

CBSE Board
Class XI Physics

Time: - 3

Marks: - 70 Marks

General Instructions

- (a) All questions are compulsory.
- (b) There are 29 questions in total. Questions 1 to 8 carry one mark each, questions 9 to 16 carry two marks each, questions 17 to 25 carry three marks each and questions 27 to 29 carry five marks each.
- (c) Question 26 is a value based question carrying four marks.
- (d) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all three questions of five marks each. You have to attempt only one of the given choices in such questions.
- (e) Use of calculator is not permitted.
- (f) You may use the following physical constants wherever necessary.

$$e = 1.6 \times 10^{-19} C$$

$$c = 3 \times 10^8 ms^{-1}$$

$$h = 6.6 \times 10^{-34} JS$$

$$\mu_o = 4\pi \times 10^{-7} NA^{-2}$$

$$k_B = 1.38 \times 10^{23} JK^{-1}$$

$$N_A = 6.023 \times 10^{23} / mole$$

$$m_n = 1.6 \times 10^{-27} kg$$

- 1. Express one parsec in terms of light year. (1)
- 2. While swimming, why does a person push the water backwards? (1)
- 3. Define angle of repose. (1)
- 4. A man whose mass is 75 kg walks up 10 steps, each 20 cm high, in 5 s. Find the power he develops. Take $g = 10 ms^{-2}$. (1)
- 5. Will it be correct to write the unit of torque as Joule? (1)
- 6. Plot a graph between the height (h) of a liquid in a capillary tube and the radius (r) of the tube. (1)
- 7. What is the relation between α , β and γ ; the coefficient of linear, superficial and cubical expansion respectively? (1)

8. Name the mode of transfer of energy in which there is no actual movement of matter along the direction of propagation. (1)

9. Force (F) and density (d) are related as

$$F = \frac{\alpha}{\beta + \sqrt{d}}$$

(i) then the dimensions of α are

(ii) then the dimensions of β are. (2)

10. Prove that the vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - \hat{j}$ are perpendicular to each other. (2)

11. What is the ratio of maximum range and height for a projectile for an angle at which range is maximum? (2)

12. A body constrained to move along the z-axis of a coordinate system is subjected to a constant force given by $F = -\hat{i} + 2\hat{j} + 3\hat{k}$ N where \hat{i}, \hat{j} and \hat{k} are unit vectors along x, y and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis? (2)

13. The equation of a transverse wave on a string is $y = 4 \sin 2\pi \left(\frac{t}{0.05} - \frac{x}{50} \right)$ with length in cm and time in seconds. Calculate the wave velocity. (2)

14. Calculate the acceleration produced in the earth when a stone of mass 6 kg falls on it. Mass of earth (M) = 6×10^{24} kg, and the acceleration due to gravity (g) = 9.8 ms^{-2} . (2)

15.

(a) Write an adiabatic relation between pressure and volume.

(b) Write an adiabatic relation between:

(i) Volume and temperature

(ii) Pressure and temperature. (2)

16. State two characteristics for a gas to be ideal or perfect. (2)

OR

A hollow sphere with a small hole at its bottom is filled with water. It is hung by a long thread and made to oscillate as the water flows out. How is the period of oscillation affected and why? (2)

17. A motor boat, with its engine on in running river and blown over by a horizontal wind is observed to travel at 20 km/hour in a direction 53° east of north. The velocity of the boat with its engine on in still water and blown over by the horizontal wind is 4 km/hr eastward and the velocity of the boat with its engine on over the running river, in the absence of wind is a 8 km/hr, due south. Calculate
- (a) the velocity of the boat in magnitude and direction over still water in the absence of wind.
- (b) the velocity of the wind in magnitude and direction. (3)

18. Derive a relation for the distance covered in n^{th} second by a uniformly accelerated body. (3)

19. A man weighing 60 kg is standing in a lift. Find his weight as recorded by the weighing machine when the lift.
- (a) moves upward with a uniform velocity of 5 ms^{-1} ,
- (b) moves upward with an acceleration of 2 ms^{-1} ,
- (c) moves downward with a uniform acceleration of 2 ms^{-1} ,
- (d) falls freely under gravity. (3)

20. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds.
- Electron mass = $9.11 \times 10^{-31} \text{ kg}$
- Proton mass = $1.67 \times 10^{-27} \text{ kg}$
- $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$ (3)

OR

A liquid is in streamlined flow through a tube of non-uniform cross-section. Prove that sum of its kinetic energy, pressure energy and potential energy per unit mass remains constant. (3)

21. A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant, (i) linear speed, (ii) angular speed, (iii) angular momentum, (iv) kinetic energy, (v) potential energy, (vi) total energy throughout its orbit?
- Neglect any mass loss of the comet when it comes very close to the sun. (3)

22.

