DAY TWENTY EIGHT

Optical Instruments

Learning & Revision for the Day

- Microscope
- Telescope
- Resolving Power of an Optical Instrument

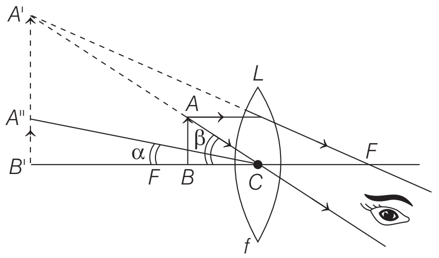
An **optical instrument** is used to enhance and analyses the light waves. The light waves are in the form of photons, hence optical instruments also determine the characteristics properties of light waves.

Microscope

It is an optical instrument which forms a magnified image of a small nearby object and thus, increases the visual angle subtended by the image at the eye, so that the object is seen to be bigger and distinct.

1. Simple Microscope (Magnifying Glass)

It consists of a single convex lens of small focal length and forms a magnified image of an object placed between the optical centre and the principal focus of the lens.



If the image is formed at the near point of eye, then $m = \left(1 + \frac{D}{f}\right)$

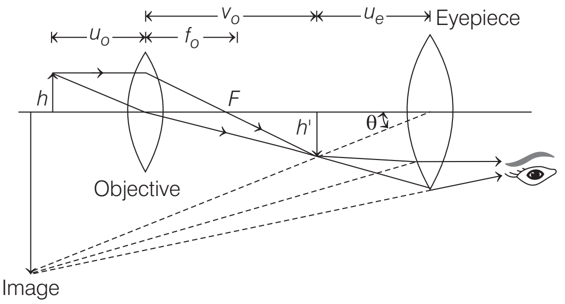
But, if the image is formed at infinity, then $m = \frac{D}{f}$

where, D = normal viewing distance (25 cm),

f = focal length of magnifying lens.

2. Compound Microscope

It consists of two lenses of small focal length and small apertures. Also, the focal length and aperture of objective lens are smaller than that of eyepiece. The image formed by the objective lens is real, inverted and magnified. This image acts as the object for the eyepiece and the final image is highly magnified, virtual and inverted w.r.t. the original object.



If m_o and m_e be the magnifications produced by the objective and the eyepiece respectively, then total magnification of microscope $m = m_o \times m_e$.

If final image is formed at the near point *(D)* of the eye, then

$$m = -\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

or

$$m = -\frac{L}{f_o}\left(1 + \frac{D}{f_e}\right) \text{(approx)}$$

If final image is formed at infinity, then

$$m = -\frac{v_o}{u_o} \cdot \frac{D}{f_e} = -\frac{L}{f_o} \cdot \frac{D}{f_e} \text{ (approx)}$$

Length of tube of microscope, $L = v_o + u_e$.

NOTE
- Huygens' eyepiece is free from chromatic and spherical aberration, but it cannot be used for measurement purposes.
- Ramsden's eyepiece can be used for precise measurement as cross wires can be fixed in this eyepiece. It slightly suffers from spherical and chromatic aberrations.

Telescope

Telescope is an optical instrument which increases, the visual angle at the eye by forming the image of a distant object at the least distance of distinct vision, so that the object is seen distinct and bigger.

Refracting Telescope

It consists of an objective lens of large focal length f_o and large aperture. The eyepiece consists of a convex lens of small aperture and small focal length f_e.

Distance between the two lenses is set as,

$$L = f_o + f_e$$

In normal adjustment, the final image is formed at infinity and magnifying power of the telescope is

$$m = -\frac{f_o}{f_e}$$

In practical adjustment, the final image is formed at the near point of the observer's eye. In this arrangement,

$$m = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Reflecting Telescope

It consists of an objective which is a large paraboloid concave mirror of maximum possible focal length f_o and the eyepiece is a convex lens of small focal length and small aperture, then

Magnifying power, $m = -\dfrac{f_o}{f_e}$

Reflecting type telescope is considered superior as it is free from spherical and chromatic aberrations, is easy to install and maintain, and can produce image of greater intensity.

