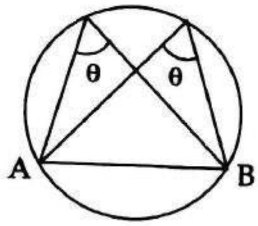
08

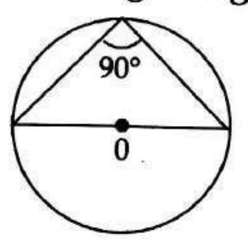
Circle and its Tangent lines

1. Main Geometric properties Related to circle

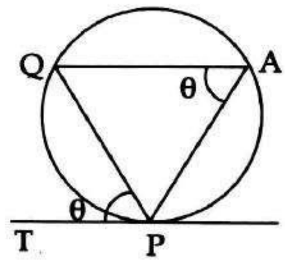
1.1. **Angles in the same segment of a circle are equal**



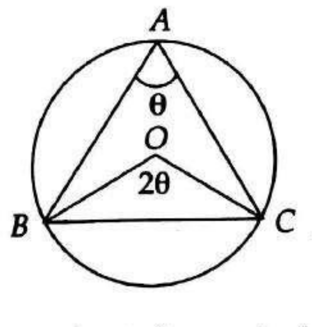
1.2. **The angle in a semicircle is right angled.**



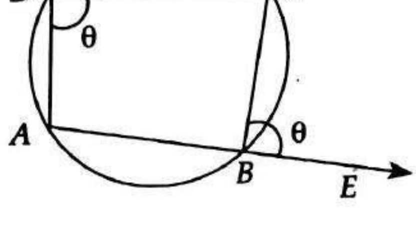
1.3. **If line *PT* touches a circle at the point *P* and a chord *PQ* is drawn from point of contact *P*, then angle made by *PQ* in the alternate segment (∠*PAQ* in figure) of the circle is equal to angle (∠*QPT* in figure) made by the tangent *PT* to the circle.**



1.4. **The angle at the centre (*O* in figure) in a circle is double the angle at the circumference standing on the same arc or same base (*BC* in figure) i.e. in the same segment.**

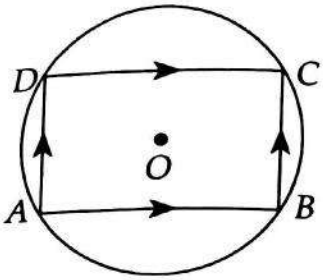


1.5. **A quadrilateral inside the circle formed by taking four points on the circumference of the circle is called a cyclic quadrilateral sum of its opposite angle is 180° (i.e. ∠*A* + ∠*C* = 180° and**

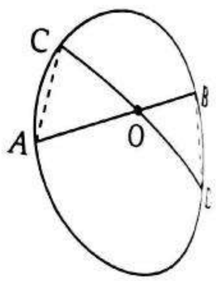


$\angle B + \angle D = 180°$). Its converse is also true. If AB is produced
then $\angle CBE = \angle D = \theta$

1.6. If a parallelogram is inscribed inside a circle, it is either a rectangle or a square.



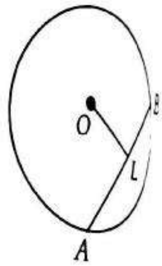
1.7. If two chords AB and CD of a circle intersect at O then ΔAOC and ΔDOB are similar i.e. $\Delta AOC \sim \Delta BOD$ (In the given figure $\angle A = \angle D$, $\angle C = \angle B$ and $\angle BOD = \angle AOC$)



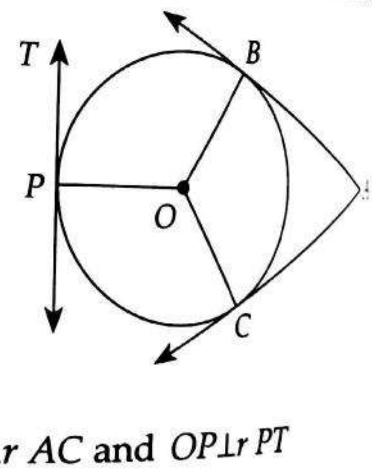
Hence, $\frac{AO}{DO} = \frac{AC}{DB} = \frac{OC}{OB}$

or, $(AO)(OB) = (OC)(OD)$

1.8. Perpendicular drawn from the centre of a circle to any chord bisects the chord. Its converse is also true. In the given figure $OL \perp r\ AB \Leftrightarrow AL = BL$



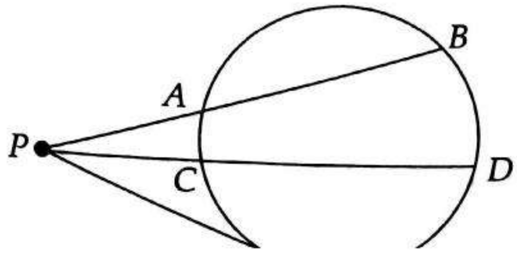
1.9. From a point on the circle, only one tangent can be drawn to the circle (PT in figure). However two tangents (AB and AC) can be drawn to a circle from an external point. Length of these two tangents are equal i.e. $AB = AC$



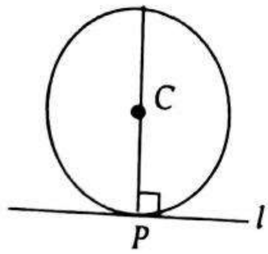
The line joining centre and point of contact of a circle is perpendicular to the tangent drawn at point of contact.

In the adjacent figure $OB \perp r\ AB$, $OC \perp r\ AC$ and $OP \perp r\ PT$

1.10. In the given figure $(PA)(PB) = PT^2 = PC.\ PD$



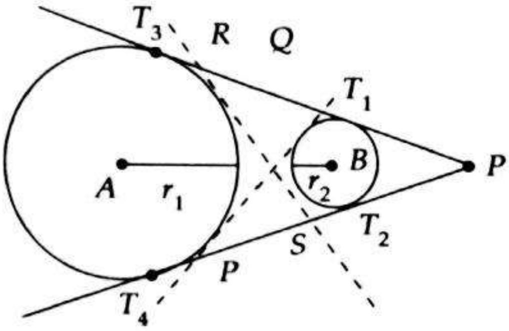
2. Tangent and Normal to a circle : **A line that touches a circle at one and only point is called a tangent line or simply tangent to the circle. In the given figure l is a tangent line to the circle that touches the circle at point P. This point is called point of contact of tangent**



A line through point P and perpendicular to tangent l is called normal to the circle. Normal to a circle always passes through its center.

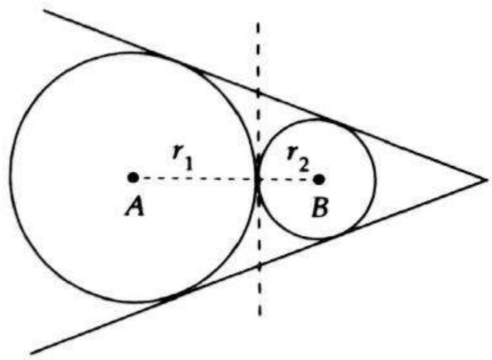
3. Number of common tangents to the two circles : **There are maximum number of four common tangents and minimum number of zero tangent to the two given circles. They are as follows.**

3.1. Four common tangents : **If distance between centres of two en circles is greater than sum their radii i.e. $AB > r_1 + r_2$ (see figure), then four common tangents can be drawn to the two circles.**



See the given figure, T_1T_3 and T_2T_4 are direct common tangent while PQ and RS are transverse common tangents.

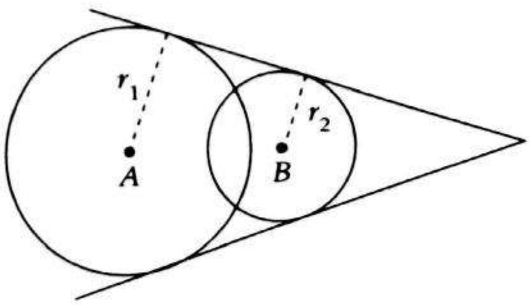
3.2. Three common tangents : **When distance between centres of two circles is equal to sum of their radii ($AB = r_1 + r_2$) then maximum of three common tangents can be drawn to the circle. In this situation two circles touch externally.**



(see the figure)

Two common tangents

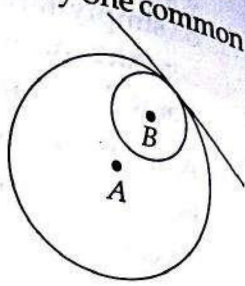
When two circles intersect each other at two distinct points then two common tangents can be drawn to the circles (see the figure). Hence distance between centres of two circles is less than sum of their radii but greater than difference of radii i.e.



$|r_1 - r_2| < AB < r_1 + r_2$

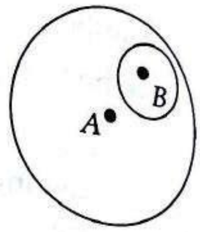
3.4. One common tangent :

When two circles touch each other internally then only one common tangent can be drawn to them (see the figure). In this situation distance between centres of the two circles is equal to difference of their radii i.e.



$AB = |r_1 - r_2|$

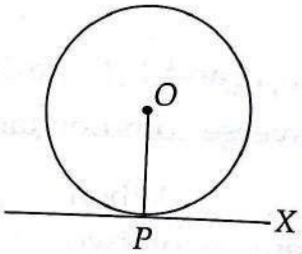
3.5. When one circle lies completely inside other circle, then common tangent cannot be drawn to the two circles (see the figure). Here distance between centres of two circles is less than difference between radii of two circles.



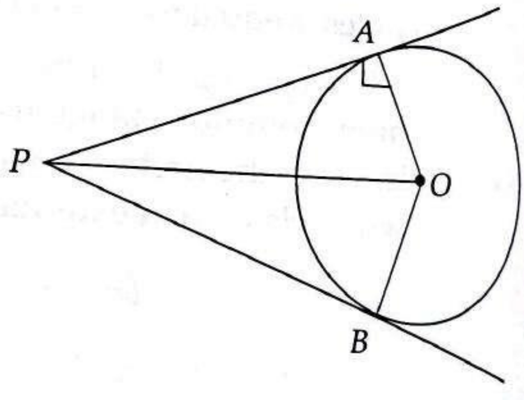
i.e. $AB < |r_1 - r_2|$

4. Some properties of tangents to a circle

4.1. Tangent drawn at any point to the circle is perpendicular to radius of the circle drawn through the point i.e. point of contact. In the given figure $\angle OPX = 90°$. Its converse is also true.



4.2. length of tangents drawn from an outside point to a given circle are equal. In the given figure, if PA and PB are tangents line then $PA = PB$



$\because$ PA is perpendicular to OA

$\therefore$ $PA^2 + OA^2 = OP^2$

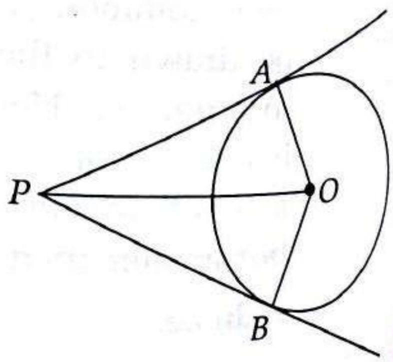
4.3. If PA and PB are tangents to a circle with centre O, then

$\angle APO = \angle BPO$

$\angle PAO = \angle PBO = 90°$

$\because$ Side PO is common

$\therefore$ $\Delta PAO \cong \Delta PBO$

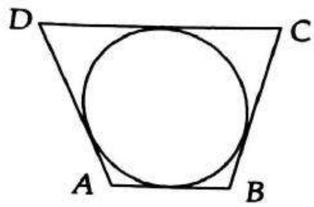


Also, $\angle AOB = 180° - \angle APB$

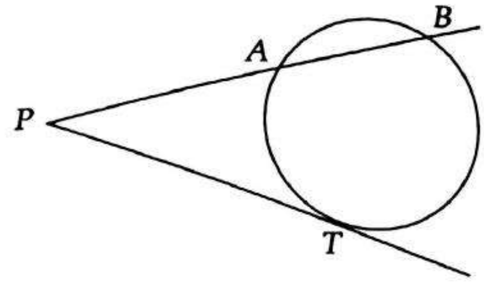
($\because$ sum of remaining two angles of quadrilateral $= 180°$)

4.4. If each side of a quadrilateral touches a given circle then sum of one pair of opposite side is equal to sum of another pair of opposite side.

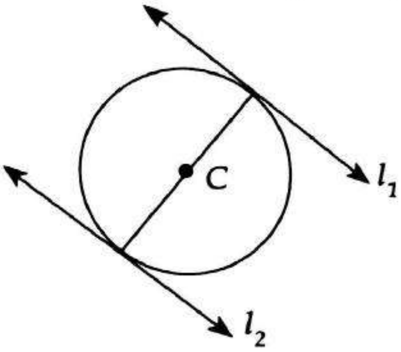
In the given figure $AB + CD = AD + BC$



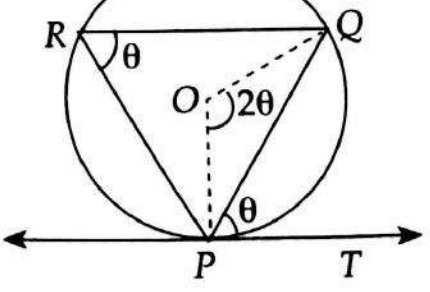
4.5. If PAB be a secant that intersects a given circle at A and B and PT is a tangent line then $PA \cdot PB = PT^2$



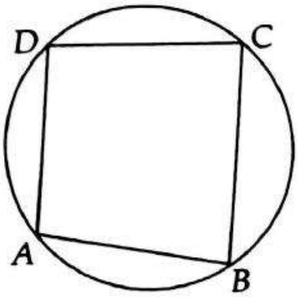
4.6. Tangents drawn at extrimities (end points) of a diameter of a given circle are parallel. In the given figure $l_1 \parallel l_2$



4.7. In the given figure, if PT is a tangent to the circle then $\angle PRQ = \angle TPQ = \theta$ and $\angle POQ = 2\theta$



5. Important properties of cyclic Quadrilateral



5.1. In the cyclic quadrilateral $ABCD$

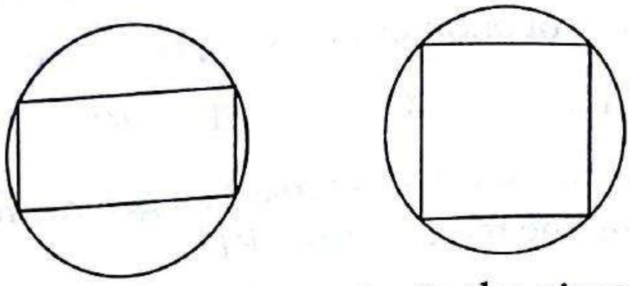
$\angle A + \angle C = 180^\circ$

and $\angle B + \angle D = 180^\circ$

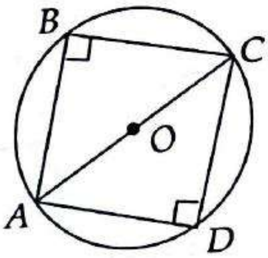
Its converse are also true i.e. in any quadrilateral if

$\angle A + \angle C = \angle B + \angle D = 180^\circ$ then $ABCD$ is a cyclic quadrilateral

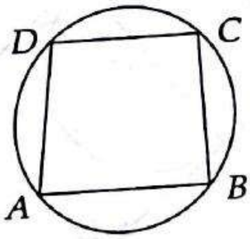
5.2. Every cyclic parallelogram is a rectangle. Every cyclic rhombus is a square



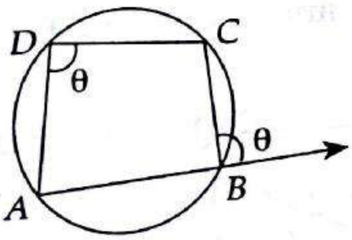
5.3. Angle in a semicircle is right angle. In the given figure if O is the centre then $\angle ABC = \angle ADC = 90^\circ$



5.4. If a trapezium *ABCD*, where *AB* | | *DC*, is inscribed in a circle then its non parallel sides are equal i.e. $BC = AD$. Thus we can say that a trapezium inscribed in a circle is always isosceles. Converse of the statement is also true.



5.5. If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.



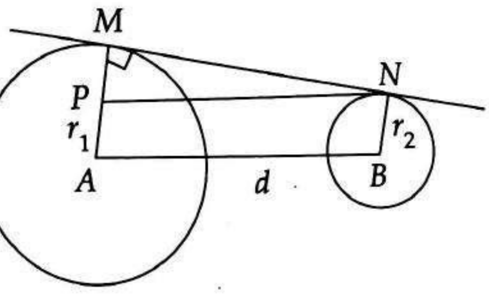
... common tangents ... from point *P* to two ...