- (i) Prove that the work done in stretching a wire = $\frac{1}{2} \times \text{tension} \times \text{extension}$.
- (ii) Prove that the work done per unit volume in stretching a wire for every type of strain = $\frac{1}{2} \times \text{stress} \times \text{strain}$. (3)

23. What does a heat engine do? Name two types of heat engines. Give one example of each. (3)

24. Two vessels of the same size are at the same temperature. One of them holds 1 kg of H_2 gas and the other holds 1 kg of N_2 gas.

- (i) Which of the vessels contains more molecules?
- (ii) Which of the vessels is under greater pressure and why?
- (iii) In which vessel is the average molecular speed greater? How many times is it greater? (3)

25. A sound wave travelling along a string is described by $y = 5 \times 10^{-3} \sin(80x - 3t)$.

Calculate

- (i) the amplitude
- (ii) the wavelength
- (iii) frequency of the wave. (3)

26.

(a) Savita was surprised to see oil spreading on to the surface of water and asked her mother to explain why oil spreads on to the surface of water. Her mother explained her daughter the reason behind it. By going through the explanation she thought of learning more about the other scientific phenomenon also. What qualities do you find in Savita?

(b) Oil spreads over the surface of water whereas water does not spread over the surface of oil. Why? (4)

27. What do you mean by the centripetal force? Derive an expression for it. (5)

OR

What is a projectile? Write the expressions for the time of flight, and maximum height for the projectile thrown upwards at an angle θ with the horizontal direction.

The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall? (5)

28. The displacement of a body is given to be proportional to the cube of time elapsed.

What is the nature of the acceleration of the body? Justify your answer.

A car accelerates from rest at a constant rate of α for some time; after which it decelerates at constant rate of β to come to rest. If the total time elapsed is T second.

- (a) Draw a velocity time graph for the motion.
- (b) Calculate maximum velocity attained in terms of α , β and T. (5)

OR

A train, standing in a station-yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with at a speed of 10 m s^{-1} . What are the frequency, wavelength, and speed of sound for an observer standing on the station's platform? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of 10 m s^{-1} ? The speed of sound in still air can be taken as 340 m s^{-1} . (5)

29. When a mass is suspended from two springs separately, the periods of vertical oscillations are T_1 and T_2 . Find the period when the same mass is suspended from two springs connected in series and in parallel. (5)

OR

Derive an expression for maximum speed a vehicle should have, to take a turn on a banked road. Hence deduce expression for angle of banking at which there is minimum wear and tear to the tyres of the vehicle. (5)

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Solution

1.

$$1 \text{ par sec} = 3.08 \times 10^{16} \text{ m}$$

$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$$

$$\therefore \frac{1 \text{ par sec}}{1 \text{ ly}} = \frac{3.08 \times 10^{16} \text{ m}}{9.46 \times 10^{15} \text{ m}}$$

$$\text{Or } 1 \text{ par sec} = \frac{3.26}{3.26} \text{ ly}$$

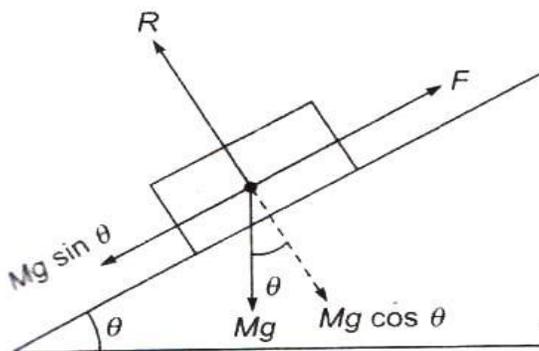
2.

While swimming, a person pushes the water backwards, and as a result he himself is pushed forward by the reaction force exerted by the water on him.

3.

The angle of repose is defined as the angle of the inclined plane at which a body placed on it, just begins to slide.

