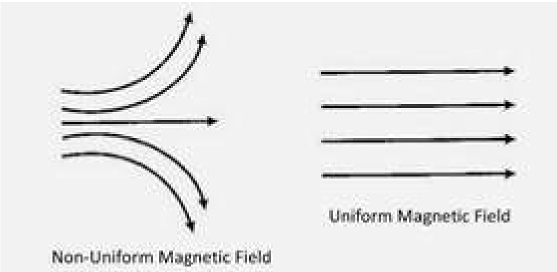
38. Electromagnetic Induction

Short Answer

Answer.1



Given that the magnetic field is non-uniform. The magnetic field will not be uniform throughout the area of the loop. Due to non-uniformity, the magnetic flux lines will be random. In random motion, the filed does not vary with time. If there is no change in the magnetic field, it can't induce emf in the loop. So if the metallic loop is placed in a non-uniform magnetic field, the filed will not induce emf in it. Thus, Non-uniform magnetic field does not induce the emf.

Answer.2

In the above circuit, the inductor is connected to a battery through a switch. In that when the switch is closed, the current will induce in the circuit. The magnetic flux will be increased. Change in flux will induce emf. When the switch has opened the drop in the current occur is more than the increase in the current when the switch is closed.

Due to this large amount of emf is induced in the inductor when the switch is closed as compared to when the switch is opened.

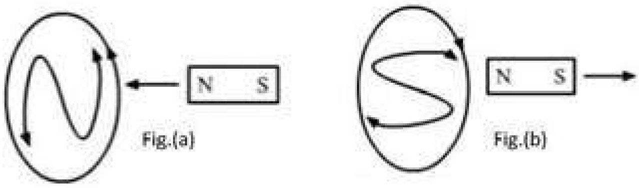
Answer.3

If the two ends of the coil of the moving-coil galvanometer is connected together, then it will act as a closed loop. thus, the coil no longer acts as an inductor. Therefore, all oscillations stop at once.

While If the ends of the coil are not connected, then the coil will act as an indicator, it will oscillate up to the current in it decays slowly.

Answer.4

Lenz's law: The direction of the induced current is such that it opposes the magnetic field that has induced it.



See fig.(a). In that, the north pole is facing towards the loop. According to Lenz's law, the direction of induced current will be anticlockwise. And the magnet is coming towards the loop. Then the flux through the loop will increase; it will create a magnetic field. This newly generated magnetic field will cancel the original magnetic field. Therefore the loops get repelled.

See fig.(b) in which the magnetic is moving away from the loop. It will lead to a decrease in the magnetic field intensity. Therefore, flux through the loop decreases. Therefore, flux through the loop decreases. According to Lenz's law in induced current produces a magnetic field in the opposite direction of the original field. Hence the loop attracts the magnet.

Answer.5

Let us consider the loops as A and B. When the battery is connected to loop A; the current will flow in a clockwise direction. So the direction of the magnetic field due to the current will be towards left as seen from the loop B. Due to a sudden change in flux through loop A, a current will induce in loop B. But it will only be induced for a moment when the current suddenly jumps from zero to a constant value. After it attained a constant value, there will be no induced current in loop B.

According to the Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it. So the induced current in loop B will be in the opposite direction to the magnetic field of loop A. So the direction of induced current in loop B will be in anticlockwise direction. And the current through loop B will end when the current through loop A becomes zero. Because the directions of the currents in the loops are opposite, they will repel each other.

Answer.6

When the battery is suddenly disconnected, a current induced in B due to a sudden change in the flux through it. But it is only induced for a moment when the current suddenly falls to zero. According to the Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it. So that the induced current is such that it increases the decrease magnetic field. Therefore, if the current in loop A is in a clockwise direction, then the induced current in loop B will also be the clockwise direction. Hence two loops will attract each other.

Conclusion: If two circular loops are placed coaxially. A battery is connected to one of the loops. After some time, the battery is disconnected then the loops will attract each other.

Answer.7

If the magnetic field is suddenly changing, it will induce eddy currents on the walls of the copper box. And because of these eddy currents there will be a magnetic field and that will be in the opposite direction. Copper is a good conductor of electricity, so a magnetic field due to eddy currents will have strong strength. This newly generated magnetic field will cancel the original magnetic field. So the magnetic field inside the box will become zero. In this way, the copper box will become a shield and protect the objects inside in it form varying magnetic fields.

Answer.8

When solid waste (metallic and non-metallic particles) allowed to slide over a permanent magnet, an emf will be induced in metallic particles. According to the Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it. So the induced emf in the metallic particles will oppose the downward notion along the inclined plane of the permanent magnet. And non-metallic particles are free from these effects. In this way, metallic particles slow down and get separated from the non-metallic particles.

Answer.9

Non-metallic or insulating materials are free of the effects of induced eddy currents or induced emf. According to the Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it. Means the induced eddy current opposes its cause. That’s why an aluminium bar will falls slowly through a small region containing a magnetic field.

Answer.10

When the circuit is on, eddy currents will produce on the surface of the metallic Bob. Eddy currents will generate thermal energy. Thermal energy comes as the loss of the kinetic energy of the Bob. Therefore, oscillations are more quickly damped when the circuit is on compared to when the circuit is of the bob.

Hence, oscillation is more quickly damped when the circuit is on compared to when the circuit is off.

Answer.11

(a) The largest mutual induction

Mutual induction will be larger when the two loops are placed coaxially. Then the flux through a loop due to another loop is large. Hence, we will get the largest mutual inductance.

(b) The mutual inductance will be small when the two loops are placed such that their axis are perpendicular to each other. The flux through the loop due to another loop will be small. Therefore, we will get the smallest mutual inductance.

Thus, we can conclude that to get the largest mutual inductance the loop has to place coaxially and to get smallest mutual inductance, the axis of the loops has to place perpendicularly.

Answer.12

Self-inductance is given by

$$L = \mu_0 n^2 A l$$

Self-inductance per unit length is given by

$$\frac{L}{l} = \mu_0 n^2 A$$

Where μ_0 is the permeability of the free space

n is the number of turns per unit length

A is Area of a cross section of the solenoid

L is the self-inductance

l is the length the wire

Therefore, self-inductance at the centre and that near its ends will be the same since the self -induction is independent of the distance of the point from the centre of the solenoid.

Answer.13

Energy density is given by

Energy density, $u = \frac{B^2}{2\mu_0}$

Where

U is the energy density of the solenoid

B is the magnetic field intensity of the solenoid

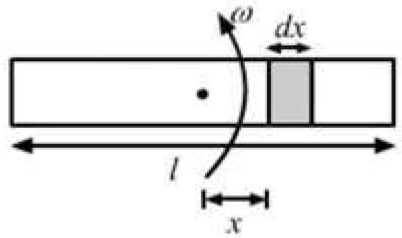
μ_0 is the permeability of the free space.

In the solenoid, energy is stored in the form of magnetic field. For the constant flow of current, the magnetic field inside the solenoid is uniform. So, the energy density in a solenoid is constant. Nothing is greater. The energy density at all the points inside a solenoid is the same.

Objective I

Answer.1

Let us consider a small element at a distance from the centre of the rod rotating with angular velocity about its axis perpendicular bisector.



Formula used: The emf induced is given by

$\epsilon = Bvl$

The emf is induced in the rod because of the small element is

$d\epsilon = Bwxdx$

Where $d\epsilon$ is the emf induced in that small element

B is the magnetic field

wx is the angular velocity of the small element

dx is the length of the small element

The emf induced across the centre and the end of the rod is found by the integration of the above relation with respect to x from limit o to $\frac{l}{2}$

$$\int d\epsilon = \int_0^{l/2} Bwxdx$$

$$\epsilon = Bw\left[\frac{x^2}{2}\right]_0^{\frac{l}{2}}$$

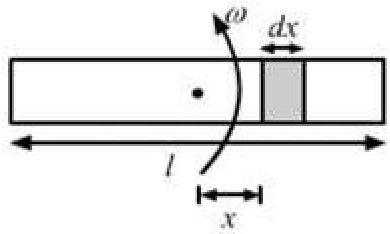
$$\epsilon = \frac{Bwl^2}{8}$$

The potential difference between the centre of the rod and an end is $\frac{Bwl^2}{8}$.

Thus, option B is correct.

Answer.2

Let us consider a small element at a distance from the center of the rod rotating with angular velocity about its axis perpendicular bisector.



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B is the magnetic field

wx is the angular velocity of the small element

dx → length of the small element

The emf induced across the centre and the end of the rod is

$$\int d\epsilon = \int_0^{l/2} Bwxdx$$

$$\epsilon = Bw\left[\frac{x^2}{2}\right]_0^{\frac{l}{2}}$$

$$\epsilon = \frac{Bwl^2}{8}$$

The potential difference between the two ends is

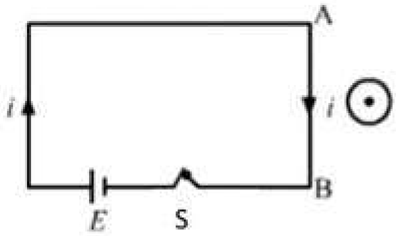
$$\frac{Bwl^2}{8} - \frac{Bwl^2}{8} = 0$$

The potential difference between the two ends of the rod is zero.

Answer.3

Let us consider tow points on the circuits A and B.

When the switch S is closed, the current will flow through A&B. Due to this magnetic field introduce in the loop.

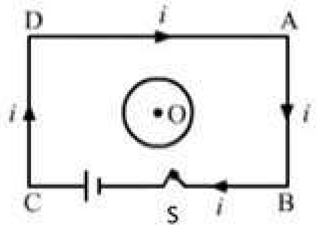


According to Lenz's law the induced current is such that it opposes the increase in the magnetic field that induces it.

So, the induced current will be in clockwise direction, and it opposes the increase in the magnetic field in the upward direction in the loop. When the switch S is opened, the current will fall due to this magnetic field in the loop will decrease. According to Lenz's law, the induced current will be in the anti-clockwise direction opposing the direction in the magnetic field in an upward direction in the loop.

If the switch is closed and after some time it is opened again, the closed loop will show a clockwise current-pulse and then an anticlockwise current-pulse

Answer.4



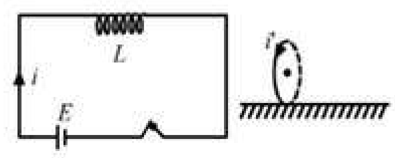
According to Lenz's law, the induced current is such that it opposes the increase in the magnetic field that induces it. So the induced current will be anticlockwise when the switch S is closed. When the switch S is open, the current will suddenly fall. Then the magnetic field at the centre of the loop will decrease. Then the induced emf will be in the clockwise direction.

If the closed loop is completely enclosed in the circuit containing the switch.

Answer.5

The flux linked with the copper tube will change because of the motion of the magnet. This will produce an eddy current in the body of the copper tube. According to the Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it. So the induced Current will be in the opposite direction to the magnet field of the bar magnet. And it will slow the fall of the magnet. Then negative acceleration nothing retarding force will act on the bar magnet. If the velocity of the magnet is increased then the retarding force will also increase. And it will happen up to this retarding force is equal to the force of gravity. Then the total force acting on the bar magnet will become zero. Then the magnet will move with almost a constant speed. So option B is correct.

Answer.6



For the circuit

Emf introduced in the solenoid, $E = -L\frac{di}{dt}$

Where

L is the self-inductance of the solenoid

I is the current in the solenoid.

The direction of the current in the loop is clockwise. When the switch is closed the current will flow in the circuit. Therefore, the current on the solenoid will be increase. Then it will induce a current in the copper ring which is placed along a axis of solenoid. According to Lenz's law the induced current is such that it opposes the increase in the magnetic field that induces it. So the induce current in the copper ring will be anticlockwise. Because of the opposite direction of the currents the ring will repel. So it will move away from the solenoid.

Horizontal solenoid connected to a battery and a switch. A copper ring is placed on a frictionless track, the axis of the ring being along the axis of the solenoid. As the switch is closed, the ring will move away fro the solenoid.

Answer.7

Formula used: emf is given by

$\epsilon = vBl ----(1)$

Where v is the speed

B is the magnetic field

l is the length

That above formula states that an emf induced by moving a conductor of length l with some velocity v in a magnetic field B.

$\epsilon = \frac{d\Phi}{dt} ----(2)$

Where Φ is the electric flux through the conductor

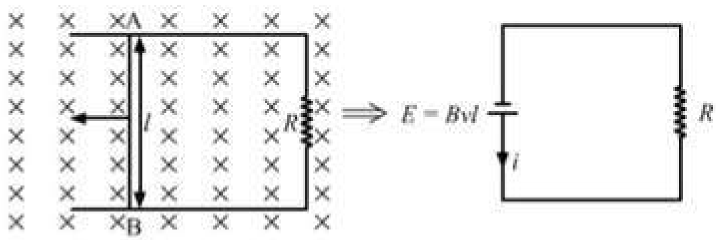
ϵ emf induced in the conductor

Formula (2) states that an emf can be induced by changing the magnetic field that causes the change in flux through a conductor in a loop.

So option A is the answer.

Answer.8

Draw the equivalent circuit



The formula used: The emf induced across A and B is

$$\epsilon = vBl$$

Where v is the speed

B is the magnetic field

l is the length

The induced emf will serve as a voltage source. The direction of the current is anticlockwise, according to Lenz's law.

According to Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it.

$$P = \frac{\epsilon^2}{R}$$

Putting the value in the above equation, we get

$$= \frac{(v^2 B^2 l^2)}{R}$$

The induced emf depends on the length of the wire but not on the shape of the wire.

If the wire is replaced by a semicircular wire, the induced emf will be same.

Answer.9

Formula used: The emf developed across the ends of the loop is given by

$$\epsilon = vBl$$

Where v is the speed

B is the magnetic field

L is the length

The power delivered to the loop is

$$P = \frac{\epsilon^2}{R}$$

Where P is the Power delivered to the loop

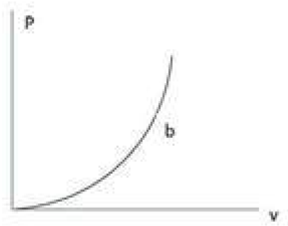
R is the resistance

ϵ is the emf induced

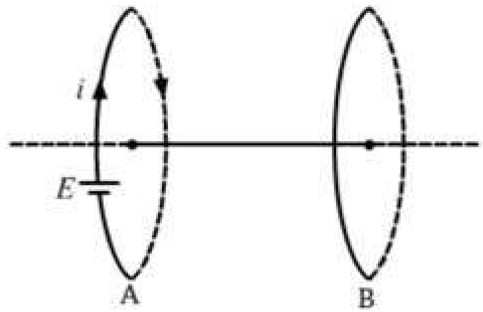
$$P = \frac{(vBl)^2}{R}$$

$$P = \frac{v^2 B^2 l^2}{R}$$

From the above equation, P is proportional to v^2. Curve b showing this relation.



Answer.10



By the right-hand screw rule, the magnetic field will be towards the left side. Due to the increase in resistance with temperature, the current through the loop will decrease with time. This change in current will induce the current in loop B. According to Lenz's law, the induced current is such that it opposes the increase in the magnetic field that induces it. So the direction of induced current in loop B will be in a clockwise direction. Because of the directions of currents, they will attract each other.

Thus, Option A is correct.

Answer.11

The magnetic field inside the solenoid is parallel to its axis. If the plane of the loop contains the axis of the solenoid, the angle between the area vector of a circular loop and the magnetic field will be zero.

The formula used: The flux through the circular loop is given by

$\Phi = BA\ cos\theta$

Where Φ is the electric flux

B is the magnetic field due to the solenoid

A is the Area of the circular loop

is the Angle between magnetic field and area vector

$\Phi = BAcos0°, \cos 0° = 1$

θ

$\Phi = BA$ =constant

Then the induced emf is given by

$$\epsilon = \frac{d\Phi}{dt}$$

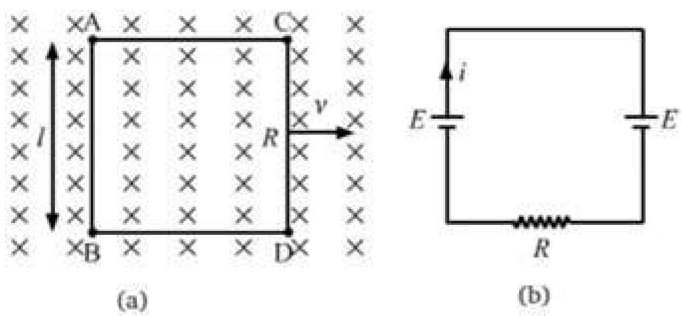
$$= \frac{d}{dt}(BA)$$

$\epsilon = 0$ Because BA is constant

The induced emf does not depend on the varying current through the solenoid. The induced emf will be zero for a constant flux through the loop. So no current will be induced in the loop.

Answer.12

if we draw the equivalent circuit, it will be look like



The formula used: The emf induces across the ends AB and CD is given by

$$E = vBl$$

Where E is the emf

V is the velocity

B is the magnetic field

l is the length

Apply KVL in the loop in fig(b).

$$E - E + iR = 0$$

$$i = 0$$

Here the current I become zero means that there is no current induced in the loop. So the current induced in the loop will be zero.

Objective II

Answer.1

A bar magnet is moving along the axis of a copper ring. The movement of a magnet there will be a magnetic field. Then the current will be induced in the copper ring. It is observed that the current is in an anticlockwise direction looking from the magnet. Then the magnetic field induced in the copper ring will be towards the observer (We are observing the current direction form the magnet).

According to Lenz's law, the induced current is such that it opposes the increase in the magnetic field that induces it. If the south pole faces the ring and the magnet is moving towards the then the current induced in the ring will be the clockwise direction. In the same case if the magnet is moving away from the ring, then the current will be in the anticlockwise direction. If the magnet is moving towards the ring and north pole is facing the ring, then the current induced in the ring will be in the anticlockwise direction. If the magnet is moving away from the ring, in this case, the induced current will be in a clockwise direction. So option B and C are correct.