NOTE
- The large aperture of telescope objective, helps in forming a brighter image.
- If diameter of pupil of human eye is *d* and that of telescope be *D*, then image formed by telescope will be $\left(\frac{D}{d}\right)^2$ times brighter than the image of the same object, seen directly by the unaided eye.

Resolving Power of an Optical Instrument

Resolving power of an optical instrument is its ability to produce distinct images of two points of an object (or two nearby objects) very close together. Resolving power of an optical instrument is inverse of its limit of resolution. Smaller the limit of resolution of a device, higher is its resolving power. Limit of resolution of a normal human eye is 1′ .

The minimum distance (or angular distance) between two points of an object whose images can be formed distinctly by the lens of an optical instrument, is called its **limit of resolution**.

Resolving Power of a Telescope

If the aperture (diameter) of the telescope objective be the *D*, then the minimum angular separation ($d\theta$) between two distant objects, whose images are just resolved by the telescope, is

$$d\theta = \frac{1.22\lambda}{D}$$

and resolving power of the telescope,

$$\text{RP} = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$

Resolving Power of a Microscope

The least distance (d) between two points, whose images are just seen distinctly by a microscope, is given by

$$d = \frac{1.22\lambda}{2n_m \sin\theta}$$

where, λ = wavelength of light used to illuminate the object,

n_m = refractive index of the medium between the object and the objective lens, and

θ = semi angle of the cone of light from the point object.

The term $n_m \sin\theta$ is generally called the numerical aperture of the microscope.

$\therefore$ Resolving power of the microscope,

$$RP = \frac{1}{d}$$

$$= \frac{2\,n_m \sin\theta}{1.22\lambda} = \frac{n_m \sin\theta}{0.61\lambda}$$

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 To obtain a magnified image at the distance of distinct vision with simple microscope, where should the object be placed?

(a) Away from the focus
(b) At focus
(c) Between the focus and the optical centre
(d) None of the above

2 To obtain the maximum magnification with a simple microscope, where should the eye be placed?

(a) Close to the lens
(b) Half way between the focus and the optical centre
(c) Close to the focus
(d) None of the above

3 A man can see clearly upto 3 m. To see upto 12 m, he must use a lens of

(a) $-\frac{3}{4}$ D (b) 3 D (c) –3 D (d) $-\frac{1}{4}$ D

4 A magnifying glass is used as the object to be viewed can be brought closer to the eye than the normal near point. This results in

(a) the formation of virtual erect image and larger angle to be subtended by the object at the eye and hence viewed in greater detail
(b) increase in field of view
(c) infinite magnification at near point
(d) a diminished but clear image

5 In a compound microscope, the objective produces a magnification of 10, while the eyepiece produces a magnification of 5, then the over all magnification achieved by a compound microscope is

(a) 2 (b) 50 (c) 0.5 (d) 25.00

6 A telescope consists of two thin lenses of focal lengths 0.3 m and 3 cm, respectively. It is focussed on moon which subtends on angle of 0.5° at the objective. Then, the angle subtended at the eye by the final image will be

(a) 5° (b) 0.25° (c) 0.5° (d) 0.35°

7 An astronomical telescope is set for normal adjustment and the distance between its objective and eyepiece is 1.05 cm. The magnifying power of the telescope is 20. What is the focal length of the objective?

(a) 2m (b) 1 m (c) 0.05 m (d) 0.25 m

8 The magnifying power of a telescope is 9. When it is adjusted for parallel rays, the distance between the objective and the eyepiece is found to be 20 cm . The focal length of lenses are

(a) 18 cm, 2 cm (b) 11 cm, 9 cm
(c) 10 cm, 10 cm (d) 15 cm, 5 cm

9 A simple telescope, consisting of an objective of focal length 60 cm and a single eye lens of focal length 5 cm is focussed on a distant object in such a way that parallel rays emerge from the eye lens. If the object subtends an angle of 2° at the objective, the angular width of the image is

(a) 10° (b) 24° (c) 50° (d) (1/6)°

10 In a laboratory four convex lenses L_1, L_2, L_3 and L_4 of focal lengths 2, 4, 6 and 8 cm respectively are available. Two of these lenses form a telescope of length 10 cm and magnifying power 4. The objective and eye lenses are respectively,

