JEE ADVANCED

Single Correct Answer Type

1. The least value of the expression $2\log_{10} x - \log_x(0.01)$, for $x > 1$, is

a. 10 **b.** 2
c. -0.01 **d.** 4 **(IIT-JEE 1980)**

2. The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$ is

a. 4 **b.** 1
c. 3 **d.** 2 **(IIT-JEE 1982)**

3. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is

a. $-4 < x \le 0$ **b.** $0 < x < 1$
c. $-100 < x < 100$ **d.** $-\infty < x < \infty$
(IIT-JEE 1982)

4. If x satisfies $|x-1| + |x-2| + |x-3| \ge 6$, then

a. $0 \le x \le 4$ **b.** $x \le -2$ or $x \ge 4$
c. $x \le 0$ or $x \ge 4$ **d.** None of these
(IIT-JEE 1983)

5. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval

a. $\left[\frac{1}{2}, 2\right]$ **b.** $[-1, 2]$
c. $\left[-\frac{1}{2}, 1\right]$ **d.** $\left[-1, \frac{1}{2}\right]$ **(IIT-JEE 1984)**

6. If $\ln(a+c)$, $\ln(a-c)$, $\ln(a-2b+c)$ are in A.P., then

a. a, b, c are in A.P. **b.** a^2, b^2, c^2 are in A.P.
c. a, b, c are in G.P. **d.** a, b, c are in H.P.
(IIT-JEE 1994)

7. The set of all real numbers x for which $x^2 - |x+2| + x > 0$ is

a. $(-\infty, -2)$ **b.** $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
c. $(-\infty, -1) \cup (1, \infty)$ **d.** $(\sqrt{2}, \infty)$
(IIT-JEE 2002)

8. Let (x_0, y_0) be the solution of the following equations:

$(2x)^{\ln 2} = (3y)^{\ln 3}$;
$3^{\ln x} = 2^{\ln y}$

Then x_0 is

a. $\frac{1}{6}$ **b.** $\frac{1}{3}$
c. $\frac{1}{2}$ **d.** 6 **(IIT-JEE 2011)**

Multiple Correct Answers Type

1. If S is the set of all real x such that $\frac{2x-1}{2x^3+3x^2+x}$ is positive, then S contains

a. $\left(-\infty, -\frac{3}{2}\right)$ **b.** $\left(-\frac{3}{2}, -\frac{1}{4}\right)$
c. $\left(-\frac{1}{4}, \frac{1}{2}\right)$ **d.** $\left(\frac{1}{2}, 3\right)$
e. None of these **(IIT-JEE 1986)**

2. The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has

a. at least one real solution

b. exactly three solutions

c. exactly one irrational solution

d. complex roots **(IIT-JEE 1989)**

3. If $3^x = 4^{x-1}$, then $x =$

a. $\dfrac{2\log_3 2}{2\log_3 2 - 1}$ b. $\dfrac{2}{2 - \log_2 3}$

c. $\dfrac{1}{1 - \log_4 3}$ d. $\dfrac{2\log_2 3}{2\log_2 3 - 1}$

(JEE Advanced 2013)

Matching Column Type

1. Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$

Match the expressions/statements in Column I with expressions/statements in Column II.

	Column I	Column II
(a)	If $-1 < x < 1$, then $f(x)$ satisfies	(p) $0 < f(x) < 1$
(b)	If $1 < x < 2$, then $f(x)$ satisfies	(q) $f(x) < 0$
(c)	If $3 < x < 5$, then $f(x)$ satisfies	(r) $f(x) > 0$
(d)	If $x > 5$, then $f(x)$ satisfies	(s) $f(x) < 1$

(IIT-JEE 2007)

2. Match the statements/expressions in Column I with the statements/expressions in Column II.

Column I	Column II
(a) The minimum value of $\dfrac{x^2+2x+4}{x+2}$ is	(p) 0
(b) Let A and B be 3×3 matrices of real numbers, where A is symmetric, B is skew symmetric, and $(A + B)(A - B) = (A - B)(A + B)$. If $(AB)' = (-1)^k AB$, where $(AB)'$ is the transpose of the matrix AB, then the possible values of k are	(q) 1
(c) Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than	(r) 2
(d) If $\sin\theta = \cos\phi$, then the possible values of $\dfrac{1}{\pi}\left(\theta \pm \phi - \dfrac{\pi}{2}\right)$ are	(s) 3

(IIT-JEE 2008)

3. Match the statements/expressions given in Column I with the values given in Column II.

Column I	Column II
(a) The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \dfrac{\pi}{2}\right)$	(p) 1
(b) Value(s) of k for which the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	(q) 2
(c) Value(s) of k for which $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x+1\rvert + \lvert x+2\rvert = 4k$ has integer solution(s)	(r) 3
(d) If $y' = y + 1$ and $y(0) = 1$, then value(s) of $y(\log_e 2)$	(s) 4
	(t) 5

(IIT-JEE 2009)

Integer Answer Type

1. The value of

$$6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\cdots}}}\right) \text{ is}$$

(IIT-JEE 2012)

Fill in the Blanks

1. The solution of the equation $\log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0$ is ________. **(IIT-JEE 1986)**

