995]**

(a) $\pm\frac{\mathbf{i}+\mathbf{j}-\mathbf{k}}{\sqrt{3}}$ (b) $\pm\frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$ (c) $\pm\frac{\mathbf{i}+\mathbf{j}-2\mathbf{k}}{\sqrt{6}}$ (d) $\pm\mathbf{k}$

Solution: (c) Let $\hat{\mathbf{d}}=\alpha\,\mathbf{i}+\beta\,\mathbf{j}+\gamma\,\mathbf{k}$

$\mathbf{a}.\hat{\mathbf{d}}=0\Rightarrow(\mathbf{i}-\mathbf{j}).(\alpha\mathbf{i}+\beta\mathbf{j}+\gamma\mathbf{k})=0\Rightarrow\alpha-\beta=0\Rightarrow\alpha=\beta$

$[\mathbf{b}\,\mathbf{c}\,\mathbf{d}]=0\Rightarrow(\mathbf{b}\times\mathbf{c}).\mathbf{d}=0\Rightarrow\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 1 & -1\\ -1 & 0 & 1\end{vmatrix}.(\alpha\,\mathbf{i}+\beta\,\mathbf{j}+\gamma\,\mathbf{k})=0\Rightarrow(\mathbf{i}+\mathbf{j}+\mathbf{k}).(\alpha\,\mathbf{i}+\beta\,\mathbf{j}+\gamma\,\mathbf{k})=0\Rightarrow\alpha+\beta+\gamma=0$

$\Rightarrow\gamma=-(\alpha+\beta)=-2\alpha$; $(\beta=\alpha)$

$|\hat{\mathbf{d}}|=1\Rightarrow\alpha^2+\beta^2+\gamma^2=1\Rightarrow\alpha^2+\alpha^2+4\alpha^2=1\Rightarrow\alpha=\pm\frac{1}{\sqrt{6}}=\beta$ and $\gamma=\mp\frac{2}{\sqrt{6}}$

$\therefore\ \hat{\mathbf{d}}=\pm\frac{1}{\sqrt{6}}(\mathbf{i}+\mathbf{j}-2\mathbf{k})$.

Example: 58 Let $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\mathbf{x}$ satisfies equation $\mathbf{p}\times|(\mathbf{x}-\mathbf{q})\times\mathbf{p}|+\mathbf{q}\times|(\mathbf{x}-\mathbf{r})\times\mathbf{q}|+\mathbf{r}\times|(\mathbf{x}-\mathbf{p})\times\mathbf{r}|=0$, then $\mathbf{x}$ is given by **[IIT 1997]**

(a) $\frac{1}{2}(\mathbf{p}+\mathbf{q}-2\mathbf{r})$ (b) $\frac{1}{2}(\mathbf{p}+\mathbf{q}+\mathbf{r})$ (c) $\frac{1}{3}(\mathbf{p}+\mathbf{q}+\mathbf{r})$ (d) $\frac{1}{3}(2\mathbf{p}+\mathbf{q}-\mathbf{r})$

Solution: (b) Let $|\mathbf{p}|=|\mathbf{q}|=|\mathbf{r}|=\mathbf{k}$

$\therefore\ \mathbf{p}=k\,\hat{\mathbf{p}}, \mathbf{q}=k\,\hat{\mathbf{q}}, \mathbf{r}=k\,\hat{\mathbf{r}}$

Let $\mathbf{x}=\alpha\,\hat{\mathbf{p}}+\beta\,\hat{\mathbf{q}}+\gamma\,\hat{\mathbf{r}}$

Now, $\mathbf{p}\times\{\mathbf{x}-\mathbf{q}\}\times\mathbf{p}\}=(\mathbf{p}.\mathbf{p})(\mathbf{x}-\mathbf{q})-\{\mathbf{p}.(\mathbf{x}-\mathbf{q})\}\mathbf{p}=|\mathbf{p}|^2(\mathbf{x}-\mathbf{q})-\{(\mathbf{p}.\mathbf{x})-\mathbf{p}.\mathbf{q}\}\mathbf{p}$

$=k^2(\mathbf{x}-\mathbf{q})-\{|\mathbf{p}|(\hat{\mathbf{p}}.\hat{\mathbf{x}})-0\}|\mathbf{p}|\hat{\mathbf{p}}=k^2(\mathbf{x}-\mathbf{q})-|\mathbf{p}|^2(\hat{\mathbf{p}}.\hat{\mathbf{x}})\hat{\mathbf{p}}=k^2\{\mathbf{x}-\mathbf{q}-\alpha\,\hat{\mathbf{p}})$

$\therefore\ \mathbf{p}\times\{(\mathbf{x}-\mathbf{q})\times\mathbf{p}\}+\mathbf{q}\times(\mathbf{x}-\mathbf{r})\times\mathbf{q}\}+\mathbf{r}\times\{(\mathbf{x}-\mathbf{p})\times\mathbf{r}\}=\mathbf{0}$

$\Rightarrow\ k^2\{\mathbf{x}-\mathbf{q}-\alpha\mathbf{p}+\mathbf{x}-\mathbf{r}-\beta\mathbf{q}+\mathbf{x}-\mathbf{p}-\gamma\mathbf{r}\}=0\Rightarrow 3\mathbf{x}-(\mathbf{p}+\mathbf{q}+\mathbf{r})-(\alpha\mathbf{p}+\beta\mathbf{q}+\gamma\mathbf{r})=0$

$\Rightarrow\ 3\mathbf{x}-(\mathbf{p}+\mathbf{q}+\mathbf{r})-\mathbf{x}=0\ \Rightarrow 2\mathbf{x}-(\mathbf{p}+\mathbf{q}+\mathbf{r})=0$

$\therefore\ \mathbf{x}=\frac{1}{2}(\mathbf{p}+\mathbf{q}+\mathbf{r})$

Example: 59 Let the unit vectors $\mathbf{a}$ and $\mathbf{b}$ be perpendicular and the unit vector $\mathbf{c}$ be inclined at an angle θ to both $\mathbf{a}$ and $\mathbf{b}$. If $\mathbf{c} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma(\mathbf{a}\times\mathbf{b})$, then **[Orissa JEE 2003]**

(a) $\alpha = \beta = \cos\theta\,,\gamma^2 = \cos 2\theta$
(b) $\alpha = \beta = \cos\theta, \gamma^2 = -\cos 2\theta$
(c) $\alpha = \cos\theta, \beta = \sin\theta, \gamma^2 = \cos 2\theta$
(d) None of these

Solution: (b) We have, $|\mathbf{a}| = |\mathbf{b}| = 1$

$\mathbf{a}\,.\,\mathbf{b} = 0$; (as $\mathbf{a}\perp\mathbf{b}$)

$\mathbf{c} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma(\mathbf{a}\times\mathbf{b})$ ......(i)

Taking dot product by $\mathbf{a}$, $\mathbf{a}\,.\,\mathbf{c} = \alpha|\,\mathbf{a}|^2 + \beta(\mathbf{a}\,.\,\mathbf{b}) + \gamma[\mathbf{a}\;\mathbf{a}\;\mathbf{b}]$

$\Rightarrow\; |\,\mathbf{a}|.|\,\mathbf{c}|\cos\theta = \alpha.1 + 0 + 0 \;\Rightarrow\; 1.|\,\mathbf{c}|.\cos\theta = \alpha$

As $|\,\mathbf{c}| = 1$; $\therefore\;\; \alpha = \cos\theta$

Taking dot product of (i) by $\mathbf{b}$

$\mathbf{b}\,.\,\mathbf{c} = \mathbf{b}\,.\,\mathbf{a} + \beta|\,\mathbf{b}|^2 + \gamma\,[\mathbf{b}\;\mathbf{a}\;\mathbf{b}] \Rightarrow |\,\mathbf{b}||\,\mathbf{c}|\cos\theta = 0 + \beta.1 + 0$

$\therefore\;\; \beta = 1.1.\cos\theta = \cos\theta$

$|\,\mathbf{c}|^2 = 1 \;\Rightarrow\; \alpha^2 + \beta^2 + \gamma^2 = 1 \;\Rightarrow\; \cos^2\theta + \cos^2\theta + \gamma^2 = 1$

$\therefore\;\; \gamma^2 = 1 - 2\cos^2\theta = -\cos 2\theta$

Hence, $\alpha = \beta = \cos\theta, \gamma^2 = -\cos 2\theta$

Example: 60 The locus of a point equidistant from two given points whose position vectors are $\mathbf{a}$ and $\mathbf{b}$ is equal to

(a) $\left[\mathbf{r} - \frac{1}{2}(\mathbf{a}+\mathbf{b})\right].(\mathbf{a}+\mathbf{b}) = 0$
(b) $\left[\mathbf{r} - \frac{1}{2}(\mathbf{a}+\mathbf{b})\right].(\mathbf{a}-\mathbf{b}) = 0$
(c) $\left[\mathbf{r} - \frac{1}{2}(\mathbf{a}+\mathbf{b})\right].\mathbf{a} = 0$
(d) $[\mathbf{r} - (\mathbf{a}+\mathbf{b})].\,\mathbf{b} = 0$

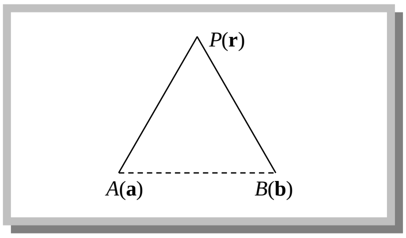
Solution: (b) Let $P\,(\mathbf{r})$ be a point on the locus.

$\therefore\;\; AP = BP$

$\Rightarrow\;\; |\,\mathbf{r}-\mathbf{a}| = |\,\mathbf{r}-\mathbf{b}| \;\Rightarrow\; |\,\mathbf{r}-\mathbf{a}|^2 = |\,\mathbf{r}-\mathbf{b}|^2 \;\Rightarrow\; (\mathbf{r}-\mathbf{a}).(\mathbf{r}-\mathbf{a}) = (\mathbf{r}-\mathbf{b}).(\mathbf{r}-\mathbf{b})$

$\Rightarrow\;\; 2\mathbf{r}.(\mathbf{a}-\mathbf{b}) = \mathbf{a}\,.\,\mathbf{a} - \mathbf{b}.\mathbf{b} \;\Rightarrow\; \mathbf{r}.(\mathbf{a}-\mathbf{b}) = \frac{1}{2}(\mathbf{a}+\mathbf{b}).(\mathbf{a}-\mathbf{b})$

$\therefore\;\; [\mathbf{r} - \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{a}+\mathbf{b})].(\mathbf{a}-\mathbf{b}) = 0$. This is the locus of *P*.



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Assignment

Modulus and Direction cosines of Vector

Basic Level

1. The perimeter of a triangle with sides $3\mathbf{i}+4\mathbf{j}+5\mathbf{k}$, $4\mathbf{i}-3\mathbf{j}-5\mathbf{k}$ and $7\mathbf{i}+\mathbf{j}$ is **[MP PET 1991]**

(a) $\sqrt{450}$ (b) $\sqrt{150}$ (c) $\sqrt{50}$ (d) $\sqrt{200}$

2. The magnitudes of mutually perpendicular forces **a, b** and **c** are 2, 10 and 11 respectively. Then the magnitude of its resultant is **[IIT 1984]**

(a) 12 (b) 15 (c) 9 (d) None of these

3. If $\mathbf{a}=2\mathbf{i}+\mathbf{j}-8\mathbf{k}$ and $\mathbf{b}=\mathbf{i}+3\mathbf{j}-4\mathbf{k}$, then the magnitude of $\mathbf{a}+\mathbf{b}=$ **[MP PET 1996]**

(a) 13 (b) $\frac{13}{3}$ (c) $\frac{3}{13}$ (d) $\frac{4}{13}$

4. The position vectors of A and B are 2**i** – 9**j** – 4**k**, and 6**i** – 3**j** + 8**k** respectively, then the magnitude of $\overrightarrow{AB}$ is **[MP PET 20**

(a) 11 (b) 12 (c) 13 (d) 14

5. If the position vectors of P and Q are $(\mathbf{i}+3\mathbf{j}-7\mathbf{k})$ and $(5\mathbf{i}-2\mathbf{j}+4\mathbf{k})$, then $|\overrightarrow{PQ}|$ is **[MP PET 2001]**

(a) $\sqrt{158}$ (b) $\sqrt{160}$ (c) $\sqrt{161}$ (d) $\sqrt{162}$

6. If **a, b, c** are mutually perpendicular unit vectors, then $|\mathbf{a}+\mathbf{b}+\mathbf{c}|=$ **[Karnataka CET 2002]**

(a) $\sqrt{3}$ (b) 3 (c) 1 (d) 0

7. Let $\mathbf{a}=\mathbf{i}+\mathbf{j}+p\mathbf{k}$ and $\mathbf{b}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, then $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}|+|\mathbf{b}|$, holds for

(a) All real p (b) No real p (c) $p=-1$ (d) $p=1$

8. For any two vectors **a** and **b**, which of the following is true

(a) $|\mathbf{a}+\mathbf{b}|\geq|\mathbf{a}|+|\mathbf{b}|$ (b) $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}|+|\mathbf{b}|$ (c) $|\mathbf{a}+\mathbf{b}|<|\mathbf{a}|+|\mathbf{b}|$ (d) $|\mathbf{a}+\mathbf{b}|\leq|\mathbf{a}|+|\mathbf{b}|$

9. If **a** and **b** are the adjacent sides of a parallelogram, then $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}-\mathbf{b}|$ is a necessary and sufficient condition for the parallelogram to be a

(a) Rhombus (b) Square (c) Rectangle (d) Trapezium

10. The direction cosines of vector $\mathbf{a}=3\mathbf{i}+4\mathbf{j}+5\mathbf{k}$ in the direction of positive axis of x, is **[MP PET 1991]**

(a) $\pm\frac{3}{\sqrt{50}}$ (b) $\frac{4}{\sqrt{50}}$ (c) $\frac{3}{\sqrt{50}}$ (d) $-\frac{4}{\sqrt{50}}$

11. A force is a

(a) Unit vector (b) Localised vector (c) Zero vector (d) Free vector

12. A zero vector has

(a) Any direction (b) No direction (c) Many directions (d) None of these

Advance Level

13. The perimeter of the triangle whose vertices have the position vectors $(\mathbf{i}+\mathbf{j}+\mathbf{k})$, $(5\mathbf{i}+3\mathbf{j}-3\mathbf{k})$ and $(2\mathbf{i}+5\mathbf{j}+9\mathbf{k})$ is given by

[MP PET 1993]

(a) $15+\sqrt{157}$ (b) $15-\sqrt{157}$ (c) $\sqrt{15}-\sqrt{157}$ (d) $\sqrt{15}+\sqrt{157}$

14. If the vectors $6\mathbf{i}-2\mathbf{j}+3\mathbf{k}$, $2\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ and $3\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ form a triangle, then it is **[Karnataka CET 1999]**

(a) Right angled (b) Obtuse angled (c) Equilateral (d) Isosceles

15. The vectors $\overrightarrow{AB}=3\mathbf{i}+4\mathbf{k}$ and $\overrightarrow{AC}=5\mathbf{i}-2\mathbf{j}+4\mathbf{k}$ are the sides of a triangle *ABC*. The length of the median through *A* is

[AIEEE 2003]

(a) $\sqrt{18}$ (b) $\sqrt{72}$ (c) $\sqrt{33}$ (d) $\sqrt{288}$

16. If **a** and **b** are two unit vectors inclined at an angle 2θ to each other, then $|\mathbf{a}+\mathbf{b}|<1$, if

(a) $\frac{\pi}{3}<\theta<\frac{2\pi}{3}$ (b) $\theta<\frac{\pi}{3}$ (c) $\theta>\frac{2\pi}{3}$ (d) $\theta=\frac{\pi}{2}$

17. If the position vectors of *A* and *B* are $\mathbf{i}+3\mathbf{j}-7\mathbf{k}$ and $5\mathbf{i}-2\mathbf{j}+4\mathbf{k}$, then the direction cosine of $\overrightarrow{AB}$ along *y*- axis is **[MNR 1**

(a) $\frac{4}{\sqrt{162}}$ (b) $-\frac{5}{\sqrt{162}}$ (c) –5 (d) 11

18. The position vectors of four points *A, B, C, D* lying in plane are **a, b, c, d** respectively. They satisfy the relation $|\mathbf{a}-\mathbf{d}|=|\mathbf{b}-\mathbf{d}|=|\mathbf{c}-\mathbf{d}|$, then the point *D* is

(a) Centroid of ΔABC (b) Circumcentre of ΔABC (c) Orthocentre of ΔABC (d) Incentre of ΔABC

19. In a parallelopiped the ratio of the sum of the squares on the four diagonals to the sum of the squares on the three coterminous edges is

(a) 2 (b) 3 (c) 4 (d) 1

Addition of Vectors

Basic Level

20. *P* is the point of intersection of the diagonals of the parallelogram *ABCD*. If *O* is any point, then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=$

[Rajasthan PET 1989]

(a) $\overrightarrow{OP}$ (b) $2\overrightarrow{OP}$ (c) $3\overrightarrow{OP}$ (d) $4\overrightarrow{OP}$

21. If $\mathbf{p}=7\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ and $\mathbf{q}=3\mathbf{i}+\mathbf{j}+5\mathbf{k}$, then the magnitude of **p** – 2**q** is **[MP PET 1987]**

(a) $\sqrt{29}$ (b) 4 (c) $\sqrt{62}-2\sqrt{35}$ (d) $\sqrt{66}$

22. If *C* is the middle point of *AB* and *P* is any point outside *AB,* then **[MNR 1991, UPSEAT 2000]**

(a) $\overrightarrow{PA}+\overrightarrow{PB}=\overrightarrow{PC}$ (b) $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$ (c) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=0$ (d) $\overrightarrow{PA}+\overrightarrow{PB}+2\overrightarrow{PC}=0$

23. If $\mathbf{a}=2\mathbf{i}+5\mathbf{j}$ and $\mathbf{b}=2\mathbf{i}-\mathbf{j}$, then the unit vector along **a** + **b** will be **[Rajasthan PET 1985, 1995]**

(a) $\frac{\mathbf{i}-\mathbf{j}}{\sqrt{2}}$ (b) $\mathbf{i}+\mathbf{j}$ (c) $\sqrt{2}(\mathbf{i}+\mathbf{j})$ (d) $\frac{\mathbf{i}+\mathbf{j}}{\sqrt{2}}$

24. What should be added in vector $\mathbf{a}=3\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ to get its resultant a unit vector **i** **[Roorkee 1977]**

(a) $-2\mathbf{i}-4\mathbf{j}+2\mathbf{k}$ (b) $-2\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ (c) $2\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ (d) None of these

25. If $\mathbf{a}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$, $\mathbf{b}=-\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=3\mathbf{i}+\mathbf{j}$, then the unit vector along its resultant is **[Roorkee 1980]**

(a) $3\mathbf{i}+5\mathbf{j}+4\mathbf{k}$ (b) $\frac{3\mathbf{i}+5\mathbf{j}+4\mathbf{k}}{50}$ **(c)** $\frac{3\mathbf{i}+5\mathbf{j}+4\mathbf{k}}{5\sqrt{2}}$ (d) None of these

26. In the triangle *ABC,* $\overrightarrow{AB}=\mathbf{a}$, $\overrightarrow{AC}=\mathbf{c}$, $\overrightarrow{BC}=\mathbf{b}$, then **[Rajasthan PET 1984]**

(a) $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$ (b) $\mathbf{a}+\mathbf{b}-\mathbf{c}=0$ (c) $\mathbf{a}-\mathbf{b}+\mathbf{c}=0$ (d) $-\mathbf{a}+\mathbf{b}+\mathbf{c}=0$

27. If **a** has magnitude 5 and points north-east and vector **b** has magnitude 5 and points north-west, then $|\mathbf{a}-\mathbf{b}|=$ **[MNR 1**

(a) 25 (b) 5 (c) $7\sqrt{3}$ (d) $5\sqrt{2}$

28. If $|\mathbf{a}|=3$, $|\mathbf{b}|=4$ and $|\mathbf{a}+\mathbf{b}|=5$, then $|\mathbf{a}-\mathbf{b}|=$ **[EAMCET 1994]**

(a) 6 (b) 5 (c) 4 (d) 3

29. If the sum of two unit vectors is a unit vector, then the angle between them is equal to

[MP PET 1999; UPSEAT 2000; Rajasthan PET 2002; Roorkee 1998]

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

30. *A, B, C, D, E* are five coplanar points, then $\overrightarrow{DA}+\overrightarrow{DB}+\overrightarrow{DC}+\overrightarrow{AE}+\overrightarrow{BE}+\overrightarrow{CE}$ is equal to **[Rajasthan PET 1999]**

(a) $\overrightarrow{DE}$ (b) $3\overrightarrow{DE}$ (c) $2\overrightarrow{DE}$ (d) $4\overrightarrow{ED}$

31. If $\mathbf{a}\neq 0, \mathbf{b}\neq 0$ and $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}-\mathbf{b}|$, then the vectors **a** and **b** are

[MNR 1988; IIT Screening 1989; MP PET 1990, 97; Rajasthan PET 1984, 90, 96, 99; Karnataka CET 1999; Roorkee 1986]

(a) Parallel to each other (b) Perpendicular to each other

(c) Inclined at an angle of 60° (d) Neither perpendicular nor parallel

32. If *ABCDEF* is a regular hexagon and $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=\lambda\overrightarrow{AD}$,then $\lambda=$ **[Rajasthan PET 1985]**

(a) 2 (b) 3 (c) 4 (d) 6

33. If *O* be the circumcentre and *O*' be the orthocentre of a triangle *ABC*, then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=$ **[MNR 1987, EAMCET 1994]**

(a) $2\overrightarrow{OO'}$ (b) $2\overrightarrow{O'O}$ (c) $\overrightarrow{OO'}$ (d) $\overrightarrow{O'O}$

34. Let $\mathbf{a}=\mathbf{i}$ be a vector which makes an angle of 120° with a unit vector **b**. Then the unit vector (**a** + **b**) is **[MP PET 1991]**

(a) $-\frac{1}{2}\mathbf{i}+\frac{\sqrt{3}}{2}\mathbf{j}$ (b) $-\frac{\sqrt{3}}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}$ (c) $\frac{1}{2}\mathbf{i}+\frac{\sqrt{3}}{2}\mathbf{j}$ (d) $\frac{\sqrt{3}}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}$

35. If θ be the angle between the unit vectors **a** and **b**, then $\cos\frac{\theta}{2}=$ **[MP PET 1998]**

(a) $\frac{1}{2}|\mathbf{a}-\mathbf{b}|$ (b) $\frac{1}{2}|\mathbf{a}+\mathbf{b}|$ (c) $\frac{|\mathbf{a}-\mathbf{b}|}{|\mathbf{a}+\mathbf{b}|}$ (d) $\frac{|\mathbf{a}+\mathbf{b}|}{|\mathbf{a}-\mathbf{b}|}$

36. If $|\mathbf{a}|=3$, $|\mathbf{b}|=4$, $|\mathbf{c}|=5$ and $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$, then the angle between **a** and **b** is **[MP PET 1989; Bihar CEE 1994]**

(a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

37. If *ABCD* is a parallelogram, $\overrightarrow{AB}=2\mathbf{i}+4\mathbf{j}-5\mathbf{k}$ and $\overrightarrow{AD}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$, then the unit vector in the direction of *BD* is **[Roorkee**

(a) $\frac{1}{\sqrt{69}}(\mathbf{i}+2\mathbf{j}-8\mathbf{k})$ (b) $\frac{1}{69}(\mathbf{i}+2\mathbf{j}-8\mathbf{k})$ (c) $\frac{1}{\sqrt{69}}(-\mathbf{i}-2\mathbf{j}+8\mathbf{k})$ (d) $\frac{1}{69}(-\mathbf{i}-2\mathbf{j}+8\mathbf{k})$

38. If **a** and **b** are unit vectors making an angle θ with each other then $|\mathbf{a}-\mathbf{b}|$ is **[BIT Ranchi 1991; Karnataka CET 2000, 01]**

