20. Co-ordinate Geometry: Area of Triangular Region

Let us Calculate 20

1 A. Question

Find the area of triangular region with vertices given below.

(2, −2), (4, 2) and (−1, 3)

Answer

Area of triangle is given by formula

Area = $\frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Here,

(x_1, y_1) is (2, −2)

⇒ $x_1 = 2$ and $y_1 = -2$

(x_2, y_2) is (4, 2)

⇒ $x_2 = 4$ and $y_2 = 2$

(x_3, y_3) is (−1, 3)

⇒ $x_3 = -1$ and $y_3 = 3$

Hence substituting values in formula for area we get

Area = $\frac{1}{2} \times [2(2 - 3) + 4(3 - (-2)) + (-1)(-2 - 2)]$

Area = $\frac{1}{2} \times [-2 + 4(5) + 4]$

Area = $\frac{1}{2} \times [20 + 2]$

Area = 11 unit^2

1 B. Question

Find the area of triangular region with vertices given below.

(8, 9) (2, 6) and (9, 2)

Answer

Area of triangle is given by formula

Area = $\frac{1}{2}$ × $[x_1(y_2 – y_3) + x_2(y_3 – y_1) + x_3(y_1 – y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Here,

(x_1, y_1) is (8, 9)

⇒ $x_1 = 8$ and $y_1 = 9$

(x_2, y_2) is (2, 6)

⇒ $x_2 = 2$ and $y_2 = 6$

(x_3, y_3) is (9, 2)

⇒ $x_3 = 9$ and $y_3 = 2$

Hence substituting values in formula for area we get

Area = $\frac{1}{2}$ × [8(6 – 2) + 2(2 – 9) + 9(9 – 6)]

Area = $\frac{1}{2}$ × [32 + (-14) + 27]

Area = $\frac{1}{2}$ × [32 + 13]

Area = $\frac{1}{2}$ × 45

Area = 22.5 $unit^2$

1 C. Question

Find the area of triangular region with vertices given below.

(1, 2), (3, 0) and origin

Answer

Area of triangle is given by formula

Area = $\frac{1}{2}$ × $[x_1(y_2 – y_3) + x_2(y_3 – y_1) + x_3(y_1 – y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Here,

(x_1, y_1) is (1, 2)

$\Rightarrow x_1 = 1$ and $y_1 = 2$

(x_2, y_2) is (3, 0)

$\Rightarrow x_2 = 3$ and $y_2 = 0$

(x_3, y_3) is origin which is (0, 0)

$\Rightarrow x_3 = 0$ and $y_3 = 0$

Hence substituting values in formula for area we get

Area = $\frac{1}{2} \times [1(0 - 0) + 3(0 - 2) + 0(2 - 0)]$

Area = $\frac{1}{2} \times [0 + (-6) + 0]$

Area = -3

As area cannot be negative

Area = 3 unit^2

2. Question

Prove that the points (3, −2), (−5, 4) and (−1, 1) are collinear.

Answer

Let the points be

A = (x_1, y_1) = (3, -2) and

B = (x_2, y_2) = (-5, 4) and

C = (x_3, y_3) = (-1, 1)

Now if the points A, B and C are collinear then the area formed by the triangle by joining these three points would be 0

Area of triangle is given by formula

Area = $\frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

So, if we prove that area of ΔABC = 0 then points A, B and C are collinear

$\Rightarrow$ Area = $\frac{1}{2} \times [3(4 - 1) + (-5)(1 - (-2)) + (-1)(-2 - 4)]$

$\Rightarrow$ Area = $\frac{1}{2}$ × [9 + (-5)(3) + 6]

$\Rightarrow$ Area = $\frac{1}{2}$ × [15 + (-15)]

$\Rightarrow$ Area = 0

Hence points (3, −2), (−5, 4) and (−1, 1) are collinear.

Method 2

Another method is that you can find distances between every pair of two points and if any distance is equal to sum of other two distances then points are collinear. Here AC + BC = AB.

3. Question

Let us write by calculating what value of K, the points (1, −1), (2, −1) and (K, −1) lie on same straight line.

Answer

Let the points be

A = (x_1, y_1) = (1, -1) and

B = (x_2, y_2) = (2, -1) and

C = (x_3, y_3) = (K, -1)

These points lie on a straight line which means points A, B and C are collinear

As these points are collinear the area of triangle formed by these points would be 0

$\Rightarrow$ Area = 0

Area of triangle is given by formula

Area = $\frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

$\Rightarrow \frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \frac{1}{2} \times [1(-1 - (-1)) + 2(-1 - (-1)) + K(-1 - (-1))] = 0$

