6.5 RADIOACTIVITY

6.214 (a) The probability of survival (i.e. not decaying) in time t is $e^{-\lambda t}$. Hence the probability of decay is $1 - e^{-\lambda t}$

(b) The probability that the particle decays in time dt around time t is the difference

$$e^{-\lambda t} - e^{-\lambda(t+dt)} = e^{-\lambda t}[1 - e^{-e\lambda dt}] = \lambda e^{-\lambda t} dt$$

Therefore the mean life time is

$$T = \int_0^\infty t\lambda e^{-\lambda t} dt \Big/ \int_0^\infty \lambda e^{-\lambda t} dt = \frac{1}{\lambda}\int_0^\infty x e^{-x} dx \Big/ \int_0^\infty e^{-x}\, dx = \frac{1}{\lambda}$$

6.215 We calculate λ first

$$\lambda = \frac{\ln 2}{T_{1/2}} = 9{\cdot}722 \times 10^{-3} \text{ per day}$$

Hence

fraction decaying in a month $= 1 - e^{-\lambda t} = 0{\cdot}253$

6.216 Here $N_0 \simeq \frac{1\,\mu g}{24\,g} \times 6{\cdot}023 \times 10^{23} = 2{\cdot}51 \times 10^{16}$

Also $\lambda = \frac{\ln 2}{T_{1/2}} = 0{\cdot}04621$ per hour

So the number of β rays emitted in one hour is

$$N_0(1 - e^{-\lambda}) = 1{\cdot}13 \times 10^{15}$$

6.217 If N_0 is the number of radionuclei present initially, then

$$N_1 = N_0(1 - e^{-t_1/\tau})$$

$$\eta N_1 = N_0(1 - e^{-t_2/\tau})$$

where $\eta = 2{\cdot}66$ and $t_2 = 3\,t_1$. Then

$$\eta = \frac{1 - e^{-t_2/\tau}}{1 - e^{-t_1/\tau}}$$

or $\eta - \eta e^{-t_1/\tau} = 1 - e^{-t_2/\tau}$

Substituting the values

$$1{\cdot}66 = 2{\cdot}66\, e^{-2/\tau} - e^{-6/\tau}$$

Put $e^{-2/\tau} = x$. Then

$$x^3 - 2{\cdot}66\,x + 1{\cdot}66 = 0$$

$$(x^2 - 1)\,x - 1{\cdot}66\,(x - 1) = 0$$

or $$(x-1)(x^2+x-1{\cdot}66) = 0$$

Now $$x \neq 1 \text{ so } x^2+x-1{\cdot}66 = 0$$

$$x = \frac{-1 \pm \sqrt{1+4\times 1{\cdot}66}}{2}$$

Negative sign has to be rejected as $x > 0$.

Thus $$x = 0{\cdot}882$$

This gives $$\tau = \frac{-2}{\ln 0{\cdot}882} = 15{\cdot}9 \text{ sec}.$$

6.218 If the half-life is T days

$$(2)^{-7/T} = \frac{1}{2{\cdot}5}$$

Hence $$\frac{7}{T} = \frac{\ln 2{\cdot}5}{\ln 2}$$

or $$T = \frac{7\ln 2}{\ln 2{\cdot}5} = 5{\cdot}30 \text{ days.}$$

6.219 The activity is proportional to the number of parent nuclei (assuming that the daughter is not radioactive). In half its half-life period, the number of parent nucli decreases by a factor

$$(2)^{-1/2} = \frac{1}{\sqrt{2}}$$

So activity decreases to $\dfrac{650}{\sqrt{2}} = 460$ particles per minute.

6.220 If the decay constant $\left(\text{in } (\text{hour})^{-1}\right)$ is λ, then the activity after one hour will decrease by a factor $e^{-\lambda}$. Hence

$$0{\cdot}96 = e^{-\lambda}$$

or $$\lambda = 1{\cdot}11\times 10^{-5}\,\text{s}^{-1} = 0{\cdot}0408 \text{ per hour}$$

The mean life time is 24·5 hour

6.221 Here $N_0 = \dfrac{1}{238}\times 6{\cdot}023\times 10^{23}$

$= 2{\cdot}531\times 10^{21}$

The activity is $$A = 1{\cdot}24\times 10^{4} \text{ dis/sec}.$$

Then $$\lambda = \frac{A}{N_0} = 4{\cdot}90\times 10^{-18} \text{ per sec}.$$

Hence the half life is

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = 4{\cdot}49\times 10^{9} \text{ years}$$

6.222 In old wooden atoms the number of C^{14} nuclei steadily decreases because of radioactive decay. (In live trees biological processes keep replenishing C^{14} nuclei maintaining a balance. This balance starts getting disrupted as soon as the tree is felled.)

