Chapter 1

Simple Stresses and Strains

CHAPTER HIGHLIGHTS

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INTRODUCTION

Simple Stresses and Strains

Strength: Resistance of a material to withstand external load.

Stiffness: Resistance of a material to withstand deformations.

NOTE

A strong material takes more loads at failure.
A stiff material undergoes less deformation at failure.

Assumptions for Calculating Strength of Materials

1. Material of a body is solid and continuous (no voids and no cracks).
2. Material is homogeneous and isotropic.

   Homogeneous: If a material has identical properties at all points in identical directions, it is called 'homogeneous'.

   Isotropic: If a material has identical elastic properties at a point in all directions, it is called 'isotropic'.
3. Self weight of material is ignored.
4. Superposition principle is valid.
5. Saint Venant's principle is valid.

STRESS

Stress: When a member is subjected to loads, resisting forces are developed.

Each member is in equilibrium under the action of the applied forces and the internal resisting forces. When a section of the member is considered, the intensity of the resisting force normal to the sectional plane is called 'the intensity of normal stress' or 'normal stress'.

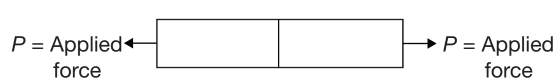
$$\text{Stress} = p = \lim_{\Delta A \to 0} \frac{\Delta R}{\Delta A} = \frac{dR}{dA}$$

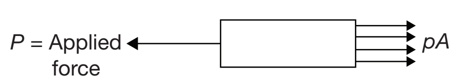
Where

R = Resisting force

A = Cross-sectional area

$$\therefore R = \int p dA, \text{ and } P = \text{Applied force}$$





NOTE

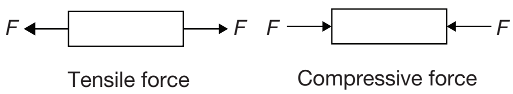
A member free to deform (or) free to move away will not develop any stress.

Units of stress: N/m^2 = Pascal

Types of Stresses

Direct Stresses

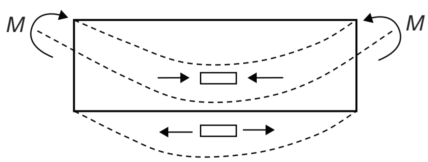
1. **Normal or direct stress:** The stress in a body due to a force perpendicular to the surface:



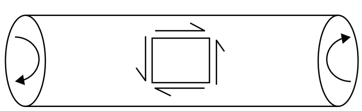
2. **Shear stress:** The tangential force per unit area, i.e., the force acting tangentially across the section.

Indirect Stresses

1. **Bending stress:** The stresses caused in the layers of a member due to bending phenomenon in a beam are called 'bending stresses'.



2. **Torsional shear stress:** Torsion or twisting of a member results in torsional shear stress.



NOTE

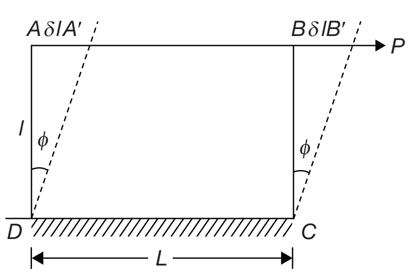
The main aim of Solid Mechanics is to determine stress resultant comprising of normal and tangential components only.

Types of Strain

Tensile strain, $e_t = \dfrac{\text{Increase in length}}{\text{Original length}}$

Compressive strain = $e_c = \dfrac{\text{Decrease in length}}{\text{Original length}}$

Shear strain is the angular deformation due to the shear forces.



Shear strain $\phi \cong \tan\phi = \dfrac{\delta l}{l}$

Volumetric strain = $e_v = \dfrac{\delta v}{v} = e_x + e_y + e_z$

= Sum of strains in the x, y, and z directions of the body.

Hooke's Law and Modulus of Elasticity

Hooke's law states that stress is proportional to strain up to the limit of proportionality in the elastic region, i.e., $p \propto e$

$$\therefore\ p = Ee,$$

Where, E = Constant of proportionality of the material

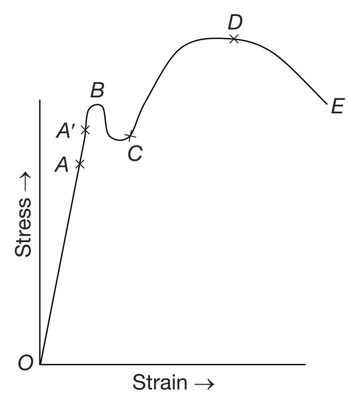
E is known as the modulus of elasticity or Young's modulus.

From Hooke's law:

$$E = \frac{p}{e} = \frac{\dfrac{P}{A}}{\dfrac{\delta L}{L}} = \frac{PL}{A\delta L}\text{, or } \delta L = \frac{PL}{AE}$$

Stress–Strain Relationship

The stress–strain relationship can be plotted by conducting a test on a specimen using the Universal Testing Machine (UTM). An extensometer can be used to measure the length variations.



In the initial portion OA, stress is directly proportional to strain. Point A is the limit of proportionality. Slightly beyond A, when the load is released, strain disappears completely and original length is regained.

This point (A') is called 'the elastic limit'. Point B is the upper yield point and point C is the lower yield point. The horizontal portion, before point C is called 'the yield plateau'.

After the point C, strain hardening occurs. Point D represents the ultimate stress which is the maximum stress the material can resist. Here, the process of necking begins. Point E is the breaking point, the stress at which the specimen fails.

In some materials, like aluminium and carbon steel, there are no specific yield points.

In brittle materials, there is no yield point. For these materials, the ultimate point and breaking point are same.

