Mensuration

Exercise – 9.1

Solution 1:

Let the side of the equilateral triangle be a cm.

Then the area of the equilateral triangle = $\frac{\sqrt{3}}{4}a^2$ cm².

$\therefore \frac{\sqrt{3}}{4}a^2 = 36\sqrt{3}$

$\therefore a^2 = 36\sqrt{3} \times \frac{4}{\sqrt{3}}$

$\therefore a^2 = 36 \times 4$

$\therefore a = 12$cm

The perimeter of an equilateral triangle

$= 3a$

$= 3 \times 12$ cm

$= 36$ cm

The perimeter of the given equilateral triangle is 36 cm.

Solution 2:

Let the side of the equilateral triangle be a cm.

The area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$\therefore \frac{\sqrt{3}}{4}a^2 = 81\sqrt{3}$

$\therefore a^2 = 81\sqrt{3} \times \frac{4}{\sqrt{3}}$

$\therefore a^2 = 81 \times 4$

$\therefore a = 18 \text{ cm} \qquad \ldots(1)$

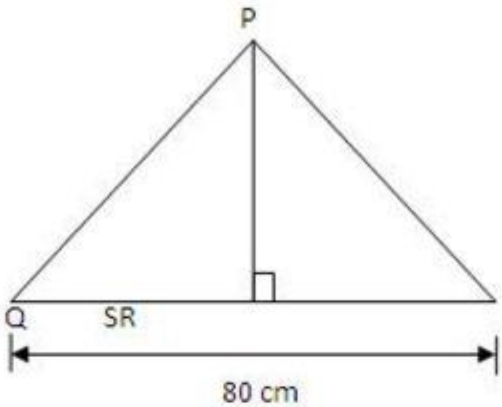
Now, the height (h) of an equilateral triangle = $\frac{\sqrt{3}}{2}a$.

$\therefore h = \frac{\sqrt{3}}{2} \times 18 \text{ cm} \qquad \ldots[\text{From}(1)]$

$= 9\sqrt{3}$ cm

The height of the given equilateral triangle is $9\sqrt{3}$cm.

Solution 3:



Let ΔPQR be an isosceles triangle with base QR = 80 cm.

PQ = PR and PS is the hieght.

$A(\Delta PQR) = 360 cm^2$ .....(Given)

$A(\Delta PQR) = \frac{1}{2} \times base \times height$

$\therefore 360 = \frac{1}{2} \times 80 \times height$

$\therefore height = \frac{360 \times 2}{80} = 9 cm$

$\therefore PS = 9$ cm .....(1)

In an isosceles triangle, the perpendicular to the base bisects the base.

$\therefore QS = \frac{1}{2} QR = \frac{1}{2} \times 80$ cm

$\therefore QS = 40$ cm .....(2)

In right angled ΔPQS, by Pythagoras' Theorem,

$PQ^2 = QS^2 + PS^2$

$= (40)^2 + (9)^2$ .....[From(2)and(4)]

$= 1600 + 81 = 1681$

$\therefore PQ = 41$ cm.

$\therefore PQ = PR = 41$ cm .....(3)

The perimeter of ΔPQR

= PQ + QR + PR

= 41 + 80 + 41 ......[From(3)and given]

= 162 cm

The perimeter of the given triangle is 162 cm.

Solution 4:

Let ΔPQR be an isosceles triangle in which

$PQ = PR = x$ cm ....(1)

$QR = 1\frac{1}{2} \times PQ = 1\frac{1}{2} \times x = 1.5x$ ....(2)

Perimeter of $\Delta PQR = PQ + PR + QR$

$\therefore 42 = x + x + 1.5x$ .....[from (1) and (2)]

$\therefore 3.5x = 42$

$\therefore x = 12$

$\therefore PQ = PR = 12$ cm, $QR = 1.5x = 18$ cm

In an isosceles triangle, the perpndicular to the base bisects the base.