6.1.1. $\because \Delta PAM \sim \Delta PBN$ (P is common angle and $\angle AMP = \angle BNP = 90°$)

$\therefore \frac{PA}{PB} = \frac{PM}{PN} = \frac{AM}{BN}$

$\Rightarrow \boxed{\frac{PA}{PB} = \frac{r_1}{r_2}}$

i.e. Point P divides the line joining the centres which is AB in the ratio $r_1 : r_2$ (externally)



6.1.2. Length of direct common tangent (MN)

From point N draw a line parallel to AB that intersects AM at P. Since $ABNP$ is a parallelogram

$\therefore \; PA = BN = r_2$

$\therefore \; PM = r_1 - r_2$

In right angled ΔPNM

$$PN^2 = PM^2 + MN^2$$

or, $AB^2 = PM^2 + MN^2 \quad (\because \; PN = AB)$

$$MN^2 = AB^2 - (PM)^2$$

$$MN = \sqrt{(AB)^2 - (PM)^2} = \sqrt{d^2 - (r_1 - r_2)^2},$$

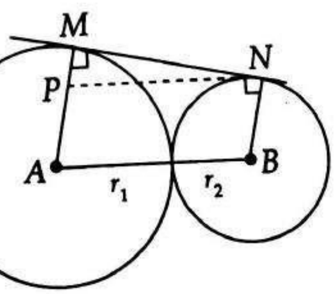
where d = distance between centres

Length of direct common tangents to two circles =

$$\sqrt{(\text{Distance between centre})^2 - (\text{Difference of radii})^2}$$

This result is also true if circles touch externally or intersect at two distinct points

6.1.3 Special case : If two circles touch externally then length of direct common tangent



$$MN = \sqrt{d^2 - (r_1 - r_2)^2}$$

$$= \sqrt{(r_1 + r_2)^2 - (r_1 - r_2)^2}$$

$$= \sqrt{4 r_1 \cdot r_2}$$

$$\text{or, } k = \frac{d}{r_1 - r_2}$$

$$\therefore \quad AP = r_1 k = \frac{r_1}{r_1 - r_2} d$$

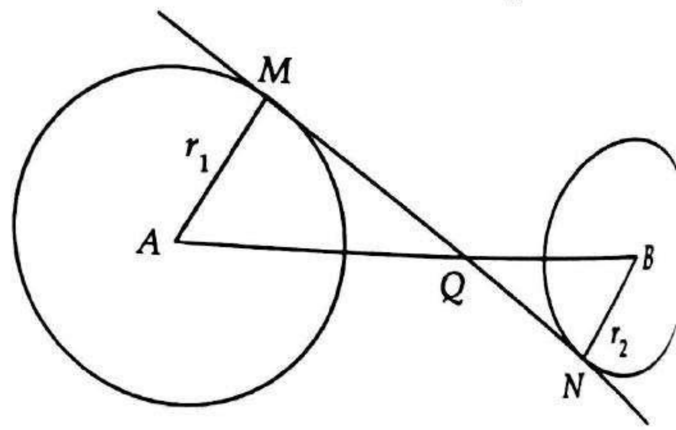
$$\text{and } BP = r_2 k = \frac{r_2}{r_1 - r_2} d$$

Thus Distance of centres from point P are respectively $\frac{r_1 d}{r_1 - r_2}$ and $\frac{r_2 d}{r_1 - r_2}$, where $r_1 > r_2$

6.1.5. Hence distance of P from N, $PN = \sqrt{BP^2 - BN^2} = \sqrt{BP^2 - r_2^2}$

Distance of P from M, $PM = \sqrt{AP^2 - AM^2} = \sqrt{AP^2 - r_1^2}$

7. Transverse Common Tangents :



In the adjacent figure MN is transverse common tangent. It touches the circle with centre A and radius r_1 at M while touches the circle with centre B and radius r_2 at N. MN and AB intersect at Q. Some important facts regarding them are as follows.

7.1. $\Delta AMQ \sim \Delta BNQ$ $\quad (\because \angle AMQ = \angle BNQ = 90^\circ \text{ and } \angle AQM = \angle BQN)$

$$\Rightarrow \quad \frac{AM}{BN} = \frac{AQ}{BQ} = \frac{MQ}{NQ},$$

$$\because \quad AM = r_1, BN = r_2$$

$$\therefore \quad \frac{AQ}{BQ} = \frac{r_1}{r_2}$$

So, Q divides line AB joining the centres ... internally

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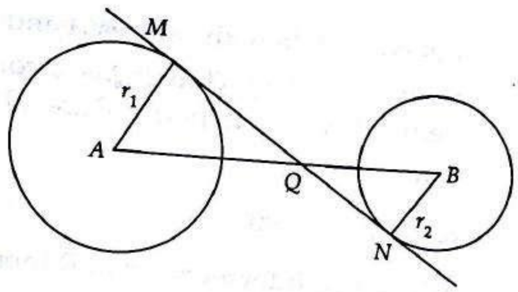
7.2. $\because \frac{AQ}{BQ} = \frac{r_1}{r_2}$

$\therefore AQ + BQ = (r_1 + r_2)k$

$\Rightarrow d = (r_1 + r_2)k$, where d is distance between centres

$\Rightarrow k = \frac{d}{r_1 + r_2}$

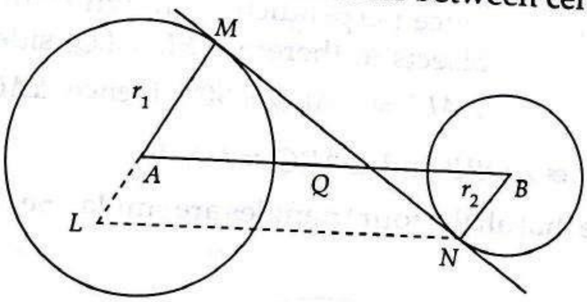
$\therefore AQ = kr_1 = \frac{r_1 d}{r_1 + r_2}$ and $BQ = kr_2 = \frac{r_2 d}{r_1 + r_2}$



Thus **Distance of centres from point Q are respectively $\frac{r_1 d}{r_1 + r_2}$ and $\frac{r_2 d}{r_1 + r_2}$**

7.3. From N draw a line parallel to AB which intersects produced part of MA at L

$\therefore ML = r_1 + r_2$ and $LN = AB = d$ = distance between centres.



In triangle MNL

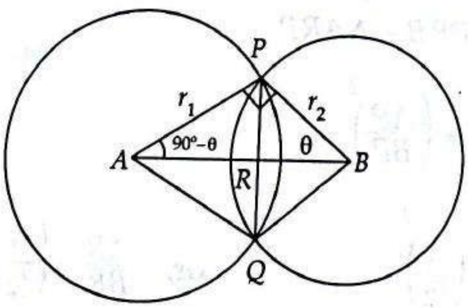
$\because \angle LMN = 90°$

$\therefore MN = \sqrt{LN^2 - LM^2} = \sqrt{d^2 - (r_1 + r_2)^2}$

Length of transverse common tangents to two circles =

$$\sqrt{(\text{Distance between centre})^2 - (\text{Sum of radii})^2}$$

8. Common chord :



Let two circle with centres A and B intersect each other at two distinct points P and Q. Thus PQ is a common chord to the two circles. If the tangents drawn from points P and Q to the two circles passes through the centres then

8.1. $\angle APB = 90^\circ$

8.2. If PQ, intersects AB at R then

$PR \perp r\ AB$ and $\Delta PAB \sim \Delta RPB \sim \Delta RAP$

Explanation : Since PA and PB are tangents and A and B are centre, thus $\angle APB = 90^\circ$

In right angled ΔPAB, Let $\angle PBA = \theta$ then $\angle PAB = 90^\circ - \theta$

In right angled ΔRPB, $\angle PRB = 90^\circ$, $\angle PBR = \theta$ and $\angle RPB = 90^\circ - \theta$

In right angled ΔRAP, $\angle PRA = 90^\circ$, $\angle RAP = 90^\circ - \theta$ and $\angle APR = \theta$

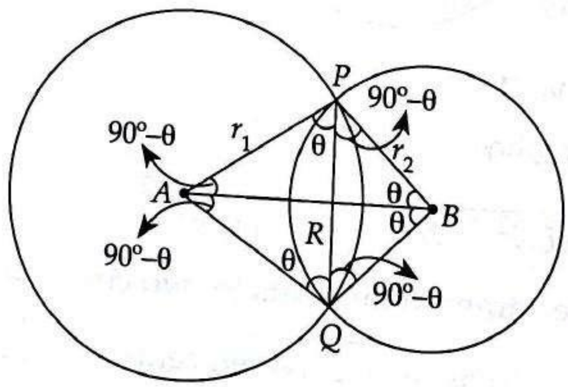
Hence $\Delta PAB \sim \Delta RPB \sim \Delta RAP$

8.3. $\Delta ARP \cong \Delta ARQ \sim \Delta PRB \cong QRB)$

Explanation : Since perpendicular drawn from centre to any chord bisects it, therefore $PL = LQ$, side AL is common and $\angle ALP = \angle ALQ = 90^\circ$. Hence $\Delta ALP \cong \Delta ALQ$

8.4. $\Delta ARP \cong \angle QRB$ and $\Delta ARQ \cong \Delta PRB$

(Note that all the four triangles are similar. See the figure and explain yourself)



8.5. $AR : RB = r_1^2 : r_2^2$

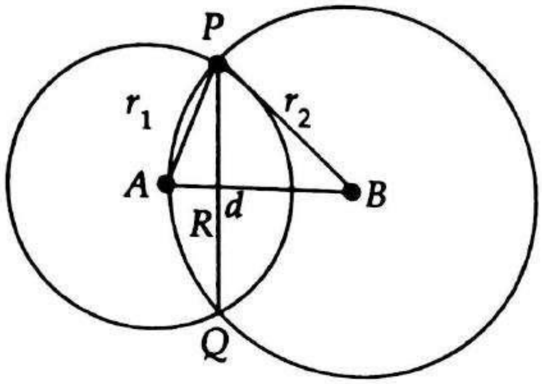
Explanation : $\because$ $\Delta PRB \sim \Delta ARP$

$$\therefore \quad \frac{\text{area of } \Delta ARP}{\text{area of } \Delta BRP} = \left(\frac{AP}{BP}\right)^2$$

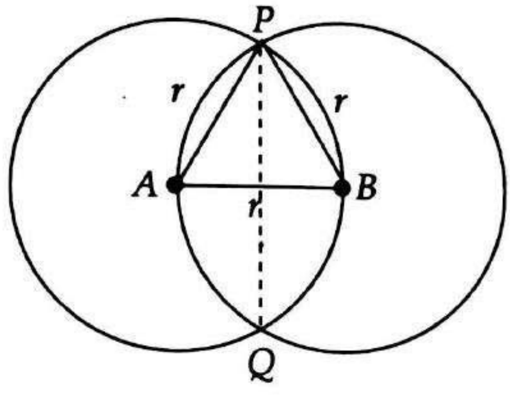
$$\Rightarrow \quad \frac{\frac{1}{2} \times AR \times PR}{\frac{1}{2} \times BR \times PR} = \left(\frac{r_1}{r_2}\right)^2 \qquad \text{or,} \quad \frac{AR}{BR} = \left(\frac{r_1}{r_2}\right)^2$$

9. Some more important facts about common chord :

9.1. If radii of two unequal circles are r_1 and r_2 and larger circle passes through centre of smaller one then $r_1^2 + r_2^2 = d^2$, where d is the distance between centres of the two circles.



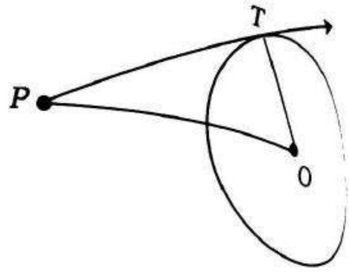
9.2. In two equal circles (circles with same radius) if one passes through the centre of the other then other must pass through centre of the former (see the figure). ΔAPB will be an equilateral triangle whose each side is equal to radius of the circle.



$\therefore$ $PQ = 2 \times$ altitude of the triangle $= 2 \times \frac{\sqrt{3}}{2} r$

length of common chord $= \sqrt{3}\, r$

2. In the figure given below O is the centre of the circle. A tangent PT is drawn from an out side point P to the circle. If radius of circle is 5 cm and $OP = 13$ cm then find the length of tangent PT.



Solution : $PT \perp OT$

$\Rightarrow \angle OTP = 90°$

$\therefore \quad PT^2 + OT^2 = OP^2$ [In right angled ΔOPT]

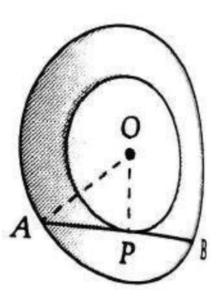
or, $PT^2 = OP^2 - OT^2$

$= 13^2 - 5^2$

$= 169 - 25 = 144 = 12^2$

$\therefore \quad PT = 12$ cm

3. Radius of two incentric circle are 5 cm and 3cm. Find out the length of arc of larger circle which touches to smaller circle ?



Solution : Let O be the common centre. AB is chord of larger circle that touches the smaller one.

Join $O - P$ then $\angle OPB = 90°$

i.e. OP is perpendicular to AB

Since perpendicular drawn from centre of a circle to any of its chord bisect the chord

$\therefore \quad AP = PB$

Now in right angled ΔAPO

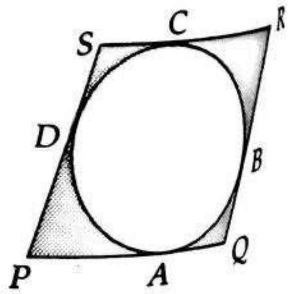
$OA^2 = AP^2 + OP^2$

$\Rightarrow \; 5^2 = AP^2 + 3^2$

$\Rightarrow \; AP = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ cm

$\therefore$ Length of chord $AB = 2AP = 2 \times 4 = 8$ cm

4. In the adjacent figure lines PQ, QR, RS and SP are tangents drawn respectively at the points A, B, C, D to the circle. If $PQ + SR = 16$ cm, then find the perimeter of the quadrilateral



Solution : If all sides of quadrilateral $PQRS$ touches a circle then $PQ + SR = PS + QR$

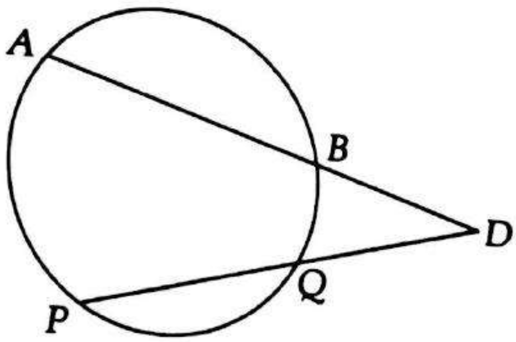
but $PQ + SR = 16$ cm (*given*)

5. A circle is drawn circumscribing a parallelogram. If length of sides of parallelogram are 3 cm and 4 cm, find its area.

Solution : Since every cyclic parallelogram is a rectangle, therefore its sides are 4 cm and 3 cm

Hence required area $= 3 \times 4 = 12\ cm^2$

6. Two chords AB and PQ of a circle mutually intersect at an outside point D. If $AD = 12$ cm, $AB = 8$ cm, $DQ = 6$ cm then find PQ and PD.



Solution : $AD = AB + BD$

or, $12 = 8 + BD$

$\therefore\ BD = 12 - 8 = 4$ cm

Now, $DB \cdot DA = DQ \cdot DP$

$\therefore\ 4 \times 12 = 6 \times DP$

or, $DP = \dfrac{4 \times 12}{6} = 8$ cm

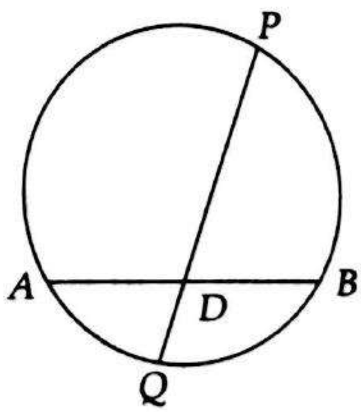
But, $DP = DQ + QP$

or, $8 = 6 + QP$

$\therefore\ QP = 8 - 6 = 2$ cm

Hence, $PQ = 2$ cm and $PD = 8$ cm.

7. Two chord AB and PQ of a circle intersect at a point D inside the circle. If $AD = 4$ cm, $DB = 6$ cm, $QD = 3$ cm, then find PD and PQ.



Solution : $AD \cdot DB = QD \cdot DP$

or, $4 \times 6 = 3 \times DP$

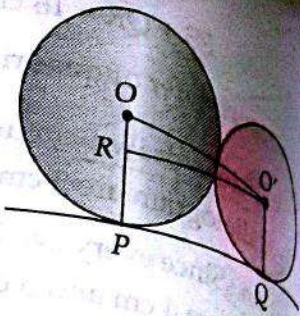
or, $24 = 3 \times DP$

$\therefore\ DP = \dfrac{24}{3} = 8$ cm

$\therefore\ PQ = PD + DQ$

$= 8$ cm $+ 3$ cm $= 11$ cm.

8. Radii of two circles are respectively 25 cm and 9 cm and their centres are 34 cm apart. Find the length of direct common tangent to the two



PQ is length of direct common tangent

Draw $O'R \parallel PQ$

$\therefore \quad RP = O'Q = 9$ cm

$\therefore \quad OR = OP - RP$

$= 25 \text{ cm} - 9 \text{ cm} = 16 \text{ cm}$

Now, in right angled $\Delta ORO'$

$$OO'^2 = OR^2 + O'R^2$$

or, $\quad 34^2 = 16^2 - RQ'^2$

$\therefore \quad RO'^2 = 34^2 - 16^2$

$= (34 + 16)(34 - 16)$

$= 50 \times 18 = 900 = 30^2$

$\therefore \quad RO' = 30$ cm

$\therefore \quad PQ = 30$ cm

Shortcut Mtd. : Length of direct common tangent $= \sqrt{d^2 - (r_1 - r_2)^2}$

$= \sqrt{34^2 - (25 - 9)^2}$

$= \sqrt{34^2 - 16^2}$

$= \sqrt{(34 + 16)(34 - 16)}$

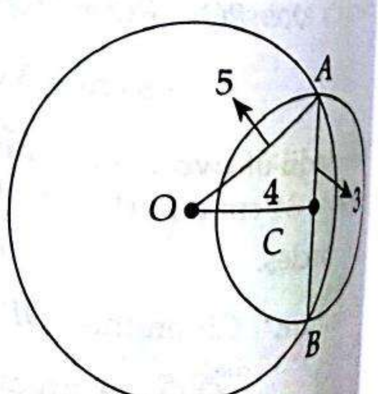
$= \sqrt{50 \times 18}$

$= \sqrt{25 \times 36}$

$= 5 \times 6 = 30$ cm

9. Two circles of radii 5 cm and 3 cm intersect at two distinct points. Their centres are 4 cm apart. Find the length of their common chord.

Solution : In the given figure two intersecting circles of 5 cm and 3 cm are shown. Their centres are respectively O and C. AB is the common chord.



According to question, $OC = 4$ cm

$\because \quad 3^2 + 4^2 = 5^2$

$\therefore \quad AC^2 + OC^2 = OA^2,$

Hence in ΔOAC, $\angle ACO = 90°$

Similarly in triangle OCB, $\angle OCB = 90°$

Now, $\because \angle OCA + \angle OCB = 90° + 90° = 180°$

Hence, AB is a straight line.

Since it is a straight line passing through centre of the smaller circle, hence it is diameter of this circle.

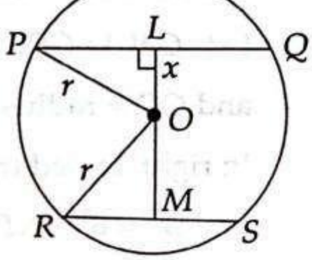
We conclude that common tangent is diameter of smaller circle

Hence, its length $= 3 \times 2 = 6$ cm

10. PQ and RS are two parallel chords of a circle. If $PQ = 30$ cm, $RS = 16$ cm and distance between PQ and RS is 23 cm, then find the radius of the circle.

Solution : See the figure, from centre O of the circle perpendicular OL is drawn to chord PQ and perpendicular OM is drawn to RS.