Factor of Safety

$$\text{Factor of safety} = \text{FS} = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

Factor of safety is used in a design process to avoid failures.
FS for steel = 1.85
FS for concrete = 3.00

Material Properties

1. **Ductile material:** It is a material which can undergo considerable deformation without rupture.
   Here, Duct = Wire
   Major portion of deformation is plastic.
   - Strong in tension
   - Moderate in compression
   - Weak in shear
2. **Malleability:** The plastic response (deformation) of a material to compressive force is known as malleability.
3. **Brittle material:** A material which fails suddenly without any plastic deformation is said to be a brittle material. It is:
   - Strong in compression
   - Moderate in shear
   - Weak in tension
4. **Elastic material:** If a material regains its original shape and size on removal of stress is said to be elastic material.
5. **Plastic material:** A material which undergoes permanent deformation without rupture is called 'plastic material'.

Elastic Constants

The elastic constants are modulus of elasticity (or Young's modulus), modulus of rigidity and bulk modulus.

Modulus of elasticity: It is already explained along with Hook's law as the ratio of linear stress to linear strain with in elastic limit. It is denoted by letter E.

Modulus of rigidity: Is the ratio of shear stress to shear strain with in elastic limit. It is denoted by letter G.
Therefore,

$$G = \frac{q}{\phi}$$

Where

q = Shear stress (some times denoted as t)

ϕ = Shear strain

As already explained, shear strain is the angular deformation due to shear forces.

Bulk modulus: It is the ratio of identical stresses p acting in three mutually perpendicular directions on a body to the corresponding volumetric strain e_v. It is denoted by the letter K.

Therefore, $K = \dfrac{p}{e_v}$

Where

$$e_v = \text{Volumetric strain}$$

$$= \frac{\Delta v}{V}$$

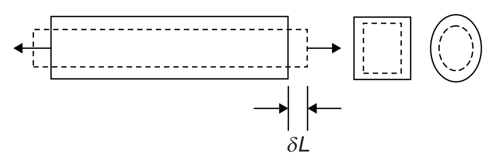
$$= \frac{\text{Change in volume}}{\text{Original volume}}$$

$$= e_x + e_y + e_z$$

Poisson's Ratio

When a force is applied, there is a change in the dimension in the direction of application of the load. A change in dimension will occur in the lateral direction also.

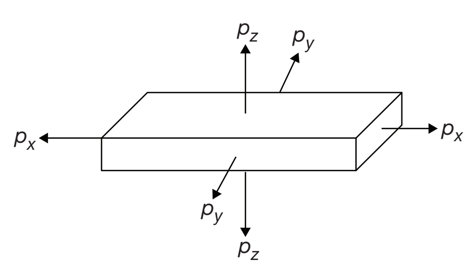
If there is an expansion axially due to force acting in that direction, there is a contraction laterally and vice versa.



Within elastic limit, there is a constant ratio between lateral strain and longitudinal strain. This is known as Poisson's ratio.

That is Poisson's ratio, $\dfrac{1}{m}$ or $\mu = \dfrac{\text{Lateral strain}}{\text{Linear strain}}$

Volumetric Strain of Rectangular Bar with Triaxial Loading



Let stresses p_x, p_y and p_z act on 3 mutually perpendicular directions x, y, z as shown in the given figure.

Change in length in the x direction is due to strain due to p_x and lateral strains due to p_y and p_z.

$$\frac{-\text{Lateral strain}}{\text{Linear strain}} = \mu$$

(Here, the '– sign' comes due to reduction in length)

Change in length due to p_x:

$$\delta l_1 = \frac{p_x}{E} \times l$$

Change in length due to p_y:

$$\delta l_2 = -\mu \frac{p_y}{E} l$$

Similarly, change in length due to p_z:

$$\delta l_3 = -\mu \frac{p_z}{E} l$$

Net change in length:

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{l}{E}(p_x - \mu p_y - \mu p_z)$$

$$e_x = \frac{\delta l}{l} = \frac{1}{E}(p_x - \mu p_y - \mu p_z)$$

Similarly, $e_y = \frac{1}{E}[-\mu p_x + p_y - \mu p_z]$

and $e_z = \frac{1}{E}[-\mu p_x - \mu p_y + p_z]$

Volumetric strain:

$$\frac{\delta V}{V} = e_v = e_x + e_y + e_z$$

$$= \frac{p_x}{E}(1-2\mu) + \frac{p_y}{E}(1-2\mu) + \frac{p_z}{E}(1-2\mu)$$

$$= (1-2\mu)\left(\frac{p_x + p_y + p_z}{E}\right)$$

In the case of uni-axial loading,

$$p_y = p_z = 0$$

$$e_v = \frac{p_x}{E}(1-2\mu)$$

When

$$p_x = p_y = p_z = p,$$

$$e_v = \frac{3p}{E}(1-2\mu)$$

From the definition of bulk modulus,

$$K = \frac{p}{e_v}$$

$$= \frac{p}{\frac{3p}{E}(1-2\mu)}$$

$$= \frac{E}{3(1-2\mu)} \quad \text{or} \quad E = 3K(1-2\mu).$$

The above equation reflects the relationship between modulus of elasticity and bulk modulus.

Another conclusion is, if $(p_x + p_y + p_z) = 0$, $\frac{dv}{v} = 0$

∴ There is no change in volume.

When $\mu = \frac{1}{2}$, $e_v = 0$ always.

Example: Rubber.

Relationship between Modulus of Elasticity and Modulus of Rigidity

E = Modulus of elasticity
G = Modulus of rigidity
μ = Poisson's ratio
$E = 2G(1 + \mu)$

Relationship between bulk modulus, modulus of elasticity and modulus of rigidity:

K = Bulk modulus

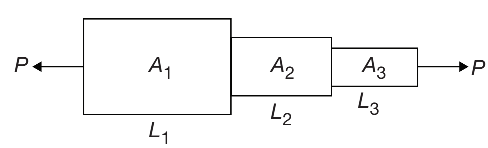
$$E = 3K(1-2\mu) = \frac{9KG}{G+3K} \Rightarrow \frac{9}{E} = \frac{3}{G} + \frac{1}{K}.$$

This is the relation connecting E, K and G.

