Chapter 5. Linear Inequations

Ex 5.1

Answer 1.

(i) x + 5 > 11

   $x + 5 > 11$

   $\Rightarrow x > 11 - 5$

   $\Rightarrow x > 6$

   But$\{x : x \in N; N < 10\}$

   Therefore, solution set $x = \{7,8,9\}$

(ii) 2x + 1 < 17

   $2x + 1 < 17$

   $\Rightarrow 2x < 17 - 1$

   $\Rightarrow x < \dfrac{16}{2}$

   $\Rightarrow x < 8$

   But$\{x : x \in N; N < 10\}$

   Therefore, solution set $x = \{1,2,3,4,5,6,7\}$

(iii) 3x - 5 ≤ 7

   $3x - 5 \leq 7$

   $\Rightarrow 3x \leq 7 + 5$

   $\Rightarrow 3x \leq 12$

   $\Rightarrow x \leq 4$

   But$\{x : x \in N; N < 10\}$

   Therefore, solution set $x = \{1,2,3,4\}$

(iv) 8 – 3x ≥ 2

   $8 - 3x \geq 2$

   $\Rightarrow -3x \geq -6$

   $\Rightarrow x \geq 2$

   But$\{x : x \in N; N < 10\}$

   Therefore, solution set $x = \{1,2\}$

(v) 5 - 2x < 11

   $5 - 2x < 11$

   $\Rightarrow -2x < 11 - 5$

   $\Rightarrow -2x < 6$

   $\Rightarrow x > -3$

   But$\{x : x \in N; N < 10\}$

   Therefore, solution set $x = \{1,2,3,4,5,6,7,8,9\}$

Answer 2.

(i) $3x > 12$

$\quad 3x > 12$

$\quad\quad x > \dfrac{12}{3}$

$\quad\quad x > 4$

$\quad$ Since, replacement set is R

$\quad$ Solution set = {x: x∈R and x > 4}

(ii) $2x - 3 < 7$

$\quad 2x - 3 < 7$

$\quad 2x < 7 + 3$

$\quad 2x < 10$

$\quad\quad x < \dfrac{10}{2}$

$\quad$ Since, replacement set is R

$\quad$ Solution set = {x : x∈ R and x < 5}

(iii) $3x + 2 \leq 11$

$\quad 3x + 2 \leq 11$

$\quad 3x \leq 11 - 2$

$\quad 3x \leq 9$

$\quad\quad x \leq \dfrac{9}{3}$

$\quad\quad x \leq 3$

$\quad$ Since, replacement set is R

$\quad$ Solution set = {x : x ∈ R and x ≤ 3}

(iv) $14 - 3x \geq 5$

$\quad 14 - 3x \geq 5$

$\quad -3x \geq 5 - 14$

$\quad 3x \leq 9$ (multiplying by -1 changes the sign)

