* Thermal stresses *

(Always a normal stress)

⇒ [Total normal stress develope on the x-S/c of a Component] equals to = [Mechanical Normal stress] + [Termal stress "Anormal stress].

Conditions

① There should be a temprature variation

② Thermal deformation should be restricted either completely (or) partially

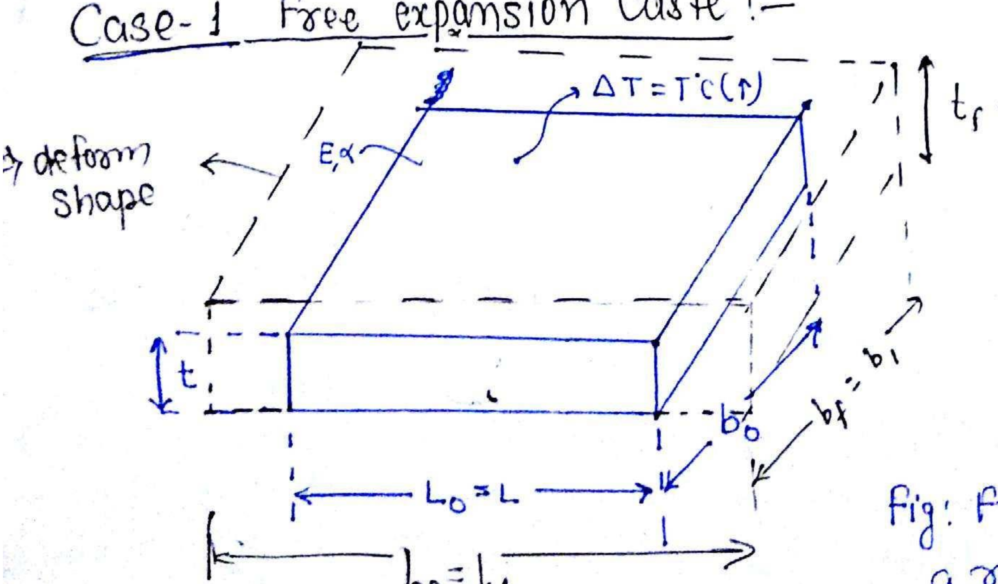
Mechanical Normal stress } — i.e. axial & bending stresses

⇒ developed due to loads acting on the x-S/c of a Component

Thermal stress → tensile stress
Thermal stress → Compressive stress

⇒ Thermal stress is equal to zero when only 1st Condition is satisfied [∵ Free expansion (or) Contraction]

Case-1 Free expansion Caste :-



α - thermal expansion Const.

Fig: Free expansion of a rect. block.

$$\left.\begin{aligned} (\delta_{th})_x &= \delta L = L_1 - L = \alpha T L \\ (\delta_{th})_y &= \delta t = t_1 - t = \alpha T t \\ (\delta_{th})_z &= \delta_b = b_1 - b = \alpha T b \end{aligned}\right\} \text{---} (1)$$

where α = linear Coefficient of thermal expansion

$$(\delta_{th})_x > (\delta_{th})_y > (\delta_{th})_z \quad \because [L > b > t]$$

In cube $L = b = t$

$(\delta_{th})_x = (\delta_{th})_y = (\delta_{th})_z$

Strain

$$(\epsilon_{th})_x = \frac{\delta L}{L} = \frac{(\delta_{th})_x}{L} = \alpha T \quad \{\text{tensile temp}\uparrow\}$$

$$\boxed{(\epsilon_{th})_x = (\epsilon_{th})_y = (\epsilon_{th})_z = \pm \alpha T} \quad \{\text{Comp. temp}\downarrow\}$$

* Deformation in all dir^n are unequal but thermal strain in all three dir^n are equal in mag. and like in nature.

* Thermal strain is tensile in nature when temp. increases and vice-versa.

* $\boxed{(\sigma_{th})_x = (\sigma_{th})_y = (\sigma_{th})_z = 0}$ } i.e. free expansion $\because R_x = R_y = R_z = 0$ (Reaction)

$\Rightarrow$ * $\boxed{\epsilon_v = \frac{\delta V}{V} = \epsilon_x + \epsilon_y + \epsilon_z = \pm 3\epsilon_{th} = \pm 3\alpha T}$

Volumetric strain

eg A Cube of side $a = 1$ cm under free expansion $\Delta T = 1^\circ C$ for expansion permitted in all three dirn Find change in Vol. due to free expansion

Soln

$$e_v = \frac{\Delta V}{V} = 3\alpha T$$

$\Delta T = 1^\circ C$

$V = 1\ cm^3$

$$\Delta V = 3\alpha\ cm^3$$

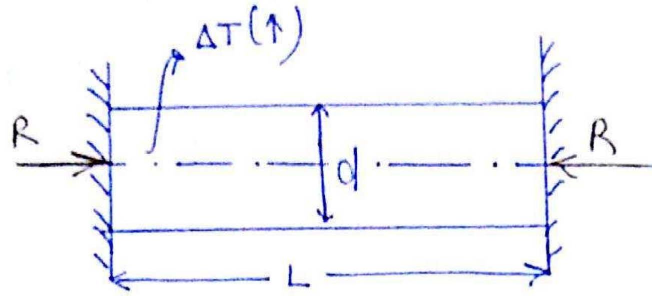
* To maintain dimension stability in presence of temp. variation ~~steel and~~ 'INVAR' (i.e. steel & Nickel alloy) should be selected as the material for the component

$$\because \alpha_{invar} = 1.2 \times 10^{-6}/^\circ C$$

Material	$\alpha/^\circ C$
Diamond	$1.1 \times 10^{-6}/^\circ C$
Invar	$1.2 \times 10^{-6}/^\circ C$
Steel	$12 \times 10^{-6}/^\circ C$
Copper	$16 \times 10^{-6}/^\circ C$
Gun Metal	$19 \times 10^{-6}/^\circ C$
Brass & Bronze	$20 \times 10^{-6}/^\circ C$
Aluminium	$23 \times 10^{-6}/^\circ C$