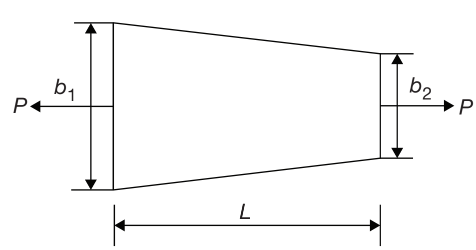
Bars of Varying Cross-sections

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3 + \ldots$$

$$= \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E} + \ldots$$



Bars with a Continuously Varying Cross-section (Width Vary from b1 to b2)



$$\delta L = \frac{PL}{tE(b_1 - b_2)} \log \frac{b_1}{b_2}$$

Where

t = Thickness

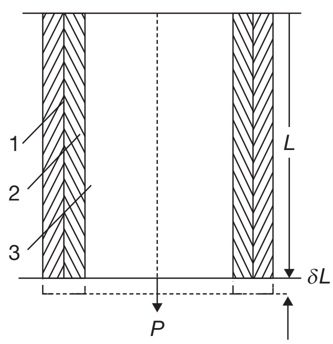
b = Breadth

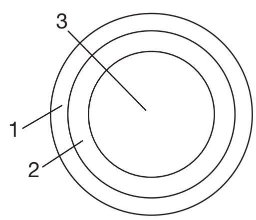
For a cylindrical rod, when the diameter changes from d_1 to d_2:

$$\delta L = \frac{4PL}{\pi E d_1 d_2}$$

Compound Bars

These consist of parts of different materials joined together and loaded commonly. Therefore, the elongation is same in all the materials.





$$\sigma_1 = \frac{P_1}{A_1}, \sigma_2 = \frac{P_2}{A_2}, \sigma_3 = \frac{P_3}{A_3}$$

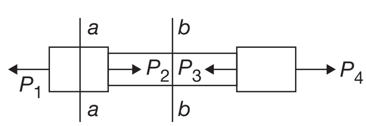
$$P = P_1 + P_2 + P_3 = \sigma_1 A_1 + \sigma_2 A_2 + \sigma_3 A_3$$

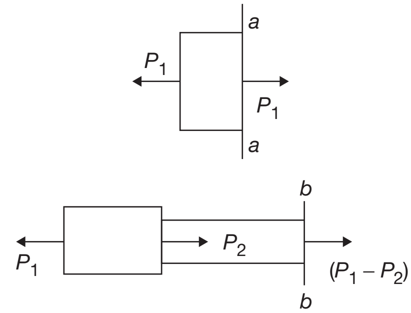
$$\text{Strain} = e = \frac{\delta L}{L} = \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} = \frac{\sigma_3}{E_3}$$

$$\therefore \frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2}, \frac{\sigma_1}{\sigma_3} = \frac{E_1}{E_3}, \frac{\sigma_2}{\sigma_3} = \frac{E_2}{E_3}$$

σ = Stress

Bars with varying loads: In this case, loads may vary from portion to portion. Loads acting on each portion are found out. By finding the elongation of each portion, the total elongation is found out.

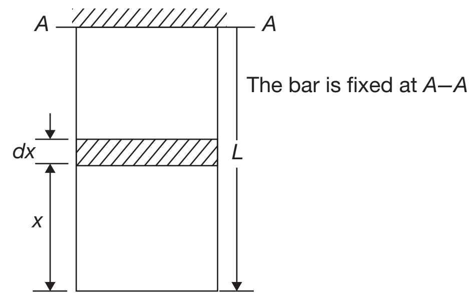




For equilibrium, $-P_1 + P_2 - P_3 + P_4 = 0$.

Elongation Due to Self Weight

1. **Bar of uniform cross-section:**



Weight below the elemental length $= wx \cdot A$

Where

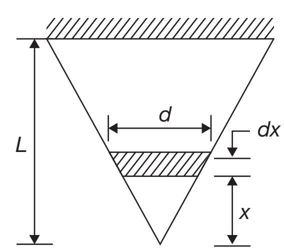
w = Specific weight

A = Area of cross-section

$$\text{Elongation of elemental length} = \frac{Pdx}{AE} = \frac{wxA \cdot dx}{AE} = \frac{wx}{E} dx$$

$$\text{Total elongation due to self weight} = \int_o^L \frac{w}{E} x dx = \frac{wL^2}{2E}$$

2. **Solid conical bar:**



Consider an element of length dx and diameter d at a distance x from the free end.

$$\text{Extension of } dx = \frac{Pdx}{AE}$$

$$= \frac{1}{3}\frac{w}{E} x dx \quad \left(\text{as } P = \frac{1}{3}\frac{\pi d^2 wx}{4} \text{ and } A = \frac{\pi d^2}{4} \right)$$

$$\text{Total extension} = \int_o^L \frac{1}{3}\frac{w}{E} x dx$$

$$= \frac{w}{3E}\left[\frac{x^2}{2}\right]_o^L = \frac{wL^2}{6E}.$$

Thermal Stresses

Materials expand on heating and contract on cooling. The change in dimension is found to be proportional to the length of the member, and also to the change in temperature, i.e., $\delta L = \alpha\, t\, L$

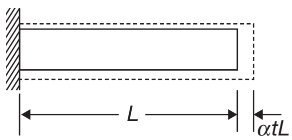
Where

t = Change in temperature,

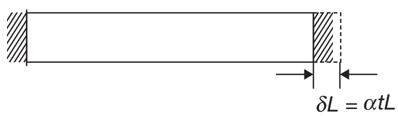
L = Length,

α = Constant of proportionality.

α is called the coefficient of linear thermal expansion.



No stresses will be developed if the bar is free to expand. But, if the free expansion is prevented, then thermal stresses will be developed.



The stress developed is compressive.