$\therefore PL = \frac{PQ}{2} = \frac{30}{2} = 15$ cm



and $RM = \frac{RS}{2} = \frac{16}{2} = 8$ cm

Let $OL = x$ cm,

then, $OM = (23 - x)$ cm

In ΔOLP, $OP^2 = PL^2 + LO^2$

or, $r^2 = 15^2 + x^2$ ... (i)

Again in ΔOMR, $OR^2 = OM^2 + RM^2$

or, $r^2 = (23 - x)^2 + 8^2$ ... (ii)

From equation (i) and (ii), $15^2 + x^2 = (23 - x)^2 + 8^2$

or, $225 + x^2 = 23^2 - 46x + x^2 + 64$

or, $225 = 529 - 46x + 64$

or, $225 = 593 - 46x$

$\therefore 46x = 368$

or, $x = \frac{368}{46} = 8$

Thus from (i) $r^2 = 15^2 + 8^2$

$= 225 + 64 = 289$

or, $r = \sqrt{289} = 17$ cm

radius = 17 cm

11. Length of one of the chord of a circle is 16 cm and it is 15 cm away from centre. Find the length of that chord of the circle which is 8 cm away from the centre.

Solution : In the given figure, O is the centre of the circle.

AB is a chord whose length is 16 cm

OM is perpendicular bisector of chord AB

$\therefore \quad MB = \frac{16}{2} = 8$ cm and $OM = 15$ cm (given)

In right angled ΔOMB, $OB^2 = OM^2 + MB^2$

or, $OB^2 = 15^2 + 8^2$

$= 225 + 64 = 289$

or, $OB = \sqrt{289} = 17$ cm

Thus radius of circle is 17 cm

Let CD be a chord of the circle at a distance of 8 cm from centre.

Let $ON \perp r\ CD$, then $ON = 8$ cm

and OD = radius of circle = 17 cm

In right angled triangle OND, $OD^2 = ON^2 + ND^2$

or, $17^2 = 8^2 + ND^2$

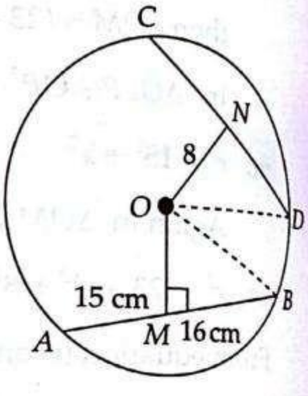
or, $ND^2 = 17^2 - 8^2 = 289 - 64 = 225$

or, $ND = \sqrt{225} = 15$ cm

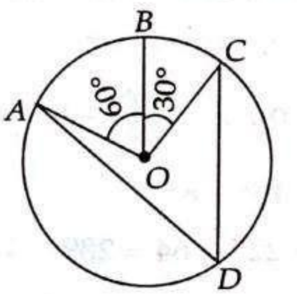
Since ON is perpendicular bisector of CD

$\therefore \quad CD = 2\ ND = 2 \times 15 = 30$ cm

Hence, length of chord which is 8 cm away from centre is 30 cm



12. In the given figure O is the centre of circle and $\angle BOC = 30°$, $\angle AOB = 60°$. If their is a point D on circle, not on arc ABC, then find $\angle ADC$.



Solution : $\angle AOC = \angle AOB + \angle BOC$

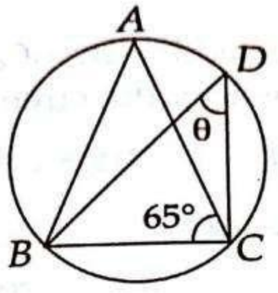
$= 60° + 30° = 90°$

∴ arc ABC subtends an angle of $90°$ on the centre

∴ arc ABC subtends an angle of $\frac{90°}{2} = 45°$ on point D.

∴ Hence, $\angle ADC = \frac{1}{2}\angle AOC = 45°$.

13. In the given figure if $AB = AC$ then find θ.



Solution : In ΔABC, $AB = AC \Rightarrow \angle B = \angle C$

$\angle B = 65°$ $\quad (\because \angle C = 65°)$

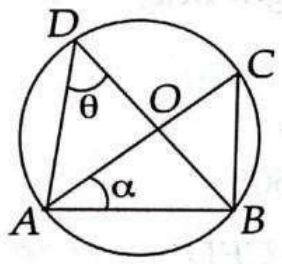
In ΔABC, $\angle A + 65° + 65° = 180°$

or, $\angle A = 180° - 65° - 65° = 50°$

Since, angle in the same segment are equal,

$\therefore \theta = \angle A = 50°$

14. In the figure given below O is the centre of the circle, if $\theta = 60°$, then find angle α



Solution : $\because$ AC and BD are passing through centre.

$\therefore$ ΔBAD and ΔABC are right angle triangle with $\angle BAD = 90°$

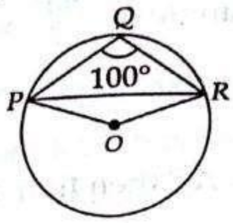
and $\angle ABC = 90°$

$\because$ θ and $\angle ACB$ are angle of the same segment

$\therefore$ $\angle ACB = \theta = 60°$

Now, in ΔABC, $\angle ACB + \angle ABC + \alpha = 180°$

15. In the given figure $\angle PQR = 100°$, where P, Q, R are points on a circle with centre O. Find the measure of $\angle OPR$



Solution : Since angle subtended by arc of a circle at the centre twice the angle subtended by it at the circumference

$\therefore$ reflex $\angle POR = 2\angle PQR = 2 \times 100° = 200°$

and $\angle POR = 360° - 200° = 160°$

Now, in ΔOPR

$OP = OR$ (radii of the circle)

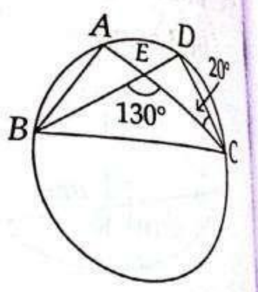
or, $\angle OPR = \angle ORP$ (angle opposite to equal sides are equal)

$\therefore$ $\angle POR + \angle OPR + \angle ORP = 180°$ (Sum of angles of a triangle)

or, $160° + 2\angle OPR = 180°$ ($\because \angle OPR = \angle ORP$)

or, $\angle OPR = \dfrac{180° - 160°}{2} = 10°$

16. In the given figure A, B, C, D are four points on a circle. AC and BD intersect at point E such that $\angle BEC = 130°$ and $\angle ECD = 20°$. Find the measure of $\angle BAC$



Solution : Since BD is a straight line,

$\therefore$ $\angle BEC + \angle CED = 180°$

or, $\angle CED = 180° - \angle BEC$

$= 180° - 130° = 50°$

Now, In ΔECD, $\angle EDC + \angle CED + \angle DCE = 180°$

or, $\angle EDC + 50° + 20° = 180°$

or, $\angle EDC = 180° - 50° - 20° = 110°$

Since angles in the same segment are equal

(here see the segment above base BC)

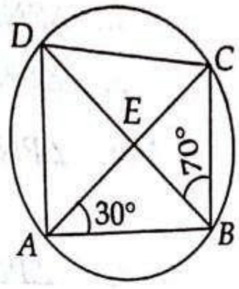
$\therefore$ $\angle BAC = \angle BDC = 110°$

17. ABCD is a cyclic quadrilateral whose diagonals intersect at E. If $\angle DBC = 70°$ and $\angle BAC = 30°$ then find $\angle BCD$. Again if $AB = BC$ then find $\angle ECD$.

Solution : In the given figure $\angle BDC = \angle BAC$

$(\because \angle BAC = 30^\circ$ is given)

$\therefore \angle BDC = 30^\circ$



In ΔBCD, $\angle BDC + \angle DBC + \angle BCD = 180^\circ$

or, $30^\circ + 70^\circ + \angle BCD = 180^\circ$

$(\because \angle DBC = 70^\circ$ is given, $\angle BDC = 30^\circ$ is evaluated)

or, $\angle BCD = 180^\circ - 30^\circ - 70^\circ = 80^\circ$

Again if $AB = BC$ then

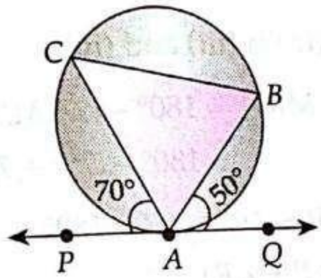
$\angle BCA = \angle BAC = 30^\circ$

(Angles opposite to equal sides of a triangle are equal)

$\therefore \angle ECD = \angle BCD - \angle BCE$

$= 80^\circ - 30^\circ = 50^\circ$

18. In the figure given below, find each angle of ΔABC.



Solution : $\because$ PQ touches circle at A.

$\therefore \angle BAQ = \angle ACB = 50^\circ$ *[Angle in the alternate segment]*

Similarly, $\angle PAC = \angle ABC = 70^\circ$

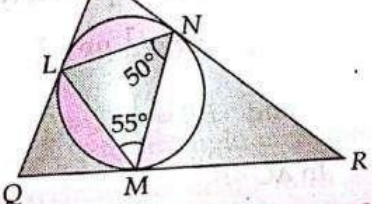
but, $\angle PAC + \angle CAB + \angle BAQ = 180^\circ$ *[$\because$ P, A, Q are collinear]*

$\therefore 70^\circ + \angle CAB + 50^\circ = 180^\circ$

$\therefore CAB = 180^\circ - (70^\circ + 50^\circ) = 60^\circ$

Hence, angles of ΔABC are, $\angle A = 60^\circ$, $\angle B = 70^\circ$ and $\angle C = 50^\circ$.

19. In the given figure PQ, QR and RP touches a given circle respectively at point L, M and N. If $\angle LMN = 55^\circ$ and $\angle MNL = 50^\circ$, then find $\angle P, \angle Q$ and $\angle R$



Solution : In ΔLMN

$\angle MLN + \angle LMN + \angle MNL = 180^\circ$

$\therefore \angle MLN + 55^\circ + 50^\circ = 180^\circ$

[Given that $\angle LMN = 55^\circ$ and $\angle MNL = 50^\circ$]

... (i)

$\therefore \angle MLN = 180^\circ - (55^\circ + 50^\circ) = 75^\circ$

In ΔPNL, $PN = PL$ [length of tangent from P to the circle]

$\therefore \quad \angle PNL = \angle PLN$

but, $\angle PNL = \angle NML = 55°$

[angle in the alternate segment on chord LN]

$\therefore \quad \angle PNL = \angle PLN = 55°$

Hence, $\angle LPN = 180° - (55 + 55°) = 70°$

[Sum of angles of ΔPLN = 180°] ...(ii)

Again, in ΔRMN, $RN = RM$ [length of tangent from R to the circle]

$\therefore \quad \angle RNM = \angle RMN$

but, $\angle PNL + \angle LNM + \angle MNR = 180°$ [P, N, R are collinear] ...(iii)

or, $55° + 30° + \angle MNR = 180°$

$\therefore \quad \angle MNR = 180° - (50° + 55°) = 75°$

$\Rightarrow \quad \angle RMN = 75°$ (from (iii) and (iv)) ...(iv)

Now in ΔRMN, $\angle MRN = 180° - (\angle RMN + \angle RNM)$

$= 180° - (75° + 75°) = 30°$

Now, In ΔPQR, $\angle P + \angle Q + \angle R = 180°$ ...(v)

$\therefore \quad \angle Q = 180° - (\angle P + \angle R)$

$= 180° - (70° + 30°) = 180° - 100° = 80°$ [from (ii) and (v)]

$\therefore \quad \angle P = 70°, \angle Q = 80°$ and $\angle R = 30°$;

20. PQ is a line segment and R is its midpoint. Semicircles are drawn at the same side of PQ taking PR, RQ and PQ as diameters. A circle of radius r and centre O is drawn touching all the three semi circles.

Prove that $r = \frac{1}{6}PQ$

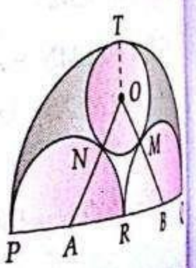
Solution : Let, $PQ = x$

$\therefore \quad PR = RQ = \frac{1}{2}PQ = \frac{x}{2}$ ... (i)

A and B are respectively midpoints of PR and RQ.

$\therefore \quad AR = \frac{1}{2}PR = \frac{x}{4}$ [from (i)]

and $\quad RB = \frac{1}{2}RQ = \frac{x}{4}$



In ΔOAB

$OA = ON + NA = r + \frac{x}{4}$

$OB = OM + MB = r + \frac{x}{4}$

$\therefore$ $OA = OB$

Thus ΔOAB is an isosceles triangle and R is midpoint of its base AB.

$\therefore$ $OR \perp AB$

Now, from right angled ΔORA,

$OA^2 = OR^2 + AR^2$

or, $(ON + NA)^2 = OR^2 + AR^2$

or, $\left(\frac{x}{4}+r\right)^2 = (RT - TO)^2 + \left(\frac{x}{4}\right)^2$ $[\because OR = RT - TO]$

or, $\frac{x^2}{16} + r^2 + \frac{1}{2}x \cdot r = \left(\frac{x}{2} - r\right)^2 + \frac{x^2}{16}$ $[\because RT = RQ = \frac{x}{2}]$

or, $\frac{x^2}{16} + r^2 + \frac{xr}{2} = \frac{x^2}{4} - rx + r^2 + \frac{x^2}{16}$

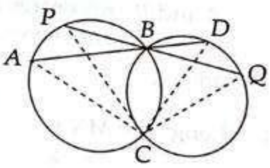
or, $\frac{3}{2}xr = \frac{x^2}{4}$

or, $3r = \frac{x}{2}$

or, $r = \frac{x}{6}$

$\therefore$ $r = \frac{1}{6}PQ$ Proved

21. In the given figure two circles intersect at two points B and C. Two line segments ABD and PBQ passing through point B, intersects circles respectively at A, D and P, Q. Prove that $\angle ACP = \angle QCD$



Solution : Since angle in the same segment are equal

$\therefore$ $\angle ACP = \angle ABP$, (take AP as base) ... (i)

$\angle QCD = \angle QBD$, (take QD as base) ... (ii)

and $\angle ABP = \angle QBD$ (vertically opposite angle) ... (iii)

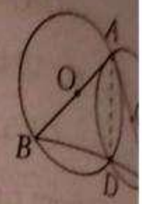
from Adding (i), (ii) and (iii)

$\angle ACP = \angle QCD$

22. If two circles are drawn taking any two sides to the triangle as diameter then prove that point of intersection of two circles lies on the third side.

Solution : Let ABC be a triangle. Two circles are taking AB and AC as diameters. Both circles intersect

Prove To : Point D lies on line BC.



Join $A-D$

Since AB and AC are diameter of two circles and angle in a semi circle is right angle.

$\therefore\ \angle ADB = 90^\circ$ and $\angle ADC = 90^\circ$

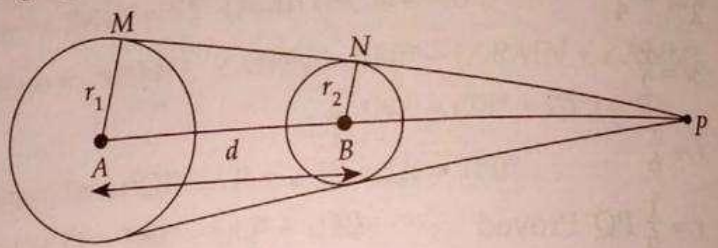
Adding $\angle ADB + \angle ADC = 90^\circ + 90^\circ = 180^\circ$

i.e. BDC is a straight line

Thus point D lies on line BC. Proved.

Exercise—8A

Instruction (1 – 6) : Answer the questions given below on the basis following figure.



A and B are centres of the circles whose radii are respectively r_1 and r_2. PNM is a direct common tangent touching the circles respectively at M and N.

1. Length of MN is

   (a) $\sqrt{d^2-(r_1-r_2)^2}$ (b) $\sqrt{d^2+(r_1-r_2)^2}$

   (c) $d^2-(r_1-r_2)^2$ (d) $d+\sqrt{(r_1-r_2)^2}$

2. Ratio $PA : PB$ equals

   (a) $r_1 : r_2$ (internal) (b) $r_1 : r_2$ (external)

   (c) $r_2 : r_1$ (internal) (d) $r_2 : r_1$ (external)

3. Length of AP is

   (a) $\frac{r_1 d}{r_1+r_2}$ (b) $\frac{r_2 d}{r_1+r_2}$ (c) $\frac{r_1 d}{r_1-r_2}$ (d) $\frac{r_2 d}{r_1-r_2}$

4. Length of BP is

   (a) $\frac{r_1 d}{r_1+r_2}$ (b) $\frac{r_2 d}{r_1+r_2}$ (c) $\frac{r_1 d}{r_1-r_2}$ (d) $\frac{r_2 d}{r_1-r_2}$

Distance between P and N is

(a) $\frac{r_2}{r_1 - r_2}\sqrt{d^2 - (r_1 - r_2)^2}$ (b) $\frac{r_1}{r_1 - r_2}\sqrt{d^2 - (r_1 - r_2)^2}$

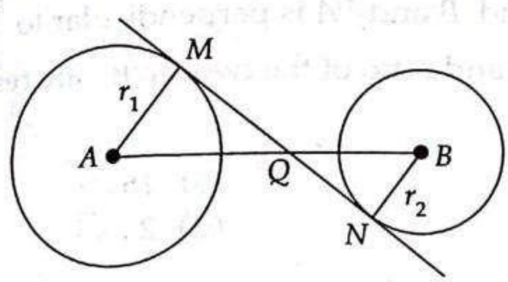
(c) $\frac{r_2}{r_1 + r_2}\sqrt{d^2(r_1 - r_2)^2}$ (d) $\frac{r_1}{r_1 + r_2}\sqrt{d^2 - (r_1 - r_2)^2}$

Distance between P and M is

(a) $\frac{r_2}{r_1 - r_2}\sqrt{d^2 - (r_1 - r_2)^2}$ (b) $\frac{r_1}{r_1 - r_2}\sqrt{d^2 - (r_1 - r_2)^2}$

(c) $\frac{r_2}{r_1 + r_2}\sqrt{d^2 - (r_1 - r_2)^2}$ (d) $\frac{r_1}{r_1 + r_2}\sqrt{d^2 - (r_1 - r_2)^2}$

Instruction (7 – 12) : Answer the questions given below on the basis of following figure



MN is a transverse common tangent. A and B are centres of the circles whose radii are respectively r_1 and r_2. Length of AB is d.