Trick = DISC GBA

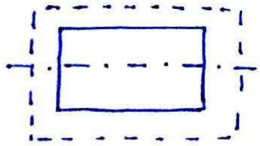
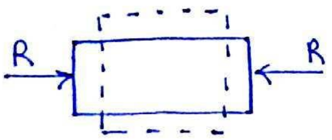
Ques



Determine
ⓐ δL change in length
ⓑ δd change in dia

Ans ⓐ $\delta L = 0$ $\because$ It is not a free expⁿ Rxn will occur

ⓑ $\delta d = \alpha T d (1 + \mu)$

Strain (→) / loading Codⁿ (↓)	x-dirⁿ	y-dirⁿ	z-dirⁿ
	αT	αT	αT
	$\epsilon_{long} = -\alpha T$	$\epsilon_{lateral} = \mu \alpha T$	$\epsilon_{lateral} = \mu \alpha T$

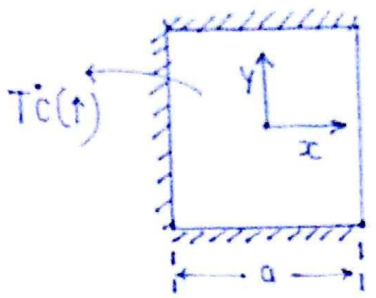
$$\left(\epsilon_{total}\right)_x = 0 \qquad \left(\epsilon_{total}\right)_{y\,\&\,z} = \alpha T (1 + \mu)$$

$$\left(\epsilon_{total}\right)_x = \frac{\delta L}{L} = 0 \Rightarrow \delta L = 0$$

$$\left(\epsilon_{total}\right)_y = \left(\epsilon_{total}\right)_z = \frac{\delta d}{d} = \alpha T (1 + \mu)$$

$$\delta d = \alpha T d (1 + \mu)$$

Q.23
workbook



$$(\epsilon_{total})_y = (\epsilon_{th})_y + (\epsilon_{long})_y = 0$$

$$\alpha T + (\epsilon_{long})_y = 0$$

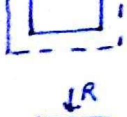
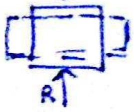
$$\boxed{(\epsilon_{long})_y = -\alpha T}$$

$$(\epsilon_{total})_x = (\epsilon_{th})_x + \cancel{(\epsilon_{long})_x} + (\epsilon_{lateral})_x$$

$$\frac{(\delta a)_x}{a} = \alpha T + \left(-\mu(\epsilon_{long})_y\right)$$

$$(\delta a)_x = \alpha T a + a\left(-\mu(-\alpha T)\right)$$

$$(\delta a)_x = \alpha T a(1+\mu)$$

strain (→) loading	x-dirn	y-dirn
	αT	αT
 R R	$\epsilon_{lateral} = \mu\alpha T$	$(\epsilon_{long})_y = -\alpha T$
	$(\epsilon_{total})_x = \alpha T(1+\mu)$	$(\epsilon_{total})_y = 0$

Case-~~3~~2 [completely restricted expansion] in one dirn.

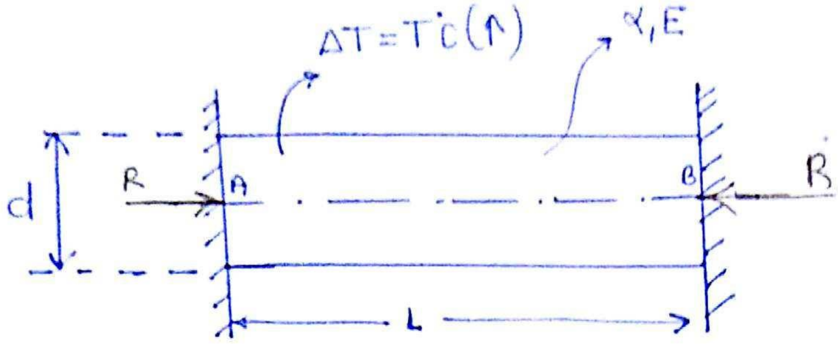


Fig - A prismatic bar made of same material rigidly held between two support.

$$(\delta_L)_{total} = (\delta_{th.}) + (\delta_{a.l.}) = 0$$

$$(\delta_L)_{total} = (\alpha T L) + \left(\frac{-RL}{AE}\right) = 0$$

$$\boxed{R = +\alpha T E A}$$

$$\sigma_{thermal} = \sigma_{axial} = \frac{R}{A} = \frac{\alpha TEA}{A} = \alpha E T$$

☆ $\boxed{\sigma_{axial} = \alpha E T}$ thermal stress "Compressive nature"

- If temp. increase, $\sigma_{th.}$ → Compressive
- If temp. decrease, σ_{th} → tensile

$$\cancel{\epsilon_{th}} \Rightarrow \epsilon_{long} = \frac{\sigma_{th}}{E} = \alpha T \text{ (Comp.)}$$

$$\epsilon_{th.} = -\epsilon_{long} = \alpha T \text{ (tensile)}$$

⇒ Temp ↑ Strain → tensile

⇒ Temp ↓ Strain → Comp.

$$\boxed{\epsilon_{th} = \pm \alpha T} \quad \text{— ①}$$

$$\boxed{\epsilon_{long} = -\epsilon_{th}}$$

$$\boxed{\sigma_{th} = \epsilon_{long} E = \pm \alpha T E} \quad \text{— ②}$$

* when temp.(↑), ϵ_{th} is tensile in nature, ϵ_{long} is comp. in nature & σ_{th} is Comp. in nature.