Answer.2

The potential difference across the two ends is given by

$\epsilon = vBl$

Where ϵ is the emf or potential difference

v→ velocity of conducting rod

B is the magnetic field

L is the length of the rod

Consider some conditions,

i. The magnetic field is in the perpendicular direction to the velocity of the rod

ii. The magnetic field is in the perpendicular direction to the length of the rod

iii. The velocity of the rod is perpendicular to the direction to the length of the rod

In the above-mentioned conditions, only the potential difference across the two ends will be non zero.

In the given conditions in the A, B and C, the potential difference across the two ends is zero. So option D is correct.

Answer.3

Conducting loop is placed in a uniform magnetic field with its plane perpendicular to the field. An emf will be induced only when the magnetic flux changes. If there is no change in magnetic flux, then induced emf will be zero. An emf will be induced in the loop only when

i. The loop is rotated about a diameter

ii. Deforming the loop.

If the loop is deformed, the area of the loop inside the magnetic field changes. So magnetic flux will change leads to induce an emf. On rotating about its axis, the magnetic flux does not change. So an emf induced in the loop is zero.

Thus, Option C and D are correct.

Answer.4

A metal sheet is placed in front of the strong magnetic pole. This pole has q strong magnetic field. A strong magnetic pole will attract the magnet if the metal is magnetic then a force is needed to hold the metal sheet. If the metal is magnetic or not if we want to move the metal sheet away from that strong magnetic pole we need some force do it. Because of the movement, eddy currents will be induced in the sheet. These eddy currents produce thermal energy. Thermal energy comes at the cost of kinetic energy. Thus, the plates slow down. So we need a force to hold the sheet if the metal is magnetic. And also the sheet needs a force to move away from that strong magnetic filed even it is nonmagnetic.

So option A, C and D are correct.

Answer.5

The iron rod has high permeability. So if we insert the iron rod in the solenoid along its axis, the magnetic flux inside the solenoid will increases. The self-inductance of the coil is proportional to the permeability of the material inside the solenoid. Because of an increase in the permeability inside the coil, the self-inductance will also increase. Thus, a constant current I will be maintained in the solenoid. Then

1. magnetic field at the centre

2. Magnetic flux linked with the solenoid

3. self-inductance of the solenoid is increased if an iron rod is inserted in the solenoid along its axis.

Thus, option A, B and C is the correct option.

Answer.6

The two solenoids are identical. Therefore, the self-inductance of the two solenoids is the same.

The formula used: energy stored in the inductor is given by

$$U = \frac{1}{2}Li^2$$

Where U is the Magnetic energy

L is the self-Inductance

I is the Current

The current in both solenoids is the same. That implies magnetic field energy is also the same.

The power dissipated as heat is given by

$$P = \frac{i^2}{R}$$

Where P is the power

I is the current

R is the Resistance

The time constant is given by

$$t = \frac{L}{R}$$

Where t is the time constant

L is the self-Inductance

R is resistance is given by

$$R = \frac{\rho l}{A}$$

Where ρ is the resistivity

l is the length of the wire

A is the Area of cross section

The two solenoids are differed by the Area of the cross section A.

=>$R_{Thick} < R_{Thin}$

=> $t_{Thick} > t_{Thin}$

=> $P_{Thick} > P_{Thin}$

Thus, the time constant and Joule heat is different for the two solenoids. And self-inductance and magnetic field are the same for the both.

Answer.7

The formula used: At time t, the current in the LR circuit is given by

$$I = \frac{\epsilon}{R}\left(1 - e^{-\frac{tR}{L}}\right)$$

Where I is the Current

ϵ is the emf

R is the resistance

L is the self-inductance

At time $t = 0$ the current is given by

$$I = \frac{\epsilon}{R}\left(1 - e^{-\frac{0 \times R}{L}}\right)$$

At t=0, the current in the circuit is zero.

The magnetic field energy in the inductor is given by

$$U = \frac{1}{2}Li^2$$

Current is zero at t=0, So magnetic field energy is also zero.

Therefore, power delivers by the battery is also zero at t=0.

At t=0, the current is on the verge to start growing in the circuit. So there will an induced emf at that time to oppose the growing current.

In the LR circuit, at time t=0 there will be an induced emf in the inductor. Exactly at t=0, the current, magnetic field and power delivered by the battery are zero.

Answer.8

Due to electromagnetic induction, the emf will be induced in the rod. Induced emf is given by the formula end A becomes more positive because the direction of the induced emf is from A to B. This is because of the magnetic field exerts a same force equal to qvB on each of the electrons, where $q = -1.6 \times 10^{-16}$ C. According to Fleming's left hand rule (If a conductor is placed in a magnetic field then the force acting on the conductor is in the perpendicular direction to the magnetic field and current direction) the, end A becomes positive and, end B becomes negatively charged.

Conclusion: A rod AB moves with a uniform velocity v in a uniform magnetic field then the end A becomes positively charged.

Answer.9

The formula used: For RC circuit the time constant is

$$\tau = RC$$

Where t is the time constant

R is the resistance

C is the capacitance

Frequency $f = \frac{1}{\tau}$

Then the frequency of the RC circuit is given by

$$f_1 = \frac{1}{RC}$$

For LR circuit the time constant is given by

$$\tau = \frac{L}{R}$$

Where τ is the time constant

L is the inductance

R is the resistance

Then the frequency is given by

$$f_2 = \frac{R}{L}$$

Let's take a multiplication of f1 and f2

$$f_1 \times f_2 = \frac{1}{RC} \times \frac{R}{L}$$

$$f_1 f_2 = \frac{1}{LC}$$

$$=> \sqrt{f_1 f_2} = \frac{1}{\sqrt{LC}}$$

This combination $\sqrt{f_1 f_2} = \frac{1}{\sqrt{LC}}$ has a dimension of frequency.

The above combination has a dimension of frequency

i. RC has a frequency component $f_1 = \frac{1}{RC}$

ii. LR has a frequency component $f_2 = \frac{R}{L}$

iii. f1f2 has a frequency component $\sqrt{f_1 f_2} = \frac{1}{\sqrt{LC}}$

Thus, option A, B and C are correct.

Answer.10

Formula used:

The charge on capacitor at time t, when the switch is closed is

$$Q = \epsilon C\left(1 - e^{-\frac{t}{RC}}\right)$$

Where Q is the Charge

ϵ is the emf

T is the Time period

R is the Resistance

C is the Capacitance

At t=0, the charge on the capacitor is

$$Q = \epsilon C\left(1 - e^{-\frac{0}{RC}}\right) = 0$$

After a long time $t_0 = \infty$, the charge on the capacitor is

$$Q = \epsilon C\left(1 - e^{-\frac{\infty}{RC}}\right)$$

$$= \epsilon C$$

The current in inductor L is given by

$$I = \frac{\epsilon}{R}\left(1 - e^{-\frac{tR}{L}}\right)$$

Where I is the current

ϵ is the voltage or potential difference

R is the Resistance

L is the inductance

T is the time period

At initial we take, t=0, the current in the inductor is given by

$$I = \frac{\epsilon}{R}\left(1 - e^{-\frac{0\times R}{L}}\right)$$

$I = 0$

After a long time t_0=∞ is

$$I = \frac{\epsilon}{R}\left(1 - e^{-\frac{t_0 \times R}{L}}\right)$$

$$I = \frac{\epsilon}{R}\left(1 - e^{-\frac{\infty \times R}{L}}\right)$$

$$I = \frac{\epsilon}{R}$$

The charge on C long after t=0 is ϵC

The current in L long after t = t_0 is $\frac{\epsilon}{R}$

Exercises

Answer.1

(a)using faraday's law of induction

$$\int \vec{E}.d\vec{l} = \epsilon$$

Where ϵ is the emf or voltage

$\int \vec{E}.dl^{\rightarrow}$ has thus the dimensions of voltage

Voltage is given by formula

$$V = \frac{W}{Q}$$

Where

W=work done

Q=charge

Dimensions of W=$[ML^2T^{-2}]$

Dimensions of Q=$[AT]$

The dimensions of voltage can be given as

$$V = \frac{[ML^2T^{-2}]}{[AT]}$$

$V = [ML^2T^{-3}A^{-1}]$……(i)

(b) vBl is the motional emf developed due to motion of conductor of length l with velocity v in a magnetic field (B)

Therefore, vBl has same dimensions as of voltage

Therefore, by eqn.(i)

$vBl = [ML^2T^{-3}A^{-1}]$

(c) by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt}$$

Where

ϵ =emf produced

ϕ =flux of magnetic field

thus $\frac{d\phi}{dt}$ has same dimensions as of emf or voltage

therefore, by eqn.(i)

$$\frac{d\phi}{dt} = [ML^2A^{-1}T^{-3}]$$

Therefore $\int \vec{E}.d\vec{l}, vBl$ and $\frac{d\phi}{dt}$ all have same dimensional formula equal to $[ML^2A^{-1}T^{-3}]$

Answer.2

Given:

flux of magnetic field ϕ = at^2 + bt + c

magnitudes of a, b and c= 0.20,0.40 and 0.60

(a) We know that,

Dimensions on both sides of an equation are equal and only terms with same dimensions can be added

Therefore

Dimensions of ϕ, at^2, bt and c are same

Thus units of a are given by

$$a = \frac{\phi}{t^2} = \frac{\frac{\phi}{t}}{t}$$

Now,

by faraday's law of electromagnetic induction

$\epsilon = -\frac{d\phi}{dt}$...(i)

Where

ϵ =emf produced

ϕ =flux of magnetic field

therefore, units of $\frac{\phi}{t}$ is same as voltage i.e. Volt

$$\therefore \boldsymbol{unit\ of\ a = Volt/sec}$$

Unit of b is given by

$$b = \frac{\phi}{t}$$

Using eqn.(i)

$$\boldsymbol{unitof\ b = Volt}$$

SI unit of c is given by

$$c = \phi$$

$$\therefore \boldsymbol{SI\ unit\ of\ c \Rightarrow SI\ unit\ of\ flux \Rightarrow Weber}$$

(b)

We know that

by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt}$$

Where

ϵ =emf produced

ϕ =flux of magnetic field

here ϵ is given by

$$\epsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(at^2 + bt + c) = -(2at + b)$$

Putting the values of a, b and t=2sec we have

$$\epsilon = 2 \times 0.2 \times 2 + 0.4 = 1.2\, Volt$$

Therefore, the induced emf at t=2sec is given by 1.2Volts

Answer.3

Given:

Area of loop=$2.0 \times 10^{-3} m^2$

We know that

by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt}$$

Where

ϵ =emf produced

ϕ =flux of magnetic field

so, the average induced emf in the conducting loop between time intervals t_1 and t_2 is given by

$\epsilon = \frac{\Delta\phi_2 - \Delta\phi_1}{t_2 - t_1}$...(ii)

Also

Magnetic flux through the circular ring of area A is given by

$\phi = \vec{B}.\vec{A}$

Since loop is placed perpendicular to the field,

$\phi = BA$

(i) $t_1 = 0 \; and \; t_2 = 10ms$

Using eqn.(ii)

$\epsilon = -\frac{\Delta\phi_2 - \Delta\phi_1}{t_2 - t_1} = -\frac{A(B_2 - B_1)}{t_2 - t_1}$. (iii)

Putting the values of $t_1, t_2, B_1 \; and \; B_2$ we get

$$\epsilon = -2 \times \frac{10^{-3}(0.01)}{10 \times 10^{-3}} = -2 \times 10^{-3} = -2mV$$

(ii) $t_1 = 10ms \; and \; t_2 = 20ms$

Using eqn.(ii)

$\epsilon = -\frac{\Delta\phi_2 - \Delta\phi_1}{t_2 - t_1} = -\frac{A(B_2 - B_1)}{t_2 - t_1}$. (iii)

Putting the values of $t_1, t_2, B_1 \; and \; B_2$ we get

$$\epsilon = -2 \times \frac{10^{-3}(0.02)}{10 \times 10^{-3}} = -4 \times 10^{-3} = -4mV$$

(iii) $t_1 = 20ms \; and \; t_2 = 30ms$

Using eqn.(ii)

$\epsilon = -\frac{\Delta\phi_2 - \Delta\phi_1}{t_2 - t_1} = -\frac{A(B_2 - B_1)}{t_2 - t_1}$. (iii)

Putting the values of $t_1, t_2, B_1 \; and \; B_2$ we get

$$\epsilon = -2 \times \frac{10^{-3}(-0.02)}{10 \times 10^{-3}} = 4 \times 10^{-3} = 4mV$$

(iv) $t_1 = 30ms \; and \; t_2 = 40ms$

Using eqn.(ii)

$\epsilon = -\frac{\Delta\phi_2 - \Delta\phi_1}{t_2 - t_1} = -\frac{A(B_2 - B_1)}{t_2 - t_1}$. (iii)

Putting the values of t_1, t_2, B_1 and B_2 we get

$$\epsilon = -2 \times \frac{10^{-3}(-0.01)}{10 \times 10^{-3}} = 2 \times 10^{-3} = 2mV$$

∴ emf across the intervals

0-10ms =-2mV

10-20ms=-4mV

20-30ms=4mV

30-40ms=2mV

From the graph we can see that flux varies in a nonlinear fashion between time Intervals 10-20 ms and 20-30 ms and hence the derivative of flux wrt time is not constant in the given interval

∴ emf is not constant in time intervals 10-20 ms and 20-30ms

Answer.4

Given:

Radius if circular loop=5.0cm

Magnetic field intensity=0.50T

Area of circular loop $A = \pi r^2$

Initial magnetic flux through the loop is given by

$$\phi_1 = BAcos0° = BA$$

(As loop is placed perpendicular to magnetic field)

After loop is removed from the field after time Δt=0.50s the magnetic flux

$$\phi_2 = 0$$

(As no magnetic field passes through the loop)

We know that,

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1)/\Delta t$$

Where

$\phi_1\, and\ \phi_2$ are flux across the cross section at time intervals $t_1\, and\ t_2$ respectively.

Putting the values of $\phi_1, \phi_2\ and\ \Delta t$ we get

$$\epsilon = -\frac{0 - BA}{\Delta t} = \frac{B\pi r^2}{\Delta t} = 0.50 \times \pi \times \frac{(5 \times 10^{-2})^2}{0.50} = 25\pi \times 10^{-4}$$
$$= 7.8 \times 10^{-3} Volt$$

$\epsilon = 7.8 \times 10^{-3} Volt$

Therefore, average induced emf produced in the loop during this time interval is 7.8×10^{-3} Volts.

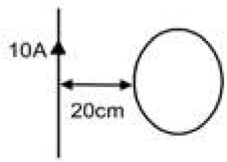
Answer.5

Given:

Area of circular loop $A\ = 1mm^2 = 1 \times 10^{-6} m^2$

Separation between wire and loop $d = 20cm = 20 \times 10^{-2} m$

Current through the wire $i = 10A$



We know that magnetic field (B) due to an infinitely long wire carrying a current i at a perpendicular distance d from the wire is given by

$$B = \frac{\mu_0 i}{2\pi d}$$

Also,

Average induced emf in time interval Δt is given by

$\epsilon = -(\phi_2 - \phi_1)/\Delta t$

Where

$\phi_1\, and\ \phi_2$ are flux across the cross section at time intervals $t_1\, and\ t_2$ respectively.

Magnetic flux due to magnetic field B through cross section area A is given by

$\phi = \vec{B}.\vec{A}$

Since magnetic field due to long wire is perpendicular to the circular loop, initial magnetic flux through the loop is given by

$\phi = BA = \frac{\mu_0 i}{2\pi d} A$

After time interval of 0.1s current in the wire becomes zero and hence magnetic field

$\therefore \phi_2 = 0$

Average induced emf is given by

$$\epsilon = -\frac{(\phi_2 - \phi_1)}{\Delta t} = -\frac{0 - BA}{\Delta t} = \frac{\frac{\mu_0 i}{2\pi d} A}{\Delta t}$$

$$= 4\pi \times 10^{-7} \times 10 \times \frac{10^{-6}}{2\pi \times 2 \times 10^{-1} \times 10^{-1}} = 10^{-10} Volt$$

$\epsilon = 10^{-10} V$

Therefore, average emf induced in the loop in time interval of 0.1s is given by 10^{-10}V

Answer.6

Given:

Length of side of square=$50cm = 0.5m$

No. of turns=50

Intensity of magnetic field =1.0T

Average induced emf in time interval Δt is given by

$\epsilon = -(\phi_2 - \phi_1)/\Delta t$ ...(i)

Where

$\phi_1\, and\, \phi_2$ are flux across the cross section at time intervals $t_1\, and\, t_2$ respectively.