Length of MN is

(a) $\sqrt{d^2 - (r_1 + r_2)^2}$ (b) $\sqrt{d^2 - (r_1 - r_2)^2}$

(c) $\sqrt{d^2 + (r_1 - r_2)^2}$ (d) $\sqrt{d^2 + (r_1 + r_2)^2}$

Ratio $AQ : QB$ equals

(a) $r_1 : r_2$ (external) (b) $r_1 : r_2$ (internal)

(c) $r_2 : r_1$ (internal) (d) $r_2 : r_1$ (external)

Length of AQ is

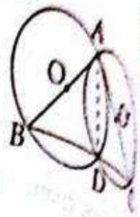
(a) $\frac{r_1 d}{r_1 - r_2}$ (b) $\frac{r_2 d}{r_1 - r_2}$ (c) $\frac{r_1 d}{r_1 + r_2}$ (d) $\frac{r_2 d}{r_1 + r_2}$

Length of BQ is

(a) $\frac{r_1 d}{r_1 - r_2}$ (b) $\frac{r_2 d}{r_1 - r_2}$

(c) $\frac{r_1 d}{r_1 + r_2}$ (d) $\frac{r_2 d}{r_1 + r_2}$

Prove To : Point D lies on line BC.



Join $A-D$

Since AB and AC are diameter of two circles and angle in a semi circle is right angle.

$\therefore \quad \angle ADB = 90^\circ$ and $\angle ADC = 90^\circ$

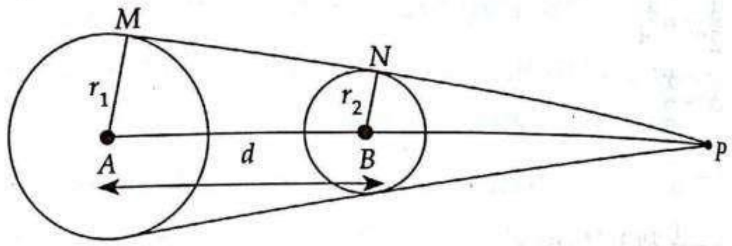
Adding $\angle ADB + \angle ADC = 90^\circ + 90^\circ = 180^\circ$

i.e. BDC is a straight line

Thus point D lies on line BC. Proved.

Exercise—8A

Instruction (1 – 6) : Answer the questions given below on the basis of following figure.



A and B are centres of the circles whose radii are respectively r_1 and r_2. PNM is a direct common tangent touching the circles respectively at M and N.

1. Length of MN is

(a) $\sqrt{d^2-(r_1-r_2)^2}$ (b) $\sqrt{d^2+(r_1-r_2)^2}$

(c) $d^2-(r_1-r_2)^2$ (d) $d+\sqrt{(r_1-r_2)^2}$

2. Ratio $PA : PB$ equals

(a) $r_1 : r_2$ (internal) (b) $r_1 : r_2$ (external)

(c) $r_2 : r_1$ (internal) (d) $r_2 : r_1$ (external)

3. Length of AP is

(a) $\frac{r_1 d}{r_1+r_2}$ (b) $\frac{r_2 d}{r_1+r_2}$ (c) $\frac{r_1 d}{r_1-r_2}$ (d) $\frac{r_2 d}{r_1-r_2}$

4. Length of BP is

(a) $\frac{r_1 d}{r_1+r_2}$ (b) $\frac{r_2 d}{r_1+r_2}$ (c) $\frac{r_1 d}{r_1-r_2}$ (d) $\frac{r_2 d}{r_1-r_2}$

5. Distance between P and N is

(a) $\frac{r_2}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$ (b) $\frac{r_1}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$

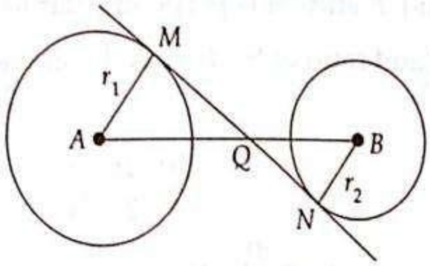
(c) $\frac{r_2}{r_1+r_2}\sqrt{d^2(r_1-r_2)^2}$ (d) $\frac{r_1}{r_1+r_2}\sqrt{d^2-(r_1-r_2)^2}$

6. Distance between P and M is

(a) $\frac{r_2}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$ (b) $\frac{r_1}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$

(c) $\frac{r_2}{r_1+r_2}\sqrt{d^2-(r_1-r_2)^2}$ (d) $\frac{r_1}{r_1+r_2}\sqrt{d^2-(r_1-r_2)^2}$

Instruction (7 – 12) : Answer the questions given below on the basis of following figure



MN is a transverse common tangent. A and B are centres of the circles whose radii are respectively r_1 and r_2. Length of AB is d.

7. Length of MN is

(a) $\sqrt{d^2-(r_1+r_2)^2}$ (b) $\sqrt{d^2-(r_1-r_2)^2}$

(c) $\sqrt{d^2+(r_1-r_2)^2}$ (d) $\sqrt{d^2+(r_1+r_2)^2}$

8. Ratio $AQ : QB$ equals

(a) $r_1 : r_2$ (external) (b) $r_1 : r_2$ (internal)

(c) $r_2 : r_1$ (internal) (d) $r_2 : r_1$ (external)

9. Length of AQ is

(a) $\frac{r_1 d}{r_1-r_2}$ (b) $\frac{r_2 d}{r_1-r_2}$ (c) $\frac{r_1 d}{r_1+r_2}$ (d) $\frac{r_2 d}{r_1+r_2}$

10. Length of BQ is

(a) $\frac{r_1 d}{r_1-r_2}$ (b) $\frac{r_2 d}{r_1-r_2}$

(c) $\frac{r_1 d}{r_1+r_2}$ (d) $\frac{r_2 d}{r_1+r_2}$

11. Distance between point Q and M is

(a) $\frac{r_1}{r_1+r_2}\sqrt{d^2-(r_1+r_2)^2}$ (b) $\frac{r_2}{r_1+r_2}\sqrt{d^2-(r_1+r_2)^2}$

(c) $\frac{r_1}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$ (d) $\frac{r_2}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$

12. Distance between point Q and N is

(a) $\frac{r_1}{r_1+r_2}\sqrt{d^2-(r_1+r_2)^2}$ (b) $\frac{r_2}{r_1+r_2}\sqrt{d^2-(r_1+r_2)^2}$

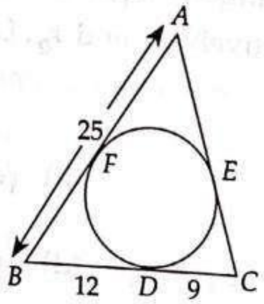
(c) $\frac{r_1}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$ (d) $\frac{r_2}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$

13. Two circles cut each other at points P and Q. Centres of two circles are respectively A and B and PA is perpendicular to PB. If AB intersects segment PQ at R and ratio of the two circles are respectively 16 : 9 then $AR : BR$ is

(a) 4 : 3 (b) 16 : 9

(c) 256 : 81 (d) $2 : \sqrt{3}$

14. In the given figure length of side AC is



(a) 20 (b) 22 (c) 21 (d) 18

15. PQ is a line segment of 12 cm whose midpoint is R. Taking PR, RQ and PQ as diameters semicircles are drawn at the same side of PQ. The area of the circle that touches all the three circles is

(a) 2π sq. cm (b) 4π sq. cm (c) 6π sq. cm (d) $\frac{9}{4}$ sq. cm

16. The difference in lengths of parallel sides of a trapezium inscribed in a circle is 6 cm; if distance between parallel sides is 4 cm then difference in lengths of its non parallel side is

(a) 10 cm (b) 5 cm

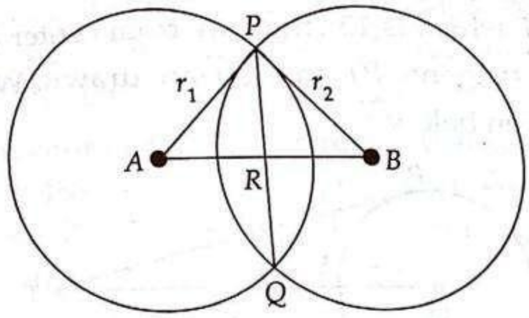
(c) 0 cm (d) Cannto be determined

17. The length of two perpendicular chords of a circle are respectively $2a$ and $2b$. If distance of its point on intersection from the centre is c, then what is the radius of circle ?

(a) $\frac{\sqrt{a^2+b^2+c^2}}{2}$ (b) $\sqrt{\frac{a^2+b^2+c^2}{2}}$

(c) $\frac{\sqrt{a^2+b^2-c^2}}{2}$ (d) $\sqrt{\frac{a^2+b^2-c^2}{2}}$

18. In the given figure A and B are centres of the circles. If $AR = a$, $RB = b$ then which of the following is equal to $a-b$?



(a) $\frac{r_1^2-r_2^2}{\sqrt{r_1^2+r_2^2}}$ (b) $\frac{r_1^2+r_2^2}{\sqrt{r_1^2-r_2^2}}$

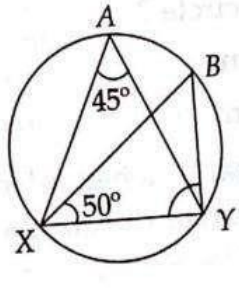
(c) $\frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}}$ (d) None of these

19. A circle with radius r has a chord PQ whose length is $2a$. The tangents drawn at points P and Q to the circle meet at T, what is the length of TP?

(a) $\frac{ar}{\sqrt{r^2-a^2}}$ (b) $\frac{2ar}{\sqrt{r^2-a^2}}$

(c) $\frac{r^2+a^2}{\sqrt{r^2-a^2}}$ (d) $\frac{ar}{r-a}$

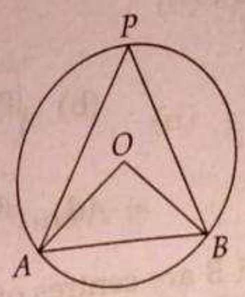
20. In the figure given below what is the measure of $\angle BYX$?



(a) 85° (b) 50°

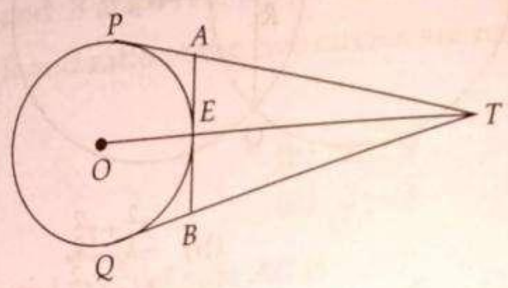
(c) 45° (d) 90°

21. In the figure given below, radius OA is equal to chord AB. What is the measure of $\angle APB$?



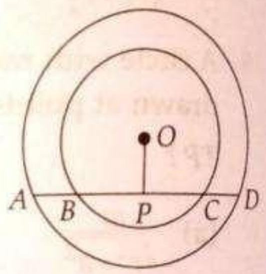
(a) 30° (b) 60° (c) 15° (d) 45°

22. From a point T which is 13 cm away from center O of a circle whose radius is 5 cm, tangents PT and QT are drawn. What is the length of AB in figure given below.



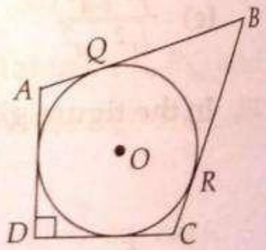
(a) $\frac{19}{3}$ cm (b) $\frac{20}{3}$ cm (c) $\frac{40}{13}$ cm (d) $\frac{22}{3}$ cm

23. In the adjacent figure AD is a straight line. OP is perpendicular to AD and O is centre of both circles. If $OA = 20$ cm, $OB = 15$ cm and $OP = 12$ cm then what is the length of AB ?



(a) 7 cm (b) 8 cm
(c) 10 cm (d) 12 cm

24. In the adjacent figure, a circle is inscribed in the quadrilateral $ABCD$. If $BC = 38$ cm, $QB = 27$ cm, $DC = 25$ cm and AD is perpendicular to DC then what is the radius of the circle ?

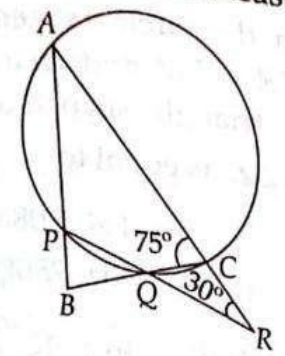


(a) 11 cm (b) 14 cm
(c) 15 cm (d) 16 cm

25. Each side of a quadrilateral touches a circle. If length of its three consecutive sides are 6 cm, 7 cm and 5 cm then what is length of its fourth side ?

(a) 3 cm (b) 4 cm
(c) 5 cm (d) 8 cm

26. In the figure given below, what is the measure of $\angle CBA$?



(a) 30° (b) 45° (c) 50° (d) 60°

27. A, B, C, D are four distinct points on a circle whose centre is O. If $\angle OBD - \angle CDB = \angle CBD - \angle ODB$ then what is the measure of $\angle A$?

(a) 45° (b) 60° (c) 120° (d) 135°

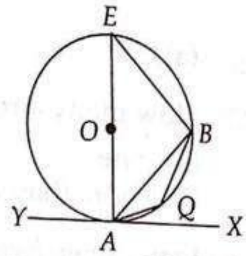
28. PQ is a common chord of the two circles. APB is a secant line joining points A and B respectively on the two circles. Two equal tangents AC and BC are drawn. If $\angle ACB = 45°$ then which is equal to $\angle AQB$?

(a) 75° (b) 90° (c) 120° (d) 135°

29. $ABCD$ is a cyclic quadrilateral. Tangents at A and C intersect at P. If $\angle ABC = 100°$ then what is the measure of $\angle APC$?

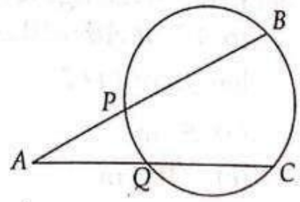
(a) 10° (b) 20° (c) 30° (d) 40°

30. In the adjacent figure, YAX is a tangent to the circle with centre O. If $\angle BAX = 70°$ and $\angle BAQ = 40°$ then what is $\angle ABQ$?



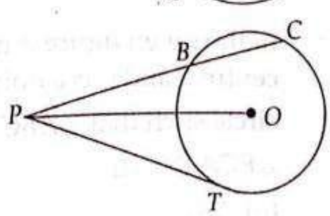
(a) 20° (b) 30°
(c) 35° (d) 40°

31. In the adjacent figure, $AP = 3$ cm, $PB = 5$ cm, $AQ = 2$ cm and $QC = x$. What is the value of x ?



(a) 6 cm (b) 8 cm
(c) 10 cm (d) 12 cm

32. In the adjacent figure PT is tangent to the circle with radius 6 cm. If distance between point P and centre O is 10 cm and $PB = 5$ cm, then is the length of chord BC ?

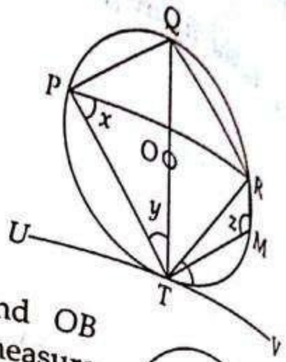


(a) 7.8 cm (b) 8.0 cm
(c) 8.4 cm (d) 9.0 cm

33. A point moves such that its distance from two fixed points A and B always remains same. What is the locus of point P ?

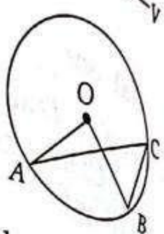
(a) a straight line which is perpendicular bisector to AB
(b) a circle whose centre is A
(c) a circle whose centre is B
(d) a straight line passing thorough either A or B.

34. In the adjacent figure O is the centre of the circle. At a point T on the circle tangent $\angle UTV$ is draw. If $\angle VTR = 52°$ and triangle PTR is an isosceles triangle such that $TP = TR$ then $\angle x + \angle y + \angle z$ is equal to—



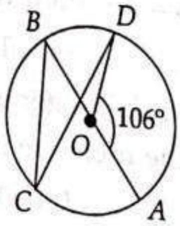
(a) 175° (b) 208°
(c) 218° (d) 250°

35. In the adjacent figure, $\angle AOB = 46°$; AC and OB mutually intersect at right angle. What is the measure of $\angle OBC$ where O is the centre of the circle.



(a) 44° (b) 46°
(c) 67° (d) 78.5°

36. In the figure given below O is the centre of the circle and $\angle AOD = 106°$. $\angle BCD$ is equal to

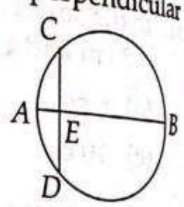


(a) 53° (b) 43° (c) 40° (d) 37°

37. How many circles can pass through a given pair of points ?

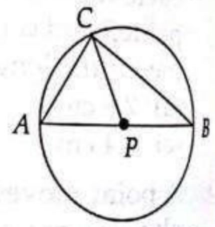
(a) one (b) only two
(c) more than two but finite (d) infinitely many

38. In the given figure AB is a diameter of the circle and CD is perpendicular to AB. If $AB = 10$ cm and $AE = 2$ cm then what is the length of ED ?



(a) 5 cm (b) 4 cm
(c) $\sqrt{10}$ cm (d) $\sqrt{20}$ cm

39. In the given figure A and B are extremities of a diameter of a circle with centre P and C is a point on the circumference of the circle such that $\angle ABC = 35°$. What is the measure of $\angle PCA$?



(a) 25° (b) 30°
(c) 35° (d) 55°

40. $ABCD$ is a quadrilateral whose sides touch a given circle. Which of the following is true regarding above statement ?

(a) $AB + AD = CB + CD$ (b) $AB : CD = AD : BC$
(c) $AB + CD = AD + BC$ (d) $AB : AD = CB : CD$

41. Suppose PAB is a secant to a circle which intersects circle at A and B and PC is a tangent. Which of the following is true ?