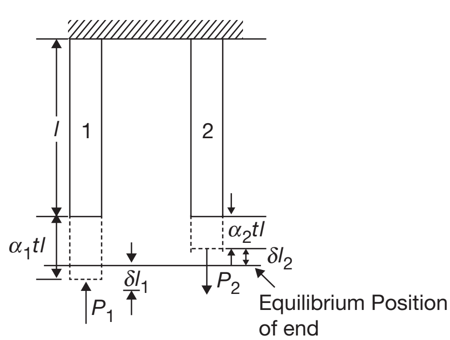
As $\delta L = \dfrac{PL}{AE}$

$\dfrac{PL}{AE} = \alpha tL$, i.e., $\dfrac{pl}{E} = l\alpha\, t$

$\therefore\ p = E \propto t$

where, $p = \dfrac{P}{A}$, the thermal stress.

Thermal Stresses in Compound Bars



Free expansion of bar 1 = $\alpha_1 tL$

Free expansion of bar 2 = $\alpha_2 tL$

As free expansion is prevented due to the compounding of the bars, the end of bars will have an equilibrium position as shown in the given figure and stresses will be developed in the bars.

At equilibrium condition, bar 1 shortens by δl_1 and bar 2 elongates by δl_2.

But, $P_1 = P_2 = P$

$$\alpha_2 tL + \delta l_2 = \alpha_1 tL - \delta l_1$$

$$\delta l_1 = \frac{PL}{A_1E_1};\ \delta l_2 = \frac{PL}{A_2E_2}$$

Solving for P, the stresses in the bars can be found out.

SOLVED EXAMPLES

Example 1

With a 30 m long steel tape and 15 mm × 0.8 mm cross-section a length was measured. The measured length was 120 m. During measurement, a force of 100 N more than the normal was applied. What is the actual length of the line? Modulus of elasticity = 2×10^5 N/mm^2.

Solution

Elongation of 30 m tape during measurement was

$$\delta L = \frac{PL}{AE}$$

$$= \frac{100 \times (30 \times 1000)}{(15 \times 0.8)(2 \times 10^5)} = 1.25\,\text{mm}$$

$\therefore$ If measured length is 30 m, the actual length is:

$$30 + \frac{1.25}{1000} = 30.00125\ \text{m}$$

If measured length is 120 m, actual length is

$$120 \times \frac{30.00125}{30} = 120.005\ \text{m}.$$

Example 2

A steel pipe is to be used to support a load of 150 kN. Pipes having outside diameter of 101.6 mm are available in different thicknesses of 3 mm, 3.5 mm, 3.65 mm, and 3.85 mm. Assuming a factor of safety of 1.8, choose the most economical thickness. (yield stress = 250 N/mm^2)

Solution

Permissible stress p

$$= \frac{250}{1.8} = 138.9 \text{ N/mm}^2; \ p = \frac{P}{A}$$

$$\therefore A = \frac{P}{p} = \frac{150}{138.9} \times 10^3 = 1080 \text{ mm}^2$$

$$A = \frac{\pi}{4}(D^2 - d^2) = 1080$$

$$D^2 - d^2 = 1080 \frac{x4}{\pi}$$

$$101 \cdot 6^2 - d^2 = 1375 \cdot 8$$

$$d^2 = 10322.56 - 1375.8 = 8946.76$$

$$d = 94.59 \text{ mm}$$

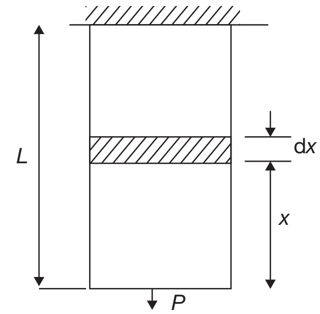
$$\therefore t = \frac{D-d}{2} = 3.505$$

∴ 3.65 mm thick pipe is sufficient.

Example 3

Find extension of a bar of length L and weight w/unit length having uniform cross-section area 'A' suspended from top, due to its self weight and a load 'P' applied at bottom. What is the extension, if P = weight of the bar.

Solution



Weight of the bar = wAL

Extension due to $P = \dfrac{PL}{AE}$

Extension of the bar due to self weight $= \dfrac{wL^2}{2E}$

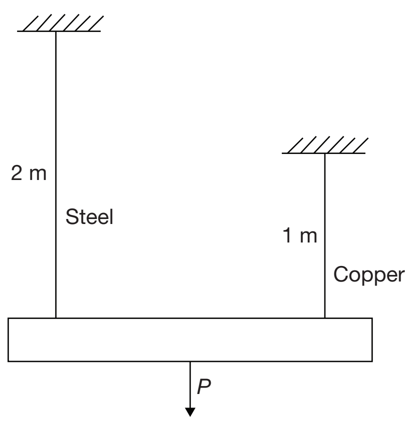
Total extension $= \dfrac{PL}{AE} + \dfrac{wL^2}{2E}$

when $P = wAL$,

Total extension $= \dfrac{3}{2}\dfrac{wL^2}{E}$.

Exercises

1. A rigid beam of negligible weight is supported in a horizontal position by two rods of steel and copper, 2 m and 1 m long having values of cross-sectional area 1 cm^2 and 2 cm^2 and E of 200 GPa and 100 GPa respectively. A load P is applied as shown in the figure below.



If the rigid beam is to remain horizontal then

(A) the forces on both sides should be equal.

(B) the forces on copper rod should be twice the force on steel.

(C) the force on the steel rod should be twice the force on copper.

(D) the force P must be applied at the centre of the beam.

2. The principle of superposition is made use of in structural computations when:

(A) The geometry of the structure changes by a finite amount during the application of the loads.

(B) The changes in the geometry of the structure during the application of the loads is too small and the strains in the structure are directly proportional to the corresponding stresses.

(C) The strains in the structure are not directly proportional to the corresponding stresses, even though the effect of changes in geometry can be neglected.