Magnetic flux due to magnetic field B through cross section area A is given by

$\phi = \vec{B}.\vec{A}$

(a) During removal

Initial magnetic flux through the loop

$\phi_1 = B.A = 50 \times 0.5 \times 0.5 = 12.5\, Tm^2$

Final magnetic flux through the loop

$\phi_2 = 0$

Time interval Δt=0.25s

Using eqn.(i) we get

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{0 - 12.5}{0.25} = 50V$$

Magnitude of average emf during removal is 50V

(b) During restoration

Initial magnetic flux through the loop

$\phi_1 = 0$

Final magnetic flux through the loop

$\phi_2 = BA = 1 \times 50 \times 0.5 \times 0.5 = 12.5\, Tesla - m^2$

Time interval Δt=0.25s

Using eqn.(i) we get

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{12.5 - 0}{0.25} = -50V$$

Magnitude of average emf during restoration is 50V

(c) During the motion

Initial magnetic flux through the loop

$\phi_1 = 0$

Final magnetic flux through the loop

$\phi_2 = 0$

Time interval Δt=0.25+0.25=0.50s

Using eqm. (i) we get

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = 0$$

Therefore, magnitude of average emf during its motion is zero

Answer.7

Given

Resistance of the coil $r = 25\Omega$

(a) during its removal

the emf induced in the $loop\ \epsilon\ = 50V$

so the current flowing through the loop is given by

$$i = \frac{\epsilon}{r} = \frac{50}{25} = 2A$$

Thermal energy developed in the coil (H) during its removal in time interval Δt=0.25s is given by

$$H = i^2 r\Delta t = 4 \times 25 \times 0.25 = 25J$$

Therefore, energy developed in the coil during removal is 25J

(b) during its restoration

The magnitude of emf induced in the loop ϵ =50V

so the current flowing through the loop is given by

$$i = \frac{\epsilon}{r} = \frac{50}{25} = 2A$$

Thermal energy developed in the coil (H) during its removal in time interval Δt=0.25s is given by

$$H = i^2 r\Delta t = 4 \times 25 \times 0.25 = 25J$$

Therefore, energy developed in the coil during restoration is 25J

(c) during motion

total thermal energy developed =energy developed during removal+ energy developed during restoration

$\Rightarrow H = 25 + 25 = 50J$

As heat energy is scalar quantity, it is algebraically added

Therefore, total energy developed in coil during its motion is 50J

Answer.8

Given:

Area of conducting loop=$5.0cm^2 = 5 \times 10^{-4} m^2$

Variation of magnetic field with time $B = B_0 sin\omega t$

Angle of field with normal to coil θ =60°

Magnetic flux due to magnetic field B through cross section area A is given by

$\phi = \vec{B}.\vec{A} = BAcos\theta$

Here flux through the loop is given by

$\phi = BAcos60° = B_0 sin\omega t \times cos60°$. (i)

Also,

by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt}$$

Where

ϵ =emf produced

ϕ =flux of magnetic field

using eqn.(i)

we get

$$\epsilon = -\frac{d}{dt}\left(\frac{B_0 A sin\omega t}{2}\right) = -\frac{B_0 A\omega cos\omega t}{2} ...(ii)$$

Since maximum value of cosωt =1

Therefore, maximum value of magnitude of emf induced in the loop is given by

$$\epsilon_{max} = \frac{B_0 A\omega}{2}$$

Putting the values of B_0 and ω we get,

$$\epsilon_{max} = 0.2 \times 5 \times 10^{-4} \times \frac{300}{2} = 0.015V$$

Therefore, maximum value of emf induced in the coil is 0.015V

(b) from eqn.(ii), we have

$$\epsilon = -\frac{B_0 A\omega cos\omega t}{2}$$

At t=π/900 s magnitude of induced emf is given by

$$\epsilon = 0.015 \times \cos\left(300 \times \frac{\pi}{900}\right) = \frac{0.015}{2} = 7.5 \times 10^{-3}V$$

Therefore magnitude of induced emf at t=π/900 is $7.5 \times 10^{-3}V$

(c) from eqn.(ii), we have

$$\epsilon = -\frac{B_0 A\omega cos\omega t}{2}$$

At t=π/600 s magnitude of induced emf is given by

$$\epsilon = 0.015 \times \cos\left(300 \times \frac{\pi}{600}\right) = 0.015 \times \cos\left(\frac{\pi}{2}\right) = 0V$$

Therefore magnitude of induced emf at t=π/600 is $0V$

Answer.9

Given:

Area of pole faces A=$1cm^2 = 10^{-4}m^2$

Magnetic field intensity =0.1T

Time taken to remove the magnet completely $\Delta t = 1s$

Initial magnetic flux through the loop is given by formula

$\phi = \vec{B}.\vec{A}$

Since magnetic field through the square loop is perpendicular to the loop above eqn. reduces to

$\phi_1 = BA$ ...(i)

When the magnet is removed after 1s the magnetic flux passing through the square loop becomes zero

$\therefore \phi_2 = 0$ ...(ii)

Average induced emf in time interval Δt is given by

$\epsilon = -(\phi_2 - \phi_1)/\Delta t$ ...(iii)

Where

$\phi_1 \, and \, \phi_2$ are flux across the cross section at time intervals $t_1 \, and \, t_2$ respectively.

Using eqns.(i), (ii) and (iii) we get,

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{0 - BA}{\Delta t} = \frac{BA}{\Delta t}$$

Putting the values of B, A and Δt in the above eqn.

$$\epsilon = 0.1 \times \frac{10^{-4}}{1} = 10^{-5}V$$

Therefore average emf induced in the square loop is $10^{-5}V$

Answer.10

Given:

Average induced emf in the loop $\epsilon = 20mV = 2 \times 10^{-2}V$

Time taken to rotate the loo $\Delta t = 0.2s$

Edge length of square loop $= 2cm = 0.02m$

Area of square loop $A = 0.02^2 = 4 \times 10^{-4}$

We know that,

Average induced emf in time interval Δt is given by

$\epsilon = -(\phi_2 - \phi_1)/\Delta t$ ...(i)

Where

$\phi_1\, and\, \phi_2$ are flux across the cross section at time intervals $t_1\, and\, t_2$ respectively.

Magnetic flux(ϕ) through the loop is given by the formula

$\phi = \vec{B}.\vec{A}$

$\phi = BAcos\theta$

Where B=magnetic field intensity

A=area of cross section

θ =angle between area vector and magnetic field

Initially, angle between area vector and magnetic field is 0°

Therefore, initial flux through the coil is

$\phi_1 = BAcos0° = BA$

When it is rotated by 180° flux passing through the coil is given by

$\phi_2 = BAcos180° = -BA$

Putting this values in eqn.(i) we get,

$$\epsilon = -\frac{(\phi_2 - \phi_1)}{\Delta \mathrm{t}} = -\frac{-BA - BA}{\Delta t} = \frac{2BA}{\Delta t}$$

Putting the values of ϵ, B and Δt in the above eqn.

$$2 \times 10^{-2} = 2 \times B \times 4 \times \frac{10^{-4}}{0.2}$$

$$B = 20 \times \frac{10^{-3}}{4 \times 10^{-3}} = 5T$$

Therefore, magnitude of magnetic field intensity is 5T

Answer.11

Given:

Face area of loop =A

Resistance of loop=R

Magnetic field intensity =B

Magnetic flux(ϕ) through the loop is given by the formula

$$\phi = \vec{B}.\vec{A}$$

$$\phi = BAcos\theta$$

Where B=magnetic field intensity

A=area of cross section

θ =angle between area vector and magnetic field

initially loop is perpendicular to the applied magnetic field hence initial flux is

$$\phi_1 = BAcos0° = BA$$

Finally, when the loop is withdrawn from the field flux is given by

$$\phi_2 = 0$$

Now,

Average induced emf in time interval Δt is given by

$\epsilon = -(\phi_2 - \phi_1)/\Delta t$ ...(i)

Where

ϕ_1 *and* ϕ_2 are flux across the cross section at time intervals t_1 *and* t_2 respectively

Using eqn.(i) we get

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{0 - BA}{\Delta t} = \frac{BA}{\Delta t}$$

Current flowing in the loop is calculated by using formula

$$i = \frac{\epsilon}{R} = \frac{BA}{R\Delta t}$$

Hence the charge (Q) flowing through the loop is

$$Q = i\Delta t = \frac{BA}{R\Delta t} \times \Delta t = \frac{BA}{R}$$

Therefore, charge flowing through any cross-section of the wire is BA/R

Answer.12

Given:

Radius of solenoid $r = 2cm = 0.02m$

No. of turns in the solenoid $n = \frac{100}{cm} = 10000m^{-1}$

Current in the solenoid $i = 5A$

Radius of second coil $r' = 1cm = 0.01m$

No. of turns in the coil $N = 100$

Resistance of the coil $R = 20\Omega$

We know that,

Magnetic field inside solenoid (B) is given by formula

$$B = \mu_0 ni$$

Where,

n=no. of turns per unit length

i=current through solenoid

Magnetic flux(ϕ) through the coil is given by the formula

$\phi = \vec{B}.\vec{A}$

$\phi = BAcos\theta$

Where B=magnetic field intensity

A=area of cross section of the coil

θ =angle between area vector and magnetic field

magnetic field inside solenoid is perpendicular to the coil

initially flux through the coil is given by

$\phi_1 = BAcos0° = \mu_0 ni \times \pi r'^2 \times N = \mu_0 Nni\pi r'^2$

When the current in the solenoid is reversed in direction of magnetic field gets reversed and flux through the coil now m=becomes

$\phi_2 = BAcos180° = -BA = -\mu_0 Nni\pi r'^2$

$\phi_2 = 0$

Now,

Average induced emf in time interval Δt is given by

$\epsilon = -(\phi_2 - \phi_1)/\Delta t$ ...(i)

Where

$\phi_1\, and\, \phi_2$ are flux across the cross section at time intervals $t_1\, and\, t_2$ respectively

Putting these values in eqn.(i) we get

$$\epsilon = \frac{2\mu_0 Nni\pi r'^2}{\Delta t}$$

Current (i) through the coil of resistance R can be calculated as

$$i = \frac{\epsilon}{R} = \frac{2\mu_0 Nni\pi r'^2}{R\Delta t}$$

Hence the charge (Q) passing through the coil in time Δt is

$$Q = i\Delta t = \frac{2\mu_0 Nni\pi r'^2}{R}$$

Putting the values of μ_0, I, N, n π r' and R in above eqn.

$$Q = 2 \times 4\pi \times 10^{-7} \times 100 \times 10^4 \times 5 \times 3.14 \times \frac{10^{-4}}{20} = 2 \times 10^{-4} C$$

Therefore flowing through the galvanometer is $2 \times 10^{-4} C$

Answer.13

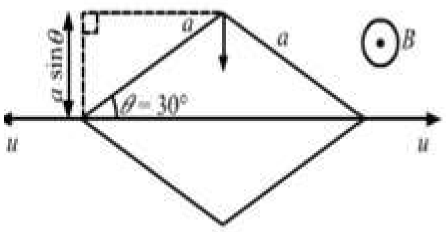
Given:

Edge length of square frame =a

Magnetic field intensity B

Speed of corners of rhombus =u

(a) when the angles at the corner reduce to 60°



We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$$\epsilon = (\vec{v} \times \vec{B}).\vec{l}$$

Motional emf is produced in each side of rhombus and the effective length of each side is the length perpendicular to velocity of corners.

From the fig. the effective length of each side is

$$l_{eff} = asin30° = \frac{a}{2}$$

Since velocity is perpendicular to magnetic field the equation of emf induced in each side given by

$$\epsilon = vBl = \frac{uBa}{2}$$

Total emf induced in all four side is

$$\epsilon = 4 \times \frac{uBa}{2} = 2uBa$$

Therefore, induced emf in the frame when the angles at the corner reduces to 60° is 2uBa

(b) total resistance of the frame =R

hence current flowing in the frame is

$$i = \frac{\epsilon}{R} = \frac{2uBa}{R}$$

Therefore, current flowing in the frame at this instant is 2uBa/R

(c) initially the frame is in form of square of side a and area a^2

at this time flux through this frame is given by

$$\phi_1 = B.A = Ba^2$$

Finally, when the frame reduces to straight line flux passing through the frame reduces to zero

$$\phi_2 = 0$$

Average emf induced in the frame in time t is given by

$$\epsilon = -\frac{\phi_2 - \phi_1}{t} = -\frac{0 - Ba^2}{t} = \frac{Ba^2}{t}$$

The current flowing through the frame is then given by

$$i = \frac{\epsilon}{R} = \frac{Ba^2}{Rt}$$

Where R is the resistance of frame

Hence the charge (Q) flowing through the side of frame in time t is

$$Q = it = \frac{Ba^2}{R}$$

Therefore total charge flowing through side of frame by the time the frame reduces to straight line is Ba^2/R

Answer.14

Given: Initial flux

$$\phi_1 = 0.35Wb$$

Final flux $\phi_2 = 0.85Wb$

Time interval $\Delta t = 0.5s$

Average induced emf in time interval Δt is given by

$\epsilon = -(\phi_2 - \phi_1)/\Delta t$ ...(i)

Where

ϕ_1 *and* ϕ_2 are flux across the cross section at time intervals t_1 *and* t_2 respectively.

Putting the values in eqn.(i) we get,

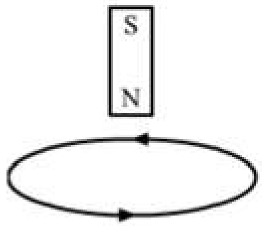
$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{0.85 - 0.35}{0.5} = -1V$$

Magnitude of induced emf =1V

From Lenz's law,

The direction of induced current is such that it opposes the change that has induced it.

According to fig.



The flux in the downward direction increases, so the current is induced such that flux in upward direction increases. Therefore, current is induced in anticlockwise direction

Therefore, average emf induced in the coil is 1V in anti-clockwise direction

Answer.15

Let the area of the wire-loop be A and magnetic field intensity be B.

When the wire rotates in its own plate the area through which flux passes remains same and B is also constant.

Hence, the flux passing through the loop remains constant and is given by

$$\phi = BA$$

Now,

by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt}$$

Where

ϵ =emf produced

ϕ =flux of magnetic field

as flux is constant (independent of time), its derivative with respect to time is zero. Therefore, emf induced in the loop is zero

therefore, zero emf is induced in the wire-loop

Answer.16

Given:

Side length of square loop=5cm$= 0.05m$

Speed of square loop$= 1cms^{-1} = 0.01ms^{-1}$

Width of magnetic field $= 20cm = 0.2m$

Magnetic field intensity =0.6T

(a) t=2s

distance moved by the loop$= 0.01 \times 2 = 0.02m$

area of the loop under magnetic field =

area of rectangle of length 0.05m and width 0.02m

$\Rightarrow 0.02 \times 0.05 = 10^{-3}m^2$

Now,

Initial magnetic flux through the loop $\phi_1 = 0$ (at t=0)

Final magnetic flux through the loop is given by

$\phi = \vec{B}.\vec{A}$

$\therefore \phi_2 = 0.6 \times 10^{-3} = 6 \times 10^{-4}Wb$

Average induced emf in time interval Δt is given by

$\epsilon = -(\phi_2 - \phi_1)/\Delta t$ ...(i)

Where

$\phi_1 \, and \, \phi_2$ are flux across the cross section at time intervals $t_1 \, and \, t_2$ respectively.

Putting the values of $\phi_1, \phi_2 \, and \, \Delta t = 2s$ in eqn.(i),

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{6 \times 10^{-4} - 0}{2} = -3 \times 10^{-4}V$$

Therefore magnitude of induced emf at t=2s is $3 \times 10^{-4}V$

(b) t=10s

distance moved by the square loop $0.01 \times 10 = 0.1m$

at this moment, square loop is completely inside the magnetic field and area of loop through which flux pass $= 0.05 \times 0.05 = 25 \times 10^{-4}m^2$

so the flux linkage does not changes with time $\therefore \Delta\phi = 0$

and thus from eqn.(i)

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = 0$$

Therefore, magnitude of induced emf in the coil at t=10s is zero

(c) t=22s

distance moved by the loop $0.01 \times 22 = 0.22m$

the loop is moving out of the field, the area of loop under the field is $= (2 \times 5 \times 10^{-4}m^2)$

the magnetic flux acting on the loop is

$= -0.6 \times 2 \times 5 \times 10^{-4}\ Tm^2$

(- sign as the flux has decreased)

The induced emf is

$$\epsilon = -\frac{\Delta\phi}{\Delta t} = -\frac{6 \times 10^{-4} - 0}{2} = -3 \times 10^{-4}V$$

Therefore magnitude of induced emf at t=22s is $3 \times 10^{-4}V$

(d) t=30s

distance moved by the square loop$= 0.01 \times 30 = 0.3m$

at this time, square loop is completely outside the magnetic field and the area of loo through which flux passes =0

hence the flux linkage through the loop remains zero $\therefore \Delta\phi = 0$

and thus from eqn.(i)

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = 0$$

Answer.17

Given:

Resistance of the loop R=4.5mΩ$4.5 \times 10^{-3}\Omega$

Time interval=30s

As heat produced is a scalar quantity total heat is found by algebraically adding heat produced in different time intervals

$\therefore H_T = H_a + H_b + H_c + H_d$

Where

H_a =heat produced during time interval 0-5s

H_b =heat produced during time interval 5-20s

H_c =heat produced during time interval 20-25s

H_d =heat produced during time interval 25-30s

Now,

(i) during time interval 0-5s emf produced in the loop is given by

$\epsilon = 3 \times 10^{-4} V$

Current in the coil is

$$i = \frac{\epsilon}{R} = 3 \times \frac{10^{-4}}{(4.5 \times 10^{-3})} = 6.7 \times 10^{-2} A$$

Heat produced in the coil is given by the formula

$H = i^2 Rt$ .... (i)

where

i=current

R=resistance

t=time interval

putting the values of i, R and t in above eqn. we get,

$H_a = (6.7 \times 10^{-2})^2 \times 4.5 \times 10^{-3} \times 5$

Emf induced in the time interval 5-20s and 25-30s =0

So current in the coil during this time

$i = 0$

Heat produced in the coil is given by eqn.(i)

$H_b = H_d = 0$

Emf induced in the time interval 20-25s is same as that induced at 5s

$\epsilon = 3 \times 10^{-4} V$

Hence the current and heat produced during this interval is same and given by

$H_c = (6.7 \times 10^{-2})^2 \times 4.5 \times 10^{-3} \times 5$

Total heat produced is given by

$H_T = H_a + H_b + H_c + H_d$

$H_T = H_a + H_c = 2 \times (6.7 \times 10^{-2})^2 \times 4.5 \times 10^{-3} \times 5\, J$

$H_T = 2 \times 10^{-4} J$

Therefore total heat produced in the loop during interval 0-30s is $2 \times 10^{-4} J$

Answer.18

Given:

Radius of cylindrical region $r = 10cm = 0.1m$

Length of wire =80cm

Resistance of the wire $R = 4\Omega$

Rate of increase of magnetic field =0.01T/s$= \frac{dB}{dt}$

Area of loop inside magnetic field =area of semicircle of radius 0.1m

$$\Rightarrow A = \frac{\pi r^2}{2}$$

We know that,

Flux (ϕ) of magnetic field (B) through the loop of cross section area A in the magnetic field is given by

$$\phi = \vec{B}.\vec{A}$$

$$\phi = BAcos\theta$$

Since magnetic field is perpendicular to the loop the flux becomes

$$\phi = BAcos0° = BA$$

Rate of change of magnetic field wrt. time is given by

$$\frac{d\phi}{dt} = \frac{d(BA)}{dt} = A.\frac{dB}{dt}$$

(since area of cross section in magnetic field does not change with time, A remains constant)

Now,

by faraday's law of electromagnetic induction

$\epsilon = -\frac{d\phi}{dt}$ ...(i)

Where

ϵ =emf produced

ϕ =flux of magnetic field

using eqn.(i) the emf induced in the loop is given by

$$\epsilon = -\frac{d\phi}{dt} = -A\frac{dB}{dt}$$

Hence the current through the loop (i) of resistance R is

$$i = \frac{\epsilon}{R} = -\frac{A}{R}\frac{dB}{dt}$$

Putting the values of A, R and $\frac{dB}{dt}$ in the above eqn. the magnitude of current is

$$i = \frac{\pi r^2}{2R}\frac{dB}{dt} = 3.14 \times 0.01 \times \frac{0.01}{2 \times 4} = 3.9 \times 10^{-5} A$$

Therefore current induced in the frame is $3.9 \times 10^{-5} A$

Answer.19

Given:

Rate of increase of magnetic field =$20mTs^{-1} = 0.02Ts^{-1}$

Side length of square loop =$1cm = 0.01m$

Resistance of each side =4Ω

Area of the coil adef = area of coil abcd =$10^{-4}m^2$

We know that,

Flux (ϕ) of magnetic field (B) through the loop of cross section area A in the magnetic field is given by

$$\phi = \vec{B}.\vec{A}$$

$$\phi = BAcos\theta$$

Since magnetic field is perpendicular to the loop the flux becomes

$$\phi = BAcos0° = BA$$

Rate of change of magnetic field wrt. time is given by

$$\frac{d\phi}{dt} = \frac{d(BA)}{dt} = A.\frac{dB}{dt}$$

(since area of cross section in magnetic field does not change with time, A remains constant)

Now,

by faraday's law of electromagnetic induction

$\epsilon = -\frac{d\phi}{dt}$ ...(i)

Where

ϵ =emf produced

ϕ =flux of magnetic field

using eqn.(i) the emf induced in the loop is given by

$$\epsilon = -\frac{d\phi}{dt} = -A\frac{dB}{dt}$$

Hence the current through the loop (i) of resistance R is

$i = \frac{\epsilon}{R} = -\frac{A}{R}\frac{dB}{dt}$ ...(ii)

(a) when the switch S_1 is closed but S_2 is open

no current flows through loop abcd

net resistance of the loop adef R =4× 4 =16Ω

area of loop adef =$10^{-4}m^2$

using eqn.(ii) current can be given by

$$i = -\frac{A}{R}\frac{dB}{dt} = \frac{10^{-4}}{16} \times 0.02 = 1.25 \times 10^{-7}$$

As the magnetic field increases, the flux of magnetic field increases in downward direction so by Lenz's law

The direction of induced current is such that it opposes the change that has induced it

Therefore, current flows in anticlockwise direction **(along ad)** to increase the magnetic flux in upward direction

(b) S_1 is open but S_2 is closed

No current flows in loop adef

Net resistance of loop abcd=4× 4=16Ω

Area of loop abcd =$10^{-4}m^2$

using eqn.(ii) current can be given by

$$i = -\frac{A}{R}\frac{dB}{dt} = \frac{10^{-4}}{16} \times 0.02 = 1.25 \times 10^{-7}$$

As the magnetic field increases, the flux of magnetic field increases in downward direction so by Lenz's law

Therefore, current flows in anticlockwise direction **(along da)** to increase the magnetic flux in upward direction.

(c) When both S_1 and S_2 is open

No current flows in both the loop adef and abcd

And hence current in wire ad is zero

$\therefore i = 0$

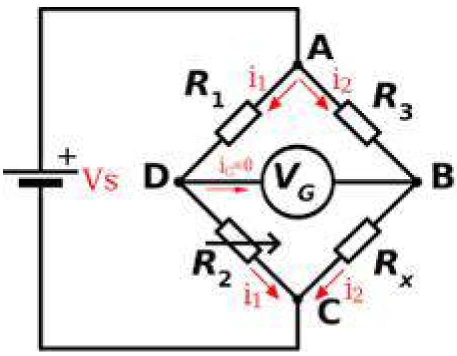
(d) When both S_1 and S_2 is closed

The circuit forms a **balanced Wheatstone Bridge** and the current flowing through the wire ad is zero

$\therefore i = 0$

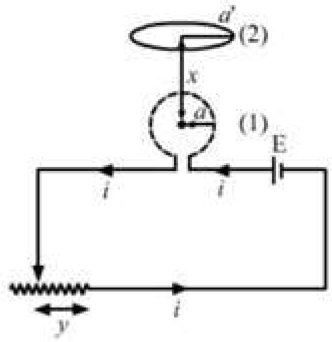
<u>Concept of wheat stone bridge:</u>

When the circuit forms a Wheatstone bridge in balanced condition then the current through galvanometer (i_G) becomes zero



Answer.20

Given:



Area of coil (2) of radius a' $=\pi a'^2$

We know that magnetic field due to coil (1) at the center of coil (2) is

$$B = \frac{\mu_0 N i a^2}{2(a^2 + x^2)^{3/2}}$$

Where

N=no. of turns in coil (1)

i= current in coil (1)

a=radius of coil (1)

x=distance of center of coil (2) from center of coil (1)

We know that,

Flux (ϕ) of magnetic field (B) through the loop of cross section area A in the magnetic field is given by

$$\phi = \vec{B}.\vec{A}$$

$$\phi = BAcos\theta$$

Since the magnetic field due to coil (1) is parallel to axis of coil (2) θ =0° and flux through the coil (2) is given by

$$\phi = BAcos0° = BA = \frac{\mu_0 N i a^2}{2(a^2 + x^2)^{3/2}}\ \pi a'^2$$

Now,

by faraday's law of electromagnetic induction

$e = -\frac{d\phi}{dt}$ ...(i)

Where

e =emf produced

ϕ =flux of magnetic field

using eqn.(i) emf induced in the coil (2) is given by

$e = \frac{\mu_0 N a^2}{2(a^2+x^2)^{3/2}} \pi a'^2 \frac{di}{dt}$. (ii)

Let y be the distance of sliding contact from its right end

Given,

Total length of rheostat =L

Total resistance of rheostat=R

When the sliding contact is at a distance y from its right end then the resistance (R') of the rheostat is given by

$$R' = \frac{R}{L} y$$

So the current i flowing through the circuit is given by

$$i = \frac{\epsilon}{R' + r} = \frac{\epsilon}{\frac{R}{L} y + r}$$

Where r is the resistance of the coil and ϵ is the emf of battery

Putting value of i in eqn.(ii) we get,

$$e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \frac{d\left(\frac{\epsilon}{\frac{R}{L} y + r}\right)}{dt}$$

$$\Rightarrow e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \left[\epsilon \times \frac{-\frac{R}{L}\frac{dy}{dt}}{\left(\frac{R}{L} y + r\right)^2}\right]$$

Since $\frac{dy}{dt} = v$

$$e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \left[\epsilon \times \frac{-\frac{R}{L} v}{\left(\frac{R}{L} y + r\right)^2}\right]$$

(a) When the contact begins to slide $y = L$

Therefore, magnitude of emf induced is

$$e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \left[\epsilon \times \frac{\frac{R}{L} v}{\left(\frac{R}{L} L + r\right)^2}\right]$$

$$\boldsymbol{e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \left[\epsilon \times \frac{\frac{R}{L} v}{(R + r)^2}\right]}$$

(b) When the contact has slid through half the length of rheostat $y = \frac{L}{2}$

Therefore, magnitude of emf induced is

$$e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \left[\epsilon \times \frac{\frac{R}{L} v}{\left(\frac{\frac{R}{L} L}{2} + r\right)^2}\right]$$

$$\boldsymbol{e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \left[\epsilon \times \frac{\frac{R}{L} v}{\left(\frac{R}{2} + r\right)^2}\right]}$$

Answer.21

Given:

Radius of coil $r = 2cm = 0.02m$

No. of turns in the coil $N = 50$

Magnetic field intensity $B = 0.2T$

We know that,

Flux (ϕ) of magnetic field (B) through the loop of cross section area A in the magnetic field is given by

$$\phi = \vec{B}.\vec{A}$$

$$\phi = NBAcos\theta$$

Where N=no. of turns in the coil

Since magnetic field is perpendicular to the loop the flux becomes

$$\phi = NBAcos0° = NBA$$

Initial flux through the coil is given by

$$\phi_1 = NBA$$

After 0.1 s the coil is rotated through an angle of 60° =θ

Finally, the flux through the coil becomes

$$\phi_2 = NBAcos60° = \frac{NBA}{2}$$

Average induced emf in time interval Δt is given by

$\epsilon = -(\phi_2 - \phi_1)/\Delta t$ ...(i)

Where

$\phi_1 \, and \, \phi_2$ are flux across the cross section at time intervals $t_1 \, and \, t_2$ respectively.

Using eqn.(i) emf induced in the coil is given by

$$\epsilon = -\frac{\frac{NBA}{2} - NBA}{\Delta t} = \frac{NBA}{2\Delta t}$$

Putting the values of N, B, A and Δt in above eqn. we get

$$\epsilon = 50 \times 0.2 \times \frac{\pi(0.02)^2}{2 \times 0.1} = 6.28 \times 10^{-3} V$$

Therefore average emf induced in the coil is $6.28 \times 10^{-3} V$

(b) the current through the coil (i) is calculated using formula

$$i = \epsilon/R$$

Hence the charge(Q) crossing the cross-section of the wire in time interval Δt is

$$Q = i \times \Delta t = \frac{\epsilon \Delta t}{R}$$

Putting the values of ϵ, R and Δt we get,

$$Q = 6.28 \times 10^{-3} \times \frac{0.1}{4} = 1.57 \times 10^{-4} C$$

Therefore charge crossing cross-section of the wire in the coil is $\mathbf{1.57 \times 10^{-4} C}$

Answer.22

Given:

No. of turns in the coil $N = 100$

Magnetic field intensity $B = 4 \times 10^{-4} T$

Angular velocity of rotation $\omega = 300 rev min^{-1} = 2\pi \times 5 = 10\pi rad s^{-1}$

Area of the coil $A = 25 cm^2 = 25 \times 10^{-4} m^2$

Resistance of the coil $R = 4\Omega$

Magnetic flux through the circular coil ϕ can be given by formula

$$\phi = \vec{B}.\vec{A}$$

$\phi = NBAcos\theta$ .... (i)

Where B=magnetic field intensity

A=area of cross section

N=no. of turns in the coil

θ =angle between area vector and magnetic field

(a) Initially, angle between area vector and magnetic field is 0°

Therefore, initial flux through the coil is

$\phi_1 = NBA$

When it is rotated by 180° flux passing through the coil is given by

$\phi_2 = NBAcos180° = -NBA$

Average induced emf in time interval Δt is given by

$\epsilon = -(\phi_2 - \phi_1)/\Delta t$ ...(i)

Where

ϕ_1 *and* ϕ_2 are flux across the cross section at time intervals t_1 *and* t_2 respectively.

Average induced emf is then given by

$\epsilon = -\frac{-NBA-NBA}{\Delta t} = \frac{2NBA}{\Delta t}$ ......(ii)

Now,

Angular velocity of coil $\omega = 10\pi rads^{-1}$

Time taken to complete half revolution i.e. rotate by π radian

$$\Delta t = \frac{1}{10}s$$

Putting the values of N, B, A and Δt in eqn.(ii)

$$\epsilon = 2 \times 100 \times 4 \times 10^{-4} \times 25 \times \frac{10^{-4}}{0.1} = 2 \times 10^{-3}V$$

Therefore average emf induced in the coil in half a turn is $2 \times 10^{-3}V$

(b) In a full term coil returns to its original position

$\therefore \phi_1 = \phi_2 = NBA$

And hence emf induce in the coil using eqn.(ii) is

$$\epsilon = -\frac{NBA - NBA}{\Delta t} = 0$$

Therefore, average emf induced in full turn in the coil is zero

(c) Emf induced in the coil in part (a) is

$\epsilon = 2 \times 10^{-3}V$

Hence the current i flowing through the coil of resistance R is

$$i = \frac{\epsilon}{R} = 2 \times \frac{10^{-3}}{4} = 5 \times 10^{-4} A$$

So the charge displaced in time interval Δt =0.1s is

$$Q = i\Delta t = 5 \times 10^{-4} \times 0.1 = 5 \times 10^{-5} C$$

Therefore net charge displaced in part (a) is $5 \times 10^{-5} C$

Answer.23

Given:

Radius of coil r=10cm

Resistance of the coil R=40Ω

No. of turns in the coil N=1000

Horizontal component of earth's magnetic field=$3.0 \times 10^{-5} T$

Angle of rotation θ =180°

Magnetic flux through the circular coil ϕ can be given by formula

$$\phi = \vec{B}.\vec{A}$$

$\phi = NBAcos\theta$ .... (i)

Where B=magnetic field intensity

A=area of cross section

N=no. of turns in the coil

θ =angle between area vector and magnetic field

Initially, angle between area vector and magnetic field is 0°

Therefore, initial flux through the coil is

$$\phi = NBA$$

When it is rotated by 180° flux passing through the coil is given by

$$\phi = NBAcos180° = -NBA$$

Now,

by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt}$$

Where

ϵ =emf produced

ϕ =flux of magnetic field

therefore, emf produced in the coil is given by

$\epsilon = -\frac{-NBA-NBA}{dt} = \frac{2NBA}{dt}$.........(ii)

Current passing through the loop (i) of resistance R is

$i = \frac{\epsilon}{R} = \frac{2NBA}{Rdt}$ using eq.(ii)

Charge flowing through the galvanometer (Q) in time dot is given by formula

$$Q = i.dt = \frac{2NBA}{Rdt} \times dt = \frac{2NBA}{R}$$

Putting the values of N, B, A and R we get,

$$Q = 2 \times 1000 \times 3 \times 10^{-5} \times \pi \times \frac{10^{-2}}{40} = 4.71 \times 10^{-5} C$$

Therefore charge which flows through the galvanometer is $4.71 \times 10^{-5} C$

Answer.24

Given:

Radius of circular coil $r = 5.0cm = 5 \times 10^{-2} m$

Magnetic field intensity $B = 0.01T$

Angular speed of coil ω =$80 rev min^{-1} = 2\pi \times \frac{80}{60}\ rad s^{-1}$

Magnetic flux through the circular coil ϕ can be given by formula

$\phi = \vec{B}.\vec{A}$

$\phi = BA\cos\theta$

$\phi = BA\cos\omega t$...(i)

As $\theta = \omega t$

Where ω =angular velocity of loop

Where θ is the angle between magnetic field and area vector of loop.

by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt}$$

Where

ϵ =emf produced

ϕ =flux of magnetic field

putting the value of eqn.(i) in above eqn. we get,

$\epsilon = -\frac{d}{dt}(BA\cos\omega t) = \omega BA\sin\omega t$...(ii)

Since maximum value of $\sin\omega t$ is equal to 1

$\therefore \epsilon_{max} = BA\omega$

Putting the values of B, A and ω we get,

$\epsilon = 0.01 \times \pi \times 25 \times 10^{-4} \times 2\pi \times \frac{80}{60} = 6.66 \times 10^{-4} V$.......... (iii)

Therefore maximum emf induced in the circular coil is $6.66 \times 10^{-4} V$

(b) from eqn.(ii) emf induced in the coil is given by

$\epsilon = BA\omega\sin\omega t$

Average value of induced emf is given by formula

$$\epsilon_{av} = \left(\int_0^T BA\omega\sin\omega t\ dt\right) / \int_0^T dt \ = BA\omega\left[-\frac{\cos\omega t}{\omega}\right]_0^T$$

Where $T = 2\pi/\omega$ is the time taken by the coil to complete one revolution

$$\Rightarrow \epsilon_{av} = -\frac{BA\omega}{\omega}[cos\omega t]_0^{\frac{2\pi}{\omega}} = 0$$

Therefore, average induced emf is zero

(c). the average of squares of induced emf is given by the formula

$$\epsilon_{av}^2 = (\int_0^T B^2A^2\omega^2 \sin^2 \omega t\,.dt) / \int_0^T dt$$

$$\Rightarrow \epsilon_{av}^2 = B^2A^2\omega^2 \int_0^T \sin^2 \omega t \;dt \;/ \int_0^T dt$$

$$\Rightarrow \epsilon_{av}^2 = \frac{B^2A^2\omega^2}{2} \int_0^T (1 - cos2\omega t).dt \;/ \int_0^T dt$$

$$\Rightarrow \epsilon_{av}^2 = \frac{B^2A^2\omega^2\left[t - \frac{sin2\omega t}{2\omega}\right]_0^T}{2T}$$

$$\Rightarrow \epsilon_{av}^2 = \frac{B^2A^2\omega^2}{2T}\left[T - \frac{sin4\pi - sin0}{2\omega}\right] = \frac{B^2A^2\omega^2}{2}$$

Putting the values of B, A and ω we get

$\epsilon_{av}^2 = \frac{(6.66\times10^{-4})^2}{2} = 2.2 \times 10^{-7}V^2$ using eqn.(iii)

Therefore average of squares of emf produced is $2.2 \times 10^{-7}V^2$

Answer.25

Given:

Resistance of the coil R=100Ω

Time period T=1min =60s

From previous question induced emf is given by

$\epsilon = BA\omega sin\omega t$

Current in the coil i is given by

$i = \frac{\epsilon}{R} = \frac{BA\omega sin\omega t}{R}$.... (i)

Heat produced in the circuit is calculated by the following formula

$$H = \int_0^T i^2 R dt$$

Using eqn.(i) we get,

$$H = \int_0^T B^2 A^2 \omega^2 \sin^2 \frac{\omega t}{R^2} R\, dt$$

$$\Rightarrow H = \frac{B^2 A^2 \omega^2}{2R} \int_0^T (1 - cos2\omega t).dt$$

$$H = \frac{B^2 A^2 \omega^2}{2R} \left[t - \frac{sin2\omega t}{2\omega} \right]_0^T$$

$$H = \frac{B^2 A^2 \omega^2 T}{2R}$$

Putting the values of B, A, ω, T and R we have

$$H = \frac{60}{2R} \times \pi^2 r^4 \times B^2 \times \left(80 \times \frac{2\pi}{60}\right)^2$$

$$H = 1.33 \times 10^{-7} J$$

Therefore heat produced in the circuit in one minute is $1.33 \times 10^{-7} J$

Answer.26

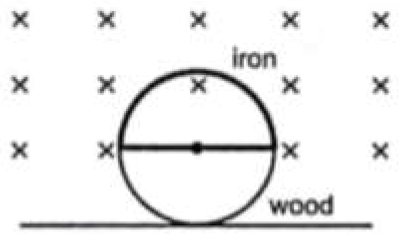
Given:

Radius of circular wheel r =10cm=0.1m

Magnetic field intensity $B = 2 \times 10^{-4} T$

Area of semicircular part $A = \frac{\pi r^2}{2}$

Initially



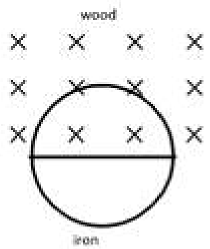
Magnetic flux through the wheel ϕ can be given by formula

$$\phi = \vec{B}.\vec{A}$$

Since area vector and magnetic field is parallel we get flux

$$\phi_1 = BA = \frac{B\pi r^2}{2} = 2 \times 10^{-4} \times 3.14 \times \frac{(0.1)^2}{2} = 3.14 \times 10^{-6} Wb$$

After time interval Δt=2s



Flux through the wheel

$$\phi_2 = 0$$

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1)/\Delta t \text{ ...(i)}$$

Where

ϕ_1 *and* ϕ_2 are flux across the cross section at time intervals t_1 *and* t_2 respectively.