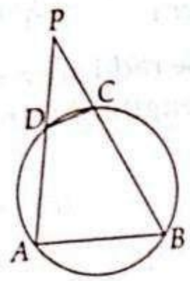
(a) Area of the rectangle with PA, PB as adjacent sides is equal to area of square whose side is PC

(b) Area of the rectangle with PA, PC as adjacent sides is equal to area of square whose each side is PB

(c) Area of the rectangle with PC, PB as adjacent sides is equal to area of square whose each side is PA

(d) Perimeter of the rectangle with PA, PB as adjacent side is equal to perimeter of the square whose each side is PC

42. In the figure given below if $\angle BAD = 60^\circ$, $\angle ADC = 105^\circ$ then $\angle DPC$ is equal to



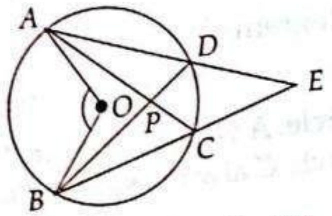
(a) 40° (b) 45° (c) 50° (d) 60°

43. In the given figure O is centre and PQ is a diameter of the circle. If $\angle ROS = 44^\circ$ and OR is bisector of $\angle PRQ$ then measure of $\angle RTS$ is

(a) 46° (b) 64°

(c) 69° (d) None of these

44. In the figure given below O is the centre of the circle while AC and BD intersect at P. If $\angle AOB = 100^\circ$ and $\angle DAP = 30^\circ$ then what is the measure of $\angle APB$?



(a) 77° (b) 80° (c) 85° (d) 90°

45. Suppose A and B are two fixed points. What is the locus of P if angle $APB = 90^\circ$?

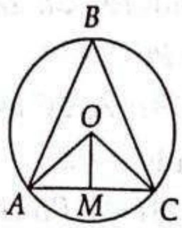
(a) line AB itself

(b) Point P itself

(c) circumference of the circle having AB as diameter

(d) perpendicular bisector to line AB

46. In the figure given below O is centre of the circle, $OA = 3cm$, $AC = 3cm$ and OM is perpendicular to AC. What is the measure of $\angle ABC$?



(a) $60°$ (b) $45°$
(c) $30°$ (d) None of these

47. Two circles touch each other internally. Their radii are respectively 4 cm and 6 cm. What is the maximum length of chord of outer circle which lies out side the inner circle.

(a) $4\sqrt{2}$ cm (b) $4\sqrt{3}$ cm (c) $6\sqrt{3}$ cm (d) $8\sqrt{2}$ cm

48. Centres of two circles whose radii are respectively 4.5 cm and 3.5 cm are 10 cm apart. What is the length of transverse common tangent to the two circle ?

(a) 8 cm (b) 7 cm (c) 6 cm (d) None of these

49. If radii of two circles are respectively 6 cm and 3 cm and length of transverse common tangent to the two circles is 8 cm, then what is the distance between centres of the two circles ?

(a) 14 cm (b) $\sqrt{145}$ cm (c) $\sqrt{155}$ cm (d) 13 cm

50. ABC is an equilateral triangle inscribed in a circle with $AB = 5$ cm. Suppose bisector of angle A meets BC at X and circle at Y, then what is the value of $3\ AX.\ AY$?

(a) 16 cm^2 (b) 20 cm^2 (c) 25 cm^2 (d) 30 cm^2

51. Two unequal circles touch each other externally at point P. If APB and CPD are two secants intersecting circles at A, B, C and D then which of the following is true ?

(a) $ACBD$ is a parallelogram (b) $ACBD$ is a trapezium
(c) $ACBD$ is a rhombus (d) None of the above

52. Suppose C is a given circle. A variable point P moves such that tangents drawn from P to the circle C always subtends an angle 60°. What is the locus of P ?

(a) a straight line
(b) a circle concentric with circle C
(c) a circle touching circle C
(d) a circle intersecting circle C at two distinct points

53. $ABCD$ is a cyclic quadrilateral and $A + B = 2(C + D)$. If $\angle C > 30°$, then which of the following is true

(a) $\angle D \geq 90°$ (b) $\angle D < 90°$ (c) $\angle D \leq 90°$ (d) $\angle D > 90°$

54. If diameters of two circles are 6 units and 10 units and their centres are 8 unit apart, how many tangents line can be drawn to the circle ?

(a) 1 (b) 2 (c) 3 (d) 4

55. Point A is situated at a distance of 6.5 cm from the centre of a circle. The length of tangent drawn from point A to the circle is 6 cm. What is the radius of the circle ?

(a) 5 cm (b) 4 cm (c) 3.5 cm (d) 2.5 cm

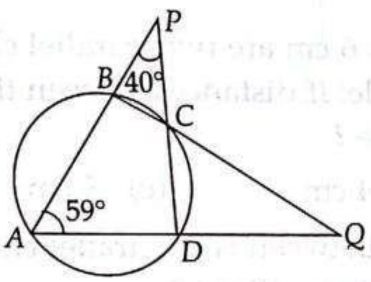
56. A circle with centre O is given and C is a point on its minor arc AB. If $\angle AOB = 100°$ then $\angle ACB$ is equal to which of the following ?

(a) 80° (b) 90° (c) 100° (d) 130°

57. ΔABC is a triangle with $AB = AC$. A circle passing through point P touches AC at D and cuts AB at P. If D is the midpoint of AC then which one of the following is true ?

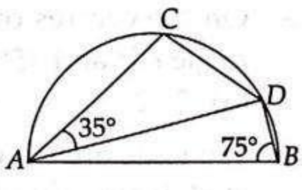
(a) $AB = 2AP$ (b) $AB = 3AP$ (c) $AB = 4AP$ (d) $2AB = 5AP$

58. In the figure given below $\angle PAQ = 59°$, $\angle APD = 40°$ then $\angle AQB$ is equal to which of following ?



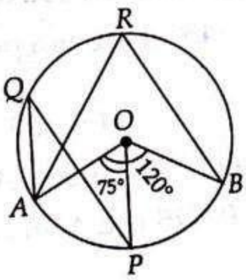
(a) 19° (b) 20° (c) 22° (d) 27°

59. In the adjacent figure C and D are two points on circumference of a semicircle having AB as diameter. Given that $\angle ABD = 75°$ and $\angle DAC = 35°$. What is the measure of $\angle BDC$?



(a) 130° (b) 110°
(c) 90° (d) 100°

60. In the figure given below, if $\angle AOP = 75°$ and $\angle AOB = 120°$, then what is $\angle AQP$?



(a) 45° (b) 37.5° (c) 30° (d) 22.5°

61. Length of two chords AB and AC of a circle are respectively 8 cm and 6 cm and $\angle BAC = 90°$. What is the radius of the circle ?
(a) 25 cm (b) 20 cm (c) 4 cm (d) 5 cm

62. The chord of circle whose radius is 5 cm touches another circle whose radius is 3 cm. If two circles are concentric, then what is length of the chord ?
(a) 10 cm (b) 12.5 cm (c) 8 cm (d) 7 cm

63. A chord of a circle is equal to its radius. Angle subtends by the chord on major arc of the circles is
(a) 30° (b) 45° (c) 60° (d) 90°

64. Radii of two concentric circles are, respectively 9 cm and 15 cm. If chord of larger circle touches the smaller one then length of the chord is
(a) 24 cm (b) 12 cm (c) 30 cm (d) 18 cm

65. Two chords AB and CD of a circle with centre O meet at an out side point P. If $\angle AOC = 50°$, $\angle BOD = 40°$ then what is the measure of $\angle BPD$?
(a) 60° (b) 40° (c) 45° (d) 5°

66. $AB = 8$ cm and $CD = 6$ cm are two parallel chords lie on same side of centre of a given circle. If distance between them is 1 cm then what is the radius of the circle ?
(a) 5 cm (b) 4 cm (c) 3 cm (d) 2 cm

67. What is the distance between two parallel chords each having length 8 cm of a circle of diameter 10 cm ?
(a) 6 cm (b) 7 cm (c) 8 cm (d) 5.5 cm

68. On the centres of two circles are of same length respectively subtend angle 60° and 75°. What is the ratio of radii of the two circles.
(a) 5 : 2 (b) 5 : 4 (c) 3 : 2 (d) 2 : 1

69. Tangents are drawn at extremities AB of a diameter of a circle centred at P. If tangents drawn at a point C on the circle meet the other two tangents respectively at Q and R then measure of $\angle QPR$ is
(a) 45° (b) 60° (c) 90° (d) 180°

70. AB is a chord to a given circle and PAT is a tangent to the circle at point A. If $\angle BAT = 75°$ and $\angle BAC = 45°$ and C is a point on the circle, then $\angle ABC$ is
(a) 40° (b) 45° (c) 60° (d) 70°

71. Consider a circle centred at O. Tangents at A and B to the circle intersects at P. In the quadrilateral $PAOB$ if $\angle AOB : \angle APB = 5 : 1$, then measure of $\angle APB$ is
(a) 30° (b) 60° (c) 45° (d) 15°

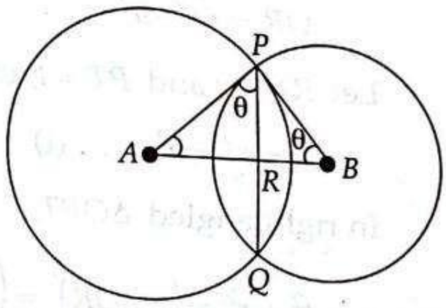
Answers—8A

1. (a)	2. (b)	3. (c)	4. (d)	5. (a)	6. (b)	7. (a)	8. (b)
9. (c)	10. (d)	11. (a)	12. (b)	13. (b)	14. (b)	15. (b)	16. (c)
17. (b)	18. (a)	19. (a)	20. (a)	21. (a)	22. (b)	23. (a)	24. (b)
25. (b)	26. (d)	27. (b)	28. (d)	29. (b)	30. (b)	31. (c)	32. (a)
33. (a)	34. (c)	35. (c)	36. (d)	37. (d)	38. (b)	39. (d)	40. (c)
41. (a)	42. (b)	43. (d)	44. (b)	45. (c)	46. (c)	47. (d)	48. (c)
49. (b)	50. (c)	51. (b)	52. (b)	53. (b)	54. (c)	55. (d)	56. (d)
57. (c)	58. (c)	59. (a)	60. (b)	61. (d)	62. (c)	63. (a)	64. (a)
65. (d)	66. (a)	67. (a)	68. (b)	69. (d)	70. (c)	71. (a)	

Explanation

For 1 to 12, See theory portion carefully.

13. (b) $\because \Delta ARP \sim PRB$



$$\frac{\text{area of }(\Delta ARP)}{\text{area of }(\Delta PRB)} = \left(\frac{AP}{PB}\right)^2$$

or, $$\frac{\frac{1}{2}\cdot AR\cdot PR}{\frac{1}{2}\cdot RB\cdot PR} = \frac{r_1^2}{r_2^2}$$

$$\Rightarrow \frac{AR}{RB} = \frac{r_1^2}{r_2^2} = \frac{16}{9}$$

14. (b) $BF = BD = 12$

$\therefore \; AF = 25 - 12 = 13 = AE$

Hence, $AC = AE + EC = 13 + 9 = 22$

15. (b) See solved example 20

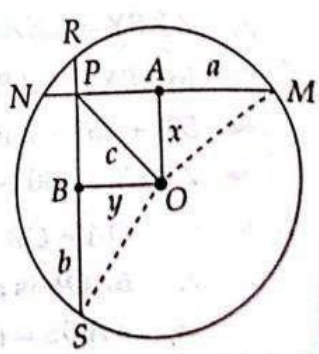
$r = \frac{PQ}{6} = \frac{12}{6} = 2$ cm $\quad \therefore \;$ Area $= \pi r^2 = 4\pi \text{ cm}^2$.

16. (c) We know that non parallel sides of a trapezium inscribed in a circle are equal. Thus required difference = 0 cm

17. (b) In figure, chord $MN = 2a$, chord $RS = 2b$

$OA \perp r \; MN \quad \Rightarrow \quad AM = a$

$OB \perp r \; RS \quad \Rightarrow \quad BS = b$



Let $OA = x$, $OB = y$

$\therefore \; PB = x$ and $PA = y$

In ΔOAM, $a^2 + x^2 = r^2$

In ΔOBS, $b^2 + y^2 = r^2$

adding $a^2 + b^2 + x^2 + y^2 = 2r^2 \qquad \ldots$ (i) but in ΔOPA, $x^2 + y^2 = c^2$

$\therefore$ from (i), $a^2 + b^2 + c^2 = 2r^2$

or, $r = \sqrt{\dfrac{a^2+b^2+c^2}{2}}$

18. (a) $r_1^2 = PR^2 + a^2$ and $r_2^2 = PR^2 + b^2$

$r_1^2 - r_2^2 = a^2 - b^2$

or, $(a-b)(a+b) = r_1^2 - r_2^2$

or, $(a-b) = \dfrac{r_1^2 - r_2^2}{a+b} = \dfrac{r_1^2 - r_2^2}{\sqrt{r_1^2 + r_2^2}}$ $\left(\because a + b = AB = \sqrt{AP^2 + BP^2}\right)$

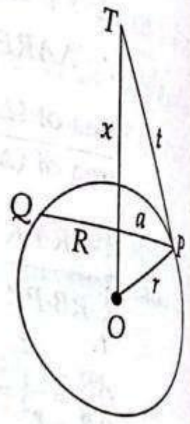
19. (a) In the given figure,

$OR = \sqrt{r^2 - a^2}$

Let $RT = x$ and $PT = t$, then in right angle ΔTRP

$t^2 = x^2 + a^2$ ... (i)

In right angled ΔOPT,



$t^2 + r^2 = (x + OR)^2 = \left(x + \sqrt{r^2 - a^2}\right)^2$

or, $t^2 + r^2 = x^2 + r^2 - a^2 + 2x\sqrt{r^2 - a^2}$

or, $r^2 + a^2 + r^2 = r^2 + r^2 - a^2 + 2x\sqrt{r^2 - a^2}$

from (i), $t^2 = x^2 + a^2$)

or, $2a^2 = 2x\sqrt{r^2 - a^2}$

or,, $x = \dfrac{a^2}{\sqrt{r^2 - a^2}}$ $\therefore$ $t^2 = x^2 + a^2 = \dfrac{a^4}{r^2 - a^2} + a^2 = \dfrac{a^4 + a^2r^2 - a^4}{r^2 - a^2}$

or, $t = \dfrac{ar}{\sqrt{r^2 - a^2}}$

20. (a) We know that angles in the same segment of a circle are equal.

$\therefore$ $\angle XBY = \angle XAY = 45°$

In ΔBXY, $\angle BXY + \angle XBY + \angle BYX = 180°$

$\Rightarrow$ $50° + 45° + \angle BYX = 180°$ $(\because \angle BXY = 50°)$

$\Rightarrow$ $\angle BYX = 180° - 95° = 85°$

21. (a) $\because$ $OA = OB = AB$ (given)

$\therefore$ ΔAOB is an equilateral triangle.

$\Rightarrow$ $\angle AOB = 60°$

We know that the angle at the centre in a circle is double the angle at circumference.

$\therefore \angle AOB = 2\angle APB \quad \Rightarrow \quad \angle APB = \frac{60^\circ}{2} = 30^\circ$

22. (b) Given, $OT = 13, OP = 5$

$\therefore PT = \sqrt{13^2 - 5^2} = 12$, $TE = OT - OE = 13 - 5 = 8$

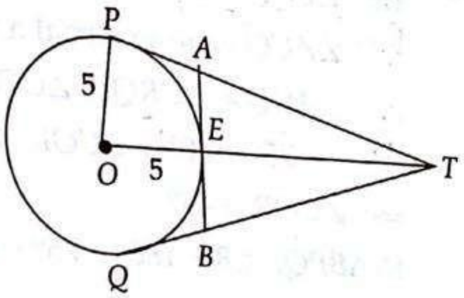
In ΔTOP and ΔTAE, angle at point T is common

and $\angle OPT = \angle AET = 90^\circ$ (angle made by tangent line)

$\therefore \Delta TOP \sim \Delta TAE \Rightarrow \frac{TO}{TA} = \frac{TP}{TE} = \frac{OP}{AE}$

$\Rightarrow \frac{13}{TA} = \frac{12}{8} = \frac{5}{AE}$

$\Rightarrow AE = \frac{5 \times 8}{12} = \frac{10}{3}$



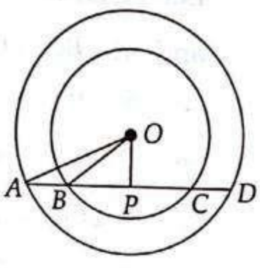
$\therefore$ By Symmetry, $AB = 2AE = \frac{20}{3}$

23. (a) In ΔOPB,

$OB^2 = OP^2 + BP^2$

$\Rightarrow (15)^2 = (12)^2 + BP^2$

$\Rightarrow BP^2 = \sqrt{15^2 - 12^2} = 9$



and In ΔAOP,

$OA^2 = OP^2 + AP^2$

$\Rightarrow (20)^2 = (12)^2 + AP^2 \Rightarrow AP = \sqrt{20^2 - 12^2} = 16$

Hence $AB = AP - BP = 16 - 9 = 7$ cm

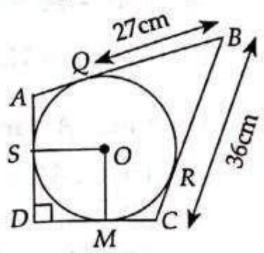
24. (b) $\because$ Length of tangents from an out side point are equal

$\therefore BQ = BR = 27$

$\Rightarrow RC = 38 - 27 = 11$cm

$\therefore RC = CM = 11$ cm

Now, $DM = 25 - 11 = 14$ cm = radius of circle



25 (b) See the figure

Let, $AP = AS = a$

$BP = BQ = b$

$CQ = CR = c$

and $DR = DS = d$

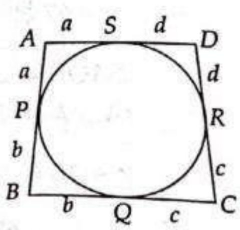
According to question,

$a + b = 6, b + c = 7, c + d = 5$

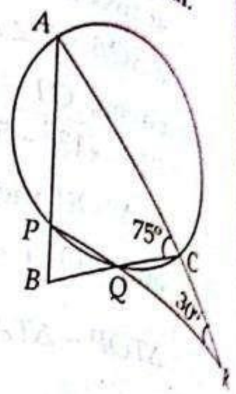
$\therefore (a + b) + (c + d) - (b + c) = 6 + 5 - 7$

or, $a + d = 4$

or, $AD = 4$



26. (d) $\because$ Sum of opposite angles of a cyclic quadrilateral are equal.