In the figure



$$R = Mg \cos \theta$$

$$F = Mg \sin \theta$$

$$\text{But } \mu = \frac{F}{R}$$

$$= \frac{Mg \sin \theta}{Mg \cos \theta} \text{ or } \mu = \tan \theta$$

Thus, the coefficient of limiting friction is equal to the tangent of the angle of repose.

4.

Mass, $m = 75 \text{ kg}$;

$$\text{Total height} = \left(\frac{10 \times 20}{100} \right) \text{m} = 2\text{m}$$

$$t = 5 \text{ s}$$

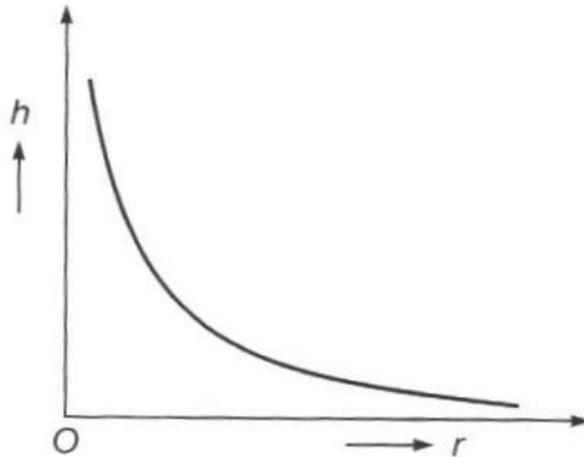
$$\begin{aligned} \therefore \text{Power} &= \frac{\text{Work}}{\text{Time}} = \frac{75 \times 10 \times 2}{5} \\ &= 300\text{W} \end{aligned}$$

5.

No. The correct unit of torque is newton-metre, and not joule. This is done to create a difference between the scalar nature of work and the vector nature of torque.

6.

The graph of h vs r is



7.

$$\alpha : \beta : \gamma :: 1 : 2 : 3$$

8.

Wave motion

9.

(i) $M^{3/2} L^{-1/2} T^{-2}$

(ii) $M^{1/2} L^{-3/2} T^0$

10.

$$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{B} = 2\hat{i} - \hat{j}$$

\vec{A} is perpendicular to \vec{B} , if $\vec{A} \cdot \vec{B} = 0$

$$\therefore (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (2\hat{i} - \hat{j}) = 2 - 2 = 0$$

Thus \vec{A} is perpendicular to \vec{B} .

11.

$$R = \frac{u^2 \sin 2\theta}{g}, R_{\max} = \frac{u^2}{g} [\text{At } \theta = 45^\circ]$$

$$H = \frac{u^2 \sin^2 \theta}{g} = \frac{u^2 (\sin 45^\circ)^2}{2g} = \frac{u^2}{4g}$$

$$\frac{R_{\max}}{H} = 4$$

12.

Work done is defined as the product of component of force along the direction of motion and displacement.

$$\vec{F} = (-\hat{i} + 2\hat{j} + 3\hat{k})$$

The body moves 4m along z-axis only.

$$\therefore \vec{S} = (0\hat{i} + 0\hat{j} + 4\hat{k})\text{m}$$

$$\text{Work done, } W = \vec{F} \cdot \vec{S}$$

$$= (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (0\hat{i} + 0\hat{j} + 4\hat{k})$$

$$= 12(\hat{k} \cdot \hat{k})\text{Nm}$$

$$= 12\text{J} \quad (\because \hat{k} \cdot \hat{k} = 1)$$

13.

$$v = \frac{\omega}{k}$$

$$= \frac{2\pi / 0.05}{2\pi / 50}$$

$$= 1000\text{cm/s} = 10\text{m/s}$$

14.

Let m be the mass of the stone.

$$a_s (= g) = Flm$$

$$\text{and } a_e = F/M$$

where M is the mass of the earth

$$\therefore \frac{a_e}{a_s} \frac{Flm}{F/M} = \frac{m}{M}$$

$$\begin{aligned} \text{Or } a_e &= \frac{m}{M} \times g = \frac{6 \times 9.8}{6 \times 10^{24}} \\ &= 9.8 \times 10^{-24} \text{ m s}^{-2} \end{aligned}$$

15.