If $T_{1/2}$ is the half life of C^{14} then $e^{-t \times \frac{\ln 2}{T_{1/2}}} = \frac{3}{5}$

Hence $$t = T_{1/2} \frac{\ln 5/3}{\ln 2} = 4105 \text{ years} \approx 4 \cdot 1 \times 10^3 \text{ years}$$

6.223 What this implies is that in the time since the ore was formed, $\frac{\eta}{1+\eta}$ U^{238} nuclei have remained undecayed. Thus

$$\frac{\eta}{1+\eta} = e^{-t \times \frac{\ln 2}{T_{1/2}}}$$

or $$t = T_{\frac{1}{2}} \frac{\ln \frac{1+\eta}{\eta}}{\ln 2}$$

Substituting $T_{\frac{1}{2}} = 4 \cdot 5 \times 10^9$ years, $\eta = 2 \cdot 8$

we get $$t = 1 \cdot 98 \times 10^9 \text{ years.}$$

6.224 The specific activity of Na^{24} is

$$\lambda \frac{N_A}{M} = \frac{N_A \ln 2}{M T_{\frac{1}{2}}} = 3 \cdot 22 \times 10^{17} \text{ dis/(gm.sec)}$$

Here M = molar weight of N_a^{24} = 24 gm, N_A is Avogadro number & $T_{\frac{1}{2}}$ is the half-life of Na^{24}.

Similarly the specific activity of U^{235} is

$$\frac{6 \cdot 023 \times 10^{23} \times \ln 2}{235 \times 10^8 \times 365 \times 86400}$$

$$= 0 \cdot 793 \times 10^5 \text{ dis/(gm–s)}$$

6.225 Let V = volume of blood in the body of the human being. Then the total activity of the blood is $A' V$. Assuming all this activity is due to the injected Na^{24} and taking account of the decay of this radionuclide, we get

$$V A' = A e^{-\lambda t}$$

Now $$\lambda = \frac{\ln 2}{15} \text{ per hour}, \quad t = 5 \text{ hour}$$

Thus $$V = \frac{A}{A'} e^{-\ln 2/3} = \frac{2 \cdot 0 \times 10^3}{(16/60)} e^{-\ln 2/3} \text{ cc} = 5 \cdot 95 \text{ litre}$$

6.226 We see that

Specific activity of the sample

$$= \frac{1}{M+M'} \{\text{Activity of } M \text{ gm of } Co^{58} \text{ in the sample}\}$$

Here M and M' are the masses of Co^{58} and Co^{59} in the sample. Now activity of M gm of Co^{58}

$$= \frac{M}{58} \times 6{\cdot}023 \times 10^{23} \times \frac{\ln 2}{71{\cdot}3 \times 86400} \text{ dis/sec}$$

$$= 1{\cdot}168 \times 10^{15} \text{ M}$$

Thus from the problem

$$1{\cdot}168 \times 10^{15} \frac{M}{M+M'} = 2{\cdot}2 \times 10^{12}$$

or $$\frac{M}{M+M'} = 1{\cdot}88 \times 10^{-3} \quad \text{i.e. } 0{\cdot}188\ \%$$

6.227 Suppose N_1, N_2 are the initial number of component nuclei whose decay constants are λ_1, λ_2 $\left(\text{in (hour)}^{-1}\right)$

Then the activity at any instant is

$$A = \lambda_1 N_1 e^{-\lambda_1 t} + \lambda_2 N_2 e^{-\lambda_2 t}$$

The activity so defined is in units dis/hour. We assume that data ln A given is of its natural logarithm. The daughter nuclei are assumed nonradioactive.