Therefore, average induced emf ϵ is

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{0 - 3.14 \times 10^{-6}}{2} = 1.57 \times 10^{-6} V$$

Therefore average induced emf in the wheel is $1.57 \times 10^{-6} V$

Answer.27

Given:

Length of rod l=20cm=0.2m

Velocity of rod v$= 10cms^{-1} = 0.1ms^{-1}$

Magnetic field intensity $B = 0.1T$

(a) we know that,

A charge q moving with velocity v inside a magnetic field B experiences a force F given by

$\vec{F} = q.\left(\vec{v} \times \vec{B}\right)$.... (i)

Since velocity of rod is perpendicular to magnetic field the above eqn. reduces to

$F = qVB$

To find the force on a free electron $q = e = 1.6 \times 10^{-19}C$

Putting the values of q, v and B we get,

$F = 1.6 \times 10^{-19} \times 0.1 \times 0.1 = 1.6 \times 10^{-21}N$

Therefore average force experienced by a free electron is $\mathbf{1.6 \times 10\text{^} - 21}N$

(b) we know that,

force experienced by a charge particle having charge q in presence of an electric field E is given by

$\vec{F} = q.\vec{E}$

To balance this force with the magnetic force, equating above eqn. with eqn.(i),

$q\vec{E} = q.\left(\vec{v} \times \vec{B}\right) = qvB$

$\Rightarrow E = vB$

Putting the values of v and B we get,

$E = 0.1 \times 0.1 = 0.01Vm^{-1}$

This electric field is created due to emf produced due to motion of conducting rod, as a result of which the free electrons in the rod experiences a force

Therefore, electric field needed to balance magnetic force is 0.01Vm^{-1}

(c) We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$\epsilon = (\vec{v} \times \vec{B}).\vec{l}$

Since $\vec{v}$ and $\vec{B}$ are perpendicular and their cross product is parallel to $\vec{l}$, eqn.(i) reduces to

$\epsilon = vBl$

Putting the values of v, B and l we get,

$\epsilon = 0.1 \times 0.1 \times 0.2 = 2 \times 10^{-3}V$

Therefore motional emf between the ends of the rod is $2 \times 10^{-3}V$

Answer.28

Given:

Velocity of meter stick $v = 2ms^{-1}$

Intensity of magnetic field $B = 0.2T$

Length of stick =1m

We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$\epsilon = (\vec{v} \times \vec{B}).\vec{l}$

Since $\vec{v}$ and $\vec{B}$ are perpendicular and their cross product is parallel to $\vec{l}$, eqn.(i) reduces to

$\epsilon = vBl$

Putting the values of v, B and l we get,

$\epsilon = 2 \times 0.2 \times 1 = 0.4V$

Therefore, emf induced between the ends of a stick is 0.4V

Answer.29

Given:

Speed of spacecraft $v=3\times 10^7 m/s$

Magnetic field intensity $B=3 \times 10^{-10} T$

Width of spacecraft l=10m

We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$\epsilon = (\vec{v} \times \vec{B}).\vec{l}$

Since $\vec{v}$ and $\vec{B}$ are perpendicular and their cross product is parallel to $\vec{l}$, eqn.(i) reduces to

$\epsilon = vBl$

Putting the values of v, B and l we get,

$\epsilon = 3 \times 10^7 \times 3 \times 10^{-10} \times 10 = 9 \times 10^{-2} V$

$\epsilon = 0.09V$

Therefore, emf induced across the width of spacecraft is 0.09V

Answer.30

Given:

Vertical component of earth's magnetic field $B=0.2\times 10^{-4}T$

Speed of train v=$180kmh^{-1} = 50ms^{-1}$

Separation between rails l=1m

We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$\epsilon = (\vec{v} \times \vec{B}).\vec{l}$.... (i)

Since $\vec{v}$ and $\vec{B}$ are perpendicular and their cross product is parallel to $\vec{l}$, eqn.(i) reduces to

$\epsilon = vBl$

Putting the values of v, B and l we get,

$\epsilon = 50 \times 0.2 \times 10^{-4} \times 1 = 10^{-3}V = 1mV$

Therefore, the reading of millivoltmeter is 1mV

Answer.31

Given:

Speed of right-angled triangle =v

We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$\epsilon = (\vec{v} \times \vec{B}).\vec{l}$.... (i)

(a) for loop abc since the flux of magnetic field through the triangle does not change hence the emf induced in loop abc is zero

$\therefore \epsilon_{abc} = 0$

Therefore, emf induced in loop abc is zero

(b) in the segment bc

bc is perpendicular to velocity. emf induced in the segment bc is given by

$$\epsilon_{bc} = (\vec{v} \times \vec{B}).\vec{l} = (v\hat{\imath} \times B\hat{k}).(bc(-\hat{\jmath})) = vB\hat{\jmath}.\hat{\jmath}(bc) = vB(bc)$$

Therefore, emf induced in segment bc is vB(bc)

(c) in the segment ac

ac is parallel to velocity. Emf induced in the segment ac is given by

$$\epsilon_{ac} = (\vec{v} \times \vec{B}).\vec{l} = (v\hat{\imath} \times B\hat{k}).(ac\hat{\imath}) = vB(-\hat{\jmath}).ac(\hat{\imath}) = 0$$

Therefore, emf induced in the segment ac is zero

(d) in the segment ab

the effective length of ab perpendicular to velocity is given by bc (-ĵ)

emf induced in the segment ab is given by

$$\epsilon_{ab} = (\vec{v} \times \vec{B}).\vec{l} = (v\hat{\imath} \times B\hat{k}).(bc(-\hat{\jmath}) + ac\hat{\imath}) = vB\hat{\jmath}.\hat{\jmath}(bc) + 0 = vB(bc)$$

Therefore, emf induced in the segment ab is vB(bc)

Answer.32

Given:

Radius of semicircular wire=r

Velocity =v

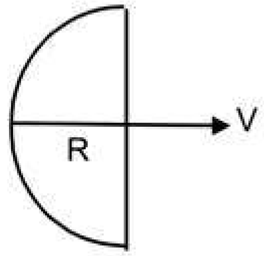
We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$\epsilon = (\vec{v} \times \vec{B}).\vec{l}$

In the case of semicircular wire, l denotes the effective length of wire perpendicular to velocity.

(a) when the velocity is perpendicular to diameter joining free ends



the effective length of wire perpendicular to velocity is given by length of diameter

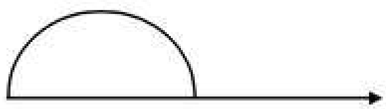
$\therefore l = 2R$

Therefore, induced emf in the wire is given by

$\epsilon = vB \times 2R = 2BvR$

Therefore, induced emf in this case is 2BvR

(b) when the velocity is parallel to the diameter



The effective length of wire parallel to velocity is zero

$\therefore l = 0$

Therefore, induced emf in the wire is given by

$\epsilon = vB \times 0 = 0$

Therefore, induced emf in this case is zero

Answer.33

Given:

Length of the wire=10cm

Angle of length of wire with velocity=60°

Magnetic field intensity=1.0T

Speed of wire v=20cm/s =0.2m/s

(a) we know that,

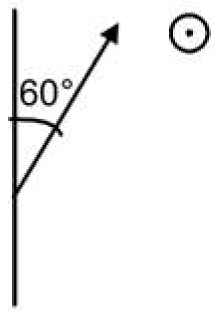
motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$$\epsilon = (\vec{v} \times \vec{B}).\vec{l}$$

It is given that plane of motion is perpendicular to electric field i.e. angle between $\vec{v}$ and $\vec{B}$ =90°

And the angle between velocity and length of wire=60°

$$\therefore \epsilon = Bvl sin60°$$



We take only that component of length vector which is perpendicular to velocity vector

Putting the values of B, v, l we get

$$\epsilon = 1 \times 0.2 \times 0.1 \times \frac{\sqrt{3}}{2} = 17.32 \times 10^{-3} V$$

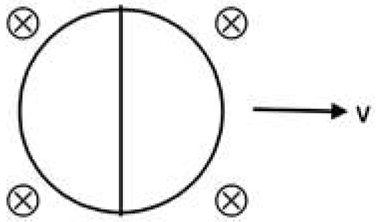
Therefore the emf induced in the rod is $1.732 \times 10^{-2} V$

Answer.34

Given:

Radius of ring =R

Velocity of ring=v



(b) we know that motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$\epsilon = (\vec{v} \times \vec{B}).\vec{l} = B^{\rightarrow}.(\vec{v} \times l^{\rightarrow})$.... (i)

This value is maximum when length between the points is perpendicular to the velocity of the rod

Thus the emf is the highest between the end points of diameter perpendicular to the velocity and the value of this emf is given by

$\epsilon = B\ v \times 2R = 2BvR$

Therefore, maximum value of emf is 2BvR

(c) the value of eqn.(i) is minimum when length between the points is parallel to the velocity of the rod

Thus the emf is lowest between the end points of diameter parallel to the velocity and this value of emf is given by

$\epsilon = Bv \times 2R \times sin0° = 0$

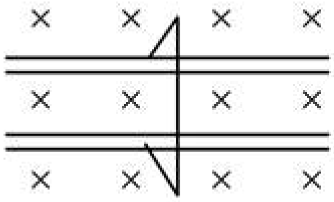
Therefore, minimum value of emf is 0

Answer.35

Given:

Velocity of wire=v

Separation between rail=l



We know that,

Force experienced by a wire of length l carrying current I in a magnetic field B and placed perpendicular to magnetic field is given by

$$F = ilB$$

Now since here there is no formation of closed circuit the circuit is open and the current flowing in the wire =0

Therefore, force experienced by wire due to magnetic field =0

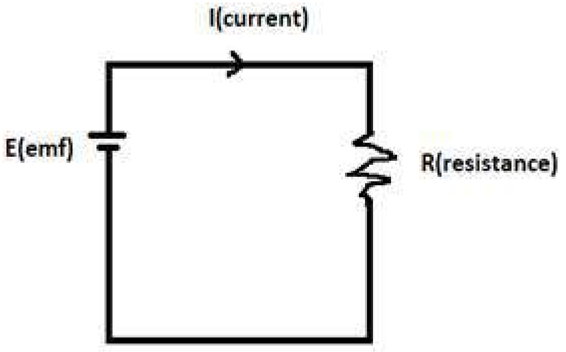
Hence net force on wire becomes zero and it moves with constant velocity v

Therefore, no external force is needed to move the wire with velocity v

Answer.36

Current will flow from the left edge to the right, that is, in the clockwise direction. Therefore, the induced emf will also flow along the clockwise direction.

Diagram showing induced emf as a battery:



Given:

Resistance per unit length = r

Length of wire = l

Velocity with which the wire moves = v

Formula used:

We know, that $\boldsymbol{E = Blv}$ ... (i), where E = emf, B = magnetic field, v = velocity with which the wire moves, l = length of wire.

Now, total resistance$(R) = r \; x \; l'$ ... (ii), where r = resistance per unit length, l' = total length of loop

Horizontal length of loop = vt, where v = velocity, t = time.

Hence, total length of loop$(l') = 2(l + vt)$

Therefore, R(total resistance) = $2r(l + vt)$ ... (iii), where r = resistance per unit length, l = length of wire

By Ohm's law, we know that $\boldsymbol{E = IR}$, where E = emf, I = current, R = total resistance. Hence, from (iii), we get, $E = 2Ir(l + vt)$ ....(iv)

Equation (i) and (iv), we get:

$Blv = 2Ir(l + vt)$

=> $I = \frac{Blv}{2r(l+vt)}$

Current in the circuit $I = \frac{Blv}{2r(l+vt)}$ (Answer)

Answer.37

Formula used:

(a) Magnetic force on a current carrying wire $\boldsymbol{F} = \boldsymbol{I}(\boldsymbol{l}\ \boldsymbol{X}\ \boldsymbol{B})$, where I = current, l = length of wire, B = magnetic field.

Since l and B are perpendicular to each other, magnetic force F becomes $F = IlB$ … (i)

Now, from the previous problem, $I = \frac{Blv}{2r(l+vt)}$ … (ii)

Now, the force needed to keep the sliding wire from moving will be equal to the magnetic force, but in the opposite direction.

Let this force be F'.

Hence, $F' = F = ILB$, where F = magnetic force, I = current, L = length of wire, B = magnetic field.

Substituting the value of I from (ii):

$$F' = \frac{Blv}{(2r(l+vt))x} \times lB = \frac{B^2 l^2 V}{2r(L+vt)}$$

Hence, force required to keep the wire from sliding = $\frac{B^2 l^2 V}{2r(L+vt)}$ (Ans)

(b) Now, just after time t = 0, the force required to stop the wire from sliding will be $F_0 = \frac{B^2 l^2 v}{2rl}$ … (i) (substituting t = 0), from the previous part of this question.

Now, let the time taken for the required force to be $\frac{F_0}{2}$ be t = T.

Hence, from the previous question, substituting t = T,

$$\frac{FF_0}{2} = \frac{BlB^2 lv^2 v}{4r} \ldots \text{(ii)}$$

Substituting the value of F_0 from (i), we get

$$\frac{BlB^2 lv^2 v}{4rl} = \frac{B^2 l^2 v}{2rl}$$

$$\Rightarrow 2r(l + vT) = 4rl$$

$\Rightarrow l + vT = 2l \Rightarrow T = \frac{l}{v}$

Time taken for the force to reduce to $\frac{F_0}{2} = \frac{l}{v}$ (Ans)

Answer.38

Given:

Mass of PQ = m

Resistance of PQ = r

Length of PQ between the two rails = l

Magnetic field = B

Resistance connected to the rails = R

Velocity with which PQ is pushed towards right at t=0 = v_0

Formula used:

(a) By Ohm's law, E = IR', where E = emf, I = current, R' = total resistance.

Hence, current $\boldsymbol{I} = \frac{\boldsymbol{E}}{\boldsymbol{R'}}$ ... (i)

Now, emf induced due to the moving road in the magnetic field $\boldsymbol{E} = \boldsymbol{Blv}$ ... (ii), where B = magnetic field, l = length of rod, v= velocity of rod

Also, total resistance $R' = r + R$ ... (iii), where r = resistance of PQ, R = resistance attached to the rails.

Hence, substituting the values of E and R' from (ii) and (iii) in (i), we get

$$I = \frac{Blv}{r + R}$$

Therefore, current in the loop when the speed of the wire PQ is $v = \frac{Blv}{r+R}$. (Ans)

(b) Now, magnetic force on a current carrying wire $\boldsymbol{F = IlB}$ ... (i), where I = current, l = length of wire, B = magnetic field.

From the previous part, the value of current at an instant when velocity = v is $v = \frac{Blv}{r+R}$... (ii)

Therefore, from (i) and (ii), magnetic force $F = \frac{B^2 l^2 v}{r+R}$ ... (iii)

According to Newton's second law of motion, $F = ma$ ... (iv), where F = force, m = mass, a = acceleration.

Hence, equating (iii) and (iv):

$$ma\ ma = \frac{B^2 l^2 v}{r+R} \Rightarrow a = \frac{B^2 l^2 v}{[m(r+R)]}$$

Therefore, acceleration of the wire at this instant = $a = \frac{B^2 l^2 v}{[m(r+R)]}$ (Ans)

(c) Velocity v' can be expressed as $\boldsymbol{v = v_0 - at}$ ... (i), where v_0 = initial velocity, a = acceleration, t = time. We put a negative sign before at since the force is opposite to velocity, and it

Now, from the previous part, we can write acceleration $a = \frac{B^2 l^2 v}{[m(r+R)]}$ ... (ii)

Hence, from (i) and (ii), we can write $v\ v = v\mathrm{v}_0 - \frac{B^2 l^2 v}{[m(r+R)]}t$

But, distance travelled x = vt, where v = velocity, t = time.