(a) $PV^\gamma = \text{constant}$, where $\gamma = C_P/C_V$

(b)

(i) $TV^{\gamma-1} = \text{constant}$

(ii) $P^{\frac{1-\gamma}{\gamma}} T = \text{constant}$

or $P^{1-\gamma} T^\gamma = \text{constant}$

16.

An ideal or a perfect gas possesses the following two characteristics:

(i) The size of the molecules of the gas is zero, i.e. each molecule is only a point mass with no dimensions.

(ii) There is no force of attraction between the molecules.

OR

$$T = 2\pi \sqrt{\frac{l}{g}} \text{ where } l \text{ is effective length.}$$

Initially as the level of water drops, the effective length of the pendulum increases and this increases the time period. When the hollow sphere is completely empty, the effective length decreases due to rise in centre of gravity and the time period reduces to its original time period.

17.

\vec{V}_B = Velocity of boat in still water when wind is not blowing

\vec{V}_W = Velocity of wind

\vec{V}_R = Velocity of river with respect to ground

$$(a) \vec{V}_B + \vec{V}_W + \vec{V}_R = 20 \text{ km/h} = (20 \sin 53^\circ)\hat{i} + (20 \cos 53^\circ)\hat{j}$$

$$\vec{V}_B + \vec{V}_W = 4 \text{ km/h due east}$$

$$= 4\hat{i} + 0\hat{j}$$

$$\vec{V}_B + \vec{V}_R = 8 \text{ km/h due south} = 0\hat{i} - 8\hat{j}$$

$$(\vec{V}_B + \vec{V}_W) + (\vec{V}_B + \vec{V}_R) = (\vec{V}_B + \vec{V}_W + \vec{V}_R) + \vec{V}_B = 4\hat{i} - 8\hat{j}$$

$$(20 \sin 53^\circ)\hat{i} + (20 \cos 53^\circ)\hat{j} + \vec{V}_B = 4\hat{i} - 8\hat{j}$$

$$\vec{V}_B = (-11.97)\hat{i} - (20.03)\hat{j}$$

$$\text{i.e. } |\vec{V}_B| = \sqrt{(11.97)^2 + (20.03)^2}$$

$$= 23.32 \text{ km/h}$$

Direction is given as

$$\tan \theta = \frac{-20.03}{-11.97}$$

$$\theta = \tan^{-1}(1.67) = 59^\circ \text{ south of west}$$

$$(b) \vec{V}_W = \vec{V}_B + \vec{V}_W - \vec{V}_B$$

$$= [4\hat{i} + 0\hat{j}] - [(-11.97)\hat{i} - (20.03)\hat{j}] = (15.97)\hat{i} + (20.03)\hat{j}$$

$$|\vec{V}_W| = \sqrt{(15.97)^2 + (20.03)^2}$$

$$= 25.61 \text{ km/h}$$

Its direction is given as

$$\tan \theta = \tan^{-1}\left(\frac{20.03}{15.97}\right) = \tan^{-1}(1.25) = 51.34^\circ \text{ north of east}$$

18.

Let S_n and S_{n-1} be the distance covered in n and $(n-1)$ seconds respectively.

$$S_n = x(n) - x(0)$$

$$= v(0)n + \frac{1}{2}an^2 \quad (\because x(t) - x(0) = v(0)t + \frac{1}{2}at^2)$$

$$\text{and } S_{n-1} = x(n-1) - x(0)$$

$$= v(0)(n-1) + \frac{1}{2}a(n-1)^2$$

$$\text{But } S = S_n - S_{n-1}$$

$$= \left[v(0)n + \frac{1}{2}an^2 \right] - \left[v(0)(n-1) + \frac{1}{2}a(n-1)^2 \right]$$

$$= v(0)n + \frac{1}{2}an^2 - v(0)n + v(0) - \frac{1}{2}an^2 - \frac{a}{2} + an$$

$$= v(0) - \frac{a}{2} + an$$

$$\text{or } S = v(0) + \frac{a}{2}(2n-1)$$

19.