2. The sum of all real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is ________. **(IIT-JEE 1997)**

True/False Type

1. For $0 < a < x$, the minimum value of the function $\log_a x + \log_x a$ is 2. **(IIT-JEE 1984)**

Subjective Type

1. Find all integers x for which $(5x - 1) < (x + 1)^2 < (7x - 3)$. **(IIT-JEE 1978)**

2. Solve for x: $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$. **(IIT-JEE 1978)**

3. Solve the following equation for x: $2\log_x a + \log_{ax} a + 3\log_{a^2x} a = 0$, $a > 0$. **(IIT-JEE 1978)**

4. Show that for any triangle with sides a, b and c, $3(ab + bc + ca) < (a + b + c)^2 < 4(bc + ca + ab)$. When are the first two expressions equal? **(IIT-JEE 1979)**

5. For what values of m, does the system of equations $3x + my = m$, $2x - 5y = 20$ has solution satisfying the conditions $x > 0$, $y > 0$. **(IIT-JEE 1980)**

6. Find the solution set of the system

$x + 2y + z = 1;$

$2x - 3y - w = 2;$

$x \ge 0;\ y \ge 0;\ z \ge 0;\ w \ge 0.$ **(IIT-JEE 1980)**

7. Let $y = \sqrt{\dfrac{(x+1)(x-3)}{(x-2)}}$. Find all the real values of x for which y takes real values. **(IIT-JEE 1980)**

8. Show that the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real solution. **(IIT-JEE 1982)**

9. Find all real values of x which satisfy $x^2 - 3x + 2 > 0$ and $x^2 - 2x - 4 \le 0$. **(IIT-JEE 1983)**

10. Find the set of all x for which $\dfrac{2x}{(2x^2+5x+2)} > \dfrac{1}{(x+1)}$. **(IIT-JEE 1987)**

11. Solve $|x^2 + 4x + 3| + 2x + 5 = 0$. **(IIT-JEE 1988)**

12. If $\log_3 2$, $\log_3 (2^x - 5)$, and $\log_3\left(2^x - \dfrac{7}{2}\right)$ are in arithmetic progression, determine the value of x. **(IIT-JEE 1990)**

Answer Key

JEE Advanced

Single Correct Answer Type

1. d. 2. a. 3. d. 4. c. 5. c.
6. d. 7. b. 8. c.

Multiple Correct Answers Type

1. a., d. 2. a., b., c. 3. a., b., c.

Matching Column Type

1. (a) – (p), (r), (s); (b) – (q), (s); (c) – (q), (s); (d) – (p), (r), (s)
2. (c) – (r), (s)
3. (c) – (q), (r), (s), (t)

Integer Answer Type

1. 4

Fill in the Blanks Type

1. 4 2. 4

True/False Type

1. False

Subjective Type

1. 3
2. $x = 3/2$
3. $\begin{cases} x > 0, \ne 1, & \text{if } a = 1 \\ x = a^{-1/2}, a^{-4/3}, & \text{if } a > 0, \ne 1 \end{cases}$
4. $a = b = c$
5. $m \in \left(-\infty, \dfrac{-15}{2}\right) \cup (30, \infty)$
6. $x = 1, y = 0, z = 0, \omega = 0$
7. $[-1, 2) \cup [3, \infty)$ 8. $[-1, 1) \cup (2, 4]$
10. $(-2, -1) \cup (-2/3, -1/2)$ 11. $-4, -1-\sqrt{3}$
12. $x = 3$

Hints and Solutions

JEE Advanced

Single Correct Answer Type

1. d. $2 \log_{10} x - \log_x 0.01$

$$= 2\log_{10}x - \frac{\log_{10}0.01}{\log_{10}x}$$

$$= 2\log_{10}x + \frac{2}{\log_{10}x}$$

$$= 2\left[\log_{10}x + \frac{1}{\log_{10}x}\right]$$

[Here, $x > 1 \Rightarrow \log_{10} x > 0$]

$$= 2\left[\left(\sqrt{\log_{10}x} - \sqrt{\frac{1}{\log_{10}x}}\right)^2 + 2\right]$$

≥ 4

2. a. $|x|^2 - 3|x| + 2 = 0$

or $(|x| - 2)(|x| - 1) = 0$

$\Rightarrow$ $|x| = 1$ or 2

$\Rightarrow$ $x = \pm 1, \pm 2$

Therefore, there are 4 real solutions.

3. d. Given expression $x^{12} - x^9 + x^4 - x + 1 = f(x)$

For $x < 0$ put $x = -y$, where $y > 0$

Then we get

$f(x) = y^{12} + y^9 + y^4 + y + 1 > 0$ for $y > 0$

For $0 < x < 1$, $x^9 < x^4 \Rightarrow -x^9 + x^4 > 0$

Also $1 - x > 0$ and $x^{12} > 0$

$\Rightarrow$ $x^{12} - x^9 + x^4 + 1 - x > 0 \Rightarrow f(x) > 0$

For $x > 1$, $f(x) = x(x^3 - 1)(x^8 + 1) + 1 > 0$

So $f(x) > 0$ for $-\infty < x < \infty$

4. c. Let $|x - 1| + |x - 2| + |x - 3| < 6$

$\therefore$ $|(x - 1) + (x - 2) + (x - 3)|$

$< |x - 1| + |x - 2| + |x - 3| < 6$

or $|3x - 6| < 6$

or $|x - 2| < 2$

or $-2 < x - 2 < 2$

or $0 < x < 4$

Hence, for $|x - 1| + |x - 2| + |x - 3| \geq 6$, $x \leq 0$ or $x \geq 4$

5. c. Given that $a^2 + b^2 + c^2 = 1$ (1)

we know $(a + b + c)^2 \geq 0$

$\Rightarrow$ $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \geq 0$

$\Rightarrow$ $2(ab + bc + ca) \geq -1$ [Using (1)]

$\Rightarrow$ $ab + bc + ca \geq -1/2$ (2)

Also we know that, $\frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] \geq 0$

$\Rightarrow$ $a^2 + b^2 + c^2 - ab - bc - ca \geq 0$

$\Rightarrow$ $ab + bc + ca \leq 1$ (3) [Using (1)]

Combining (2) and (3), we get

$-1/2 \leq ab + bc + ca \leq 1$

$\Rightarrow$ $ab + bc + ca \in [-1/2, 1]$

6. d. $\ln(a + c)$, $\ln(a - c)$, $\ln(a - 2b + c)$ are in A.P.