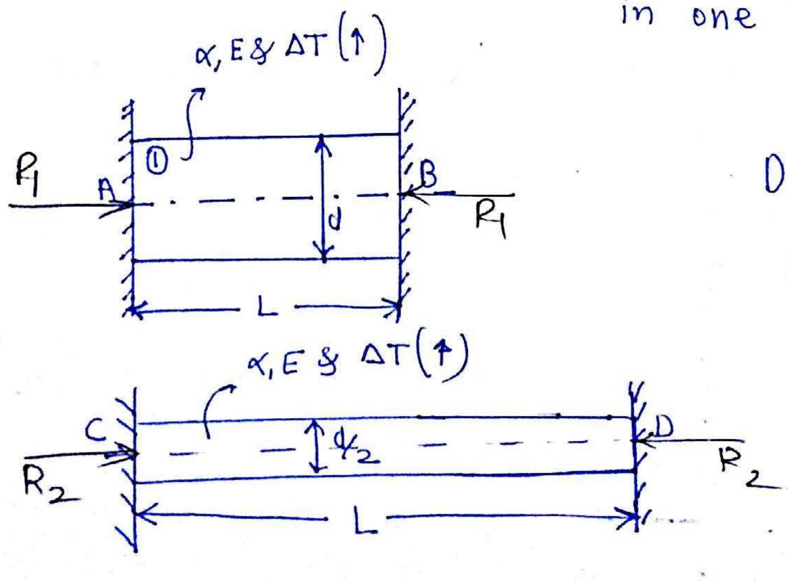
* When temp.(↓), ϵ_{th} is Comp. in nature, ϵ_{long} is tensile in nature, & σ_{th} is tensile in nature.

From equation ② $\boxed{\sigma_{th} \propto f[\alpha, E \,\&\, \Delta T]}$

i.e. Thermal stress (σ_{th}) is independent of dimension of the member.

Conditions for ② eqn. — ⓐ Prismatic bar
ⓑ Same material
ⓒ Completely restricted expansion in one direction.

Ques



Determine

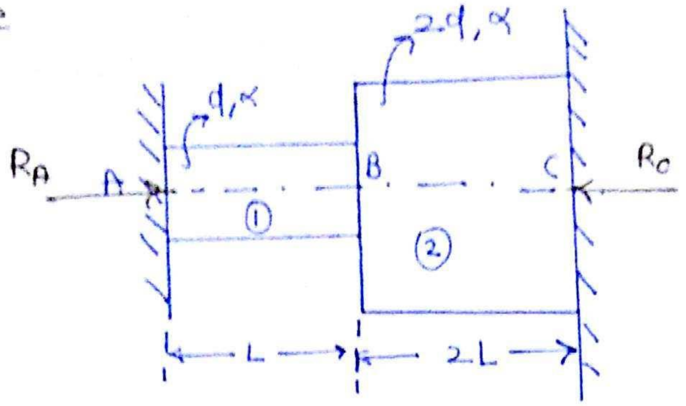
ⓐ $R_1/R_2 = ?$

ⓑ $\dfrac{\sigma_1}{\sigma_2} = ?$

Ans $\sigma = \alpha T E$ $\quad R = \alpha T E A$

$$\sigma_1/\sigma_2 = 1, \quad \frac{R_1}{R_2} = A_1/A_2 = \frac{\frac{\pi d^2}{4}}{\frac{\pi}{4} \times (d/2)^2} = 4$$

Que



(a) $\frac{R_A}{R_C} = ?$

(b) $\frac{\sigma_1}{\sigma_2} = ?$

} Due to temp. Rise.

Ans

$$\Sigma H = 0$$

$$-R_A + R_C = 0 \Rightarrow R_A = R_C = R$$

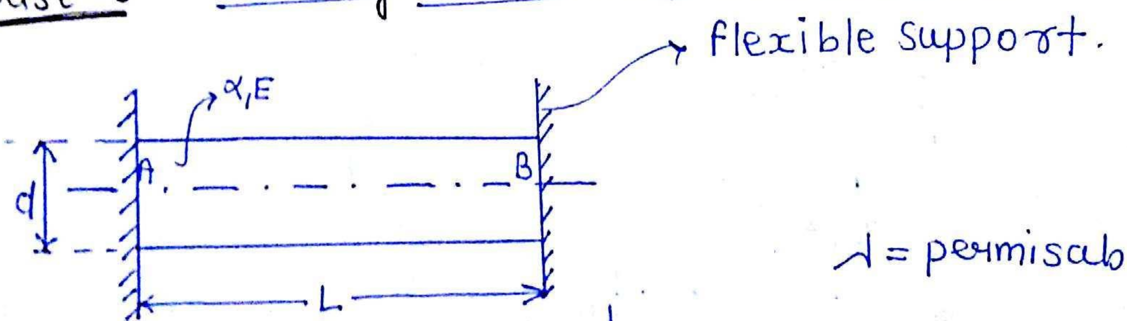
$$P_1 = -R \ ; \ P_2 = -R$$

$$\frac{\sigma_1}{\sigma_2} = \frac{P_1/A_1}{P_2/A_2} = \frac{A_2}{A_1} = \frac{(2d)^2}{(d)^2} = 4$$

$$R_A = R\ (\longleftarrow)$$
$$R_C = R\ (\longrightarrow)$$
$$\Rightarrow \frac{R_A}{R_C} = -1$$

$$\text{Ratio of } \frac{(\text{Axial load})_1}{(\text{Axial load})_2} = 1$$

Case-3 Partially restricted expansion in one direction:



λ = permisable expansion

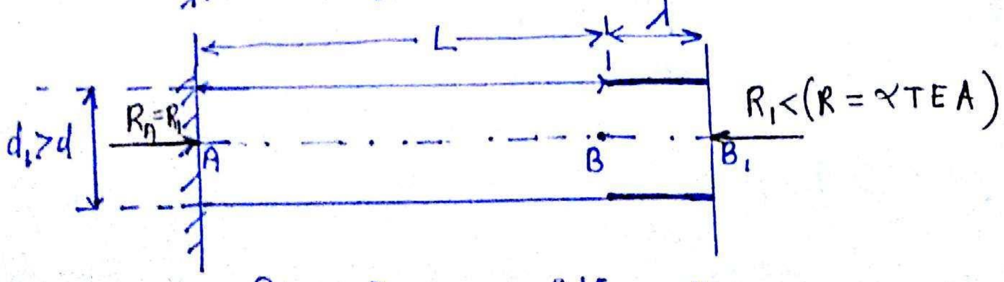


Fig:- Final position of bar

$$(\delta L)_{total} = (\delta_{th.}) + (\delta_{ad}) = \lambda$$

$$(\alpha T L) + \left(\frac{-R_1 L}{A E} \right) = \lambda$$

$$\because \frac{R_1}{A} = \sigma_{th}$$

$$\Rightarrow \frac{\sigma_{th} L}{E} = \alpha T L - \lambda$$

$$\sigma_{th} = \frac{(\alpha T L - \lambda) E}{L} \quad (\text{Comp.})$$

$$\boxed{\sigma_{th} = \pm \left[\frac{\delta_{th} - \lambda}{L} \right] E}$$

$(\alpha T L - \lambda)$ or $(\delta_{th} - \lambda)$ is Restricted expansion

where λ = change in length of bar

= expansion permitted by flexible Support

= Yielding of support

= Gap between two rails

= Spring deflection

= zero for completedly restricted expansion

= δ_{th} or $\alpha T L$ for free expansion

Above equation Valid for

(a) Prismatic bar

(b) Same material.

Ques A prismatic bar held b/w two rigidly support at a temp. of 10°c Initial stress in the bar 10 MPa (tensile) (that is at a temp. of 10°c) Determine stress develope on the x-s/c of the bar, when temp. of bar raised to 15°C assume $L = 1\,m$; $d = 50\,mm$; $\alpha = 10\times10^{-6}/°c$

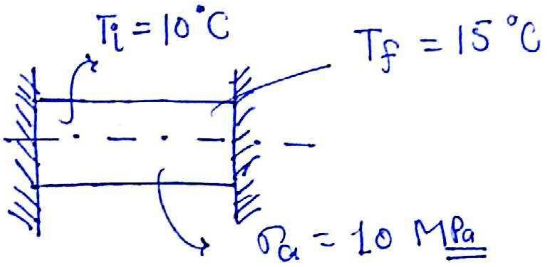
$E = 200\ GPa$

Solution

$T_i = 10°c$

$T_f = 15°c$

$\sigma_i = 10\ MPa$



$$(\sigma_{total}) = \sigma_{mech.} + \sigma_{thermal}$$

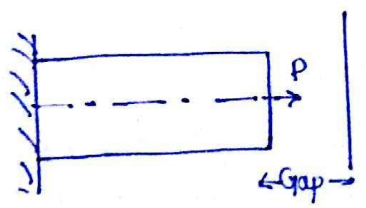
$$= (\sigma_{mech}) \pm (\alpha T E)$$

$$= 10\ MPa + (-10\times10^{-6}\times5\times200\times10^{3})$$

$$= 10 - 10$$

$$\sigma_{total} = 0$$

$\Rightarrow$ &



$$\frac{PL}{AE} = Gap \ \ \& \ \ P = ?$$

$$\sigma_{mech} = P/A \ \ ---$$

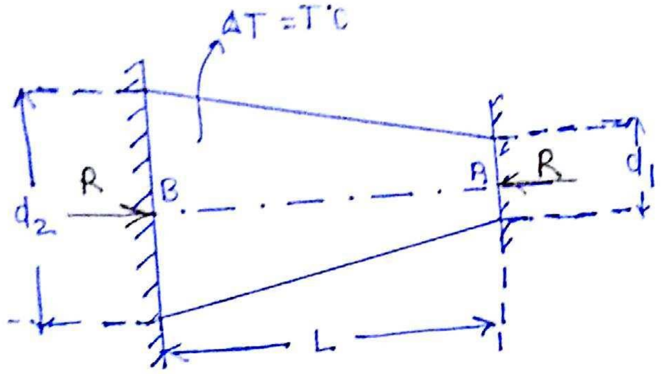
Que A tappered bar rigidly ~~support~~ held between two supports. Determine following when temp. raise to T°C

i) Rxn at support

ii) Max. stress develope on x-s/c of bar.

iii) Ratio of mini & max. stress develope on the x-s/c of bar.

Soln



total change in length = zero

$$(\delta L)_{total} = (\delta_{th}) + (\delta_{ad}) = 0$$

$$(\alpha T L) + \left[\frac{-4RL}{\pi d_1 d_2 E}\right] = 0$$

$$\boxed{R = \alpha T E\left(\frac{\pi d_1 d_2}{4}\right)}$$

$$\sigma_{max} = \sigma_A = \frac{R}{A_{min}} = \frac{4R}{\pi d_1^2} = \alpha T E\left[\frac{d_2}{d_1}\right]$$

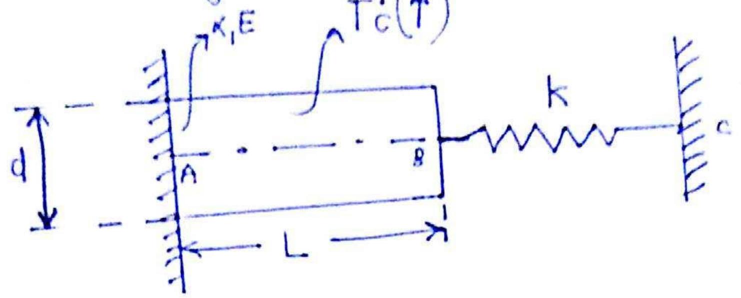
$$\sigma_{min} = \sigma_B = \frac{R}{A_{max}} = \frac{4R}{\pi d_2^2}$$

$$\frac{\sigma_{max}}{\sigma_{min}} = \frac{4R}{\pi d_1^2} \times \frac{\pi d_2^2}{4R} = \frac{d_2^2}{d_1^2} = \left[\frac{d_{larger}}{d_{smaller}}\right]^2$$

Ques For the bar as shown in fig Determine following

1) thermal stress.