$\quad\quad x \leq \dfrac{9}{3}$

$\quad\quad x \leq 3$

$\quad$ Since, replacement set is R

$\quad$ Solution set = {x : x ∈ R and x ≤ 3}

(v) $7x + 11 > 16 - 3x$

$7x + 11 > 16 - 3x$

$7x + 3x > 16 - 11$

$10x > 5$

$x > \dfrac{5}{10} = 0.5$

Since, replacement set is R

Solution set = $\{x : x \in$ R and $x > 0.5\}$

(vi) $3x + 25 < 8x - 10$

$3x + 25 < 8x - 10$

$25 + 10 < 8x - 3x$

$35 < 5x$

$x > 7$

Since, replacement set is R

Solution set = $\{x : x \in$ R and $x > 7\}$

(vii) $2(3x - 5) \leq 8$

$2(3x - 5) \leq 8$

$6x - 10 \leq 8$

$6x \leq 8 + 10$

$6x \leq 18$

$x \leq 3$

Since, replacement set is R

Solution set = $\{x : x \in$ R and $x \leq 3\}$

(viii) $x + 7 \geq 15 + 3x$

$x + 7 \geq 15 + 3x$

$x - 3x \geq 15 - 7$

$-2x \geq 8$

$2x \leq -8$

$x \leq -4$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x \leq -4\}$

(ix) $2x - 7 \geq 5x + 8$

$2x - 7 \geq 5x + 8$

$2x - 5x \geq 8 + 7$

$-3x \geq 15$

$3x \leq -15$

$x \leq -5$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x \leq -5\}$

(x) $9 - 4x \leq 15 - 7x$

$9 - 4x \leq 15 - 7x$

$9 - 15 \leq 4x - 7x$

$-6 \leq -3x$

$6 \geq 3x$ (multiplying by -1 changes the sign)

$x \leq 2$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x \leq 2\}$

Answer 3.

$6 - 10x < 36$

$-10x < 36 - 6$

$-10x < 30$

$10x > -30$

$x > -3$

Solution set = $\{-2, -1, 0, 1, 2\}$

Answer 4.

$3 - 2x \geq x - 12$

$3 + 12 \geq x + 2x$

$15 \geq 3x$

$x \leq 5$

Solution set = $\{1,2,3,4,5\}$

Answer 5.

$5x - 9 \leq 15 - 7x$

$5x + 7x \leq 15 + 9$

$12x \leq 24$

$x \leq 2$

Solution set = $\{0,1,2\}$

Answer 6.

$7 + 5x > x - 13$

$-x + 5x > -13 - 7$

$4x > -20$

$x > -5$

Solution set = $\{-4, -3, -2, -1\}$

Answer 7

$5x - 14 < 18 - 3x$

$5x + 3x < 18 + 14$

$8x < 32$

$x < 4$

Solution set = $\{0,1,2,3\}$

Answer 8.

$2x + 7 \geq 5x - 14$

$2x - 5x \geq -14 - 7$

$-3x \geq -21$

$3x \leq 21$

$x \leq 7$

Solution set = {2,3,5,7}

Answer 9.

$\dfrac{x}{4} + 3 \leq \dfrac{x}{3} + 4$

$\dfrac{x + 12}{4} \leq \dfrac{x + 12}{3}$

$3x + 36 \leq 4x + 48$

$3x - 4x \leq 48 - 36$

$-x \leq 12$

$x \geq -12$

Solution set = {-11, -9, -7, -5, -3, -1}

Answer 10.

$\dfrac{x + 3}{3} \leq \dfrac{x + 8}{4}$

$4x + 12 \leq 3x + 24$

$4x - 3x \leq 24 - 12$

$x \leq 12$

Solution set = {2,4,6,8,10,12}

Answer 11.

(i) $x \in Z$

$x + 17 \leq 4x + 9$

$x - 4x \leq 9 - 17$

$-3x \leq -8$

$3x \geq 8$

$x \geq \dfrac{8}{3}$

Since $x \in Z$

Smallest value of $x = \lceil 3 \rceil$

(ii) $x \in R$

$x + 17 \leq 4x + 9$

$x - 4x \leq 9 - 17$

$-3x \leq -8$

$3x \geq 8$

$x \geq \dfrac{8}{3}$

Since $x \in R$

Smallest value of $x = \left\lceil 2\dfrac{2}{3} \right\rceil$

Answer 12.

$\dfrac{2}{x^2} - \dfrac{5}{x} + 2 = 0$

$2 - 5x + 2x^2 = 0$

$2x^2 - 5x + 2 = 0$

$x^2 - \dfrac{5}{2}x + 1 = 0$

$x^2 - 2x - \dfrac{1}{2}x + 1 = 0$

$x(x - 2) - \dfrac{1}{2}(x - 2) = 0$

$(x - 2)(x - \dfrac{1}{2}) = 0$

$(x - 2) = 0, (x - \dfrac{1}{2}) = 0$

$x = 2, x = \dfrac{1}{2}$

Answer 13.