(a) L_2, L_3 (b) L_1, L_4 (c) L_1, L_2 (d) L_4, L_1

11 The magnifying power of an astronomical telescope is 8 and the distance between the two lenses is 54 cm . The focal length of eye lens and objective lens will be respectively

(a) 6 cm and 48 cm (b) 48 cm and 6 cm
(c) 8 cm and 64 cm (d) 6 cm and 60 cm

12 In a refracting astronomical telescope, the final image is

(a) real, inverted and magnified
(b) real, erect and magnified
(c) virtual, erect and magnified
(d) virtual, inverted and magnified

13 The magnifying power of a telescope is high, if

(a) both the objective and eyepiece have short focal lengths
(b) both the objective and the eyepiece have long focal lengths
(c) the objective has a short focal length and the eyepiece has a long focal length
(d) the objective has a long focal length and the eyepiece has a short focal length

14 An astronomical telescope has a large aperture to

(a) reduce spherical aberration
(b) have high resolution
(c) increase span of observation
(d) have low dispersion

15 Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye?

(take, wavelength of light = 500 nm)

(a) 5 m (b) 1 m
(c) 6 m (d) 3 m

16 Wavelength of light used in an optical instrument are $\lambda_1 = 4000$ Å and $\lambda_2 = 5000$ Å, then ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is

(a) 16 : 25
(b) 9 : 1
(c) 4 : 5
(d) 5 : 4

Direction (Q. Nos. 17-20) *Each of these questions contains two statements Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

17 **Statement I** Very large size telescopes are reflecting telescopes instead of refracting telescopes.

Statement II It is easier to provide mechanical support to large size mirrors than large size lenses.

→ JEE Main (Online) 2013

18 **Statement I** The resolving power of a telescope is more if the diameter of the objective lens is more.

Statement II Objective lens of large diameter collects more light.

19 **Statement I** The focal length of the objective of telescope is larger than that of eyepiece.

Statement II The resolving power of telescope increases when the aperture of objective is small.

20 **Statement I** Resolving power of an optical instrument is reciprocal to its limit of resolution.

Statement II Smaller the distance between two point objects the instrument can resolve, higher is its resolving power.

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 An observer looks at a distance tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears **→ JEE Main 2016 (Offline)**

(a) 10 times taller (b) 10 times nearer
(c) 20 times taller (d) 20 times nearer

2 Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is **→ JEE Main 2015**

(a) 30 μm (b) 100 μm (c) 300 μm (d) 1 μm

3 For compound microscope $f_o = 1$ cm, $f_e = 2.5$ cm. An object is placed at distance 1.2 cm from objective lens. What should be the length of microscope for normal adjustment?

(a) 8.5 cm (b) 8.3 cm (c) 6.5 cm (d) 6.3 cm

4 A telescope consists of two lenses of focal length 10 cm and 1 cm. The length of the telescope when an object is kept at a distance of 60 cm from the objective and the final image is formed at least distant of distinct vision is

(a) 15.05 cm
(b) 12.96 cm
(c) 13.63 cm
(d) 14.44 cm

5 A light source is placed at a distance b from a screen. The power of the lens required to obtain k-fold magnified image is

(a) $\dfrac{k+1}{kb}$ (b) $\dfrac{(k+1)^2}{kb}$

(c) $\dfrac{kb}{k+1}$ (d) $\dfrac{kb}{(k-1)^2}$

6 Two white dots are 1 mm apart on a black paper. They are viewed by naked eye of pupil size 3 mm diameter. Upto what distance, the dots are clearly and separately visible?

(a) 3 m (b) 6 m
(c) 1 m (d) 5 m

7 Resolution of human eye is about 1 min. From what distance a normal human eye can just resolve two objects which are 3m apart.