$\Rightarrow \frac{1}{2} \times [1(-1 + 1) + 2(-1 + 1) + K(-1 + 1)] = 0$

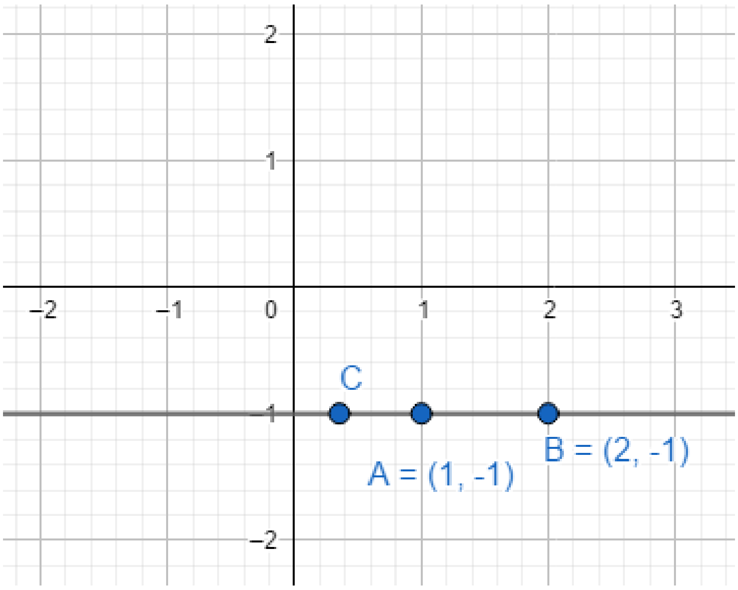
$\Rightarrow$ K × 0 = 0 … (a)

Here we can put any value for K from negative infinity to infinity and our equation (a) is satisfied

Hence K can take any value for the points (1, −1), (2, −1) and (K, −1) to lie on same straight line.

Method 2

If we plot the given points we can observe that K can take any value since the y-coordinate for all the points A, B and C is the same



4. Question

Let us prove that the line joining two points (1, 2) and (−2, −4) passes through origin.

Answer

Let the points be

A = (x_1, y_1) = (1, 2) and

B = (x_2, y_2) = (-2, -4)

Let O be the origin

O = (x_3, y_3) = (0, 0)

Now if the area of triangle formed by joining point A, O and B i.e. ΔAOB is zero then we can say that points A, O and B are collinear which means they lie on the straight line which would imply that line passing through A and B will pass through origin O

So we have to prove that area(ΔAOB) = 0

Area of triangle is given by formula

Area = $\frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Substituting values

$\Rightarrow$ Area = $\frac{1}{2} \times [1(-4 - 0) + (-2)(0 - 2) + 0(2 - (-4))]$

$\Rightarrow$ Area = $\frac{1}{2} \times [-4 + 4 + 0]$

$\Rightarrow$ Area = 0

Hence, line joining two points (1, 2) and (–2, –4) passes through origin.

5. Question

Let us prove that the midpoint of line segment joining two points (2, 1) and (6, 5) lie on the line joining two points (–4, –5) and (9, 8).

Answer

Le the points be M = (2, 1) and N = (6, 5)

Midpoint of line segment joining two points is given by $\left(\frac{M_x+N_x}{2}, \frac{M_y+N_y}{2}\right)$

Where (M_x, M_y) are x and y coordinates of point M and (N_x, N_y) are x and y coordinates of point N

$\Rightarrow$ Midpoint = $\left(\frac{2+6}{2}, \frac{1+5}{2}\right)$

$\Rightarrow$ Midpoint = (4, 3)

Let this point be A (4, 3)

Let the points be B and C as follows

B = (–4, –5) and C = (9, 8)

Now, we have to prove that the midpoint i.e. A lies on line joining points B and C

If we prove that area of triangle formed by joining points A, B and C is 0 then A, B and C will be collinear and thus we can say that point A lies on line joining points B and C

Thus the 3 vertices of triangle here are

A = (x_1, y_1) = (4, 3)

B = (x_2, y_2) = (-4, -5)

C = (x_3, y_3) = (9, 8)

So we have to prove that area(ΔACB) = 0

Area of triangle is given by formula

Area = $\frac{1}{2}$ × [$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$]

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Substituting values

⇒ Area = $\frac{1}{2}$ × [4(-5 – 8) + (-4)(8 – 3) + 9(3 – (-5))]

⇒ Area = $\frac{1}{2}$ × [-52 + (-20) + 72]

⇒ Area = $\frac{1}{2}$ × [- 72 + 72]

⇒ Area = 0

Hence midpoint of (2, 1) and (6, 5) lie on the line joining two points (−4, −5) and (9, 8)

6 A. Question

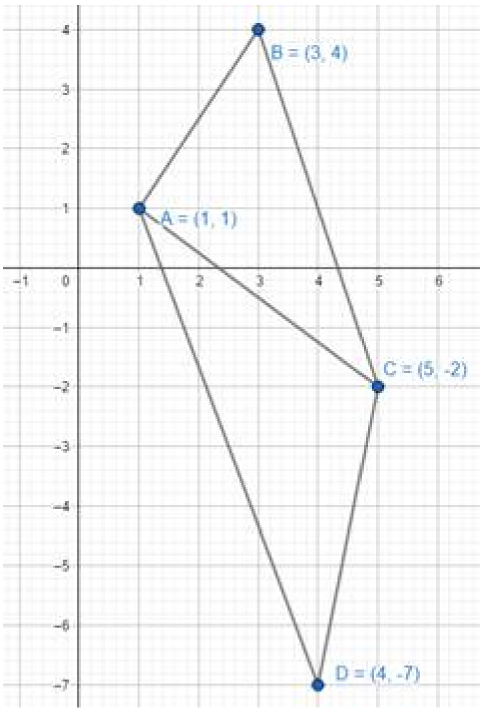
Let us find the area of quadrilateral region formed by the line joining four given points each.