(i) $2x - 11 \le 7 - 3x$, $x \in N$

$2x - 11 \le 7 - 3x$

$2x + 3x \le 7 + 11$

$5x \le 18$

$x \le \dfrac{18}{5}$

$x \le 3.6$

Since $x \in N$

Solution set $= \{1, 2, 3\}$

(ii) $3(5x + 3) \ge 2(9x - 17)$, $x \in W$
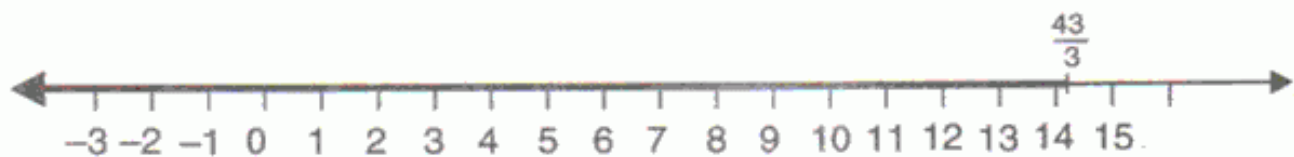
$3(5x + 3) \ge 2(9x - 17)$

$15x + 9 \ge 18x - 34$

$15x - 18x \ge -34 - 9$

$-3x \ge -43$

$3x \le 43$

$x \le \dfrac{43}{3}$

Solution set $= \left[x \le \dfrac{43}{3} \right]$

(iii) $2(3x - 5) > 5(13 - 2x)$, $x \in W$

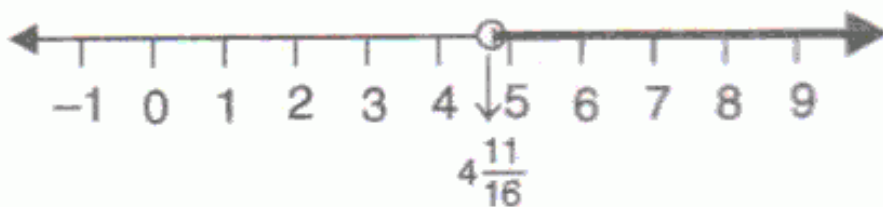$2(3x - 5) > 5(13 - 2x)$

$6x - 10 > 65 - 10x$

$6x + 10x > 65 + 10$

$16x > 75$

$x > \dfrac{75}{16}$

$x > 4\dfrac{11}{16}$

Solution set $= \left[x > 4\frac{11}{16} \right]$

$4\frac{11}{16}$

(iv) $3x - 9 \le 4x - 7 < 2x + 5, \ x \in R$

$3x - 9 \le 4x - 7$	$4x - 7 < 2x + 5$
$3x - 4x \le -7 + 9$	$4x - 2x < 5 + 7$
$-x \le 2$ and	$2x < 12$
$x \ge -2$	$x < 6$

Solution set $= [-2 \le x < 6]$

(v) $2x - 7 < 5x + 2 \le 3x + 14, \ x \in R$

$2x - 7 < 5x + 2$	$5x + 2 \le 3x + 14$
$2x - 5x < 2 + 7$	$5x - 3x \le 14 - 2$
$-3x < 9$ and	$2x \le 12$
$x > -3$	$x \le 6$

Solution set $= [-3 < x \le 6]$

(vi) $-3 \le \frac{1}{2} - \frac{2x}{3} \le 2\frac{2}{3}, x \in N$

$-3 \le \frac{1}{2} - \frac{2x}{3}$	$\frac{1}{2} - \frac{2x}{3} \le 2\frac{2}{3}$
$-3 \le \frac{3 - 4x}{6}$	$\frac{3 - 4x}{6} \le \frac{8}{3}$
$-18 \le 3 - 4x$	$9 - 12x \le 48$
$-18 - 3 \le -4x$ and	$-12x \le 39$
$-21 \le -4x$	$12x \ge -39$
$x \le \frac{21}{4}$	$x \ge -3\frac{1}{4}$
$x \le 5\frac{1}{4}$	