We see from the data that at large t the change in ln A per hour of elapsed time is constant and equal to $-0{\cdot}07$. Thus

$$\lambda_2 = 0{\cdot}07 \text{ per hour}$$

We can then see that the best fit to data is obtained by

$$A(t) = 51{\cdot}1\, e^{-0.66\, t} + 10{\cdot}0\, e^{-0{\cdot}07\, t}$$

[To get the fit we calculate $A(t)\, e^{0{\cdot}07\, t}$. We see that it reaches the constant value 10·0 at $t = 7, 10, 14, 20$ very nearly. This fixes the second term. The first term is then obtained by subtracting out the constant value 10·0 from each value of $A(t)\, e^{0{\cdot}07\, t}$ in the data for small t.]

Thus we get $\lambda_1 = 0{\cdot}66$ per hour

$$\left.\begin{aligned} T_1 &= 1{\cdot}05 \text{ hour} \\ T_2 &= 9{\cdot}9 \text{ hours} \end{aligned}\right\} \text{half–lives}$$

Ratio $$\frac{N_1}{N_2} = \frac{51{\cdot}1}{10{\cdot}0} \times \frac{\lambda_2}{\lambda_1} = 0{\cdot}54$$

The answer given in the book is misleading.

6.228 Production of the nucleus is governed by the equation

$$\frac{dN}{dt} = \underset{\substack{\uparrow \\ \text{supply}}}{g} - \underbrace{\lambda N}_{\text{decay}}$$

We see that N will approach a constant value $\frac{g}{\lambda}$. This can also be proved directly. Multiply by $e^{\lambda t}$ and write

$$\frac{dN}{dt} e^{\lambda t} + \lambda e^{\lambda t} N = g e^{\lambda t}$$

Then $$\frac{d}{dt}(N e^{\lambda t}) = g e^{\lambda t}$$

or $$N e^{\lambda t} = \frac{g}{\lambda} e^{\lambda t} + \text{const}$$

At $t = 0$ when the production is starteed, $N = 0$

$$0 = \frac{g}{\lambda} + \text{constant}$$

Hence $$N = \frac{g}{\lambda}(1 - e^{-\lambda t})$$

Now the activity is

$$A = \lambda N = g(1 - e^{-\lambda t})$$

From the problem

$$\frac{1}{2{\cdot}7} = 1 - e^{-\lambda t}$$

This gives $\lambda t = 0{\cdot}463$

so $$t = \frac{0{\cdot}463}{\lambda} = \frac{0{\cdot}463 \times T}{0{\cdot}693} = 9{\cdot}5 \text{ days}.$$

Algebraically $$t = -\frac{T}{\ln 2} \ln\left(1 - \frac{A}{g}\right)$$

6.229 (a) Suppose N_1 and N_2 are the number of two radionuclides A_1, A_2 at time t. Then

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \qquad (1)$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \qquad (2)$$

From (1)

$$N_1 = N_{10} e^{-\lambda_1 t}$$

where N_{10} is the initial number of nuclides A_1 at time $t = 0$

From (2)

$$\left(\frac{dN_2}{dt} + \lambda_2 N_2\right) e^{\lambda_2 t} = \lambda_1 N_{10}\, e^{-(\lambda_1 - \lambda_2) t}$$

or $$(N_2\, e^{\lambda_2 t}) = \text{const}\ \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2} e^{-(\lambda_1 - \lambda_2) t}$$

since $$N_2 = 0 \quad \text{at} \quad t = 0$$

Constant $$N_2 = \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2}$$

Thus $$= \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2} (e^{-\lambda_2 t} - e^{-\lambda_1 t})$$

(b) The activity of nuclide A_2 is $\lambda_2 N_2$. This is maximum when N_2 is maximum. That happens when $\frac{dN_2}{dt} = 0$

This requires $$\lambda_2\, e^{-\lambda_2 t_m} = \lambda_1 e^{-\lambda_1 t_m}$$

or $$t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$$

6.230 (a) This case can be obtained from the previous one on putting

$$\lambda_2 = \lambda_1 - \varepsilon$$

where ε is very small and letting $\varepsilon \to 0$ at the end. Then

$$N_2 = \frac{\lambda_1 N_{10}}{\varepsilon} (e^{\varepsilon t} - 1)\, e^{-\lambda_1 t} = \lambda_1 t\, e^{-\lambda_1 t} N_{10}$$

or dropping the subscript 1 as the two values are equal

$$N_2 = N_{10}\, \lambda\, t\, e^{-\lambda t}$$

(b) This is maximum when

$$\frac{dN_2}{dt} = 0 \quad \text{or} \quad t = \frac{1}{\lambda}$$

6.231 Here we have the equations

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \text{and} \quad \frac{dN_3}{dt} = \lambda_2 N_2$$