(1, 1), (3, 4), (5, −2), (4, −7)

Answer

Let the points be A (1, 1), B (3, 4), C (5, −2) and D (4, −7)

Plot the points we get the quadrilateral as shown



Divide the quadrilateral in two triangles by joining points A and C thus by observing figure we can conclude that

area(ABCD) = area(ΔABC) + area(ΔACD)

let us find area(ΔABC)

vertices are

$A = (x_1, y_1) = (1, 1)$

$B = (x_2, y_2) = (3, 4)$

$C = (x_3, y_3) = (5, -2)$

Area of triangle is given by formula

$\text{Area} = \frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Substituting values

$\Rightarrow \text{area}(\Delta ABC) = \frac{1}{2} \times [1(4 - (-2)) + 3(-2 - 1) + 5(1 - 4)]$

$\Rightarrow \text{area}(\Delta ABC) = \frac{1}{2} \times [6 + (-9) + (-15)]$

$\Rightarrow \text{area}(\Delta ABC) = \frac{1}{2} \times [6 - 24]$

$\Rightarrow \text{area}(\Delta ABC) = -9$

As area cannot be negative

$\Rightarrow \text{area}(\Delta ABC) = 9 \text{ unit}^2$

Let us find area(ΔACD)

Vertices are

$A = (x_1, y_1) = (1, 1)$

$C = (x_2, y_2) = (5, -2)$

$D = (x_3, y_3) = (4, -7)$

$\Rightarrow \text{area}(\Delta ACD) = \frac{1}{2} \times [1(-2 - (-7)) + 5(-7 - 1) + 4(1 - (-2))]$

$\Rightarrow \text{area}(\Delta ACD) = \frac{1}{2} \times [5 + (-40) + 12]$

$\Rightarrow \text{area}(\Delta ACD) = \frac{1}{2} \times [-40 + 17]$

⇒ area(ΔACD) = 11.5 $unit^2$

Thus, area(ABCD) = area(ΔABC) + area(ΔACD)

⇒ area(ABCD) = 9 + 11.5

⇒ area(ABCD) = 20.5 $unit^2$

Therefore, area of quadrilateral region is 20.5 $unit^2$

6 B. Question

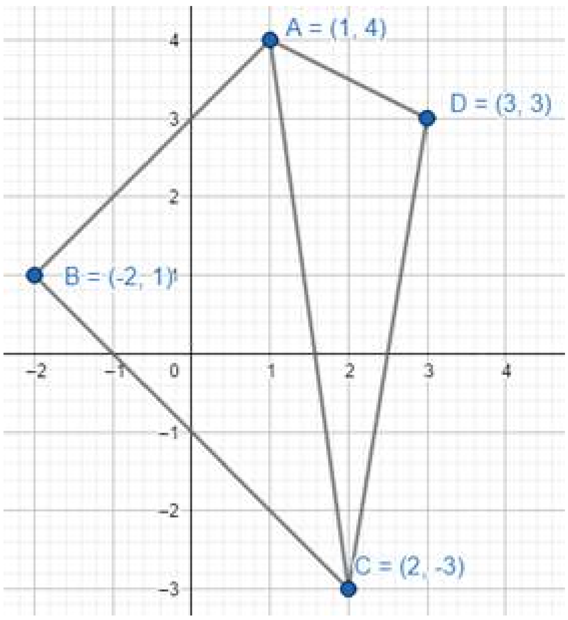
Let us find the area of quadrilateral region formed by the line joining four given points each.

(1, 4), (−2, 1), (2, −3), (3, 3)

Answer

Let the points be A (1, 4), B (-2, 1), C (2, −3) and D (3, 3)

Plot the points we get the quadrilateral as shown



Divide the quadrilateral in two triangles by joining points A and C thus by observing figure we can conclude that

area(ABCD) = area(ΔABC) + area(ΔACD)

let us find area(ΔABC)

vertices are

A = (x_1, y_1) = (1, 4)

B = (x_2, y_2) = (-2, 1)

C = (x_3, y_3) = (2, -3)

Area of triangle is given by formula

Area = $\frac{1}{2}$ × [$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$]

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Substituting values

⇒ area(ΔABC) = $\frac{1}{2}$ × [1(1 – (-3)) + (-2)(-3 – 4) + 2(4 – 1)]

⇒ area(ΔABC) = $\frac{1}{2}$ × [4 + 14 + 6]

⇒ area(ΔABC) = $\frac{1}{2}$ × 24

⇒ area(ΔABC) = 12

⇒ area(ΔABC) = 12 unit^2

Let us find area(ΔACD)

Vertices are

A = (x_1, y_1) = (1, 4)

C = (x_2, y_2) = (2, -3)

D = (x_3, y_3) = (3, 3)

⇒ area(ΔACD) = $\frac{1}{2}$ × [1(-3 – 3) + 2(3 – 4) + 3(4 – (-3))]

⇒ area(ΔACD) = $\frac{1}{2}$ × [(-6) + (-2) + 21]

⇒ area(ΔACD) = $\frac{1}{2}$ × 13

⇒ area(ΔACD) = 6.5 unit^2

Thus, area(ABCD) = area(ΔABC) + area(ΔACD)

⇒ area(ABCD) = 12 + 6.5

⇒ area(ABCD) = 18.5 unit^2

Therefore, area of quadrilateral region is 18 unit^2

7. Question

The co-ordinates of three points A, B, C are (3, 4) (–4, 3) and (8, –6) respectively. Let us find the area of triangle and the perpendicular length drawn from the point A on BC.

