Some Applications of Trigonometry

Selected NCERT Questions

1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.

Sol. Let AB be the vertical pole and AC be the long rope tied to point C.

In right ΔABC, we have [Fig. 10.7]

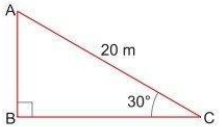


Fig. 10.7

$$\sin 30° = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{AB}{20} \Rightarrow \frac{20}{2} = AB$$

$$\Rightarrow \quad AB = 10 \text{ m}$$

Therefore, height of the pole is 10 m.

2. A tree breaks due to storm and the broken part bends, so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Sol. In right angle ΔABC, AC is the broken part of the tree (Fig. 10.8).

So, the total height of tree $= (AB + AC)$

Now in right angle ΔABC,

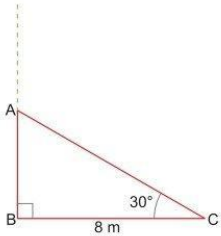


Fig. 10.8

$$\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8} \Rightarrow AB = \frac{8}{\sqrt{3}}$$

Again, $\cos 30° = \frac{BC}{AC}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{8}{AC} \Rightarrow AC = \frac{16}{\sqrt{3}}$$

Hence, the height of the tree $= AB + AC$

$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3} \text{ m}$$

3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Sol. Let AC be a steep slide for elder children and DE be a slide for younger children. Then $AB = 3$ m and $DB = 1.5$ m (Fig. 10.9).

Now, in right angle ΔDBE, we have

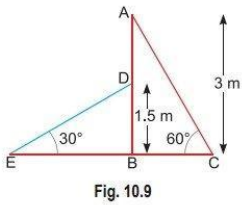


Fig. 10.9

$$\sin 30° = \frac{BD}{DE} = \frac{1.5}{DE}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1.5}{DE} \qquad \therefore \quad DE = 2 \times 1.5 = 3 \text{ m}$$

$\therefore$ Length of slide for younger children = 3 m

Again, in right angle ΔABC, we have

$$\sin 60° = \frac{AB}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{AC}$$

$$\Rightarrow \qquad AC = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3} \text{ m}$$

So, the length of slide for elder children is $2\sqrt{3}$ m.

4. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Sol. Let AB be the horizontal ground and K be the position of the kite and its height from the ground is 60 m and let length of string AK be x m. (Fig. 10.10)

$\therefore \quad \angle KAB = 60°$

Now, in right angle ΔABK we have

$$\sin 60° = \frac{BK}{AK} = \frac{60}{x} \quad \Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{x} \quad \Rightarrow \quad \sqrt{3}x = 120$$

$$\therefore \qquad x = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3} \text{ m}$$

So, the length of string is $40\sqrt{3}$ m.

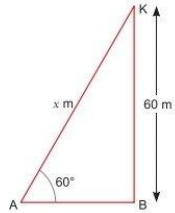


Fig. 10.10

5. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Sol. Let AB be the pedestal of height h metres and BC be the statue of height 1.6 m.

Let D be any point on the ground such that,

$\angle BDA = 45°$ and $\angle CDA = 60°$

Now, in ΔBDA, we have

$$\tan 45° = \frac{AB}{DA} = \frac{h}{DA} \qquad \Rightarrow \qquad 1 = \frac{h}{DA}$$

$$\therefore \qquad DA = h \qquad \ldots(i)$$

Again in ΔADC, we have

$$\tan 60° = \frac{AC}{AD} = \frac{AB + BC}{AD}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{h + 1.6}{h} \qquad \text{(From equation } (i)\text{)}$$

$$\Rightarrow \qquad \sqrt{3}h = h + 1.6 \qquad \Rightarrow \qquad (\sqrt{3} - 1)h = 1.6$$

$$\therefore \qquad h = \frac{1.6}{\sqrt{3} - 1} = \frac{1.6}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{1.6\,(\sqrt{3} + 1)}{3 - 1} = \frac{1.6\,(\sqrt{3} + 1)}{2} = 0.8 \times (\sqrt{3} + 1) \text{ m}$$

Hence, height of the pedestal is $0.8\,(\sqrt{3} + 1)$ m.

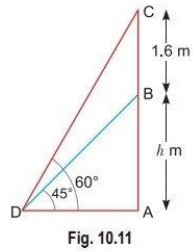


Fig. 10.11

6. Two poles of equal heights are standing opposite to each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles. **[*CBSE* 2019 (30/1/2)]**

Sol. Let AB and CD be two poles of equal height h metre and let P be any point between the poles, such that $\angle APB = 60°$ and $\angle DPC = 30°$.

The distance between two poles is 80m. (Given)

Let $AP = x$ m, then $PC = (80 - x)$ m

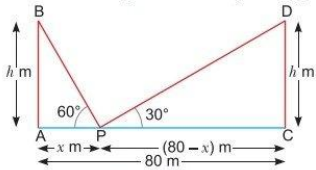


Fig. 10.12

Now, in ΔAPB, we have

$$\tan 60° = \frac{AB}{AP} = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad \Rightarrow \quad h = \sqrt{3}x \qquad ...(i)$$

Again in ΔCPD, we have

$$\tan 30° = \frac{DC}{PC} = \frac{h}{(80 - x)}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{80 - x} \quad \Rightarrow \quad h = \frac{80 - x}{\sqrt{3}} \qquad ...(ii)$$

From (i) and (ii), we have

$$\sqrt{3}x = \frac{80 - x}{\sqrt{3}} \quad \Rightarrow \quad 3x = 80 - x$$

$$\Rightarrow \quad 4x = 80 \quad \Rightarrow \quad x = \frac{80}{4} = 20 \text{ m}$$

Now, putting the value of x in equation (i), we have

$$h = \sqrt{3} \times 20 = 20\sqrt{3} \text{ m}$$

Hence, the height of the pole is $20\sqrt{3}$ m and the distance of the point from first pole is 20 m and that of the second pole is 60 m.

7. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (Fig. 10.13). Find the height of the tower and the width of the canal.

Sol. Let height of the tower be h metres and width of the canal be x metres, so $AB = h$ m and $BC = x$ m

Now in ΔABC, we have

$$\tan 60° = \frac{h}{x} \quad \Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad \Rightarrow \quad h = \sqrt{3}x \qquad ...(i)$$

Now, in ΔADB, we have

$$\tan 30° = \frac{AB}{DB} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{20 + x} \quad \Rightarrow \quad 20 + x = \sqrt{3}h \qquad ...(ii)$$

From (i) and (ii), we have

$$20 + x = \sqrt{3} \times \sqrt{3}x \quad \Rightarrow \quad 20 + x = 3x$$

$$\Rightarrow \quad 20 = 3x - x = 2x \quad \Rightarrow \quad x = \frac{20}{2} = 10 \text{ m}$$

Now, putting the value of x in equation (i), we have

$$h = \sqrt{3} \times 10 = 10\sqrt{3} \quad \Rightarrow \quad h = 10\sqrt{3} \text{ m}$$

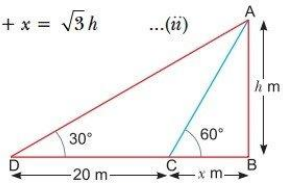


Fig. 10.13

Hence, height of the tower is $10\sqrt{3}$ m and width of the canal is 10 m.

8. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower. [*CBSE Delhi* 2017]

Sol. Let PQ be the building of height 7 metres and AB be the cable tower. Now it is given that the angle of elevation of the top A of the tower observed from the top P of building is 60° and the angle of depression of the base B of the tower observed from P is 45°. (Fig. 10.14)

So, $\angle APR = 60°$ and $\angle QBP = 45°$

Let $QB = x$ m, $AR = h$ m then, $PR = x$ m

Now, in ΔAPR, we have

$$\tan 60° = \frac{AR}{PR} \Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3}x = h \quad \Rightarrow \quad h = \sqrt{3}x \qquad ...(i)$$

Again, in ΔPBQ we have

$$\tan 45° = \frac{\hat{P}Q}{QB} \Rightarrow 1 = \frac{7}{x} \Rightarrow x = 7 \qquad ...(ii)$$

Putting the value of x in equation (i), we have

$$h = \sqrt{3} \times 7 = 7\sqrt{3}$$

i.e., $AR = 7\sqrt{3}$ metres

So, the height of tower $= AB = AR + RB = 7\sqrt{3} + 7 = 7(\sqrt{3} + 1)$ m.