The weight recorded = Reaction on weighing machine

(a) When the lift moves upwards with a uniform velocity,

$$Ma = R - Mg; a = 0 \text{ for uniform velocity}$$

$$\therefore R = Mg = 60 \times 9.8$$

$$= 588 \text{ N}$$

\therefore the reading shown by the machine

$$= \frac{588}{9.8} = 60 \text{ kgf}$$

(b) When the lift is moving upwards with a uniform acceleration of 2 ms^{-2} , we have,

$$Ma = R - Mg$$

$$R = M(g + a)$$

$$= 60(9.8 + 2)$$

$$= 60 \times 11.8 \text{ N}$$

$$= 708 \text{ N}$$

The reading shown by the machine

$$= \frac{708}{9.8} = 72.24 \text{ kgf}$$

(c) When the lift moves downward with a uniform acceleration of 2ms^{-2} , we have

$$Ma = Mg - R$$

$$R = M(g - a)$$

$$= 60(9.8 - 2)$$

$$= 468\text{N}$$

The reading shown by the machine is

$$\frac{468}{9.8} = 47.8\text{kgf}$$

(d) When the lift falls freely, $a = g$

$$Ma = Mg - R$$

$$R = M(g - a)$$

$$= M(g - g)$$

$$R = 0$$

The reading shown by the machine is zero i.e. the body appears to be weightless.

20.

Kinetic energy of electron,

$$E_e = 10 \text{ keV} = 10^4 \times 1\text{eV}$$

$$= 10^4 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 1.6 \times 10^{-15} \text{ J}$$

Kinetic energy of proton,

$$E_p = 100 \text{ keV} = 10^5 \times 1\text{eV}$$

$$= 10^5 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 1.6 \times 10^{-14} \text{ J}$$

$$E_e = \frac{1}{2} m_e v_e^2$$

$$E_p = \frac{1}{2} m_p v_p^2$$

Where the symbols have their usual meanings.

$$\begin{aligned} \therefore v_e &= \sqrt{\frac{2E_e}{m_e}} \\ &= \sqrt{\frac{2 \times 1.6 \times 10^{-15}}{9.11 \times 10^{-31}}} \\ &= 5.93 \times 10^7 \text{ ms}^{-1} \end{aligned}$$

and

$$v_p = \sqrt{\frac{2E_p}{m_p}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-14}}{1.67 \times 10^{-27}}}$$

$$= 4.38 \times 10^6 \text{ ms}^{-1}$$

The required ratio $\frac{v_e}{v_p} = \frac{5.93 \times 10^7}{4.38 \times 10^6} = 13.5$

OR

$$(P_1 - P_2) Av \Delta t = Av \rho \Delta t g (h_2 - h_1) + \frac{1}{2} Av \Delta t \rho (v_2^2 - v_1^2)$$

$$\therefore P_1 - P_2 = \rho g (h_2 - h_1) + \frac{\rho}{2} (v_2^2 - v_1^2)$$

$$\text{(i.e.) } P_1 + \rho g h_1 + \frac{\rho}{2} v_1^2 = P_2 + \rho g h_2 + \frac{\rho}{2} v_2^2$$

$$\Rightarrow \frac{P_1}{\rho} + g h_1 + \frac{1}{2} v_1^2 = \frac{P_2}{\rho} + g h_2 + \frac{1}{2} v_2^2$$

$$\therefore \frac{P}{\rho} + g h + \frac{1}{2} v^2 = \text{constant}$$

21.

(i) No. The linear speed of the comet changes when it orbits the sun in a highly elliptical orbit, because the linear speed $v = R \omega$

(ii) No.

(iii) Yes, as the comet moves under the effect of a pure radial force.

(iv) No, when the comet is closer to the sun its kinetic energy increases because of the increase in its speed.

(v) No, as the distance keeps on varying from the sun, its potential energy also keeps on varying.

(vi) The total energy of the comet always remains constant. (3: ½ mark each point)

22.

(i) Let L be the original length of a wire of cross-section A and Young's modulus Y.

Let the extension in the wire be l.

$$\therefore Y = \frac{FL}{Al}$$

$$\text{or } F = \frac{YAl}{L}$$

Let dl be the further extension in the wire. Then small work done dW in the extension gets stored in the wire and raises its energy.

$$dW = F dl$$

$$= \frac{Y A l}{L} dl,$$

$$\text{or } \int_a^w dW = \frac{YA}{L} \int_b^l dl$$

$$\begin{aligned} \text{or } W &= \frac{YA}{L} \times \frac{l^2}{2} \\ &= \frac{1}{2} \times \frac{YA l}{L} \times l \end{aligned}$$

$$\text{or } W = \frac{1}{2} \times \text{Stretching force} \times \text{Extension}$$

$$\begin{aligned} \text{(ii) } \frac{\text{Work done}}{\text{volume}} &= \frac{1}{2} \frac{Fl}{AL} \\ &= \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L} \\ &= \frac{1}{2} \times \text{Stress} \times \text{Strain} \end{aligned}$$

23.

