Introduction to Euclid's Geometry

Solve each of the following question using appropriate Euclid's axiom :

1) Two salesmen make equal sales during the month of August. In September, each salesman doubles his sale of the month of August. Compare their sales in September.

2) It is known that $x + y = 10$ and that $x = z$. Show that $z + y = 10$?

3) Look at the Fig. 5.3. Show that length AH > sum of lengths of AB + BC + CD.

A B C D E F G H

Fig. 5.3

4) In the Fig.5.4, we have
   AB = BC, BX = BY. Show that AX = CY.

5) In the Fig.5.5, we have
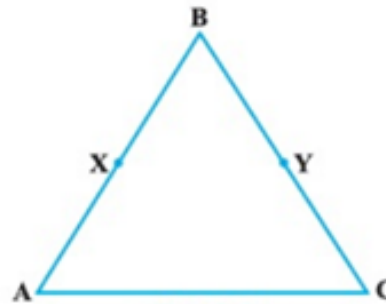   X and Y are the mid-points of AC and BC and
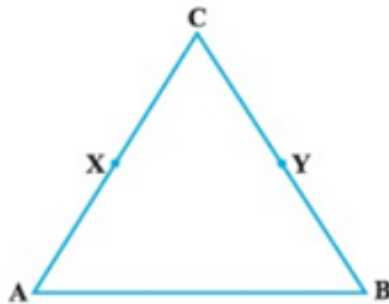   AX = CY. Show that AC = BC.

Fig. 5.4

Fig. 5.5

6) In the Fig.5.6, we have

   $BX = \dfrac{1}{2} AB$

   $BY = \dfrac{1}{2} BC$ and AB = BC. Show that

   BX = BY.

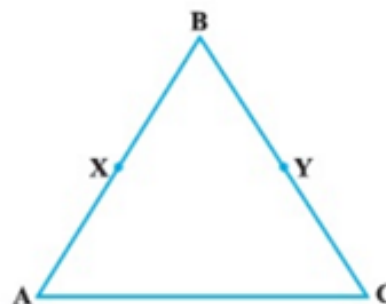Fig. 5.6

7) In the Fig.5.7, we have
$\angle 1 = \angle 2$, $\angle 2 = \angle 3$. Show that $\angle 1 = \angle 3$.

8) In the Fig. 5.8, we have
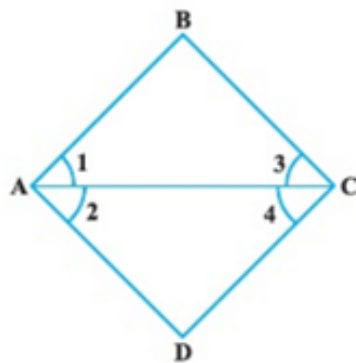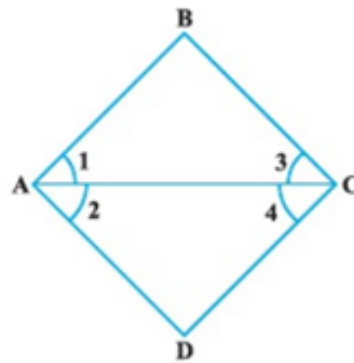$\angle 1 = \angle 3$ and $\angle 2 = \angle 4$. Show that $\angle A = \angle C$.

Fig. 5.7

Fig. 5.8

9) In the Fig. 5.9, we have
$\angle ABC = \angle ACB$, $\angle 3 = \angle 4$. Show that $\angle 1 = \angle 2$.