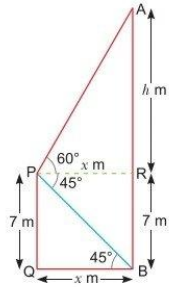


Fig. 10.14

9. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Sol. Let AB be the lighthouse of height 75 m and P, Q be the position of the two ships whose angles of depression are 45° and 30°, respectively (Fig. 10.15).

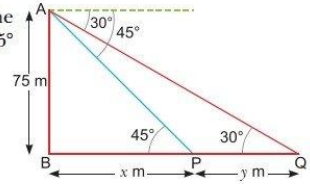


Fig. 10.15

Let $BP = x$ m and $PQ = y$ m, we have

$\angle APB = 45°$ and $\angle AQB = 30°$

Now, in ΔABP we have

$$\tan 45° = \frac{AB}{BP} \Rightarrow 1 = \frac{75}{x}$$

$$\Rightarrow \quad x = 75 \text{ m} \qquad ...(i)$$

Again, in ΔABQ we have

$$\tan 30° = \frac{AB}{BQ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{x + y} \Rightarrow x + y = 75\sqrt{3} \qquad ...(ii)$$

From (i) and (ii), we have

$$75 + y = 75\sqrt{3}$$

$$y = 75\sqrt{3} - 75 \Rightarrow y = 75(\sqrt{3} - 1)$$

Hence, the distance between two ships is $75(\sqrt{3} - 1)$ metres.

10. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (Fig. 10.16). Find the distance travelled by the balloon during the interval.

Sol. Let A and B be two positions of the balloon

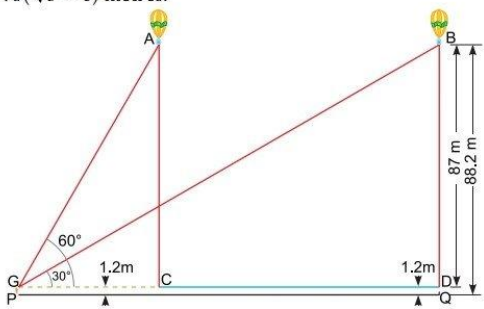


Fig. 10.16

and G be the point of observation. (eyes of the girl)

Now, we have

$AC = BD = BQ - DQ = 88.2 \text{ m} - 1.2 \text{ m} = 87 \text{ m}$

$\angle AGC = 60°$, $\angle BGD = 30°$

Now, in ΔAGC, we have

$$\tan 60° = \frac{AC}{GC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{87}{GC}$$

$$\Rightarrow \quad CG = \frac{87}{\sqrt{3}} = \frac{87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{87 \times \sqrt{3}}{3}$$

$$\Rightarrow \quad GC = 29 \times \sqrt{3} \qquad \ldots(i)$$

Again, in ΔBGD, we have

$$\tan 30° = \frac{BD}{GD} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{87}{GD}$$

$$\Rightarrow \quad GD = 87 \times \sqrt{3} \qquad \ldots(ii)$$

From (i) and (ii), we have

$$CD = 87 \times \sqrt{3} - 29 \times \sqrt{3}$$
$$= \sqrt{3}\ (87 - 29) = 58\sqrt{3}$$

Hence, the balloon travels $58\sqrt{3}$ metres.

11. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point. [*CBSE Delhi* 2017 (C)]

Sol. Let OA be the tower of height h, and P be the initial position of the car when the angle of depression is 30°.

After 6 seconds, the car reaches to Q such that the angle of depression at Q is 60°. Let the speed of the car be v metre per second. Then,

$$PQ = 6v \qquad (\because \text{ Distance} = \text{speed} \times \text{time})$$

and let the car take t seconds to reach the tower OA from Q (Fig. 10.17). Then, $OQ = vt$ metres.

Now, in ΔAQO, we have

$$\tan 60° = \frac{OA}{QO}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{vt} \quad \Rightarrow \quad h = \sqrt{3}\,vt \qquad \ldots(i)$$

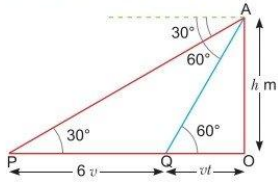


Fig. 10.17

Now, in ΔAPO, we have

$$\tan 30° = \frac{OA}{PO}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{6v + vt} \quad \Rightarrow \quad \sqrt{3}\,h = 6v + vt \qquad \ldots(ii)$$

Now, substituting the value of h from (i) into (ii), we have

$$\sqrt{3} \times \sqrt{3}\,vt = 6v + vt$$

$$\Rightarrow \quad 3vt = 6v + vt \quad \Rightarrow \quad 2vt = 6v \quad \Rightarrow \quad t = \frac{6v}{2v} = 3$$

Hence, the car will reach the tower from Q in 3 seconds.

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the Sun's elevation is

[*NCERT Exemplar*]

(*a*) 60° (*b*) 45° (*c*) 30° (*d*) 90°

2. The length of a string between a kite and a point on the ground is 85 m, if the string makes an angle θ with the ground level such that $\tan\theta = \frac{15}{8}$, then at what height is the kite from the ground?

[*Competency Based Question*]

(*a*) 75 m (*b*) 79.41 m (*c*) 80 m (*d*) 72.5 m

3. A ladder 18 m long makes an angle of 60° with a wall. The height of the point where the ladder reaches the wall is

(*a*) $9\sqrt{3}$ m (*b*) $18\sqrt{3}$ m (*c*) 18 m (*d*) 9 m

4. The angle of depression of a car parked on the road from the top of 150 m high tower is 30°. The distance of the car from the tower (in metres) is

(*a*) $50\sqrt{3}$ (*b*) $150\sqrt{3}$ (*c*) $150\sqrt{2}$ (*d*) 75

5. If the angle of elevation of top of a tower from a point on the ground which is $20\sqrt{3}$ m away from the foot of the tower is 30°, then the height of the tower is

(*a*) 60 m (*b*) 30 m (*c*) 25 m (*d*) 20 m

6. If the angles of elevation of the top of a tower from the two points at a distance of 2 m and 8 m from the base of the tower and in the same straight line with it are 60° and 30°, then the height of the tower is

(*a*) 3 m (*b*) 4 m (*c*) 5 m (*d*) 6 m

7. If a 30 m ladder is placed against a 15 m wall such that it just reaches the top of the wall, then the elevation of the wall is equal to

(*a*) 45° (*b*) 30° (*c*) 60° (*d*) 50°

8. A 1.6 m tall girl stands at distance of 3.2 m from a lamp post and casts shadow of 4.8 m on the ground, then the height of the lamp post is

(*a*) 8 m (*b*) 4 m (*c*) 6 m (*d*) $\frac{8}{3}$ m

9. An observer 1.6 m tall is $20\sqrt{3}$ m away from a tower. The angle of elevation from his eye to the top of the tower is 30°. The height of the tower is

(*a*) 21.6 m (*b*) 23.2 m (*c*) 24.72 m (*d*) None

10. From the top of a 42 m high light house, the angles of depression of two ships that are in a straight line are 60° and 45° respectively. What is the distance between the two ships? (Use $\sqrt{3} = 1.73$)

(*a*) 12.4 m (*b*) 17.8 m (*c*) 24.2 m (*d*) 32.39 m

Answers

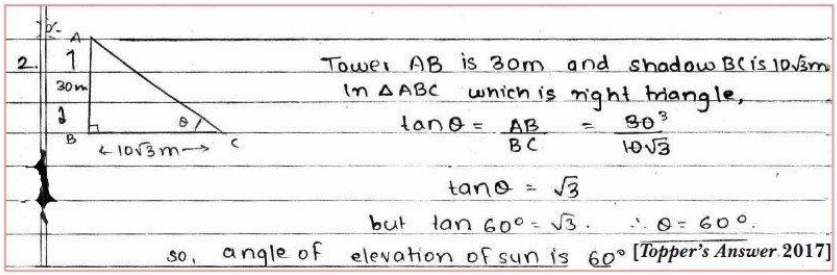
1. (*a*) **2.** (*a*) **3.** (*a*) **4.** (*b*) **5.** (*d*) **6.** (*b*) **7.** (*b*)
8. (*d*) **9.** (*a*) **10.** (*b*)

Very Short Answer Questions

Each of the following questions are of 1 mark.

1. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun ? **[*CBSE (AI)* 2017]**

Sol.