Therefore, velocity v as a function of x is $v\ v = v\mathrm{v}_0 - \frac{B^2 l^2 v}{[m(r+R)]}t$(Ans)

(d) We know that,$a\ a = \frac{dv}{dt} v \frac{dv}{dx} = \frac{B^2 l^2 v}{[m(r+R)]} t \frac{dv}{dx} X v \frac{dv}{dx}$, where a = acceleration, v = velocity, x = distance, t = time.

Now, from the part (b), acceleration as a function of time

$$dx = \frac{dvm(R + r)}{B^2 l^2}$$

$$\Rightarrow x = \frac{m(R+r)v_0}{B^2 l^2}$$

Now, the wire can travel maximum distance when its velocity is v_0.

Hence, integrating on both sides, we get

$$\Rightarrow x = \frac{m(r+R)v_0}{B^2 l^2}$$

Therefore, maximum distance travelled by the wire

$x = \frac{m(r+R)v_0}{B^2 l^2}$ (Ans)

Answer.39

Given:

Length ab = cd = 30 cm = 0.3 m

Length bc = ad = 80 cm = 0.8 m

Total resistance R = 2 Ω

Magnetic field B = 0.02 T

Force F = 3.2×10^{-6} N

Formula used:

(a) Magnetic force on a current carrying wire $\boldsymbol{F = IlB}$ … (i),where I = current, l = length of wire, B = magnetic field.

Hence, current $I = \frac{F}{lB}$ … (ii)

Now, emf $E = Blv$ … (iii), where B = magnetic field, l = length of wire, v = constant velocity with which it is moving

Also, by Ohm's law, $E = IR$ ... (iv), where I = current,

R = resistance.

Hence, equating (iii) and (iv) and substituting I from (ii), we get

$Blv = \frac{FR}{lB}$, where B = magnetic field, v = velocity, F = force, R = resistance, l = length of wire.

Here, since the force is applied on the side cd, we consider

l =30 cm = 0.3 m (the shorter length).

Hence,

Substituting the given values, we get

$v = \frac{(3.2 \times 10^{-6})^2}{(0.3 \times 0.02)^2}$ ms^{-1} = 0.18 ms^{-1}

Constant speed with which the frame moves = **0.18 ms^{-1}**(Ans)

(b) Emf induced in the loop $\boldsymbol{E = Blv}$, where B = magnetic field, l = length of the wire which is moving, v = velocity

Hence, $E = (0.02 \times 0.3 \times 0.18)\, V$ = 0.001 V

Emf induced in loop = **0.001 V** (Ans)

Answer.40

Given:

Resistance of wire(R) = 0.2 Ω

Length of wire(l) = 20 cm = 0.2 m

Current(I) = 2 μA = 2 x 10^{-6} A

Velocity with which the wire moves(v) = 20 cms^{-1} = 0.2 ms^{-1}

Horizontal component of earth's magnetic field(B_H) = 3.0×10^{-5} T

Formula used:

Angle of dip $\delta = tan^{-1}(\frac{B_V}{B_H})$ ... (i), where B_V = vertical component of earth's magnetic field, B_H = horizontal component of earth's magnetic field.

Now, emf induced in the wire $E = B_V lv$ ... (ii), where B_V = vertical component of earth's magnetic field, l = length of wire, v = velocity with which the wire moves

Also, by Ohm's law, $E = IR$ ... (iii), where E = emf, I = current, v = velocity.

Equating (ii) and (iii) we get

$B_V l_v$= IR ⇒ $B_V = \frac{IR}{l_v}$

Substituting the given values, we get

$B_V = \frac{2 X 10^{-6} X 0.2}{0.2 X 0.2} T$ = 10^{-5} T

Hence, angle of dip ⇒ $\delta = tan^{-1}\left(\frac{1}{3}\right)$ (Ans)

Answer.41

Given:

Length of ab = l

Mass = m

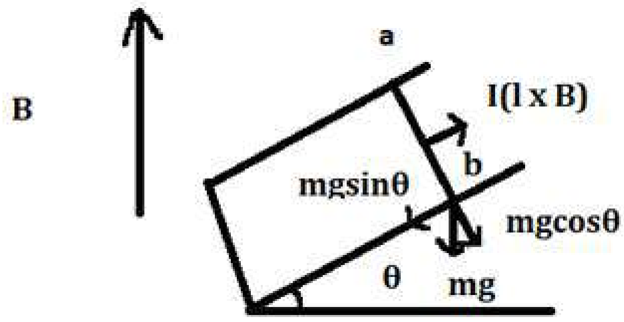
Resistance of ab = R

Angle between the plane of the rails and the horizontal = θ

Magnetic field = B

Velocity with which the wire slides along the rails = v

Diagram:



Formula used:

Emf induced in the wire ab $E = Bl'v$ ... (i), where

E = emf, B = magnetic field, l' = component of l perpendicular to the magnetic field = l cosθ, v = velocity

Magnetic force on ab **F = I (l X B)**, where I = current, l = length of ab, B = magnetic field

Now, the angle between l and B is 90^0 - θ.

Therefore, F = IlBsin (90^0 - **θ**) = IlBcosθ ... (ii)

This force will be equal to the sinθ component of the weight of the wire.

Hence, $F = mgsin\theta$ ... (iii)

Now, $I = \frac{E}{R} = Bl'\frac{v}{R} = \frac{Blvcos\theta}{R}$ ... (iv), where I = current, E = emf, R = resistance, B = magnetic field, v = velocity, l = length of wire, θ = angle between plane of rod and horizontal

Therefore, substituting this in (ii), we get

F = $\frac{B^2 l^2 vcos^2\theta}{R}$ ... (v)

Hence, equating (iii) and (v), we get

$$mgsin\theta = \frac{B^2 l^2 vcos^2\theta}{R}$$

⇒ Magnetic field B $B = \sqrt{\frac{mgRsin\theta mgsin\theta R}{vll^2 vcos^2\theta\theta}}$ (proved)

Answer.42

Given:

Velocity with which the wires move(v) = 5 cm s^{-1} = 0.05 m s^{-1}

Resistance(R) = 19 Ω

Resistance of each of the wires(r) = 2 Ω

Length of wire(l) = 4 cm = 0.04 m

Magnetic field(B) = 1 T

Formula used:

Emf induced $\boldsymbol{E = Blv}$ ... (i) where B = magnetic field, l = length of wire, v = velocity

Also, by Ohm's law, emf $\boldsymbol{E = IR}$ ... (ii), where I = current, R = total resistance.

(a) When the wires slide in the same direction, we have two parallel sources of emf with current flowing in the same direction.

Hence, from (i),net emf $E = (1\ X\ 0.04\ X\ 0.05)\ V = 2\ X\ 10^{-3} V$

Net parallel resistance of the 2 Ω wires = $\frac{2\times 2}{2+2}$ Ω = 1 Ω

Hence, total resistance(R) = (1 + 19) Ω = 20 Ω

Therefore, substituting these values in (ii), we get

Current $I = \frac{2\times 10^{-3}}{20}$ A = $\mathbf{10^{-4}}$ **A** (Ans)

(b) When the wires slide in opposite directions, the two parallel sources of emf have opposing directions. Hence, the net emf is 0.

Therefore, the net current is also 0. (Ans)

Answer.43

(a) When the 19Ω resistor is removed and the wires in move in the same direction, their polarity remains the same. Hence, the circuit remains incomplete and the current through P_2Q_2 is 0. (Ans)

(b) When the wires slide in opposite directions, the polarity of one of the wires reverses and current flows.

In this case, emf $E = Blv$, where B = magnetic field, l = length, v = velocity.

Hence, $E = (1\ X\ 0.04\ X\ 0.05)$ V = $2 \times 10^{-3}\ V$

But here, resistance R = 2 Ω only

Therefore, Current flowing through the wire $I = \frac{E}{R}$ = $\mathbf{10^{-3}}$ **A** (Ans)

Answer.44

Given:

Speed(v) = 5 cms^{-1} = 0.05 ms^{-1}

External resistance(R) = 10Ω

Magnetic field(B) = 1 T

Formula used:

Induced emf $\boldsymbol{E = Blv}$ ... (i), where B = magnetic field, l = length of sliding wire, v = velocity

(a) When the switch S is thrown to the middle rail, length of sliding wire l = 2 cm = 0.02 m

Hence, induced emf in this case from (i) is

E = (1 x 0.02 x 0.05) V = 10^{-3} V

Given resistance R = 10Ω

Therefore, current flowing through the resistor $\boldsymbol{I} = \frac{\boldsymbol{E}}{\boldsymbol{R}}$

where E = emf, R = resistance.

$I = \frac{10^{-3}}{10} A$ = **10^{-4} A = 0.1 mA** (Ans)

(b) When the switch S is thrown to the bottom rail, length of sliding wire(l') = 4 cm = 0.04 m

Hence, induced emf $E' = Bl'v$ = (1 x 0.04 x 0.05) V = 2 x 10^{-3} V, where B = magnetic field, l' = length of sliding wire, v = velocity

Resistance R = 10Ω

Therefore, current flowing through the resistor I' = E'/R, where E' = emf, R = resistance

$I = \frac{2 \times 10^{-3}}{10}$A = **2 x 10^{-4} A= 0.2 mA** (Ans)

Answer.45

Given:

Initial current passing through the circuit = i

Velocity with which the wire ab moves = v

Resistance of each wire = r

Formula used:

Induced emf due to moving of wire ab $\boldsymbol{E' = Blv}$ … (i), where B = magnetic field, l = length of sliding wire, v = velocity

Initial emf in the wire E_0 = ir, where i = current, r = resistance of wire ab.

Hence, net emf$(E) = E_0 - E' = ir - Blv$ ... (ii)

Now, net resistance = 2r

Hence, current passing through the wire cd = $\frac{E}{2r} = \frac{ir - Blv}{2r}$ (Ans)

Answer.46

Given:

Initial current = i

Length of sliding wire ab = l

Mass = m

Magnetic field = B

Formula used:

Magnetic force on the wire ab $\boldsymbol{F = ilB}$ ... (i), where i = current, l = length of sliding wire, B = magnetic field

Now, velocity v can be written as $\boldsymbol{v = u + at}$ ... (ii), where u = initial velocity = 0(in this case), a = acceleration, t = time.

Hence, acceleration a = v/t ... (iii)

Now, according to Newton's 2nd law of motion, $\boldsymbol{F = ma}$ ... (iv), where F = force, m = mass, a = acceleration.

Substituting (iii) in (iv) and equating (i) and (iv), we get

$$ilB = \frac{mv}{t} \Rightarrow v = \frac{ilBt}{m}$$

Hence, velocity of the wire as a function of time is $v = \frac{ilBt}{m}$ (Ans)

Answer.47

Given:

Initial mass = m

Magnetic field = B

Length of sliding wire = l

Formula used:

Magnetic force $\boldsymbol{F} = \boldsymbol{ilB}$ … (i), where i = current, l = length of sliding wire, B = magnetic field.

At equilibrium, this magnetic force balances the weight of the wire mg acting downward, where m = mass, g = acceleration due to gravity.

Therefore, $mg = ilB$ … (ii)

Now, when the wire ab is replaced by another wire of mass 2m, the weight acting downward will be 2mg, where g = acceleration due to gravity.

Hence, net force = 2mg - ilB … (iii) where 2m = mass, g = acceleration due to gravity, i = current, l = length of sliding wire, B = magnetic field.

According to Newton's law of motion, $\boldsymbol{F} = \boldsymbol{m'a}$ … (iv), where F = net force, m' = mass, acceleration

In this case, m' = 2m.

Therefore, equating (iii) and (iv), we get

2mg - ilB = 2ma

$\Rightarrow a = \frac{2mg - ilB}{2m}$ … (v)

Now, distance travelled can be expressed as $s\ S = ut\ ut + \frac{1}{2}at\frac{1}{2}at^{2^2}$ ... (vi), where s = distance travelled, u = initial velocity = 0(in this case), t = time, a = acceleration.

Now, distance travelled = l (given)

Therefore, from (v), (vi) becomes:

$$l = \frac{1}{2} \times \frac{2mg - Blv}{2m} t^2$$

$$\Rightarrow t\ t = \sqrt{\frac{4mmll}{2mg - BlV\ ilB}}$$

But, from (i), $ilB = mg$.

Therefore, $t\ t = \sqrt{\frac{4mmll}{2mg - mg\ ilB}} \Rightarrow t = \sqrt{\frac{2l}{g}}$

Required time taken = $t = \sqrt{\frac{2l}{g}}$ (Ans)

Answer.48

Given:

Length of sliding wire = width of frame = d

Mass = m

Resistance = R

Magnetic field = B

Initial force = F

Formula used:

(a) Induced emf(when it attains a speed v) $\boldsymbol{E} = \boldsymbol{Bdv}$ ... (i), where B = magnetic field, d = width of frame, v = velocity

Therefore, induced current $\boldsymbol{I} = \frac{\boldsymbol{E}}{\boldsymbol{R}}$, where E = induced emf, R = resistance ⇒ $I = \frac{Bdv}{R}$ ... (ii)

Now, magnetic force acting on the wire $\boldsymbol{F'} = \boldsymbol{IdB}$ ... (iii), where I = current, d = length of sliding wire = width of frame, B = magnetic field

Substituting (ii) in (iii), $F' = \frac{B^2d^2v}{R}$ ... (iv)

Now, as the magnetic force is in opposite direction to applied force, net force = $F' - F = F - \frac{B^2d^2v}{R}$ ... (v)

But, from Newton's 2nd law of motion, **net force = ma** ... (vi), where m = mass, a = acceleration

Equating (v) and (vi):

$$ma = F - \frac{B^2d^2v}{R} \Rightarrow a = F - \frac{B^2d^2v}{Rm}$$

Acceleration of the frame at speed $v = F - \frac{B^2d^2v}{Rm}$ (Ans)

(b) For the velocity to be constant, acceleration needs to be 0.

Hence, from previous part,

$\frac{F}{m} - \frac{B^2d^2v_0}{mR} = 0$ where F = external force, m = mass, B = magnetic field, d = width of frame, v_0 = constant velocity, R = resistance

=> $v_0 = \frac{FR}{B^2d^2}$

Constant velocity $v_0 = \frac{FR}{B^2d^2}$ (Ans)

(c) From part (a), acceleration $a = \frac{F}{m} - \frac{B^2d^2v}{mR}$

Now, acceleration a = dv/dt, where v= velocity, t = time

Hence, $\frac{dv}{dt} = \frac{F}{m} - \frac{B^2d^2v}{mR} = \frac{FR - B^2d^2v}{mR}$

=> $\frac{dv}{FR - B^2d^2v} = \frac{dt}{mR}$

Integrating with proper limits, we get

$$\int_0^v \frac{dv}{FR - B^2d^2v} = \int_0^t \frac{dt}{mR}$$

=> $\left[\frac{-1}{B^2d^2} ln|FR - B^2d^2v|\right]_0^v = \frac{t}{mR}$ => $ln|FR - B^2d^2v| - ln(RF) \frac{(-B^2d^2t)}{mR}$

=> $\frac{RF - B^2d^2v}{RF} = e^{-\frac{B^2d^2t}{mR}}$

=> $B^2d^2v = RF\left(1 - e^{-\frac{B^2d^2t}{mR}}\right)$

=> $v = \frac{RF}{B^2d^2}\left(1 - e^{-\frac{B^2d^2t}{mR}}\right)$

But, from previous part (b), we found out that $v_0 = \frac{FR}{B^2d^2} \Rightarrow$

$$F = \frac{v_0B^2d^2}{R}$$

Hence, $v = \frac{Rxv_0B^2d^2}{(B^2d^2R)\left(1 - e^{-\frac{B^2d^2t}{mR}}\right)}$

=> $v = v_0\left(1 - e^{-\frac{B^2d^2t}{mR}}\right) = v_0(1 - e^{-\frac{Ft}{Mv_0}})$ (proved)

Answer.49

Given:

Emf of the battery = ϵ

Length of sliding wire = l

Resistance = r

Magnetic field = B

Velocity = v

Formula used:

(a) Induced emf $\boldsymbol{E'} = \boldsymbol{Blv}$ … (i), where B = magnetic field, l = length of sliding wire, v = velocity

Therefore, net emf = $\epsilon - Blv$

Hence, current in the wire$(i) = \frac{\epsilon - Blv}{r}$(Ans)

(b) Magnetic force acting on the wire $\boldsymbol{F} = \boldsymbol{ilB}$ … (ii), where i = current, l = length of sliding wire, B = magnetic field.

Hence, from (a), $i = \frac{\epsilon - Blv}{r}$

Therefore, force $F = \frac{(\epsilon - Blv)lB}{r}$ (Ans)

(c) At constant velocity, net force will be 0.

Hence, $\frac{(\epsilon - Blv)lB}{r}$ = 0

$\Rightarrow \epsilon - Blv = 0 \Rightarrow v = \frac{\epsilon}{Bl}$

Hence, value of velocity = $\frac{\epsilon}{Bl}$ (Ans)

Answer.50

Given:

Length of sliding wire ab = l

Resistance = r

Mass = m

Magnetic field = B

Speed = v

Formula used:

(a) Induced emf in the loop $E = \boldsymbol{Blv}$ (Ans), where B = magnetic field, l = length, v = velocity

(b) Induced current in the wire $I = \frac{E}{r} = \frac{\boldsymbol{Blv}}{\boldsymbol{r}}$ (Ans) , where E = emf, r = resistance

As the wire is moving, the magnetic flux is increasing. Hence, the direction of the current will be such as to oppose the increase in flux. Hence, the current will move from b to a.