$\therefore \quad \angle ACQ + \angle APQ = 180°$

$\Rightarrow \quad 75° + \angle APQ = 180°$

$\Rightarrow \quad \angle APQ = 105°$

$\because \quad \angle APQ + \angle BPQ = 180°$

$\therefore \quad 105° + \angle BPQ = 180°$

or, $\angle BPQ = 180° - 105° = 75°$

$\because \quad \angle ACQ$ is an external angle of ΔRCQ

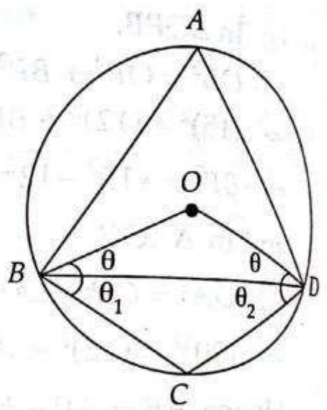
$\therefore \quad \angle ACQ = \angle CRQ + \angle COR$

$\Rightarrow \quad 75° = 30° + \angle COR$

$\Rightarrow \quad \angle COR = 45°$

In ΔBPQ, $\angle B = 180° - 75° - 45° = 60°$

27. (b) Given, $\angle OBD + \angle ODB = \angle CBD + \angle CDB$



Let $\angle OBD = \angle ODB = \theta$

and $\angle DBC = \theta_1, \angle BDC = \theta_2$

$\therefore \quad \theta + \theta = \theta_1 + \theta_2$

$\Rightarrow \quad 2\theta = \theta_1 + \theta_2 \quad \ldots$ (i)

$\therefore \quad \angle BOD = 180° - 2\theta$

$\Rightarrow \quad \angle BCD = \dfrac{360° - (180° - 2\theta)}{2}$

$\Rightarrow \quad 180° - (\theta_1 + \theta_2) = 90° + \theta$

$\Rightarrow \quad 180° - 2\theta = 90° + \theta \quad \Rightarrow \quad \theta = 30°$

$\therefore \quad \angle BOD = 120° \quad \Rightarrow \quad \angle BAD = 60°$

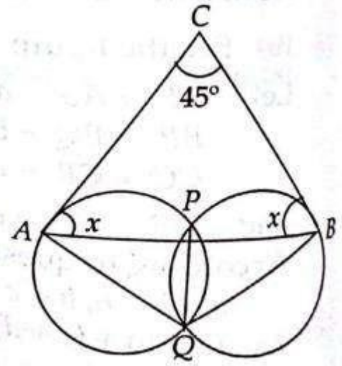
28. (d) Since AC and BC are equal, therefore $\angle CAB = \angle CBA$

let $\angle CAB = \angle CBA = x$

$\therefore \quad 45° + x + x = 180°$

$\Rightarrow \quad 2x = 180° - 45°$

$\Rightarrow \quad x = 67\frac{1}{2}°$



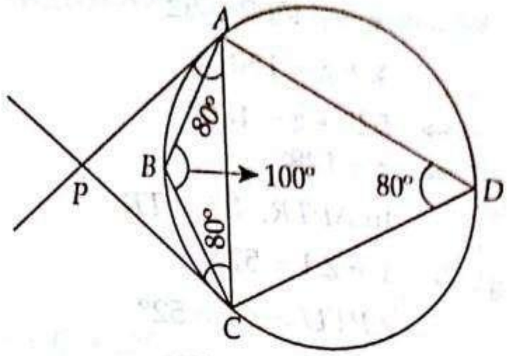
$\angle AQP = \angle x = \angle BQP = 67\frac{1}{2}°$

(Angle at the alternate segment)

$\Rightarrow \quad \angle AQB = \angle AQP + \angle BQP$

$= 67\frac{1}{2}° + 67\frac{1}{2}° = 135°$

(a) We know that sum of opposite angles of a cyclic quadrilateral is 180°



$\therefore \angle B + \angle D = 180°$

$\Rightarrow 100 + \angle D = 180°$

$\angle D = 80$

$\Rightarrow \angle ACP = \angle PAC = 80°$

$\therefore$ (Angle at the alternate segment)

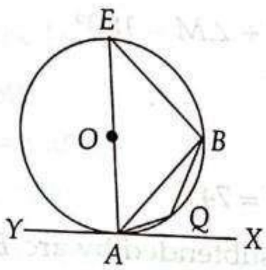
In ΔPAC,

$\angle P + \angle PAC + \angle PCA = 180°$

$\Rightarrow \angle P + 80° + 80° = 180°$

$\Rightarrow \angle P = 180° - 160° = 20°$

(b) $\angle QAX = 70° - 40° = 30°$



$\therefore \angle QAX = \angle ABQ = 30°$ [From theorem]

31. (c) See result of article 1.10, we have

$AB \times AP = AC \times AQ$

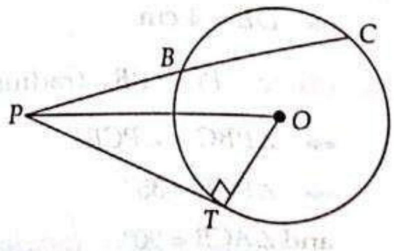
$\Rightarrow 8 \times 3 = (2 + x) \times 2$

$\Rightarrow \frac{8 \times 3}{2} = 2 + x$

$\Rightarrow x = 10$ cm

32. (a) $PO = 10$ cm, radius $OT = 6$ cm, $PB = 5$ cm

In ΔOTP, $OP^2 = PT^2 + OT^2$



$\Rightarrow 10^2 = PT^2 + 6^2$

$\Rightarrow PT = 8$ cm

But, $PT^2 = PB \times PC$

$\Rightarrow 8^2 = 5 \times (BC + PB)$

$\Rightarrow 64 = 5\,(BC + 5)$

$\Rightarrow 5BC = 39 \quad \Rightarrow BC = 7.8$ cm

33. (a) P will be perpendicular bisector of AB.

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34. (c) $x = \angle VTR = 52^\circ$

$x + z = 180^\circ$ (∵ $PTMR$ is a cyclic quadrilateral)

$\Rightarrow 52^\circ + z = 180^\circ$

$\Rightarrow z = 128^\circ$

In ΔPTR, $PT = TR$

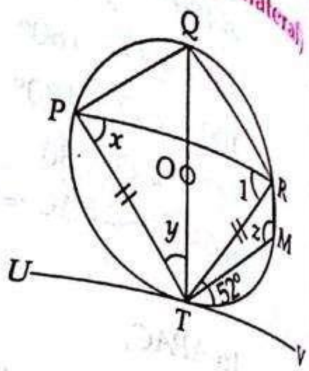
$\therefore x = \angle 1 = 52^\circ$

$\angle PTU = \angle 1 = 52^\circ$

$\angle QTU = y + 52^\circ$

$\Rightarrow 90^\circ = y + 52^\circ \Rightarrow y = 38^\circ$

$\therefore x + y + z = 52^\circ + 38^\circ + 128^\circ = 218^\circ$



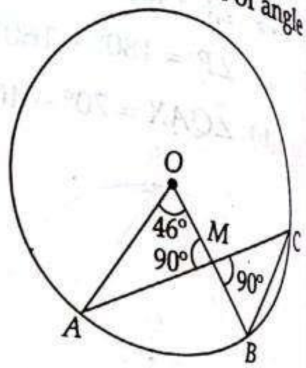
35. (c) Since angle subtended by are at circumference is half that of angle subtended at centre.

$\because \angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 46^\circ = 23^\circ$

In ΔMCB, $\angle C + \angle B + \angle M = 180^\circ$

$\Rightarrow 23 + \angle B + 90^\circ = 180^\circ$

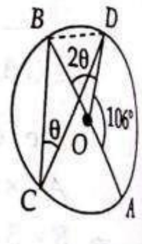
$\Rightarrow \angle B = 67^\circ$



36. (d) $\angle BOD = 180^\circ - 106^\circ = 74^\circ$

$\because$ $\angle BOD$, is the angle subtended by arc BD at centre and $\angle BCD$, is the angle subtended by arc BD at circumference

$\therefore \angle BCD = \frac{1}{2} \angle BOD = \frac{1}{2} \times 74^\circ = 37^\circ$



37. (d) For two given points, infinite circles can pass

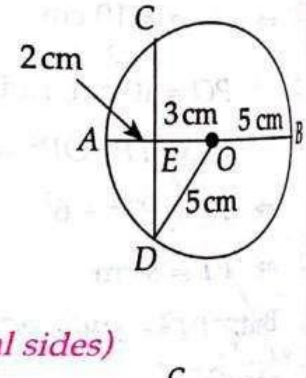
38. (b) In ΔOED,

$(OD)^2 = (DE)^2 + (EO)^2$

$\Rightarrow (5)^2 = (DE)^2 + (3)^2$

$\Rightarrow (DE)^2 = 25 - 9 = 16$

$\Rightarrow DE = 4$ cm



39. (d) $\because PC = PB$ (radius)

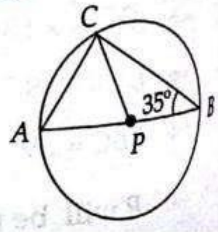
$\Rightarrow \angle PBC = \angle PCB$ *(angle opposite to equal sides)*

$\Rightarrow \angle PCB = 35^\circ$

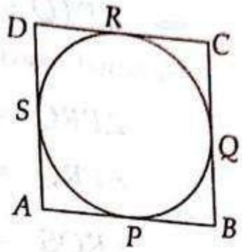
and $\angle ACB = 90^\circ$ *(angle in a semicircle)*

$\Rightarrow \angle PCA + \angle PCB = 90^\circ$

$\Rightarrow \angle PCA = 90^\circ - 35^\circ = 55^\circ$



(c) We know that length of tangents drawn from an outside point to a given circle are equal.



$\therefore$ $AP = AS$

$BP = BQ$

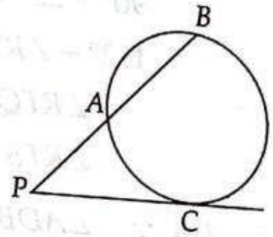
$CR = CQ$

$DR = DS$

$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow AB + CD = AD + BC$

(a) $\therefore\ PC^2 = PA \times PB$



Area of rectangle whose adjacent sides are PA and PB is equal to area of square whose each side is PC.

42. (b) $\because\ \angle BAD = 60^\circ,\ \ \angle ADC = 105^\circ$

In cyclic quadrilateral $ABCD$,

$\angle BAD + \angle BCD = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 60^\circ = 120^\circ$

Now, $\angle BCD + \angle DCP = 180^\circ$ *(linear pair of angles)*

$\Rightarrow \angle DCP = 180^\circ - 120^\circ = 60^\circ$

and $\angle ADC + \angle CDP = 180^\circ$ *(linear pair of angles)*

$\Rightarrow 105^\circ + \angle CDP = 180^\circ \quad \therefore\ \angle CDP = 75^\circ$

Hence in ΔCPD, $\angle DCP + \angle CDP + \angle DPC = 180^\circ$

$\Rightarrow 60^\circ + 75^\circ + \angle DPC = 180^\circ$

$\Rightarrow \angle DPC = 180^\circ - 135^\circ = 45^\circ$

43. (d) $\because$ Line OR is bisector of $\angle PRQ$

$\therefore\ \angle PRO = \angle ORQ = 45^\circ$

Also $OP = OR$ *(radius)*

$\therefore\ \angle OPR = 45^\circ$

In ΔORS, $OR = OS$

$\Rightarrow \angle ORS = \angle OSR = \dfrac{180^\circ - 44^\circ}{2} = 68^\circ$

$\therefore\ \angle MRS = 68^\circ - 45^\circ = 23^\circ$

$\Rightarrow \angle PRS = 90^\circ + 23^\circ = 113^\circ$

Since sum of opposite angle of a cyclic quadrilateral is 180°

$\angle PRS + \angle PQS = 180^\circ$

$\Rightarrow \angle PQS = 180^\circ - 113^\circ = 67^\circ$

In ΔPTQ, $\angle QPT + \angle PQT + \angle PTQ = 180^\circ$

$\Rightarrow$ $\angle PTQ = 180^\circ - 45^\circ - 67^\circ = 68^\circ$

Second method—

$\angle PRQ = 90^\circ$ (Angle in a semicircle)

$\angle PRQ = \angle QRT = 90^\circ$

$\angle RQS = \frac{1}{2}\angle ROS = \frac{1}{2} \times 44^\circ = 22^\circ$

In ΔRTQ, $\angle QRT + \angle RQT + \angle RTQ = 180^\circ$

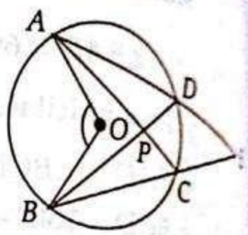
$90^\circ + 22^\circ + \angle RTQ = 180^\circ$

$112^\circ + \angle RTQ = 180^\circ$

$\angle RTQ = 68^\circ$

$\angle RTS = 68^\circ$

44. (b) $\because$ $\angle ADB = \frac{1}{2}\angle AOB = 50^\circ$



In ΔDPA, $\angle DAP + \angle ADP + \angle DPA = 180^\circ$

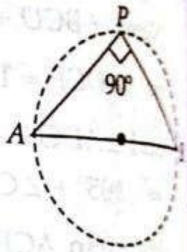
$\Rightarrow$ $30^\circ + 50^\circ + \angle DPA = 180^\circ$

$\Rightarrow$ $\angle DPA = 100^\circ$

and $\angle DPA + \angle APB = 180^\circ$

$\Rightarrow$ $\angle APB = 180^\circ - 100 = 80^\circ$

45. (c) Locus of point P is a circle with AB as one of the diameter because when P moves on the circle $\angle APB$ remain 90°. (Due to the fact that angle of a semicircle is 90°)



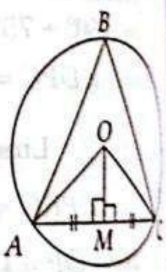
46. (c) $OA = OC = 3$ cm *(Radus of the circle)*

$\therefore$ $OA = OC = AC = 3$ cm

$\therefore$ ΔAOC is an equilatral triangle.

$\angle AOC = 60^\circ$

$\angle ABC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 60^\circ = 30^\circ$



47. (d) Since PQ is tangent to the internal circle, therefore $PQ \perp AB$. Clearly $OP = OQ$ and two chords of outer circle are AB and PQ.

$\therefore$ $OA \times OB = OP \times OQ$

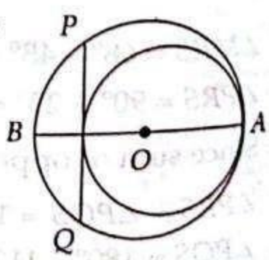
$\because$ OA = diameter of internal circle = 8 cm.

$OB = AB - OA = 12 - 8 = 4$ cm

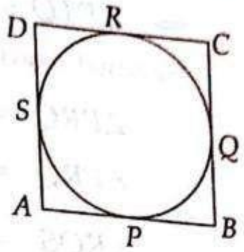
$\therefore$ $4 \times 8 = OP^2$

$\Rightarrow$ $OP = \sqrt{32} = 4\sqrt{2}$ cm

$\therefore$ $PQ = 2 \times 4\sqrt{2} = 8\sqrt{2}$ cm



(c) We know that length of tangents drawn from an outside point to a given circle are equal.



$\therefore$ $AP = AS$

$BP = BQ$

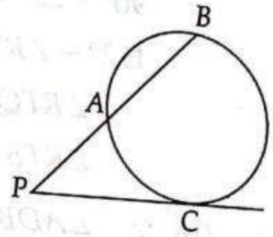
$CR = CQ$

$DR = DS$

$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow AB + CD = AD + BC$

(a) $\therefore PC^2 = PA \times PB$



Area of rectangle whose adjacent sides are PA and PB is equal to area of square whose each side is PC.

42. (b) $\because$ $\angle BAD = 60°$, $\angle ADC = 105°$

In cyclic quadrilateral $ABCD$,

$\angle BAD + \angle BCD = 180°$

$\Rightarrow \angle BCD = 180° - 60° = 120°$

Now, $\angle BCD + \angle DCP = 180°$ *(linear pair of angles)*

$\Rightarrow \angle DCP = 180° - 120° = 60°$

and $\angle ADC + \angle CDP = 180°$ *(linear pair of angles)*

$\Rightarrow 105° + \angle CDP = 180°$ $\therefore \angle CDP = 75°$

Hence in ΔCPD, $\angle DCP + \angle CDP + \angle DPC = 180°$

$\Rightarrow 60° + 75° + \angle DPC = 180°$

$\Rightarrow \angle DPC = 180° - 135° = 45°$

43. (d) $\because$ Line OR is bisector of $\angle PRQ$

$\therefore$ $\angle PRO = \angle ORQ = 45°$

Also $OP = OR$ *(radius)*

$\therefore$ $\angle OPR = 45°$

In ΔORS, $OR = OS$

$\Rightarrow \angle ORS = \angle OSR = \frac{180° - 44°}{2} = 68°$

$\therefore$ $\angle MRS = 68° - 45° = 23°$

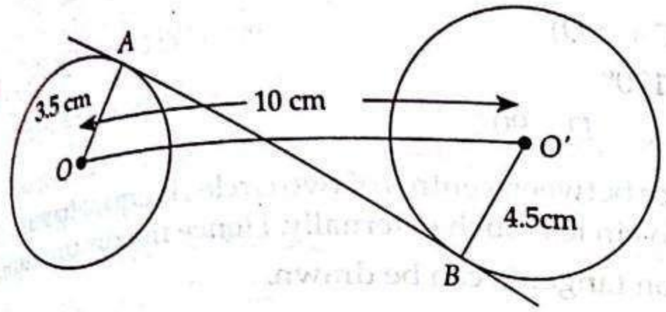
$\Rightarrow \angle PRS = 90° + 23° = 113°$

Since sum of opposite angle of a cyclic quadrilateral is 180°

$\angle PRS + \angle PQS = 180°$

$\Rightarrow \angle PQS = 180° - 113° = 67°$

(c)



length of transverse common tangent

$= \sqrt{(\text{distance between centres of circles})^2 - (\text{sum of radii})^2}$

$= \sqrt{10^2 - (4.5+3.5)^2} = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$ cm

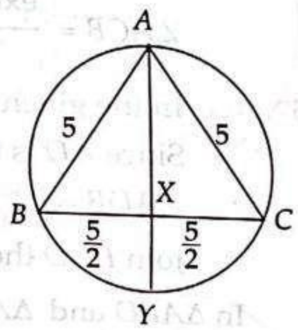
(b) length of transverse common tangent $= \sqrt{d^2 - (r_1 + r_2)^2}$

or, $8 = \sqrt{d^2 - (6+3)^2}$

or, $64 = d^2 - 81$

$\Rightarrow d^2 = 81 + 64 = 145 \quad \Rightarrow d = \sqrt{145}$

(c) In ΔABC,



$BX = \frac{5}{2}$ cm, $CX = \frac{5}{2}$ cm

and $AX = \frac{\sqrt{3}}{2} \times 5 = \frac{5\sqrt{3}}{2}$ cm

$\because$ AY and BC are two chords of circle

$\therefore$ $AX \times XY = BX \times CX$

$\frac{5\sqrt{3}}{2} \times XY = \frac{5}{2} \times \frac{5}{2}$

$\Rightarrow XY = \frac{5}{2\sqrt{3}}$

$\therefore$ $AX \cdot AY = \left(\frac{5\sqrt{3}}{2} + \frac{5}{2\sqrt{3}}\right) \times \frac{5}{2\sqrt{3}} = \frac{25}{3}$ cm^2

51. (b) $ABCD$ will be a trapezium

52. (b) Locus of P is a circle which is concentric with circle 'C'.