$\therefore QS = \frac{1}{2} \times QR = \frac{1}{2} \times 18 = 9\,\text{cm}$

In ΔPQS using Pythagoras' theorem

$PQ^2 = QS^2 + PQ^2$

$\therefore (12)^2 = (9)^2 + h^2$

$\therefore h^2 = (12)^2 - (9)^2$

$\therefore h^2 = 63$

$\therefore h = 3\sqrt{7}$ cm

Area of $\Delta PQR = \frac{1}{2} \times \text{base} \times \text{height}$

$$A(\Delta PQR) = \frac{1}{2} \times QR \times PS$$

$$= \frac{1}{2} \times 18 \times 3\sqrt{7}$$

$$= 27\sqrt{7}\ \text{cm}^2$$

The length of the congruent sides of the triangle, $PQ = PR = 12$ cm.

The height of the triangle, $PS = 3\sqrt{7}$ cm.

The area of the triangle $= 27\sqrt{7}\ \text{cm}^2$.

Solution 5:

Let ΔPQR be an isosceles right angled triangle in which

$PQ = QR = 10$ cm. $m\angle Q = 90°$, $QS \perp PR$.

$$A(\Delta PQR) = \frac{1}{2} \times PQ \times QR$$

$$= \frac{1}{2} \times 10 \times 10$$

$\therefore A(\Delta PQR) = 50 cm^2$ .....(1)

In right angled ΔPQR, by Phythagoras' Theorem,

$$PR^2 = PQ^2 + QR^2$$

$$= (10)^2 + (10)^2$$

$$PR^2 = 200$$

$\therefore PR = 10\sqrt{2}$ cm .....(2)

Now, Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$$\therefore A(\Delta PQR) = \frac{1}{2} \times PR \times QS$$

$$\therefore 50 = \frac{1}{2} \times 10\sqrt{2} \times QS$$

$$\therefore QS = \frac{50 \times 2}{10\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}$$

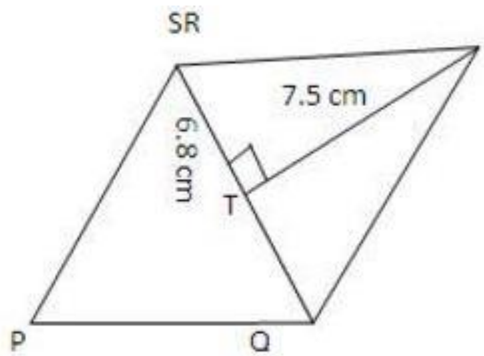
The length of the altitude on the hypotenuse is $5\sqrt{2}$ cm.

Exercise – 9.2

Solution 1:

Let PQRS be the parallelogram having SQ as one diagonal of length 6.8 cm and RT of length 7.5 cm perpendicular to SQ.

The diagonal of the parallelogram divides it into two congruent triangles.



Area of one triangle

$$=\frac{1}{2} \times \text{base (diagonal)} \times \text{height}$$

$$=\frac{1}{2} \times 6.8 \times 7.5 = 25.5 \text{ cm}^2$$

Area of the parallelogram

$= 2 \times$ Area of one traingle

$= 2 \times 25.5 = 51 \text{ cm}^2$.

Area of the parallelogram is 51 cm^2.

Solution 2:

Let the length and breadth of the rectangle be l cm and b cm respectively.

The area of the rectangle $= l \times b$

$\therefore l \times b = 192\text{cm}^2$

$$l = \frac{192}{b} \qquad \ldots.(1)$$

The perimeter of a rectangle $= 2(l + b)$

$\therefore 56 \text{ cm} = 2(l + b)$

$\therefore l + b = 28 \text{ cm} \qquad \ldots.(2)$

Substitute equation (1) in eq (2)

$$\frac{192}{b} + b = 28\text{cm}$$

$b^2 - 28b + 192 = 0$

$\therefore (b - 16)(b - 12) = 0$

$\therefore b = 16$ or $b = 12$

Substituting b = 16 in equation (2)

$l + 16 = 28$

$\therefore l = 12$

Substituting b = 12 in equation (2)

$l + 12 = 28$

$\therefore l = 16$

The dimensions of the rectangle are length = 16 cm and breadth = 12 cm.

Solution 3:

The area of the square-shaped field
= (side)2
= (300)2
= 90000 m2
Cost of levelling the field per square metre = Rs. 1.25

∴The cost of levelling = rate × area
= Rs. 1.25 × 90000
= Rs. 1,12,500
The cost of levelling the field is Rs. 1,12,500.