[*Topper's Answer* 2017]

2. If the angle of elevation of a tower from a distance of 100 m from its foot is 60°, then what will be the height of the tower? **[*NCERT Exemplar*]**

Sol. Let h be the height of the tower. (Fig. 10.18)

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{100}$$

$$h = 100\sqrt{3} \text{ m}$$

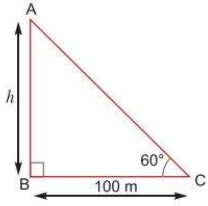


Fig. 10.18

3. If a pole 12 m high casts a shadow $4\sqrt{3}$ m long on the ground, find the Sun's elevation. **[*NCERT Exemplar*]**

Sol. $\tan \theta = \dfrac{12}{4\sqrt{3}} = \dfrac{3}{\sqrt{3}} = \sqrt{3}$

$\Rightarrow \theta = 60°$

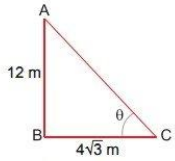
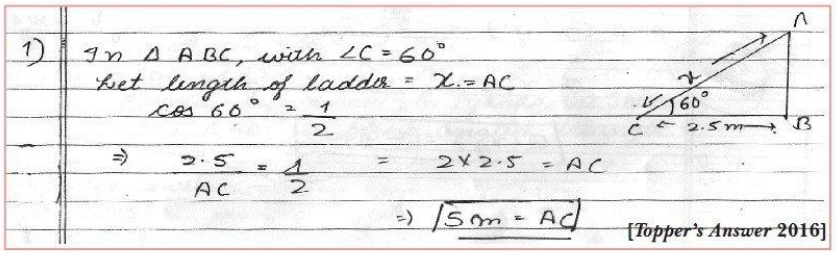


Fig. 10.19

4. A ladder, leaning against a wall, makes an angle 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder. **[*CBSE (AI)* 2016(30/2)]**

Sol.



[*Topper's Answer* 2016]

5. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30°. Find the height of tower. **[*CBSE (F)* 2016]**

Sol. Let AB be the height of tower and DE be the height of observer.

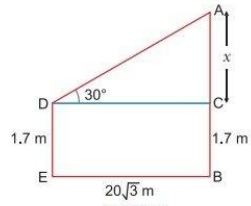


Fig. 10.20

Then in ΔACD, $\frac{AC}{DC} = \tan 30°$.

$$\Rightarrow \quad \frac{x}{20\sqrt{3}} = \tan 30° = \frac{1}{\sqrt{3}} \quad \Rightarrow x = 20 \text{ m}$$

$$\therefore \quad AB = 20 + 1.7 = 21.7 \text{ m}$$

6. In Fig. 10.21, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower. [*CBSE* 2020(30/5/1)]

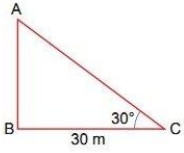


Fig. 10.21

Sol. We have,

$$\tan 30° = \frac{AB}{BC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{30}$$

$$\Rightarrow \quad AB = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3} \text{ m}$$

$\therefore$ Height of the tower $= 10\sqrt{3}$ m

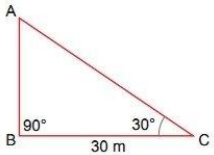


Fig. 10.22

7. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}$: 1. What is the angle of elevation of the sun? [*CBSE Delhi* 2017]

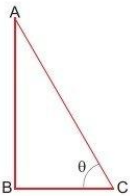


Fig. 10.23

Sol. Given: $\frac{AB}{BC} = \frac{\sqrt{3}}{1}$

Then, $\tan \theta = \frac{AB}{BC} = \sqrt{3}$

$$\Rightarrow \quad \theta = 60°$$

Short Answer Questions–I

Each of the following questions are of* 2 *marks.

1. 'Skysails' is that genre of engineering science that uses extensive utilization of wind energy to move a vessel in the sea water. The 'Skysails' technology allows the towing kite to gain a height of anything between 100–300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the questions:

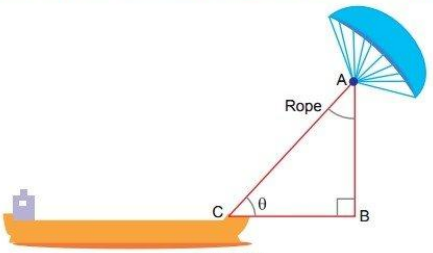


Fig. 10.24

(i) In the given figure, if $\cos(90 - \theta) = \cos(3\theta - 30°)$, where θ and $3\theta - 30°$ are acute angles, then find the value of θ.

(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200 m?

Sol. (i) $\cos(90° - \theta) = \cos(3\theta - 30°)$

$\Rightarrow 90° - \theta = 3\theta - 30° \Rightarrow \theta = 30°$

(ii) $\frac{AB}{AC} = \sin 30°$

$\frac{200}{AC} = \frac{1}{2} \Rightarrow AC = 400$ m

$\therefore$ Length of rope $= AC = 400$ m

2. The height of a tower is 12 m. What is the length of its shadow when Sun's altitude is 45°?

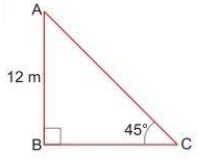


Fig. 10.25

Sol. Let AB be the tower [Fig. 10.25].

Then, $\angle C = 45°$, $AB = 12$ m

$$\tan 45° = \frac{AB}{BC} = \frac{12}{BC} \quad \Rightarrow \quad 1 = \frac{12}{BC} \quad \Rightarrow \quad BC = 12 \text{ m}$$

$\therefore$ The length of the shadow is 12 m.

3. The angle of elevation of the top of a tower from a point on the ground, which is 20 m away from the foot of the tower, is 60°. Find the height of the tower.

Sol. Let BC be the tower whose height is h metres and A be the point at a distance of 20 m from the foot of the tower. The angle of elevation of the top of the tower from point A is given to be 60°.

Now, in right angle ΔCBA we have,

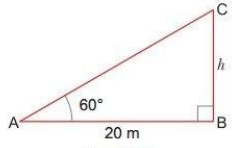


Fig. 10.26

$$\tan 60° = \frac{BC}{AB} = \frac{h}{20} \Rightarrow \sqrt{3} = \frac{h}{20}$$

$$\Rightarrow \quad h = 20\sqrt{3} \text{ m}$$

Hence, the height of the tower is $20\sqrt{3}$ m.

Short Answer Questions–II

Each of the following questions are of 3 marks.

1. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 12 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary and angle of elevation at C is 60° then find the height of the tower.

Sol. It is given that $\angle ACB$ and $\angle ADB$ are complementary and $\angle ACB = 60°$.

$\therefore \quad \angle ADB = 90° - 60° = 30°$

Now, in right ΔABC,

$$\tan 60° = \frac{AB}{BC} = \frac{h}{4}$$

$$\tan 60° = \frac{h}{4}$$

$$\Rightarrow \quad \frac{h}{4} = \sqrt{3}$$

$$\Rightarrow \quad h = 4\sqrt{3} \text{ m}$$

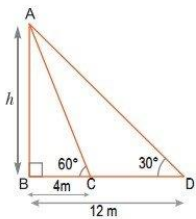


Fig. 10.27 (a)

Alternately

In right ΔABD,

$$\tan 30° = \frac{AB}{BD} = \frac{h}{12}$$

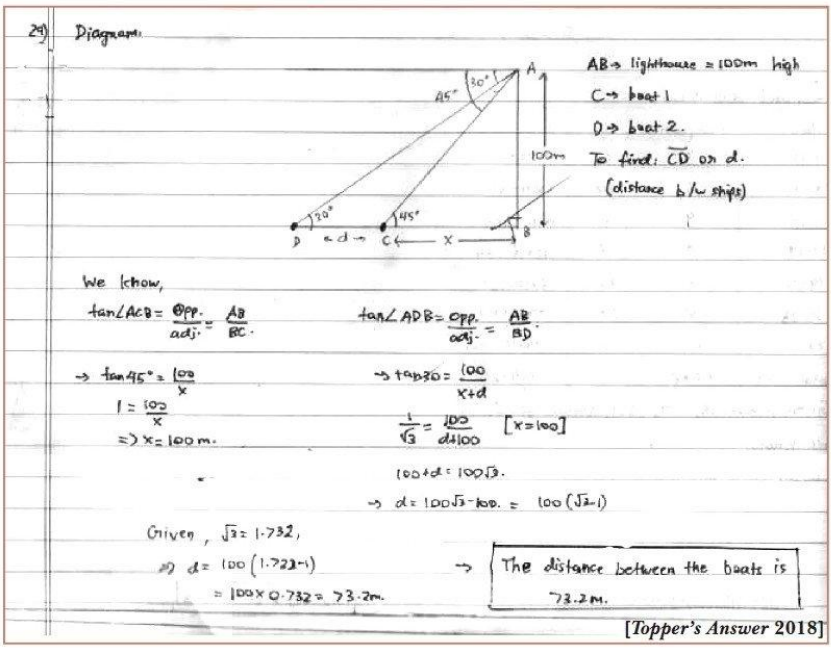
$$\frac{1}{\sqrt{3}} = \frac{h}{12}$$

$$\sqrt{3} = \frac{12}{h} \Rightarrow h = \frac{12}{\sqrt{3}} = 4\sqrt{3} \text{ m}$$

$\therefore$ Height of tower is $4\sqrt{3}$ m.

2. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$] [*CBSE* 2018(30/1)]

Sol.



[*Topper's Answer* 2018]

3. Two men on either side of a 75 m high building and in line with base of building observe the angle of elevation of the top of the building as 30° and 60°. Find the distance between the two men. (Use $\sqrt{3} = 1.73$) [*CBSE (F)* 2016]

Sol. Let AB be the building having height 75 m and the angles of elevation are 30° and 60° from the point M_1 and M_2 respectively; [Fig. 10.27 (b)]

In ΔABM_1, $\frac{AB}{BM_1} = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad BM_1 = 75\sqrt{3}$ m

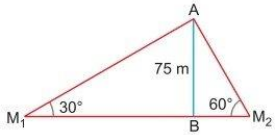


Fig. 10.27 (b)

In ΔABM_2,

$$\frac{AB}{BM_2} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad BM_2 = \frac{75}{\sqrt{3}} = 25\sqrt{3} \text{ m}$$

$$\therefore \quad M_1M_2 = M_1B + BM_2$$

$$= 75\sqrt{3} + 25\sqrt{3} = 100\sqrt{3} \text{ m} = 173 \text{ m}$$

$\therefore$ Distance between two men = 173 m

4. Determine the height of a mountain if the elevation of its top at an unknown distance from the base is 30° and at a distance 10 km further off from the mountain, along the same line, the angle of elevation is 15°. (Use tan 15° = 0.27)

Sol. Let AB be the mountain of height h kilometres. Let C be a point at a distance of x km, from the base of the mountain such that the angle of elevation of the top at C is 30°. Let D be a point at a distance of 10 km from C such that angle of elevation at D is of 15°.

In ΔABC (Fig. 10.28), we have

$$\tan 30° = \frac{AB}{AC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

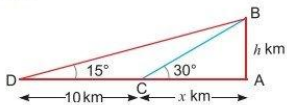


Fig. 10.28

$$\Rightarrow \quad x = \sqrt{3}h$$

In ΔADB, we have

$$\tan 15° = \frac{AB}{AD} \quad \Rightarrow \quad 0.27 = \frac{h}{x + 10}$$

$$\Rightarrow \quad 0.27\,(x + 10) = h \qquad \ldots(i)$$

Substituting $x = \sqrt{3}h$ in equation (i), we get

$$0.27\,(\sqrt{3}h + 10) = h$$

$$\Rightarrow \quad 0.27 \times \sqrt{3}h + 0.27 \times 10 = h$$

$$\Rightarrow \quad 2.7 = h - 0.27 \times \sqrt{3}h \quad \Rightarrow \quad 2.7 = h\,(1 - 0.27 \times \sqrt{3})$$

$$\Rightarrow \quad 2.7 = h\,(1 - 0.46) \quad \Rightarrow \quad h = \frac{2.7}{0.54} = 5$$

Hence, the height of the mountain is 5 km.

5. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.

Sol. In Fig. 10.29, A and B represent points on the bank on opposite sides of the river, so that AB is the width of the river. P is a point on the bridge at a height of 3 m, *i.e.*, DP = 3m. We are interested to determine the width of the river, which is the length of the side AB of the ΔAPB.

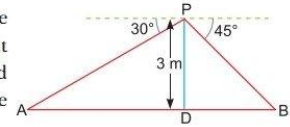


Fig. 10.29

In right ΔADP, $\angle A = 30°$

So, $\tan 30° = \dfrac{PD}{AD}$

i.e., $\dfrac{1}{\sqrt{3}} = \dfrac{3}{AD}$ or $AD = 3\sqrt{3}$ m

Also, in right ΔPDB,

$$\frac{PD}{DB} = \tan 45° \Rightarrow \frac{3}{DB} = 1$$

$\therefore$ $DB = 3$m

Now, $AB = BD + AD = 3 + 3\sqrt{3} = 3(1 + \sqrt{3})$ m

Therefore, the width of the river is $3(\sqrt{3} + 1)$ m.

6. The angle of elevation of a aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of $3600\sqrt{3}$ metres then find the speed of the aeroplane.

[*Competency Based Question*] [*CBSE* 2019 (30/5/1)]

Sol. Let the position of plane be at C, after 30 seconds, it will be at C' (See figure 10.30)

Let $AB = x$ m and $AB' = y$ m

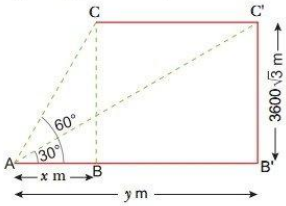


Fig. 10.30

In ΔABC, we have

$$\tan 60° = \frac{BC}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{3600\sqrt{3}}{x} \quad \Rightarrow \quad x = 3600 \text{ m}$$

Now,
In $\Delta AB'C'$, we have

$$\tan 30° = \frac{B'C'}{AB'} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{y}$$

$\Rightarrow$ $y = 3600 \times 3 = 10800$ m

$\therefore$ Distance covered in 30 seconds $= y - x$

$= (10800 - 3600)$ m $= 7200$ m

$\therefore$ Speed of the plane $= \dfrac{7200}{30}$

$\Rightarrow$ Speed of the plane $= 240$ m/sec. $= 240 \times \dfrac{18}{5}$ km/h $= 864$ km/h

7. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/h.

[*CBSE Delhi* 2017]

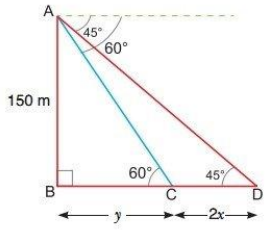


Fig. 10.31

Sol. Let the speed of boat be x m/min.

$\therefore$ $CD = 2x$

In ΔABC,

$$\frac{150}{y} = \tan 60° \quad \Rightarrow \quad y = \frac{150}{\sqrt{3}} = 50\sqrt{3}\,\text{m}$$

In ΔABD

$$\frac{150}{y + 2x} = \tan 45° \quad \Rightarrow \quad 150 = 50\sqrt{3} + 2x$$

$(\because y = 50\sqrt{3}$ m)

$\Rightarrow \quad x = 25(3 - \sqrt{3})$

Speed $= 25(3 - \sqrt{3})$ m/min

$\therefore \quad = 25 \times 60\,(3 - \sqrt{3})$ m/h $= 1500\,(3 - \sqrt{3})$ m/h

8. Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of two ships as observed from the top of the light house are 60° and 45°. If the height of the light house is 200 m, find the distance between the two ships. [Use $\sqrt{3} = 1.73$] [*CBSE Delhi* 2014]

Sol. Let the distance between the two ships be d metres.

Let the distance of one ship from the light house be x metres. Then, the distance of the other ship from the light house will be $(d - x)$ metres.

In ΔACO,

$$\tan 45° = \frac{OC}{AC} = \frac{200}{x}$$

$$1 = \frac{200}{x} \quad \Rightarrow \quad x = 200 \text{ m} \quad ...(i)$$

In ΔBCO, $\tan 60° = \frac{OC}{BC}$

$$\sqrt{3} = \frac{200}{d - x} \quad \Rightarrow \quad d - x = \frac{200}{\sqrt{3}}$$

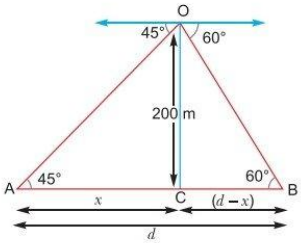


Fig. 10.32

Substituting the value of x from (i)

$$d - 200 = \frac{200}{\sqrt{3}}$$

$$d = \frac{200}{\sqrt{3}} + 200 = 200\left(\frac{1}{\sqrt{3}} + 1\right)$$

$$= 200\left(\frac{1 + \sqrt{3}}{\sqrt{3}}\right) = 200 \times \frac{(1 + \sqrt{3})}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 200\,\frac{(\sqrt{3} + 3)}{3} = 200 \times \frac{(1.73 + 3)}{3} = 200 \times \frac{4.73}{3} = \frac{946}{3} = 315.33 \text{ m}$$

9. A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 5 m. From a point on the ground the angles of elevation of the top and bottom of the flagstaff are 60° and 30° respectively. Find the height of the tower and the distance of the point from the tower. (Take $\sqrt{3} = 1.732$) [*CBSE (F)* 2016]

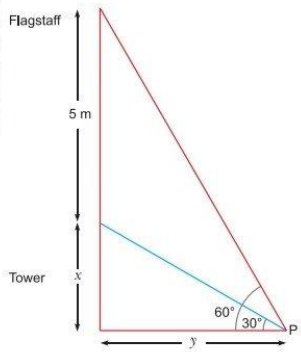


Fig. 10.33

Sol. Let height of tower be x m and

distance of point from tower be y m.