From problem 229

$$N_1 = N_{10}\, e^{-\lambda_1 t}$$

$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2} (e^{-\lambda_2 t} - e^{-\lambda_1 t})$$

Then $$\frac{dN_3}{dt} = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} N_{10}\, (e^{-\lambda_2 t} - e^{-\lambda_1 t})$$

or $$N_3 = \text{Const} - \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}\left(\frac{e^{-\lambda_2 t}}{\lambda_2} - \frac{e^{-\lambda_1 t}}{\lambda_1}\right) N_{10}$$

since $N_3 = 0$ initially

$$\text{Const} = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} N_{10}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

So $$N_3 = \frac{\lambda_1 \lambda_2 N_{10}}{\lambda_1 - \lambda_2}\left[\frac{1}{\lambda_2}(1 - e^{-\lambda_2 t}) - \frac{1}{\lambda_1}(1 - e^{-\lambda_1 t})\right]$$

$$= N_{10}\left[1 + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1}\right]$$

6.232 We have the chain

$$\underset{A_1}{B_i^{210}} \xrightarrow[\lambda_1]{\beta} \underset{A_2}{P_0^{210}} \xrightarrow[\lambda_2]{\alpha} \underset{A_3}{P_b^{206}}$$

of the previous problem initially

$$N_{10} = \frac{10^{-3}}{210} \times 6{\cdot}023 \times 10^{23} = 2{\cdot}87 \times 10^{18}$$

A month after preparation

$$N_1 = 4{\cdot}54 \times 10^{16}$$

$$N_2 = 2{\cdot}52 \times 10^{18}$$

using the results of the previous problem.

Then $$A_\beta = \lambda_1 N_1 = 0{\cdot}725 \times 10^{11} \text{ dis/sec}$$

$$A_\alpha = \lambda_2 N_2 = 1{\cdot}46 \times 10^{11} \text{ dis/sec}$$

6.233 (a) Ra has $Z = 88$, $A = 226$

After 5 α emission and 4 β (electron) emission

$$A = 206$$

$$Z = 88 + 4 - 5 \times 2 = 82$$

The product is ${}^{82}Pb^{206}$

(b) We require

$$-\Delta Z = 10 = 2n - m$$

$$-\Delta A = 32 = n \times 4$$

Here n = no. of α emissions

m = no. of β emissions

Thus $n = 8$, $m = 6$

6.234 The momentum of the α-particle is

$\sqrt{2 M_\alpha T}$. This is also the recoil momentum of the daughter nuclear in opposite direction.

The recoil velocity of the daughter nucleus is

$$\frac{\sqrt{2 M_\alpha T}}{M_d} = \frac{2}{196}\sqrt{\frac{2T}{M_p}} = 3{\cdot}39 \times 10^5 \text{ m/s}$$

The energy of the daughter nucleus is $\frac{M_\alpha}{M_d} T$ and this represents a fraction

$$\frac{M_\alpha / M_d}{1 + \frac{M_\alpha}{M_d}} = \frac{M_\alpha}{M_\alpha + M_d} = \frac{4}{200} = \frac{1}{50} = 0{\cdot}02$$

of total energy. Here M_d is the mass of the daughter nucleus.

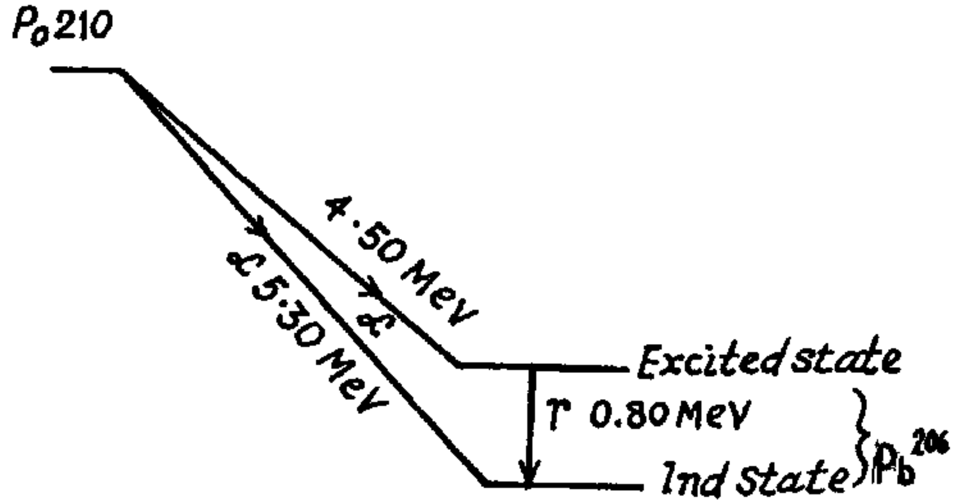
6.235 The number of nuclei initially present is