$\left[-3\frac{1}{4} \le x \le 5\frac{1}{4} \right]$

Solution set $= \left[-3\frac{1}{4} \le x \le 5\frac{1}{4} \right]$
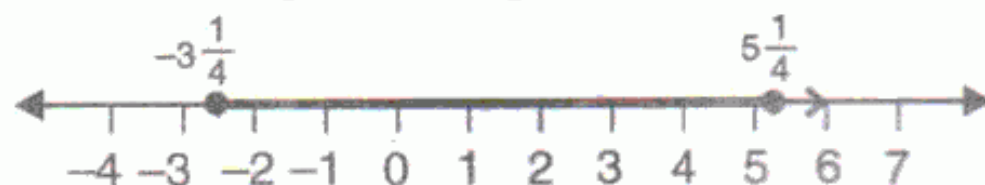
(vii) $4\frac{3}{4} \ge x + \frac{5}{6} > \frac{1}{3}, x \in R$

$$4\frac{3}{4} \ge x + \frac{5}{6} \qquad\qquad x + \frac{5}{6} > \frac{1}{3}$$

$$\frac{19}{4} \ge \frac{6x + 5}{6} \qquad\qquad \frac{6x + 5}{6} > \frac{1}{3}$$

$114 \ge 24x + 20 \quad$ and $\quad 18x + 15 > 6$

$114 - 20 \ge 24x \qquad\qquad 18x > 6 - 15$

$94 \ge 24x \qquad\qquad\qquad 18x > -9$

$x \le 3\frac{11}{12} \qquad\qquad\qquad x > -\frac{1}{2}$
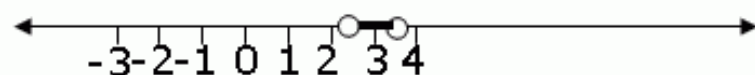
Solution set $= \left[-\frac{1}{2} < x \le 3\frac{11}{12} \right]$

(viii) $\frac{1}{3}(2x - 1) < \frac{1}{4}(x + 5) < \frac{1}{6}(3x + 4), x \in R$

$$\frac{1}{3}(2x - 1) < \frac{1}{4}(x + 5) \qquad \frac{1}{4}(x + 5) < \frac{1}{6}(3x + 4)$$

$4(2x - 1) < 3(x + 5) \qquad\quad 6(x + 5) < 4(3x + 4)$

$8x - 4 < 3x + 15 \qquad\qquad 6x + 30 < 12x + 16$

$8x - 3x < 15 + 4 \quad$ and $\quad 6x - 12x < 16 - 30$

$5x < 19 \qquad\qquad\qquad\quad -6x < -14$

$x < 3\frac{4}{5} \qquad\qquad\qquad\quad x > 2\frac{1}{3}$

Solution set $= \left[2\frac{1}{3} < x < 3\frac{4}{5} \right]$

(ix) $\frac{1}{3}(5x - 8) \ge \frac{1}{2}(4x - 7), x \in R$

$\frac{1}{3}(5x - 8) \ge \frac{1}{2}(4x - 7)$

$2(5x - 8) \ge 3(4x - 7)$

$10x - 16 \ge 12x - 21$

$10x - 12x \ge -21 + 16$

$-2x \ge -5$

$x \le \frac{5}{2}$

$x \le 2\frac{1}{2}$

Solution set $= \left[x \le 2\frac{1}{2} \right]$

(x) $\frac{5}{4}x > 1 + \frac{1}{3}(4x - 1), x \in R$

$\frac{5}{4}x > 1 + \frac{1}{3}(4x - 1)$
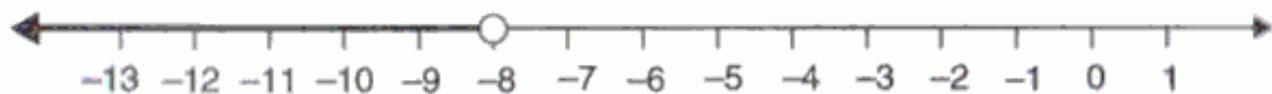
$\frac{5}{4}x > \frac{3 + (4x - 1)}{3}$