(a) 1 (b) 0 (c) $\cos\frac{\theta}{2}$ (d) $2\sin\frac{\theta}{2}$

39. If the moduli of the vectors **a, b, c** are 3, 4, 5 respectively and **a** and **b** + **c**, **b** and **c** + **a**, **c** and **a** + **b** are mutually perpendicular, then the modulus of **a** + **b** + **c** is **[IIT 1981]**

(a) $\sqrt{12}$ (b) 12 (c) $5\sqrt{2}$ (d) 50

40. If **a** and **b** are unit vectors and **a** – **b** is also a unit vector, then the angle between **a** and **b** is **[Rajasthan PET 1991; MP PE**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

41. If in a triangle $\overrightarrow{AB}=\mathbf{a}$, $\overrightarrow{AC}=\mathbf{b}$ and *D, E* are the mid-points of *AB* and *AC* respectively, then $\overrightarrow{DE}$ is equal to **[Rajasthan P**

(a) $\frac{\mathbf{a}}{4}-\frac{\mathbf{b}}{4}$ (b) $\frac{\mathbf{a}}{2}-\frac{\mathbf{b}}{2}$ (c) $\frac{\mathbf{b}}{4}-\frac{\mathbf{a}}{4}$ (d) $\frac{\mathbf{b}}{2}-\frac{\mathbf{a}}{2}$

42. *ABCDE* is a pentagon. Forces $\overrightarrow{AB}, \overrightarrow{AE}, \overrightarrow{DC}, \overrightarrow{ED}$ act at a point. Which force should be added to this system to make the resultant $2\overrightarrow{AC}$ **[MNR 1984]**

(a) $\overrightarrow{AC}$ (b) $\overrightarrow{AD}$ (c) $\overrightarrow{BC}$ (d) $\overrightarrow{BD}$

43. In a regular hexagon *ABCDEF*, $\overrightarrow{AE} =$ **[MNR 1984]**

(a) $\overrightarrow{AC}+\overrightarrow{AF}+\overrightarrow{AB}$ (b) $\overrightarrow{AC}+\overrightarrow{AF}-\overrightarrow{AB}$ (c) $\overrightarrow{AC}+\overrightarrow{AB}-\overrightarrow{AF}$ (d) None of these

44. $3\overrightarrow{OD}+\overrightarrow{DA}+\overrightarrow{DB}+\overrightarrow{DC}=$ **[IIT 1988]**

(a) $\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$ (b) $\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{BD}$ (c) $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}$ (d) None of these

45. In a triangle *ABC*, if $2\overrightarrow{AC}=3\overrightarrow{CB}$, then $2\overrightarrow{OA}+3\overrightarrow{OB}$ equals **[IIT 1988]**

(a) $5\overrightarrow{OC}$ (b) $-\overrightarrow{OC}$ (c) $\overrightarrow{OC}$ (d) None of these

46. If $|\overrightarrow{AO}+\overrightarrow{OB}|=|\overrightarrow{BO}+\overrightarrow{OC}|$, then *A, B, C* form **[IIT 1983]**

(a) Equilateral triangle (b) Right angled triangle (c) Isosceles triangle (d) Line

47. Three forces of magnitudes 1, 2, 3 dynes meet in a point and act along diagonals of three adjacent faces of a cube. The resultant force is **[MNR 1987]**

(a) 114 dynes (b) 6 dynes (c) 5 dynes (d) None of these

48. If $\mathbf{p}+\mathbf{q}+\mathbf{r}=0, |\mathbf{p}|=3, |\mathbf{q}|=5, |\mathbf{r}|=7$. Then angle between **p** and **q** is

[UPSEAT 2001; Kurukshetra CEE 1998; AIEEE 2002, MP PET 2002]

(a) $\frac{\pi}{16}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

49. If *A, B, C* are the vertices of a triangle whose position vectors are **a, b, c** and **G** is the centroid of the ΔABC, then $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}$ is

[Karnataka CET 2000; MP PET 1997]

(a) 0 (b) $\vec{A}+\vec{B}+\vec{C}$ (c) $\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$ (d) $\frac{\mathbf{a}-\mathbf{b}-\mathbf{c}}{3}$

50. If $\mathbf{a}=3\mathbf{i}-2\mathbf{j}+\mathbf{k}$, $\mathbf{b}=2\mathbf{i}-4\mathbf{j}-3\mathbf{k}$ and $\mathbf{c}=-\mathbf{i}+2\mathbf{j}+2\mathbf{k}$, then $\mathbf{a}+\mathbf{b}+\mathbf{c}$ is **[MP PET 2001]**

(a) $3\mathbf{i}-4\mathbf{j}$ (b) $3\mathbf{i}+4\mathbf{j}$ (c) $4\mathbf{i}-4\mathbf{j}$ (d) $4\mathbf{i}+4\mathbf{j}$

51. If **x** and **y** are two unit vectors and π is the angle between them, then $\frac{1}{2}|\mathbf{x}-\mathbf{y}|$ is equal to **[UPSEAT 2001]**

(a) 0 (b) $\pi/2$ (c) 1 (d) $\pi/4$

52. If *D, E, F* are respectively the mid points of *AB, AC* and *BC* in ΔABC, then $\overrightarrow{BE}+\overrightarrow{AF}=$ **[EAMCET 2003]**

(a) $\overrightarrow{DC}$ (b) $\frac{1}{2}\overrightarrow{BF}$ (c) $2\overrightarrow{BF}$ (d) $\frac{3}{2}\overrightarrow{BF}$

53. If *ABCD* is a rhombus whose diagonals cut at the origin *O*, then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$ equals

(a) $\overrightarrow{AB}+\overrightarrow{AC}$ (b) $\vec{O}$ (c) $2(\overrightarrow{AB}+\overrightarrow{BC})$ (d) $\overrightarrow{AC}+\overrightarrow{BD}$

54. *ABCD* is a parallelogram with *AC* and *BD* as diagonals. Then $\overrightarrow{AC}-\overrightarrow{BD}=$ **[EAMCET 2001]**

(a) $4\overrightarrow{AB}$ (b) $3\overrightarrow{AB}$ (c) $2\overrightarrow{AB}$ (d) $\overrightarrow{AB}$

55. The vectors **b** and **c** are in the direction of north-east and north-west respectively and $|\mathbf{b}|=|\mathbf{c}|=4$. The magnitude and direction of the vector $\mathbf{d}=\mathbf{c}-\mathbf{b}$, are **[Roorkee 2000]**

(a) $4\sqrt{2}$, towards north (b) $4\sqrt{2}$, towards west (c) 4, towards east (d) 4, towards south

56. Let **a** and **b** be two unit vectors inclined at an angle θ, then $\sin(\theta/2)$ is equal to **[UPSEAT 2002]**

(a) $\frac{1}{2}|\mathbf{a}-\mathbf{b}|$ (b) $\frac{1}{2}|\mathbf{a}+\mathbf{b}|$ (c) $|\mathbf{a}-\mathbf{b}|$ (d) $|\mathbf{a}+\mathbf{b}|$

57. If **a, b, c** are three vectors such that $\mathbf{a}=\mathbf{b}+\mathbf{c}$ and the angle between **b** and **c** is $\pi/2$, then **[EAMCET 2003; Bihar CEE 198**

(a) $a^2=b^2+c^2$ (b) $b^2=c^2+a^2$ (c) $c^2=a^2+b^2$ (d) $2a^2-b^2=c^2$

(**Note :** Here $a=|\mathbf{a}|$, $b=|\mathbf{b}|$, $c=|\mathbf{c}|$)

Advance Level

58. If $\mathbf{a},\mathbf{b},\mathbf{c}$ are three vectors of equal magnitude and the angle between each pair of vectors is $\frac{\pi}{3}$ such that $|\mathbf{a}+\mathbf{b}+\mathbf{c}|=\sqrt{6}$ then $|\mathbf{a}|$ is equal to

(a) 2 (b) –1 (c) 1 (d) $\frac{1}{3}\sqrt{6}$

59. Let $\mathbf{a},\mathbf{b},\mathbf{c}$ be three unit vectors such that $|\mathbf{a}+\mathbf{b}+\mathbf{c}|=1$ and $\mathbf{a}\perp\mathbf{b}$. If **c** makes angles α,β with **a, b** respectively then $\cos\alpha+\cos\beta$ is equal to

(a) $\frac{3}{2}$ (b) 1 (c) –1 (d) None of these

60. A vector of magnitude 2 along a bisector of the angle between the two vectors $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$ is

(a) $\frac{2}{\sqrt{10}}(3\mathbf{i}-\mathbf{k})$ (b) $\frac{1}{\sqrt{26}}(\mathbf{i}-4\mathbf{j}+3\mathbf{k})$ (c) $\frac{2}{\sqrt{26}}(\mathbf{i}-4\mathbf{j}+3\mathbf{k})$ (d) None of these

61. The vector $\mathbf{i}+x\mathbf{j}+3\mathbf{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\mathbf{i}+(4x-2)\mathbf{j}+2\mathbf{k}$. The value of x is

(a) $-\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 2

62. If I is the centre of a circle inscribed in a triangle ABC, then $|\overrightarrow{BC}|\overrightarrow{IA}+|\overrightarrow{CA}|\overrightarrow{IB}+|\overrightarrow{AB}|\overrightarrow{IC}$ is

(a) 0 (b) $\overrightarrow{IA}+\overrightarrow{IB}+\overrightarrow{IC}$ (c) $\frac{\overrightarrow{IA}+\overrightarrow{IB}+\overrightarrow{IC}}{3}$ (d) None of these

63. If the vector $-\mathbf{i}+\mathbf{j}-\mathbf{k}$ bisects the angle between the vector $\mathbf{e}$ and the vector $3\mathbf{i}+4\mathbf{j}$, then the unit vector in the direction of **e** is

(a) $\frac{1}{15}(11\mathbf{i}+10\mathbf{j}+2\mathbf{k})$ (b) $-\frac{1}{15}(11\mathbf{i}-10\mathbf{j}+2\mathbf{k})$ (c) $-\frac{1}{15}(11\mathbf{i}+10\mathbf{j}-2\mathbf{k})$ (d) $-\frac{1}{15}(11\mathbf{i}+10\mathbf{j}+2\mathbf{k})$

64. The sides of a parallelogram are $2\mathbf{i}+4\mathbf{j}-5\mathbf{k}$, $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$,then the unit vector parallel to one of the diagonals

(a) $\frac{1}{7}(3\mathbf{i}+6\mathbf{j}-2\mathbf{k})$ (b) $\frac{1}{7}(3\mathbf{i}-6\mathbf{j}-2\mathbf{k})$ (c) $\frac{1}{7}(-3\mathbf{i}+6\mathbf{j}-2\mathbf{k})$ (d) $\frac{1}{7}(3\mathbf{i}+6\mathbf{j}+2\mathbf{k})$

65. A point O is the centre of a cricle circumscribed about a triangle ABC. Then $\overrightarrow{OA}\sin 2A+\overrightarrow{OB}\sin 2B+\overrightarrow{OC}\sin 2C$ is equal to

(a) $(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC})\sin 2A$ (b) $3.\overrightarrow{OG}$, where G is the centroid of triangle ABC

(c) $\vec{O}$ (d) None of these

66. If $\mathbf{a}+\mathbf{b}+\mathbf{c}=\alpha\mathbf{d}$, $\mathbf{b}+\mathbf{c}+\mathbf{d}=\beta\mathbf{a}$ and **a, b, c** are non-coplanar, then the sum of $\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}=$ **[DCE 1997]**

(a) 0 (b) $(\beta-1)\mathbf{d}+(\alpha-1)\mathbf{a}$ (c) $(\alpha-1)\mathbf{d}-(\beta-1)\mathbf{a}$ (d) $(\alpha-1)\mathbf{d}+(\beta-1)\mathbf{a}$

67. Let **a** and **b** be two non-parallel unit vectors in a plane. If the vectors $(\alpha\mathbf{a}+\mathbf{b})$ bisects the internal angle between **a** and **b,** then α is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 4

68. The horizontal force and the force inclined at an angle 60° with the vertical, whose resultant is in vertical direction of P kg, are **[IIT 1983]**

(a) P, $2P$ (b) $P, P\sqrt{3}$ (c) $2P, P\sqrt{3}$ (d) None of these

69. If the resultant of two forces is of magnitude P and equal to one of them and perpendicular to it, then the other force is **[MNR 1986]**

(a) $P\sqrt{2}$ (b) P (c) $P\sqrt{3}$ (d) None of these

70. ABC is an isosceles triangle right angled at A. Forces of magnitude $2\sqrt{2}, 5$ and 6 act along $\overrightarrow{BC}, \overrightarrow{CA}$ and $\overrightarrow{AB}$ respectively. The magnitude of their resultant force is **[Roorkee 1999]**

(a) 4 (b) 5 (c) $11+2\sqrt{2}$ (d) 30

71. If the resultant of two forces of magnitudes P and Q acting at a point at an angle of 60° is $\sqrt{7}Q$, then P/Q is **[Roorkee 1999]**

(a) 1 (b) 3/2 (c) 2 (d) 4

72. Five points given by A, B, C, D, E are in a plane. Three forces $\overrightarrow{AC}, \overrightarrow{AD}$ and $\overrightarrow{AE}$ act at A and three forces $\overrightarrow{CB}, \overrightarrow{DB}, \overrightarrow{EB}$ act at B. Then their resultant is **[AMU 2001]**

(a) $2\overrightarrow{AC}$ (b) $3\overrightarrow{AB}$ (c) $3\overrightarrow{DB}$ (d) $2\overrightarrow{BC}$

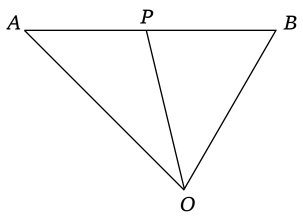
Position Vectors

Basic Level

73. If **a, b, c** are the position vectors of the vertices A, B, C of the triangle ABC, then the centroid of ΔABC is **[MP PET 1987**

(a) $\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$ (b) $\frac{1}{2}\left(\mathbf{a}+\frac{\mathbf{b}+\mathbf{c}}{2}\right)$ (c) $\mathbf{a}+\frac{\mathbf{b}+\mathbf{c}}{2}$ (d) $\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{2}$

74. If in the given figure $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$ and $AP:PB=m:n$, then $\overrightarrow{OP}=$ **[Rajasthan PET 1981; MP PET 1988]**



(a) $\frac{m\mathbf{a}+n\mathbf{b}}{m+n}$ (b) $\frac{n\mathbf{a}+m\mathbf{b}}{m+n}$ (c) $m\mathbf{a}-n\mathbf{b}$ (d) $\frac{m\mathbf{a}-n\mathbf{b}}{m-n}$

75. The position vectors of A and B are $\mathbf{i}-\mathbf{j}+2\mathbf{k}$ and $3\mathbf{i}-\mathbf{j}+3\mathbf{k}$. The position vector of the middle point of the line AB is **[MP PET 1988]**

(a) $\frac{1}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}+\mathbf{k}$ (b) $2\mathbf{i}-\mathbf{j}+\frac{5}{2}\mathbf{k}$ (c) $\frac{3}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}+\frac{3}{2}\mathbf{k}$ (d) None of these

76. If the position vectors of the points A and B are $\mathbf{i}+3\mathbf{j}-\mathbf{k}$ and $3\mathbf{i}-\mathbf{j}-3\mathbf{k}$, then what will be the position vector of the mid point of AB **[MP PET 1992]**

(a) $\mathbf{i}+2\mathbf{j}-\mathbf{k}$ (b) $2\mathbf{i}+\mathbf{j}-2\mathbf{k}$ (c) $2\mathbf{i}+\mathbf{j}-\mathbf{k}$ (d) $\mathbf{i}+\mathbf{j}-2\mathbf{k}$

77. The position vectors of two points A and B are $\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}+\mathbf{k}$ respectively. Then $|\overrightarrow{AB}|=$ **[Ranchi BIT 1992]**

(a) 2 (b) 3 (c) 4 (d) 5

78. The position vector of the points which divides internally in the ratio 2 : 3 the join of the points $2\mathbf{a}-3\mathbf{b}$ and $3\mathbf{a}-2\mathbf{b}$, is

[Al CBSE 1985]

(a) $\frac{12}{5}\mathbf{a}+\frac{13}{5}\mathbf{b}$ (b) $\frac{12}{5}\mathbf{a}-\frac{13}{5}\mathbf{b}$ (c) $\frac{3}{5}\mathbf{a}-\frac{2}{5}\mathbf{b}$ (d) None of these

79. If **a** and **b** are P.V. of two points *A, B,* and *C* divides *AB* in ratio 2 : 1, then P.V. of *C* is **[Rajasthan PET 1996]**

(a) $\frac{\mathbf{a}+2\mathbf{b}}{3}$ (b) $\frac{2\mathbf{a}+\mathbf{b}}{3}$ (c) $\frac{\mathbf{a}+2}{3}$ (d) $\frac{\mathbf{a}+\mathbf{b}}{2}$

80. If three points *A, B, C* whose position vector are respectively $\mathbf{i}-2\mathbf{j}-8\mathbf{k}, 5\mathbf{i}-2\mathbf{k}$ and $11\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ are collinear, then the ratio in which *B* divides *AC* is **[Rajasthan PET 1999]**

(a) 1 : 2 (b) 2 : 3 (c) 2 : 1 (d) 1 : 1

81. If *O* is the origin and *C* is the mid point of *A* (2, –1) and *B* (–4, 3). Then value of $\overrightarrow{OC}$ is **[Rajasthan PET 2001]**

(a) $\mathbf{i}+\mathbf{j}$ (b) $\mathbf{i}-\mathbf{j}$ (c) $-\mathbf{i}+\mathbf{j}$ (d) $-\mathbf{i}-\mathbf{j}$

82. If the position vectors of *P* and *Q* are $\mathbf{i}+3\mathbf{j}-7\mathbf{k}$ and $5\mathbf{i}-2\mathbf{j}+4\mathbf{k}$ respectively, then $\overrightarrow{PQ}$ is equal to **[MP PET 2003]**

(a) $-4\mathbf{i}+5\mathbf{j}-11\mathbf{k}$ (b) $4\mathbf{i}-5\mathbf{j}+11\mathbf{k}$ (c) $\mathbf{i}+\mathbf{j}+\mathbf{k}$ (d) None of these

83. The position vectors of two vertices and the centroid of a triangle are $\mathbf{i}+\mathbf{j}, 2\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{k}$ respectively. The position vector of the third vertex of the triangle is

(a) $-3\mathbf{i}+2\mathbf{k}$ (b) $3\mathbf{i}-2\mathbf{k}$ (c) $\mathbf{i}+\frac{2}{3}\mathbf{k}$ (d) None of these

84. The position vector of three consecutive vertices of a parallelogram are $\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ and $7\mathbf{i}+9\mathbf{j}+11\mathbf{k}$ respectively. The position vector of the fourth vertex is

(a) $7(\mathbf{i}+\mathbf{j}+\mathbf{k})$ (b) $5(\mathbf{i}+\mathbf{j}+\mathbf{k})$ (c) $6\mathbf{i}+8\mathbf{j}+10\mathbf{k}$ (d) None of these

Advance Level

85. If **a** and **b** are the position vectors of *A* and *B* respectively, then the position vector of a point *C* on *AB* produced such that $\overrightarrow{AC}=3\overrightarrow{AB}$ is **[MNR 1980; MP PET 1995, 1999]**

(a) 3**a** – **b** (b) 3**b** – **a** (c) 3**a** – 2**b** (d) 3**b** – 2**a**

86. If the position vectors of the points *A, B, C, D* be $2\mathbf{i}+3\mathbf{j}+5\mathbf{k}$, $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$, $-5\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ and $\mathbf{i}+10\mathbf{j}+10\mathbf{k}$ respectively, then

[MNR 1982]

(a) $\overrightarrow{AB}=\overrightarrow{CD}$ (b) $\overrightarrow{AB}\parallel\overrightarrow{CD}$ (c) $\overrightarrow{AB}\perp\overrightarrow{CD}$ (d) None of these

87. The position vector of a point *C* with respect to *B* is $\mathbf{i}+\mathbf{j}$ and that of *B* with respect to *A* is $\mathbf{i}-\mathbf{j}$. The position vector of *C* with respect to *A* is **[MP PET 1989]**

(a) 2**i** (b) 2**j** (c) –2**j** (d) –2**i**

88. *A* and *B* are two points. The position vector of *A* is 6**b** – 2**a**. A point *P* divides the line *AB* in the ratio 1 : 2. If **a** – **b** is the position vector of *P* , then the position vector of *B* is given by **[MP PET 1993]**

(a) $7\mathbf{a}-15\mathbf{b}$ (b) $7\mathbf{a}+15\mathbf{b}$ (c) $15\mathbf{a}-7\mathbf{b}$ (d) $15\mathbf{a}+7\mathbf{b}$

89. The points *D, E, F* divide *BC, CA* and *AB* of the triangle *ABC* in the ratio 1 : 4, 3 : 2 and 3 : 7 respectively and the point *K* divides *AB* in the ratio 1 : 3, then $(\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}):\overrightarrow{CK}$ is equal to **[MNR 1987]**

(a) 1 : 1 (b) 2 : 5 (c) 5 : 2 (d) None of these

90. The point B divides the arc AC of a quadrant of a circle in the ratio 1 : 2. If O is the centre and $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, then the vector $\overrightarrow{OC}$ is **[MNR 1988]**

(a) $\mathbf{b} - 2\mathbf{a}$ (b) $2\mathbf{a} - \mathbf{b}$ (c) $3\mathbf{b} - 2\mathbf{a}$ (d) None of these

91. The point having position vectors $2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, $3\mathbf{i}+4\mathbf{j}+2\mathbf{k}$, $4\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ are the vectices of **[EAMCET 1988]**

(a) Right angled triangle (b) Isosceles triangle (c) Equilateral triangle (d) Collinear

92. Let p and q be the position vectors of P and Q respectively with respect to O and $|\mathbf{p}| = p$, $|\mathbf{q}| = q$. The points R and S divide PQ internally and externally in the ratio 2 : 3 respectively. If $\overrightarrow{OR}$ and $\overrightarrow{OS}$ are perpendicular,then **[IIT Sc**

(a) $9p^2 = 4q^2$ (b) $4p^2 = 9q^2$ (c) $9p = 4q$ (d) $4p = 9q$

93. The position vectors of the points A, B, C are $(2\mathbf{i}+\mathbf{j}-\mathbf{k})$, $(3\mathbf{i}-2\mathbf{j}+\mathbf{k})$ and $(\mathbf{i}+4\mathbf{j}-3\mathbf{k})$ respectively. These points **[Haryana CET 2002]**

(a) Form an isosceles triangle (b) Form a right-angled triangle (c) Are collinear (d) Form a scalene triangle

94. $ABCDEF$ is a regular hexagon where centre O is the origin. If the position vectors of A and B are $\mathbf{i}-\mathbf{j}+2\mathbf{k}$ and $2\mathbf{i}+\mathbf{j}-\mathbf{k}$ respectively, then $\overrightarrow{BC}$ is equal to

(a) $\mathbf{i}-\mathbf{j}+2\mathbf{k}$ (b) $-\mathbf{i}+\mathbf{j}-2\mathbf{k}$ (c) $3\mathbf{i}+3\mathbf{j}-4\mathbf{k}$ (d) None of these

95. Let $\overrightarrow{AB} = 3\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $\overrightarrow{AC} = \mathbf{i}-\mathbf{j}+3\mathbf{k}$. If the point P on the line segment BC is equidistant from AB and AC, then $\overrightarrow{AP}$ is

(a) $2\mathbf{i}-\mathbf{k}$ (b) $\mathbf{i}-2\mathbf{k}$ (c) $2\mathbf{i}+\mathbf{k}$ (d) None of these

96. If $4\mathbf{i}+7\mathbf{j}+8\mathbf{k}$, $2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, and $2\mathbf{i}+5\mathbf{j}+7\mathbf{k}$ are the position vectors of the vertices A, B and C respectively of triangle ABC. The position vector of the point where the bisector of angle A meets BC, is

(a) $\frac{2}{3}(-6\mathbf{i}-8\mathbf{j}-6\mathbf{k})$ (b) $\frac{2}{3}(6\mathbf{i}+8\mathbf{j}+6\mathbf{k})$ (c) $\frac{1}{3}(6\mathbf{i}+13\mathbf{j}+18\mathbf{k})$ (d) $\frac{1}{3}(5\mathbf{j}+12\mathbf{k})$