(*i*) From the Fig. 10.33, $\frac{x}{y} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow y = \sqrt{3}x$

(*ii*) $\frac{x+5}{y} = \tan 60° = \sqrt{3}$ or $\frac{x+5}{\sqrt{3}x} = \sqrt{3}$ $(\because y = \sqrt{3}x)$

$\Rightarrow x + 5 = 3x \Rightarrow x = \frac{5}{2} = 2.5$

Height of tower = 2.5 m

Distance of point from tower $= y = \sqrt{3}x$

$= (2.5 \times 1.732)$ or 4.33 m

10. A spherical balloon of radius r subtends an angle θ at the eye of an observer. If the angle of elevation of its centre is ϕ, find the height of the centre of the balloon.

[*Competency Based Question*] [*NCERT Exemplar*]

Sol. In Fig. 10.34, O is the centre of balloon, whose radius $OP = r$ and $\angle PAQ = \theta$. Also, $\angle OAB = \phi$. Let the height of the centre of the balloon be h. Thus $OB = h$.

In ΔOAP, we have

$$\sin\frac{\theta}{2} = \frac{r}{d}, \text{ where } OA = d \qquad ...(i)$$

Also in ΔOAB,

$$\sin\phi = \frac{h}{d} \qquad ...(ii)$$

From (i) and (ii), we get

$$\frac{\sin\phi}{\sin\frac{\theta}{2}} = \frac{\frac{h}{d}}{\frac{r}{d}} = \frac{h}{r} \quad \text{or} \quad h = r\sin\phi\,\text{cosec}\,\frac{\theta}{2}$$

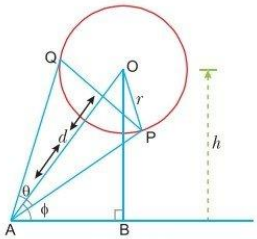


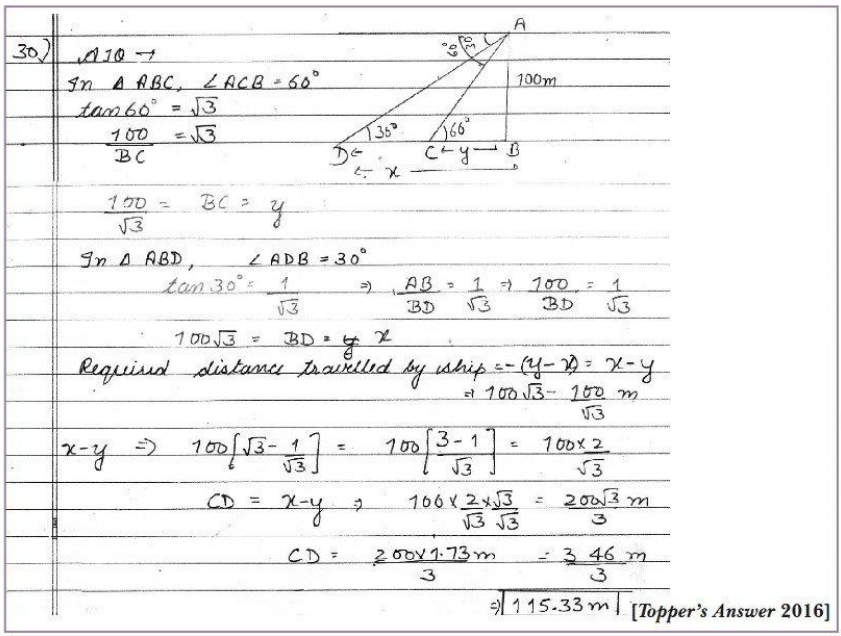
Fig. 10.34

Long Answer Questions

Each of the following questions are of 5 marks.

1. As observed from the top of a light house, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it, changes from 30° to 60°. Find the distance travelled by the ship during the period of observation. (Use $\sqrt{3}$ =1.73) **[*CBSE (AI)* 2016(30/2)]**

Sol.



[*Topper's Answer* 2016]

2. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

[*CBSE* 2020(30/2/1)]

Sol. Let AB be the building of height 20 m, BC be the transmission tower of height h m and O be any point on the ground.

Let $OA = x$ m

Now, In ΔOAB, we have

$$\tan 45° = \frac{AB}{OA} \Rightarrow 1 = \frac{20}{x}$$

$\Rightarrow$ $x = 20$ m ...(i)

In ΔOAC, we have

$$\tan 60° = \frac{AC}{OA} \Rightarrow \sqrt{3} = \frac{h + 20}{x}$$

$\Rightarrow$ $h + 20 = 20\sqrt{3} \Rightarrow h = 20\sqrt{3} - 20$

$\Rightarrow$ $h = 20(\sqrt{3} - 1) = 20(1.73 - 1)$

$\Rightarrow$ $h = 20 \times 0.73 = 14.6$ m

$\therefore$ Height of the tower = 14.6 metres

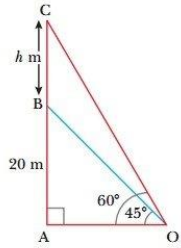


Fig. 10.35

3. If the angle of elevation of a cloud from a point 10 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°, find the height of the cloud from the surface of lake.

[*CBSE* 2020(30/3/1)]

Sol. Let the position of the cloud be E (Fig. 10.36), and F be the image of the cloud in the lake.

In ΔBDE, we have

$$\tan 30° = \frac{h}{x}$$

$\Rightarrow$ $\frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow$ $x = \sqrt{3}\,h$ ...(i)

In ΔBDF, we have

$$\tan 60° = \frac{FD}{BD} = \frac{10 + (h + 10)}{x}$$

$\Rightarrow$ $\sqrt{3} = \frac{h + 20}{\sqrt{3}\,h}$ [From (i)]

$\Rightarrow$ $3h = h + 20 \Rightarrow 2h = 20 \Rightarrow h = 10$ m

$\therefore$ Height of the cloud from the surface of lake = (10 + 10) m

= 20 m

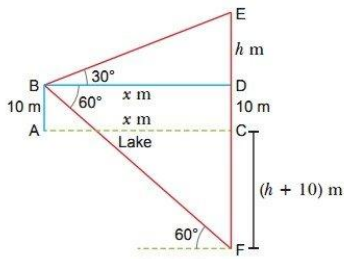


Fig. 10.36

4. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. (Use $\sqrt{3} = 1.732$]) [*Competency Based Question*] [*CBSE* 2019 (30/1/2)]

Sol. Let AB be the light house of height = 100 m.

Also let the distance $DC = x$ metres and $CB = y$ metres.

In ΔABC, we have

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{100}{y} \Rightarrow y = \frac{100}{\sqrt{3}} \text{ m} \quad ...(i)$$

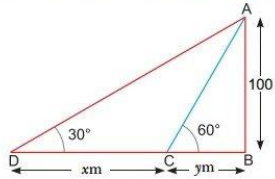


Fig. 10.37

In ΔABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{x+y} \Rightarrow x + y = 100\sqrt{3} \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$x + \frac{100}{\sqrt{3}} = 100\sqrt{3} \Rightarrow \sqrt{3}x + 100 = 300 \Rightarrow x = \frac{200}{\sqrt{3}} \text{m}$$

Time taken to travel from C to D = 2 minutes

As we know that speed = $\frac{\text{distance}}{\text{Time}}$

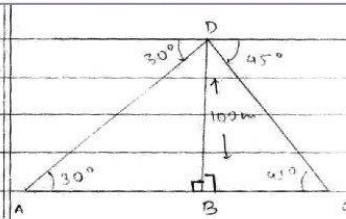
$$= \frac{\frac{200}{\sqrt{3}}}{2} = \frac{200}{2\times\sqrt{3}} = \frac{100}{1.732} = 57.74 \text{ m/min (approrx.)}$$

Hence, the speed of boat is 57.74 m/ min.