(D) None of these

3. The maximum value of Poisson's ratio for an elastic material is

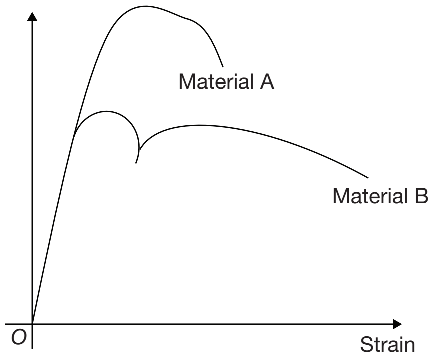
(A) 0.25 (B) 0.5

(C) 0.75 (D) 1.0

4. A cantilever beam of tubular section consists of 2 materials copper as outer cylinder and steel as inner cylinder.

It is subjected to a temperature rise of 20 C and $\alpha_{copper} > \alpha_{steel}$. The stresses developed in the tubes will be
(A) compression in steel and tension in copper.
(B) tension in steel and compression in copper.
(C) no stress in both.
(D) tension in both the materials.

5. The stress–strain diagram for two materials A and B is shown below:



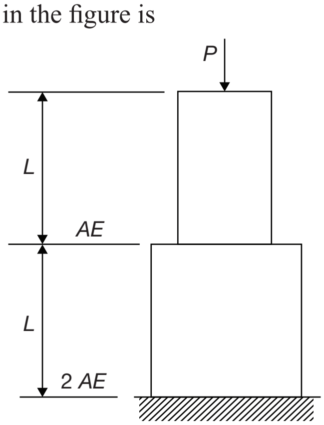
The following statements are made based on this diagram:

I. Material A is more brittle than material B.
II. The ultimate strength of material B is more than that of A.

With reference to the above statements, which of the following applies?
(A) Both the statements are false
(B) Both the statements are true
(C) I is true but II is false
(D) I is false but II is true

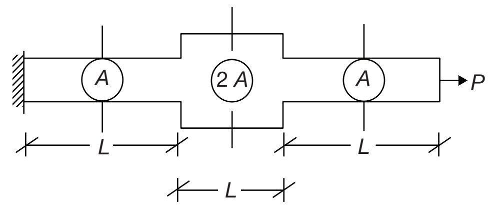
6. The shear modulus (G), modulus of elasticity (E) and the Poisson's ratio (μ) of a material are related as,
(A) $G = E/[2(1 + \mu)]$
(B) $E = G/[2(1 + \mu)]$
(C) $G = E/[2(1 - \mu)]$
(D) $G = E/[2(\mu - 1)]$

7. The axial movement of top surface of stepped column as shown in the figure is



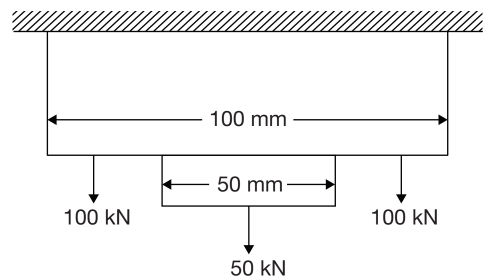
(A) 2.5 PL/AE (B) 3 PL/AE
(C) 1.5 PL/AE (D) 2 PL/AE

8. The total elongation of the structural element fixed, at one end, free at the other end, and of varying cross-section as shown in the figure when subjected to a force P at free end is given by



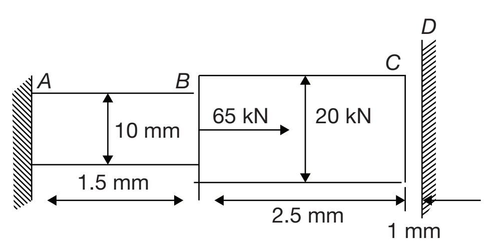
(A) PL/AE (B) 3 PL/AE
(C) 2.5 PL/AE (D) 2 PL/AE

9. A bar of varying square cross-section is loaded symmetrically as shown in the figure. Loads shown are placed on one of the axes of symmetry of cross-section. Ignoring self weight, the maximum tensile stress anywhere in N/mm^2 is



(A) 16.0 (B) 20.0
(C) 25.0 (D) 30.0

10. A steel bar ABC is placed on a smooth horizontal table is fixed at its left end A as shown in the figure. Its right end C is 1 mm away from another support D. A load of 65 kN is applied axially at the cross section B and acts from left to right. AB is 1 cm in diameter and 1.5 m long and BC is 2 cm in diameter and 2.5 m long. Young's modulus $E = 20 \times 10^6$ N/cm^2. The stress in portion BC will be



(A) 123 N/mm^2 (B) 228 N/mm^2
(C) 308 N/mm^2 (D) 417 N/mm^2

11. A bar length L tapers uniformly from diameter 1.1D at one end to 0.9D at the other end. The elongation due to axial pull is computed using mean diameter D. What is the approximate error in computed elongation?
(A) 10% (B) 5%
(C) 1% (D) 0.5%

12. In a tensile test, near the elastic limit zone
(A) tensile stress increases at a faster rate.
(B) tensile stress decreases at a faster rate.
(C) tensile stress increases in linear proportion to the stress.
(D) tensile stress decreases in linear proportion to the stress.

13. If Poisson's ratio of a material is 0.5, modulus of elasticity of the material is
(A) $\frac{1}{3}$ times the shear modulus.
(B) 3 times the shear modulus.
(C) 4 times the shear modulus.
(D) equal to the shear modulus.