$\Rightarrow$ $2\ln(a - c) = \ln(a + c) + \ln(a - 2b + c)$

$\Rightarrow$ $(a - c)^2 = (a + c)(a - 2b + c)$

$= (a + c)^2 - 2b(a + c)$

or $2b(a + c) = (a + c)^2 - (a - c)^2$

$= 4ac$

or $b = \frac{2ac}{a + c}$

Hence, a, b, c are in H.P.

7. b. $x^2 - |x + 2| + x > 0$

Case (i): $x + 2 \geq 0$ or $x \geq -2$

$\therefore$ $x^2 - x - 2 + x > 0$

or $x^2 - 2 > 0$

$\Rightarrow$ $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$\Rightarrow$ $x \in [-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ $(\because x \geq -2)$ (1)

Case (ii): $x + 2 < 0$ or $x < -2$

$\therefore$ $x^2 + x + 2 + x > 0$

or $x^2 + 2x + 2 > 0$, which is true $\forall x \in R$

$\therefore$ $x < -2$ (2)

From (1) and (2), $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Alternative solution:

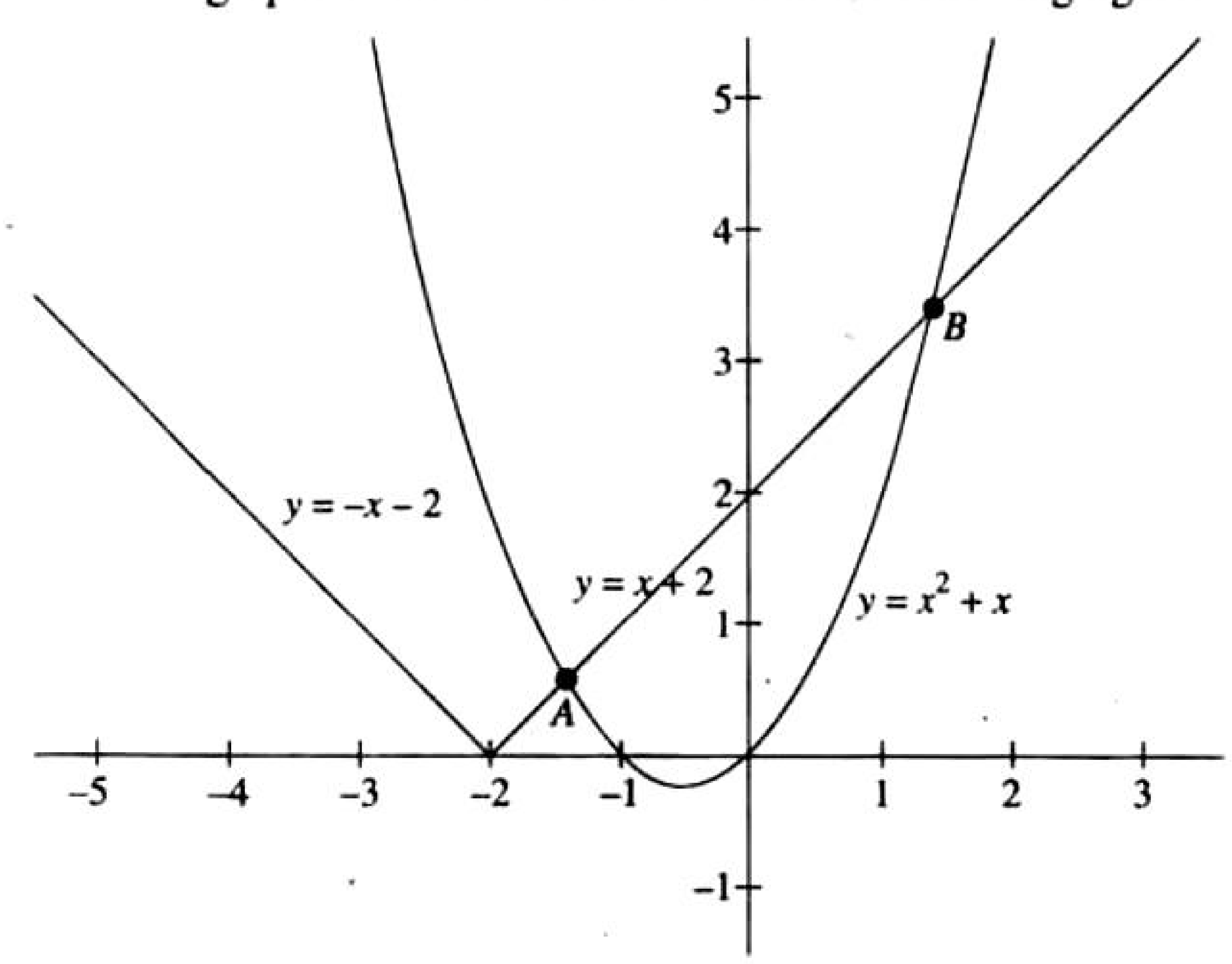
We have $x^2 - |x + 2| + x > 0$

or $x^2 + x > |x + 2|$

Now draw the graphs of $y = x^2 + x$ and $y = |x + 2|$

Graph of $y = x^2 + x = x(x + 1)$ is upward parabola which intersects x-axis at $(0, 0)$ and $(-1, 0)$.