Collinear and Parallel Vectors

Basic Level

97. If $\mathbf{a} = \mathbf{i}-2\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i}+\lambda\mathbf{j}$ are parallel, then λ is **[Rajasthan PET 1996]**

(a) 4 (b) 2 (c) –2 (d) – 4

98. The vectors $3\mathbf{i}+\mathbf{j}-5\mathbf{k}$ and $a\mathbf{i}+b\mathbf{j}-15\mathbf{k}$ are collinear, if **[Rajasthan PET 1986; MP PET 1988]**

(a) $a = 3,\ b = 1$ (b) $a = 9,\ b = 1$ (c) $a = 3,\ b = 3$ (d) $a = 9,\ b = 3$

99. If $\mathbf{a}$ = (1, –1) and $\mathbf{b}$ = (–2, m) are two collinear vectors, then m = **[MP PET 1998]**

(a) 4 (b) 3 (c) 2 (d) 0

100. If **a**, **b**, **c** are the position vectors of three collinear points, then the existence of x, y, z is such that

(a) $x\mathbf{a}+y\mathbf{b}+z\mathbf{c} = 0,\ x+y+z \neq 0$ (b) $x\mathbf{a}+y\mathbf{b}+z\mathbf{c} \neq 0,\ x+y+z = 0$

(c) $x\mathbf{a}+y\mathbf{b}+z\mathbf{c} \neq 0,\ x+y+z \neq 0$ (d) $x\mathbf{a}+y\mathbf{b}+z\mathbf{c} = 0,\ x+y+z = 0$

101. If **a** and **b** are two non-collinear vectors, then $x\mathbf{a}+y\mathbf{b}$ = 0 **[Rajasthan PET 2001]**

(a) x = 0, but y is not necessarily zero (b) y = 0, but x is not necessarily zero

(c) x = 0, y = 0 (d) None of these

102. If **a** and **b** are two non-collinear vectors, then $x\mathbf{a}+y\mathbf{b}$ (where x and y are scalars) represents a vector which is **[MP PET**

(a) Parallel to **b** (b) Parallel to **a** (c) Coplanar with **a** and **b** (d) None of these

103. If **a, b, c** are non-collinear vectors such that for some scalars x, y, z, $x\mathbf{a}+y\mathbf{b}+z\mathbf{c} = 0$, then **[Rajasthan PET 2002]**

(a) $x=0, y=0, z=0$ (b) $x\neq 0, y\neq 0, z=0$ (c) $x=0, y\neq 0, z\neq 0$ (d) $x\neq 0, y\neq 0, z\neq 0$

104. If the position vectors of the points *A, B, C* be **a**, **b**, 3**a** – 2**b** respectively, then the points *A, B, C* are **[MP PET 1989]**

(a) Collinear (b) Non-collinear

(c) Form a right angled triangle (d) None of these

105. If two vertices of a triangle are $\mathbf{i}-\mathbf{j}$ and $\mathbf{j}+\mathbf{k}$, then the third vertex can be **[Roorkee 1995]**

(a) $\mathbf{i}+\mathbf{k}$ (b) $\mathbf{i}-2\mathbf{j}-\mathbf{k}$ (c) $\mathbf{i}-\mathbf{k}$ (d) $2\mathbf{i}-\mathbf{j}$

106. If the vectors $3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $6\mathbf{i}-4x\mathbf{j}+y\mathbf{k}$ are parallel, then the value of x and y will be **[Rajasthan PET 1985, 1986]**

(a) –1, –2 (b) 1, –2 (c) –1, 2 (d) 1, 2

107. The position vectors of four points *P, Q, R, S* are $2\mathbf{a}+4\mathbf{c}$, $5\mathbf{a}+3\sqrt{3}\mathbf{b}+4\mathbf{c}$, $-2\sqrt{3}\mathbf{b}+\mathbf{c}$ and $2\mathbf{a}+c$ respectively, then **[MP PET 1997]**

(a) $\overrightarrow{PQ}$ is parallel to $\overrightarrow{RS}$ (b) $\overrightarrow{PQ}$ is not parallel to $\overrightarrow{RS}$

(c) $\overrightarrow{PQ}$ is equal to $\overrightarrow{RS}$ (d) $\overrightarrow{PQ}$ is parallel and equal to $\overrightarrow{RS}$

108. The vectors $2\mathbf{i}+3\mathbf{j}$, $5\mathbf{i}+6\mathbf{j}$ and $8\mathbf{i}+\lambda\mathbf{j}$ have their initial points at (1, 1). The value of λ so that the vectors terminate on one straight line, is

(a) 0 (b) 3 (c) 6 (d) 9

109. The points with position vectors $20\mathbf{i}+p\mathbf{j}$, $5\mathbf{i}-\mathbf{j}$ and $10\mathbf{i}-13\mathbf{j}$ are collinear. The value of *p* is **[Pb. CET 1999]**

(a) 7 (b) – 37 (c) – 7 (d) 37

Advance Level

110. Three points whose position vectors are $\mathbf{a}+\mathbf{b}$, $\mathbf{a}-\mathbf{b}$ and $\mathbf{a}+k\mathbf{b}$ will be collinear, if the value of *k* is **[IIT 1984]**

(a) Zero (b) Only negative real number (c) Only positive real number (d) Every real number

111. The points with position vectors $10\mathbf{i}+3\mathbf{j}$, $12\mathbf{i}-5\mathbf{j}$ and $a\mathbf{i}+11\mathbf{j}$ are collinear, If *a* = **[MNR 1992; Kurukshetra CEE 2002]**

(a) – 8 (b) 4 (c) 8 (d) 12

112. Let the value of $\mathbf{p}=(x+4y)\mathbf{a}+(2x+y+1)\mathbf{b}$ and $\mathbf{q}=(y-2x+2)\mathbf{a}+(2x-3y-1)\mathbf{b}$, where **a** and **b** are non-collinear vectors. If $3\mathbf{p}=2\mathbf{q}$, then the value of *x* and *y* will be **[Rajasthan PET 1984; MNR 1984]**

(a) – 1, 2 (b) 2, –1 (c) 1, 2 (d) 2, 1

113. If $(x, y, z)\neq(0, 0, 0)$ and $(\mathbf{i}+\mathbf{j}+3\mathbf{k})x+(3\mathbf{i}-3\mathbf{j}+\mathbf{k})y+(-4\mathbf{i}+5\mathbf{j})z=\lambda(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})$, then the value of λ will be **[IIT 1982; Rajasthan PET 1984]**

(a) – 2, 0 (b) 0, – 2 (c) – 1, 0 (d) 0, – 1

114. The vectors $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$, $\lambda\mathbf{i}+4\mathbf{j}+7\mathbf{k}$, $-3\mathbf{i}-2\mathbf{j}-5\mathbf{k}$ are collinear, if λ equals **[Kurukshetra CEE 1996]**

(a) 3 (b) 4 (c) 5 (d) 6

115. If three points *A, B* and *C* have position vectors (1, *x*, 3), (3, 4, 7) and (*y*, – 2, – 5) respectively and if they are collinear, then (*x*, *y*) = **[EAMCET 2002]**

(a) (2, – 3) (b) (– 2, 3) (c) (2, 3) (d) (– 2, – 3)

116. The position vectors of three points are $2\mathbf{a}-\mathbf{b}+3\mathbf{c}$, $\mathbf{a}-2\mathbf{b}+\lambda\mathbf{c}$ and $\mu\mathbf{a}-5\mathbf{b}$ where **a**, **b**, **c** are non-coplanar vectors. The points are collinear when

(a) $\lambda=-2,\ \mu=\frac{9}{4}$ (b) $\lambda=-\frac{9}{4},\ \mu=2$ (c) $\lambda=\frac{9}{4},\ \mu=-2$ (d) None of these

117. Three points whose position vectors are **a, b, c** will be collinear if

(a) $\lambda\mathbf{a} + \mu\mathbf{b} = (\lambda + \mu)\mathbf{c}$ (b) $\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a} = 0$ (c) $[\mathbf{a}\ \ \mathbf{b}\ \ \mathbf{c}] = 0$ (d) None of these

118. If $\mathbf{p} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{q} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, then a vector along **r** which is linear combination of **p** and **q** and also perpendicular to **q** is

[MNR 1986]

(a) $\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$ (b) $\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}$ (c) $-\frac{1}{2}(\mathbf{i} + 5\mathbf{j} - 4\mathbf{k})$ (d) None of these

119. If **a** and **b** are two non zero and non-collinear vectors, then **a** + **b** and **a** – **b** are **[MP PET 1997]**

(a) Linearly dependent vectors (b) Linearly independent vectors

(c) Linearly dependent and independent vectors (d) None of these

120. If **p, q** are two non-collinear and non-zero vectors such that $(\mathbf{b} - \mathbf{c})\mathbf{p}\times\mathbf{q} + (\mathbf{c} - \mathbf{a})\mathbf{p} + (\mathbf{a} - \mathbf{b})\mathbf{q} = 0$, where **a, b, c** are the lengths of the sides of a triangle, then the triangle is

(a) Right angled (b) Obtuse angled (c) Equilateral (d) Isosceles

121. If $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k},\ \mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k},\ \mathbf{b} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{c} = -2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ such that $r = \lambda\,\mathbf{a} + \mu\,\mathbf{b} + \nu\,\mathbf{c}$ then

(a) $\mu,\ \frac{\lambda}{2},\ \nu$ are in A.P. (b) $\lambda,\ \mu,\ \nu$ are in A.P. (c) $\lambda,\ \mu,\ \nu$ are in H.P. (d) $\mu,\ \lambda,\ \nu$ are in G.P.

122. Let **a, b, c** are three non-coplanar vectors such that $\mathbf{r}_1 = \mathbf{a} - \mathbf{b} + \mathbf{c},\ \mathbf{r}_2 = \mathbf{b} + \mathbf{c} - \mathbf{a},\ \mathbf{r}_3 = \mathbf{c} + \mathbf{a} + \mathbf{b},\ \mathbf{r} = 2\mathbf{a} - 3\mathbf{b} + 4\mathbf{c}$. If $\mathbf{r} = \lambda_1\mathbf{r}_1 + \lambda_2\mathbf{r}_2 + \lambda_3\mathbf{r}_3$, then

(a) $\lambda_1 = 7$ (b) $\lambda_1 + \lambda_3 = 3$ (c) $\lambda_1 + \lambda_2 + \lambda_3 = 4$ (d) $\lambda_3 + \lambda_2 = 2$

123. If $\mathbf{c} = 2\mathbf{a} - 3\mathbf{b}$ and $2\mathbf{c} = 3\mathbf{a} + 4\mathbf{b}$ then **c** and **a** are

(a) Like parallel vectors (b) Unlike parallel vectors (c) Are at right angles (d) None of these

124. The sides of a triangle are in A.P., then the line joining the centroid to the incentre is parallel to

(a) The largest side (b) The smaller side (c) The middle side (d) None of the sides

125. In a trapezoid the vector $\overrightarrow{BC} = \lambda\overrightarrow{AD}$. We will then find that $\mathbf{p} = \overrightarrow{AC} + \overrightarrow{BD}$ is collinear with *AD*. If $\mathbf{p} = \mu\overrightarrow{AD}$, then

(a) $\mu = \lambda + 1$ (b) $\lambda = \mu + 1$ (c) $\lambda + \mu = 1$ (d) $\mu = 2 + \lambda$

Scalar or Dot Product of Two Vectors

Basic Level

126. The angle between the vectors $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $2\mathbf{i} - \mathbf{j} - \mathbf{k}$ is **[MNR 1990]**

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

127. If $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k},\ \mathbf{b} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{c} = 3\mathbf{i} + \mathbf{j}$, then $\mathbf{a} + t\mathbf{b}$ is perpendicular to **c** if t = **[MNR 1979; MP PET 2002]**

(a) 2 (b) 4 (c) 6 (d) 8

128. The angle between the vectors $3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ is **[MP PET 1990]**

(a) $\cos^{-1}\frac{2}{\sqrt{7}}$ (b) $\sin^{-1}\frac{2}{\sqrt{7}}$ (c) $\cos^{-1}\frac{2}{\sqrt{5}}$ (d) $\sin^{-1}\frac{2}{\sqrt{5}}$

129. If **a, b, c** are non zero-vectors such that **a . b = a . c,** then which statement is true **[Rajasthan PET 2001]**

(a) $\mathbf{b} = \mathbf{c}$ (b) $\mathbf{a} \perp (\mathbf{b} - \mathbf{c})$ (c) $\mathbf{b} = \mathbf{c}$ or $\mathbf{a} \perp (\mathbf{b} - \mathbf{c})$ (d) None of these

130. The vector $2\mathbf{i} + a\mathbf{j} + \mathbf{k}$ is perpendicular to the vector $2\mathbf{i} - \mathbf{j} - \mathbf{k}$, if a = **[MP PET 1987]**

(a) 5 (b) – 5 (c) – 3 (d) 3

131. The vector $2\mathbf{i}+\mathbf{j}-\mathbf{k}$ is perpendicular to the vector $\mathbf{i}-4\mathbf{j}+\lambda\mathbf{k}$, if $\lambda =$ **[MNR 1983; MP PET 1988]**

(a) 0 (b) – 1 (c) – 2 (d) – 3

132. If the vectors $a\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ and $3\mathbf{i}+6\mathbf{j}-5\mathbf{k}$ are perpendicular to each other, then *a* is given by **[MP PET 1993]**

(a) 9 (b) 16 (c) 25 (d) 36

133. The value of λ for which the vectors $2\lambda\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $2\mathbf{j}+\mathbf{k}$ are perpendicular, is **[MP PET 1992]**

(a) None (b) – 1 (c) 1 (d) Any

134. The angle between the vectors $\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}+\mathbf{k}$ is **[Ranchi BIT 1991]**

(a) $\cos^{-1}\left(\frac{1}{\sqrt{15}}\right)$ (b) $\cos^{-1}\left(\frac{4}{\sqrt{15}}\right)$ (c) $\cos^{-1}\left(\frac{4}{15}\right)$ (d) $\frac{\pi}{2}$

135. If λ is a unit vector perpendicular to plane of vector **a** and **b** and angle between them is θ, then **a . b** will be **[Rajastha**

(a) $|\mathbf{a}|\,|\mathbf{b}|\sin\theta\vec{\lambda}$ (b) $|\mathbf{a}|\,|\mathbf{b}|\cos\theta\vec{\lambda}$ (c) $|\mathbf{a}|\,|\mathbf{b}|\cos\theta$ (d) $|\mathbf{a}|\,|\mathbf{b}|\sin\theta$

136. If the vectors $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ and $p\mathbf{i}+q\mathbf{j}+r\mathbf{k}$ are perpendicular, then **[Rajasthan PET 1989]**

(a) $(a+b+c)(p+q+r)=0$ (b) $(a+b+c)(p+q+r)=1$ (c) $ap+bq+cr=0$ (d) $ap+bq+cr=1$

137. If θ be the angle between two vectors **a** and **b**, then $\mathbf{a}\,.\,\mathbf{b}\ge 0$ if **[MP PET 1995]**

(a) $0\le\theta\le\pi$ (b) $\frac{\pi}{2}\le\theta\le\pi$ (c) $0\le\theta\le\frac{\pi}{2}$ (d) None of these

138. If $\mathbf{a}=2\mathbf{i}+4\mathbf{j}+2\mathbf{k}$ and $\mathbf{b}=8\mathbf{i}-3\mathbf{j}+\lambda\mathbf{k}$ and $\mathbf{a}\perp\mathbf{b}$, then value of λ will be **[Rajasthan PET 1995]**

(a) 2 (b) – 1 (c) – 2 (d) 1

139. If **a** and **b** are mutually perpendicular vectors, then $(\mathbf{a}+\mathbf{b})^2 =$ **[MP PET 1994]**

(a) $\mathbf{a}+\mathbf{b}$ (b) $\mathbf{a}-\mathbf{b}$ (c) $\mathbf{a}^2-\mathbf{b}^2$ (d) $(\mathbf{a}-\mathbf{b})^2$

140. If $\mathbf{a}=\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ and $\mathbf{b}=3\mathbf{i}-\mathbf{j}+2\mathbf{k}$, then the angle between the vectors $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$ is **[Karnataka CET 1994]**

(a) 30° (b) 60° (c) 90° (d) 0°

141. $\mathbf{a}\,.\,\mathbf{b}=0$, then **[Rajasthan PET 1995]**

(a) $\mathbf{a}\perp\mathbf{b}$ (b) $\mathbf{a}\parallel\mathbf{b}$

(c) Angle between **a** and **b** is 60° (d) None of these

142. The angle between the vectors $(2\mathbf{i}+6\mathbf{j}+3\mathbf{k})$ and $(12\mathbf{i}-4\mathbf{j}+3\mathbf{k})$ is **[MP PET 1996]**

(a) $\cos^{-1}\left(\frac{1}{10}\right)$ (b) $\cos^{-1}\left(\frac{9}{11}\right)$ (c) $\cos^{-1}\left(\frac{9}{91}\right)$ (d) $\cos^{-1}\left(\frac{1}{9}\right)$

143. If the vectors $a\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $-\mathbf{i}+5\mathbf{j}+a\mathbf{k}$ are perpendicular to each other, then *a* = **[MP PET 1996]**

(a) 6 (b) – 6 (c) 5 (d) – 5

144. If the angle between two vectors $\mathbf{i}+\mathbf{k}$ and $\mathbf{i}-\mathbf{j}+a\mathbf{k}$ is $\pi/3$, then the value of *a* = **[MP PET 1997]**

(a) 2 (b) 4 (c) –2 (d) 0

145. (**a . b**) **c** and (**a . c**) **b** are **[Rajasthan PET 2000]**

(a) Two like vectors (b) Two equal vectors

(c) Two vectors in direction of **a** (d) None of these

146. The angle between the vector $2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}-\mathbf{k}$ is **[UPSEAT 2000]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) 0

147. If **a** = (1, –1, 2), **b** = (– 2, 3, 5), **c** = (2, – 2, 4) and **i** is the unit vector in the *x*-direction, then $(\mathbf{a}-2\mathbf{b}+3\mathbf{c}).\mathbf{i} =$

[Karnataka CET 2001]

(a) 11 (b) 15 (c) 18 (d) 36

148. If $a\mathbf{i}+6\mathbf{j}-\mathbf{k}$ and $7\mathbf{i}-3\mathbf{j}+17\mathbf{k}$ are perpendicular vectors, then the value of a is **[Karnataka CET 2001]**

(a) 5 (b) – 5 (c) 7 (d) $\frac{1}{7}$

149. If $\mathbf{a}+\mathbf{b}\perp\mathbf{a}$ and $|\mathbf{b}|=\sqrt{2}|\mathbf{a}|$ then

(a) $(2\mathbf{a}+\mathbf{b})\parallel\mathbf{b}$ (b) $(2\mathbf{a}+\mathbf{b})\perp\mathbf{b}$ (c) $(2\mathbf{a}-\mathbf{b})\perp\mathbf{b}$ (d) $(2\mathbf{a}+\mathbf{b})\perp\mathbf{a}$

150. If **a** and **b** are adjacent sides of a rhombus, then **[Rajasthan PET 2001]**

(a) $\mathbf{a}\,.\,\mathbf{b}=0$ (b) $\mathbf{a}\times\mathbf{b}=\mathbf{0}$ (c) $\mathbf{a}\,.\,\mathbf{a}=\mathbf{b}\,.\,\mathbf{b}$ (d) None of these

151. If $|\mathbf{a}|=|\mathbf{b}|$, then $(\mathbf{a}+\mathbf{b}).(\mathbf{a}-\mathbf{b})$ is **[MP PET 2002]**

(a) Positive (b) Negative (c) Zero (d) None of these

152. If $4\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $3\mathbf{i}+m\mathbf{j}+2\mathbf{k}$ are at right angle, then m = **[Karnataka CET 2002]**

(a) – 6 (b) – 8 (c) – 10 (d) – 12

153. If the vectors $3\mathbf{i}+\lambda\mathbf{j}+\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}+8\mathbf{k}$ are perpendicular, then λ is **[Kerala (Engg.) 2002]**

(a) – 14 (b) 7 (c) 14 (d) $\frac{1}{7}$

154. $(\mathbf{a}\,.\,\mathbf{i})^2+(\mathbf{a}\,.\,\mathbf{j})^2+(\mathbf{a}\,.\,\mathbf{k})^2$ is equal to

(a) $\mathbf{a}^2$ (b) 3 (c) $|\mathbf{a}.(\mathbf{i}+\mathbf{j}+\mathbf{k})|^2$ (d) None of these

155. If the vectors $\mathbf{i}-2x\mathbf{j}-3y\mathbf{k}$ and $\mathbf{i}+3x\mathbf{j}+2y\mathbf{k}$ are orthogonal to each other, then the locus of the point (x, y) is

(a) A circle (b) An ellipse (c) A parabola (d) A straight line

156. If $\vec{A}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$, $\vec{B}=-\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\vec{C}=3\mathbf{i}+\mathbf{j}$, then the value of t such that $\vec{A}+t\vec{B}$ is at right angle to vector $\vec{C}$, is **[Rajasthan PET 2002]**

(a) 3 (b) 4 (c) 5 (d) 6

157. If **a** and **b** are two perpendicular vectors, then out of the following four statements

(i) $(\mathbf{a}+\mathbf{b})^2=(\mathbf{a})^2+(\mathbf{b})^2$ (ii) $(\mathbf{a}-\mathbf{b})^2=(\mathbf{a})^2+(\mathbf{b})^2$ (iii) $|\mathbf{a}+\mathbf{b}|^2=|\mathbf{a}|^2+|\mathbf{b}|^2$

(iv) $(\mathbf{a}+\mathbf{b})^2=(\mathbf{a}-\mathbf{b})^2$

(a) Only one is correct (b) Only two are correct (c) Only three are correct (d) All the four are correct

Advance Level

158. If **a, b, c** are unit vectors such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$, then $\mathbf{a}\,.\,\mathbf{b}+\mathbf{b}\,.\,\mathbf{c}+\mathbf{c}\,.\,\mathbf{a}=$ **[MP PET 1988; Karnataka CET 2000; UPSEAT 2003]**

(a) 1 (b) 3 (c) $-\frac{3}{2}$ (d) $\frac{3}{2}$

159. A unit vector in the xy-plane which is perpendicular to $4\mathbf{i}-3\mathbf{j}+\mathbf{k}$ is **[Rajasthan PET 1991]**

(a) $\frac{\mathbf{i}+\mathbf{j}}{\sqrt{2}}$ (b) $\frac{1}{5}(3\mathbf{i}+4\mathbf{j})$ (c) $\frac{1}{5}(3\mathbf{i}-4\mathbf{j})$ (d) None of these

160. The vectors $2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$ and $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ are perpendicular, when **[MNR 1982; MP PET 1988, 2002]**

(a) $a=2, b=3, c=-4$ (b) $a=4, b=4, c=5$ (c) $a=4, b=4, c=-5$ (d) None of these

161. The unit normal vector to the line joining $\mathbf{i}-\mathbf{j}$ and $2\mathbf{i}+3\mathbf{j}$ and pointing towards the origin is **[MP PET 1989]**

(a) $\frac{4\mathbf{i}-\mathbf{j}}{\sqrt{17}}$ (b) $\frac{-4\mathbf{i}+\mathbf{j}}{\sqrt{17}}$ (c) $\frac{2\mathbf{i}-3\mathbf{j}}{\sqrt{13}}$ (d) $\frac{-2\mathbf{i}+3\mathbf{j}}{\sqrt{13}}$

162. The position vector of coplanar points *A, B, C, D* are **a, b, c** and **d** respectively, in such a way that $(\mathbf{a}-\mathbf{d}).(\mathbf{b}-\mathbf{c})=(\mathbf{b}-\mathbf{d}).(\mathbf{c}-\mathbf{a})=0$, then the point *D* of the triangle *ABC* is **[IIT 1984]**