Answer

Area of triangle is given by formula

Area = $\frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Here,

A = (x_1, y_1) = (3, 4)

B = (x_2, y_2) = (-4, 3)

C = (x_3, y_3) = (8, -6)

Substituting values

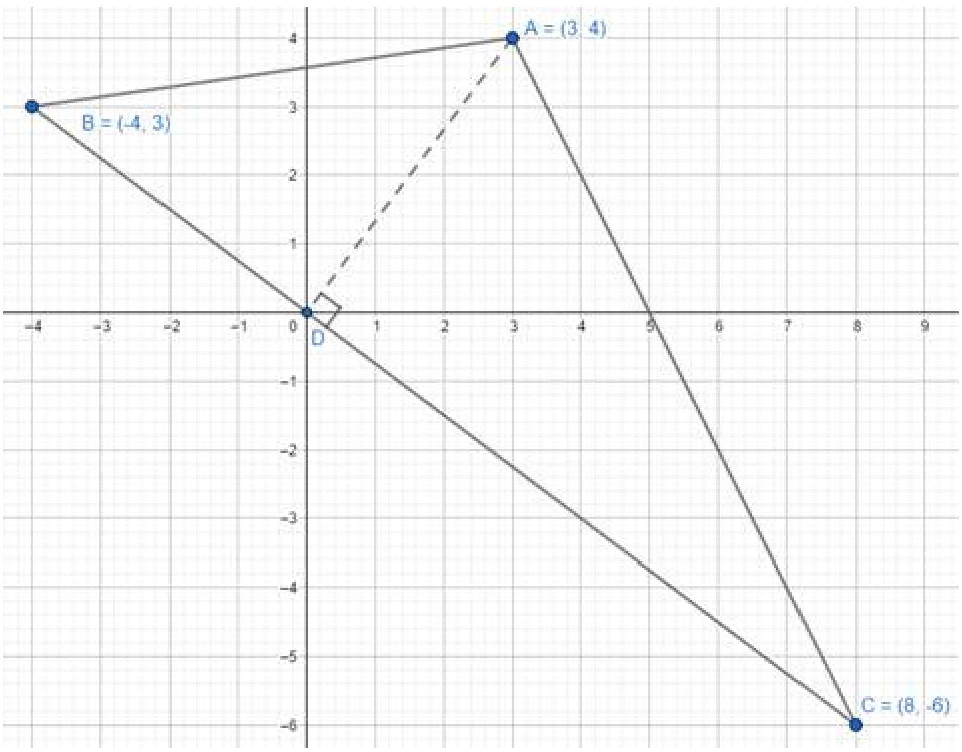
⇒ Area = $\frac{1}{2} \times [3(3 - (-6)) + (-4)(-6 - 4) + 8(4 - 3)]$

⇒ Area = $\frac{1}{2} \times [27 + 40 + 8]$

⇒ Area = $\frac{1}{2} \times 75$

⇒ Area = 37.5 unit^2 … (i)

Consider AD is the perpendicular dropped on BC as shown



So, for ΔABC BC is the base then AD is the height

Area of triangle = $\frac{1}{2} \times$ base × height … (ii)

Length of BC can be calculated by distance formula because coordinates of B and C are known

⇒ Distance BC = $\sqrt{(8-(-4))^2+(-6-3)^2}$

= $\sqrt{12^2+9^2}$

= $\sqrt{144+81}$

= $\sqrt{225}$

= 15 units

Distance BC = 15 units

Using (i) and (ii)

⇒ $37.5 = \frac{1}{2} \times BC \times AD$

⇒ $37.5 = \frac{1}{2} \times 15 \times AD$

⇒ $75 = 15 \times AD$

⇒ AD = 3 units

area of triangle is 37.5 $unit^2$ and the perpendicular length drawn from the point A on BC i.e. AD = 3 units

8. Question

In triangle ABC, co-ordinate of A is (2, 5) and the centroid of triangle is (–2, 1), let us find the co-ordinate of mid point of BC.