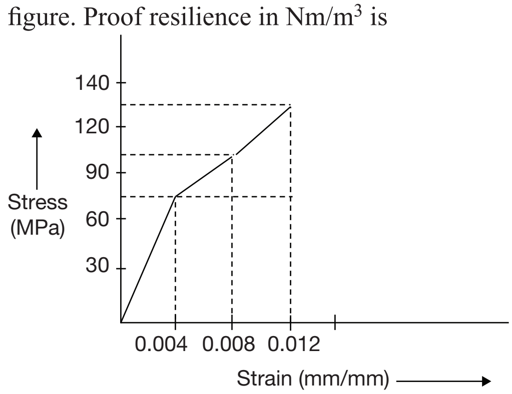
14. A 2 m long mild steel bar of 2000 mm^2 cross-sectional area is subjected to an axial load of 40 kN. If Young's modulus for the shaft is 2×10^5 N/mm^2, extension of the shaft in mm is
(A) 0.5 mm (B) 1 mm
(C) 0.2 mm (D) 2 mm

15. A steel bar of 1 m length is heated from 30°C to 60°C. Coefficient of linear expansion is 12×10^{-6}/°C and Young's modulus is 2×10^5 MN/m^2. Stress developed in the bar is
(A) 18 N/mm^2 (B) zero
(C) 36 N/mm^2 (D) 72 N/mm^2

16. Relationship between modulus of elasticity E, modulus of rigidity G and bulk modulus K is
(A) $E = \frac{6KG}{3K+G}$ (B) $E = \frac{9KG}{3K+G}$
(C) $E = \frac{3K+G}{6KG}$ (D) $E = \frac{3K+G}{9KG}$

17. A bar of 3 m in length 30 mm breadth and 20 mm thickness is subjected to a compressive stress of 50 kN/m^2. What will be the final volume of the bar if the Poisson's ratio is 0.30 and modulus of rigidity is 90 GN/m^2?
(A) Will increase by 0.4615 mm^3
(B) Will decrease by 0.5625 mm^3
(C) Will decrease by 0.4615 mm^3
(D) Will increase by 0.5625 mm^3

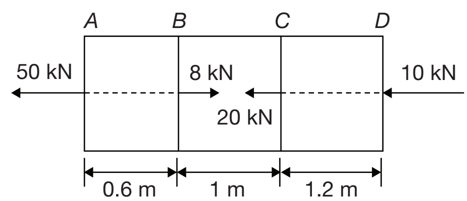
18. Stress–strain behaviour of a material is shown in the figure. Proof resilience in Nm/m^3 is



(A) 10×10^4 (B) 15×10^4
(C) 76×10^4 (D) 130×10^4

19. The extension of a circular bar tapering uniformly from diameter d_1 to d_2 is same as that of a uniform circular bar of same length, under same load. Diameter of the uniform bar is
(A) $\sqrt{d_1 d_2}$ (B) $\sqrt{d_1^2 - d_2^2}$
(C) $\frac{d_1 + d_2}{2}$ (D) $\frac{d_1 - d_2}{2}$

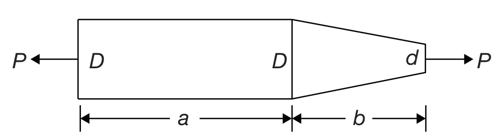
20. A brass bar having a cross-sectional area of 1000 mm^2 is subjected to axial forces as shown in the figure. The total change in length of the bar is.
Take $E = 1.05 \times 10^5$ N/mm^2.



(A) –0.15 mm (B) +0.15 mm
(C) –0.1143 mm (D) +0.1143 mm

21. A solid metal tube with modulus of elasticity E and Poisson's ratio μ is constrained on all faces. It is heated so that temperature rises uniformly. If coefficient of thermal expansion is α, the compressive stress developed in the cube due to the heating is
(A) $\frac{E\alpha\Delta T}{2(1-2\mu)}$ (B) $\frac{E\alpha\Delta T}{(1-2\mu)}$
(C) $\frac{2E\alpha\Delta T}{(1-2\mu)}$ (D) $\frac{E\alpha\Delta T}{3(1-2\mu)}$

22. A bar of length L, breadth b and thickness t is subjected to an axial pull of P. If e_x is the strain in the direction of pull, volumetric strain produced is (m = Poisson's ratio)
(A) $e_x(1 + 2\mu)$ (B) $e_x(1 - 2\mu)$
(C) $e_x(1 + \mu)$ (D) $e_x(1 - \mu)$

23.


A bar is having uniform diameter D for a length a and tapering diameter from D to d for a length b as shown in the figure. If the bar is subjected to an axial pull P, the extension produced is
(A) $\frac{4P}{\pi DE}\left(\frac{a}{D}+\frac{b}{d}\right)$ (B) $\frac{4P}{\pi dE}\left(\frac{a}{D}+\frac{b}{d}\right)$
(C) $\frac{2P}{\pi DE}\left(\frac{a}{D}+\frac{b}{d}\right)$ (D) $\frac{2P}{\pi dE}\left(\frac{a}{D}+\frac{b}{d}\right)$

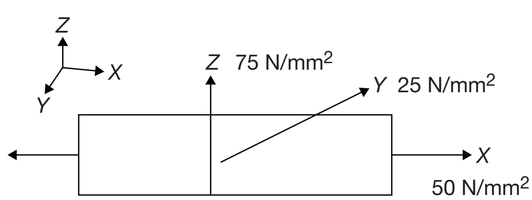
24. When a body is permanently deformed under a load, it is said to have undergone?
(A) Elastic deformation
(B) Limit of elastic deformation
(C) Plastic deformation
(D) Uniform deformation

25. A bar of uniform cross-section A and length L is suspended from top. If E is the Young's modulus and W the weight of the bar, extension produced due to self weight is
(A) $\dfrac{WL}{2AE}$ (B) $\dfrac{WL}{AE}$
(C) $\dfrac{3}{2}\dfrac{WL}{AE}$ (D) $\dfrac{WL}{3AE}$

26. When a load was suddenly applied on a bar of cross-section area A and Length L, stress produced is p. If e is the strain, produced strain energy stored is
(A) $\dfrac{1}{2}peAL$ (B) $peAL$
(C) $\dfrac{3}{2}peAL$ (D) $2peAL$

27. Coefficient of linear expansion of a solid is α. A cube of volume V of this solid is heated by 1°. Then change in volume of the cube is
(A) $V\alpha$ (B) $3V\alpha$
(C) 3α (D) $\dfrac{V\alpha}{3}$

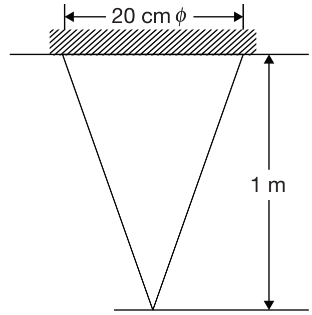
28. A steel bar 35 cm long, 6 cm × 6 cm in cross-section is subjected to the loading as shown in the figure.