The graphs of the curves are shown in the following figure.



As shown in figures graphs intersect at points A and B.

For point of intersection, we solve $x^2 + x = x + 2$

which gives $x = \pm\sqrt{2}$

For $x^2 + x > |x + 2|$, graph of $y = x^2 + x$ must lie above the graph of $y = |x + 2|$.

From the figure, this occurs for $x <$ abscissa of point A or $x >$ abscissa of point B.

Thus, $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

8. c. $(2x)^{\ln 2} = (3y)^{\ln 3}$ (i)

$3^{\ln x} = 2^{\ln y}$ (ii)

In Eq. (ii) taking log on both sides, we get

$\Rightarrow$ $(\log x)(\log 3) = (\log y)\log 2$

$$\Rightarrow \quad \log y = \frac{(\log x)(\log 3)}{\log 2} \qquad \text{(iii)}$$

In Eq. (i), taking log on both sides, we get

$(\log 2)\{\log 2 + \log x\} = \log 3\{\log 3 + \log y\}$

$$(\log 2)^2 + (\log 2)(\log x) = (\log 3)^2 + \frac{(\log 3)^2(\log x)}{\log 2}$$

[from (iii)]

$$\text{or} \quad (\log 2)^2 - (\log 3)^2 = \frac{(\log 3)^2 - (\log 2)^2}{\log 2}(\log x)$$

or $\quad -\log 2 = \log x$

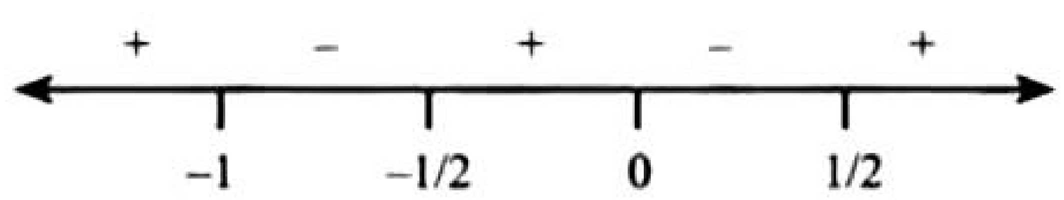
$$\Rightarrow \quad x = \frac{1}{2} \Rightarrow x_0 = \frac{1}{2}.$$

Multiple Correct Answers Type

1. a., d. We have $f(x) = \dfrac{2x-1}{2x^3+3x^2+x} = \dfrac{2x-1}{x(2x+1)(x+1)}$

For critical points, $x = 1/2, 0, -1/2, -1$

On the number line by sign scheme method, we have



For $f(x) > 0$, when $x \in (-\infty, -1) \cup (-1/2, 0) \cup (1/2, \infty)$.

Clearly, S contains $(-\infty, -3/2)$ and $(1/2, 3)$.

Therefore, (a) and (d) are the correct answers.

2. a., b., c. $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$

Taking log on both sides with base 2.

$$\therefore \quad (\log_2 x)\left(\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}\right) = \log_2\sqrt{2}$$

$$\text{or} \quad t\left(\frac{3}{4}t^2 + t - \frac{5}{4}\right) = \frac{1}{2}$$

or $\quad 3t^3 + 4t^2 - 5t - 2 = 0$

or $\quad (t-1)(3t^2 + 7t + 2) = 0$

or $\quad (t-1)(3t+1)(t+2) = 0$

or $\quad t = \log_2 x = 1, -2, -1/3$

$\Rightarrow \quad x = 2, 2^{-2}, 2^{-1/3}$

3. a., b., c. $3^x = 4^{x-1} \Rightarrow \log_2 3^x = (x-1)\log_2 4 = 2(x-1)$

or $\quad x \log_2 3 = 2x - 2$

$$\text{or} \quad x = \frac{2}{2 - \log_2 3}$$

Rearranging, we get

$$x = \frac{2}{2 - \dfrac{1}{\log_3 2}} = \frac{2\log_3 2}{2\log_3 2 - 1}$$

Rearranging again, we get

$$x = \frac{\log_3 4}{\log_3 4 - 1} = \frac{\dfrac{1}{\log_4 3}}{\dfrac{1}{\log_4 3} - 1} = \frac{1}{1 - \log_4 3}.$$

Matching Column Type

1. (a) – (p), (r), (s)

We have $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6} = \dfrac{(x-5)(x-1)}{(x-2)(x-3)}$

If $-1 < x < 1$ then $f(x) = \dfrac{(-\text{ve})(-\text{ve})}{(-\text{ve})(-\text{ve})} = +\text{ve} \therefore f(x) > 0$

Also $f(x) - 1 = \dfrac{-x-1}{x^2 - 5x + 6} = -\dfrac{(x+1)}{(x-2)(x-3)}$

for $-1 < x < 1, f(x) - 1 = \dfrac{-(+\text{ve})}{(-\text{ve})(-\text{ve})} = -\text{ve}$

$\Rightarrow f(x) - 1 < 0 \Rightarrow f(x) < 1$

$\therefore 0 < f(x) < 1$

(b) – (q), (s)

if $1 < x < 2$ then $f(x) = \dfrac{(-\text{ve})(+\text{ve})}{(-\text{ve})(-\text{ve})} = -\text{ve}$