5. From the top of a tower, 100 m high, a man observes two cars on the opposite sides of the tower and in same straight line with its base, with angles of depression 30° and 45°. Find the distance between the cars. [Take $\sqrt{3}$ = 1.732] **[*CBSE* 2017 (30/3)]**

Sol.

30.



To find : AC

Solution:

In ΔABD, $\angle DAB = 30°$

In ΔBDC, $\angle BCD = 45°$,

also, $BD = 100$m.

In right ΔABD,

$$\tan 30° = \frac{DB}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{AB}$$

$$AB = 100\sqrt{3} \quad 100 \times 1.732 = 173.2 \text{ m}$$

In right ΔDBC,

$$\tan 45° = \frac{DB}{BC}$$

$$1 = \frac{100}{BC} \Rightarrow BC = 100\text{m}.$$

Now, $AC = AB + BC = 100 + 173.2$ m $= 273.2$ m

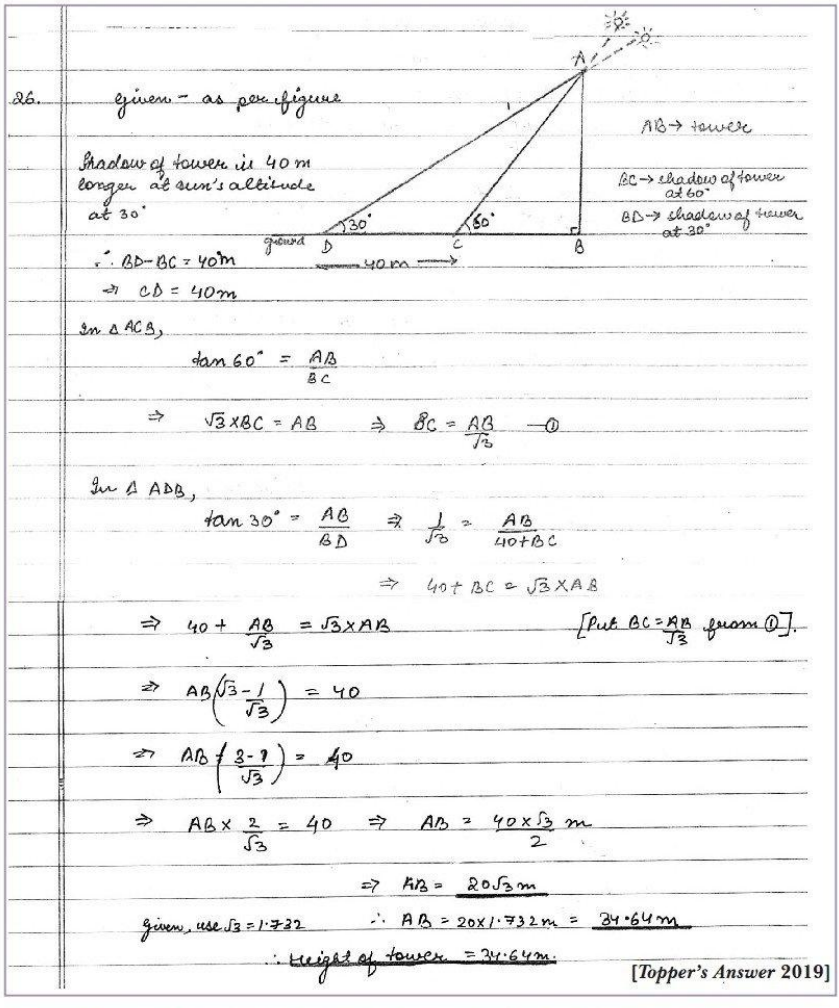
or $100(\sqrt{3}+1)$ m

[*Topper's Answer* 2017]

6. **The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower. (Use $\sqrt{3} = 1.732$])**

[*CBSE* 2019(30/3/1)]

Sol.



[*Topper's Answer* 2019]

7. **A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag staff of height *h*. At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of *h*.** **[*CBSE* 2020(30/3/1)]**

Sol. Let AB be the vertical tower and BC be the flag staff.

Let $AD = x$ m

In ΔABD, we have

$$\tan 45° = \frac{AB}{AD}$$

$$\Rightarrow \quad 1 = \frac{20}{x} \quad \Rightarrow \quad x = 20 \text{ m}$$

In ΔACD, we have

$$\tan 60° = \frac{AC}{AD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h + 20}{x} \quad \Rightarrow \quad \sqrt{3} = \frac{h + 20}{20}$$

$$\Rightarrow \quad 20\sqrt{3} = h + 20 \quad \Rightarrow \quad h = 20\sqrt{3} - 20$$

$$\Rightarrow \quad h = 20(\sqrt{3} - 1) \text{ m}$$

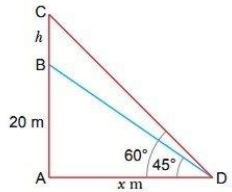


Fig. 10.38

8. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of 20 metre high building, finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on opposite sides of the bird. Find the distance of bird from the girl. [Given $\sqrt{2} = 1.414$] **[*CBSE* 2019(30/5/1)]**

Sol. Let B be the position of bird. O and P be the positions of boy and girl respectively and PQ be the building.

We have, $\angle AOB = 30°$ and, $\angle BPM = 45°$

In ΔAOB, we have

$$\sin 30° = \frac{AB}{100} \quad \Rightarrow \quad \frac{1}{2} = \frac{AB}{100}$$

$$\Rightarrow \quad AB = 50 \text{ m}$$

$$\therefore \quad BM = AB - AM = 50 - 20 = 30 \text{ m}$$

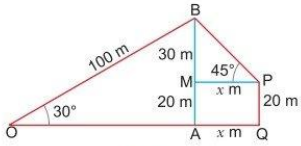


Fig. 10.39

In ΔPBM, we have

$$\sin 45° = \frac{BM}{BP} \quad \Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{30}{BP} \quad \Rightarrow \quad BP = 30\sqrt{2} \text{ m}$$

Hence, distance of bird from girl is $30\sqrt{2}$ m.

9. A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.

[*Competency Based Question*]

Sol. In Fig. 10.40, let C represents the position of the man on the deck of the ship, A represents the top of hill and D its base.

Now in right-angled triangle CWD,

$$\tan 30° = \frac{10}{WD} \quad \Rightarrow \quad WD = \frac{10}{\tan 30°}$$

$$\Rightarrow \quad WD = \frac{10}{\frac{1}{\sqrt{3}}} = 10\sqrt{3} = 17.3 \text{ m}$$

Also, in right-angled triangle ABC, we have

$$\tan 60° = \frac{AB}{BC} \text{ or } \frac{AB}{WD} \quad \text{(From fig. } BC = WD\text{)}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AB}{10\sqrt{3}} \quad \Rightarrow \quad AB = 10\sqrt{3} \times \sqrt{3} = 30 \text{ m}$$

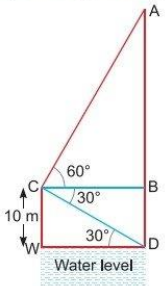
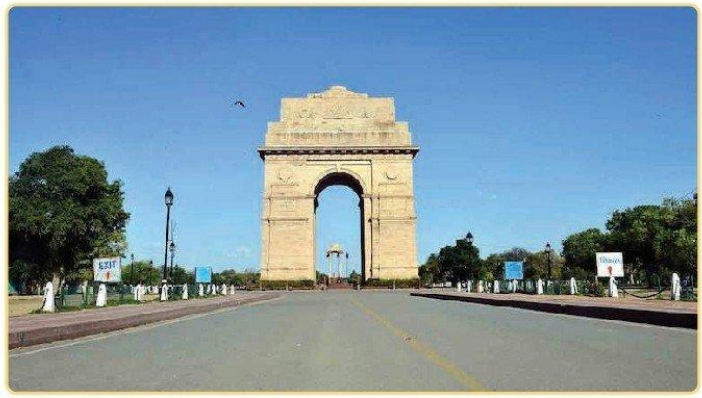


Fig. 10.40



(*i*) What is the angle of elevation if they are standing at a distance of 42 m away from the monument?

(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 0°

(*ii*) They want to see the tower at an angle of 60°. So, they want to know the distance where they should stand and hence the distance is

(*a*) 24.24 m (*b*) 20.12 m (*c*) 42 m (*d*) 24.64 m

(*iii*) If the altitude of the Sun is at 60° then the height of the vertical tower that will cast a shadow of length 20 m, is

(*a*) $20\sqrt{3}$ m (*b*) $\frac{20}{\sqrt{3}}$ m (*c*) $\frac{15}{\sqrt{3}}$ m (*d*) $15\sqrt{3}$ m

(*iv*) The ratio of the length of a rod and its shadow is 1:1. The angle of elevation of the Sun is

(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90°

(*v*) The angle formed by the line of sight with the horizontal when the object view is below the horizontal level is

(*a*) corresponding angle (*b*) angle of elevation

(*c*) angle of depression (*d*) complete angle

Sol. (*i*) Let θ be the angle of elevation.