Answer

The ΔABC with coordinates of A as (2, 5) and assuming B and C coordinates to be (x_2, y_2) and (x_3, y_3) respectively as shown

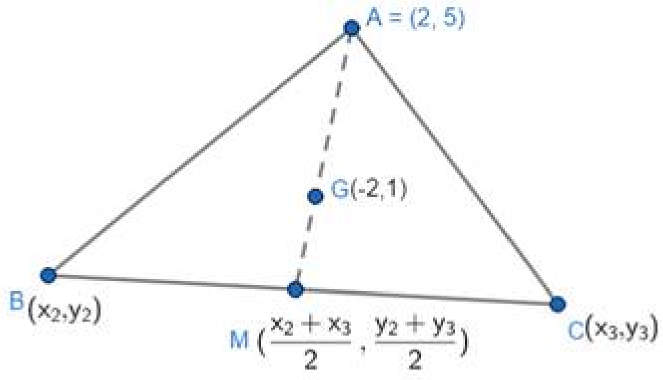
M is the midpoint of segment BC with coordinates as shown and G is the centroid

The vertices of ΔABC with their coordinates are

A = (x_1, y_1) = (2, 5) and

B = (x_2, y_2) and

C = (x_3, y_3)



G = (-2, 1)

Centroid of a triangle is given by

$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\Rightarrow (-2, 1) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\Rightarrow (-2, 1) = \left(\frac{2+x_2+x_3}{3}, \frac{5+y_2+y_3}{3}\right)$

Equate x-coordinate and y-coordinate

$\Rightarrow \frac{2+x_2+x_3}{3} = -2$ and $\frac{5+y_2+y_3}{3} = 1$

$\Rightarrow 2 + x_2 + x_3 = -6$ and $5 + y_2 + y_3 = 3$

$\Rightarrow x_2 + x_3 = -8$ and $y_2 + y_3 = -2$

Divide by 2

$\Rightarrow \frac{x_2+x_3}{2} = -4$ and $\frac{y_2+y_3}{2} = -1$

Thus midpoint M of BC is (-4, -1)

9. Question

The co-ordinates of vertices of a triangle are (4, −3), (−5, 2) and (x, y); let us find the value of x and y, if the centroid of triangle is at origin.

Answer

Let the points of triangle be

$A = (x_1, y_1) = (4, -3)$ and

$B = (x_2, y_2) = (-5, 2)$ and

$C = (x_3, y_3) = (x, y)$

Centroid of triangle is origin hence G be the centroid G (0, 0)

Centroid of a triangle is given by

$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Substituting values

$\Rightarrow (0, 0) = \left(\frac{4+(-5)+x}{3}, \frac{-3+2+y}{3}\right)$

Equate x-coordinate and y-coordinate

$\Rightarrow \frac{4+(-5)+x}{3} = 0$ and $\frac{-3+2+y}{3} = 0$

$\Rightarrow x - 1 = 0$ and $y - 1 = 0$

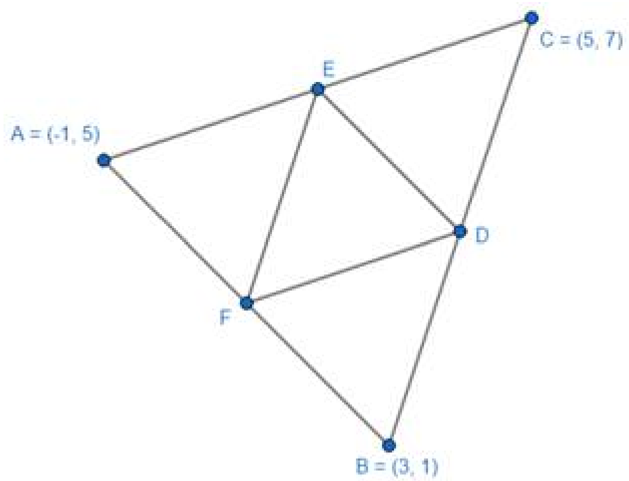
$\Rightarrow x = 1$ and $y = 1$

Hence (x, y) is (1, 1)

10. Question

The vertices at ΔABC are A(−1, 5), B(3, 1) and C(5, 7). D, E, F are the mid points of BC, CA and AB respectively. Let us find the area of triangular region ΔDEF and prove that ΔABC = 4 ΔDEF.

Answer



Let us find area of ΔABC

Area of triangle is given by formula

$\text{Area} = \frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Here,

$A = (x_1, y_1) = (-1, 5)$

$B = (x_2, y_2) = (3, 1)$

$C = (x_3, y_3) = (5, 7)$

Substituting values

$\Rightarrow$ Area(ΔABC) $= \frac{1}{2} \times [(-1)(1 - 7) + 3(7 - 5) + 5(5 - 1)]$

$\Rightarrow$ Area(ΔABC) $= \frac{1}{2} \times [6 + 6 + 20]$

$\Rightarrow$ Area(ΔABC) $= \frac{1}{2} \times 32$

$\Rightarrow$ Area(ΔABC) = 16 unit^2 … (i)

Let us now find the midpoints of BC, AC and AB i.e. points D, E and F respectively

Point F is the midpoint of AB

$\Rightarrow F = \left(\frac{-1+3}{2}, \frac{5+1}{2}\right)$

$\Rightarrow F = (1, 3)$

Point D is the midpoint of BC

$\Rightarrow D = \left(\frac{3+5}{2}, \frac{1+7}{2}\right)$

$\Rightarrow D = (4, 4)$

Point E is the midpoint of AC

$\Rightarrow E = \left(\frac{-1+5}{2}, \frac{5+7}{2}\right)$

$\Rightarrow E = (2, 6)$

Let us find area of ΔDEF

Area $= \frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Here,

$D = (x_1, y_1) = (1, 3)$

$E = (x_2, y_2) = (4, 4)$

$F = (x_3, y_3) = (2, 6)$

Substituting values

$\Rightarrow$ Area(ΔDEF) = $\frac{1}{2}$ × [1(4 – 6) + 4(6 – 3) + 2(3 – 4)]