ii) spring force



Data $A = 200\ mm^2$

$E = 200\ GPa$

$L = 2\ m$

$\alpha = 12 \times 10^{-6} /°C$

$\Delta T = 100°C\ (\downarrow)$

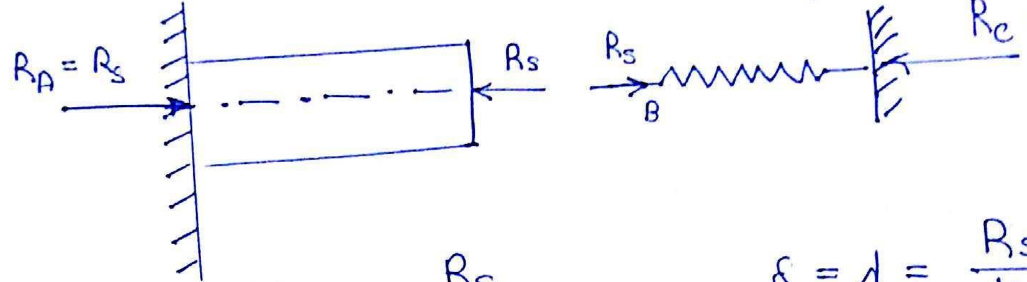
$K = 1000\ N/mm$

$$\Rightarrow K_{bar} = \frac{AE}{L} = \frac{200 \times 200 \times 10^3}{2000}$$

$$\boxed{K_{bar} = 2 \times 10^4\ N/mm}$$

$R_C = R_S$

Sol^n



$$\sigma_{th} = \frac{R_S}{A}$$

$$R_S = \sigma_{th} A \quad -(\mathrm{I})$$

$$\delta_s = \lambda = \frac{R_S}{K}$$

$$R_S = K\delta_s \text{ or } K\lambda$$

$\downarrow$ spring deflection.

$$(\mathrm{I}) = (\mathrm{II})$$

$$\sigma_{th} A = K[\lambda]$$

$$\lambda = \frac{\sigma_{th} A}{K}$$

$$\sigma_{th} = \left(\frac{\delta_{th} - \lambda}{L}\right) E$$

$$\sigma_{th} = \left[\frac{\alpha T L - \frac{\sigma_{th} A}{K}}{L}\right] E$$

$$\sigma_{th} = \alpha T E - \frac{\sigma_{th} A E}{K L}$$

$$\sigma_{th}\left[1+\frac{AE}{KL}\right] = \alpha TE$$

$$\sigma_{th} = \frac{\alpha TE}{\left(1+\frac{AE/L}{K}\right)} \quad \text{Compressive}$$

$$\boxed{\sigma_{th} = \frac{\alpha TE}{1+\left[\frac{K_{bar}}{K_{spring}}\right]}} \quad \text{Compressive}$$

where :- $K_{bar} = \frac{AE}{L}$ = Axial stifness of bar.

$$\boxed{R_s = \sigma_{th}A = \frac{\alpha TEA}{\left[1+\frac{K_{bar}}{K_{spring}}\right]}}$$

Stifness of bar

$$\delta_a = \frac{PL}{AE}$$

$$\boxed{K_{bar} = \frac{P}{\delta_a} = \frac{AE}{L}}$$ ⇒ load require for 1 unit deformation

Torsion stifness of shaft

$$\boxed{K_t = K_{shaft} = \frac{T}{\theta} = \frac{GJ}{L}}$$ ⇒ Torque require for 1 radian angle of twist.

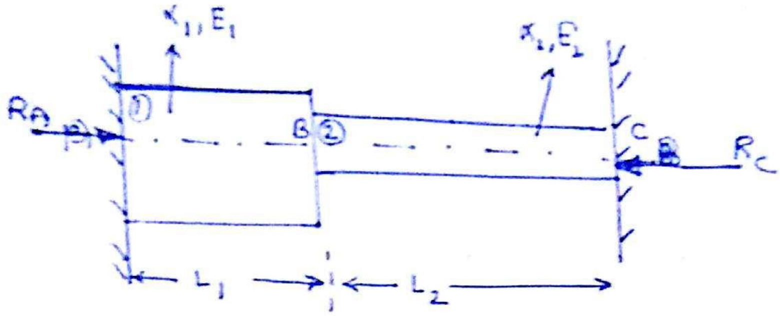
So $\sigma_{th} = \frac{12\times10^{-6}\times100\times200\times10^3}{1+\frac{2\times10^4}{1000}} = \frac{240}{21}$

$\sigma_{th} = 11.42\,MPa$ ⇒ $R_s = \sigma_{th}A$ ⇒ $R_s = 11.42\times200$

$R_s = 2.285\,kN$

Thermal stress in Compound bars:- (i.e. Bar in Series)

Bar in Series:-



$$\Sigma H = 0 \Rightarrow -R_A + R_C = 0$$

$$R_A = R_C = R$$

$$P_1 = -R \; ; \quad P_2 = -R$$

Total chang in length of Compound bar $= 0$

$$(\delta L)_{C.B.} = (\delta L)_1 + (\delta L)_2$$

$$(\delta L)_{C.B.} = \left[(\alpha_1 T L_1) + \left(\frac{-R_1 L_1}{A_1 E_1}\right)\right] + \left[\alpha_2 T L_2 + \left(\frac{-R_2 L_2}{A_2 E_2}\right)\right] = 0$$

$$[\alpha_1 T L_1 + \alpha_2 T L_2] = \frac{R L_1}{A_1 E_1} + \frac{R L_2}{A_2 E_2} = 0$$

$$(\alpha_1 T L_1 + \alpha_2 T L_2) = \left[\frac{\sigma_1 L_1}{E_1} + \frac{\sigma_2 L_2}{E_2}\right] \quad -①$$

$$\left[\text{Sum of thermal def}^n\right] = \left[\text{Sum of Axial def}^n\right]$$

$$\boxed{\Sigma \delta_{thermal} = \Sigma \delta_{axial}}$$

$$\boxed{\frac{\sigma_1}{\sigma_2} = \frac{A_2}{A_1}} \quad \text{--- (2)} \quad [\because P_1 = P_2 = -R]$$

by solving equation (1) & (2)

$\sigma_1 = \underline{\ ?\ }$ MPa (Comp.)
$\sigma_2 = \underline{\ ?\ }$ MPa (Comp.) } when temp. of C.B increase.