(a) 10 km (b) 15 km (c) 20 km (d) 30 km

8 A student make a compound microscope by using two lenses of focal lengths 1.5 cm and 6.25 cm. She kept an object at 2 cm from objective and forms its final image at 25 cm from eye lens. Distance between two lenses is

(a) 6 cm (b) 7 cm
(c) 9 cm (d) 11 cm

9 Orbital radius of moon is 3.8×10^5 km and its diameter is 3.5×10^2 km.It is seen from a telescope with lenses of 4 m and 10 m. Angle subtended by the moon's image on eye of observer will be

(a) 15° (b) 20°
(c) 30° (d) 35°

10 Match the following column I with column II.

Column I	Column II
A. Magnification of simple microscope	1. $\frac{-v_0}{u_0}\left(1+\frac{D}{f_e}\right)$
B. Magnification of compound microscope when image formed at least distance of vision	2. $\left(\frac{D}{1.22\lambda}\right)$
C. Tube length of telescope	3. $\left(1+\frac{D}{f}\right)$
D. Resolving power of a telescope	4. $f_0 + f_e$

Codes

	A	B	C	D		A	B	C	D
(a)	1	2	3	4	(b)	2	1	4	3
(c)	3	1	4	2	(d)	3	2	4	1

ANSWERS

SESSION 1	**1** (c)	**2** (a)	**3** (d)	**4** (a)	**5** (b)	**6** (a)	**7** (b)	**8** (a)	**9** (b)	**10** (d)
	11 (a)	**12** (d)	**13** (d)	**14** (b)	**15** (a)	**16** (d)	**17** (a)	**18** (a)	**19** (c)	**20** (c)
SESSION 2	**1** (c)	**2** (a)	**3** (b)	**4** (b)	**5** (b)	**6** (d)	**7** (a)	**8** (d)	**9** (b)	**10** (c)

Hints and Explanations

SESSION 1

1 Magnification is obtained in a simple microscope when the object is close to the lens between the focus and the optical centre, as lens is converging.

2 The magnification of simple microscope is given by

$$m = 1 + \frac{D - a}{f}$$

where, D = least distance of vision,
f = focal length and
a = distance of lens from eye

So, lesser the distance between eye and lens, greater is the magnification.

3 Using, $f = \frac{xy}{x - y}$

$$\Rightarrow f = \frac{3 \times 12}{3 - 12} = -4 \text{ m}$$

Hence, power of lens required,

$$P = \frac{1}{f} = \frac{-1}{4} \text{ D}$$

4 This results in the formation of virtual, erect image and the object subtends a larger angle at the eye and the image is viewed in greater detail.

5 $m = m_o \times m_e = 10 \times 5 = 50$

6 For a telescope, $\frac{\beta}{\alpha} = \frac{f_o}{f_e}$

$$\therefore \frac{\beta}{0.5^\circ} = \frac{0.3}{0.03} \Rightarrow \beta = 5^\circ$$

7. Here, $m = f_o / f_e$

and $L = f_o + f_e = 1.05$

or $f_e = 1.05 - f_o$

and $20 = f_o/(1.05 - f_o)$

This gives $f_o = 1\text{ m}$ and $f_e = 0.05\text{ m}$.

8 $f_o + f_e = 20\text{ cm}$,

$$m = f_o/f_e = 9.$$

This gives

$f_e = 2\text{ cm}$ and $f_o = 18$ cm.

9 It is a case of normal adjustment.

Hence, $m = f_o / f_e$

Also, $m = \beta/\alpha$

Therefore, $\frac{\beta}{\alpha} = \frac{f_o}{f_e}$

Here, $f_o = 60$ cm, $f_e = 5\text{ cm}, \alpha = 2^\circ$
Hence, $\beta = 24^\circ$.

10 Length of tube = 10 cm

$$f_o + f_e = 10\text{ cm} \quad \ldots\text{(i)}$$

Magnification, $m = \frac{f_o}{f_e} = 4$

$$f_o = 4 f_e$$

On putting in Eq. (i), we get

$$5f_e = 10\text{ cm}$$

or $f_e = 2$ cm

and $f_o = 8$ cm

Hence, L_4 and L_1 will be used.

Note *In telescope, objective always have larger focal length than eyepiece.*

11 As $L = f_o + f_e = 54$ cm

and $|m| = \dfrac{f_o}{f_e} = 8$

On simplification, we get

$f_e = +\ 6$ cm

and $f_o = +\ 48$ cm

12 The image formed by the objective lens is real, inverted and larger object and the eyepiece forms a second image but virtual, inverted and larger than the first.

13 The magnifying power of a telescope (if the object is at infinity) is given by

$$M = \frac{f_o}{f_e} \cdot \frac{D + f_e}{D}$$

where, D = least distance of distinct vision, where the final image is formed.