$$\frac{10^{-3}}{210} \times 6{\cdot}023 \times 10^{23} = 2{\cdot}87 \times 10^{18}$$

In the mean life time of these nuclei the number decaying is the fraction $1 - \frac{1}{e} = 0{\cdot}632$. Thus the energy released is

$2{\cdot}87 \times 10^{18} \times 0{\cdot}632 \times 5{\cdot}3 \times 1{\cdot}602 \times 10^{-13}$ J.= 1·54 M J

6.236 We neglect all recoil effects. Then the following diagram gives the energy of the gamma ray



6.237 (a) For an alpha particle with initial K.E. 7·0 MeV, the initial velocity is

$$v_0 = \sqrt{\frac{2T}{M_\alpha}}$$

$$= \sqrt{\frac{2 \times 7 \times 1{\cdot}602 \times 10^{-6}}{4 \times 1{\cdot}672 \times 10^{-24}}}$$

$$= 1{\cdot}83 \times 10^9 \text{ cm/sec}$$

Thus $\qquad R = 6{\cdot}02$ cm

(b) Over the whole path the number of ion pairs is

$$\frac{7 \times 10^6}{34} = 2{\cdot}06 \times 10^5$$

Over the first half of the path :- We write the formula for the mean path as $R \alpha E^{3/2}$ where E is the initial energy. Thus if the energy of the α–particle after traversing the first half of the path is E_1 *then*

$$R_0 E_1^{3/2} = \frac{1}{2} R_0 E_0^{3/2} \quad \text{or} \quad E_1 = 2^{-2/3} E_0$$

Hence number of ion pairs formed in the first half of the path length is

$$\frac{E_0 - E_1}{34\, e\, V} = (1 - 2^{-2/3}) \times 2{\cdot}06 \times 10^5 = 0{\cdot}76 \times 10^5$$

6.238 In β^- decay

$$_Z X^A \rightarrow {}_{Z+1}Y^A + e^- + Q$$

$$Q = (M_x - M_y - m_e)\, c^2$$

$$= [(M_x + Z\, m_e) - (M_y + Z\, m_e + m_e)]\, c^2$$

$$= (M_p - M_d)\, c^2$$

since M_p, M_d are the masses of the atoms. The binding energy of the electrons in ignored.

In K capture

$$e_K^- + {}_Z X^A \rightarrow {}_{Z-1}Y^A + Q$$

$$Q = (M_X - M_Y)\, c^2 + m_e c^2$$

$$= (M_x^{c^2} + Z\, m_e c^2) - (M_Y c^2 + (Z-1)\, m_e c^2)$$

$$= c^2 (M_p - M_d)$$

In β^+ decay $\quad {}_Z X^A \rightarrow {}_{Z-1}Y^A + e^+ + Q$

Then $\quad Q = (M_x - M_y - m_e)\, c^2$

$$= [M_x + Z\, m_e]\, c^2 - [M_y + (Z-1)\, m_e]\, c^2 - 2\, m_e c^2$$

$$= (M_p - M_d - 2\, m_e)\, c^2$$

6.239 The reaction is $\quad Be^{10} \rightarrow B^{10} + e^- + \bar{\nu}_e$

For maximum K.E. of electrons we can put the energy of $\bar{\nu}_e$ to be zero. The atomic masses are

$$Be^{10} = 10{\cdot}016711 \text{ amu}$$

$$B^{10} = 10{\cdot}016114 \text{ amu}$$

So the K.E. of electrons is (see previous problem)

$$597 \times 10^{-6} \text{ amu} \times c^2 = 0{\cdot}56 \text{ MeV}$$

The mòmentum of electrons with this K.E. is $0{\cdot}941\,\frac{MeV}{c}$

and the recoil energy of the daughter is

$$\frac{(0{\cdot}941)^2}{2\times M_d c^2} = \frac{(0{\cdot}941)^2}{2\times 10\times 938}\,\text{MeV} = 47{\cdot}2\,\text{eV}$$