If Young's modulus of elasticity is 150 kN/mm^2 and Poisson's ratio is 0.25, then what will be the strain in 'y;' direction?
(A) Elongation by 0.0413 mm
(B) Compression by 0.0413 microns
(C) Zero strain
(D) None of these

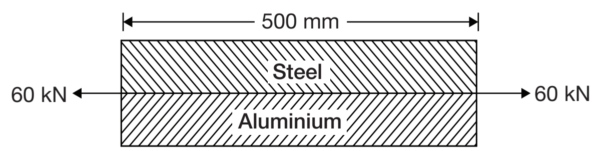
29. A bar of 3 m in length 30 mm breadth and 20 mm thickness is subjected to a compressive stress of 50 kN/m^2. What will be the final volume of the bar if the Poisson's ratio is 0.30 and modulus of rigidity is 90 GN/m^2.
(A) Will increase by 0.4615 mm^3
(B) Will decrease by 0.5625 mm^3
(C) Will decrease by 0.4615 mm^3
(D) Will increase by 0.5625 mm^3

30. A solid conical bar of uniformly varying cross-section is hung vertically as shown.



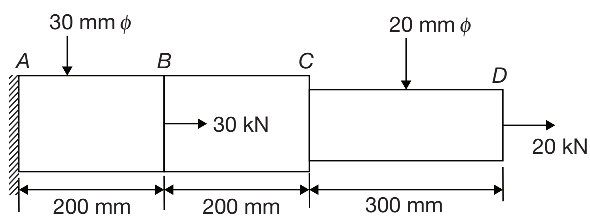
If specific weight is 80000 N/m^3 and modulus of elasticity is, $E = 2 \times 10^5$ N/mm^2, then extension of its length due to self weight is
(A) 6.67×10^{-5} mm
(B) 1.33×10^{-4} mm
(C) 1×10^{-4} mm
(D) 4.45×10^{-5} mm

31.



A compound bar of steel and aluminium of length 500 mm is subjected to an axial load of 60 kN. Area of cross-section of steel is 750 mm^2 and aluminium is 1000 mm^2. Modulus of elasticity of steel and aluminium are 2×10^5 N/mm^2 and 1×10^5 N/mm^2 respectively. Load shared by steel and aluminium is in the ratio
(A) 2.25 (B) 2.00
(C) 1.5 (D) 1.25

32.



In the composite bar shown above, stress in the portion BC is
(A) 22.64 N/mm^2
(B) 33.95 N/mm^2
(C) 28.3 N/mm^2
(D) 36.23 N/mm^2

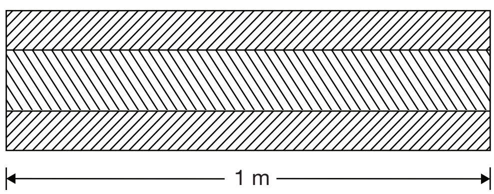
33. For elastic materials, ratio of Young's modulus to bulk modulus is given by
(A) $2(1 + \mu)$ (B) $3(1 - 2\mu)$
(C) $2(1 - \mu)$ (D) $3(1 + 2\mu)$

34. Two mild steel rods of same length are subjected to a force P. If the diameter of the second rod is twice that of the first rod and the stresses developed are in the limit of proportionality, the correct statement among the following is:

(A) The elongation of second rod is $\frac{1}{4}$ times that of first rod.
(B) The elongation of the second rod is half of the elongation of the first rod.
(C) The elongation of the first rod is $\frac{1}{4}$ times that of the second rod.
(D) Elongation of both rods are equal.

35. Modulus of rigidity and bulk modulus of a material are 0.5×10^5 MPa and 0.8×10^5 MPa respectively. Value of modulus of elasticity (in MPa) is
(A) 0.765×10^5 (B) 0.932×10^5
(C) 1.034×10^5 (D) 1.241×10^5

36.



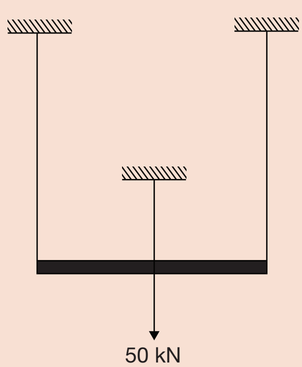
A compound bar is made with a brass rod of 30 mm outside diameter and a steel tube of 30 mm internal diameter and 50 mm outside diameter. Length of the bar is 1 m. Modulus of elasticity of steel and brass are 200 kN/mm^2 and 100 kN/mm^2 respectively. Coefficient of thermal expansion for steel and brass are 11.6×10^{-6}/°C and 18.7×10^{-6}/°C respectively. If the compound bar is heated from 30°C to 90°C, stress developed in the brass rod (in N/mm^2) is ______.