$\therefore f(x) < 0$ and so $f(x) < 1$

(c) – (q), (s)

If $3 < x < 5$ then

$$f(x) = \frac{(-\text{ve})(+\text{ve})}{(+\text{ve})(+\text{ve})} = -\text{ve}$$

$\therefore \ f(x) < 0$ and so $f(x) < 1$

(d) – (p), (r), (s)

For $x > 5, f(x) > 0$

Also $f(x) - 1 = \dfrac{-(x+1)}{(x-2)(x-5)} < 0$ for $x > 5$

$\Rightarrow f(x) < 1 \therefore 0 < f(x) < 1$

2. (c) – (r), (s)

$a = \log_3 \log_3 2$

$\Rightarrow \quad 3^a = \log_3 2$

$\Rightarrow \quad 3^{-a} = \log_2 3$

Now, $1 < 2^{(-k+3^{-a})} < 2$

$\Rightarrow \quad 1 < 2^{-k}2^{3^{-a}} < 2$

$\Rightarrow \quad 1 < 2^{-k}2^{\log_2 3} < 2$

$\Rightarrow \quad 1 < 3 \cdot 2^{-k} < 2$

$$\Rightarrow \quad \frac{1}{3} < 2^{-k} < \frac{2}{3}$$

$$\Rightarrow \quad \frac{3}{2} < 2^k < 3$$

$$\Rightarrow \quad \log_2\left(\frac{3}{2}\right) < k < \log_2 3$$

$\Rightarrow \quad k = 1$ or $k < 2$ and $k < 3$

Note: Solutions for the remaining parts are given in their respective chapters.

3. (c) – (q), (r), (s), (t)

We have $f(x) = |x-1| + |x-2| + |x+1| + |x+2| = 4k$

Clearly, for any integral value of x, $f(x)$ takes even integral value.

Also, least value of $f(x)$ is 6 for $-1 \le x \le 1$.

Thus, the possible values of k are 2, 3, 4 and 5.

Note: Solutions for the remaining parts are given in their respective chapters.

Integer Answer Type

1. (4) Let $\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\cdots}}} = y$

So, $4-\frac{1}{3\sqrt{2}}y = y^2 \qquad (y > 0)$

or $\quad y^2+\frac{1}{3\sqrt{2}}y-4=0$

or $\quad y=\frac{8}{3\sqrt{2}}$

So, the required value of expression

$= 6+\log_{3/2}\left(\frac{1}{3\sqrt{2}}\times\frac{8}{3\sqrt{2}}\right)$

$= 6+\log_{\frac{3}{2}}\frac{4}{9} = 6-2=4.$

Fill in the Blanks Type

1. $\log_7\log_5(\sqrt{x+5}+\sqrt{x})=0$

or $\quad \log_5(\sqrt{x+5}+\sqrt{x})=1$

or $\quad \sqrt{x+5}+\sqrt{x}=5$

or $\quad x+5=25+x-10\sqrt{x}$

or $\quad 2=\sqrt{x}$

or $\quad x=4$, which satisfies the given equation.

2. $|x-2|^2+|x-2|-2=0$

$\Rightarrow \quad (|x-2|+2)(|x-2|-1)=0$

$\Rightarrow \quad |x-2|-1=0$

$\Rightarrow \quad x-2=\pm 1$

$\Rightarrow \quad x=1, 3$

Therefore, the sum of the roots is $3+1=4$

True/False Type

1. **False**

Given that $0 < a < x$

Let $f(x)=\log_a x+\log_x a=\log_a x+\frac{1}{\log_a x}$

Consider $g(y)=y+\frac{1}{y}$, where $\log_a x=y$

$\therefore \quad y+\frac{1}{y}=\left(\sqrt{y}-\frac{1}{\sqrt{y}}\right)^2+2\ge 2$

But equality holds when $y=1$.

$\Rightarrow \quad x=a$ which is not possible

$\therefore \quad y+\frac{1}{y}>2$

i.e., g_{min} can not be 2.

Hence, f_{min} can not be 2.

Therefore, statement is false.

Subjective Type

1. There are two parts: $(5x-1)<(x+1)^2$ and $(x+1)^2<(7x-3)$

Taking first part, we have

$(5x-1)<(x+1)^2$

or $\quad 5x-1<x^2+2x+1$

or $\quad x^2-3x+2>0$

or $\quad (x-1)(x-2)>0$

$\Rightarrow \quad x<1$ or $x>2 \qquad (1)$

Taking second part, we have

$(x+1)^2<(7x-3)$

or $\quad x^2-5x+4<0$

or $\quad (x-1)(x-4)<0$

or $\quad 1<x<4 \qquad (2)$

From (1) and (2) taking common values of x, we get $2<x<4$.

Then integral value of x is 3 only.