$\Rightarrow$ Area(ΔDEF) = $\frac{1}{2}$ × [-2 + 12 + (-2)]

$\Rightarrow$ Area(ΔDEF) = $\frac{1}{2}$ × 8

$\Rightarrow$ Area(ΔDEF) = 4 unit^2 … (ii)

From (i) and (ii) we can conclude that area(ΔABC) = 4 × area(ΔDEF)

11 A. Question

The area of triangular region formed by three points (0, 4), (0, 0) and (–6, 0) is

A. 24 sq. unit

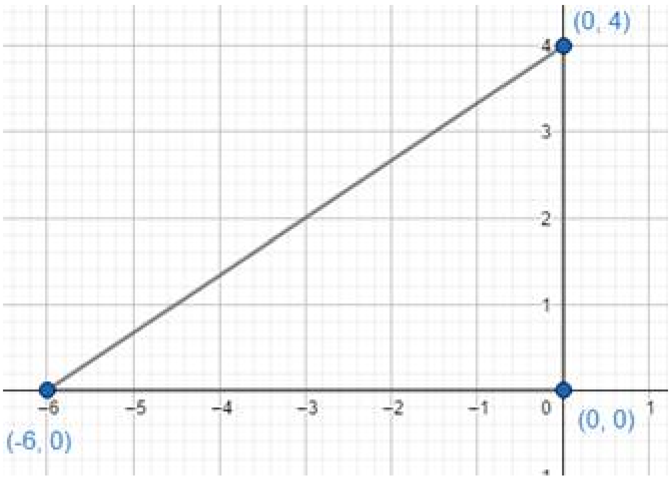
B. 12 sq. unit

C. 6 sq. unit

D. 8 sq. unit

Answer

If we plot these points roughly on the coordinate axis we can see that it's a right angled triangle with base = 6 units and height = 4 units



Thus, area = $\frac{1}{2}$ × base × height

$\Rightarrow$ area = $\frac{1}{2}$ × 6 × 4 = 12 unit^2

Another method is to use the formula.

Area = $\frac{1}{2}$ × [$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$]

11 B. Question

The co-ordinates of centroid of a triangle formed by the three points (7, –5), (–2, 5) and (4, 6) is

A. (3, –2)

B. (2, 3)

C. (3, 2)

D. (2, –3)

Answer

Let the points of triangle be

$(x_1, y_1) = (7, -5)$ and

$(x_2, y_2) = (-2, 5)$ and

$(x_3, y_3) = (4, 6)$

Centroid of a triangle is given by

$$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Substituting values

$$\Rightarrow G = \left(\frac{7+(-2)+4}{3}, \frac{-5+5+6}{3}\right)$$

$\Rightarrow G = (3, 2)$

11 C. Question

ABC is a right-angled triangle of which ∠ABC = 90°, co-ordinates of A and C are (0, 4) and (3, 0) respectively, then the area of triangle ACB is

A. 12 sq. unit

B. 6 sq. unit

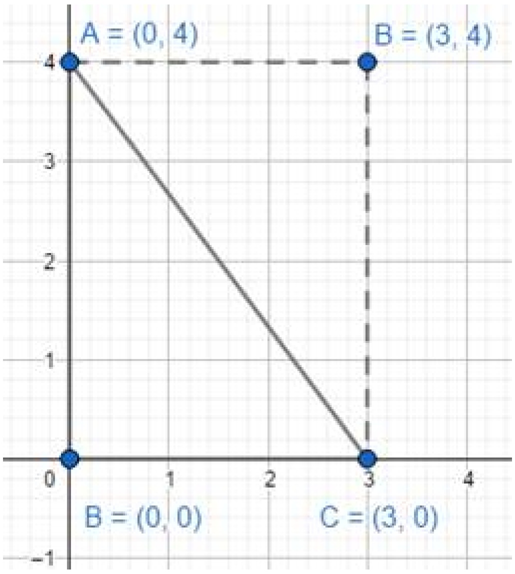
C. 24 sq. unit

D. 8 sq. unit

Answer

Point B can either be origin or (3, 4) as shown

Consider B to be the origin



Base of ΔABC = BC = 3 units

Height of ΔABC = AB = 4 units

Thus, area = $\frac{1}{2}$ × base × height

⇒ area = $\frac{1}{2} \times 3 \times 4 = 6$ unit^2

11 D. Question

If (0, 0), (4, −3) and (x, y) are collinear then

A. x = 8, y = −6

B. x = 8, y = 6

C. x = 4, y = −6

D. x = −8, y = 6

Answer

Let A = (0, 0), B = (4, -3) and C = (x, y)

As the points are collinear area of triangle formed by these points is 0

Area = $\frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Here,

$(x_1, y_1) = (0, 0)$

$(x_2, y_2) = (4, -3)$

$(x_3, y_3) = (x, y)$

Substituting values

⇒ $0 = \frac{1}{2} \times [0(-3 - y) + 4(y - 0) + x(0 - (-3))]$

⇒ $0 = 0 + 4y + 3x$

⇒ $4y + 3x = 0$

Here there are infinite values for x and y which will satisfy the equation $4y + 3x = 0$ which means there are infinite points

This can also be seen geometrically that is if line is passing through (0, 0) and (4, -3) there are infinite points on this line.