37. A steel rail of 12 m length was laid at a temperature of 24°C. Modulus of elasticity and coefficient of thermal expansion of the rail are 2×10^5 N/mm^2 and 12×10^{-6}/°C respectively. At a temperature of 42°C a stress of 18 N/mm^2 is developed at the joint. Gap between the rails (in mm) at 24°C is
(A) 1.232
(B) 1.368
(C) 1.512
(D) 1.747

PREVIOUS YEARS' QUESTIONS

1. A metal bar of length 100 mm is inserted between two rigid supports and its temperature is increased by 10°C. If the coefficient of thermal expansion is 12×10^{-6} per C and the Young's modulus is 2×10^5 MPa, the stress in the bar is **[GATE, 2007]**
(A) zero (B) 12 MPa
(C) 24 MPa (D) 2400 MPa

2. A rigid bar is suspended by three rods made of the same material as shown in the figure. The area and length of the central rod are $3A$ and L respectively, while that of the two outer rods are $2A$ and $2L$ respectively. If a downward force of 50 kN is applied to the rigid bar, the forces in the central and each of the outer rods will be **[GATE, 2007]**



(A) 16.67 kN each
(B) 30 kN and 15 kN
(C) 30 kN and 10 kN
(D) 21.4 kN and 14.3 kN

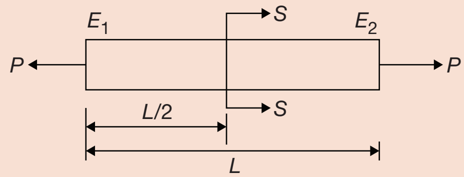
3. A rod of length L and diameter D is subjected to a tensile load P. Which of the following is sufficient to calculate the resulting change in diameter? **[GATE, 2008]**
(A) Young's modulus
(B) Shear modulus
(C) Poisson's ratio
(D) Both Young's modulus and shear modulus

4. The number of independent elastic constants for a linear elastic isotropic and homogeneous material is **[GATE, 2010]**
(A) 4 (B) 3
(C) 2 (D) 1

5. The Poisson's ratio is defined as **[GATE, 2012]**
(A) $\left|\frac{\text{Axial stress}}{\text{Lateral stress}}\right|$ (B) $\left|\frac{\text{Lateral strain}}{\text{Axial stress}}\right|$
(C) $\left|\frac{\text{Lateral stress}}{\text{Axial stress}}\right|$ (D) $\left|\frac{\text{Axial strain}}{\text{Lateral strain}}\right|$

6. A solid steel cube constrained on all six faces is heated so that the temperature rises uniformly by ΔT. If the thermal coefficient of the material is α, Young's modulus is E and the Poisson's ratio is υ, the thermal stress developed in the cube due to heating is **[GATE, 2012]**
(A) $-\frac{\alpha(\Delta T)E}{(1-2v)}$ (B) $-\frac{2\alpha(\Delta T)E}{(1-2v)}$
(C) $-\frac{3\alpha(\Delta T)E}{(1-2v)}$ (D) $-\frac{\alpha(\Delta T)E}{3(1-2v)}$

7. A rod of length L having uniform cross-sectional area A is subjected to a tensile force P as shown in the figure below. If the Young's modulus of the material varies linearly from E_1 to E_2 along the length of the rod, the normal stress developed at the section-SS is **[GATE, 2013]**



(A) $\frac{P}{A}$ (B) $\frac{P(E_1 - E_2)}{A(E_1 + E_2)}$

(C) $\frac{PE_2}{AE_1}$ (D) $\frac{PE_1}{AE_2}$

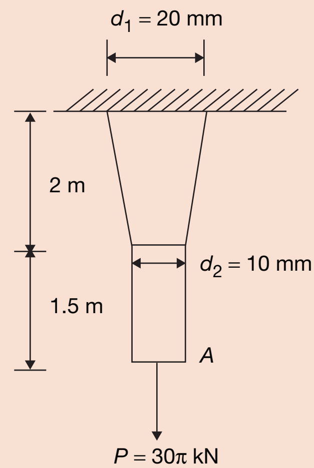
8. A 200 mm long, stress free rod at room temperature is held between two immovable rigid walls. The temperature of the rod is uniformly raised by 250°C. If the Young's modulus and coefficient of thermal expansion are 200 GPa and 1×10^{-5}/°C respectively, the magnitude of the longitudinal stress (in MPa) developed in the rod is ______. **[GATE, 2014]**

9. A steel cube, with all faces free to deform, has Young's modulus E, Poisson's ratio ϑ, and coefficient of thermal expansion α. The pressure (hydrostatic stress) developed within the cube, when it is subjected to a uniform increase in temperature ΔT, is given by **[GATE, 2014]**

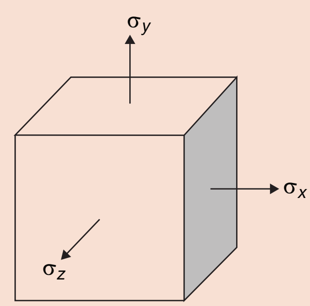
(A) 0 (B) $\frac{\alpha(\Delta T)E}{1-2\vartheta}$

(C) $-\frac{\alpha(\Delta T)E}{1-2\vartheta}$ (D) $\frac{\alpha(\Delta T)E}{3(1-2\vartheta)}$

10. A tapered circular rod of diameter varying from 20 mm to 10 mm is connected to another uniform circular rod of diameter 10 mm as shown in the following figure. Both bars are made of same material with the modulus of elasticity, $E = 2 \times 10^5$ MPa. When subjected to a load $P = 30\pi$ kN, the section at point A is ________ mm. **[GATE, 2015]**



11. An elastic isotropic body is in a hydrostatic state of stress as shown in the figure. For no change in the volume to occur, what should be its Poisson's ratio? **[GATE, 2016]**



(A) 0.00 (B) 0.25
(C) 0.50 (D) 1.00

Answer Keys

Exercises

1. B	**2.** B	**3.** B	**4.** B	**5.** C	**6.** A	**7.** C	**8.** C	**9.** C	**10.** A
11. C	**12.** B	**13.** B	**14.** C	**15.** B	**16.** B	**17.** C	**18.** B	**19.** A	**20.** C
21. B	**22.** B	**23.** A	**24.** C	**25.** A	**26.** B	**27.** B	**28.** B	**29.** C	**30.** A
31. C	**32.** C	**33.** B	**34.** A	**35.** D	**36.** 33 to 33.75		**37.** C		

Previous Years' Questions

1. C	**2.** C	**3.** D	**4.** C	**5.** B	**6.** A	**7.** A	**8.** 499 to 501	**9.** A
10. 15	**11.** C							