$\sigma_1 = \underline{\quad}$ MPa (tensile)
$\sigma_2 = \underline{\quad}$ MPa (tensile) } when temp. of C.B decrease

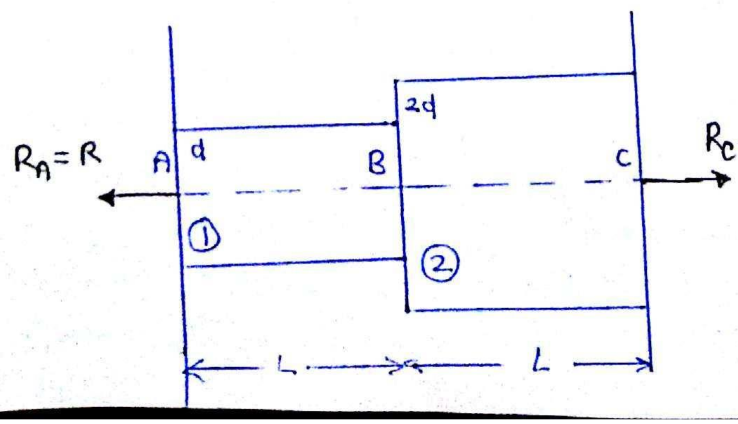
* when temp. of Compound bar increase compressive ~~them~~ thermal stress develope wheather bars are made of different or same material.

* when temp. of C.B. decrease tensile thermal stress develope wheather bars are made of diff. material or same material.

Question for the compound bar as shown in fig. determine when temp. of compound bar decrease by 80 °C

Assume $\alpha = 12 \times 10^{-6} /°C$, $E = 200$ GPa

i) stress develope in both the bars
ii) Reactions at the supports if $d = 40$ mm
iii) Deformation at B if $L = 500$ mm



$$\Sigma \delta_{thermal} = \Sigma \delta_{axial}$$

$$(\alpha T L) + (\alpha T L) = \frac{\sigma_1 L}{E} + \frac{\sigma_2 L}{E}$$

$$\sigma_1 + \sigma_2 = 2 \alpha T E$$

$$= 12 \times 10^{-6} \times 80 \times 200 \times 10^{3} \times 2$$

$$\sigma_1 + \sigma_2 = 192 \times 2$$

$$\sigma_1 + \sigma_2 = 384 \text{ MPa} \quad -(1)$$

$$\frac{\sigma_1}{\sigma_2} = \frac{A_2}{A_1} = \left(\frac{d_2}{d_1}\right)^2 = 4$$

$$\sigma_1 = 4 \sigma_2 \quad -(2)$$

From (1) and (2)

$$\sigma_1 = 307.2 \text{ MPa}$$

$$\sigma_2 = 76.8 \text{ MPa}$$

Reaction $R_1 = \sigma_1 A_1$ (or) $\sigma_2 A_2 = R$

$$R = (307.2) \frac{\pi}{4} (40)^2$$

$$R = 386.038 \text{ KN}$$

$$\delta_B = \delta_{BA} \text{ (or) } \delta_{BC}$$

$$\delta_B = \delta_1 \text{ (or) } \delta_2$$

$$\delta_B = \delta_1 = -\alpha_1 T L_1 + \frac{\sigma_1 L_1}{E_1}$$

$$\delta_B = (-12 \times 10^{-6} \times 80 \times 500) + \left(\frac{301.2 \times 500}{200 \times 10^3}\right)$$

$$\delta_B = 0.288 \text{ MM} \quad (\longrightarrow)$$

$$\delta_B = \delta_2 = -\alpha_2 T L_2 + \frac{\sigma_2 L_2}{E_2}$$

$$= -0.288 \text{ MM}$$

$$= 0.288 \ (\longrightarrow)$$

Q.2
workbook

$$\alpha_1 T L_1 + \alpha_2 T L_2 + \alpha_3 T L_3 = \frac{\sigma_1 L_1}{E_1} + \frac{\sigma_2 L_2}{E_2} + \frac{\sigma_3 L_3}{E_3}$$

$$3(\alpha T \cancel{L}) = \frac{\cancel{L}}{E}(\sigma_1 + \sigma_2 + \sigma_3)$$

$$\sigma_1 + \sigma_2 + \sigma_3 = 3 \alpha T E$$

$$= 3 \times 12 \times 10^{-6} \times 50 \times 200 \times 10^3$$

$$\sigma_1 + \sigma_2 + \sigma_3 = 360 \text{ MPa} \quad -(1)$$

$$\sigma_1 A_1 = \sigma_2 A_2 = \sigma_3 A_3$$

$$\frac{\sigma_1}{\sigma_2} = \frac{A_2}{A_1} \ ; \ \frac{\sigma_1}{\sigma_3} = \frac{A_3}{A_1}$$

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{10}{20}\right)^2 \ ; \ \frac{\sigma_1}{\sigma_3} = \left(\frac{20}{20}\right)^2 \Rightarrow \sigma_1 = \sigma_3 \quad -(3)$$

$$\sigma_2 = 4\sigma_1 \quad -(2)$$

From eq^n (1), (2) & (3)