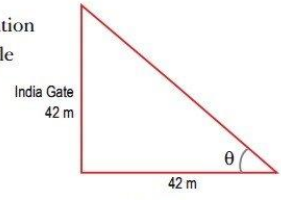


Fig. 10.42

$\therefore \quad \tan\theta = \frac{42}{42} = 1$

$\Rightarrow \quad \tan\theta = 1 = \tan 45° \quad \Rightarrow \quad \theta = 45°$

$\therefore$ Angle of elevation is 45°.

$\therefore$ Option (*b*) is correct.

(*ii*) Let x m be the required distance.

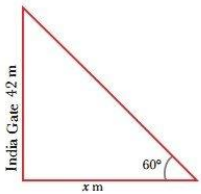


Fig. 10.43

$\therefore \quad \tan 60° = \frac{42}{x}$

$\Rightarrow \quad \sqrt{3} = \frac{42}{x} \quad \Rightarrow \quad x = \frac{42}{\sqrt{3}}$

$\Rightarrow \quad x = \frac{42}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{42\sqrt{3}}{3} = 14\sqrt{3}$

$= 14 \times 1.732 = 24.24$ m

$\therefore$ Option (*a*) is correct.

(*iii*) Let h m be the height of the vertical tower.

We have,

$$\therefore \quad \tan 60° = \frac{h}{20}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{20} \quad \Rightarrow \quad h = 20\sqrt{3} \text{ m}$$

$\therefore$ Option (*a*) is correct.

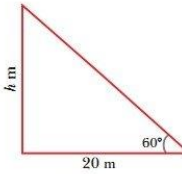


Fig. 10.44

(*iv*) Let θ be the angle of elevation.

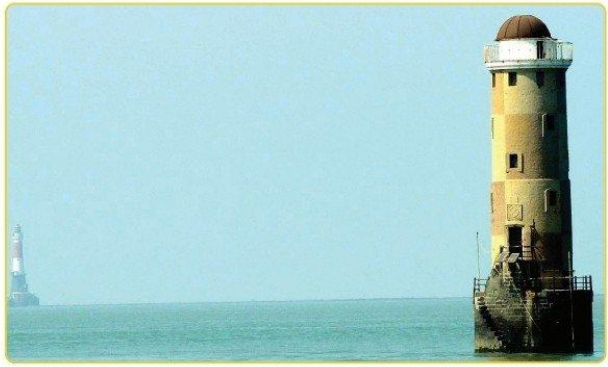
$$\therefore \quad \tan\theta = \frac{1}{1} = 1 \quad \Rightarrow \quad \tan\theta = 1 = \tan 45° \quad \Rightarrow \quad \theta = 45°$$

$\therefore$ Option (*b*) is correct.

(*v*) Angle of depression

$\therefore$ Option (*c*) is correct.

2. A guard, stationed at the top of a 300 m tower, observed an unidentified boat coming towards it. A clinometer or inclinometer is an instrument used for measuring angles or slopes(tilt). The guard used the clinometer to measure the angle of depression of the boat coming towards the lighthouse and found it to be 30°.



Based on the above information answer the following questions.

(*i*) Make a labelled figure on the basis of the given information and calculate the distance of the boat from the foot of the observation tower.

(*ii*) After 10 minutes, the guard observed that the boat was approaching the tower and its distance from tower is reduced by $300(\sqrt{3}-1)$ m. He immediately raised the alarm. What was the new angle of depression of the boat from the top of the observation tower?

Sol. (*i*)

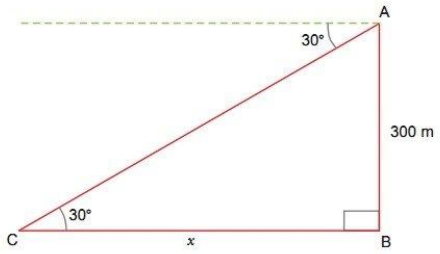


Fig. 10.45

In ΔABC, $\tan 30° = \frac{300}{x} \Rightarrow x = 300\sqrt{3}$ m

(*ii*) Distance of boat from tower $= 300\sqrt{3} - 300(\sqrt{3} - 1) = 300$ m

Let the angle of depression $= \theta$

$\tan\theta = \frac{300}{300} = 1 \Rightarrow \theta = 45°$

PROFICIENCY EXERCISE

■ Objective Type Questions: [1 mark each]

1. Choose and write the correct option in each of the following questions.

(*i*) Which are the angles of depression from the observing positions A and C respectively of the object at E in Fig. 10.46?

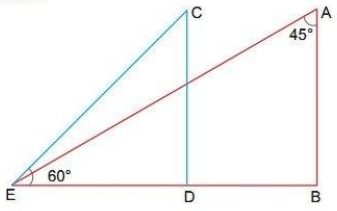


Fig. 10.46

(*a*) 30°, 60° (*b*) 45°, 30° (*c*) 45°, 60° (*d*) 15°, 30°

(*ii*) The length of the ladder making an angle of 45° with a wall and whose foot is 7 m away from the wall is

(*a*) $\frac{7\sqrt{2}}{2}$ m (*b*) $7\sqrt{2}$ m (*c*) $14\sqrt{2}$ m (*d*) 14 m

(*iii*) A tower casts a shadow 90 m long and at the same time another tower casts a shadow of 120 m on the ground. If the height of the second tower is 80 m, then the height of the first tower is

(*a*) 60 m (*b*) 55 m (*c*) 50 m (*d*) 40 m

(*iv*) If the angles of elevation of a tower from two points distant a and b where $a > b$ from its foot and in the same straight line from it are 30° and 60°, then the height of the tower is

[*Competency Based Question*]

(*a*) $\sqrt{a+b}$ (*b*) $\sqrt{ab}$ (*c*) $\sqrt{a-b}$ (*d*) $\sqrt{\frac{a}{b}}$

(*v*) The angles of elevation of two cars, from the car to the top of a building are 45° and 30°. If the cars are on the same side of the building and are 50 m apart, what is the height of the building?

(*a*) $25(\sqrt{3}+1)$ m (*b*) $25(\sqrt{3}-1)$ m (*c*) $50(\sqrt{3}+1)$ m (*d*) $50(\sqrt{3}-1)$ m

■ Very Short Answer Questions: [1 mark each]

2. The ratio of the length of a tree and its shadow is $1 : \frac{1}{\sqrt{3}}$. What is the Sun's angle of elevation?

3. If two towers of height h_1 and h_2 subtend angle of 60° and 30° respectively at the mid-point of the line joining their feet, then find $h_1 : h_2$. **[*CBSE Delhi* 2015]**

4. The height of the tower is 100 m. When the angle of elevation of Sun is 30°, then what is the length of shadow of the tower?

5. The tops of two poles of height 16 m and 10 m are connected by a wire of length l metres. If the wire makes an angle of 30° with the horizontal, then find l.

6. An observer, 1.5 m tall, is 28.5 m away from a 30 m high tower. Determine the angle of elevation of the top of the tower from the eye of the observer. **[*CBSE Delhi* 2017 (*C*)]**

7. The angle of elevation of the top of a tower from a point P on the ground is α. After walking a distance d meter towards the foot of the tower, angle of elevation is found to be β. Which angle of elevation is greater?

■ Short Answer Questions–I: [2 marks each]

8. A tree is broken by wind. The top struck the ground at an angle of 30° and at a distance of 30 m from the root. Find the height of the whole tree.

9. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.

■ Short Answer Questions–II: [3 marks each]

10. From a window 15 m high above the ground in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 30° and 45°, respectively. Show that the height of the opposite house is 23.66 m.**[*Competency Based Question*]**

11. The length of a string between a kite and a point on the ground is 90 metres. If the string makes an angle θ with the ground level such that $\tan\theta = \frac{15}{8}$, how high is the kite? Assume that there is no slack in the string.