(a) Incentre (b) Circumcentre (c) Orthocentre (d) None of these

163. If $\vec{F_1}=\mathbf{i}-\mathbf{j}+\mathbf{k}, \vec{F_2}=-\mathbf{i}+2\mathbf{j}-\mathbf{k}, \vec{F_3}=\mathbf{j}-\mathbf{k}$, $\vec{A}=4\mathbf{i}-3\mathbf{j}-2\mathbf{k}$ and $\vec{B}=6\mathbf{i}+\mathbf{j}-3\mathbf{k}$, then the scalar product of $\vec{F_1}+\vec{F_2}+\vec{F_3}$ and $\overrightarrow{AB}$ will be **[Roorkee 1980]**

(a) 3 (b) 6 (c) 9 (d) 12

164. If the moduli of **a** and **b** are equal and angle between them is 120° and **a . b** = – 8, then | **a** | is equal to **[Rajasthan PET 1**

(a) – 5 (b) – 4 (c) 4 (d) 5

165. The position vector of vertices of a triangle *ABC* are $4\mathbf{i}-2\mathbf{j}, \mathbf{i}+4\mathbf{j}-3\mathbf{k}$ and $-\mathbf{i}+5\mathbf{j}+\mathbf{k}$ respectively, then $\angle ABC=$ **[Rajasthan PET 1988, 1997]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

166. *A, B, C, D* are any four points, then $\overrightarrow{AB}.\overrightarrow{CD}+\overrightarrow{BC}.\overrightarrow{AD}+\overrightarrow{CA}.\overrightarrow{BD}=$ **[MNR 1986]**

(a) $2\overrightarrow{AB}.\overrightarrow{BC}.\overrightarrow{CD}$ (b) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$ (c) $5\sqrt{3}$ (d) 0

167. If $|\mathbf{a}|=3, |\mathbf{b}|=1, |\mathbf{c}|=4$ and $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$, then $\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a}=$ **[MP PET 1995; Rajasthan PET 2000]**

(a) – 13 (b) – 10 (c) 13 (d) 10

168. The value of *c* so that for all real *x*, the vectors $cx\mathbf{i}-6\mathbf{j}+3\mathbf{k}, x\mathbf{i}+2\mathbf{j}+2cx\mathbf{k}$ make an obtuse angle are **[EAMCET 1994]**

(a) c < 0 (b) $0<c<\frac{4}{3}$ (c) $-\frac{4}{3}<c<0$ (d) $c>0$

169. The vector $\frac{1}{3}(2\mathbf{i}-2\mathbf{j}+\mathbf{k})$ is **[IIT Screening 1994]**

(a) A unit vector (b) Makes an angle $\frac{\pi}{3}$ with the vector $2\mathbf{i}-4\mathbf{j}+3\mathbf{k}$

(c) Parallel to the vector $-\mathbf{i}+\mathbf{j}-\frac{1}{2}\mathbf{k}$ (d) Perpendicular to the vector $3\mathbf{i}+2\mathbf{j}-2\mathbf{k}$

170. The value of *x* for which the angle between the vectors $\mathbf{a}=-3\mathbf{i}+x\mathbf{j}+\mathbf{k}$ and $\mathbf{b}=x\mathbf{i}+2x\mathbf{j}+\mathbf{k}$ is acute and the angle between **b** and *x*-axis lies between $\frac{\pi}{2}$ and π satisfy **[Kurukshetra CEE 1996]**

(a) $x>0$ (b) $x<0$ (c) $x>1$ only (d) $x<-1$ only

171. If the scalar product of the vector $\mathbf{i}+\mathbf{j}+\mathbf{k}$ with a unit vector parallel to the sum of the vectors $2\mathbf{i}+4\mathbf{j}-5\mathbf{k}$ and $\lambda\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ be 1, then $\lambda=$ **[Roorkee 1985, 95; Kurukshetra CEE 1998; UPSEAT 1992, 2000]**

(a) 1 (b) – 1 (c) 2 (d) – 2

172. If **a** is any vector in space, then **[MP PET 1997]**

(a) $\mathbf{a}=(\mathbf{a}.\mathbf{i})\mathbf{i}+(\mathbf{a}.\mathbf{j})\mathbf{j}+(\mathbf{a}.\mathbf{k})\mathbf{k}$ (b) $\mathbf{a}=(\mathbf{a}\times\mathbf{i})+(\mathbf{a}\times\mathbf{j})+(\mathbf{a}\times\mathbf{k})$

(c) $\mathbf{a}=\mathbf{j}(\mathbf{a}.\mathbf{i})+\mathbf{k}(\mathbf{a}.\mathbf{j})+\mathbf{i}(\mathbf{a}.\mathbf{k})$ (d) $\mathbf{a}=(\mathbf{a}\times\mathbf{i})\times\mathbf{i}+(\mathbf{a}\times\mathbf{j})\times\mathbf{j}+(\mathbf{a}\times\mathbf{k})\times\mathbf{k}$

173. If **a, b** and **c** are unit vectors, then $|\mathbf{a}-\mathbf{b}|^2+|\mathbf{b}-\mathbf{c}|^2+|\mathbf{c}-\mathbf{a}|^2$ does not exceed **[IIT 1995, 2001]**

(a) 4 (b) 9 (c) 8 (d) 6

174. If **a** and **b** are two unit vectors, such that $\mathbf{a}+2\mathbf{b}$ and $5\mathbf{a}-4\mathbf{b}$ are perpendicular to each other then the angle between **a** and **b** is

[IIT Screening 2002]

(a) 45° (b) 60° (c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{2}{7}\right)$

175. **a, b, c** are three vectors, such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$, $|\mathbf{a}|=1, |\mathbf{b}|=2, |\mathbf{c}|=3$, then $\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a}$ is equal to **[AIEEE 2003]**

(a) 0 (b) – 7 (c) 7 (d) 1

176. A unit vector in *xy*-plane that makes an angle 45° with the vectors $(\mathbf{i}+\mathbf{j})$ and an angle of 60° with the vector $(3\mathbf{i}-4\mathbf{j})$ is

[Kurukshetra CEE 2002]

(a) $\mathbf{i}$ (b) $\frac{1}{\sqrt{2}}(\mathbf{i}-\mathbf{j})$ (c) $\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ (d) None of these

177. The angle between the vectors $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$, when **a** = (1, 1, 4) and **b** = (1, – 1, 4) is **[Karnataka CET 2003]**

(a) 90° (b) 45° (c) 30° (d) 15°

178. Let $\mathbf{u}=\mathbf{i}+\mathbf{j}$, $\mathbf{v}=\mathbf{i}-\mathbf{j}$ and $\mathbf{w}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$. If **n** is a unit vector such that **u . n** = 0 and **v . n** = 0, then $|\mathbf{w}.\mathbf{n}|$ is equal to

[AIEEE 2003]

(a) 0 (b) 1 (c) 2 (d) 3

179. If *a, b, c* are the p^{th}, q^{th}, r^{th} terms of an *HP* and $\mathbf{u}=(q-r)\mathbf{i}+(r-p)\mathbf{j}+(p-q)\mathbf{k}$, $\mathbf{v}=\frac{\mathbf{i}}{a}+\frac{\mathbf{j}}{b}+\frac{\mathbf{k}}{c}$, then

(a) **u, v** are parallel vectors (b) **u, v** are orthogonal vectors (c) **u . v** = 1 (d) $\mathbf{u}\times\mathbf{v}=\mathbf{i}+\mathbf{j}+\mathbf{k}$

180. *ABC* is an equilateral triangle of side *a*. The value of $\overrightarrow{AB}.\overrightarrow{BC}+\overrightarrow{BC}.\overrightarrow{CA}+\overrightarrow{CA}.\overrightarrow{AB}$ is equal to

(a) $\frac{3a^2}{2}$ (b) $3a^2$ (c) $-\frac{3a^2}{2}$ (d) None of these

181. If $\mathbf{e}_1=(1,1,1)$ and $\mathbf{e}_2=(1,1,-1)$ and **a** and **b** are two vectors such that $\mathbf{e}_1=2\mathbf{a}+\mathbf{b}$ and $\mathbf{e}_2=\mathbf{a}+2\mathbf{b}$ then angle between **a** and **b** is

(a) $\cos^{-1}\left(\frac{7}{9}\right)$ (b) $\cos^{-1}\left(\frac{7}{11}\right)$ (c) $\cos^{-1}\left(-\frac{7}{11}\right)$ (d) $\cos^{-1}\left(\frac{6\sqrt{2}}{11}\right)$

182. A vector whose modulus is $\sqrt{51}$ and makes the same angle with $\mathbf{a}=\frac{\mathbf{i}-2\mathbf{j}+2\mathbf{k}}{3}$, $\mathbf{b}=\frac{-4\mathbf{i}-3\mathbf{k}}{5}$ and $\mathbf{c}=\mathbf{j}$, will be **[Roorkee**

(a) $5\mathbf{i}+5\mathbf{j}+\mathbf{k}$ (b) $5\mathbf{i}+\mathbf{j}-5\mathbf{k}$ (c) $5\mathbf{i}+\mathbf{j}+5\mathbf{k}$ (d) $\pm(5\mathbf{i}-\mathbf{j}-5\mathbf{k})$

183. In a right angled triangle *ABC*, the hypotenues *AB* = *p*, then $\overrightarrow{AB}.\overrightarrow{AC}+\overrightarrow{BC}.\overrightarrow{BA}+\overrightarrow{CA}.\overrightarrow{CB}$ is equal to

(a) $2p^2$ (b) $\frac{p^2}{2}$ (c) p^2 (d) None of these

184. If the vectors $\mathbf{b}=\left(\tan\alpha, -1, 2\sqrt{\sin\frac{\alpha}{2}}\right)$ and $\mathbf{c}=\left(\tan\alpha, \tan\alpha, -\frac{3}{\sqrt{\sin\frac{\alpha}{2}}}\right)$ are orthogonal and a vector $\mathbf{a}=(1, 3, \sin 2\alpha)$ makes an obtuse angle with the z-axis, then the value of α is

(a) $\alpha=(4n+1)\pi-\tan^{-1}2$ (b) $\alpha=(4n+2)\pi-\tan^{-1}2$ (c) $\alpha=(4n+1)\pi+\tan^{-1}2$ (d) $\alpha=(4n+2)\pi+\tan^{-1}2$

185. If the vectors $\mathbf{a}=(2,\log_3 x, a)$ and $\mathbf{b}=(-3, a\log_3 x, \log_3 x)$ are inclined at an acute angle, then

(a) $a=0$ (b) $a<0$ (c) $a>0$ (d) None of these

186. The value of x for which the angle between the vectors $\mathbf{a} = x\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 2x\mathbf{i} + x\mathbf{j} - \mathbf{k}$ is acute and the angle between the vector **b** and y-axis lies between $\frac{\pi}{2}$ and π are **[DCE 2001]**

(a) < 0 (b) > 0 (c) – 2, – 3 (d) 1, 2

187. If **a, b, c** are linearly independent vectors and $\Delta = \begin{vmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \\ \mathbf{a}.\mathbf{a} & \mathbf{a}.\mathbf{b} & \mathbf{a}.\mathbf{c} \\ \mathbf{a}.\mathbf{c} & \mathbf{b}.\mathbf{c} & \mathbf{c}.\mathbf{c} \end{vmatrix}$, then

(a) $\Delta = 0$ (b) $\Delta = 1$ (c) $\Delta =$ any non-zero value (d) None of these

188. The position vectors of the points A, B and C are $\mathbf{i} + \mathbf{j} + \mathbf{k}, \mathbf{i} + 5\mathbf{j} - \mathbf{k}$ and $2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ respectively.The greatest angle of the triangle ABC is

(a) 135^o (b) 90° (c) $\cos^{-1}\left(\frac{2}{3}\right)$ (d) $\cos^{-1}\left(\frac{5}{7}\right)$

Component of Vector

Basic Level

189. If **a** and **b** are two non-zero vectors, then the component of **b** along **a** is **[MP PET 1991]**

(a) $\frac{(\mathbf{a}.\mathbf{b})\mathbf{a}}{\mathbf{b}.\mathbf{b}}$ (b) $\frac{(\mathbf{a}.\mathbf{b})\mathbf{b}}{\mathbf{a}.\mathbf{a}}$ (c) $\frac{(\mathbf{a}.\mathbf{b})\mathbf{b}}{\mathbf{a}.\mathbf{b}}$ (d) $\frac{(\mathbf{a}.\mathbf{b})\mathbf{a}}{\mathbf{a}.\mathbf{a}}$

190. Projection of the vector $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ in the direction of the vector $4\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$ will be

[Rajasthan PET 1990; MNR 1980; MP PET 2002; UPSEAT 2002]

(a) $\frac{5\sqrt{6}}{10}$ (b) $\frac{9}{19}$ (c) $\frac{19}{9}$ (d) $\frac{\sqrt{6}}{19}$

191. If $\mathbf{a} = 4\mathbf{i} + 6\mathbf{j}$ and $b = 3\mathbf{j} + 4\mathbf{k}$, then the component of **a** along **b** is **[IIT Screening 1989; MNR 1983, 87; UPSEAT 2000]**

(a) $\frac{18}{10\sqrt{3}}(3\mathbf{j} + 4\mathbf{k})$ (b) $\frac{18}{25}(3\mathbf{j} + 4\mathbf{k})$ (c) $\frac{18}{\sqrt{3}}(3\mathbf{j} + 4\mathbf{k})$ (d) $(3\mathbf{j} + 4\mathbf{k})$

192. The projection of vector $2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ on the vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ will be **[Rajasthan PET 1984, 90, 97, 99]**

(a) $\frac{1}{\sqrt{14}}$ (b) $\frac{2}{\sqrt{14}}$ (c) $\frac{3}{\sqrt{14}}$ (d) $\sqrt{14}$

193. If vector $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ and vector $\mathbf{b} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, then $\frac{\text{Projection of vector } \mathbf{a} \text{ on vector } \mathbf{b}}{\text{Projection of vector } \mathbf{b} \text{ on vector } \mathbf{a}} =$ **[MP PET 1994, 1999]**

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) 3 (d) 7

194. The projection of **a** along **b** is **[Rajasthan PET 1995]**

(a) $\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|}$ (b) $\frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a}|}$ (c) $\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{b}|}$ (d) $\frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{b}|}$

195. If $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, then the projection of **b** on **a** is **[Karnataka CET 2002]**

(a) 3 (b) 4 (c) 5 (d) 6

196. The projection of the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$ along the vector **j** is **[Kerala (Engg.) 2002]**

(a) 1 (b) 0 (c) 2 (d) – 1

197. If $\hat{\mathbf{a}}$ is a unit vector and **b**, a non-zero vector not parallel to $\hat{\mathbf{a}}$, then the vector $\mathbf{b} - (\hat{\mathbf{a}}.\mathbf{b})\hat{\mathbf{a}}$ is

(a) Parallel to **b** (b) At right angles to $\hat{\mathbf{a}}$ (c) Parallel to $\hat{\mathbf{a}}$ (d) At right angles to **b**

Advance Level

198. If $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$, then a vector in the direction of $\mathbf{a}$ and having magnitude as $|\mathbf{b}|$ is **[IIT 1983]**

(a) $7(\mathbf{i}+\mathbf{j}+\mathbf{k})$ (b) $\frac{7}{3}(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$ (c) $\frac{7}{9}(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$ (d) None of these

199. The vector $\mathbf{b} = 3\mathbf{j} + 4\mathbf{k}$ is to be written as the sum of a vector $\mathbf{b}_1$ parallel to $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and a vector $\mathbf{b}_2$ perpendicular to $\mathbf{a}$. Then $\mathbf{b}_1 =$ **[MNR 1993; UPSEAT 2000]**

(a) $\frac{3}{2}(\mathbf{i}+\mathbf{j})$ (b) $\frac{2}{3}(\mathbf{i}+\mathbf{j})$ (c) $\frac{1}{2}(\mathbf{i}+\mathbf{j})$ (d) $\frac{1}{3}(\mathbf{i}+\mathbf{j})$

200. The components of a vector $\mathbf{a}$ along and perpendicular to the non-zero vector $\mathbf{b}$ are respectively **[IIT 1988]**

(a) $\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|}, \frac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}|}$ (b) $\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{b}|}, \frac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{b}|}$ (c) $\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|}, \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|}$ (d) $\frac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}|}, \frac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{b}|}$

201. Let $\mathbf{b} = 4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{c}$ be two vectors perpendicular to each other in the xy-plane. All vectors in the same plane having projections 1 and 2 along $\mathbf{b}$ and $\mathbf{c}$ respectively, are given by **[IIT 1987]**

(a) $2\mathbf{i}-\mathbf{j}, \frac{2}{5}\mathbf{i}+\frac{11}{5}\mathbf{j}$ (b) $2\mathbf{i}+\mathbf{j}, -\frac{2}{5}\mathbf{i}+\frac{11}{5}\mathbf{j}$ (c) $2\mathbf{i}+\mathbf{j}, -\frac{2}{5}\mathbf{i}-\frac{11}{5}\mathbf{j}$ (d) $2\mathbf{i}-\mathbf{j}, -\frac{2}{5}\mathbf{i}+\frac{11}{5}\mathbf{j}$

202. Let $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}, \mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{c} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ be three vectors. A vector in the plane of $\mathbf{b}$ and $\mathbf{c}$ whose projection on $\mathbf{a}$ is of magnitude $\sqrt{2/3}$ is **[IIT 1993]**

(a) $2\mathbf{i}+3\mathbf{j}-3\mathbf{k}$ (b) $2\mathbf{i}+3\mathbf{j}+3\mathbf{k}$ (c) $-2\mathbf{i}-\mathbf{j}+5\mathbf{k}$ (d) $2\mathbf{i}+\mathbf{j}+5\mathbf{k}$

Work done by a Force

Basic Level

203. If the position vectors of A and B be $6\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $4\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$, then the work done by the force $\vec{F} = \mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ in displacing a particle from A to B is **[MP PET 1987]**

(a) 15 units (b) 17 units (c) – 15 units (d) None of these

204. If the force $\vec{F} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ moves from $\mathbf{i} + \mathbf{j} - \mathbf{k}$ to $2\mathbf{i} - \mathbf{j} + \mathbf{k}$, then work done will be represented by **[Ranchi BIT 1992]**

(a) 3 (b) 4 (c) 5 (d) 6

205. The work done by the force $\vec{F} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ in displacing a particle from the point (3, 4, 5) to the point (1, 2, 3) is

[MP PET 1994; Kurukshetra CEE 2002]

(a) 2 units (b) 3 units (c) 4 units (d) 5 units

206. The work done in moving an object along the vector $3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$, if the applied force is $\vec{F} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$, is **[MP PET 1997, 2**

(a) 7 (b) 8 (c) 9 (d) 10

207. A force $\vec{F} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ acts at a point A whose position vector is $2\mathbf{i} - \mathbf{j}$. If point of application of $\vec{F}$ moves from A to the point B with position vector $2\mathbf{i} + \mathbf{j}$, then work done by $\vec{F}$ is **[Pb. CET 2000]**

(a) 4 (b) 20 (c) 2 (d) None of these

Advance Level

208. Force $3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ are acting on a particle and displace it from the point $2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ to the point $4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$, then work done by the force is **[MP PET 1995]**

(a) 30 units (b) 36 units (c) 24 units (d) 18 units

209. A force of magnitude 5 units acting along the vector $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$ displaces the point of application from (1, 2, 3) to (5, 3, 7), then the work done is **[Kerala (Engg.) 2002]**

(a) 50/7 (b) 50/3 (c) 25/3 (d) 25/4

210. If forces of magnitudes 6 and 7 units acting in the directions $\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ and $2\mathbf{i}-3\mathbf{j}-6\mathbf{k}$ respectively act on a particle which is displaced from the point $P(2, -1, -3)$ to $Q(5, -1, 1)$, then the work done by the forces is

(a) 4 units (b) - 4 units (c) 7 units (d) - 7 units

Vector or Cross Product of Two Vectors

Basic Level

211. If $\mathbf{u}=2\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $\mathbf{v}=6\mathbf{i}-3\mathbf{j}+2\mathbf{k}$, then a unit vector perpendicular to both **u** and **v** is **[MP PET 1987]**

(a) $\mathbf{i}-10\mathbf{j}-18\mathbf{k}$ (b) $\frac{1}{\sqrt{17}}\left(\frac{1}{5}\mathbf{i}-2\mathbf{j}-\frac{18}{5}\mathbf{k}\right)$ (c) $\frac{1}{\sqrt{473}}(7\mathbf{i}-10\mathbf{j}-18\mathbf{k})$ (d) None of these

212. $(\mathbf{a}-\mathbf{b})\times(\mathbf{a}+\mathbf{b})=$ **[MP PET 1987]**

(a) $2\mathbf{a}\times\mathbf{b}$ (b) $\mathbf{a}\times\mathbf{b}$ (c) $\mathbf{a}^2-\mathbf{b}^2$ (d) None of these

213. If $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$, then which relation is correct **[Rajasthan PET 1985; Roorkee 1981; AIEEE 2002]**

(a) $\mathbf{a}=\mathbf{b}=\mathbf{c}=0$ (b) $\mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{c}=\mathbf{c}.\mathbf{a}$ (c) $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}$ (d) None of these

214. If θ be the angle between the vectors **a** and **b** and $|\mathbf{a}\times\mathbf{b}|=\mathbf{a}.\mathbf{b}$, then $\theta=$ **[Rajasthan PET 1990; MP PET 1990; UPSEAT 200**

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) 0

215. If **a** and **b** are two vectors such that **a** . **b** = 0 and $\mathbf{a}\times\mathbf{b}=0$, then **[MNR 1988; IIT 1989; UPSEAT 2000, 01]**

(a) **a** is parallel to **b** (b) **a** is perpendicular to **b**

(c) Either **a** or **b** is a null vector (d) None of these

216. $(2\hat{\mathbf{a}}+3\hat{\mathbf{b}})\times(5\hat{\mathbf{a}}+7\hat{\mathbf{b}})=$ **[MP PET 1988]**

(a) $\hat{\mathbf{a}}\times\hat{\mathbf{b}}$ (b) $\hat{\mathbf{b}}\times\hat{\mathbf{a}}$ (c) $\hat{\mathbf{a}}+\hat{\mathbf{b}}$ (d) $7\hat{\mathbf{a}}+10\hat{\mathbf{b}}$

217. Which of the following is not a property of vectors **[MP PET 1987]**

(a) $\mathbf{u}\times\mathbf{v}=\mathbf{v}\times\mathbf{u}$ (b) $\mathbf{u}.\mathbf{v}=\mathbf{v}.\mathbf{u}$ (c) $(\mathbf{u}\times\mathbf{v})^2=\mathbf{u}^2.\mathbf{v}^2-(\mathbf{u}.\mathbf{v})^2$ (d) $\mathbf{u}^2=|\mathbf{u}|^2$

218. The number of vectors of unit length perpendicular to vectors **a** = (1, 1, 0) and **b** = (0, 1, 1) is **[Ranchi BIT 1991; IIT 1987; Kurukshetra CEE 1998; DCE 2000; MP PET 2002]**

(a) Three (b) One (c) Two (d) Infinite

219. If $\mathbf{a}\neq 0, \mathbf{b}\neq 0, \mathbf{c}\neq 0$, then true statement is **[MP PET 1991]**

(a) $\mathbf{a}\times(\mathbf{b}+\mathbf{c})=(\mathbf{c}+\mathbf{b})\times\mathbf{a}$ (b) $\mathbf{a}.(\mathbf{b}+\mathbf{c})=-(\mathbf{b}+\mathbf{c}).\mathbf{a}$ (c) $\mathbf{a}\times(\mathbf{b}-\mathbf{c})=(\mathbf{c}-\mathbf{b})\times\mathbf{a}$ (d) $\mathbf{a}.(\mathbf{b}-\mathbf{c})=(\mathbf{c}-\mathbf{b}).\mathbf{a}$

220. A unit vector which is perpendicular to $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$ and to $-\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ is **[MP PET 1992]**

(a) $\frac{1}{\sqrt{5}}(2\mathbf{i}-\mathbf{k})$ (b) $\frac{1}{\sqrt{5}}(-2\mathbf{i}+\mathbf{k})$ (c) $\frac{1}{\sqrt{5}}(2\mathbf{i}+\mathbf{j}+\mathbf{k})$ (d) $\frac{1}{\sqrt{5}}(2\mathbf{i}+\mathbf{k})$