Solution 4:

Let the length and breadth of the hall be l metres and b metres.
The length of the rectangular hall is 5 m more than its breadth.
∴l = b + 5 ………(1)
The area of the rectangular hall = l × b
∴l × b = 750 ………(2)
Substitute (1) in (2) we get
∴(b + 5) × b = 750
$b^2 + 5b - 750 = 0$
$b^2 - 25b + 30b - 750 = 0$
$b(b - 25) + 30(b - 25) = 0$
$(b + 30)(b - 25) = 0$
$b + 30 = 0$ or $b - 25 = 0$
$b = -30$ or $b = 25$
But breadth of the hall cannot be negative.
∴ b = 25 m
Substitute b = 25 in equation (1) we get
l = 25 + 5 = 30m
The perimeter of the rectangular hall
= 2 (l + b)
= 2 (30 + 25)
= 110
The perimeter of the rectangular hall is 110 m.

Solution 5:

$m\angle APB = 90^\circ$

$\therefore$ In right angled ΔABP, by Phythagoras' Theorem,

$AB^2 = AP^2 + BP^2$

$\therefore (10)^2 = AP^2 + (6)^2$

$\therefore AP^2 = (10)^2 - (6)^2$

$\therefore AP^2 = 100 - 36$

$\therefore AP^2 = 64$

$\therefore AP = 8$

Seg AP and Seg DQ are perpendicular to the same parallel lines, line AD and line BC.

$\therefore$ seg DQ = seg AP = 8 ....(1)

In right angled ΔDQC, by Phythagoras' theorem,

$DC^2 = DQ^2 + QC^2$

$\therefore (17)^2 = (8)^2 + QC^2$ [from(1)]

$\therefore QC^2 = (17)^2 - (8)^2$

$\therefore QC^2 = 225$

$\therefore QC = 15$ .....(2)

In $\square$ADQP,

$AP \parallel DQ$, $AP \perp BC$, $DQ \perp BC$

$\therefore \square$ADQP is a rectangle,

$\therefore AD = PQ = 7$ ...(3)

$BC = BP + PQ + QC$ ......(B-P-Q-C)

$= 6 + 7 + 15$ ....[from(2) and(3)]

$\therefore BC = 28$

Area of trapezium ABCD

$= \frac{1}{2} \times (\text{Sum of lengths of parallel sides}) \times (\text{height})$

$= \frac{1}{2} \times (AD + BC) \times AP$

$= \frac{1}{2} \times (7 + 28) \times 8$

$= 140$

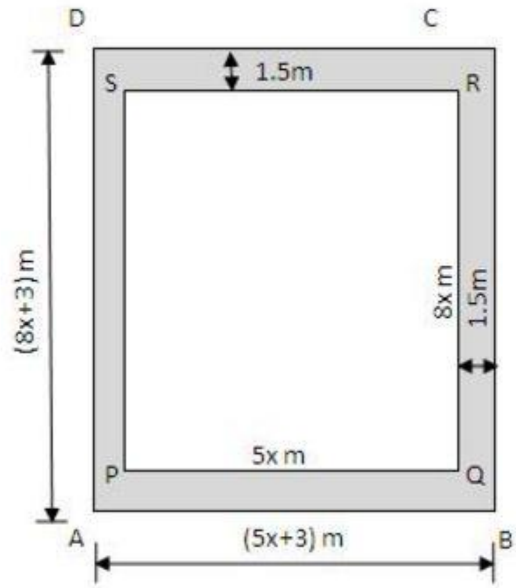
$\therefore$ BC is 28 units;

$\therefore$ The area of $\square$ABCD is 140 sq. units.

Solution 6:

The ratio of the length and given breath is 8 : 5

.....(Given)



□PQRS is a park;

The shaded region is the path

The length and breadth of the rectangular park are in the ratio of 8 : 5.

Let the length be 8x m and the breadth be 5x m.

For □ABCD,

$AB = 5x + 1.5 + 1.5 = (5x + 3)$ m,

$BC = 8x + 1.5 + 1.5 = (8x + 3)$ m.