It is a device used for converting heat energy into mechanical energy.

- (i) External combustion engine: where fuel is burnt in a separate unit, outside the working portion of the engine, e.g. steam engine.
- (ii) Internal combustion engine: where fuel is burnt within the working portion of the engine, e.g petrol engine and diesel engine.

24.

(i) Hydrogen

As 2 g of hydrogen contains N molecules, 1kg of hydrogen contains

$$\frac{N}{2} \times 1000 = 500N \text{ molecules, where } N = 6.023 \times 10^{23}$$

In case of N_2 , 28 g of nitrogen contains N molecules.

Therefore, 1 kg of nitrogen contains

$$\frac{N}{28} \times 1000 = 36 N \text{ molecules}$$

(ii) Hydrogen

$$\text{As } P = \frac{1}{3} \frac{M}{V} c^2, P \propto C^2$$

Since M and V are the same in both the cases, $C_{H_2} > C_{N_2}$, therefore, the pressure exerted by hydrogen is more than that by nitrogen.

$$\text{(iii) } \frac{C_{H_2}}{C_{N_2}} = \sqrt{\frac{\rho_{N_2}}{\rho_{H_2}}} = \sqrt{\frac{14}{1}} = 3.74$$

$$\therefore C_{H_2} = 3.74 C_{N_2}$$

25.

(i) Amplitude, $A = 5 \times 10^{-3} m$

ii) $k = 80$; $k = \frac{2\pi}{\lambda}$ find $\lambda = \frac{\pi}{40} m$

iii) $\omega = 3$; $\omega = 2\pi f$ find $f = \frac{3}{2\pi} Hz$

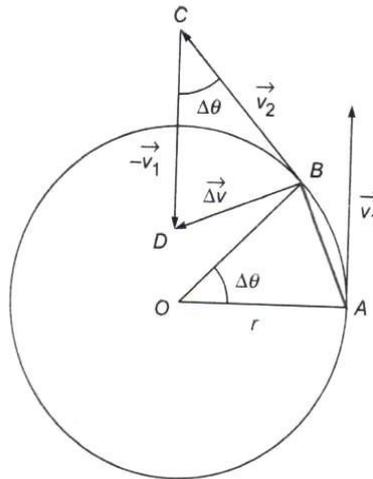
26.

(a) She has inquisitiveness; she wants know the scientific reason behind the various phenomena.

(b) The surface tension of the water is more than that of oil, therefore when oil is poured over water, the greater value of surface tension of water, pulls the oil in all directions. On the other hand, when water is poured over oil, it does not spread over it because surface tension of oil is less than that of water.

27.

Centripetal force is that force which is required to move a body in circular path with uniform speed. This force acts on the body along the radius and towards the centre.



Consider a body of mass m moving along a circular path of radius r. A and B denote the position of the body at times t and (t + Δt) and the angular displacement is $\Delta\theta$ in time Δt .

The velocities at A and B are represented by \vec{v}_1 and \vec{v}_2 respectively.

As the speed is uniform,

$$|\vec{v}_1| = |\vec{v}_2| = v$$

Change in velocity in going from A to B = $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

where $\vec{CD} = -\vec{v}_1$

From $\triangle BCD$, $\vec{BD} = \vec{BC} + \vec{CD}$

or $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

Thus, when the body moves from A to B the change in velocity is represented by \vec{BD}

From similar $\triangle AOB$ and $\triangle BCD$, $\frac{BD}{AB} = \frac{BC}{OA}$

$$\text{or } |\Delta\vec{v}| = \frac{AB \times BC}{OA} = \frac{r \Delta\theta \times v}{r} \quad \left[\because \Delta\theta = \frac{\widehat{AB}}{r} \approx \frac{AB}{r} \text{ and } |\vec{BC}| = BC = |\vec{v}_2| = v \right]$$

or $\Delta v = v \Delta\theta$

Dividing by Δt on both sides we get,

$$\frac{\Delta v}{\Delta t} = v \frac{\Delta\theta}{\Delta t}$$

$$\text{or } \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = v \left[\lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} \right]$$

$$\text{or } \frac{dv}{dt} = v\omega$$

$$\text{Or } a_c = v\omega$$

where a_c is called the centripetal acceleration.