6.240 The masses are

$$Na^{24} = 24 - 0{\cdot}00903\,\text{amu} \quad \text{and } Mg^{24} = 24 - 0{\cdot}01496\,\text{amu}$$

The reaction is

$$Na^{24} \rightarrow Mg^{24} + e^- + \bar{\nu}_e$$

The maximum K.E. of electrons is

$$0{\cdot}00593 \times 93\,\text{MeV} = 5{\cdot}52\,\text{MeV}$$

Average K.E. according to the problem is then $\frac{5{\cdot}52}{3} = 1{\cdot}84\,\text{MeV}$

The initial number of Na^{24} is

$$\frac{10^{-3}\times 6{\cdot}023\times 10^{23}}{24} = 2{\cdot}51\times 10^{19}$$

The fraction decaying in a day is

$$1-(2)^{-24/15} = 0{\cdot}67$$

Hence the heat produced in a day is

$0{\cdot}67\times 2{\cdot}51\times 10^{19}\times 1{\cdot}84\times 1{\cdot}602\times 10^{-13}$ Joul $= 4{\cdot}95$ MJ

6.241 We assume that the parent nucleus is at rest. Then since the daughter nucleus does not recoil, we have

$$\vec{p} = -\vec{p}_\nu$$

i.e. positron & ν mometum are equal and opposite. On the other hand

$\sqrt{c^2p^2+m_e^2c^4} + cp = Q$ = total energy released. (Here we have used the fact that energy of the neutrino is $c\,|\vec{p}_\nu| = cp$)

Now $\quad Q = [\,(\text{Mass of } C^{11} \text{ nucleus}) - (\text{Mass of } B^{11} \text{nucleus})\,]\,c^2$

$= [\,\text{Mass of } C^{11} \text{ atom} - \text{Mass of } B^{11} \text{ atom} - m_e\,]\,c^2$

$= 0{\cdot}00213\,\text{amu}\times c^2 - m_e c^2$

$= (0{\cdot}00213\times 931 - 0{\cdot}511)\,\text{MeV} = 1{\cdot}47\,\text{MeV}$

Then $\quad c^2p^2+(0{\cdot}511)^2 = (1{\cdot}47-cp)^2 = (1{\cdot}47)^2 - 2{\cdot}94\,cp + c^2p^2$

Thus $cp = 0{\cdot}646$ MeV = energy of neutrino

Also K.E. of electron $= 1{\cdot}47 - 0{\cdot}646 - 0{\cdot}511 = 0{\cdot}313$ MeV

6.242 The K.E. of the positron is maximum when the energy of neutrino is zero. Since the recoil energy of the nucleus is quite small, it can be calculated by successive approximation. The reaction is

$$N^{13} \rightarrow C^{13} + e^+ + \nu_e .$$

The maximum energy available to the positron (including its rest energy) is

$$c^2 \,(\text{Mass of } N^{13} \text{nucleus - Mass of } C^{13} \text{ nucleus})$$
$$= c^2 \,(\text{Mass of } N^{13} \text{ atom - Mass of } C^{13} \text{ atom } - m_e)$$
$$= 0{\cdot}00239\, c^2 - m_e c^2$$
$$= (0{\cdot}00239 \times 931 - 0{\cdot}511)\text{ MeV}$$
$$= 1{\cdot}71 \text{ MeV}$$

The momentum corresponding to this energy is $1{\cdot}636\text{ MeV/c}$.