Now, the area of the path

$= A(□ABCD) - A(□PQRS)$

$= (8x + 3)(5x + 3) - 8x \times 5x$

But area of the path is 594 m^2

$\therefore (8x + 3)(5x + 3) - 8x \times 5x = 594$

$\therefore 40x^2 + 39x + 9 - 40x^2 = 594$

$\therefore 39x = 585$

$\therefore x = 15$

The length $= 8x = 8 \times 15 = 120$ m and breadth $= 5x = 5 \times 15 = 75$ m

The dimensions of the park are 120 m and 75 m.

Solution 7:

Let the length of the lawn = 75 m and its breadth = 60m.
Road ABCD is parallel to the length and road EFGH is parallel to the breadth.
AB = FG = 4 m
AD = 75 m
EF = 60 m
The area of the road ABCD
= AD × AB
= 75 × 4
= 300 m2 …(1)
The area of the road EFGH
= EF× FG
= 60 × 4
= 240 m2 …(2)
□UVWX which is 4 × 4, is common to both the roads.
Area of □UVWX = 4 × 4 = 16 m2 …(3)
∴The area of the road
= A(□ABCD) + A(□EFGH) – A(□UVWX)
= (300 + 240 -16) m2…..[From (1),(2) and (3)]
= 524 m2
The total cost of gravelling the road
= rate × area
= Rs. 4.50 × 524
= Rs. 2358
The total cost of gravelling the road is Rs.2358.

Exercise – 9.3

Solution 1:

A horse is tethered at a point by a 10 m long rope.
The horse can graze around that point in a circular path.
The horse can graze in a circular path with the point where horse is tethered as the centre and radius equal to the length of the rope, i.e. 10 m.
So the area of the region where the horse can graze is area of the circle.
The area of circle = πr2
= 3.14 × (10)2
= 3.14 × 100
= 314 m2
The area of the region where the horse can graze is 314 m2.

Solution 2:

The diameter of the circle = 2r = 20 cm
The diameter of the circle is equal to the side of the square.
∴ The side of the square is 20 cm.
Area of a square= $(side)^2$ = 400 cm^2
Area of a circle = πr^2
= 3.14 × $(10)^2$
= 314 cm^2
Area of the shaded portion
= Area of the square – Area of the circle
=(400 – 314)cm^2
= 86 cm^2
The area of the shaded portion is 86 cm^2.

Solution 3:

Area of the circle with radius 3 m
= πr^2
= $\pi (3)^2$
= 9π m^2 …(1)
Area of the circle with radius 4 m
= πr^2
= $\pi (4)^2$
= 16 π m^2 …(2)
∴Area of the required circle
= 9 π + 16 π … [From (1) and (2)]
= 25π m^2 …(3)
Let the radius of the required circle be R.
Then its area = πR^2 = 25 π …[From (3)]
∴ R^2 = 25 ∴ R= 5 m
The radius of the required circle is 5 m.

Solution 4:

The distance travelled by a wheel in 1 revolution
= the circumfernce of the wheel.
$=2\pi r$
$=2\times\frac{22}{7}\times 45\ldots(\text{diameter} = 90\text{ cm} \quad \therefore r = 45\text{ cm})$
$\therefore$ The distance travelled by the wheel in 210 revolutions
$=2\times\frac{22}{7}\times 45\times 210$
$= 2\times 22\times 45\times 30$ cm
= The distance travelled by the wheel in 1 minute.

$\therefore$ The distace travelled by the wheel in 1 hour
$=2\times 22\times 45\times 30\times 60$ cm
$=\frac{2\times 22\times 45\times 30\times 60}{100\times 1000}$ km
$= 35.64$ km
The speed of the bus is 35.64 km/hr.

Solution 5:

Area of the circle with radius 14 cm
$= \pi(14)^2$
$= \frac{22}{7}\times 14\times 14$
$= 616\text{ cm}^2$

$\therefore$ The area of the other circle
$= (770 - 616) = 154\text{ cm}^2 \qquad \ldots(1)$
Let the radius of the circle be r cm
The its area $= \pi r^2 = 154 \qquad \ldots[\text{From }(1)]$
$\therefore \frac{22}{7}r^2 = 154$
$\therefore r^2 = 49$
$\therefore r = 7$ cm

The radius of the other circle is 7 cm.