2. $4^x-3^{x-\frac{1}{2}}=3^{x+\frac{1}{2}}-2^{2x-1}$

or $\quad 4^x-\frac{3^x}{\sqrt{3}}=3^x\sqrt{3}-\frac{4^x}{2}$

or $\quad \frac{3}{2}\times 4^x=3^x\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)$

or $\quad \frac{3}{2}\times 4^x=3^x\times\frac{4}{\sqrt{3}}$

or $\quad \frac{4^{x-1}}{4^{1/2}}=\frac{3^{x-1}}{\sqrt{3}}$

or $\quad 4^{x-3/2}=3^{x-3/2}$

or $\quad \left(\frac{4}{3}\right)^{x-3/2}=1$

or $\quad x-\frac{3}{2}=0$

or $\quad x=3/2$

3. Given $a>0$, so we have two cases:

$a\ne 1$ and $a=1$. Also, it is clear that

$x>0$ and $x\ne 1$, $ax\ne 1$, $a^2x\ne 1$

Case I: If $a>0, \ne 1$, then given equation can be simplified as

$$\frac{2}{\log_a x}+\frac{1}{1+\log_a x}+\frac{3}{2+\log_a x}=0$$

Putting $\log_a x=y$, we get

$2(1+y)(2+y)+y(2+y)+3y(1+y)=0$

or $\quad 6y^2+11y+4=0$

$\Rightarrow \quad y=-4/3$ and $-1/2$

Now $\log_a x=-4/3$ and $\log_a x=-1/2$

$\Rightarrow \quad x=a^{-4/3}$ and $x=a^{-1/2}$

Case II: If $a=1$, then equation becomes

$2\log_x 1+\log_x 1+3\log_x 1=5\log_x 1=0$

This is true $\forall\, x>0, \ne 1$.

Hence, solution is $\begin{cases} x>0, \ne 1, & \text{if } a=1 \\ x=a^{-1/2}, a^{-4/3}, & \text{if } a>0, \ne 1 \end{cases}$

4. We know that

$(a-b)^2\ge 0$

$\Rightarrow \quad a^2+b^2\ge 2ab \qquad (1)$

Similarly, $b^2+c^2\ge 2bc \qquad (2)$

And $\quad c^2+a^2\ge 2ca \qquad (3)$

Adding the three inequations, we get

$2(a^2+b^2+c^2)\ge 2(ab+bc+ca)$

$\Rightarrow \quad a^2 + b^2 + c^2 \geq ab + bc + ca$

Adding $2(ab + bc + ca)$ to both sides, we get

$(a + b + c)^2 \geq 3(ab + bc + ca)$

Or $\quad 3(ab + bc + ca) \leq (a + b + c)^2$ (A)

Also, $c < a + b$ (triangle inequality)

$\Rightarrow \quad c^2 < ac + bc$ (4)

Similarly $b^2 < ab + bc$ (5)

$a^2 < ab + ca$ (6)

Adding (4), (5) and (6), we get

$a^2 + b^2 + c^2 < 2\,(ab + bc + ca)$

Adding $2(ab + bc + ca)$ to both sides we get

$\Rightarrow \quad (a + b + c)^2 < 4\,(ab + bc + ca)$ (B)

Combining (A) and (B), we get

$3(ab + bc + ca) \leq (a + b + c)^2 < 4\,(ab + bc + ca)$

First two expressions which are $3(ab + bc + ca)$ and $(a + b + c)^2$ will be equal if $a = b = c$.

5. The given equations are $3x + my - m = 0$ and

$2x - 5y - 20 = 0$

Solving these equations, we get

$$x = \frac{25m}{2m + 15}, \; y = \frac{2m - 60}{2m + 15}$$

For $x > 0$, $\dfrac{25m}{2m + 15} > 0$

$\Rightarrow \quad m < -\dfrac{15}{2}$ or $m > 0$ (1)

For $y > 0$, $\dfrac{2(m - 30)}{2m + 15} > 0$

$\Rightarrow \quad m < -\dfrac{15}{2}$ or $m > 30$ (2)

Combining (1) and (2), we get the common values of m.

i.e., $\quad m < -\dfrac{15}{2}$ or $m > 30$

$\therefore \quad m \in \left(-\infty, \dfrac{-15}{2}\right) \cup (30, \infty)$

6. The given system is

$x + 2y + z = 1$ (1)

$2x - 3y - w = 2$ (2)

where $x, y, z, w \geq 0$

Multiplying equation (1) by 2 and subtracting from (2), we get

$7y + 2z + w = 0$

$\Rightarrow \quad w = -(7y + 2z)$

Now, if $y, z < 0$, $w < 0$ (not possible)

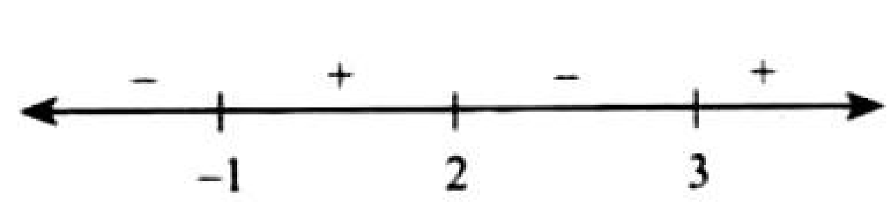
If $y = 0$, $z = 0$ then $x = 1$ and $w = 0$.