12. The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 10 m vertically above the first, its angle of elevation is 45°. Find the height of the tower. **[*NCERT Exemplar*]**

13. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (Use $\sqrt{3} = 1.73$). **[*CBSE Delhi* 2016]**

14. From the top of a 120 m high tower, a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of depression as 60° and 45°. Find the distance between the two cars. (Take $\sqrt{3} = 1.732$) **[*CBSE Delhi* 2017 (*C*)]**

15. The angles of depression of two ships from an aeroplane flying at the height of 7500 m are 30° and 45°. If both the ships are in the same line and on the same side of the aeroplane such that one ship is exactly behind the other, find the distance between the ships. [Use $\sqrt{3} = 1{\cdot}73$] **[*CBSE* (*F*) 2017]**

16. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of depression from the top of tower to the foot of hill is 30°. If tower is 50 metre high, find the height of the hill. **[*CBSE* 2018 (*C*) (30/1)]**

17. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3} = 1.732$] **[*CBSE* 2019(30/1/1)]**

18. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also the height of the other tower. **[*NCERT Exemplar*]**

■ Long Answer Questions: [5 marks each]

19. Amit, standing on a horizontal plane, finds a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, finds the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak. **[*Competency Based Question*] [*CBSE* 2019(30/2/1)]**

20. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/min. **[*CBSE* 2019(30/4/2)]**

21. There are two poles, one each on either bank of a river just opposite to each other. One pole is 60 m high. From the top of this pole, the angle of depression of the top and foot of the other pole are 30° and 60° respectively. Find the width of the river and height of the other pole. **[*CBSE* 2019(30/4/2)]**

22. The angles of depression of the top and bottom of a 8 m tall building from the top of a tower are 30° and 45° respectively. Find the height of the tower and the distance between the tower and the building. **[*CBSE* 2019(*C*)(30/1/1)]**

23. As observed from the top of a lighthouse, 75 m high from the sea level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. **[*CBSE* 2019(*C*)(30/1/1)]**

24. The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant, the angles of elevation of a balloon from these windows are observed to be 60° and 30°, respectively. Find the height of the balloon above the ground. **[*NCERT Exemplar*]**

25. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, what is the height of the hill? **[*CBSE Delhi* 2017]**

26. The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud from the surface of the lake. **[*CBSE Delhi* 2017]**

27. A bird is sitting on the top of a tree, which is 80 m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird. **[*CBSE Delhi* 2016]**

28. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. (Use $\sqrt{3} = 1.73$) **[*CBSE* (*AI*) 2016]**

29. From a point on the ground, the angle of elevation of the top of a tower is observed to be 60°. From a point 40 m vertically above the first point of observation, the angle of elevation of the top of the tower is 30°. Find the height of the tower and its horizontal distance from the point of observation. **[*CBSE* (*AI*) 2016]**

30. Two points A and B are on the same side of a tower and in the same straight line with its base. The angles of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points. **[*CBSE Delhi* 2017]**

Answers

1. (*i*) (*c*) (*ii*) (*b*) (*iii*) (*a*) (*iv*) (*b*) (*v*) (*a*)

2. 60° **3.** 3 : 1 **4.** $100\sqrt{3}$ m **5.** 12 m **6.** 45° **7.** β is greater

8. $30\sqrt{3}$ m **9.** 19 m **11.** 79.41 m **12.** 23.66 m **13.** 118.25 m, 68.25 m

14. 189.28 m **15.** 5475 m **16.** height of hill = 150 m **17.** speed of boat = 57.73 m/min

18. $10\sqrt{3}$ m, 10 m **19.** $50\sqrt{2}$ m **20.** Speed of boat is $(75 - 25\sqrt{3})$ m/min or 31.7 m/min

21. width of river = $20\sqrt{3}$ m and height of pole = 40 m

22. height of tower = $12 + 4\sqrt{3}$ m = distance between tower and building

23. $75(\sqrt{3}-1)$ m **24.** 8 m **25.** 150 m **26.** 120 m **27.** 29.28 m/s

28. 54.64 m, 94.64 m **29.** 60 m; $20\sqrt{3}$ m **30.** $5(3-\sqrt{3})$ m

Self-Assessment

Time allowed: 1 hour **Max. marks: 40**

SECTION A

1. Choose and write the correct option in the following questions. **(3 × 1 = 3)**

(*i*) Observe the figure below.

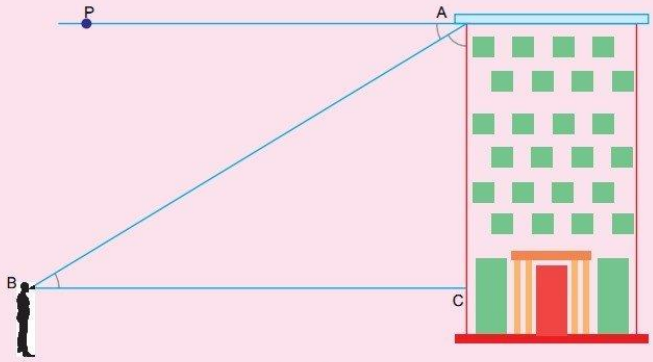


Fig. 10.47

If AB is line of sight, which of these angles represents the angle of elevation?

[*Competency Based Question*]

(*a*) $\angle ABC$ (*b*) $\angle BAP$ (*c*) $\angle ACB$ (*d*) $\angle BAC$

(*ii*) A boy is standing at the top of the tower and another boy is at the ground at some distance from the foot of the tower, then the angle of elevation and depression between the boys when both boys look at each other will be

(*a*) equal

(*b*) angle of elevation will be greater

(*c*) cannot be predicted

(*d*) angle of depression will be greater

(*iii*) The angle of elevation of a ladder leaning against a wall is 60° the foot of the ladder is 12.4 m away from the wall. The length of the ladder is

(*a*) 14.8 m (*b*) 6.2 m (*c*) 12.4 m (*d*) 24.8 m

2. Solve the following questions. **(2 × 1 = 2)**

(*i*) An observer 1.5 m tall is 20.5 m away from a tower 22 m high. Determine the angle of elevation of the top of the tower from the eye of the observer.

(*ii*) If the angle of elevation of top of a tower from a point on the ground which is $20\sqrt{3}$ m away from the foot of the tower is 30°, find the height of the tower.

SECTION B

■ **Solve the following questions.** **(4 × 2 = 8)**

3. If the length of the ladder placed against a wall is twice the distance between the foot of the ladder and the wall. Find the angle made by the ladder with the horizontal.

4. The shadow of a tower standing on a level plane is found to be 50 m longer when Sun's elevation is 30° than when it is 60°. Find the height of the tower. **[*NCERT Exemplar*]**

5. The angles of elevation of the top of a tower from two points distant s and t from its foot are 30° and 60°. Prove that the height of the tower is $\sqrt{st}$. **[*NCERT Exemplar*]**

6. In Fig. 10.48, height of a building is $h + 2$ and point C is h m from the foot of the building. Find the angle of elevation of the top of the building from a point 2 m from point C.

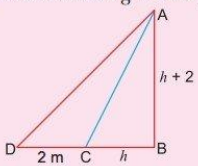


Fig. 10.48

■ **Solve the following questions.** **(4 × 3 = 12)**

7. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill. **[*CBSE (AI)* 2016]**

8. An observer finds the angle of elevation of the top of a tower from certain point is 30°. If the observer moves 20 m towards the tower, the angle of elevation of the top increases by 15°. Find the height of the tower. **[*NCERT Exemplar, CBSE Delhi* 2017]**

9. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (Use $\sqrt{3} = 1.73$). **[*CBSE Delhi* 2016]**

10. A man observes a car from the top of a tower, which is moving towards the tower with a uniform speed. If the angle of depression of the car changes from 30° to 45° in 12 minutes, find the time taken by the car now to reach the tower. **[*CBSE (AI)* 2017]**

■ **Solve the following questions.** **(3 × 5 = 15)**

11. The angles of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 60°, respectively. Find the height of the tower and also the horizontal distances between the building and the tower.

12. An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° and 60° respectively. Find the width of the river. [Use $\sqrt{3} = 1{\cdot}732$] **[*CBSE (AI)* 2017]**

13. A ladder rests against a vertical wall at an inclination α to the horizontal. Its foot is pulled away from the wall through a distance p so that its upper end slides a distance q down the wall and then the ladder makes an angle β to the horizontal. Show that $\frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$.

[*Competency Based Question*] [*NCERT Exemplar*]

Answers

1. (*i*) (*a*) (*ii*) (*a*) (*iii*) (*d*)

2. (*i*) 45° (*ii*) 20 m

3. 60° **4.** $25\sqrt{3}$ m **6.** 45° **7.** $10\sqrt{3}$ m, 40 m **8.** $10(\sqrt{3}+1)$ m

9. 118.25 m, 68.25 m **10.** 16 minutes 23 seconds

11. $25\sqrt{3}$ m, 75 m **12.** 473.2 m