We can select the correct option by substituting the values given in option in equation $4y + 3x = 0$ and if it satisfies then that is the correct option

x = 8, y = -6 satisfies the equation $4y + 3x = 0$

x = −8, y = 6 satisfies the equation $4y + 3x = 0$

Now, C = (8, -6) or C = (-8, 6), but for A, B and C to be collinear

AB + BC = ACWe know, By distance formula, Distance between two points $X(x_1, y_1)$ and $Y(x_2, y_2)$ is

$$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Case I: C = (8, -6)

AB $= \sqrt{(4 - 0)^2 + (-3 - 0)^2}$

$= \sqrt{16 + 9} = \sqrt{25} = 5$ units

BC $= \sqrt{(8 - 4)^2 + (-6 + 3)^2}$

$= \sqrt{16 + 9} = \sqrt{25} = 5$ units

AC $= \sqrt{(8 - 0)^2 + (-6 - 0)^2}$

$= \sqrt{64 + 36} = \sqrt{100} = 10$ units

Clearly, AB + BC = AC

Hence, (0, 0), (4, -3) and (8, -6) are collinear

Case II: C = (-8, 6)

AB $= \sqrt{(4 - 0)^2 + (-3 - 0)^2}$

$= \sqrt{16 + 9} = \sqrt{25} = 5$ units

$BC = \sqrt{(-8-4)^2 + (6+3)^2}$

$= \sqrt{144+81} = \sqrt{225} = 15$ units

$AC = \sqrt{(-8-0)^2 + (6-0)^2}$

$= \sqrt{64+36} = \sqrt{100} = 10$ units

Clearly, AB + BC ≠ AC

Hence, (0, 0), (4, -3) and (8, -6) are not collinear.

[In this case, BA + AC = BC, ⇒ B, A and C are collinear]

Hence, Correct option is (a)

11 E. Question

If in triangle ABC, the co-ordinates of vertex A is (7, –4) and centroid of triangle is (1, 2), then the co-ordinates a mid point of BC is

A. (–2, –5)

B. (–2, 5)

C. (2, –5)

D. (5, –2)

Answer

The ΔABC with coordinates of A as (7, -4) and assuming B and C coordinates to be (x_2, y_2) and (x_3, y_3) respectively as shown

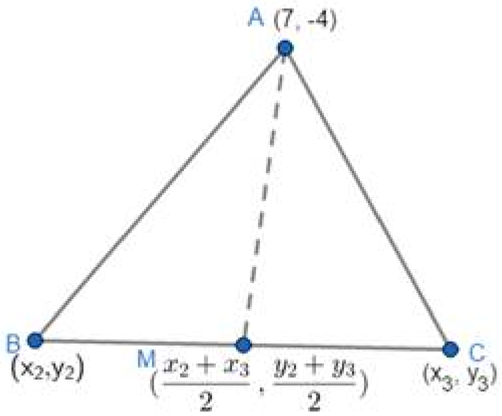
M is the midpoint of segment BC with coordinates as shown and G is the centroid

The vertices of ΔABC with their coordinates are

$A = (x_1, y_1) = (7, -4)$ and

$B = (x_2, y_2)$ and

$C = (x_3, y_3)$



G = (1, 2)

Centroid of a triangle is given by

G = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

⇒ (1, 2) = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

⇒ (1, 2) = $\left(\frac{7+x_2+x_3}{3}, \frac{-4+y_2+y_3}{3}\right)$

Equate x-coordinate and y-coordinate

⇒ $\frac{7+x_2+x_3}{3}$ = 1 and $\frac{-4+y_2+y_3}{3}$ = 2

⇒ 7 + x_2 + x_3 = 3 and – 4 + y_2 + y_3 = 6

⇒ x_2 + x_3 = -4 and y_2 + y_3 = 10

Divide by 2

⇒ $\frac{x_2+x_3}{2}$ = -2 and $\frac{y_2+y_3}{2}$ = 5

Thus midpoint M of BC is (-2, 5)

12 A. Question

The co-ordinates of mid-points of the sides of a triangle ABC are (0, 1), (1, 1) and (1, 0); let us find the co-ordinates of its centroid.