But, $v = r\omega$

$$\therefore a_c = r\omega^2 = \frac{v^2}{r}$$

If F is the magnitude of the centripetal force,

$$F = ma_c = \frac{mv^2}{r} = mr\omega^2$$

$$\text{Since } \vec{F} = m\vec{a}_c = m \left[\lim_{\Delta t \rightarrow 0} \frac{\Delta\vec{v}}{\Delta t} \right]$$

The direction of the centripetal force is same as $\Delta\vec{v}$, when $\Delta t \rightarrow 0$.

When $\Delta t \rightarrow 0$, $\Delta\theta \rightarrow 0$ and $B \rightarrow A$

In such a case, $\Delta\vec{v}$ points along \vec{BO} , i.e. the radius of the circle.

Hence, \vec{F} points along the radius and towards the centre of the circle.

OR

A body thrown up in space and allowed to fall under the effect of gravity alone is called a projectile.

The expressions for maximum height and time of flight are:

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$T = \frac{2u \sin \theta}{g}$$

Given that speed of the ball, $u = 40 \text{ m/s}$

Maximum height, $H = 25 \text{ m}$

In projectile motion, the maximum height reached by a body projected at an angle θ , is given by the relation:

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$25 = \frac{(40)^2 \sin^2 \theta}{2 \times 9.8}$$

$$\sin^2 \theta = 0.30625$$

$$\sin \theta = 0.5534$$

$$\therefore \theta = \sin^{-1}(0.5534) = 33.60^\circ$$

Horizontal range, R

$$= \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{(40)^2 \times \sin 2 \times 33.60}{9.8}$$

$$= \frac{1600 \times \sin 67.2}{9.8}$$

$$= \frac{1600 \times 0.922}{9.8} = 150.53 \text{ m}$$

28.

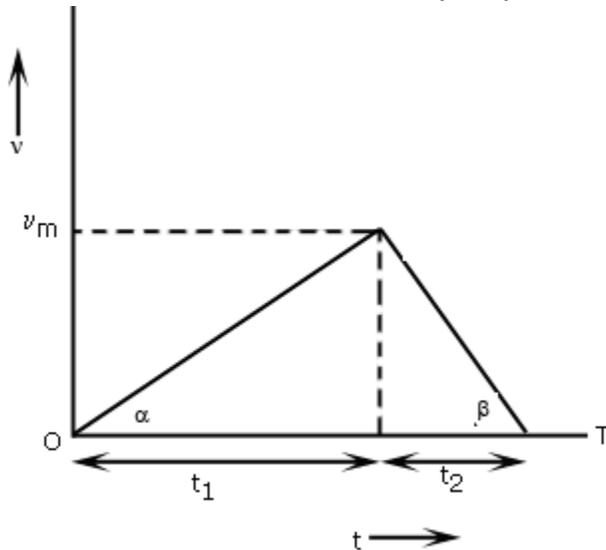
$$x \propto t^3$$

$$\Rightarrow x = kt^3$$

$$\Rightarrow v = \frac{dx}{dt} = 3kt^2$$

$$\Rightarrow a = \frac{dv}{dt} = 6kt$$

Therefore, acceleration is non-uniform ($a \propto t$)



Slope of $v-t$ graph = acceleration

$$\text{Therefore, } \alpha = \frac{v_m}{t_1}, \beta = \frac{v_m}{t_2}$$

$$\frac{1}{\alpha} = \frac{t_1}{v_m}, \frac{1}{\beta} = \frac{t_2}{v_m}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{t_1 + t_2}{v_m} = \frac{\alpha + \beta}{\alpha\beta}$$

$$v_m = \frac{(t_1 + t_2)\alpha\beta}{\alpha + \beta} = \frac{\alpha\beta T}{\alpha + \beta}$$

OR

For the stationary observer: 400 Hz; 0.875 m; 350 m/s

For the running observer: Not exactly identical

For the stationary observer:

Frequency of the sound produced by the whistle, $\nu = 400$ Hz

Speed of sound = 340 m/s

Velocity of the wind, $\nu = 10$ m/s

As there is no relative motion between the source and the observer, the frequency of the sound heard by the observer will be the same as that produced by the source, i.e., 400 Hz. The wind is blowing toward the observer. Hence, the effective speed of the sound increases by 10 units, i.e.,

Effective speed of the sound, $v_e = 340 + 10 = 350$ m/s

The wavelength (λ) of the sound heard by the observer is given by the relation:

$$\lambda = \frac{v_e}{\nu} = \frac{350}{400} = 0.875 \text{ m}$$

For the running observer:

Velocity of the observer, $v_o = 10$ m/s

The observer is moving toward the source. As a result of the relative motions of the source and the observer, there is a change in frequency (ν').