$$\sigma_1 + 4\sigma_1 + \sigma_1 = 360$$

$$6\sigma_1 = 360$$

$$\boxed{\sigma_1 = 60 \text{ MPa}}$$

$$\sigma_2 = 4 \times \sigma_1$$

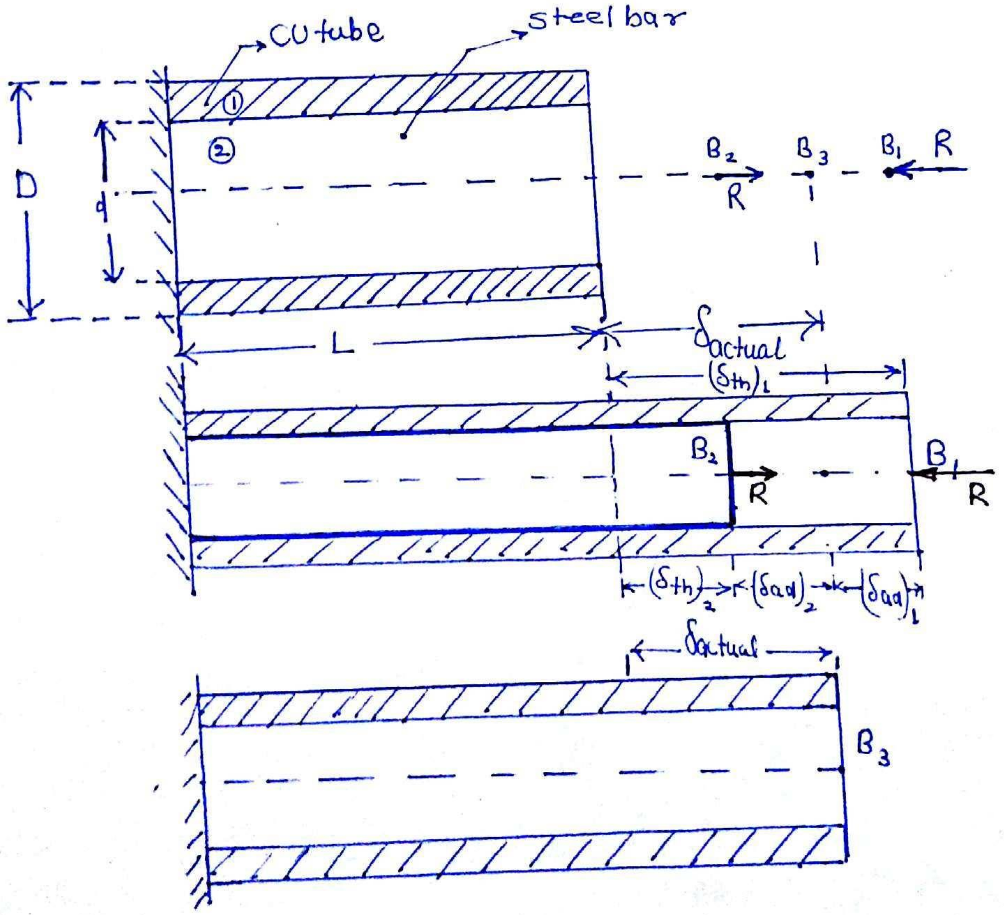
$$\sigma_2 = 4 \times 60$$

$$\boxed{\sigma_2 = 240 \text{ MPa}}$$

σ_1, σ_2 both are Compressive bcoz temp $\uparrow$,

Thermal stress in Compound bar. (i.e. Bars in Parallel)

Bars In Parallel $\qquad \alpha_{cu} > \alpha_{st.}$



$BB_1 = (\delta_{th})_1 =$ Thermal defn of Cu tube $= \alpha_1 T_1$

$BB_2 = (\delta_{th})_2 =$ — " — steel bar $= \alpha_2 T_2$

$B_2B_3 = (\delta_{ad})_2 =$ axial defn of Cu tube $= \dfrac{\sigma_2 L_2}{E_2}$

$B_1B_2 = (\delta_{ad})_1 =$ — " — steel bar $= \dfrac{\sigma_1 L_1}{E_1}$

$(BB_3) = (\delta_{actual})_{C.B.} =$ Actual defn of Composite bar

$$(\delta_{th})_2 < (\delta_{actual})_{C.B.} < (\delta_{th})_1$$

$$BB_3 = BB_2 + B_2B_3 = (\delta_{th})_2 + (\delta_{ad})_2 \quad -(1)$$

$$BB_3 = BB_1 - B_1B_2 = (\delta_{th})_1 - (\delta_{ad})_1 \quad -(2)$$

For (1) = (2)

$$(\delta_{th})_2 + (\delta_{ad})_2 = (\delta_{th})_1 - (\delta_{ad})_1$$

$$\boxed{(\delta_{ad})_1 + (\delta_{ad})_2 = (\delta_{th})_1 - (\delta_{th})_2}$$

{ Sum of axial defns = Diff. of thermal defns }

$$\frac{\sigma_1 L}{E_1} + \frac{\sigma_2 L}{E_2} = \alpha_1 T L - \alpha_2 T L$$

$$\frac{\sigma_1}{E_1} + \frac{\sigma_2}{E_2} = T(\alpha_1 - \alpha_2)$$

$$\sigma_1 + \sigma_2 = \text{—— MPa} \quad -(3)$$

$$\Rightarrow \frac{\sigma_1}{A_1} = \frac{\sigma_2}{A_2} \quad -(4)$$

by solving eq^ns (1) & (2)

$\sigma_1 = \_\_$ MPa (Comp.)
$\sigma_2 = \_\_$ MPa (Tensile)

$\left.\right\}$ when temp. increase

$\sigma_1 = \_\_$ MPa (tensile)
$\sigma_2 = \_\_$ MPa (comp.)

$\left.\right\}$ when

* when temp. of comoposite bar increase Compresive thermal stress is develope in a bar with higher Coefficient of thermal expansion (α) and tensile thermal Stress develope in a bar with lower coefficient of thermal expansion

* when temp. of Composite bar decreases tensile thermal stress is develope in a bar with higher coefficient of thermal expansion. and Compressive thermal stress develope in a bar with lower coefficient of thermal expansion.

Question

Parameter	Cu tube (1)	Steel bar (2)
ΔT	100°C (↓)	100°C (↓)
L (mm)	750	750
E (GPa)	100	200
α/°C	16×10^{-6}	12×10^{-6}
Diameter	$D_o = 200$ mm $D_i = 100$ mm	d = 100 mm.