Answer

Let the vertices of ΔABC be

A = (x_1, y_1) and

B = (x_2, y_2) and

C = (x_3, y_3)

Centroid of a triangle is given by

$$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Let (0, 1), (1, 1) and (1, 0) be midpoints of AB, BC and AC respectively

Point (0, 1) is the midpoint of AB

$$\Rightarrow (0, 1) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Equating x and y coordinates

$$\Rightarrow \frac{x_1+x_2}{2} = 0 \text{ and } \frac{y_1+y_2}{2} = 1$$

$\Rightarrow x_1 + x_2 = 0$ … (a) and $y_1 + y_2 = 2$ … (i)

Point (1, 1) is the midpoint of BC

$$\Rightarrow (1, 1) = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

Equating x and y coordinates

$$\Rightarrow \frac{x_2+x_3}{2} = 1 \text{ and } \frac{y_2+y_3}{2} = 1$$

$\Rightarrow x_2 + x_3 = 2$ … (b) and $y_2 + y_3 = 2$ … (ii)

Point (1, 0) is the midpoint of AC

$$\Rightarrow (1, 0) = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$$

Equating x and y coordinates

$$\Rightarrow \frac{x_1+x_3}{2} = 1 \text{ and } \frac{y_1+y_3}{2} = 0$$

$\Rightarrow x_1 + x_3 = 2$ … (c) and $y_1 + y_3 = 0$ … (iii)

Adding equations (a), (b) and (c) & adding equations (i), (ii) and (iii) we get

$\Rightarrow x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 4$ and $y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 4$

$\Rightarrow 2\ (x_1 + x_2 + x_3) = 4$ and $2\ (y_1 + y_2 + y_3) = 4$

$\Rightarrow x_1 + x_2 + x_3 = 2$ and $y_1 + y_2 + y_3 = 2$

Divide by 3

$$\Rightarrow \frac{x_1+x_2+x_3}{3} = \frac{2}{3} \text{ and } \frac{y_1+y_2+y_3}{3} = \frac{2}{3}$$

Hence centroid is $G = \left(\frac{2}{3}, \frac{2}{3}\right)$

12 B. Question

The co-ordinates of centroid of triangle is (6, 9) and two vertices are (15, 0) and (0, 10); let us find the co-ordinates of third vertex.

Answer

Let the vertices of triangle be

A = (x_1, y_1) = (15, 0) and

B = (x_2, y_2) = (0, 10) and

C = (x_3, y_3)

Centroid G = (6, 9)

Centroid of a triangle is given by

G = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

⇒ (6, 9) = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Substituting values

⇒ (6, 9) = $\left(\frac{15+0+x_3}{3}, \frac{0+10+y_3}{3}\right)$

Equating x and y coordinates

⇒ $\frac{15+x_3}{3}$ = 6 and $\frac{10+y_3}{3}$ = 9

⇒ 15 + x_3 = 18 and 10 + y_3 = 27

⇒ x_3 = 3 and y_3 = 17

Hence the coordinates of third vertex is (3, 17)

12 C. Question

If the three points (a, 0), (0, b) and (1, 1) are collinear then let us show that

$$\frac{1}{a}+\frac{1}{b}=1.$$

Answer

Let the collinear points be

A = (x_1, y_1) = (a, 0) and

B = (x_2, y_2) = (0, b) and

$C = (x_3, y_3) = (1, 1)$

The area of triangle formed by joining points A, B and C will be 0 because A, B and C are collinear

Area of triangle is given by

Area $= \frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

$\Rightarrow 0 = \frac{1}{2} \times [a(b - 1) + 0(1 - 0) + 1(0 - b)]$

$\Rightarrow 0 = ab - a + (-b)$

$\Rightarrow a + b = ab$

Divide throughout by ab

$\Rightarrow \frac{a}{ab} + \frac{b}{ab} = \frac{ab}{ab}$

$\Rightarrow \frac{1}{b} + \frac{1}{a} = 1$

Hence proved

12 D. Question

Let us calculate the area of triangular region formed by the three points (1, 4), (−1, 2) and (−4, 1)

Answer

Area of triangle is given by formula

Area $= \frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of triangle

Here,

(x_1, y_1) is (1, 4)

$\Rightarrow x_1 = 1$ and $y_1 = 4$

(x_2, y_2) is (-1, 2)

$\Rightarrow x_2 = -1$ and $y_2 = 2$

(x_3, y_3) is (-4, 1)

$\Rightarrow x_3 = -4$ and $y_3 = 1$

Hence substituting values in formula for area we get

Area = $\frac{1}{2}$ × [1(2 – 1) + (-1)(1 – 4) + (-4)(4 – 2)]

Area = $\frac{1}{2}$ × [1 + 3 + (-8)]

Area = $\frac{1}{2}$ × (-4)

Area = -2

Area cannot be negative

Area = 2 unit^2

12 E. Question

Let us write the co-ordinates of centroid of triangle formed by the three points (x – y, y – z), (–x, –y) and (y, z).

Answer

Let the vertices of triangle be

A = (x_1, y_1) = (x – y, y – z) and

B = (x_2, y_2) = (–x, –y) and

C = (x_3, y_3) = (y, z)

Centroid of a triangle is given by

$$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Substituting values

$$\Rightarrow G = \left(\frac{(x-y)+(-x)+y}{3}, \frac{(y-z)+(-y)+z}{3}\right)$$

$$\Rightarrow G = \left(\frac{x-y-x+y}{3}, \frac{y-z-y+z}{3}\right)$$

⇒ G = (0, 0)

Centroid is (0, 0)