14 An astronomical telescopic as a large aperture to have high resolution.

15 We know that,

$$\frac{y}{D} \geq 1.22\frac{\lambda}{d}$$

$$\Rightarrow \quad D \leq \frac{yd}{1.22\ \lambda}$$

$$\therefore \quad D \leq \frac{10^{-3} \times 3 \times 10^{-3}}{1.22 \times 5 \times 10^{-7}} = 5\text{ m}$$

$$\Rightarrow \quad D_{\max} = 5\text{m}$$

16 Resolving power of an optical instrument is inversely proportional to λ, i.e.

$$\text{RP} \propto \frac{1}{\lambda}$$

$$\therefore \frac{\text{Resolving power at } \lambda_1}{\text{Resolving power at } \lambda_2} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{4000} = 5:4$$

17 As very large size telescope needs mechanical support to large size mirror than size of lens. So, in order to fulfil this mechanical support telescope is reflecting instead of refracting telescope.

18 Resolving power of telescope is

$$= \frac{a}{1.22\lambda}$$

where, a is the diameter of objective lens and λ is the wavelength of light used. It is obvious that on increasing a, more light is collected by objective lens and so, the image formed is more bright. Thus, resolving power of telescope increases.

19 The magnifying power of telescope in relaxed state is

$$m = \frac{f_o}{f_e}$$

So, for high magnification, the focal length of objective length should be larger than that of eyepiece.

Resolving power of a telescope

$$= \frac{d}{1.22\lambda}$$

For high resolving power, diameter (d) of objective should be higher.

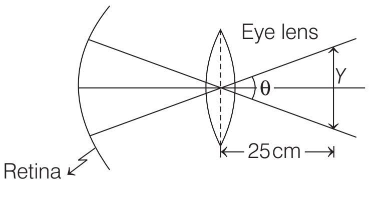
20 Resolving power of an optical instrument is its ability to produce distinct image of two points of an object very close together. Resolving power of an optical instrument is inverse of its limit of resolution. Smaller the limit of resolution of a device higher is its resolving power.

SESSION 2

1 Height of image depends upon the magnifying power to see a 20 times taller object, as the angular magnification should be 20 and we observe angular magnification. Option (c) would not be very correct as the telescope can be adjusted to form the image anywhere between infinity and least distance for distinct vision.

Suppose that the image is formed at infinity. Then, the observer will have to focus the eyes at infinity to observe the image. Hence, it is incorrect to say that the image will be appear nearer to the observer.

2 We can write resolving angle of naked eye as



Assuming human pupil to have a radius $r = 0.25$ cm or diameter $d = 2r = 2 \times 0.25 = 0.5$ cm, the wavelength of light $\lambda = 500$ nm $= 5 \times 10^{-7}$ m

We have the formula, $\sin\theta = \dfrac{1.22\ \lambda}{d}$

$$\therefore \sin\theta = \frac{1.22 \times 5 \times 10^{-7}}{0.5 \times 10^{-2}} = \frac{1.22 \times 5 \times 10^{-7}}{5 \times 10^{-3}} = 1.22 \times 10^{-4}$$

The distance of comfortable viewing is $D = 25\text{ cm} = 0.25$ m

Let Y be the minimum separation between two objects that human eye can resolve i.e. $\sin\theta = \dfrac{Y}{D}$

$$\therefore \ y = D\sin\theta = 0.25 \times 1.22 \times 10^{-4}$$

$$= 3 \times 10^{-5}\text{ m} = 30\ \mu\text{m}$$

3 When final image is formed at normal adjustment, then length of compound microscope,

$$L = v_o + u_e = \frac{u_o f_o}{(u_o + f_o)} + \frac{f_e D}{f_e + D}$$

$$= \frac{-1.2 \times 1}{-1.2 + 1} + \frac{2.5 \times 25}{2.5 + 25}$$

$$= 6 + 2.27$$

$$= 8.27 \approx 8.3\text{ cm}$$

4 Two lenses used are eyepiece and objective.

For eyepiece, $f_e = 1$ cm,

$D = v_e = 25$ cm

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\Rightarrow \quad -\frac{1}{25} - \frac{1}{u_e} = 1$$

$$\Rightarrow \quad u_e = -\frac{25}{26}\text{ cm}$$

For objective $u_o = -\ 60$ cm,

$f_o = 10$ cm

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\frac{1}{v_o} + \frac{1}{60} = \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{v_o} = \frac{1}{10} - \frac{1}{60}$$