(c) Magnetic force on the wire(upward) $\boldsymbol{F} = \boldsymbol{IlB}$ … (i), where I = current, l = length of sliding wire, B = magnetic field.

Weight acting downward = mg

Hence, net downward force = $IlB - mg$ … (ii)

According to Newton's 2nd law of motion, net force = ma … (iii), where m = mass, a = acceleration.

Hence, equating (ii) and (iii), $ma = mg - IlB$

But, I = E/r, where I = current, E = emf, r = resistance => I = Blv/r, where B = magnetic field, l = length of sliding wire, v = velocity

=> acceleration $a = \left(g - \frac{B^2l^2v}{mr}\right)$ (Ans)

(d) When the wire will move with constant velocity (let it be v_0), acceleration will be 0.

Hence, from part (d) of this question,

$$g - \frac{B^2l^2v_0}{mr} = 0$$

=> constant velocity $v_0 = \frac{mrg}{B^2l^2}$ (Ans)

(e) From part (c),

$$a = g - \frac{B^2l^2v_0}{mr}$$

But, a = dv/dt, where v = velocity, t = time

Therefore, $\frac{dv}{dt} = g - \frac{B^2l^2v_0}{mr}$

$$\Rightarrow \frac{dv}{mrg - B^2l^2v} = \frac{dt}{mr}$$

Integrating with proper limits, we get

$$\int_0^v \frac{dv}{\frac{mrg}{B^2l^2} - v} = \int_0^t \frac{dt}{mr}$$

$$\Rightarrow \left[-ln\left|\frac{mrg}{B^2l^2} - v\right|\right]_0^v = \frac{t}{mr} {>} \frac{B^2l^2v}{mrg} = 1 - e^{-\frac{B^2l^2t}{mr}}$$

$$\Rightarrow v = \frac{mrg}{(B^2l^2)\left(1 - e^{-\frac{B^2l^2t}{mr}}\right)}$$

But, from the previous part, $v_0 = \frac{mrg}{B^2l^2v}$

Therefore, velocity as a function of time : $v = v_0\left(1 - e^{-\frac{B^2l^2t}{mr}}\right)$(Ans)

(f) Now, v can be written as dx/dt, where x = position, t = time

Therefore, from previous part, $v = \frac{dx}{dt} = v_0\left(1 - e^{-\frac{B^2l^2t}{mr}}\right)$

$$\Rightarrow dx = v_0\left(1 - e^{-\frac{B^2l^2t}{mr}}\right)dt$$

Integrating with suitable limits, we get:

$$\int_0^x dx = \int_0^t v_0\left(1 - e^{-\frac{B^2 l^2 t}{mr}}\right) dt$$

$$=> x = v_0\left[t + \frac{v_0}{g\left(e^{-\frac{gt}{v_0}} - 1\right)}\right]$$

=> Displacement as a function of time $x = v_0 t - \frac{v_0^2}{g\left(1 - e^{-\frac{gt}{v_0}}\right)}$ (Ans)

(g) Then, $\frac{d}{dt}mgs = mg\frac{ds}{dt} = mgv_m\left(1 - e^{\frac{-gt}{v_m}}\right)$

$$\frac{dH}{dt} = i^2R = R\left(\frac{lBv}{R}\right)^2$$

$$\frac{dH}{dt} = \frac{l^2B^2}{R}\left(v_m\left(1 - e^{\frac{-gt}{v_m}}\right)\right)^2$$

After steady state, $t \to \infty$

$$\frac{d}{dt}mgs = mgv_m \quad \frac{dH}{dt} = \frac{l^2B^2}{R}v_m^2$$

$$\frac{dH}{dt} = \frac{l^2B^2}{R}v_m \times \frac{mgR}{l^2B^2}$$

$$\frac{dH}{dt} = mgv_m$$

Hence after steady state,

$$\frac{dH}{dt} = \frac{d}{dt}mgs$$

So, the rate of heat developed in the wire is equal to the rate at which the gravitational potential energy is decreased after steady state is reached.

Answer.51

Given:

Angular speed(w) = 100 revolutions/minute x 2π = 100 revolutions/60 sec x 2π= 10π/3 revolutions/sec

Length of each spoke(l) = 30 cm = 0.3 m

Magnetic field(B) = 2.0×10^{-5} T

Formula used:

Induced emf $\boldsymbol{E = Blv}$ ... (i), where B = magnetic field, l = length of spoke, v = velocity

Now, linear speed of the spoke v = ωr, where ω = angular speed, r = distance from the axis to the outer end.

Here, $r = \frac{l}{2}$, where l = length of spoke

Hence, $v = \frac{wl}{2}$

Therefore, emf induced $E = \frac{Bwl^2}{2}$ (from (i))

⇒ $E = \left(\frac{2\times10^{-5}\times10\frac{\pi}{3}\times(0.3)^2}{2}\right)$V = **9.42 x 10^{-6} V** (Ans)

Answer.52

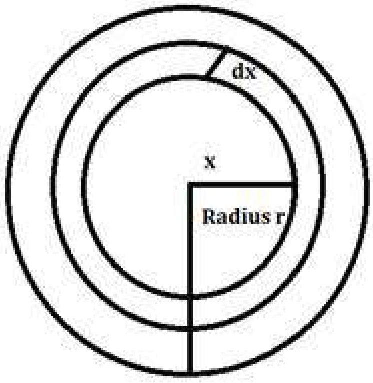
Given:

Radius = r

Angular velocity = w

Magnetic field = B

Diagram:



Formula used:

In this case, the velocity will increase radially.

Let us consider a strip of width dx at a distance x from the centre.

Hence, induced emf of this portion will be $\boldsymbol{dE = Blv = Bdx \times xw}$, where B = magnetic field, dx = width of the element, x = distance of the element from the centre, w = angular velocity

Hence, integrating on both sides using proper limits, we get

$$\int_0^E dE = \int_0^r Bxwdx$$

=> Total motional emf $\boldsymbol{E = \frac{Bwr^2}{2}}$ (Ans)

Answer.53

Given:

Resistance(R) = 10Ω

Radius(r) = 5 cm = 0.05 m

Angular speed(w) = 10 rad/s

Magnetic field(B) = 0.4 T

Formula used:

We consider a rod of length 5 cm from the centre and rotating with same w.

Hence, length of sliding rod(l) = 5 cm = 0.05 m.

Now, induced emf $\boldsymbol{E} = \boldsymbol{Blv}$ ... (i), where B = magnetic field, l = length of sliding rod, v = velocity

Now, velocity $v = \frac{l}{2} x\ w$ ... (ii), where l = length or rod, w = angular velocity

From Ohm's law, current through resistor R(I) = E/R = $Bl^2w/2R$ (from (i) and (ii)), where E = emf, R = resistance

Hence, $I = \frac{0.4 \times 0.05^2 \times 10}{2 \times 10}$ A = **0.5 mA** (Ans)

Since the disc is rotating anticlockwise, the emf induced is such that the centre is at a higher potential than the periphery. Hence, the current leaves from the centre.

Answer.54

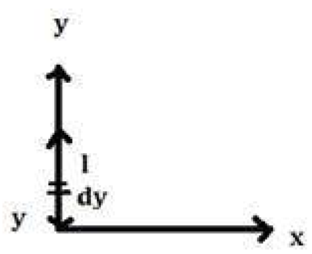
Given:

$$\vec{B} = k\frac{\vec{B}_0}{L}y$$

Where B = magnetic field, y = distance from origin on y axis, L = fixed length.

Velocity of rod = v_0 i

Diagram:



Formula used:

Now, we consider a small element dy at a distance y from the origin.

Emf induced in the element dE = Bvdy, where B = magnetic field, v = velocity, dy = length of element

$\Rightarrow dE = \frac{B_0}{L} y v_0 dy$

Integrating on both sides with proper limits, we get

$$\int_0^E dE = \int_0^L \frac{B_0}{L} y v_0 dy$$

=> Total emf $E = \frac{B_0 v_0 L^2}{2L}$

$E = \frac{B_0 v_0 L}{2}$ (Ans)

Answer.55

Given:

Current = i

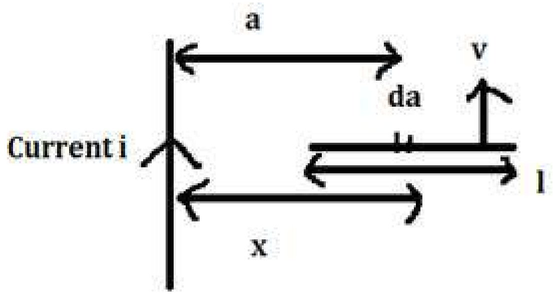
Length of rod = l

Velocity = v

Distance of centre of rod from wire = x

Hence, the two ends of the rod are at distances $x - \frac{l}{2}$ and $x + \frac{l}{2}$ from the wire

Diagram:



Formula used:

We consider an element of length da at a distance of ‘a’ from the wire.

Now, magnetic field due to an infinite current carrying wire at a distance a $(\boldsymbol{B}) = \frac{\mu_0 i}{2\pi a}$ ... (i), where μ_0 = magnetic permeability of vacuum, i = current, a = distance from wire.

Therefore, emf induced in the element $da = dE = Bdav$ ... (ii), where B = magnetic field, da = element, v = velocity

Hence, putting (i) in (ii), we get

$$dE = \frac{\mu_0 ixdaxv}{2\pi a}$$

Integrating on both sides and putting suitable limits, we get

$$\int_0^E dE = \int_{x-\frac{l}{2}}^{x+\frac{l}{2}} \frac{\mu_0 idav}{2\pi a}$$

$$\Rightarrow E = \frac{\mu_0 iv}{2\pi}[ln|a|]_{x-\frac{l}{2}}^{x+\frac{l}{2}} E = \frac{\mu_0 iv}{2\pi} ln\left|\frac{x+\frac{l}{2}}{x-\frac{l}{2}}\right| \text{(ans)}$$

Answer.56

Given:

Resistance = R

Constant velocity = v

Formula used:

From previous question, induced emf

$E = \frac{\mu_0 iv}{2\pi} ln \left|\frac{x+\frac{l}{2}}{x-\frac{l}{2}}\right| = \frac{\mu_0 iv}{2\pi} ln \left|\frac{2x+l}{2x-l}\right|$ w, induced current $I = \frac{E}{R} = \frac{\mu_0 iv}{2\pi R} ln\left|\frac{x+l/2}{x-l/2}\right|$ … (i), where E = emf, R = resistance, μ_0 = magnetic permeability of vacuum, i = current in the wire, v = velocity of sliding rod, x = distance of centre of rod from wire, l = length of rod.

Now, magnetic force on element $da = dF = IdaB$ … (ii), where I = induced current, da = element, B = magnetic field due to infinitely straight wire

$B = \frac{\mu_0 i}{2\pi a}$ … (iii), where μ_0 = magnetic permeability of vacuum, i = current in wire, a = distance from wire.

Hence, (ii) becomes

dF = $\frac{\frac{\mu_0 iv}{2\pi R} ln\left|\frac{x+\frac{l}{2}}{x-\frac{l}{2}}\right| da \times \frac{\mu_0 i}{2\pi a\left(\frac{\mu_0 i}{2\pi}\right)^2 X ln\left|\frac{x+\frac{l}{2}}{x-\frac{l}{2}}\right| X} v}{R} \times \frac{da}{a}$ integrating with suitable limits, we get

$$\int_0^F dF = \frac{\int_{x-\frac{l}{2}}^{\left(x+\frac{l}{2}\right)} \left(\frac{\mu_0 i}{2\pi}\right)^2 X ln\left|\frac{x+\frac{l}{2}}{x-\frac{l}{2}}\right| \times v}{R} \times \frac{da}{a}$$

=>Force needed to keep the wire sliding at constant velocity v

$F = \left(\frac{\mu_0 i}{2\pi}\right)^2 \left(ln\left|\frac{x+\frac{l}{2}}{x-\frac{l}{2}}\right|\right)^2 \times \frac{v}{R}$ns)

(b) Current I = E/R = $\frac{\mu_0 iv}{2\pi R} ln\left|\frac{x+\frac{l}{2}}{x-\frac{l}{2}}\right|$ here E = emf, R = resistance, μ_0 = magnetic permeability of vacuum, i = current in the wire, v = velocity of sliding rod, x = distance of centre of rod from wire, l = length of rod. (Ans)

(c) Rate of heat developed in the resistor = Power(P) = I^2R, where I = current, R = resistance

From previous part, I = $\frac{\mu_0 iv}{2\pi R} ln\left|\frac{x+\frac{l}{2}}{x-\frac{l}{2}}\right|$ therefore, rate of heat developed =

$\left(\frac{\mu_0 iv}{2\pi R} ln\left|\frac{x+\frac{l}{2}}{x-\frac{l}{2}}\right|\right)^2 R \frac{1}{R\left(\frac{\mu_0 iv}{2\pi} ln\left|\frac{x+\frac{l}{2}}{x-\frac{l}{2}}\right|\right)^2}$ (ans)

(d) Power delivered by external agent = rate of heat developed in resistor =

$\frac{1}{R\left(\frac{\mu_0 iv}{2\pi} ln\left|\frac{x+\frac{l}{2}}{x-\frac{l}{2}}\right|\right)^2}$ (ans)

Answer.57

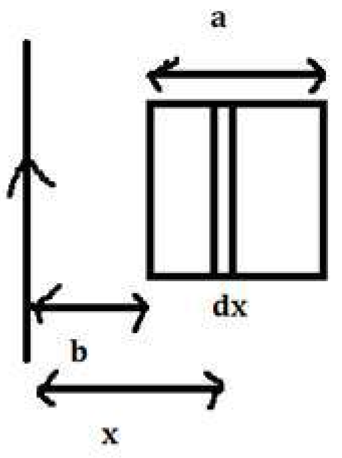
Given:

Current in wire i = i_0 sin ωt

Length of each side of square loop = a

Distance of one edge from wire = b

Diagram:



Formula used:

Magnetic flux $\Phi = \int B.da$ ... (i), where Φ = magnetic flux, B = magnetic field, da = area element

Magnetic field due to a long current carrying wire at distance x $B = \frac{\mu_0 i}{2\pi x}$ ... (ii), where μ_0 = magnetic permeability of vacuum, i = current, x = distance from wire

We consider a strip of width dx at a distance x from the wire.

Now, area element da = a dx, where a = length of loop, dx = width element

Hence, from (i) and (ii),

Flux $\Phi = \int_{b}^{a+b} B.da = \int_{b}^{a+b} \frac{\mu_0 iadx}{2\pi x} = \frac{\mu_0 ia}{2\pi} ln\left(\frac{a+b}{b}\right)$ (ans)

(b) Emf induced in frame $E = \frac{d\Phi}{dt}$, where Φ = flux, t = time

From previous part, $\Phi = \frac{\mu_0 ia}{2\pi} ln\left(\frac{a+b}{b}\right)$re, $i = i_0 \sin \omega t$

$\Rightarrow E = \frac{d\Phi}{dt} = \frac{\mu_0 a}{2\pi} ln\left(\frac{a+b}{b}\right) i_0 \omega cos\omega t$ (Ans)

(c)Heat developed in wire$(\boldsymbol{H}) = \boldsymbol{i^2 rt}$, where i = current through frame, r = resistance, t = time

From previous i = E/r = $\frac{\mu_0 a}{2\pi r} ln\left(\frac{a+b}{b}\right) i_0 \omega cos\omega t$

where E = emf, r = resistance

Hence H = $\frac{\mu_0 a}{2\pi r} ln\left(\frac{a+b}{b}\right) i_0 \omega cos\omega t$

Now, Given: $t = \frac{20\pi}{\omega}$

Hence, H = $\frac{\mu_0 ar}{2\pi} ln\left(\frac{a+b}{b}\right) i_0 \omega X \frac{20\pi cos(20\pi)}{\omega} = \frac{5\mu_0^2 a^2 i_0^2 \omega}{2\pi r} \ln^2\left(\frac{b+a}{b}\right)$

Answer.58

Given: Length of the rectangular loop=l

Breadth of the rectangular loop=b

Current in the wire=i

Speed of loop=v

We have to find the emf induced in the loop by Faradays's Law.

Faraday's law states that whenever the magnetic flux through a closed surface changes, there will be an induced emf prodced in the loop that encloses the surface. The mathematical relation is

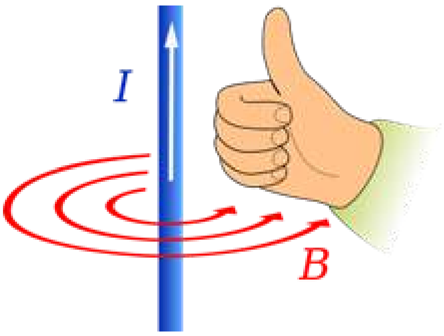
$$\mathcal{E} = -\frac{d(\phi_B)}{dt}$$

where $\mathcal{E}$ is the magnitude of the induced emf, ϕ_B is the the magnetic flux through the surface. The negative sign arises because the induced emf will be produced such that it will oppose the change of magnetic flux. The magnetic flux is given by

$$\phi_B = \oint \vec{B} \cdot \vec{dS}$$

where B is the magnetic field and dS is a small area element on the surface.

The wire which is near the loop is responsible for the magnetic field . The wire is carrying current in the upward direction so by Fleming's right hand thumb rule, the magnetic field will be perpendicular to the plane of the paper in the inward direction.