Thus, the only solution is

$x = 1, y = 0, z = 0, w = 0$

7. $y = \sqrt{\dfrac{(x+1)(x-3)}{(x-2)}}$, y will take all real values if

$$\frac{(x+1)(x-3)}{(x-2)} \geq 0$$



From the sign scheme of $\dfrac{(x+1)(x-3)}{(x-2)}$,

$x \in [-1, 2) \cup [3, \infty)$

8. $e^{\sin x} - e^{-\sin x} - 4 = 0$

Let $e^{\sin x} = y$. Then equation becomes

$$y - \frac{1}{y} - 4 = 0$$

or $\quad y^2 - 4y - 1 = 0$

or $\quad y = 2 + \sqrt{5}, 2 - \sqrt{5}$

But y is real +ve number. Therefore,

$y \neq 2 - \sqrt{5}$

if $\quad y = 2 + \sqrt{5}$

$\Rightarrow \quad e^{\sin x} = 2 + \sqrt{5}$

$\Rightarrow \quad \sin x = \log_e (2 + \sqrt{5})$

But $2 + \sqrt{5} > e$

$\Rightarrow \quad \log_e (2 + \sqrt{5}) > \log_e e$

or $\quad \log_e (2 + \sqrt{5}) > 1$

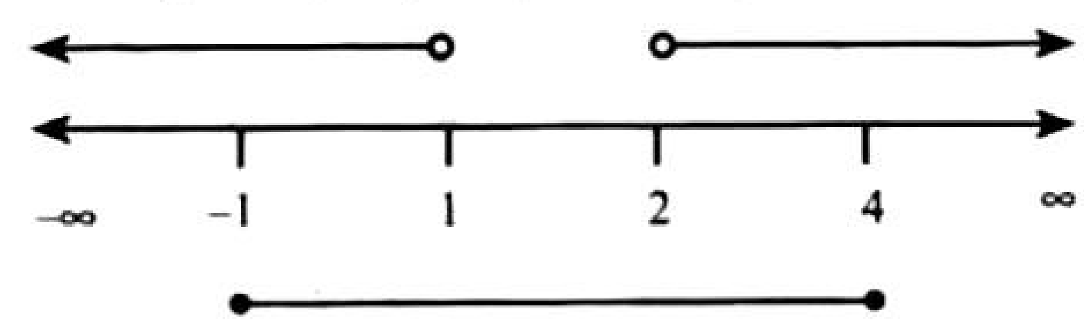
$\therefore \quad \sin x > 1$, which is not possible

Therefore, given equation has no real solution.

9. $x^2 - 3x + 2 > 0$, $x^2 - 3x - 4 \leq 0$

$\Rightarrow \quad (x - 1)(x - 2) > 0$ and $(x - 4)(x + 1) \leq 0$

$\Rightarrow \quad x \in (-\infty, 1) \cup (2, \infty)$ and $x \in [-1, 4]$



Therefore, common solution is $[-1, 1) \cup (2, 4]$.

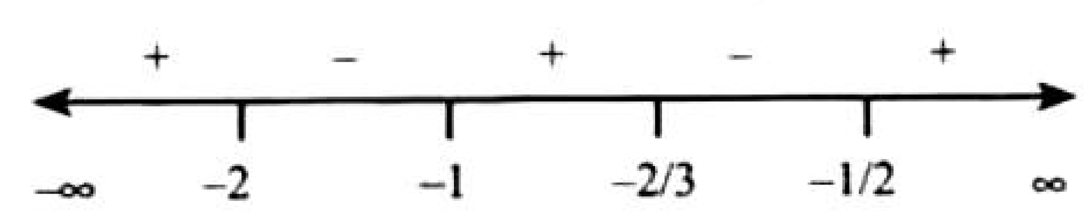
10. We are given $\dfrac{2x}{2x^2 + 5x + 2} > \dfrac{1}{x + 1}$

or $\quad \dfrac{2x}{2x^2 + 5x + 2} - \dfrac{1}{x + 1} > 0$

or $\quad \dfrac{2x^2 + 2x - 2x^2 - 5x - 2}{(2x^2 + 5x + 2)(x + 1)} > 0$

or $\quad \dfrac{-3x - 2}{(2x + 1)(x + 1)(x + 2)} > 0$

or $\quad \dfrac{(3x + 2)}{(x + 1)(x + 2)(2x + 1)} < 0$



From the sign scheme, solution is $x \in (-2, -1) \cup (-2/3, -1/2)$

11. The given equation is $|x^2 + 4x + 3| + 2x + 5 = 0$

Case I: $x^2 + 4x + 3 \geq 0$

or $\quad (x + 1)(x + 3) \geq 0$

$\Rightarrow \quad x \in (-\infty, -3] \cup [-1, \infty)$ (1)

Then given equation becomes,

$x^2 + 6x + 8 = 0$

or $\quad (x + 4)(x + 2) = 0$

$\Rightarrow \quad x = -4, -2$

But $x = -2$ does not satisfy (1), hence rejected.

Therefore, $x = -4$ is the only solution.

Case II: $x^2 + 4x + 3 < 0$

or $\quad (x + 1)(x + 3) < 0$

$\Rightarrow \quad x \in (-3, -1)$ (2)

Then given equation becomes $-(x^2 + 4x + 3) + 2x + 5 = 0$

or $\quad -x^2 - 2x + 2 = 0$

or $\quad x^2 + 2x - 2 = 0$

or $\quad x = \dfrac{-2 \pm \sqrt{4+8}}{2}$

$\Rightarrow \quad x = -1 + \sqrt{3}, -1 - \sqrt{3}$

Out of which $x = -1 - \sqrt{3}$ satisfies (2).

Therefore, $x = -1 - \sqrt{3}$ is the only solution.