221. The unit vector perpendicular to the $3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $12\mathbf{i}+5\mathbf{j}-5\mathbf{k}$, is **[Roorkee 1979; Rajasthan PET 1989, 1991]**

(a) $\frac{5\mathbf{i}-3\mathbf{j}+9\mathbf{k}}{\sqrt{115}}$ (b) $\frac{5\mathbf{i}+3\mathbf{j}-9\mathbf{k}}{\sqrt{115}}$ (c) $\frac{-5\mathbf{i}+3\mathbf{j}-9\mathbf{k}}{\sqrt{115}}$ (d) $\frac{5\mathbf{i}+3\mathbf{j}+9\mathbf{k}}{\sqrt{115}}$

222. The sine of the angle between the two vectors $3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $12\mathbf{i}+5\mathbf{j}-5\mathbf{k}$ will be **[Roorkee 1978]**

(a) $\frac{\sqrt{115}}{\sqrt{14}\sqrt{194}}$ (b) $\frac{51}{\sqrt{14}\sqrt{144}}$ (c) $\frac{\sqrt{64}}{\sqrt{14}\sqrt{194}}$ (d) None of these

223. For any two vectors **a** and **b**, if $\mathbf{a}\times\mathbf{b}=\mathbf{0}$, then **[Roorkee 1984]**

(a) **a** = **0** (b) **b** = **0** (c) Not parallel (d) None of these

224. For any vectors **a, b, c.** $\mathbf{a}\times(\mathbf{b}+\mathbf{c})+\mathbf{b}\times(\mathbf{c}+\mathbf{a})+\mathbf{c}\times(\mathbf{a}+\mathbf{b})=$ **[Roorkee 1981; Kerala (Engg.) 2002]**

(a) 0 (b) $\mathbf{a}+\mathbf{b}+\mathbf{c}$ (c) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (d) $\mathbf{a}\times\mathbf{b}\times\mathbf{c}$

225. If $|\mathbf{a}|=2, |\mathbf{b}|=5$ and $|\mathbf{a}\times\mathbf{b}|=8$, then $\mathbf{a}\,.\,\mathbf{b}$ is equal to **[Rajasthan PET 1991; AI CBSE 1984]**

(a) 0 (b) 2 (c) 4 (d) 6

226. If $\mathbf{a}=2\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $\mathbf{b}=6\mathbf{i}-3\mathbf{j}+2\mathbf{k}$,then the value of $\mathbf{a}\times\mathbf{b}$ is **[MNR 1978; Rajasthan PET 2001]**

(a) $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$ (b) $6\mathbf{i}-3\mathbf{j}+2\mathbf{k}$ (c) $\mathbf{i}-10\mathbf{j}-18\mathbf{k}$ (d) $\mathbf{i}+\mathbf{j}+\mathbf{k}$

227. A unit vector perpendicular to the vector $4\mathbf{i}-\mathbf{j}+3\mathbf{k}$ and $-2\mathbf{i}+\mathbf{j}-2\mathbf{k}$ is **[MNR 1995]**

(a) $\frac{1}{3}(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$ (b) $\frac{1}{3}(-\mathbf{i}+2\mathbf{j}+2\mathbf{k})$ (c) $\frac{1}{3}(2\mathbf{i}+\mathbf{j}+2\mathbf{k})$ (d) $\frac{1}{3}(2\mathbf{i}-2\mathbf{j}+2\mathbf{k})$

228. A unit vector perpendicular to each of the vector $2\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $3\mathbf{i}+4\mathbf{j}-\mathbf{k}$ is equal to **[MP PET 2003]**

(a) $\frac{(-3\mathbf{i}+5\mathbf{j}+11\mathbf{k})}{\sqrt{155}}$ (b) $\frac{(3\mathbf{i}-5\mathbf{j}+11\mathbf{k})}{\sqrt{155}}$ (c) $\frac{(6\mathbf{i}-4\mathbf{j}-\mathbf{k})}{\sqrt{53}}$ (d) $\frac{(5\mathbf{i}+3\mathbf{j})}{\sqrt{34}}$

229. If $\mathbf{a}=\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ and $\mathbf{b}=3\mathbf{i}+\mathbf{j}+2\mathbf{k}$, then the unit vector perpendicular to **a** and **b** is **[MP PET 1996]**

(a) $\frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$ (b) $\frac{\mathbf{i}-\mathbf{j}+\mathbf{k}}{\sqrt{3}}$ (c) $\frac{-\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$ (d) $\frac{\mathbf{i}-\mathbf{j}-\mathbf{k}}{\sqrt{3}}$

230. If θ is the angle between the vectors **a** and **b**, then $\frac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}.\mathbf{b}|}$ equal to **[Karnataka CET 1999]**

(a) $\tan\theta$ (b) $-\tan\theta$ (c) $\cot\theta$ (d) $-\cot\theta$

231. A vector perpendicular to both of the vectors $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+\mathbf{j}$ is **[Rajasthan PET 2000]**

(a) $\mathbf{i}+\mathbf{j}$ (b) $\mathbf{i}-\mathbf{j}$ (c) $c(\mathbf{i}-\mathbf{j}), c$ is a scalar (d) None of these

232. A unit vector perpendicular to the plane of $\mathbf{a}=2\mathbf{i}-6\mathbf{j}-3\mathbf{k}$, $\mathbf{b}=4\mathbf{i}+3\mathbf{j}-\mathbf{k}$ is **[MP PET 2000]**

(a) $\frac{4\mathbf{i}+3\mathbf{j}-\mathbf{k}}{\sqrt{26}}$ (b) $\frac{2\mathbf{i}-6\mathbf{j}-3\mathbf{k}}{7}$ (c) $\frac{3\mathbf{i}-2\mathbf{j}+6\mathbf{k}}{7}$ (d) $\frac{2\mathbf{i}-3\mathbf{j}-6\mathbf{k}}{7}$

233. The unit vector perpendicular to the both the vectors $\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}-\mathbf{k}$ is **[DCE 2001]**

(a) $\frac{1}{\sqrt{3}}(-\mathbf{i}+\mathbf{j}+\mathbf{k})$ (b) $(-\mathbf{i}+\mathbf{j}+\mathbf{k})$ (c) $\frac{(\mathbf{i}+\mathbf{j}-\mathbf{k})}{\sqrt{3}}$ (d) None of these

234. The unit vector perpendicular to the vectors $\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $2\mathbf{i}+3\mathbf{j}-\mathbf{k}$ is **[Karnataka CET 2001]**

(a) $\frac{-2\mathbf{i}+3\mathbf{j}+5\mathbf{k}}{\sqrt{30}}$ (b) $\frac{-2\mathbf{i}+5\mathbf{j}+6\mathbf{k}}{\sqrt{38}}$ (c) $\frac{-2\mathbf{i}+3\mathbf{j}+5\mathbf{k}}{\sqrt{38}}$ (d) $\frac{-2\mathbf{i}+4\mathbf{j}+5\mathbf{k}}{\sqrt{38}}$

235. If $\mathbf{a}=2\mathbf{i}-3\mathbf{j}-\mathbf{k}$ and $\mathbf{b}=\mathbf{i}+4\mathbf{j}-2\mathbf{k}$, then $\mathbf{a}\times\mathbf{b}$ is **[MP PET 2001]**

(a) $10\mathbf{i}+2\mathbf{j}+11\mathbf{k}$ (b) $10\mathbf{i}+3\mathbf{j}+11\mathbf{k}$ (c) $10\mathbf{i}-3\mathbf{j}+11\mathbf{k}$ (d) $10\mathbf{i}-3\mathbf{j}-10\mathbf{k}$

236. If $|\mathbf{a}|=4, |\mathbf{b}|=2$ and the angle between **a** and **b** is $\frac{\pi}{6}$, then $(\mathbf{a}\times\mathbf{b})^2$ is equal to **[AIEEE 2002]**

(a) 48 (b) 16 (c) **a** (d) None of these

237. $\mathbf{a}=3\mathbf{i}-5\mathbf{j}$ and $\mathbf{b}=6\mathbf{i}+3\mathbf{j}$ are two vectors and **c** is a vector such that $\mathbf{c}=\mathbf{a}\times\mathbf{b}$, then $|\mathbf{a}|:|\mathbf{b}|:|\mathbf{c}|$ is **[AIEEE 2002]**

(a) $\sqrt{34}:\sqrt{45}:\sqrt{39}$ (b) $\sqrt{34}:\sqrt{45}:39$ (c) 34 : 39 : 45 (d) 39 : 35 : 34

238. $3\lambda\mathbf{c}+2\mu(\mathbf{a}\times\mathbf{b})=0$, then **[AIEEE 2002]**

(a) $3\lambda+2\mu=0$ (b) $3\lambda=2\mu$ (c) $\lambda=\mu$ (d) $\lambda+\mu=0$

239. If $\mathbf{a}=2\mathbf{i}+4\mathbf{j}-5\mathbf{k}$ and $\mathbf{b}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$, then $|\mathbf{a}\times\mathbf{b}|$ is **[UPSEAT 2002]**

(a) $11\sqrt{5}$ (b) $11\sqrt{3}$ (c) $11\sqrt{7}$ (d) $11\sqrt{2}$

240. The unit vector perpendicular to both **i** + **j** and **j** + **k** is **[Kerala (Engg.) 2002]**

(a) $\mathbf{i}-\mathbf{j}+\mathbf{k}$ (b) $\mathbf{i}+\mathbf{j}+\mathbf{k}$ (c) $\frac{\mathbf{i}+\mathbf{j}-\mathbf{k}}{\sqrt{3}}$ (d) $\frac{\mathbf{i}-\mathbf{j}+\mathbf{k}}{\sqrt{3}}$

241. If $|\mathbf{a}\times\mathbf{b}|=4$ and $|\mathbf{a}.\mathbf{b}|=2$, then $|\mathbf{a}|^2|\mathbf{b}|^2=$ **[Karnataka CET 2003]**

(a) 2 (b) 6 (c) 8 (d) 20

242. If $(\mathbf{a}\times\mathbf{b})^2+(\mathbf{a}.\mathbf{b})^2=144$ and $|\mathbf{a}|=4$, then $|\mathbf{b}|=$ **[EAMCET 1994]**

(a) 16 (b) 8 (c) 3 (d) 12

243. The unit vector perpendicular to both the vectors $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{b}=2\mathbf{i}-\mathbf{j}+3\mathbf{k}$ and making an acute angle with the vector $\mathbf{k}$ is

(a) $-\frac{1}{\sqrt{26}}(4\mathbf{i}-\mathbf{j}-3\mathbf{k})$ (b) $\frac{1}{\sqrt{26}}(4\mathbf{i}-\mathbf{j}-3\mathbf{k})$ (c) $\frac{1}{\sqrt{26}}(4\mathbf{i}-\mathbf{j}+3\mathbf{k})$ (d) None of these

244. The angle between $3(\mathbf{a}\times\mathbf{b})$ and $\frac{1}{2}(\mathbf{b}-(\mathbf{a}.\mathbf{b})\hat{\mathbf{a}})$ is **[Pb. CET 1996]**

(a) 30° (b) 60° (c) 90° (d) $\cos^{-1}\left(\frac{3}{4}\right)$

Advance Level

245. If the vectors **a, b** and **c** are represented by, the sides *BC, CA* and *AB* respectively of the ΔABC, then **[IIT Screening 2000]**

(a) $\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a}=0$ (b) $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}$ (c) $\mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{c}=\mathbf{c}.\mathbf{a}$ (d) $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}=0$

246. $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}\neq 0$, where **a**, **b** and **c** are coplanar vectors, then for some scalar **k** **[Roorkee 1985; Rajasthan PET 1997]**

(a) $\mathbf{a}+\mathbf{c}=k\mathbf{b}$ (b) $\mathbf{a}+\mathbf{b}=k\mathbf{c}$ (c) $\mathbf{b}+\mathbf{c}=k\mathbf{a}$ (d) None of these

247. If $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}\neq\mathbf{0}$ and $\mathbf{a}+\mathbf{c}\neq\mathbf{0}$, then **[Rajasthan PET 1999]**

(a) $(\mathbf{a}+\mathbf{c})\perp\mathbf{b}$ (b) $(\mathbf{a}+\mathbf{c})\parallel\mathbf{b}$ (c) $(\mathbf{a}+\mathbf{c})=\mathbf{b}$ (d) None of these

248. If **a** and **b** are two vectors, then $(\mathbf{a}\times\mathbf{b})^2$ equals **[Roorkee 1975, 1979, 1981, 1985]**

(a) $\begin{vmatrix}\mathbf{a}.\mathbf{b} & \mathbf{a}.\mathbf{a}\\ \mathbf{b}.\mathbf{b} & \mathbf{b}.\mathbf{a}\end{vmatrix}$ (b) $\begin{vmatrix}\mathbf{a}.\mathbf{a} & \mathbf{a}.\mathbf{b}\\ \mathbf{b}.\mathbf{a} & \mathbf{b}.\mathbf{b}\end{vmatrix}$ (c) $\begin{vmatrix}\mathbf{a}.\mathbf{b}\\ \mathbf{b}.\mathbf{a}\end{vmatrix}$ (d) None of these

249. Given $\mathbf{a}=\mathbf{i}+\mathbf{j}-\mathbf{k}, \mathbf{b}=-\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=-\mathbf{i}+2\mathbf{j}-\mathbf{k}$. A unit vector perpendicular to both $\mathbf{a}+\mathbf{b}$ and $\mathbf{b}+\mathbf{c}$ is **[Karnataka CET 1993]**

(a) $\mathbf{i}$ (b) $\mathbf{j}$ (c) $\mathbf{k}$ (d) $\frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$

250. For any two vectors **a** and **b**, $(\mathbf{a}\times\mathbf{b})^2$ is equal to **[Roorkee 1975, 1979, 1981, 1985]**

(a) a^2-b^2 (b) a^2+b^2 (c) $a^2b^2-(\mathbf{a}.\mathbf{b})^2$ (d) None of these

251. If vectors $\vec{A}=2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, $\vec{B}=\mathbf{i}+\mathbf{j}+5\mathbf{k}$ and $\vec{C}$ form a left handed system, then $\vec{C}$ is **[Roorkee 1999]**

(a) $11\mathbf{i}-6\mathbf{j}-\mathbf{k}$ (b) $-11\mathbf{i}+6\mathbf{j}+\mathbf{k}$ (c) $11\mathbf{i}-6\mathbf{j}+\mathbf{k}$ (d) $-11\mathbf{i}+6\mathbf{j}-\mathbf{k}$

252. $(\mathbf{r}.\mathbf{i})(\mathbf{r}\times\mathbf{i})+(\mathbf{r}.\mathbf{j})(\mathbf{r}\times\mathbf{j})+(\mathbf{r}.\mathbf{k})(\mathbf{r}\times\mathbf{k})$ is equal to

(a) $3\mathbf{r}$ (b) $\mathbf{r}$ (c) **0** (d) None of these

253. If **a, b, c** are noncoplanar vectors such that $\mathbf{b}\times\mathbf{c}=\mathbf{a}$, $\mathbf{a}\times\mathbf{b}=\mathbf{c}$ and $\mathbf{c}\times\mathbf{a}=\mathbf{b}$, then

(a) $|\mathbf{a}|=1$ (b) $|\mathbf{b}|=1$ (c) $|\mathbf{a}|+|\mathbf{b}|+|\mathbf{c}|=3$ (d) None of these

254. If $\overrightarrow{AB}=\mathbf{b}$ and $\overrightarrow{AC}=\mathbf{c}$, then the length of the perpendicular from *A* to the line *BC* is

(a) $\frac{|\mathbf{b} \times \mathbf{c}|}{|\mathbf{b} + \mathbf{c}|}$ (b) $\frac{|\mathbf{b} \times \mathbf{c}|}{|\mathbf{b} - \mathbf{c}|}$ (c) $\frac{1}{2}\frac{|\mathbf{b} \times \mathbf{c}|}{|\mathbf{b} - \mathbf{c}|}$ (d) None of these

Area of parallelogram and Triangle

Basic Level

255. The area of a parallelogram whose two adjacent sides are represented by the vector $3\mathbf{i} - \mathbf{k}$ and $\mathbf{i} + 2\mathbf{j}$ is **[MNR 1981]**

(a) $\frac{1}{2}\sqrt{17}$ (b) $\frac{1}{2}\sqrt{14}$ (c) $\sqrt{41}$ (d) $\frac{1}{2}\sqrt{7}$

256. The area of the parallelogram whose diagonals are $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ is

[MNR 1985; MP PET 1988,93; Tamilnadu Engg. 2002]

(a) $10\sqrt{3}$ (b) $5\sqrt{3}$ (c) 8 (d) 4

257. The area of a parallelogram whose diagonals coincide with the following pair of vectors is $5\sqrt{3}$. The vectors are

[Kurukshetra CEE 1993]

(a) $3\mathbf{i} + 2\mathbf{j} - \mathbf{k},\ 3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ (b) $\frac{3}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} - \mathbf{k},\ 2\mathbf{i} - 6\mathbf{j} + 8\mathbf{k}$ (c) $3\mathbf{i} + \mathbf{j} - 2\mathbf{k},\ \mathbf{i} - 3\mathbf{j} + 8\mathbf{k}$ (d) None of these

258. If $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ represents the adjacent sides of a parallelogram, then the area of this parallelogram is

[Roorkee 1978, 1979; MP PET 1990; Rajasthan PET 1988, 1989, 1991]

(a) $4\sqrt{3}$ (b) $6\sqrt{3}$ (c) $8\sqrt{3}$ (d) $16\sqrt{3}$

259. If the vectors $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k},\ -\mathbf{i} + 2\mathbf{j}$ represents the diagonals of a parallelogram, then its area will be **[Roorkee 1976]**

(a) $\sqrt{21}$ (b) $\frac{\sqrt{21}}{2}$ (c) $2\sqrt{21}$ (d) $\frac{\sqrt{21}}{4}$

260. The area of a parallelogram whose adjacent sides are $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, is **[MP PET 1996, 2000]**

(a) $5\sqrt{3}$ (b) $10\sqrt{3}$ (c) $5\sqrt{6}$ (d) $10\sqrt{6}$

261. If the diagonals of a parallelogram are represented by the vectors $3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, then its area in square units is

[MP PET 1998]

(a) $5\sqrt{3}$ (b) $6\sqrt{3}$ (c) $\sqrt{26}$ (d) $\sqrt{42}$

262. The area of a parallelogram whose adjacent sides are given by the vectors $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $-3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ (in square units) is

[Karnataka CET 2001]

(a) $\sqrt{180}$ (b) $\sqrt{140}$ (c) $\sqrt{80}$ (d) $\sqrt{40}$

263. The area of the parallelogram whose diagonals are $\frac{3}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} - \mathbf{k}$ and $2\mathbf{i} - 6\mathbf{j} + 8\mathbf{k}$ is **[UPSEAT 2002]**

(a) $5\sqrt{3}$ (b) $5\sqrt{2}$ (c) $25\sqrt{3}$ (d) $25\sqrt{2}$

264. The area of the triangle whose two sides are given by $2\mathbf{i} - 7\mathbf{j} + \mathbf{k}$ and $4\mathbf{j} - 3\mathbf{k}$ is **[EAMCET 1990]**

(a) 17 (b) $\frac{17}{2}$ (c) $\frac{17}{4}$ (d) $\frac{1}{2}\sqrt{389}$

265. If $3\mathbf{i} + 4\mathbf{j}$ and $-5\mathbf{i} + 7\mathbf{j}$ are the vector sides of any triangle, then its area is given by **[Rajasthan PET 1987, 1990]**

(a) 41 (b) 47 (c) $\frac{41}{2}$ (d) $\frac{47}{2}$

Advance Level

266. Let **a, b, c** be the position vectors of the vertices of a triangle *ABC*. The vector area of triangle *ABC* is **[MP PET 1990; EAMCET 2003]**

(a) $\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}$ (b) $\frac{1}{4}(\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a})$ (c) $\frac{1}{2}(\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a})$ (d) $\mathbf{b}\times\mathbf{a}+\mathbf{c}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$

267. Consider a tetrahedron with faces F_1, F_2, F_3, F_4. Let $\vec{V_1}, \vec{V_2}, \vec{V_3}, \vec{V_4}$ be the vectors whose magnitudes are respectively equal to areas of F_1, F_2, F_3, F_4 and whose directions are perpendicular to these faces in outward direction. Then $|\vec{V_1}+\vec{V_2}+\vec{V_3}+\vec{V_4}|$ equals

(a) 1 (b) 4 (c) 0 (d) None of these

268. A unit vector perpendicular to the plane determined by the points (1, –1, 2), (2, 0, –1) and (0, 2, 1) is **[IIT 1983; MNR 19**

(a) $\pm\frac{1}{\sqrt{6}}(2\mathbf{i}+\mathbf{j}+\mathbf{k})$ (b) $\frac{1}{\sqrt{6}}(\mathbf{i}+2\mathbf{j}+\mathbf{k})$ (c) $\frac{1}{\sqrt{6}}(\mathbf{i}+\mathbf{j}+\mathbf{k})$ (d) $\frac{1}{\sqrt{6}}(2\mathbf{i}-\mathbf{j}-\mathbf{k})$

269. The position vectors of the points *A, B* and *C* are $\mathbf{i}+\mathbf{j}$, $\mathbf{j}+\mathbf{k}$ and $\mathbf{k}+\mathbf{i}$ respectively. The vector area of the $\Delta ABC = \pm\frac{1}{2}\boldsymbol{\alpha}$, where $\vec{\alpha} =$

[MP PET 1989]

(a) $-\mathbf{i}+\mathbf{j}+\mathbf{k}$ (b) $\mathbf{i}-\mathbf{j}+\mathbf{k}$ (c) $\mathbf{i}+\mathbf{j}-\mathbf{k}$ (d) $\mathbf{i}+\mathbf{j}+\mathbf{k}$

270. The area of the triangle having vertices as $\mathbf{i}-2\mathbf{j}+3\mathbf{k}, -2\mathbf{i}+3\mathbf{j}+\mathbf{k}, 4\mathbf{i}-7\mathbf{j}+5\mathbf{k}$ is **[MP PET 2004]**

(a) 26 (b) 11 (c) 36 (d) 0

271. Let $\overrightarrow{OA}=\mathbf{a}, \overrightarrow{OB}=10\mathbf{a}+2\mathbf{b}$ and $\overrightarrow{OC}=\mathbf{b}$, where *O, A* and *C* are noncollinear points. Let *p* denote the area of the quadrilateral *OABC,* and *q* denote the area of the parallelogram with *OA* and *OC* as adjacent sides. Then $\frac{p}{q}$ is equal to

(a) 4 (b) 6 (c) $\frac{1}{2}\frac{|\mathbf{a}-\mathbf{b}|}{|\mathbf{a}|}$ (d) None of these

272. The adjacent sides of a parallelogram are along $\mathbf{a}=\mathbf{i}+2\mathbf{j}$ and $\mathbf{b}=2\mathbf{i}+\mathbf{j}$. The angles between the diagonals are

(a) 30^o and 150^o (b) 45^o and 135^o (c) 90^o and 90^o (d) None of these

273. Four points with position vectors $7\mathbf{i}-4\mathbf{j}+7\mathbf{k}, \mathbf{i}-6\mathbf{j}+10\mathbf{k}, -\mathbf{i}-3\mathbf{j}+4\mathbf{k}$ and $5\mathbf{i}-\mathbf{j}+\mathbf{k}$ form a

(a) Rhombus (b) Parallelogram but not rhombus

(c) Rectangle (d) Square

274. In a ΔABC, $\overrightarrow{AB}=r\mathbf{i}+\mathbf{j}$, $\overrightarrow{AC}=s\mathbf{i}-\mathbf{j}$. If the area of triangle is of unit magnitude, then **[DCE 1996]**

(a) $|r-s|=2$ (b) $|r+s|=1$ (c) $|r+s|=2$ (d) $|r-s|=1$

Moment of a Force

Basic Level

275. The moment of the force $\vec{F}$ acting at a point *P*, about the point *C* is **[MP PET 1987]**

(a) $\vec{F}\times\overrightarrow{CP}$ (b) $\overrightarrow{CP}.\vec{F}$

(c) A vectors having the same direction as $\vec{F}$ (d) $\overrightarrow{CP} \times \vec{F}$