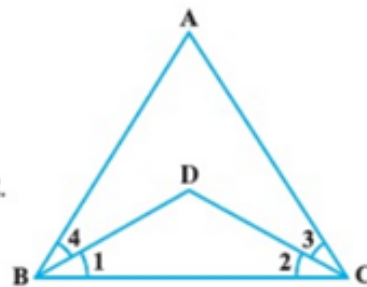10) In the Fig. 5.10, we have
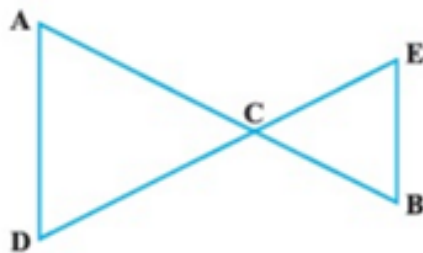$AC = DC$, $CB = CE$. Show that $AB = DE$.

Fig. 5.9

Fig. 5.10

11) In the Fig. 5.11, if $OX = \dfrac{1}{2}XY$, $PX = \dfrac{1}{2}XZ$
and $OX = PX$, show that $XY = XZ$.
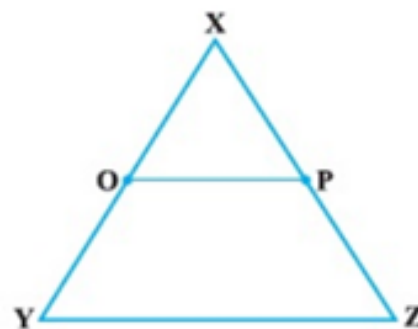
Fig. 5.11

12) In the Fig.5.12 :

(i) $AB = BC$, M is the mid-point of AB and N is the mid- point of BC. Show that $AM = NC$.

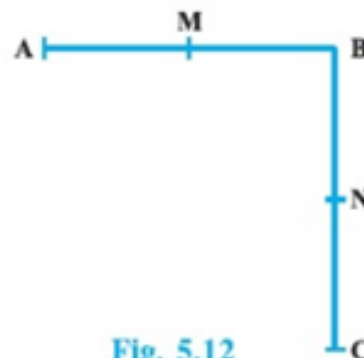(ii) $BM = BN$, M is the mid-point of AB and N is the mid-point of BC. Show that $AB = BC$.

Fig. 5.12

Write whether the following statements are **True** or **False**? Justify your answer :

13) Euclidean geometry is valid only for curved surfaces.

14) The boundaries of the solids are curves.

15) The edges of a surface are curves.

16) The things which are double of the same thing are equal to one another.

17) If a quantity B is a part of another quantity A, then A can be written as the sum of B and some third quantity C.

18) The statements that are proved are called axioms.

"For every line l and for every point P not lying on a given line l, there exists a unique line m passing through P and parallel to l " is known as Playfair's axiom.

19) Two distinct intersecting lines cannot be parallel to the same line.

20) Attempts to prove Euclid's fifth postulate using the other postulates and axioms led to the discovery of several other geometries.

21) Read the following statement :

An equilateral triangle is a polygon made up of three line segments out of which two line segments are equal to the third one and all its angles are 60° each.

Define the terms used in this definition which you feel necessary. Are there any undefined terms in this? Can you justify that all sides and all angles are equal in a equilateral triangle.

22) Study the following statement:

"Two intersecting lines cannot be perpendicular to the same line".

Check whether it is an equivalent version to the Euclid's fifth postulate. [Hint : Identify the two intersecting lines l and m and the line n in the above statement.]

23) Read the following statements which are taken as axioms :

(i) If a transversal intersects two parallel lines, then corresponding angles are not necessarily equal.

(ii) If a transversal intersect two parallel lines, then alternate interior angles are equal.

Is this system of axioms consistent? Justify your answer.

24) Read the following two statements which are taken as axioms :

(i) If two lines intersect each other, then the vertically opposite angles are not equal.

(ii) If a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180°.

Is this system of axioms consistent? Justify your answer.

25) Read the following axioms:

(i) Things which are equal to the same thing are equal to one another.

(ii) If equals are added to equals, the wholes are equal.

(iii) Things which are double of the same thing are equal to one another.

Check whether the given system of axioms is consistent or inconsistent.