We have to now calculate the magnetic field acting across the loop. Let us consider a small rectangular element of length b and width dx at a distance of x from wire. The magnteic field on this element due to the current carrying wire is given by Ampere circuital law.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i$$

$$\oint \vec{B} \cdot \vec{dl} = \oint Bdl cos\theta = \oint Bdl$$

(cos θ=1 because the $\overrightarrow{dl}$ vector and $\overrightarrow{B}$vector are both acting in ward in the plane of paper so θ=0°, here dl is a small current carrying element of the circular amperian loop)

$$\oint Bdl = B\oint dl = B \times 2\pi x$$

$$B \times 2\pi x = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi x}$$

The magnetic flux through this element will be

$$d\phi_B = \oint \overrightarrow{B} \cdot \overrightarrow{dS}$$

$$d\phi_B = \oint BdS\, cos\theta$$

$$d\phi_B = \oint BdS = Bbdx$$

(cosθ=1 because the magnetic field across the area and the normal

vector of this area element both point in the same direction so θ=0°

so cosθ=1)

The total magnetic flux through the loop will be the flux through

Infinite such elementss from x=a to x=a+l

$$\phi_B = \int_{x=a}^{x=a+l} d\phi_B$$

$$\phi_B = \int_{x=a}^{x=a+l} Bbdx$$

$$\phi_B = \int_{x=a}^{x=a+l} \frac{\mu_0 i}{2\pi x} bdx$$

$\phi_B = \frac{\mu_0 ib}{2\pi}\int_{x=a}^{x=a+l}\frac{1}{x}dx$ (taking constants out of the integral)

$$\phi_B = \frac{\mu_0 ib}{2\pi}[lnx]_{a}^{a+l}$$

$$\phi_B = \frac{\mu_0 ib}{2\pi}[\ln(a+l) - \ln(a)]$$

$$\phi_B = \frac{\mu_0 ib}{2\pi}[\ln\frac{a+l}{a}]$$

The flux through the loop is $\frac{\mu_0 ib}{2\pi}[\ln\frac{a+l}{a}]$

The emf induced will be $\mathcal{E} = -\frac{d\left(\frac{\mu_0 ib}{2\pi}[\ln\frac{a+l}{a}]\right)}{dt}$

$$\mathcal{E} = -\frac{d\left(\frac{\mu_0 ib}{2\pi}[\ln\frac{a+l}{a}]\right)}{dt}$$

$\mathcal{E} = -\left(\frac{\mu_0 ib}{2\pi}\right)\frac{d\left([\ln\frac{a+l}{a}]\right)}{dt}$ (taking constants out of the differential)

$$\mathcal{E} = -\left(\frac{\mu_0 ib}{2\pi}\right)\left(\frac{a}{a+l}\right) \times \frac{(a+l)\frac{da}{dt} - a\frac{d(l+a)}{dt}}{a^2}$$

$$\mathcal{E} = -\left(\frac{\mu_0 ib}{2\pi}\right)\left(\frac{a}{a+l}\right) \times \frac{(a+l)v - av}{a^2}$$

(because $\frac{da}{dt} = v$ as the rate of change of Distance from wire is the loop speed)

$$\mathcal{E} = -\left(\frac{\mu_0 ib}{2\pi}\right)\left(\frac{1}{a+l}\right) \times \frac{lv}{a}$$

$$\mathcal{E} = \frac{-\mu_0 iblv}{2\pi a(a+l)}$$

The magnitude of the induced emf is $\frac{\mu_0 iblv}{2\pi a(a+l)}$.

Answer.59

Given: Radius =

a

Magnetic field = B

Resistance = R

Angular velocity = w

Formula used:

Let us consider an element of length dr at a distance r from the centre.

Hence, induced emf on this portion $dE = Blv = Bdrwr$ ... (i), where B = magnetic field, dr = length of element, w = angular velocity, r = distance from centre (since v = ωr)

Hence, integrating on both sides with suitable limits, we get

$\int_0^E dE = \int_0^a Bwrdr \Rightarrow E = \frac{Bwa^2}{2}$

Now, current $I = \frac{E}{R}$, where E = emf, resistance = R

=> $I = \frac{Bwa^2}{2R}$

Hence, force on the rod $F = IlB = IaB$(where I = current, a = length of rod, B = magnetic field) ⇒ $\boldsymbol{F} = \frac{\boldsymbol{B^2wa^3}}{\boldsymbol{2R}}$ (Ans)

Answer.60

Given:

Resistance of circular loop = R

∠AOC = 90°

Angular velocity = w

Formula used:

From the previous problem, emf $E = \frac{Bwa^2}{2R}$ ... (i), where B = magnetic field, w = angular velocity, a = radius

Now, since ∠AOC = 90°, the major and minor segments of the arc AC consist of parallel combination of resistances of R/4 and 3R/4 respectively (since the resistance is divided in the ratio of the angle at the centre).

Hence, equivalent resistance $R' = \frac{\frac{R}{4}X\frac{3R}{4}}{R} = \frac{3R}{16}$

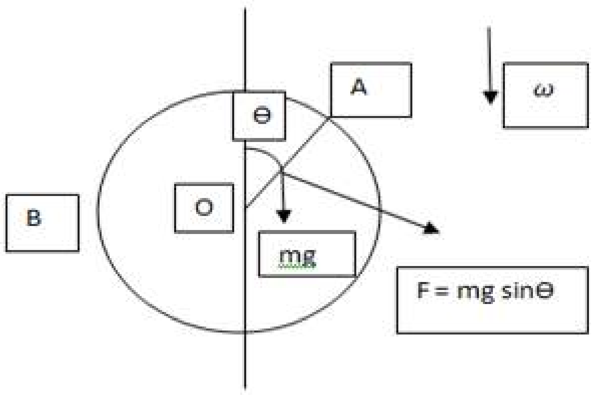
Therefore, current through the rod $I = \frac{E}{R'}$, where E = emf, R' = equivalent resistance

$\Rightarrow I = \frac{\frac{Bwa^2}{2}}{\frac{3R}{16}} = \frac{8Bwa^2}{3R}$ (Ans)

Answer.61

When the circular loop is in the vertical plane, it tends to rotate in the clockwise direction because of its weight.

Let the force applied be F and its direction be perpendicular to the rod.The component of mg along F is $mg \sin\theta$.The magnetic force is in perpendicular and opposite direction to $mg \sin\theta$.



Now, Current in the rod will be

$$I = \frac{Ba^2\omega}{2R}$$

The force on the rod will be

$$F_B = IBl = \frac{B^2a^2\omega}{2R}$$

So, the net force will be

$$F - \frac{B^2a^2\omega}{2R} + mg\sin\theta$$

The net force passes through the centre of mass of the rod.Net torque on the rod about the centre O will be

$$\tau = \left(F - \frac{B^2 a^2 \omega}{2R} + mg\sin\theta\right)\frac{OA}{2}$$

Because the rod rotates with a constant angular velocity, the net torque on it is zero.

Thus, $\tau = 0$

$$\left(F - \frac{B^2 a^2 \omega}{2R} + mg\sin\theta\right)\frac{OA}{2} = 0$$

$$F = \frac{B^2 a^2 \omega}{2R} + mg\sin\theta$$

Answer.62

Given:

Emf = ϵ

Resistance = r

Angle made by rod = θ

Angular velocity = w

Formula used:

From the previous questions, induced emf $e = \frac{Bwa^2}{2}$ ... (i), where B = magnetic field, w = angular velocity, a = radius

Hence, Total emf = $E + e = E + \frac{Bwa^2}{2}$

Total current i = total emf/resistance = $\frac{E + \frac{Bwa^2}{2}}{R} = \frac{Bwa^2 + 2E}{2R}$ ... (ii), where R = resistance

'Now, net force on the rod $F = (mgcos\theta - ilB)$ ... (iii),

where mgcosθ = component of weight along rod, where m = mass, g = acceleration due to gravity, θ = angle made by rod with horizontal, and ilB = Magnetic force, where i = current, l = length of rod, B= magnetic field.

Since the rod rotates with uniform angular velocity, net torque about O = 0.

Hence, torque = net force x distance from line of action = $(mgcos\theta - ilB)\left(\frac{a}{2}\right) = 0$, where a = radius of rod

Therefore, $mgcos\theta = ilB$ ... (iii)

Hence, $R = \frac{Bwa^2 + 2E}{2R}(a \times B) \Rightarrow R = \frac{Bwa^2 + 2EaB}{2mgcos\theta}$(Ans)

Answer.63

Given:

Mass = m

Length = l

Magnetic field = B

Capacitance = C

Formula used:

Induced emf $E = Blv$ ... (i), where B = magnetic field, l = length, v = velocity

Also, we know that $E = \frac{q}{c}$ ... (ii), where q = charge, C = capacitance.

Hence, $Blv = \frac{q}{c} \Rightarrow q = CBlv$ .. (ii)

Therefore, current $i = \frac{dq}{dt}$, where q = charge, t = time

=> $i = \frac{CBldv}{dt} = CBla$ ... (iii), where a = acceleration

Therefore, net force on rod = weight - magnetic force = mg - ilB .. (iv), where m = mass, g = acceleration due to gravity, i = current, l = length, B = magnetic force

From newton's second law of motion, F = ma ... (v), where f= force, m = mass, a = acceleration

Therefore, $ma = mg - ilB = mg - CB^2l^2a$ (from (iii))

$\Rightarrow a(m + CB^2l^2) = mg$

=>acceleration $a = \frac{mg}{m + CB^2l^2}$ (Ans)

Answer.64

Given:

Magnetic field = B

Rate of increase of magnetic field = dB/dt

Radius = r

Formula used:

(a) Induced emf $E' = \frac{d\Phi}{dt}$ ... (i), where Φ = magnetic flux, t = time

Now, Φ = B.A where B = magnetic field, A = area

Hence, $E' = \frac{d(B.A)}{dt} = \frac{AdB}{dt}$ ... (ii)

For the circular loop, $A = \pi r2$ ... (iii), where A = area, r = radius

Let the electric field be E

Hence, $\int E.dr = E'$ ... (iv) ,where dr = element of length, E' = emf

Hence, for this loop, $\int dr = 2\pi r$, where r = radius

$\Rightarrow E \times 2\pi r = \pi r^2 \frac{dB}{dt}$

$\Rightarrow E = \frac{r}{2}\frac{dB}{dt}$ (Ans)

(b) When the square is considered, A = $(2r)^2 = 4r^2$, where A = area, r = radius

In this case, $\int dr = 2r \times 4$(perimeter of square)

Hence, from $\int E.dr = E'$, where E = electric field, dr = length element, E' = emf, we get

$E \times 2r \times 4 = \frac{dB}{dt} \times 4r^2$

=> electric field $E = \frac{r}{2}\frac{dB}{dt}$ (Ans)

Answer.65

Given:

Rate of variation of current $\left(\frac{dI}{dt}\right)$ = 0.01 A s^{-1}.

No of turns/m (n) = 2000

Radius(r) = 6 cm = 0.06 m

Formula used:

(a) Radius of circle(r') = 1 cm = 0.01 m

Time(t) = 2 s

For two seconds, change of current Δi = (2 x 0.01 A.) = 0.02 A

Magnetic flux $\Phi = B.A$, where B = magnetic field, A = area

Area of circle$(A) = \pi(0.01)^2 = \pi \times 10^{-4} m^2$

Magnetic field of a solenoid $B = \mu_0 n\Delta i$, where μ_0 = magnetic permeability of vacuum, n = number of turns per unit length, Δi = change in current

Hence, flux $\Phi = B.A = \mu_0 n\Delta i \times A$

$\Rightarrow \Phi = (4\pi \times 10^{-7} \times 2000 \times 0.02) \times \pi \times 10^{-4} Tm^2 = 1.6 \times 10^{-8}$ Wb

Hence, $\frac{d\Phi}{dt}$ in 1 second = **0.785×10^{-8} Wb** (Ans)

(b) $\int E.dr = E' = \frac{d\Phi}{dt}$, where E = electric field, dr = line element, E' = emf, Φ = flux, t = time

Hence, in this case, this becomes

$E \times 2\pi r = \frac{d\Phi}{dt}$, where r = radius of circle

$\Rightarrow E = \frac{0.785 \times 10^{-8}}{2\pi \times 10^{-2}} = \mathbf{1.2 \times 10^{-7}\ Vm^{-1}}$(Ans)

(c) For the point located outside,

$\frac{d\Phi}{dt} = \frac{\mu_0 ndi}{dt} \times A = (4\pi \times 10^{-7} \times 2000 \times 0.01 \times \pi \times 0.06^2)$ Wbs^{-1}

Φ = flux, t = time, μ_0 = magnetic permeability of vacuum, n = number of turns per unit length, di/dt = rate of change in current

$\int E.dl = \frac{d\Phi}{dt} \Rightarrow E = \frac{\frac{d\Phi}{dt}}{2\pi r}$, where E = electric field, r = radius of circle(since $\int dl = 2\pi r$)

Hence, $E = \frac{(4\pi \times 10^{-7} \times 2000 \times 0.01 \times \pi \times 0.06^2)}{\pi \times 0.082} \times \frac{dB}{dt}$

$\Rightarrow E = \mathbf{5.64 \times 10^{-7}}$ Vm^{-1} (Ans)

Answer.66

Given:

Emf(E) = 20 V

Rate of change of current$\left(\frac{di}{dt}\right) = \frac{2.5-(-2.5)}{0.1}$ A/s = 50 As^{-1}

Formula used:

Emf $E = \frac{Ldi}{dt}$, where L = self inductance, di/dt = rate of change of current

Substituting the values, we get

$20 = L\, x\, 50$

=> self inductance $L = \mathbf{0.4\ H}$ (Ans)

Answer.67

Given:

Magnetic flux(Φ) = 8 × 10^{-4} Wb

Number of turns(n) = 200

Current(i) = 4 A ω

Formula used:

$L = \left(\frac{n\Phi}{i}\right)$, where L = self inductance, n = number of turns, Φ = flux, i = current

Putting the values, we get

$L = \frac{200 \times 8 \times 10^{-4}}{4} H = \mathbf{0.04\ H}$ (Ans)

Answer.68

Given:

Number of turns(n) = 240

Length of solenoid(l) = 12 cm = 0.12 m

Radius(r) = 2 cm = 0.02 m

Rate of change of current(di/dt) = 0.8 As^{-1}

Formula used:

L = $\mu_0 n^2 A/l$, where L = self inductance, μ_0 = magnetic permeability of vacuum n = number of turns, A = area, l = length

Now, emf $E = \frac{Ldi}{dt}$, where L = self inductance, $\frac{di}{dt}$ = rate of change of current

Putting the values:

$$E = \frac{4\pi \times 10^{7} \times 240^{2} \times \pi \times 0.02^{2}\ 0.8}{0.12} = \mathbf{6 \times 10^{-4}\ V} \text{ (Ans)}$$

Answer.69

For a series LR circuit, the current across the inductor varies as a

function of time. The current across the inductor at time t will

be

$$i = i_0\left(1 - e^{\frac{-tR}{L}}\right) \text{...(i)}$$

where i_0 is the current at time t=0(also called the steady state value), R is the resistance of the resistor and L is the inductance of the inductor.

We can define a quantity called the time constant for a series LR circuit. It is given as

$\tau = L/R$

So equation(i) becomes

$i = i_0\left(1 - e^{\frac{-t}{\tau}}\right)$ ...(ii)

We have to find the values of $\frac{t}{\tau}$ for three different values of current

(a)- when i is 90% of i_0

90% of i_0 is $\frac{90}{100}$ i_0, so $i = \frac{9}{10} i_0$

Putting these values in eq(ii)

$$\frac{9}{10} i_0 = i_0\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$\frac{9}{10} = \left(1 - e^{\frac{-t}{\tau}}\right)$$

$$e^{\frac{-t}{\tau}} = 1 - \frac{9}{10}$$

$$e^{\frac{-t}{\tau}} = \frac{1}{10}$$

Taking natural logarithm on both sides

$$-\frac{t}{\tau} = \ln\frac{1}{10}$$

$$-\frac{t}{\tau} = \ln(1) - \ln(10)$$

$-\frac{t}{\tau} = -\ln(10)$ (as ln (1) is equal to 0)

$$\frac{t}{\tau} = \ln(10) = 2.30$$

The value of $\frac{t}{\tau}$ for which the current is 90% of steady state

value is 2.3.

(b)- when i is 99% of i_0

99% of i_0 is $\frac{99}{100}$ i_0, so $i = \frac{99}{100} i_0$

Putting these values in eq(ii)

$$\frac{99}{100} i_0 = i_0 \left(1 - e^{\frac{-t}{\tau}}\right)$$

$$\frac{99}{100} = \left(1 - e^{\frac{-t}{\tau}}\right)$$

$$e^{\frac{-t}{\tau}} = 1 - \frac{99}{100}$$

$$e^{\frac{-t}{\tau}} = \frac{1}{100}$$

Taking natural logarithm on both sides

$$-\frac{t}{\tau} = \ln\frac{1}{100}$$

$$-\frac{t}{\tau} = \ln(1) - \ln(100)$$

$$-\frac{t}{\tau} = -\ln(100) \text{ (as ln (1) is equal to 0)}$$

$$\frac{t}{\tau} = \ln(100) = 4.60$$

The value of $\frac{t}{\tau}$ for which the current is 99% of steady state

value is 4.6.

(c)- when i is 90% of i_0

99.9% of i_0 is $\frac{999}{1000} i_0$, so $i = \frac{999}{1000} i_0$

Putting these values in eq(ii)

$$\frac{999}{1000} i_0 = i_0 \left(1 - e^{\frac{-t}{\tau}}\right)$$

$$\frac{999}{1000} = \left(1 - e^{\frac{-t}{\tau}}\right)$$

$$e^{\frac{-t}{\tau}} = 1 - \frac{999}{1000}$$

$$e^{\frac{-t}{\tau}} = \frac{1}{1000}$$

Taking natural logarithm on both sides

$$-\frac{t}{\tau} = \ln\frac{1}{1000}$$

$$-\frac{t}{\tau} = \ln(1) - \ln(1000)$$

$-\frac{t}{\tau} = -\ln(1000)$ (as ln (1) is equal to 0)

$$\frac{t}{\tau} = \ln(1000) = 6.90$$

The value of $\frac{t}{\tau}$ for which the current is 99.9% of steady state

value is 6.9.