276. A force $\vec{F} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ acts at a point A, whose position vector is $2\mathbf{i} - \mathbf{j}$. The moment of $\vec{F}$ about the origin is **[Karnataka C**

(a) $\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ (b) $\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ (c) $\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ (d) $\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$

Advance Level

277. Let the point *A, B,* and *P* be (–2, 2, 4), (2, 6, 3) and (1, 2, 1) respectively. The magnitude of the moment of the force represented by $\overrightarrow{AB}$ and acting at *A* about *P* is **[MP PET 1987]**

(a) 15 (b) $3\sqrt{41}$ (c) $3\sqrt{57}$ (d) None of these

278. The moment about the point *M*(–2, 4, –6) of the force represented in magnitude and position by $\overrightarrow{AB}$ where the points *A* and *B* have the coordinates (1, 2, –3) and (3, –4, 2) respectively, is **[MP PET 2000]**

(a) $8\mathbf{i} - 9\mathbf{j} - 14\mathbf{k}$ (b) $2\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$ (c) $-3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ (d) $-5\mathbf{i} + 8\mathbf{j} - 8\mathbf{k}$

279. A force of 39 *kg. wt* is acting at a point *P* (–4, 2, 5) in the direction of $12\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$. The moment of this force about a line through the origin having the direction of $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ is

(a) 76 units (b) –76 units (c) $42\mathbf{i} + 144\mathbf{j} - 24\mathbf{k}$ (d) None of these

280. If the magnitude of moment about the point $\mathbf{j} + \mathbf{k}$ of a force $\mathbf{i} + \alpha\mathbf{j} - \mathbf{k}$ acting through the point $\mathbf{i} + \mathbf{j}$ is $\sqrt{8}$, then the value of α is

[Tamilnadu (Engg.) 2002]

(a) 1 (b) 2 (c) 3 (d) 4

Scalar Triple Product

Basic Level

281. $\mathbf{a}.[(\mathbf{b} + \mathbf{c}) \times (\mathbf{a} + \mathbf{b} + \mathbf{c})]$ is equal to **[Rajasthan PET 1988, 2002; IIT 1981; UPSEAT 2003; MP PET 2004]**

(a) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (b) $2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (c) $3[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (d) 0

282. If **a, b, c** are three non-coplanar vector, then $\dfrac{\mathbf{a}.\mathbf{b} \times \mathbf{c}}{\mathbf{c} \times \mathbf{a}.\mathbf{b}} + \dfrac{\mathbf{b}.\mathbf{a} \times \mathbf{c}}{\mathbf{c}.\mathbf{a} \times \mathbf{b}} =$ **[IIT 1985, 86; UPSEAT 2003]**

(a) 0 (b) 2 (c) –2 (d) None of these

283. If **i, j, k** are the unit vectors and mutually perpendicular, then $[\mathbf{i}\ \mathbf{k}\ \mathbf{j}]$ is equal to **[Rajasthan PET 1986]**

(a) 0 (b) –1 (c) 1 (d) None of these

284. If $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{c} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, then $\mathbf{a}.(\mathbf{b} \times \mathbf{c}) =$ **[Rajasthan PET 1989, 2001]**

(a) 6 (b) 10 (c) 12 (d) 24

285. If $\mathbf{a}.\mathbf{b} = \mathbf{b}.\mathbf{c} = \mathbf{c}.\mathbf{a} = 0$ or **a, b, c** are a right handed triad of mutually perpendicular vectors, then $[\mathbf{a}\ \ \mathbf{b}\ \ \mathbf{c}] =$

[MP PET 1994; Tamilnadu Engg. 2001]

(a) $|\mathbf{a}|\ |\mathbf{b}|\ |\mathbf{c}|$ (b) 1 (c) –1 (d) A non-zero vector

286. $\mathbf{i}.(\mathbf{j} \times \mathbf{k}) + \mathbf{j}.(\mathbf{k} \times \mathbf{i}) + \mathbf{k}.(\mathbf{i} \times \mathbf{j}) =$ **[Karnataka CET 1994]**

(a) 1 (b) 3 (c) –3 (d) 0

287. If $\mathbf{a} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, then $\mathbf{a} \times (\mathbf{a}.\mathbf{b}) =$ **[Karnataka CET 1994]**

(a) 3**a** (b) $3\sqrt{14}$ (c) 0 (d) None of these

288. If $\mathbf{a}.\mathbf{i} = 4$, then $(\mathbf{a} \times \mathbf{j}).(2\mathbf{j} - 3\mathbf{k}) =$ **[EAMCET 1994]**

(a) 12 (b) 2 (c) 0 (d) – 12

289. $\mathbf{a}.(\mathbf{a}\times\mathbf{b})=$ **[MP PET 1996]**

(a) **b . b** (b) a^2b (c) 0 (d) a^2+ab

290. For three vectors **u, v, w** which of the following expressions is not equal to any of the remaining three **[IIT 1998; Oriss**

(a) $\mathbf{u}.(\mathbf{v}\times\mathbf{w})$ (b) $(\mathbf{v}\times\mathbf{w}).\mathbf{u}$ (c) $\mathbf{v}.(\mathbf{u}\times\mathbf{w})$ (d) $(\mathbf{u}\times\mathbf{v}).\mathbf{w}$

291. Which of the following expressions are meaningful **[IIT 1998; Rajasthan PET 2001]**

(a) $\mathbf{u}.(\mathbf{v}\times\mathbf{w})$ (b) $(\mathbf{u}.\mathbf{v}).\mathbf{w}$ (c) $(\mathbf{u}.\mathbf{v})\mathbf{w}$ (d) $\mathbf{u}\times(\mathbf{v}.\mathbf{w})$

292. Given vectors **a, b, c** such that $\mathbf{a}.(\mathbf{b}\times\mathbf{c})=\lambda\neq 0$, the value of $(\mathbf{b}\times\mathbf{c}).(\mathbf{a}+\mathbf{b}+\mathbf{c})/\lambda$ is **[AMU 1999]**

(a) 3 (b) 1 (c) -3λ (d) $\frac{3}{\lambda}$

293. If $\mathbf{a}=3\mathbf{i}-2\mathbf{j}+2\mathbf{k}, \mathbf{b}=6\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ and $\mathbf{c}=3\mathbf{i}-2\mathbf{j}-4\mathbf{k}$, then $\mathbf{a}.(\mathbf{b}\times\mathbf{c})$ is **[Karnataka CET 2001]**

(a) 122 (b) – 144 (c) 120 (d) – 120

294. $\mathbf{a}.(\mathbf{b}\times\mathbf{c})$ is equal to **[Rajasthan PET 2001]**

(a) $\mathbf{b}.(\mathbf{a}\times\mathbf{c})$ (b) $\mathbf{c}.(\mathbf{b}\times\mathbf{a})$ (c) $\mathbf{b}.(\mathbf{c}\times\mathbf{a})$ (d) None of these

295. $[\mathbf{i}\ \mathbf{k}\ \mathbf{j}]+[\mathbf{k}\ \mathbf{j}\ \mathbf{i}]+[\mathbf{j}\ \mathbf{k}\ \mathbf{i}]$ **[UPSEAT 2002]**

(a) 1 (b) 3 (c) – 3 (d) – 1

296. If $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=1$, then $\frac{\mathbf{a}.\mathbf{b}\times\mathbf{c}}{\mathbf{c}\times\mathbf{a}.\mathbf{b}}+\frac{\mathbf{b}.\mathbf{c}\times\mathbf{a}}{\mathbf{a}\times\mathbf{b}.\mathbf{c}}+\frac{\mathbf{c}.\mathbf{a}\times\mathbf{b}}{\mathbf{b}\times\mathbf{c}.\mathbf{a}}$ is equal to

(a) 3 (b) 1 (c) – 1 (d) None of these

297. If the vectors $2\mathbf{i}-3\mathbf{j}$, $\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $3\mathbf{i}-\mathbf{k}$ form three concurrent edges of a parallelopiped, then the volume of the parallelopiped is

[IIT 1983; Rajasthan PET 1995; DCE 2001; Kurukshetra CEE 1998; MP PET 2001]

(a) 8 (b) 10 (c) 4 (d) 14

298. If three vectors $\mathbf{a}=12\mathbf{i}+4\mathbf{j}+3\mathbf{k}$, $\mathbf{b}=8\mathbf{i}-12\mathbf{j}-9\mathbf{k}$ and $\mathbf{c}=33\mathbf{i}-4\mathbf{j}-24\mathbf{k}$ represents a cube, then its volume will be

[Roorkee 1988]

(a) 616 (b) 308 (c) 154 (d) None of these

299. Volume of the parallelopiped whose coterminous edges are $2\mathbf{i}-3\mathbf{j}+4\mathbf{k}$, $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$, $3\mathbf{i}-\mathbf{j}+\mathbf{k}$, is **[EAMCET 1993]**

(a) 5 cubic units (b) 6 cubic units (c) 7 cubic units (d) 8 cubic units

300. If $\mathbf{a}=-3\mathbf{i}+7\mathbf{j}+5\mathbf{k}$, $\mathbf{b}=-3\mathbf{i}+7\mathbf{j}-3\mathbf{k}$ and $\mathbf{c}=7\mathbf{i}-5\mathbf{j}-3\mathbf{k}$ are the three coterminous edges of a parallelopiped, then its volume is

[MP PET 1996]

(a) 108 (b) 210 (c) 272 (d) 308

301. Three concurrent edges *OA, OB, OC* of a parallelopiped are represented by three vectors $2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $-3\mathbf{i}-\mathbf{j}+\mathbf{k}$, the volume of the solid so formed in cubic units is **[Kurukshetra CEE 1998]**

(a) 5 (b) 6 (c) 7 (d) 8

302. What will be the volume of that parallelopiped whose sides are $\mathbf{a}=\mathbf{i}-\mathbf{j}+\mathbf{k}$, $\mathbf{b}=\mathbf{i}-3\mathbf{j}+4\mathbf{k}$ and $\mathbf{c}=2\mathbf{i}-5\mathbf{j}+3\mathbf{k}$ **[UPSEAT 1**

(a) 5 unit (b) 6 unit (c) 7 unit (d) 8 unit

303. The volume of the parallelopiped whose coterminous edges are $\mathbf{i}-\mathbf{j}+\mathbf{k}$, $2\mathbf{i}-4\mathbf{j}+5\mathbf{k}$ and $3\mathbf{i}-5j+2\mathbf{k}$ is **[Kerala (Engg.) 2**

(a) 4 (b) 3 (c) 2 (d) 8

304. The volume of the parallelopiped whose edges are represented by $-12\mathbf{i}+\alpha\mathbf{k}$, $3\mathbf{j}-\mathbf{k}$ and $2\mathbf{i}+\mathbf{j}-15\mathbf{k}$ is 546, then $\alpha=$

[IIT Screening 1989; MNR 1987]

(a) 3 (b) 2 (c) – 3 (d) – 2

Advance Level

305. $|(\mathbf{a}\times\mathbf{b}).\mathbf{c}| = |\mathbf{a}||\mathbf{b}||\mathbf{c}|$, if **[Ranchi BIT 1990; IIT 1982; AMU 2002]**

(a) $\mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{c}=0$ (b) $\mathbf{b}.\mathbf{c}=\mathbf{c}.\mathbf{a}=0$ (c) $\mathbf{c}.\mathbf{a}=\mathbf{a}.\mathbf{b}=0$ (d) $\mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{c}=\mathbf{c}.\mathbf{a}=0$

306. If **a, b, c** be any three non-coplanar vectors, then $[\mathbf{a}+\mathbf{b}, \mathbf{b}+\mathbf{c}, \mathbf{c}+\mathbf{a}]=$

[Rajasthan PET 1988; MP PET 1990, 2002; Kerala (Engg.) 2002]

(a) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (b) $2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (c) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2$ (d) $2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2$

307. If **a, b, c** are three non-coplanar vectors and **p, q, r** are defined by the relations $\mathbf{p}=\frac{\mathbf{b}\times\mathbf{c}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$, $\mathbf{q}=\frac{\mathbf{c}\times\mathbf{a}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$, $\mathbf{r}=\frac{\mathbf{a}\times\mathbf{b}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$, then $(\mathbf{a}+\mathbf{b}).\mathbf{p}+(\mathbf{b}+\mathbf{c}).\mathbf{q}+(\mathbf{c}+\mathbf{a}).\mathbf{r}=$ **[IIT 1988; BIT Ranchi 1996; AMU 2002]**

(a) 0 (b) 1 (c) 2 (d) 3

308. If $\mathbf{p}=\frac{\mathbf{b}\times\mathbf{c}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$, $\mathbf{q}=\frac{\mathbf{c}\times\mathbf{a}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$, $\mathbf{r}=\frac{\mathbf{a}\times\mathbf{b}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$, where **a, b, c** are three non-coplanar vectors, then the value of $(\mathbf{a}+\mathbf{b}+\mathbf{c}).(\mathbf{p}+\mathbf{q}+\mathbf{r})$ is given by **[MNR 1992; UPSEAT 2000]**

(a) 3 (b) 2 (c) 1 (d) 0

309. The value of $[\mathbf{a}-\mathbf{b}\ \ \mathbf{b}-\mathbf{c}\ \ \mathbf{c}-\mathbf{a}]$, where $|\mathbf{a}|=1$, $|\mathbf{b}|=5$ and $|\mathbf{c}|=3$ is **[Rajasthan PET 1988, 2000; IIT 1989]**

(a) 0 (b) 1 (c) 2 (d) 4

310. If **a, b** and **c** are three non-coplanar vectors, then $(\mathbf{a}+\mathbf{b}+\mathbf{c}).[(\mathbf{a}+\mathbf{b})\times(\mathbf{a}+\mathbf{c})]$ is equal to **[IIT 1995]**

(a) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (b) $2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (c) $-[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (d) 0

311. If **a, b, c** are three coplanar vectors, then $[\mathbf{a}+\mathbf{b}\ \ \mathbf{b}+\mathbf{c}\ \ \mathbf{c}+\mathbf{a}]=$ **[MP PET 1995]**

(a) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (b) $2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (c) $3[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (d) 0

312. If **b** and **c** are any two non-collinear unit vectors and **a** is any vector, then $(\mathbf{a}.\mathbf{b})\mathbf{b}+(\mathbf{a}.\mathbf{c})\mathbf{c}+\frac{\mathbf{a}.(\mathbf{b}\times\mathbf{c})}{|\mathbf{b}\times\mathbf{c}|}(\mathbf{b}\times\mathbf{c})=$ **[IIT 1996]**

(a) **a** (b) **b** (c) **c** (d) 0

313. If three coterminous edges of a parallelopiped are represented by $\mathbf{a}-\mathbf{b}$, $\mathbf{b}-\mathbf{c}$ and $\mathbf{c}-\mathbf{a}$, then its volume is **[MP PET 19**

(a) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (b) $2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (c) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2$ (d) 0

314. If **a, b** and **c** are unit coplanar vectors then the scalar triple product $[2\mathbf{a}-\mathbf{b}\ \ 2\mathbf{b}-\mathbf{c}\ \ 2\mathbf{c}-\mathbf{a}]$ is equal to **[IIT Screening 2000**

(a) 0 (b) 1 (c) $-\sqrt{3}$ (d) $\sqrt{3}$

315. Let $\mathbf{a}=\mathbf{i}-\mathbf{k}$, $\mathbf{b}=x\mathbf{i}+\mathbf{j}+(1-x)\mathbf{k}$ and $\mathbf{c}=y\mathbf{i}+x\mathbf{j}+(1+x-y)\mathbf{k}$, then $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ depends on **[IIT Screening 2001]**

(a) Only x (b) Only y (c) Neither x nor y (d) Both x and y

316. $(\mathbf{a}+\mathbf{b}).(\mathbf{b}+\mathbf{c})\times(\mathbf{a}+\mathbf{b}+\mathbf{c})=$ **[EAMCET 2002]**

(a) $-[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (b) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (c) 0 (d) $2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$

317. Let $\mathbf{V}=2\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $\mathbf{W}=\mathbf{i}+3\mathbf{k}$ if **U** is a unit vector, then the maximum value of the scalar triple product [**U V W**] is

[IIT Screening 2002]

(a) – 1 (b) $\sqrt{10}+\sqrt{6}$ (c) $\sqrt{59}$ (d) $\sqrt{60}$

318. If **a, b** are non-zero and non-collinear vectors then $[\mathbf{a}\ \mathbf{b}\ \mathbf{i}]\mathbf{i}+[\mathbf{a}\ \mathbf{b}\ \mathbf{j}]\mathbf{j}+[\mathbf{a}\ \mathbf{b}\ \mathbf{k}]\mathbf{k}$ is equal to

(a) $\mathbf{a}+\mathbf{b}$ (b) $\mathbf{a}\times\mathbf{b}$ (c) $\mathbf{a}-\mathbf{b}$ (d) $\mathbf{b}\times\mathbf{a}$

319. If **a, b, c** are three non-coplanar nonzero vectors then $(\mathbf{a}.\mathbf{a})\mathbf{b}\times\mathbf{c}+(\mathbf{a}.\mathbf{b})\mathbf{c}\times\mathbf{a}+(\mathbf{a}.\mathbf{c})\mathbf{a}\times\mathbf{b}$ is equal to

(a) [**b c a**]**a** (b) [**c a b**]**b** (c) [**a b c**]**c** (d) None of these

320. Let **a, b, c** be three unit vectors and $\mathbf{a}.\mathbf{b} = \mathbf{a}.\mathbf{c} = 0$. If the angle between **b** and **c** is $\frac{\pi}{3}$, then $|[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]|$ is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) None of these

321. If **a, b, c** are three non-coplanar vectors represented by concurrent edges of a parallelopiped of volume 4, then $(\mathbf{a}+\mathbf{b}).(\mathbf{b}\times\mathbf{c})+(\mathbf{b}+\mathbf{c}).(\mathbf{c}\times\mathbf{a})+(\mathbf{c}+\mathbf{a}).(\mathbf{a}\times\mathbf{b})$ is equal to

(a) 12 (b) 4 (c) ±12 (d) 0

322. The three concurrent edges of a parallelopiped represent the vectors **a, b, c** such that $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = \lambda$. Then the volume of the parallelopiped whose three concurrent edges are the three concurrent diagonals of three faces of the given parallelopiped is

(a) 2λ (b) 3λ (c) λ (d) None of these

323. If **a, b, c** are non-coplanar non-zero vectors and **r** is any vector in space then [**b c r**]**a**+[**c a r**]**b**+[**a b r**]**c** is equal to

(a) 3[**a b c**]**r** (b) [**a b c**]**r** (c) [**b c a**]**r** (d) None of these

324. If the vertices of a tetrahedron have the position vectors **0, i + j, 2j – k** and $\mathbf{i}+\mathbf{k}$ then the volume of the tetrahedron is

(a) $\frac{1}{6}$ (b) 1 (c) 2 (d) None of these

325. The three vectors $\mathbf{i}+\mathbf{j}, \mathbf{j}+\mathbf{k}, \mathbf{k}+\mathbf{i}$ taken two at a time form three planes. The three unit vectors drawn perpendicular to three planes form a parallelopiped of volume

(a) $\frac{1}{3}$ cubic units (b) 4 cubic units (c) $\frac{3\sqrt{3}}{4}$ cubic units (d) $\frac{4}{3\sqrt{3}}$ cubic units

326. The volume of the tetrahedron whose vertices are the points with position vectors $\mathbf{i}-6\mathbf{j}+10\mathbf{k}, -\mathbf{i}-3\mathbf{j}+7\mathbf{k}, 5\mathbf{i}-\mathbf{j}+\lambda\mathbf{k}$ and $7\mathbf{i}-4\mathbf{j}+7\mathbf{k}$ is 11 cubic units if the value of λ is

(a) – 1 (b) 1 (c) – 7 (d) 7

327. Let **a, b** and **c** be three non-zero and non-coplanar vectors and **p, q** and **r** be three vectors given by $\mathbf{p}=\mathbf{a}+\mathbf{b}-2\mathbf{c}$, $\mathbf{q}=3\mathbf{a}-2\mathbf{b}+\mathbf{c}$ and $\mathbf{r}=\mathbf{a}-4\mathbf{b}+2\mathbf{c}$. If the volume of the parallelopiped determined by **a**, **b**, and **c** is V_1 and that of the parallelopiped determined by **p, q,** and **r** is V_2, then $V_2 : V_1 =$

(a) 2 : 3 (b) 5 : 7 (c) 15 : 1 (d) 1 : 1

328. If **a, b, c** are any three vectors and their inverse are $\mathbf{a}^{-1}, \mathbf{b}^{-1}, \mathbf{c}^{-1}$ and $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] \neq 0$, then $[\mathbf{a}^{-1}\ \mathbf{b}^{-1}\ \mathbf{c}^{-1}]$ will be **[Roorkee 1989**

(a) Zero (b) One (c) Non-zero (d) [**a b c**]

329. **a, b, c** are three non-zero, non-coplanar vectors and **p, q, r** are three other vectors such that $\mathbf{p}=\frac{\mathbf{b}\times\mathbf{c}}{\mathbf{a}.\mathbf{b}\times\mathbf{c}}$, $\mathbf{q}=\frac{\mathbf{c}\times\mathbf{a}}{\mathbf{a}.\mathbf{b}\times\mathbf{c}}$, $\mathbf{r}=\frac{\mathbf{a}\times\mathbf{b}}{\mathbf{a}.\mathbf{b}\times\mathbf{c}}$. Then [**p q r**] equals **[Kurukshetra CEE 1993]**

(a) $\mathbf{a}.\mathbf{b}\times\mathbf{c}$ (b) $\frac{1}{\mathbf{a}.\mathbf{b}\times\mathbf{c}}$ (c) 0 (d) None of these

Coplanarity of Vectors

Basic Level

330. If the vectors $2\mathbf{i}-\mathbf{j}+\mathbf{k}$, $\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ and $3\mathbf{i}+\lambda\mathbf{j}+5\mathbf{k}$ be coplanar, then $\lambda =$

[Roorkee 1986; Rajasthan PET 1999, 2002; Kurukshetra CEE 2002]

(a) – 1 (b) – 2 (c) – 3 (d) – 4

331. If $\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{c} = 3\mathbf{i} + p\mathbf{j} + 5\mathbf{k}$ are coplanar then the value of *p* will be **[Rajasthan PET 1985, 86, 88, 91]**

(a) – 6 (b) – 2 (c) 2 (d) 6

332. A unit vector which is coplanar to vector $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and perpendicular to $\mathbf{i} + \mathbf{j} + \mathbf{k}$, is **[IIT 1992; Kurukshetra CEE 2002]**

(a) $\frac{\mathbf{i} - \mathbf{j}}{\sqrt{2}}$ (b) $\pm\left(\frac{\mathbf{j} - \mathbf{k}}{\sqrt{2}}\right)$ (c) $\frac{\mathbf{k} - \mathbf{i}}{\sqrt{2}}$ (d) $\frac{\mathbf{i} + \mathbf{j} + \mathbf{k}}{\sqrt{3}}$

333. If the vectors $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}, \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $x\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ are coplanar, then *x* = **[EAMCET 1994]**

(a) $\frac{8}{5}$ (b) $\frac{5}{8}$ (c) 0 (d) 1

334. If the vectors $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $3\mathbf{i} + 2\mathbf{j} + x\mathbf{k}$ are coplanar ,then the value of *x* is **[Karnataka CET 2000]**

(a) – 2 (b) 2 (c) 1 (d) 3

335. $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 4\mathbf{k}$, $\mathbf{c} = \mathbf{i} + \lambda\mathbf{j} + 3\mathbf{k}$ are coplanar, then the value of λ is **[MP PET 2000]**

(a) $\frac{5}{2}$ (b) $\frac{3}{5}$ (c) $\frac{7}{3}$ (d) None of these

336. $\vec{A} = \mathbf{i} + \mathbf{j} + \mathbf{k}, \vec{B} = \mathbf{i}, \vec{C} = C_1\mathbf{i} + C_2\mathbf{j} + C_2\mathbf{k}$. If $C_2 = -1$ and $C_3 = 1$, then to make three vectors coplanar **[AMU 2000]**