$15x > 12 + 16x - 4$

$15x - 16x > 8$

$-x > 8$

$x < -8$

Solution set $= [x < -8]$

Answer 14.
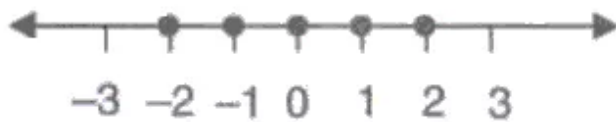
$P = \{x : -3 < x \le 7, x \in R\}$ and $Q = \{x : -7 \le x < 3, x \in R\}$
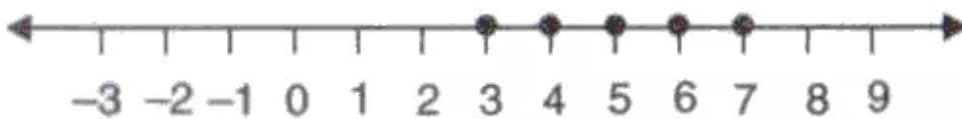$P = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$ and $Q = \{-7, -6, -5, -4, -3, -2, -1, 0, 1, 2\}$

(i) $P \cap Q$

$P \cap Q = \{-2, -1, 0, 1, 2\}$

(ii) $Q' \cap P$

$Q' \cap P = \{3, 4, 5, 6, 7\}$

(iii) P-Q

$P-Q = \{3, 4, 5, 6, 7\}$

Answer 15.

P = {x : 7x – 2 > 4x + 1, x ∈ R}   Q = {x : 9x – 45 ≥ 5(x – 5), x ∈ R}

7x – 2 > 4x + 1   9x – 45 ≥ 5x – 25

7x – 4x > 1 + 2   9x – 5x ≥ –25 + 45

3x > 3   and   4x ≥ 20

x > 1   x ≥ 5

P = {2, 3, 4, 5..........}   Q = {5, 6, 7, 8, 9, ...}

(i) P ∩ Q

P ∩ Q = {2, 3, 4, 5}

(ii) P-Q

P-Q = {6,7,8,9..........}
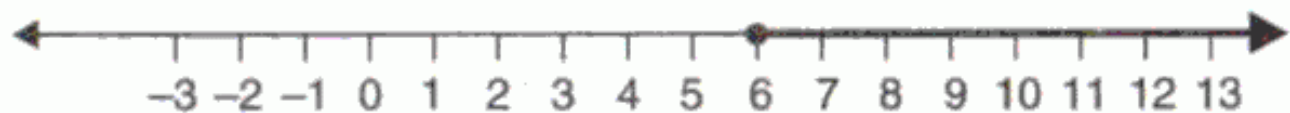
(iii) P ∩ Q'

P ∩ Q' = {6, 7, 8, 9, ...}

Answer 16.

$P = \{x : 7x - 4 > 5x + 2, x \in R\}$ $Q = \{x : x - 19 \geq 1 - 3x, x \in R\}$

$7x - 4 > 5x + 2$ $x - 19 \geq 1 - 3x$

$7x - 5x > 2 + 4$ $x + 3x \geq 1 + 19$

$2x > 6$ and $4x \geq 20$

$x > 3$ $x \geq 5$

$P = \{4, 5, 6, 7, \ldots \ldots \ldots \}$ $Q = \{5, 6, 7, 8, 9 \ldots \ldots \}$

(i) $P \cap Q$

    $P \cap Q = \{5, 6, 7, 8, \ldots \ldots \}$

(ii) $P' \cap Q$

    $P' \cap Q = \{\emptyset\}$