53. (b) In cyclic quadrilateral $ABCD$,

$\angle A + \angle C = 180° \Rightarrow \angle A = 180° - \angle C$ ... (i)

and $\angle B + \angle D = 180° \Rightarrow \angle B = 180° - \angle D$ ... (ii)

It is given that

$\angle A + \angle B = 2(\angle C + \angle D)$

$\Rightarrow 180° - \angle C + 180° - \angle D = 2(\angle C + \angle D)$ (from equation (i) and (ii))

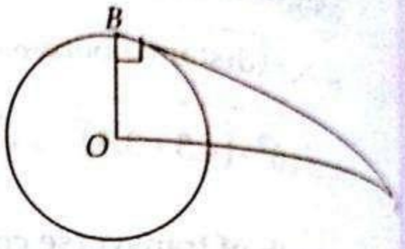
$\Rightarrow\ 360° = 3(\angle C + \angle D)$

$\Rightarrow\ \angle C + \angle D = 120°$

But, $C > 30°$ $\quad\therefore\quad D < 90°$

54. **(c)** Since distance between centres of two circles is equal to sum of the radii, the two circles touch externally. Hence maximum number of three common tangents can be drawn.

55. **(d)** $\because\ AB = 6$ cm and $OA = 6.5$ cm



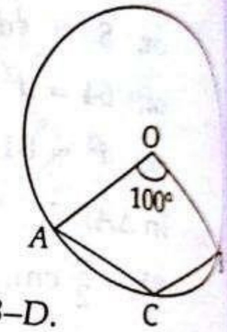
$\therefore$ In ΔOAB,

$$OB = \sqrt{OA^2 - AB^2}$$

$$= \sqrt{(6.5)^2 - (6)^2}$$

$$= \sqrt{42.25 - 36} = \sqrt{6.25} = 2.5 \text{ cm}$$

56. **(d)** $\because\ \angle AOB = 100°$



$\therefore$ external $\angle AOB = 360° - \angle AOB$

$$= 360° - 100° = 260°$$

$$\angle ACB = \frac{\text{external } \angle AOB}{2} = \frac{260°}{2} = 130°$$

57. **(c)** In the given figure D is midpoint of AC. Join B–D.

Since AD is tangent line and BD is diameter

$\therefore\ \angle ADB = 90°$

Join P–D then $\angle APD = 90°$ *(Angle in a semicircle is right angle)*

In ΔAPD and ΔADB, $\angle A$ = common,

$\angle ADB = 90° = \angle APD$

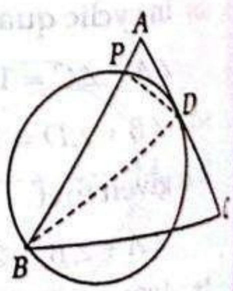
$\therefore\ \Delta APD \sim \Delta ADB$

$$\Rightarrow \frac{AP}{AD} = \frac{AD}{AB}$$

$$\Rightarrow \frac{AP}{AB} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$\therefore\ (\because\ AD = \frac{1}{2} AC = \frac{1}{2} AB) \Rightarrow 4AP = AB$

58. **(c)** Given, $\angle PAD = 59°$ and $\angle APD = 40°$



In ΔAPD, $\angle PAD + \angle APD + \angle ADP = 180°$

$\Rightarrow\ 59° + 40° + \angle ADP = 180°$

$\Rightarrow\ \angle ADP = 81°$

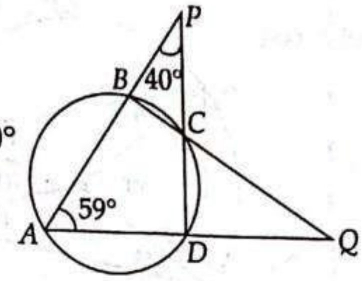
and $\angle ABC + \angle ADC = 180°$

$\Rightarrow\ \angle ABC = 180° - 81° = 99°$

$\therefore \angle CDQ = \angle ABC = 99^\circ$

and $\angle QCD = \angle BAD = 59^\circ$

In ΔCQD, $\angle CQD + \angle CDQ + \angle QCD = 180^\circ$



$\Rightarrow \angle CQD = 180^\circ - 59^\circ - 99^\circ$

$= 180^\circ - 158^\circ = 22^\circ$

$\Rightarrow \angle AQB = \angle CQD = 22^\circ$

59. (a) $\angle ADB = 90^\circ$ *(Angle in a semicircle)*

$\therefore \angle DAB = 15^\circ$

$\angle CAB = 35^\circ + 15^\circ = 50^\circ$

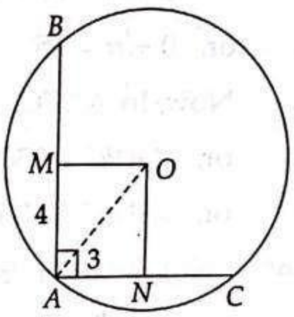
Hence, $\angle BDC = 180^\circ - 50^\circ = 130^\circ$

(opposite angle of a cyclic quadrilateral)

60. (b) $\angle AQP = \frac{1}{2} \times \angle AOP$

$= \frac{75^\circ}{2} = 37.5^\circ$

61. (d) See the figure



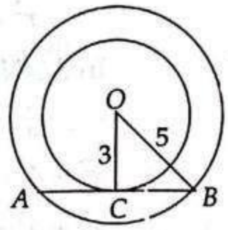
$AM = ON = \frac{8}{2} = 4$ cm

$AN = OM = \frac{6}{2} = 3$ cm

$\therefore$ radius of circle, $AO = \sqrt{(AN)^2 + (ON)^2}$

$= \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$ cm

62. (c) See the figure,

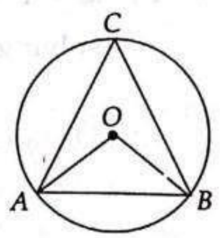


$BC = \sqrt{(OB)^2 - (OC)^2}$

$= \sqrt{(5)^2 - (3)^2} = \sqrt{25 - 9} = 4$

Hence, $AB = 2 \times BC = 2 \times 4 = 8$ cm

63. (a) See the figure,

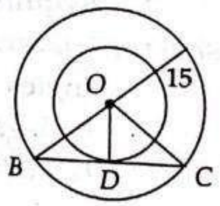


$\because AO = OB = AB =$ radius

$\therefore \angle AOB = 60^\circ$

$\therefore \angle ACB = 30^\circ$

64. (a) $\because BO = OC = 15$ cm



and $OD = 9$ cm

$\therefore BD = \sqrt{15^2 - 9^2} = 12$ cm

$\therefore BC = 2 \times 12 = 24$ cm

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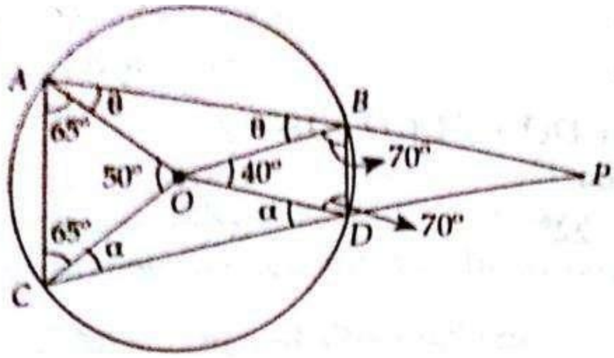
65. (d)



∵ ΔAOC is an isosceles triangle ∴ $\angle OAC = \angle OCB = 65°$

∵ ΔBOD is an isosceles triangle ∴ $\angle OBD = \angle ODB = 70°$

Let $\angle OAB = \angle OBA = \theta$ and $\angle OCD = \angle ODC = \alpha$

∴ In quadrilateral $ABCD$,

$2(\theta + 65° + \alpha + 70°) = 360°$

$\theta + \alpha + 135° = 180°$

or, $\theta + \alpha = 45°$

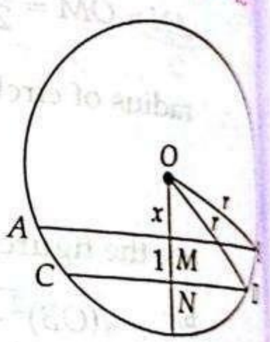
Now, In ΔAPC, $\angle APC + \theta + 65° + 65° + \alpha = 180°$

or, $\angle APC + 130° + 45° = 180°$ (∵ $\theta + \alpha = 45°$)

or, $\angle APC = 180° - 175° = 5° = \angle BPD$.

66. (a) Let larger chord is x. cm away from centre.

see the figure



In ΔOMB,

$r^2 = x^2 + 4^2$

In ΔOND,

$r^2 = (x + 1)^2 + 3^2$

∴ $x^2 + 4^2 = (x + 1)^2 + 3^2$

∴ $x^2 + 4^2 = (x + 1)^2 + 3^2$

solving $x = 3$, Hence radius $= \sqrt{3^2 + 4^2} = 5$

67. (a) Distance of 8 cm chord from centre $= \sqrt{r^2 - \left(\frac{8}{2}\right)^2}$

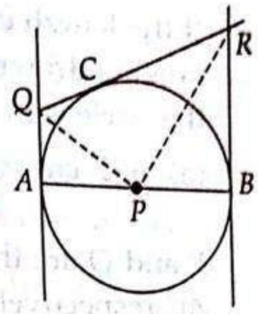
$= \sqrt{5^2 - 4^2} = 3$

∴ Required distance $= 3 \times 2 = 6$ cm

68. (b) Trick : Ratio of radii is inversely proportional to corresponding angles

∴ $\frac{r_1}{r_2} = \frac{75°}{60°} = 5 : 4$

(*d*) **shortcut :** Note that QP is bisector of $\angle AQC$ and $\angle PR$ is bisector of $\angle BRC$



But $\angle AQC + \angle BRC = 180^\circ$

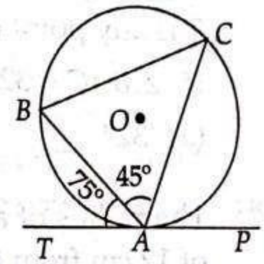
($\because$ In quadrilateral $ABQC$, $\angle QAB = \angle ABR = 90^\circ$)

Hence in ΔQPC,

$\angle QPC = 180^\circ - (\angle AQC + \angle BRC)$

$= 180^\circ - 90^\circ = 90^\circ$

(*c*) Since angle in the alternate segment are equal

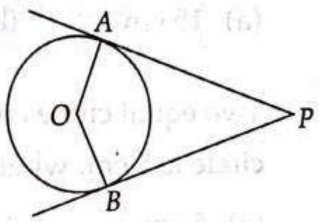


$\angle ACB = \angle BAT = 75^\circ$

$\therefore \angle ABC = 180^\circ - 45^\circ - 75^\circ = 60^\circ$

(*a*) $\because \angle AOB + \angle APB = 360^\circ - \angle OAB - \angle OBA$

$= 360^\circ - 90^\circ - 90^\circ = 180^\circ$



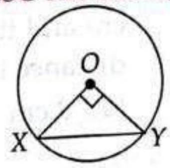
$\because \angle AOB : \angle APB = 5 : 1$

Hence, $\angle APB = \frac{180}{6} \times 1 = 30^\circ$

Exercise—8B

1. Two tangents are drawn from a point P to a circle at A and B. O is the centre of the circle. If $\angle AOP = 60^\circ$, then $\angle APB$ is
   (a) 60° (b) 30° (c) 120° (d) 90°
   [SSC Tier-I 2012]

2. In the following figure, O is the centre of the circle and XO is perpendicular to OY. If the area of the triangle XOY is 32, then the area of the circle is



   (a) 16π (b) 32π
   (c) 64π (d) 256π

3. Two circles of radii 4 cm and 9 cm respectively touch each other externally at a point and a common tangent touches them at the points P and Q respectively. Then the area of a square with one side PQ, is
   (a) 72 sq. cm (b) 144 sq. cm (c) 97 sq. cm (d) 194 sq. cm
   [SSC Tier-I 2012]

4. The tangents drawn at the points A and B of a circle centred at O meet at P. If $\angle AOB = 120^\circ$ then $\angle APB : \angle APO$ is :
   (a) $2 : 5$ (b) $3 : 2$ (c) $4 : 1$ (d) $2 : 1$
   [SSC Tier-I 2012]

5. If the length of a chord of a circle, which makes an angle 45° with the tangent drawn at one end point of the chord, is 6 cm, then the radius of the circle is :

(a) $6\sqrt{2}$ cm (b) 5 cm (c) $3\sqrt{2}$ cm (d) 6 cm

[SSC Tier-I 2012]

6. P and Q are the middle points of two chords (not diameters) AB and AC respectively of a circle with centre at a point O. The lines OP and OQ are produced to meet the circle respectively at the points R and S. T is any point on the major arc between the points R and S of the circle. If $\angle BAC = 32°$, $\angle RTS = ?$

(a) 32° (b) 74° (c) 106° (d) 64°

[SSC Tier-I 2012]

7. The radius of a circle is 13 cm and AB is a chord which is at a distance of 12 cm from the centre. The length of the chord is :

(a) 15 cm (b) 12 cm (c) 10 cm (d) 20 cm

[SSC Tier-I 2012]

8. Two equal circles pass through each other's centre. If the radius of each circle is 5 cm, what is the length of the common chord ?

(a) 5 cm (b) $5\sqrt{3}$ cm (c) $10\sqrt{3}$ cm (d) $\frac{5\sqrt{3}}{2}$ cm

[SSC Tier-I 2012]

9. ABC is a triangle. The internal bisector of the angles $\angle A$, $\angle B$ and $\angle C$ intersect the circumcircle at X, Y and Z respectively. If $\angle A = 50°$, $\angle CZY = 30°$, then $\angle BYZ$ will be

(a) 45° (b) 55° (c) 35° (d) 30°

[SSC Tier-I 2012]

10. If a circle with radius of 10 cm has two parallel chords 16 cm and 12 cm and they are one the same side of the centre of the circle, then the distance between the two parallel chords is

(a) 2 cm (b) 3 cm (c) 5 cm (d) 8 cm

[SSC Tier-I 2012]

11. Two circles of radii 8 cm and 2 cm respectively touch each other externally at the point A. PQ is the direct common tangent of those two circles of centres O_1 and O_2 respectively. Then length of PQ is equal to

(a) 2 cm (b) 3 cm (c) 4 cm (d) 8 cm

[SSC Tier-I 2012]

12. $ABCD$ is a cyclic quadrilateral. Sides AB and DC, when produced meet at the point P and sides AD and BC, when produced meet at the point Q. If $\angle ADC = 85°$ and $\angle BPC = 40°$, then $\angle CQD$ is equal to

(a) 30° (b) 40° (c) 55° (d) 85°

[SSC Tier-I 2012]

13. If a square is inscribed in a circle whose area is 314 sq. cm, then the length of each side of the square is [Given $\pi = 3.14$]

(a) $5\sqrt{2}$ cm (b) $20\sqrt{2}$ cm (c) 10 cm (d) $10\sqrt{2}$ cm

14. Two circles with same radius r intersect each other and one passes through the centre of the other. Then the length of the common chord is

(a) r (b) $\sqrt{3}r$ (c) $\frac{\sqrt{3}}{2}r$ (d) $\sqrt{5}r$

15. Two circles intersect each other at P and Q. PA and PB are two diameter. Then $\angle AQB$ is

(a) 120° (b) 135° (c) 160° (d) 180°

[SSC Tier-I 2012]

16. A and B are centres of the two circles whose radii are 5 cm and 2 cm respectively. The direct common tangents to the circles meet AB extended at P. Then P divides AB.

(a) externally in the ratio 5 : 2 (b) internally in the ratio 2 : 5
(c) internally in the ratio 5 : 2 (d) externally in the ratio 7 : 2

[SSC Tier-I 2012]

17. AC and BC are two equal chords of a circle. BA is produced to any point P and CP, when joined cuts the circle at T. Then

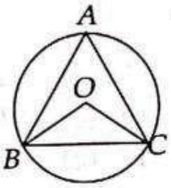
(a) $CT : TP = AB : CA$ (b) $CT : TP = CA : AB$
(c) $CT : CB = CA : CP$ (d) $CT : CB = CP : CA$

18. PQ is a direct common tangent of two circles of radii r_1 and r_2 touching each other externally at A. Then the value of PQ^2 is

(a) r_1r_2 (b) $2r_1r_2$ (c) $3r_1r_2$ (d) $4r_1r_2$

[SSC Tier-I 2012]

19. BC is the chord of a circle with centre O. A is a point on major arc BC as shown in the figure. What is the value of $\angle BAC + \angle OBC$?