(a) $C_1 = 0$ (b) $C_1 = 1$

(c) $C_1 = 2$ (d) No value of C_1 can be found

337. The vector **a** lies in the plane of vectors **b** and **c,** which of the following is correct **[Roorkee 1990]**

(a) $\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = 0$ (b) $\mathbf{a}.\mathbf{b} \times \mathbf{c} = 1$ (c) $\mathbf{a}.\mathbf{b} \times \mathbf{c} = -1$ (d) $\mathbf{a}.\mathbf{b} \times \mathbf{c} = 3$

Advance Level

338. If the vectors $\mathbf{r}_1 = \sec^2 A, 1, 1;\ \mathbf{r}_2 = 1, \sec^2 B, 1\ ;\ \mathbf{r}_3 = 1, 1, \sec^2 C$ are coplanar, then $\cot^2 A + \cot^2 B + \cot^2 C$ is equal to

(a) 0 (b) 1 (c) 2 (d) Not defined

339. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and $\mathbf{a} = (1, a, a^2)$, $\mathbf{b} = (1, b, b^2)$ and $\mathbf{c} = (1, c, c^2)$ are non-coplanar vectors, then *abc* is equal to **[IIT 1985; AIEEE 2003]**

(a) – 1 (b) 0 (c) 1 (d) 4

340. Let *a, b, c* be distinct non-negative numbers. If the vectors $a\mathbf{i} + a\mathbf{j} + c\mathbf{k}, \mathbf{i} + \mathbf{k}$ and $c\mathbf{i} + c\mathbf{j} + b\mathbf{k}$ lie in a plane, then **c** is **[IIT 19**

(a) The airthmetic mean of *a* and *b* (b) The geometric mean of *a* and *b*

(c) The harmonic mean of *a* and *b* (d) Equal to zero

341. If the vectors $a\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{i} + b\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} + c\mathbf{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$ **[BIT Ranchi 1988; Rajasthan PET 1987; IIT 1987; DCE 2001]**

(a) – 1 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 1

342. If **a, b, c** are position vector of vertices of a triangle *ABC,* then unit vector perpendicular to its plane is **[Rajasthan PET**

(a) $\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}$ (b) $\frac{\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}}{|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}|}$ (c) $\frac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}\times\mathbf{b}|}$ (d) None of these

343. If **a, b, c** are non-coplanar vectors and $\mathbf{d}=\lambda\mathbf{a}+\mu\mathbf{b}+\nu\mathbf{c}$, then λ is equal to **[Roorkee 1999]**

(a) $\frac{[\mathbf{d}\ \mathbf{b}\ \mathbf{c}]}{[\mathbf{b}\ \mathbf{a}\ \mathbf{c}]}$ (b) $\frac{[\mathbf{b}\ \mathbf{c}\ \mathbf{d}]}{[\mathbf{b}\ \mathbf{c}\ \mathbf{a}]}$ (c) $\frac{[\mathbf{b}\ \mathbf{d}\ \mathbf{c}]}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$ (d) $\frac{[\mathbf{c}\ \mathbf{b}\ \mathbf{d}]}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$

344. If the points whose position vectors are $3\mathbf{i}-2\mathbf{j}-\mathbf{k},\ 2\mathbf{i}+3\mathbf{j}-4\mathbf{k},\ -\mathbf{i}+\mathbf{j}+2\mathbf{k}$ and $4\mathbf{i}+5\mathbf{j}+\lambda\mathbf{k}$ lie on a plane, then $\lambda=$ **[IIT 1986]**

(a) $-\frac{146}{17}$ (b) $\frac{146}{17}$ (c) $-\frac{17}{146}$ (d) $\frac{17}{146}$

345. Vector coplanar with vectors **i+j** and **j+k** and parallel to the vector $2\mathbf{i}-2\mathbf{j}-4\mathbf{k}$, is **[Roorkee 2000]**

(a) $\mathbf{i}-\mathbf{k}$ (b) $\mathbf{i}-\mathbf{j}-2\mathbf{k}$ (c) $\mathbf{i}+\mathbf{j}-\mathbf{k}$ (d) $3\mathbf{i}+3\mathbf{j}-6\mathbf{k}$

346. Let $\boldsymbol{\lambda}=\mathbf{a}\times(\mathbf{b}+\mathbf{c}),\ \boldsymbol{\mu}=\mathbf{b}\times(\mathbf{c}+\mathbf{a})$ and $\mathbf{v}=\mathbf{c}\times(\mathbf{a}+\mathbf{b})$. Then

(a) $\boldsymbol{\lambda}+\boldsymbol{\mu}=\mathbf{v}$ (b) $\boldsymbol{\lambda},\boldsymbol{\mu},\mathbf{v}$ are coplanar (c) $\boldsymbol{\lambda}+\mathbf{v}=2\boldsymbol{\mu}$ (d) None of these

347. Let $\mathbf{a}=\mathbf{i}-2\mathbf{j}+3\mathbf{k},\ \mathbf{b}=2\mathbf{i}+3\mathbf{j}-\mathbf{k}$ and $\mathbf{c}=\lambda\mathbf{i}+\mathbf{j}+(2\lambda-1)\mathbf{k}$. If **c** is parallel to the plane of the vectors **a** and **b** then λ is

(a) 1 (b) 0 (c) – 1 (d) 2

348. The vectors $\mathbf{a}=x\mathbf{i}+(x+1)\mathbf{j}+(x+2)\mathbf{k},\ \mathbf{b}=(x+3)\mathbf{i}+(x+4)\mathbf{j}+(x+5)\mathbf{k}$ and $\mathbf{c}=(x+6)\mathbf{i}+(x+7)\mathbf{j}+(x+8)\mathbf{k}$ are coplanar for

(a) All values of x (b) $x<0$ (c) $x>0$ (d) None of these

349. Given a cube $ABCD\,A_1B_1C_1D_1$ with lower base $ABCD$, upper base $A_1B_1C_1D_1$ and the lateral edges AA_1, BB_1, CC_1 and DD_1; M and M_1 are the centres of the faces $ABCD$ and $A_1B_1C_1D_1$ respectively. O is a point on line MM_1, such that $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=\overrightarrow{OM_1}$, then $\overrightarrow{OM}=\lambda.\overrightarrow{OM_1}$, if $\lambda=$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{8}$

Vector Triple Product

Basic Level

350. $\mathbf{i}\times(\mathbf{j}\times\mathbf{k})$ is equal to **[Rajasthan PET 1988; MP PET 1997]**

(a) **0** (b) **i** (c) **j** (d) **k**

351. If $\mathbf{a}=\mathbf{i}+2\mathbf{j}-2\mathbf{k},\ \mathbf{b}=2\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=\mathbf{i}+3\mathbf{j}-\mathbf{k}$, then $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is equal to **[Rajasthan PET 1989]**

(a) $20\mathbf{i}-3\mathbf{j}+7\mathbf{k}$ (b) $20\mathbf{i}-3\mathbf{j}-7\mathbf{k}$ (c) $20\mathbf{i}+3\mathbf{j}-7\mathbf{k}$ (d) None of these

352. If $\mathbf{a}=\mathbf{i}+\mathbf{j}-\mathbf{k},\ \mathbf{b}=\mathbf{i}-\mathbf{j}+\mathbf{k},\ \mathbf{c}=\mathbf{i}-\mathbf{j}-\mathbf{k}$, then $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is **[MP PET 2000]**

(a) $\mathbf{i}-\mathbf{j}+\mathbf{k}$ (b) $2\mathbf{i}-2\mathbf{j}$ (c) $3\mathbf{i}-\mathbf{j}+\mathbf{k}$ (d) $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$

353. If **a, b, c** are any three vectors then $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is a vector

(a) Perpendicular to $\mathbf{a}\times\mathbf{b}$ (b) Coplanar with **a** and **b** (c) Parallel to **c** (d) Parallel to either **a** or **b**

354. If **a** and **b** are two unit vectors, then the vector $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}\times\mathbf{b})$ is parallel to the vector **[DCE 2001]**

(a) $\mathbf{a}+\mathbf{b}$ (b) $\mathbf{a}-\mathbf{b}$ (c) $2\mathbf{a}+\mathbf{b}$ (d) $2\mathbf{a}-\mathbf{b}$

355. If $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=0$, then **[Rajasthan PET 1995]**

(a) $|\mathbf{a}|=|\mathbf{b}|.|\mathbf{c}|=1$ (b) $\mathbf{b}\parallel\mathbf{c}$ (c) $\mathbf{a}\parallel\mathbf{b}$ (d) $\mathbf{b}\perp\mathbf{c}$

356. Which of the following is a true statement **[Kurukshetra CEE 1996]**

(a) $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is coplanar with **c**
(b) $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is perpendicular to **a**
(c) $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is perpendicular to **b**
(d) $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is perpendicular to **c**

357. If $\mathbf{u}=\mathbf{i}\times(\mathbf{a}\times\mathbf{i})+\mathbf{j}\times(\mathbf{a}\times\mathbf{j})+\mathbf{k}\times(\mathbf{a}\times\mathbf{k})$, then **[Rajasthan PET 1989, 97; MNR 1986, 93; MP PET 1987, 98, 99, 2004; UPSEAT 2000, 2002; Kerala (Engg.) 2002]**

(a) $\mathbf{u}=0$ (b) $\mathbf{u}=\mathbf{i}+\mathbf{j}+\mathbf{k}$ (c) $\mathbf{u}=2\mathbf{a}$ (d) $\mathbf{u}=\mathbf{a}$

358. A unit vector perpendicular to vector **c** and coplanar with vectors **a** and **b** is **[MP PET 1999]**

(a) $\dfrac{\mathbf{a}\times(\mathbf{b}\times\mathbf{c})}{|\mathbf{a}\times(\mathbf{b}\times\mathbf{c})|}$ (b) $\dfrac{\mathbf{b}\times(\mathbf{c}\times\mathbf{a})}{|\mathbf{b}\times(\mathbf{c}\times\mathbf{a})|}$ (c) $\dfrac{\mathbf{c}\times(\mathbf{a}\times\mathbf{b})}{|\mathbf{c}\times(\mathbf{a}\times\mathbf{b})|}$ (d) None of these

359. $\mathbf{i}\times(\mathbf{j}\times\mathbf{k})+\mathbf{j}\times(\mathbf{k}\times\mathbf{i})+\mathbf{k}\times(\mathbf{i}\times\mathbf{j})$ equals **[Rajasthan PET 1999]**

(a) **i** (b) **j** (c) **k** (d) **0**

360. Given three unit vectors **a, b, c** such that $\mathbf{a}\perp\mathbf{b}$ and $\mathbf{a}\parallel\mathbf{c}$, then $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is **[AMU 1999]**

(a) **a** (b) **b** (c) **c** (d) **0**

361. $\vec{A}=\mathbf{i}-2\mathbf{j}-3\mathbf{k},\ \vec{B}=2\mathbf{i}+\mathbf{j}-\mathbf{k},\ \vec{C}=\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, then $(\vec{A}\times\vec{B})\times\vec{C}$ is **[MP PET 2001]**

(a) $5(-\mathbf{i}+3\mathbf{j}+4\mathbf{k})$ (b) $4(-\mathbf{i}+3\mathbf{j}+4\mathbf{k})$ (c) $5(-\mathbf{i}-3\mathbf{j}-4\mathbf{k})$ (d) $4(\mathbf{i}+3\mathbf{j}+4\mathbf{k})$

362. If **i, j, k** are unit vectors, then **[MP PET 2001]**

(a) $\mathbf{i}.\mathbf{j}=1$ (b) $\mathbf{i}.\mathbf{i}=1$ (c) $\mathbf{i}\times\mathbf{j}=1$ (d) $\mathbf{i}\times(\mathbf{j}\times\mathbf{k})=1$

363. If **a, b, c** are any vectors, then the true statement is **[Rajasthan PET 1988]**

(a) $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ (b) $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{a}$ (c) $\mathbf{a}.(\mathbf{b}\times\mathbf{c})=\mathbf{a}.\mathbf{b}\times\mathbf{a}.\mathbf{c}$ (d) $\mathbf{a}.(\mathbf{b}-\mathbf{c})=\mathbf{a}.\mathbf{b}-\mathbf{a}.\mathbf{c}$

364. $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is equal to **[Rajasthan PET 1995; Kurukshetra CEE 1998; MP PET 2003]**

(a) $(\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{a}.\mathbf{a})\mathbf{b}$ (b) $(\mathbf{a}.\mathbf{c})\mathbf{a}-(\mathbf{b}.\mathbf{c})\mathbf{a}$ (c) $(\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{a}.\mathbf{b})\mathbf{c}$ (d) $(\mathbf{a}.\mathbf{b})\mathbf{c}-(\mathbf{a}.\mathbf{c})\mathbf{b}$

365. $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})+\mathbf{b}\times(\mathbf{c}\times\mathbf{a})+\mathbf{c}\times(\mathbf{a}\times\mathbf{b})=$ **[Rajasthan PET 2003]**

(a) **0** (b) $2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (c) $\mathbf{a}+\mathbf{b}+\mathbf{c}$ (d) $3[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$

Advance Level

366. If **a, b, c** are non-coplanar unit vectors such that $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=\dfrac{\mathbf{b}+\mathbf{c}}{\sqrt{2}}$, then the angle between **a** and **b** is **[IIT 1995]**

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{3\pi}{4}$ (d) π

367. Let **a, b, c** be three vectors from $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$, if **[Orissa JEE 2003]**

(a) $\mathbf{b}\times(\mathbf{a}\times\mathbf{c})=\mathbf{0}$ (b) $\mathbf{a}.(\mathbf{b}\times\mathbf{c})=0$ (c) $\mathbf{c}\times\mathbf{a}=\mathbf{a}\times\mathbf{b}$ (d) $\mathbf{c}\times\mathbf{b}=\mathbf{b}\times\mathbf{a}$

368. If **a, b, c** are any three vectors such that $(\mathbf{a}+\mathbf{b}).\mathbf{c}=(\mathbf{a}-\mathbf{b}).\mathbf{c}=0$, then $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is

(a) **0** (b) **a** (c) **b** (d) None of these

369. If three unit vectors **a, b, c** are such that $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=\dfrac{\mathbf{b}}{2}$, then the vector **a** makes with **b** and **c** respectively the angles **[MP PET 1998]**

(a) $40^\circ, 80^\circ$ (b) $45^\circ, 45^\circ$ (c) $30^\circ, 60^\circ$ (d) $90^\circ, 60^\circ$

370. $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}),\ \mathbf{b}\times(\mathbf{c}\times\mathbf{a})$ and $\mathbf{c}\times(\mathbf{a}\times\mathbf{b})$ are

(a) Linearly dependent (b) Equal vectors (c) Parallel vectors (d) None of these

371. **a** and **b** are two given vectors. On these vectors as adjacent sides a parallelogram is constructed. The vector which is the altitude of the parallelogram and which is perpendicular to **a** is

(a) $\frac{(\mathbf{a}\,.\,\mathbf{b})}{|\,\mathbf{a}\,|^2}\mathbf{a}-\mathbf{b}$ (b) $\frac{1}{|\,\mathbf{a}\,|^2}\left\{|\,\mathbf{a}\,|^2\,\mathbf{b}-(\mathbf{a}\,.\,\mathbf{b})\mathbf{a}\right\}$ (c) $\frac{\mathbf{a}\times(\mathbf{a}\times\mathbf{b})}{|\,\mathbf{a}\,|^2}$ (d) $\frac{\mathbf{a}\times(\mathbf{b}\times\mathbf{a})}{|\,\mathbf{b}\,|^2}$

Scalar and Vector Product of Four or more Vectors

Basic Level

372. If $\boldsymbol{\alpha}=2\mathbf{i}+3\mathbf{j}-\mathbf{k}$, $\boldsymbol{\beta}=-\mathbf{i}+2\mathbf{j}-4\mathbf{k}$ and $\boldsymbol{\gamma}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, then $(\boldsymbol{\alpha}\times\boldsymbol{\beta}).(\boldsymbol{\alpha}\times\boldsymbol{\gamma})$ is equal to **[MNR 1984; UPSEAT 2000]**

(a) 60 (b) 64 (c) 74 (d) – 74

373. $(\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a})=$ **[MP PET 1997]**

(a) [**b c a**]**a** (b) [**c a b**]**b** (c) [**a b c**]**c** (d) [**a c b**]**b**

374. If **a, b, c, d** are coplanar vectors, then $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})=$ **[MP PET 1998]**

(a) $|\,\mathbf{a}\times\mathbf{c}\,|^2$ (b) $|\,\mathbf{a}\times\mathbf{d}\,|^2$ (c) $|\,\mathbf{b}\times\mathbf{c}\,|^2$ (d) 0

375. If **.** and × represent dot product and cross product respectively then which of the following is meaningless

(a) $(\mathbf{a}\times\mathbf{b}).(\mathbf{c}\times\mathbf{d})$ (b) $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})$ (c) $(\mathbf{a}\,.\,\mathbf{b})\,(\mathbf{c}\times\mathbf{d})$ (d) $(\mathbf{a}\,.\,\mathbf{b})\times(\mathbf{c}\times\mathbf{d})$

376. Two planes are perpendicular to one another. One of them contains vectors **a** and **b** and the other contains vectors **c** and **d**, then $(\mathbf{a}\times\mathbf{b}).(\mathbf{c}\times\mathbf{d})$ equals

(a) 1 (b) 0 (c) [**a b c**] (d) [**b c d**]

Advance Level

377. **a, b, c, d** are any four vectors then $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})$ is a vector

(a) Perpendicular to **a, b, c, d**

(b) Along the line of intersection of two planes, one containing **a, b** and the other containing **c, d**

(c) Equally inclined to both $\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}\times\mathbf{d}$

(d) None of these

378. If **a, b, c** are non-coplanar non-zero vectors then $(\mathbf{a}\times\mathbf{b})\times(\mathbf{a}\times\mathbf{c})+(\mathbf{b}\times\mathbf{c})\times(\mathbf{b}\times\mathbf{a})+(\mathbf{c}\times\mathbf{a})\times(\mathbf{c}\times\mathbf{b})$ is equal to

(a) $[\mathbf{a}\;\mathbf{b}\;\mathbf{c}]^2\,(\mathbf{a}+\mathbf{b}+\mathbf{c})$ (b) $[\mathbf{a}\;\mathbf{b}\;\mathbf{c}]\,(\mathbf{a}+\mathbf{b}+\mathbf{c})$ (c) **0** (d) None of these

379. If **a, b, c** are three non-coplanar non-zero vectors and **r** is any vector is space then $(\mathbf{a}\times\mathbf{b})\times(\mathbf{r}\times\mathbf{c})+(\mathbf{b}\times\mathbf{c})\times(\mathbf{r}\times\mathbf{a})+(\mathbf{c}\times\mathbf{a})\times(\mathbf{r}\times\mathbf{b})$ is equal to

(a) 2[**a b c**]**r** (b) 3[**a b c**]**r** (c) [**a b c**]**r** (d) None of these

380. If $\mathbf{a}\parallel\mathbf{b}\times\mathbf{c}$ then $(\mathbf{a}\times\mathbf{b}).(\mathbf{a}\times\mathbf{c})$ is equal to

(a) $\mathbf{a}^2(\mathbf{b}\,.\,\mathbf{c})$ (b) $\mathbf{b}^2(\mathbf{a}\,.\,\mathbf{c})$ (c) $\mathbf{c}^2(\mathbf{a}\,.\,\mathbf{b})$ (d) None of these

381. $(\mathbf{a}\times\mathbf{b}).(\mathbf{c}\times\mathbf{d})$ is equal to

(a) $\mathbf{a}.\{\mathbf{b}\times(\mathbf{c}\times\mathbf{d})\}$ (b) $(\mathbf{a}.\mathbf{c})(\mathbf{b}.\mathbf{d})-(\mathbf{a}.\mathbf{d})(\mathbf{b}.\mathbf{c})$ (c) $\{(\mathbf{a}\times\mathbf{b})\times\mathbf{c}\}.\mathbf{d}$ (d) $(\mathbf{d}\times\mathbf{c}).(\mathbf{b}\times\mathbf{a})$

382. $(\mathbf{a}\times\mathbf{b})\times(\mathbf{a}\times\mathbf{c}).\,\mathbf{d}$ equals

(a) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}](\mathbf{b}.\mathbf{d})$ (b) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}](\mathbf{a}.\mathbf{d})$ (c) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}](\mathbf{c}.\mathbf{d})$ (d) None of these

383. $[(\mathbf{a}\times\mathbf{b})\times(\mathbf{b}\times\mathbf{c})\ (\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a})\ (\mathbf{c}\times\mathbf{a})\times(\mathbf{a}\times\mathbf{b})]$ is equal to

(a) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2$ (b) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^3$ (c) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^4$ (d) None of these

384. If **a, b, c** are coplanar vectors, then **[IIT 1989]**

(a) $\begin{vmatrix}\mathbf{a} & \mathbf{b} & \mathbf{c}\\ \mathbf{b} & \mathbf{c} & \mathbf{a}\\ \mathbf{c} & \mathbf{a} & \mathbf{b}\end{vmatrix}=0$ (b) $\begin{vmatrix}\mathbf{a} & \mathbf{b} & \mathbf{c}\\ \mathbf{a}.\mathbf{a} & \mathbf{a}.\mathbf{b} & \mathbf{a}.\mathbf{c}\\ \mathbf{b}.\mathbf{a} & \mathbf{b}.\mathbf{b} & \mathbf{b}.\mathbf{c}\end{vmatrix}=0$ (c) $\begin{vmatrix}\mathbf{a} & \mathbf{b} & \mathbf{c}\\ \mathbf{c}.\mathbf{a} & \mathbf{c}.\mathbf{b} & \mathbf{c}.\mathbf{c}\\ \mathbf{b}.\mathbf{a} & \mathbf{b}.\mathbf{c} & \mathbf{b}.\mathbf{b}\end{vmatrix}=0$ (d) $\begin{vmatrix}\mathbf{a} & \mathbf{b} & \mathbf{c}\\ \mathbf{a}.\mathbf{b} & \mathbf{a}.\mathbf{a} & \mathbf{a}.\mathbf{c}\\ \mathbf{c}.\mathbf{a} & \mathbf{c}.\mathbf{c} & \mathbf{c}.\mathbf{b}\end{vmatrix}=0$

385. For any three non-zero vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$, $\begin{vmatrix}\mathbf{r}_1.\mathbf{r}_1 & \mathbf{r}_1.\mathbf{r}_2 & \mathbf{r}_1.\mathbf{r}_3\\ \mathbf{r}_2.\mathbf{r}_1 & \mathbf{r}_2.\mathbf{r}_2 & \mathbf{r}_2.\mathbf{r}_3\\ \mathbf{r}_3.\mathbf{r}_1 & \mathbf{r}_3.\mathbf{r}_2 & \mathbf{r}_3.\mathbf{r}_3\end{vmatrix}=0$. Then which of the following is false **[AMU 2000]**

(a) All the three vectors are parallel to one and the same plane (b) All the three vectors are linearly dependent

(c) This system of equation has a non-trivial solution (d) All the three vectors are perpendicular to each other

386. $[\mathbf{b}\ \ \mathbf{c}\ \ \mathbf{b}\times\mathbf{c}]+(\mathbf{b}.\mathbf{c})^2$ is equal to

(a) $|\mathbf{b}|^2|\mathbf{c}|^2$ (b) $(\mathbf{b}+\mathbf{c})^2$ (c) $|\mathbf{b}|^2+|\mathbf{c}|^2$ (d) None of these

387. If **a, b, c** are vectors such that [**a b c**] = 4, then $[\mathbf{a}\times\mathbf{b}\ \ \mathbf{b}\times\mathbf{c}\ \ \mathbf{c}\times\mathbf{a}]=$ **[AIEEE 2002]**

(a) 16 (b) 64 (c) 4 (d) 8

Vector Equations

Basic Level

388. If position vector of points *A, B, C* are respectively **i, j, k** and *AB = CX,* then position vector of point *X* is **[MP PET 1994]**

(a) $-\mathbf{i}+\mathbf{j}+\mathbf{k}$ (b) $\mathbf{i}-\mathbf{j}+\mathbf{k}$ (c) $\mathbf{i}+\mathbf{j}-\mathbf{k}$ (d) $\mathbf{i}+\mathbf{j}+\mathbf{k}$