This is given by the relation:

$$\nu' = \left(\frac{v + v_o}{v} \right) \nu$$

$$= \left(\frac{340 + 10}{340} \right) \times 400 = 411.76 \text{ Hz}$$

Since the air is still, the effective speed of sound = $340 + 0 = 340$ m/s

The source is at rest. Hence, the wavelength of the sound will not change, i.e., λ remains 0.875 m.

Hence, the given two situations are not exactly identical.

29.

Let K_1 and K_2 be the spring constant of the two springs. Then,

$$T_1 = 2\pi \sqrt{\frac{m}{K_1}}$$

$$\text{and } T_2 = 2\pi \sqrt{\frac{m}{K_2}}$$

$$\therefore K_1 = \frac{4\pi^2 m}{T_1^2} \text{ and } K_2 = \frac{4\pi^2 m}{T_2^2}$$

When the springs are in series,

$$\begin{aligned}
T &= 2\pi \sqrt{\frac{m}{(K_1 K_2)}} \\
&= 2\pi \sqrt{\frac{m(K_1 + K_2)}{(K_1 K_2)}} \\
K_1 + K_2 &= 4\pi^2 m \left[\frac{1}{T_1^2} + \frac{1}{T_2^2} \right] \\
&= 4\pi^2 m \left(\frac{T_1^2 + T_2^2}{T_1^2 T_2^2} \right) \\
\text{and } K_1 K_2 &= \frac{16\pi^4 m^2}{T_1^2 T_2^2} \\
\therefore \frac{(K_1 + K_2)}{K_1 K_2} &= 4\pi^2 m \left(\frac{T_1^2 + T_2^2}{T_1^2 T_2^2} \right) \times \frac{T_1^2 T_2^2}{16\pi^4 m^2} \\
&= \frac{T_1^2 + T_2^2}{4\pi^2 m^2} \\
T &= 2\pi \sqrt{m \left(\frac{T_1^2 + T_2^2}{4\pi^2 m} \right)} \\
&= \sqrt{T_1^2 + T_2^2}
\end{aligned}$$

Similarly, when connected in parallel

$$T = 2\pi \sqrt{\frac{m}{K_1 + K_2}} = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$$

OR

From the forces acting on the vehicle on a banked curve,

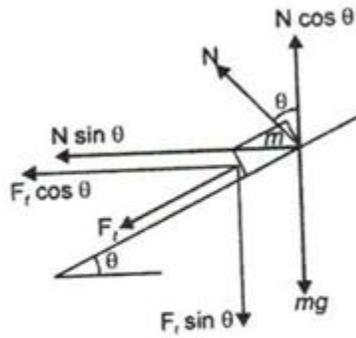
$$N \cos \theta - F_c \sin \theta = mg$$

$$N \sin \theta + F_c \cos \theta = \frac{m v^2}{r} \quad (F_c = \mu N)$$

Dividing the equation, we have,

$$\frac{v^2}{rg} = \frac{N \sin \theta + \mu N \cos \theta}{N \cos \theta - \mu N \sin \theta}$$

$$v^2 = rg \left[\frac{\tan \theta + \mu}{1 - \mu \tan \theta} \right] \text{ [dividing each term of right side by } N \cos \theta \text{]}$$



$$v = \sqrt{rg \left[\frac{\mu + \tan \theta}{1 - \mu \tan \theta} \right]}$$

If $\mu = 0$ i.e., banked road is perfectly smooth. Then from above

$$v_o = (rg \tan \theta)^{1/2}$$

$$v_o^2 = rg \tan \theta$$

Or $\tan \theta = \frac{v_o^2}{rg}$

$$\theta = \tan^{-1} \frac{v_o^2}{rg}$$