Thus, $x = -4, -1 - \sqrt{3}$

Alternative solution:

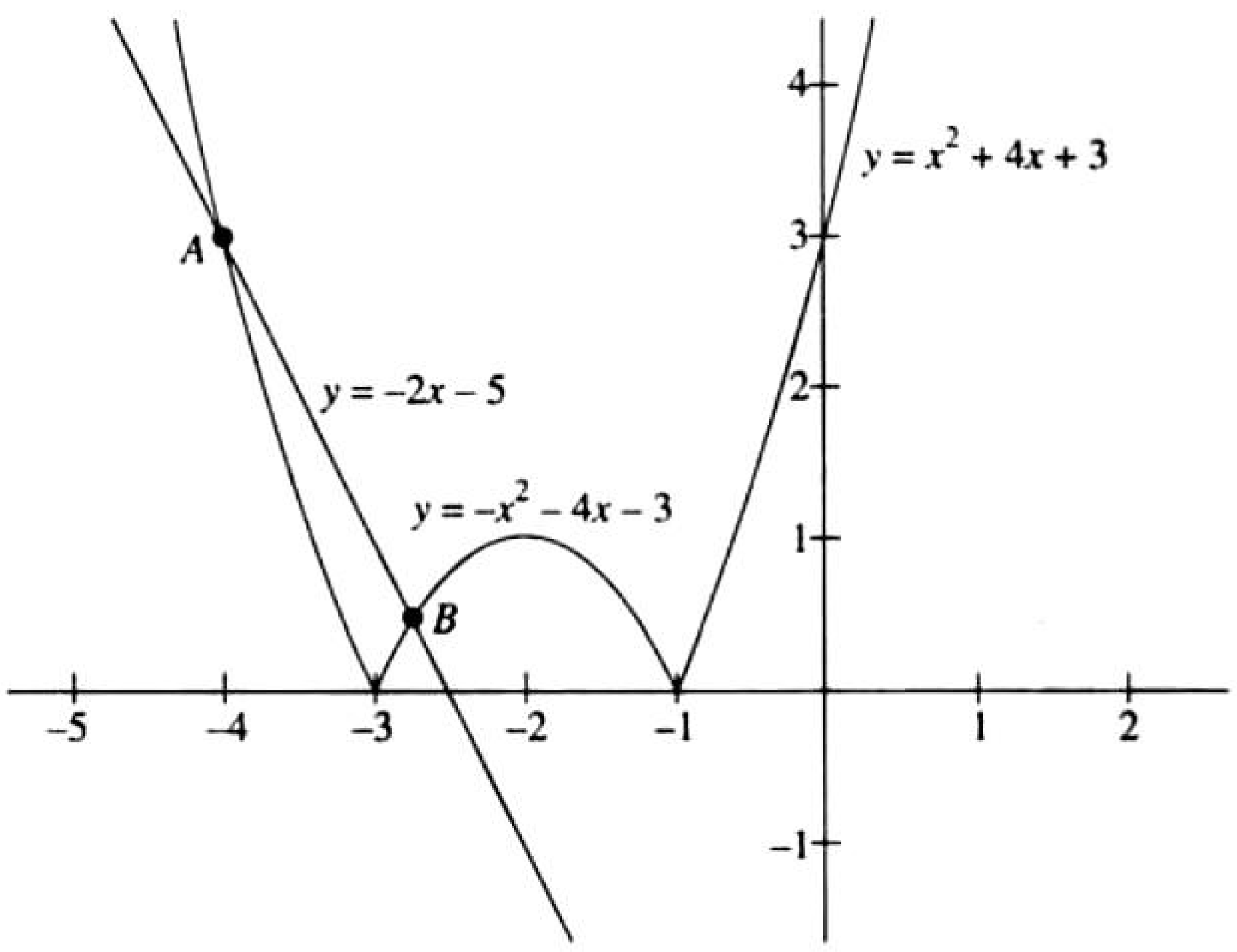
We have $|x^2 + 4x + 3| + 2x + 5 = 0$

or $\quad |x^2 + 4x + 3| = -2x - 5$

Now draw the graphs of $y = |x^2 + 4x + 3|$ and $y = -2x - 5$

To solve the equation, we locate the point of intersection of above two curves.

The graphs of the curves are shown in the following figure.



As shown in the figure, graphs intersect at two points A and B.

For point A, we solve $x^2 + 4x + 3 = y = -2x - 5$

or $\quad x^2 + 6x + 8 = 0$

or $\quad x = -2, -4.$

From the figure $x = -4$ only

For point B, we solve $-x^2 - 4x - 3 = y = -2x - 5$

or $\quad x^2 + 2x - 2 = 0$

or $\quad x = -1 \pm \sqrt{3}$

From the graph $x = -1 - \sqrt{3}$.

12. Given that $\log_3 2, \log_3 (2^x - 5), \log_3 (2^x - 7/2)$ are in A.P.

$\Rightarrow \quad 2 \log_3 (2^x - 5) = \log_3 (2^x - 7/2) + \log_3 2$

or $\quad (2^x - 5)^2 = 2\left(2^x - \dfrac{7}{2}\right)$

or $\quad (2^x)^2 - 10 \times 2^x + 25 - 2 \times 2^x + 7 = 0$

or $\quad (2^x)^2 - 12 \times 2^x + 32 = 0$

Let $2^x = y$. Then we get,

$\quad y^2 - 12y + 32 = 0$

or $\quad (y - 4)(y - 8) = 0$

$\Rightarrow \quad y = 4$ or 8

$\Rightarrow \quad 2^x = 2^2$ or 2^3

$\Rightarrow \quad x = 2$ or 3

But for $\log_3(2^x - 5)$ and $\log_3(2^x - 7/2)$ to be defined,

$\quad 2^x - 5 > 0$ and $2^x - 7/2 > 0$

$\Rightarrow \quad 2^x > 5$ and $2^x > 7/2$

$\Rightarrow \quad 2^x > 5$

$\Rightarrow \quad x \neq 2$ and therefore $x = 3$.