389. If $\mathbf{a}.\mathbf{i}=\mathbf{a}.(\mathbf{i}+\mathbf{j})=\mathbf{a}.(\mathbf{i}+\mathbf{j}+\mathbf{k})$, then **a** = **[EAMCET 2002]**

(a) **i** (b) **k** (c) **j** (d) $\mathbf{i}+\mathbf{j}+\mathbf{k}$

390. If $\mathbf{a}=\mathbf{i}+\mathbf{j}$, $\mathbf{b}=2\mathbf{j}-\mathbf{k}$ and $\mathbf{r}\times\mathbf{a}=\mathbf{b}\times\mathbf{a}$, $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$, then $\frac{\mathbf{r}}{|\mathbf{r}|}$ is equal to

(a) $\frac{1}{\sqrt{11}}(\mathbf{i}+3\mathbf{j}-\mathbf{k})$ (b) $\frac{1}{\sqrt{11}}(\mathbf{i}-3\mathbf{j}+\mathbf{k})$ (c) $\frac{1}{\sqrt{3}}(\mathbf{i}-\mathbf{j}+\mathbf{k})$ (d) None of these

391. Given that the vectors **a** and **b** are non-collinear, the values of x and y for which the vector equality $2\mathbf{u}-\mathbf{v}=\mathbf{w}$ holds true if $\mathbf{u}=x\mathbf{a}+2y\mathbf{b},\ \mathbf{v}=-2y\mathbf{a}+3x\mathbf{b},\ \mathbf{w}=4\mathbf{a}-2\mathbf{b}$ are

(a) $x=\frac{4}{7}, y=\frac{6}{7}$ (b) $x=\frac{10}{7}, y=\frac{4}{7}$ (c) $x=\frac{8}{7}, y=\frac{2}{7}$ (d) $x=2, y=3$

392. If $\mathbf{a}\times\mathbf{b}=\mathbf{c}\times\mathbf{d}$ and $\mathbf{a}\times\mathbf{c}=\mathbf{b}\times\mathbf{d}$, then

(a) $(\mathbf{a}-\mathbf{d})=\lambda(\mathbf{b}-\mathbf{c})$ (b) $\mathbf{a}+\mathbf{d}=\lambda(\mathbf{b}+\mathbf{c})$ (c) $(\mathbf{a}-\mathbf{b})=\lambda(\mathbf{c}+\mathbf{d})$ (d) None of these

393. If $\mathbf{r}.\mathbf{a}=\mathbf{r}.\mathbf{b}=\mathbf{r}.\mathbf{c}=0$ where **a, b, c** are non-coplanar, then

(a) $\mathbf{r}\perp\mathbf{c}\times\mathbf{a}$ (b) $\mathbf{r}\perp\mathbf{a}\times\mathbf{b}$ (c) $\mathbf{r}\perp\mathbf{b}\times\mathbf{c}$ (d) $\mathbf{r}=0$

394. If **i, j, k** are unit orthonormal vectors and **a** is a vector, if $\mathbf{a}\times\mathbf{r}=\mathbf{j}$, then $\mathbf{a}.\mathbf{r}$ is **[EAMCET 1990]**

(a) 0 (b) 1 (c) – 1 (d) Arbitrary scalar

395. $\mathbf{a}\times\mathbf{b}=\mathbf{a}\times\mathbf{c}$ where $(\mathbf{a}\neq 0)$ implies that **[Kurukshetra CEE 1996]**

(a) **b** = **c** (b) **a** and **b** are parallel

(c) **a, b, c** are mutually perpendicular (d) **a, b, c** are coplanar

396. The scalars l and m such that $l\mathbf{a}+m\mathbf{b}=\mathbf{c}$, where **a, b** and **c** are given vectors, are equal to

(a) $l=\frac{(\mathbf{c}\times\mathbf{b}).(\mathbf{a}\times\mathbf{b})}{(\mathbf{a}\times\mathbf{b})^2}, m=\frac{(\mathbf{c}\times\mathbf{a}).(\mathbf{b}\times\mathbf{a})}{(\mathbf{b}\times\mathbf{a})^2}$ (b) $l=\frac{(\mathbf{c}\times\mathbf{b}).(\mathbf{a}\times\mathbf{b})}{(\mathbf{a}\times\mathbf{b})}, m=\frac{(\mathbf{c}\times\mathbf{a}).(\mathbf{b}\times\mathbf{a})}{(\mathbf{b}\times\mathbf{a})}$

(c) $l=\frac{(\mathbf{c}\times\mathbf{b})\times(\mathbf{a}\times\mathbf{b})}{(\mathbf{a}\times\mathbf{b})^2}, m=\frac{(\mathbf{c}\times\mathbf{a})\times(\mathbf{b}\times\mathbf{a})}{(\mathbf{b}\times\mathbf{a})}$ (d) None of these

397. If **a** is a vector perpendicular to the vectors $\mathbf{b}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $\mathbf{c}=-2\mathbf{i}+4\mathbf{j}+\mathbf{k}$ and satisfies the condition $\mathbf{a}.(\mathbf{i}-2\mathbf{j}+\mathbf{k})=-6$, then **a** =

(a) $5\mathbf{i}+\frac{7}{2}\mathbf{j}-4\mathbf{k}$ (b) $10\mathbf{i}+7\mathbf{j}-8\mathbf{k}$ (c) $5\mathbf{i}-\frac{7}{2}\mathbf{j}+4\mathbf{k}$ (d) None of these

398. If **a** = (1, – 1, 1) and **c** = (– 1, – 1, 0), then the vector **b** satisfying $\mathbf{a}\times\mathbf{b}=\mathbf{c}$ and $\mathbf{a}.\mathbf{b}=1$ is **[MP PET 1989]**

(a) (1, 0, 0) (b) (0, 0, 1) (c) (0, – 1, 0) (d) None of these

399. If **a** = (1, 1, 1), **c** = (0, 1, – 1) are two vectors and **b** is a vector such that $\mathbf{a}\times\mathbf{b}=\mathbf{c}$ and $\mathbf{a}.\mathbf{b}=3$, then **b** is equal to

[IIT 1985, 1991]

(a) $\left(\frac{2}{3},\frac{2}{3},\frac{5}{3}\right)$ (b) $\left(\frac{2}{3},\frac{5}{3},\frac{2}{3}\right)$ (c) (5, 2, 2) (d) $\left(\frac{5}{3},\frac{2}{3},\frac{2}{3}\right)$

400. If $\mathbf{a}=2\mathbf{i}+\mathbf{k}, \mathbf{b}=\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=4\mathbf{i}-3\mathbf{j}+7\mathbf{k}$. If $\mathbf{d}\times\mathbf{b}=\mathbf{c}\times\mathbf{b}$ and $\mathbf{d}.\mathbf{a}=0$, then **d** will be **[IIT 1990]**

(a) $\mathbf{i}+8\mathbf{j}+2\mathbf{k}$ (b) $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ (c) $-\mathbf{i}+8\mathbf{j}-\mathbf{k}$ (d) $-\mathbf{i}-8\mathbf{j}+2\mathbf{k}$

401. If $\mathbf{a}.\mathbf{b}=\mathbf{a}.\mathbf{c},\ \mathbf{a}\times\mathbf{b}=\mathbf{a}\times\mathbf{c}$ and $\mathbf{a}\neq 0$, then **[Rajasthan PET 1990]**

(a) **b** = **0** (b) $\mathbf{b}\neq\mathbf{c}$ (c) $\mathbf{b}=\mathbf{c}$ (d) None of these

402. If $\mathbf{x}.\mathbf{a}=0,\ \mathbf{x}.\mathbf{b}=0$ and $\mathbf{x}.\mathbf{c}=0$ for some non-zero vector **x**, then the true statement is **[IIT 1983; Karnataka CET 2002]**

(a) [**a b c**] = 0 (b) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]\neq 0$ (c) [**a b c**] = 1 (d) None of these

403. A unit vector **a** makes an angle $\frac{\pi}{4}$ with z-axis. If $\mathbf{a}+\mathbf{i}+\mathbf{j}$ is a unit vector, then **a** is equal to **[IIT 1988]**

(a) $\frac{\mathbf{i}}{2}+\frac{\mathbf{j}}{2}+\frac{\mathbf{k}}{\sqrt{2}}$ (b) $\frac{\mathbf{i}}{2}+\frac{\mathbf{j}}{2}-\frac{\mathbf{k}}{\sqrt{2}}$ (c) $-\frac{\mathbf{i}}{2}-\frac{\mathbf{j}}{2}+\frac{\mathbf{k}}{\sqrt{2}}$ (d) None of these

404. If $[3\mathbf{a}+5\mathbf{b}\ \mathbf{c}\ \mathbf{d}]=p[\mathbf{a}\ \mathbf{c}\ \mathbf{d}]+q[\mathbf{b}\ \mathbf{c}\ \mathbf{d}]$, then $p+q=0$

(a) 8 (b) – 8 (c) 2 (d) 0

Advance Level

405. Given the following simultaneous equations for vectors **x** and **y**

$\mathbf{x}+\mathbf{y}=\mathbf{a}$ ....(i) $\mathbf{x}\times\mathbf{y}=\mathbf{b}$ ....(ii) $\mathbf{x}.\mathbf{a}=1$ ....(iii). Then **x** =............., **y** = ... **[Roorkee 1994]**

(a) **a**, **a** – **x** (b) **a** – **b**, **b** (c) **b**, **a** – **b** (d) None of these

406. $\mathbf{r}\times\mathbf{a}=\mathbf{b}\times\mathbf{a}$; $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$; $\mathbf{a}\neq 0$; $\mathbf{b}\neq 0$; $\mathbf{a}\neq\lambda\mathbf{b}$, **a** is not perpendicular to **b,** then **r** = **[EAMCET 1993]**

(a) **a** – **b** (b) **a** +**b** (c) $\mathbf{a}\times\mathbf{b}+\mathbf{a}$ (d) $\mathbf{a}\times\mathbf{b}+\mathbf{b}$

407. Let $\mathbf{a}=2\mathbf{i}+\mathbf{j}-2\mathbf{k}$ and $\mathbf{b}=\mathbf{i}+\mathbf{j}$. If **c** is a vector such that $\mathbf{a}.\mathbf{c}=|\mathbf{c}|$, $|\mathbf{c}-\mathbf{a}|=2\sqrt{2}$ and the angle between $(\mathbf{a}\times\mathbf{b})$ and **c** is 30°, then $|(\mathbf{a}\times\mathbf{b})\times\mathbf{c}|=$ **[IIT 1999]**

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 2 (d) 3

408. Let $\mathbf{a}=2\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{b}=\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and a unit vector **c** be coplanar. If **c** is perpendicular to **a,** then **c** = **[IIT 1999]**

(a) $\frac{1}{\sqrt{2}}(-\mathbf{j}+\mathbf{k})$ (b) $\frac{1}{\sqrt{3}}(-\mathbf{i}-\mathbf{j}-\mathbf{k})$ (c) $\frac{1}{\sqrt{5}}(\mathbf{i}-2\mathbf{j})$ (d) $\frac{1}{\sqrt{3}}(\mathbf{i}-\mathbf{j}-\mathbf{k})$

409. Let **a** and **b** be two non-collinear unit vectors. If $\mathbf{u}=\mathbf{a}-(\mathbf{a}.\mathbf{b})\mathbf{b}$ and $\mathbf{v}=\mathbf{a}\times\mathbf{b}$, then $|\mathbf{v}|$ is **[IIT 1999]**

(a) $|\mathbf{u}|$ (b) $|\mathbf{u}|+|\mathbf{u}.\mathbf{a}|$ (c) $|\mathbf{u}|+|\mathbf{u}.\mathbf{b}|$ (d) $|\mathbf{u}|+\mathbf{u}.(\mathbf{a}+\mathbf{b})$

410. Let **a, b, c** be three vectors such that $\mathbf{a}\neq\mathbf{0}$ and $\mathbf{a}\times\mathbf{b}=2\mathbf{a}\times\mathbf{c}$, $|\mathbf{a}|=|\mathbf{c}|=1$, $|\mathbf{b}|=4$ and $|\mathbf{b}\times\mathbf{c}|=\sqrt{15}$. If $\mathbf{b}-2\mathbf{c}=\lambda\mathbf{a}$, then λ equal to **[Orissa JEE 2004]**

(a) 1 (b) – 4 (c) 4 (d) – 2

411. Unit vectors **a**, **b** and **c** are coplanar. A unit vector **d** is perpendicular to them. If $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})=\frac{1}{6}\mathbf{i}-\frac{1}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}$ and the angle between **a** and **b** is 30°, then **c** is **[Roorkee Qualifying 1998]**

(a) $\frac{(\mathbf{i}-2\mathbf{j}+2\mathbf{k})}{3}$ (b) $\frac{(2\mathbf{i}+\mathbf{j}-\mathbf{k})}{3}$ (c) $\frac{(-\mathbf{i}+2\mathbf{j}-2\mathbf{k})}{3}$ (d) $\frac{(-\mathbf{i}+2\mathbf{j}+\mathbf{k})}{3}$

412. If vectors **a, b, c** satisfy the condition $|\mathbf{a}-\mathbf{c}|=|\mathbf{b}-\mathbf{c}|$, then $(\mathbf{b}-\mathbf{a}).\left(\mathbf{c}-\frac{\mathbf{a}+\mathbf{b}}{2}\right)$ is equal to **[AMU 1999]**

(a) 0 (b) – 1 (c) 1 (d) 2

413. Let **r** be a vector perpendicular to $\mathbf{a}+\mathbf{b}+\mathbf{c}$, where $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=2$. If $\mathbf{r}=l(\mathbf{b}\times\mathbf{c})+m(\mathbf{c}\times\mathbf{a})+n(\mathbf{a}\times\mathbf{b})$, then $l+m+n$ is

(a) 2 (b) 1 (c) 0 (d) None of these

414. Let **a**, **b** and **c** be three vectors having magnitudes 1, 1 and 2 respectively. If $\mathbf{a}\times(\mathbf{a}\times\mathbf{c})+\mathbf{b}=\mathbf{0}$, the acute angle between **a** and **c** is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) None of these

415. If $\mathbf{a}=\mathbf{i}+\mathbf{j}-\mathbf{k}, \mathbf{b}=\mathbf{i}-\mathbf{j}+\mathbf{k}$ and **c** is a unit vector perpendicular to the vector **a** and coplanar with **a** and **b**, then a unit vector **d** perpendicular to both **a** and **c** is

(a) $\frac{1}{\sqrt{6}}(2\mathbf{i}-\mathbf{j}+\mathbf{k})$ (b) $\frac{1}{\sqrt{2}}(\mathbf{j}+\mathbf{k})$ (c) $\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ (d) $\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{k})$

416. If **a** is perpendicular to **b** and **r** is a non-zero vector such that $p\mathbf{r}+(\mathbf{r}.\mathbf{b})\mathbf{a}=\mathbf{c}$, then $\mathbf{r}=$

(a) $\frac{\mathbf{c}}{p}-\frac{(\mathbf{b}.\mathbf{c})\mathbf{a}}{p^2}$ (b) $\frac{\mathbf{a}}{p}-\frac{(\mathbf{c}.\mathbf{a})\mathbf{b}}{p^2}$ (c) $\frac{\mathbf{b}}{p}-\frac{(\mathbf{a}.\mathbf{b})\mathbf{c}}{p^2}$ (d) $\frac{\mathbf{c}}{p^2}-\frac{(\mathbf{b}.\mathbf{c})\mathbf{a}}{p}$

417. Given three vectors **a, b, c** such that $\mathbf{b}.\mathbf{c}=3,\ \mathbf{a}.\mathbf{c}=\frac{1}{3}$. The vector **r** which satisfies $\mathbf{r}\times\mathbf{a}=\mathbf{b}\times\mathbf{a}$ and $\mathbf{r}.\mathbf{c}=0$ is

(a) $\mathbf{b}+9\mathbf{a}$ (b) $\mathbf{a}+9\mathbf{b}$ (c) $\mathbf{b}-9\mathbf{a}$ (d) None of these

418. If $\mathbf{a}=\mathbf{i}+\mathbf{j}$ and $\mathbf{b}=2\mathbf{i}-\mathbf{k}$ are two vectors ,then the point of intersection of two lines $\mathbf{r}\times\mathbf{a}=\mathbf{b}\times\mathbf{a}$ and $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$ is

[IIT 1992; Rajasthan PET 2000]

(a) $\mathbf{i}+\mathbf{j}-\mathbf{k}$ (b) $\mathbf{i}-\mathbf{j}+\mathbf{k}$ (c) $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ (d) $3\mathbf{i}-\mathbf{j}+\mathbf{k}$

419. A line passes through the points whose position vectors are $\mathbf{i}+\mathbf{j}-2\mathbf{k}$ and $\mathbf{i}-3\mathbf{j}+\mathbf{k}$. The position vector of a point on it at a unit distance from the first point is

(a) $\frac{1}{5}(5\mathbf{i}+\mathbf{j}-7\mathbf{k})$ (b) $\frac{1}{5}(5\mathbf{i}+9\mathbf{j}-13\mathbf{k})$ (c) $\mathbf{i}-4\mathbf{j}+3\mathbf{k}$ (d) None of these

420. The projection of the vector $\mathbf{i}+\mathbf{j}+\mathbf{k}$ on the line whose vector equation is $\mathbf{r}=(3+t)\mathbf{i}+(2t-1)\mathbf{j}+3t\mathbf{k}$, t being the scalar parameter, is

(a) $\frac{1}{\sqrt{14}}$ (b) 6 (c) $\frac{6}{\sqrt{14}}$ (d) None of these

421. If $\mathbf{a}\times(\mathbf{a}\times\mathbf{b})=\mathbf{b}\times(\mathbf{b}\times\mathbf{c})$ and $\mathbf{a}.\mathbf{b}\neq 0$, then $[\mathbf{a}\ \ \mathbf{b}\ \ \mathbf{c}]$ is equal to

(a) 0 (b) 1 (c) 2 (d) None of these

422. If $\mathbf{c}=\mathbf{a}\times\mathbf{b}$ and $\mathbf{b}=\mathbf{c}\times\mathbf{a}$ then

(a) $\mathbf{a}.\mathbf{b}=\mathbf{c}^2$ (b) $\mathbf{c}.\mathbf{a}=\mathbf{b}^2$ (c) $\mathbf{a}\perp\mathbf{b}$ (d) $\mathbf{a}\parallel\ \mathbf{b}\times\mathbf{c}$

423. If **r** satisfies the equation $\mathbf{r}\times(\mathbf{i}+2\mathbf{j}+\mathbf{k})=\mathbf{i}-\mathbf{k}$, then for any scalar *m*, **r** is equal to

(a) $\mathbf{i}+m(\mathbf{i}+2\mathbf{j}+\mathbf{k})$ (b) $\mathbf{j}+m(\mathbf{i}+2\mathbf{j}+\mathbf{k})$

(c) $\mathbf{k}+m(\mathbf{i}+2\mathbf{j}+\mathbf{k})$ (d) $\mathbf{i}-\mathbf{k}+m(\mathbf{i}+2\mathbf{j}+\mathbf{k})$

424. If $\mathbf{a}=(-1,1,1)$ and $\mathbf{b}\equiv(2,0,1)$, then the vector $\vec{X}$ satisfying the conditions

(i) That it is coplanar with **a** and **b** (ii) That it is perpendicular to **b** (iii) That $\mathbf{a}.\vec{X}=7$ is

(a) $-3\mathbf{i}+4\mathbf{j}+6\mathbf{k}$ (b) $-\frac{3}{2}\mathbf{i}+\frac{5}{2}\mathbf{j}+3\mathbf{k}$ (c) $3\mathbf{i}+16\mathbf{j}-6\mathbf{k}$ (d) None of these

425. If the non-zero vectors **a** and **b** are perpendicular to each other, then the solution of the equation, $\mathbf{r}\times\mathbf{a}=\mathbf{b}$ is given by

(a) $\mathbf{r}=x\mathbf{a}+\frac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}|^2}$ (b) $\mathbf{r}=x\mathbf{b}-\frac{\mathbf{a}\times\mathbf{b}}{|\mathbf{b}|^2}$

(c) $\mathbf{r}=x(\mathbf{a}\times\mathbf{b})$ (d) $\mathbf{r}=x(\mathbf{b}\times\mathbf{a})$

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Answer Sheet

Vector Algebra *Assignment (Basic and Advance level)*

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
a	b	a	d	d	a	d	d	c	c	b	a	a	b	c	a	b	b	c	d
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
d	b	d	a	c	b	d	b	d	b	b	b	c	c	b	d	c	d	c	b
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
d	c	c	c	a	c	c	d	a	c	c	a	c	c	b	a	a	c	c	a,c
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
a,d	a	d	a	c	a	b	c	a	b	c	b	a	b	b	b	b	b	a	b
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
c	b	a	a	d	b	a	a	b	c	c	a	c	b	c	c	d	d	c	d
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
c	c	a	a	a,b,c,d	a	a	d	b	d	c	b	d	a	a	c	b	a	b	c
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
a	b	a	c	a	d	d	b	c	d	c	a	a	d	c	c	c	c	d	c
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
a	c	d	d	d	a	a	a	b	c	c	c	c	a	a	c	d	c	b	b
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
b	c	c	c	d	d	a	c	a,c,d	b	a	a	b	b	b	d	a	d	b	c
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
c	d	c	b	d	a	c	b	d	c	b	b	b	c	a	a	b	b	a	b
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
d	c	a	a	a	c	c	c	b	a	b	a	c	c	c	b	a	c	c	d
221	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**	**239**	**240**
c	a	d	a	d	c	b	a	c	a	c	c	a	c	b	b	b	b	a	d

241	**242**	**243**	**244**	**245**	**246**	**247**	**248**	**249**	**250**	**251**	**252**	**253**	**254**	**255**	**256**	**257**	**258**	**259**	**260**
d	c	a	c	b	a	b	b	c	c	b	c	a,b,c	b	c	b	b	c	b	c
261	**262**	**263**	**264**	**265**	**266**	**267**	**268**	**269**	**270**	**271**	**272**	**273**	**274**	**275**	**276**	**277**	**278**	**279**	**280**
d	a	a	d	c	c	c	a	d	d	b	c	a	c	d	c	b	a	b	b
281	**282**	**283**	**284**	**285**	**286**	**287**	**288**	**289**	**290**	**291**	**292**	**293**	**294**	**295**	**296**	**297**	**298**	**299**	**300**
d	a	b	c	a	b	d	d	c	c	c	b	b	c	d	a	c	d	c	c
301	**302**	**303**	**304**	**305**	**306**	**307**	**308**	**309**	**310**	**311**	**312**	**313**	**314**	**315**	**316**	**317**	**318**	**319**	**320**
a	c	d	c	d	b	d	a	a	c	d	a	d	a	c	b	c	b	a	a
321	**322**	**323**	**324**	**325**	**326**	**327**	**328**	**329**	**330**	**331**	**332**	**333**	**334**	**335**	**336**	**337**	**338**	**339**	**340**
c	a	b,c	a	d	b,d	c	c	b	d	a	b	a	b	d	d	a	d	a	b
341	**342**	**343**	**344**	**345**	**346**	**347**	**348**	**349**	**350**	**351**	**352**	**353**	**354**	**355**	**356**	**357**	**358**	**359**	**360**
d	b	b	a	b	b	b	b	a	a	a	b	a,b	b	b	d	c	c	d	b
361	**362**	**363**	**364**	**365**	**366**	**367**	**368**	**369**	**370**	**371**	**372**	**373**	**374**	**375**	**376**	**377**	**378**	**379**	**380**
a	b	d	c	a	c	a	a	d	a	a,b,c	d	c	d	d	b	b,c	b	a	a
381	**382**	**383**	**384**	**385**	**386**	**387**	**388**	**389**	**390**	**391**	**392**	**393**	**394**	**395**	**396**	**397**	**398**	**399**	**400**
a,b,c,d	b	c	b	a	a	a	a	a	a	b	b	d	d	a	a	a	b	d	d
401	**402**	**403**	**404**	**405**	**406**	**407**	**408**	**409**	**410**	**411**	**412**	**413**	**414**	**415**	**416**	**417**	**418**	**419**	**420**
c	a	c	a	d	b	b	a	a,c	b,c	a,c	a	c	c	a	a	c	c	a,b	c
421	**422**	**423**	**424**	**425**															
a	c,d	b	b	a															