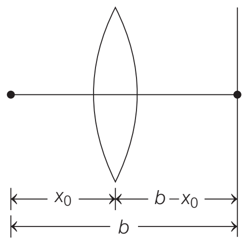
$$\Rightarrow \quad \frac{1}{v_o} = \frac{5}{60}$$

$$\Rightarrow \quad v_o = \frac{60}{5} = 12\text{ cm}$$

Length of telescope,

$$L = v_o + u_e = 12 + \frac{25}{26} = 12.96\text{ cm}$$

5 Distance of light source from lens is x_0 and distance of screen from lens is $(b - x_0)$.



$\therefore$ Image is formed on screen, hence $(b - x_0)$ is also the image distance. Image is formed on the screen, so m will be negative.

$$m = -k = \frac{v}{u}$$

$$\therefore \quad v = -ku$$

Here, $u = -x_0$, $v = b - x_0$

$$\therefore \quad b - x_0 = k x_0, \quad x_0 = \frac{b}{1 + k}$$

From lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{b - x_0} - \frac{1}{-x_0} = p$$

$$\Rightarrow \quad p = \frac{b}{x_0(b - x_0)}$$

Putting the value of x_0, we have

$$p = \frac{(k+1)^2}{kb}$$

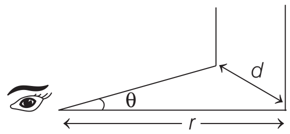
6 Using, $\frac{1.22\lambda}{a} = \frac{x}{d}$

$$\Rightarrow \quad d = \frac{xa}{1.22\lambda}$$

$$= \frac{1 \times 10^{-3} \times 3 \times 10^{-3}}{1.22 \times 500 \times 10^{-9}} \approx 5\,\text{m}$$

Note *In case, wavelength of light is not given, we take mean value of visible region ≈ 500* nm.

7 Using, $\theta = \frac{d}{r}$



We have, $r = \frac{d}{\theta}$

$$\left[\begin{array}{l}\text{where}, \theta \text{ is in radians} \\ = 1' = \left(\frac{1}{60}\right)^\circ = \frac{1}{60} \times \frac{\pi}{180}\end{array}\right]$$

$$= \frac{3}{\left(\frac{1}{60}\right) \times \frac{\pi}{180}}$$

$$= \frac{3 \times 60 \times 180}{\pi}$$

$$\approx 10.3\text{ km} \approx 10\text{ km}$$

8 As, $f_o < f_e$, for a compound microscope.

So, $f_o = 1.5\,\text{cm}$ and $f_e = 6.25\,\text{cm}$

Now, length of tube

= distance between lenses

$$= \frac{u_o f_o}{u_o - f_o} + \frac{f_e D}{f_e + D}$$

$$= \frac{2 \times 1.5}{2 - 1.5} + \frac{6.25 \times 25}{6.25 + 25} = 11\text{ cm}$$

9 Angle subtended by moon on the objective of telescope is,

$$\alpha = \frac{3.5 \times 10^3}{3.8 \times 10^5}$$

$$= \frac{3.5}{3.8} \times 10^{-2}\text{ rad}$$

As, $m = \frac{f_o}{f_e} = \frac{\beta}{\alpha}$

$$\therefore \quad \beta = \frac{400}{10} \times \alpha$$

$$= 40\alpha = 40 \times \frac{3.5}{3.8} \times 10^{-2} \times \frac{180}{\pi} \approx 20^\circ$$

10 A - Magnification of simple microscope is given by,

$$m = \left(1 + \frac{D}{F}\right)$$

B - Magnification of compound microscope, when image is formed at least distance of vision is,

$$m = \frac{-v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

C - Tube length of telescope,

$$L = f_0 + f_e$$

D - Resolving power of telescope

$$= \left(\frac{D}{1.22\lambda}\right)$$