The recoil energy of the nucleus is then

$$E = \frac{p^2}{2M} = \frac{(1{\cdot}636)^2}{2 \times 13 \times 931} = 111 \text{ eV} = 0{\cdot}111 \text{ keV}$$

on using $Mc^2 = 13 \times 931 \text{ MeV}$

6.243 The process is

$$e_k^- + Be^7 \rightarrow Li^7 + \nu$$

The energy available in the process is

$$Q = c^2 (\text{Mass of } Be^7 \text{ atom - Mass of } Li^7 \text{ atom})$$
$$= 0{\cdot}00092 \times 931 \text{ MeV} = 0{\cdot}86 \text{ MeV}$$

The momentum of a K electron is negligible. So in the rest frame of the Be^7 atom, most of the energy is taken by neutrino whose momentum is very nearly $0{\cdot}86\text{ MeV/c}$

The momentum of the recoiling nucleus is equal and opposite. The velocity of recoil is

$$\frac{0{\cdot}86\text{ MeV/c}}{M_{Li}} = c \times \frac{0{\cdot}86}{7 \times 931} = 3{\cdot}96 \times 10^6 \text{ cm/s}$$

6.244 In internal conversion, the total energy is used to knock out K electrons. The K.E. of these electrons is energy available-B.E. of K electrons

$$= (87 - 26) = 61 \text{ keV}$$

The total energy including rest mass of electrons is $0{\cdot}511 + 0{\cdot}061 = 0{\cdot}572\text{ MeV}$

The momentum corresponding to this total energy is

$\sqrt{(0{\cdot}572)^2 - (0{\cdot}511)^2} \,/\, c = 0257\text{ MeV/c}.$

The velocity is then $\quad \dfrac{c^2 p}{E} = c \times \dfrac{0{\cdot}257}{0{\cdot}572} = 0{\cdot}449\, c$

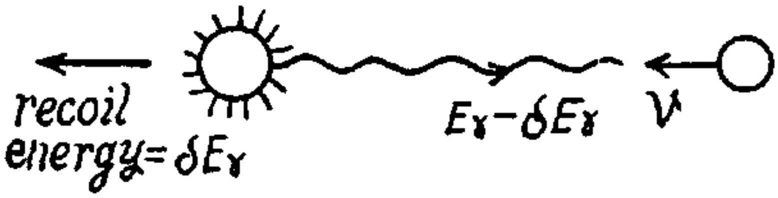
6.245 With recoil neglected, the γ–quantrum will have 129 keV energy. To a first approximation, its momentrum will be 129 keV/c and the energy of recoil will be

$$\frac{(0{\cdot}129)^2}{2\times191\times931}\text{MeV} = 4{\cdot}18\times10^{-8}\,\text{MeV}$$

In the next approximation we therefore write $E_\gamma \simeq {\cdot}129 - 8{\cdot}2\times10^{-8}\,\text{MeV}$

Therefore
$$\frac{\delta E_\gamma}{E_\gamma} = 3{\cdot}63\times10^{-7}$$

6.246 For maximum (resonant) absorption, the absorbing nucleus must be moving with enough speed to cancel the momentum of the oncoming photon and have just right energy (ε = 129 keV) available for transition to the excited state.



Since $\delta E_\gamma \simeq \dfrac{\varepsilon^2}{2Mc^2}$ and momentum of the photon is $\dfrac{\varepsilon}{c}$, these condition can be satisfied if the velocity of the nucleus is

$$\frac{\varepsilon}{Mc} = c\frac{\varepsilon}{Mc^2} = 218\,\text{m/s} = 0{\cdot}218\,\text{k m/s}$$

6.247 Because of the gravitational shift the frequency of the gamma ray at the location of the absorber is increased by

$$\frac{\delta\omega}{\omega} = \frac{gh}{c^2}$$

For this to be compensated by the Doppler shift (assuming that resonant absorption is possible in the absence of gravitational field) we must have

$$\frac{gh}{c^2} = \frac{v}{c} \quad \text{or} \quad v = \frac{gh}{c} = 0{\cdot}65\,\mu\,\text{m/s}$$

6.248 The natural life time is

$$\Gamma = \frac{\hbar}{\tau} = 4{\cdot}7\times10^{-10}\,\text{eV}$$

Thus the condition $\delta E_\gamma \geq \Gamma$ implies $\dfrac{gh}{c^2} \geq \dfrac{\Gamma}{\varepsilon} = \dfrac{\hbar}{\tau\varepsilon}$

or
$$h \geq \frac{c^2\hbar}{\tau\varepsilon g} = 4{\cdot}64\,\text{metre}$$

(h here is height of the place, not planck's constant.)