Soln

$$\frac{\sigma_c}{E_c} + \frac{\sigma_s}{E_s} = T(\alpha_c - \alpha_s)$$

$$\frac{\sigma_c}{100 \times 10^3} + \frac{\sigma_s}{200 \times 10^3} = 100\left[16 \times 10^{-6} - 12 \times 10^{-6}\right]$$

$$\sigma_c + \frac{\sigma_s}{2} = 40$$

$$2\sigma_c + \sigma_s = 80 \text{ MPa} \quad -(1)$$

$$\frac{\sigma_c}{\sigma_s} = \frac{A_c}{A_s} = \frac{d^2}{D^2 - d^2}$$

$$\frac{\sigma_c}{\sigma_s} = \frac{100^2}{200^2 - 100^2}$$

$$\frac{\sigma_c}{\sigma_s} = \frac{10000}{30000}$$

$$\sigma_s = 3\sigma_c \quad -(2)$$

From equation (1) & (2)

$$2\sigma_c + 3\sigma_c = 80$$

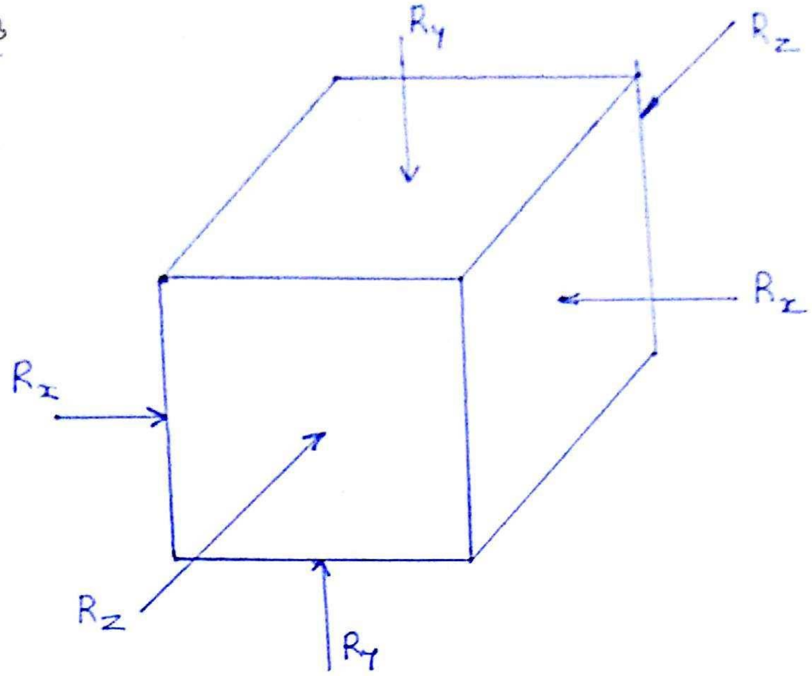
$$\sigma_c = 16 \text{ MPa}$$

$$\sigma_s = 48 \text{ MPa}$$

Question 28

workbook

Pg. 7



Constrained in All direction

$$R_x = R_y = R_z = -R$$

$$A_x = A_y = A_z = A$$

$$\sigma_x = \sigma_y = \sigma_z = \frac{-R}{A} = -\sigma = ?$$

Constrained in All direction So change in Vol. $(\Delta V) = 0$

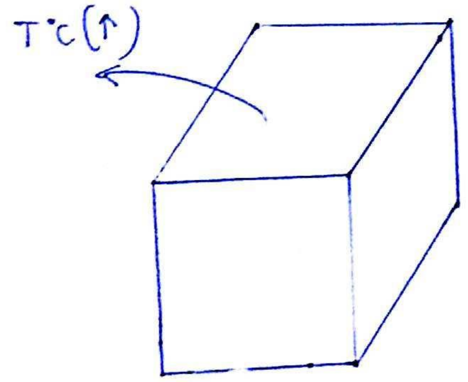


Fig 1. Increase in Vol. of Cube due to temp. rise

+

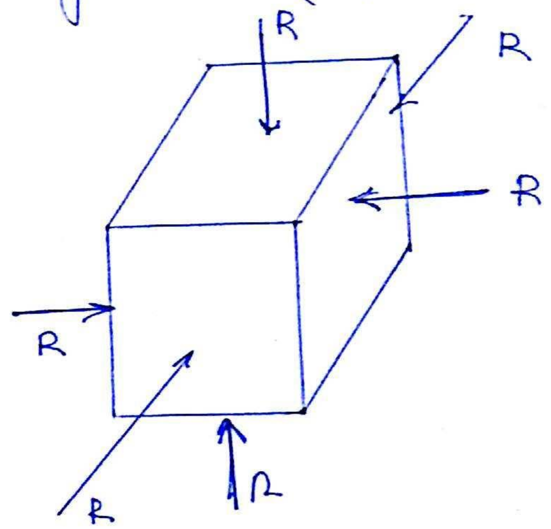


Fig. Decrease in Vol. of cube due to rens

$$(\epsilon_v)_1 = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= 3\epsilon_{th}$$

$$(\epsilon_v)_1 = 3\alpha T \quad \text{—(I)}$$

$$(\epsilon_v) = \left(\frac{1-2\mu}{E}\right)(\sigma_x + \sigma_y + \sigma_z)$$

$$\epsilon_v = \frac{1-2\mu}{E}(-3\sigma) \quad \text{—(II)}$$

$$(\epsilon_v)_{total} = (\epsilon_v)_{I} + (\epsilon_v)_{II} = 0$$

$$\Rightarrow 3\alpha T + \left(\frac{-3(1-2\mu)\sigma}{E}\right) = 0$$

$$\Rightarrow \sigma = \frac{\alpha T E}{(1-2\mu)} \quad \underline{Ans}$$

<u>OR</u> $(\epsilon_x) = (\epsilon_y) = (\epsilon_z) = 0$

Four strain in each dir^n ⇒ 1 long, 2 lateral, 1 thermo

$$-\frac{\sigma}{E} + \frac{\mu\sigma}{E} + \frac{\mu\sigma}{E} + \alpha T = 0$$

$$\alpha T E = (1-2\mu)\sigma$$

$$\sigma = \frac{\alpha T E}{1-2\mu} \quad Ans$$