(a) 120° (b) 60°
(c) 90° (d) 180°

[SSC Tier-I 2012]

20. Two circles with radii 5 cm and 8 cm touch each other externally at a point A. If a straight line through the point A cuts the circles at points P and Q respectively, then $AP : AQ$ is

(a) 8 : 5 (b) 5 : 8 (c) 3 : 4 (d) 4 : 3

[SSC Tier-I 2012]

21. Perimeter of a semi-circular bow is 72 cm. Diameter of the bow (in cm) is

(a) 7 cm (b) 14 cm (c) 28 cm (d) 21 cm

[SSC Tier-I 2012]

22. R and r are the radius of two circles ($R > r$). If the distance between the centre of the two circles be d, then length of common tangent of two circles is

(a) $\sqrt{d^2-(R-r)^2}$ (b) $\sqrt{(R-r)^2-d^2}$

(c) $\sqrt{R^2-d^2}$ (d) $\sqrt{r^2-d^2}$

23. AB is a diameter of circle with centre O. CD is a chord equal to the radius of the circles. AC and BD are produced to meet at P. Then the measure of $\angle APB$ is [SSC Tier-I 2012]

(a) 30° (b) 60° (c) 90° (d) 120°

24. P is a point outside a circle and is 13 cm away from its centre. A secant drawn from the point P intersects the circle at points A and B in such a way that $PA = 9$ cm and $AB = 7$ cm. The radius of the circle is [SSC Tier-I 2012]

(a) 5 cm (b) 4 cm (c) 4.5 cm (d) 5.5 cm

25. The area of the largest triangle that can be inscribed in a semi circle of radius x in square unit is. [SSC Tier-I 2012]

(a) x^2 (b) $2x^2$ (c) $3x^2$ (d) $4x^2$

26. The length of the common chord of two circles of radii 15 cm and 20 cm whose centres are 25 cm apart is (in cm). [SSC Tier-I 2012]

(a) 24 (b) 25 (c) 15 (d) 20

27. SR is transverse common tangent of two circles whose radii are respectively 8 cm and 3 cm and centres are 13 cm apart. If S and R are points of contact, then the length of SR is [SSC Tier-I 2012]

(a) 17 cm (b) 10 cm (c) 12 cm (d) 11 cm

[SSC Tier-I 2012]

Answers—8B

1. (a)	2. (c)	3. (b)	4. (d)	5. (c)	6. (b)	7. (c)	8. (b)
9. (c)	10. (a)	11. (d)	12. (a)	13. (d)	14. (b)	15. (d)	16. (a)
17. (c)	18. (d)	19. (c)	20. (b)	21. (c)	22. (a)	23. (b)	24. (a)
25. (a)	26. (a)	27. (c)					

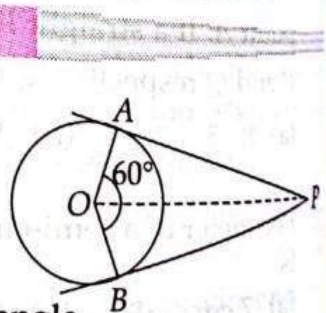
Explanation

1. (a) $\angle APO = 180^\circ - \angle OAP - \angle AOP$

$= 180^\circ - 90^\circ - 60^\circ = 30^\circ$

$\therefore \angle APB = 2\angle APO$

$= 2 \times 30^\circ = 60^\circ$



2. (c) $OX \perp OY \Rightarrow \Delta XOY$ is a right angled triangle

If $OX = OY = r$ = radius of circle

Area of triangle $= \frac{1}{2} \times r \times r = 32 \quad \Rightarrow \quad r = \sqrt{64} = 8$

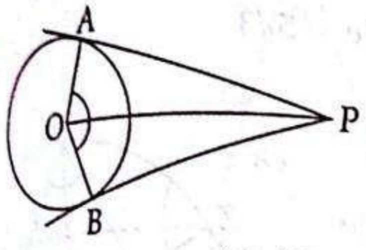
$\therefore$ Area of circle $= \pi r^2 = 64\pi$

(b) Using $PQ = \sqrt{d^2 - (R-r)^2}$

∴ Area of square $= PQ^2 = d^2 - (R-r)^2$

$= 13^2 - (9-4)^2 = 144$ sq. cm^2

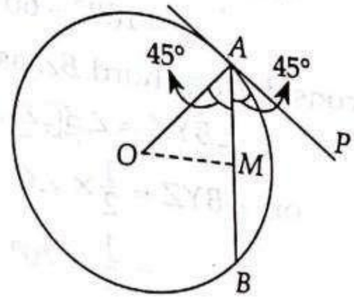
(d)



∵ OP is bisector of $\angle APB$

∴ $\angle APB : \angle APO = 2 : 1$

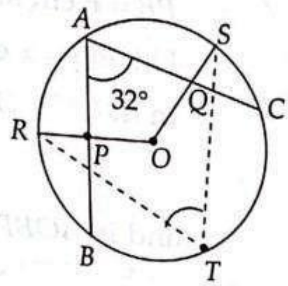
(c) AB is a chord which makes 45° with tangent line AP.



If O is centre of circle and M is midpoint of chord then $\angle OAM = 90° - 45° = 45°$

In ΔAOM, $\cos 45° = \frac{AM}{OA} = \frac{3}{r} \Rightarrow r = 3\sqrt{2}$ cm

(b) Since OP and OQ are respectively bisector of chord AB and AC therefore,



$\angle APO = 90°, \quad \angle AQO = 90°$

Hence in quadrilateral $APOQ$

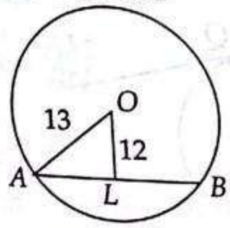
$\angle POQ = 180° - 32° = 148°$

Hence we cay say the chord RS (not drawn in figure, draw yourself) subtends angle 148° on centre O.

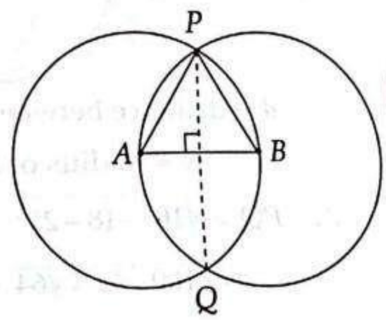
∴ Chord RS subtands $\frac{148°}{2} = 74°$ on circumference

(c) See the figure, $AL = \sqrt{13^2 - 12^2}$

∴ $AB = 5 \times 2 = 10$ cm



(b) A and B are respectively centres of two circles and PQ is the common chord.



cleraly $AB = 5$ radius of circle $= 5$

$AP = BP =$ radius of circle $= 5$

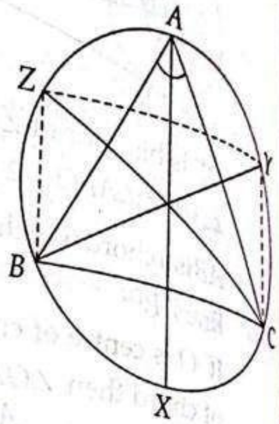
Hence ΔAPB is an equilateral triangle

Altitude of triangle $= \frac{\sqrt{3}}{2} \times \text{side}$

$= \frac{\sqrt{3}}{2} \times 5$

$\therefore$ Common chord $PQ = 2 \times \text{altitude} = 5\sqrt{3}$

9. (c) Considering CY as base,



$\angle CBY = \angle CZY = 30°$

$\therefore \angle B = 2 \times \angle CBY = 2 \times 30° = 60°$

and $\angle C = 180° - \angle A - \angle B$

$= 180° - 60° - 50° = 70°$

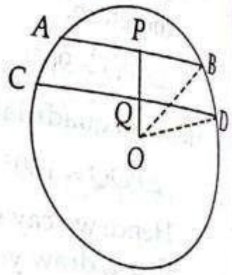
considering chord BZ as base

$\angle BYZ = \angle BCZ$

or, $\angle BYZ = \frac{1}{2} \times \angle C$

$= \frac{1}{2} \times 70° = 35°$

10. (a) In fig. $AB = 12$ cm and $CD = 16$ cm



$\therefore PB = 6$ cm and $QD = 8$ cm

Let $PQ = x$ cm then

In ΔOQD, $OQ = \sqrt{OD^2 - QD^2}$

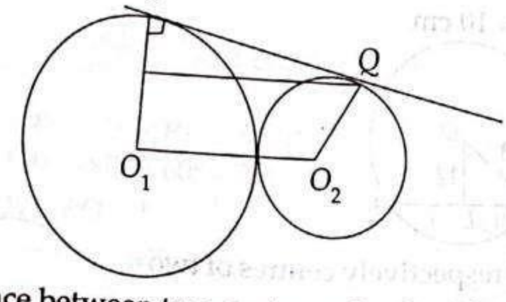
$= \sqrt{100 - 64} = \sqrt{36} = 6$ cm

and In ΔOBP, $OP = \sqrt{OB^2 - PB^2}$

$= \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = 8$ cm

$\therefore x = PQ = OP - OQ = 8 - 6 = 2$ cm

11. (d) $PQ = \sqrt{d^2 - (R - r)^2}$



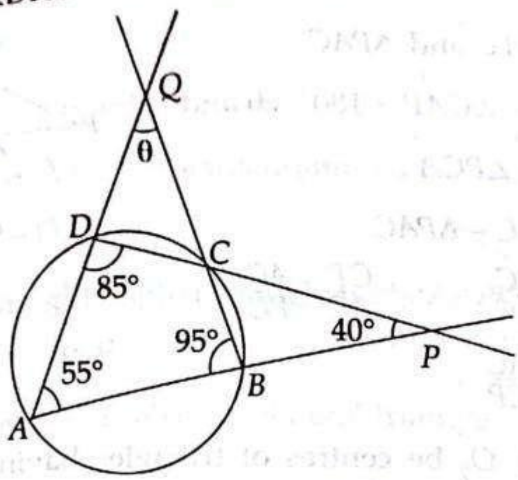
d = distance between two centres = $8 + 2 = 10$ cm

R = Radius of bigger circle and r = radius of smaller circle.

$\therefore PQ = \sqrt{10^2 - (8 - 2)^2}$

$= \sqrt{100 - 36} = \sqrt{64} = 8$ cm

13. (a) In ΔAPD, $\angle DAB + 85° + 40° = 180°$



$\Rightarrow \angle DAB = 55°$

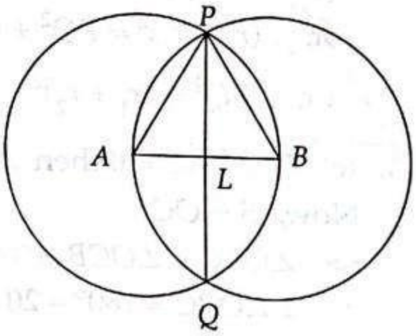
In cyclic quadrilateral, $\angle ABC = 180° - 85° = 95°$

Let $\angle CQD = \theta$

In ΔABQ, $\theta + 55° + 95° = 180°$

$\Rightarrow \theta = 180° - 150° = 30°$

14. (b) See the figure $AB = AP = BP =$ radius



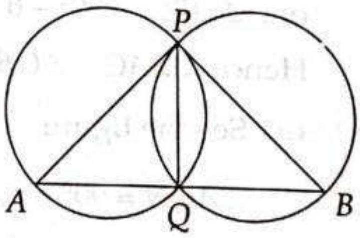
$\therefore$ ΔAPB is an quadrilateral triangle

$PQ =$ common chord $= 2 \times AL$

$= 2 \times$ altitade

$= 2 \times \frac{\sqrt{3}}{2} r = \sqrt{3} r$ (Learn)

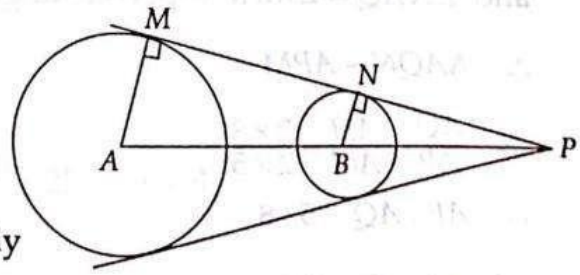
15. (d) See the figure, Since angle in a semicircle is right angled



$\therefore \angle AQP = 90°$ and $\angle BQP = 90°$

Hence $\angle AQB = 90° + 90° = 180°$

16. (a) $\because$ $\Delta AMP \sim \Delta BNP$



$\therefore$ $\frac{AP}{BP} = \frac{AM}{BN}$

or, $\frac{AP}{BP} = \frac{5}{2}$

$\because$ P is out side AB

$\therefore$ If divides AB externally

17. (c) $\because$ $BC = AC$

$\therefore$ $\angle CBA = \angle CAB = \theta$ (Say)

Join $A - T$

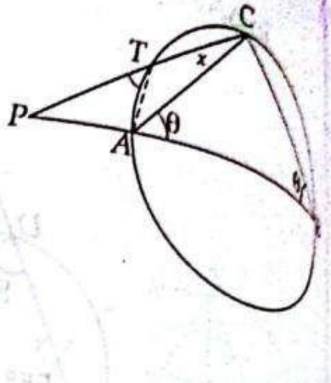
$ABCT$ is a cyclic quadrilateral

$\therefore\ \angle ATC = 180° - \theta$

Now, In ΔATC and ΔPAC

$\angle ATC = \angle CAP = 180° - \theta$ and

$\angle TCA = \angle PCA$ (common angle)



Hence, $\Delta ATC \sim \Delta PAC$

or, $\frac{AC}{TC} = \frac{PC}{AC}$ $\therefore\ \frac{CT}{AC} = \frac{AC}{PC}$ 2

or, $\frac{CT}{BC} = \frac{AC}{CP}$ (∵ AC = BC

18. (d) Let O_1 and O_2 be centres of triangles having radii r_1 and r_2 respectively

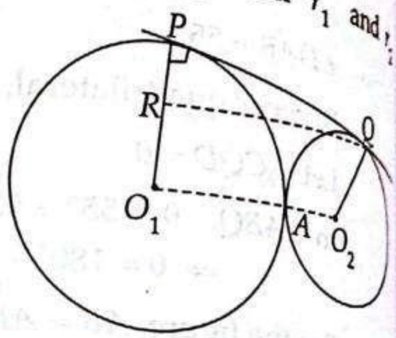
Draw $QR \mid\mid O_1O_2$

In right angled ΔRPQ

$QR^2 = PQ^2 + PR^2$ 2

or, $(r_1 + r_2)^2 = PQ^2 + (r_1 - r_2)^2$

or, $PQ^2 = (r_1 + r_2)^2 - (r_1 - r_2)^2 = 4r_1r_2$



19. (c) If $\angle BAC = \theta$ then $\angle BOC = 2\theta$

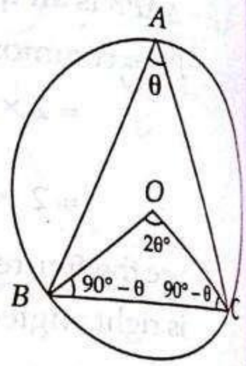
Now, $B = OC$

$\Rightarrow\ \angle OBC = \angle OCB$ 2

$\therefore\ 2\angle OBC = 180° - 2\theta$

or,, $\angle OBC = 90° - \theta$

Hence $\angle BAC + \angle OBC = \theta + 90° - \theta = 90°$



20. (b) See the figure

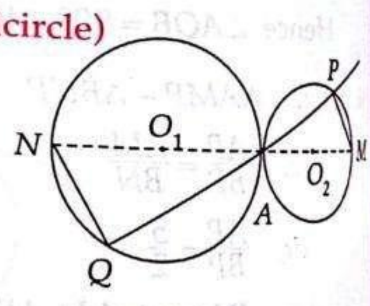
$\angle AQN = 90° = \angle APM$ (angle in a semicircle)

and $\angle NAQ = \angle MAP$ (opposite angle)

$\therefore\ \Delta AQN \sim APM$

$\Rightarrow\ \frac{AQ}{AP} = \frac{AN}{AM} = \frac{2 \times 8}{2 \times 5}$

$\therefore\ AP : AQ = 5 : 8$



21. (c) $\pi r + 2r = 72$ 2

$\Rightarrow\ \frac{22}{7}r + 2r = 72 \Rightarrow \frac{36r}{7} = 72$

$\Rightarrow\ r = 2 \times 7 = 14$ cm

$\therefore$ Diameter $= 2 \times 14 = 28$ cm

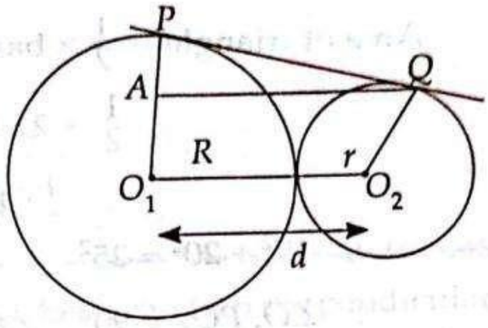
22. (b) $\because$ $O_1P \perp r\ PQ$, $O_2Q \perp r\ PQ$

and $QA \perp r\ OP_1$

$\therefore$ AO_1O_2Q is a parallelogram

$\therefore$ $PA = O_1P - OA_1$

$= O_1P - O_2Q = R - r$



ΔAPQ is a right angled triangle with $\angle APQ = 90°$

$\therefore$ $AQ^2 = AP^2 + PQ^2$

ΔAOC and ΔBOD also equilateral triangle

In ΔABP,

$d^2 = (R-r)^2 + PQ^2 \quad PQ = \sqrt{d^2 - (R-r)^2}$

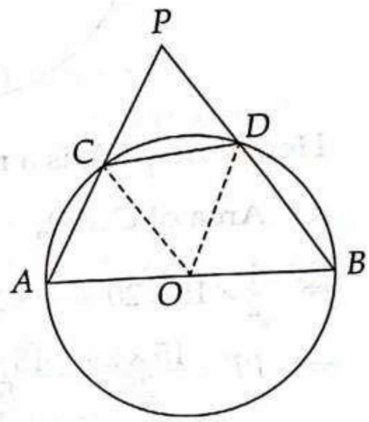
23. (b) ΔOCD is an equilateral triangle as length of CD is equal to radius

OCD is an equilateral triangle

$\Rightarrow$ all its angle are $60°$

In ΔABP,

$\angle APB = 180° - 60° - 60° = 60°$

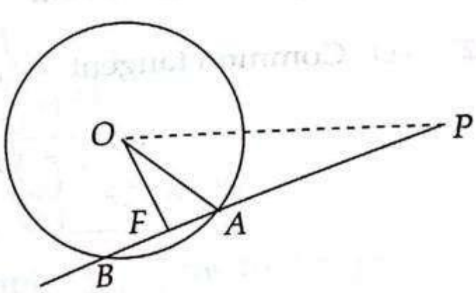


24. (a) Draw $OF \perp r\ AB$

ΔOFP is an equilateral triangle $(\angle F = 90°)$

in which $OP = 12$, $PF = PA + AF$

$= 9 + \frac{7}{2} = \frac{25}{2}$



$\therefore$ $OF^2 = OP^2 - PF^2$

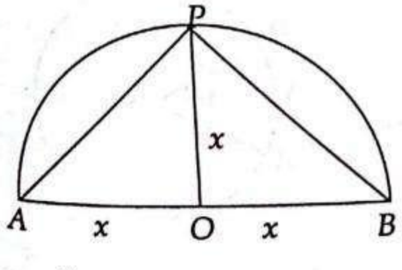
$= 169 - \left(\frac{25}{2}\right)^2$

$= \frac{676 - 625}{4} = \frac{51}{4}$

In ΔOFA, $r^2 = OF^2 + FA^2$

$= \frac{51}{4} + \left(\frac{7}{2}\right)^2 = \frac{51}{4} + \frac{49}{4} = \frac{100}{4} = 25 \ \therefore\ r = 5$

25. (a)



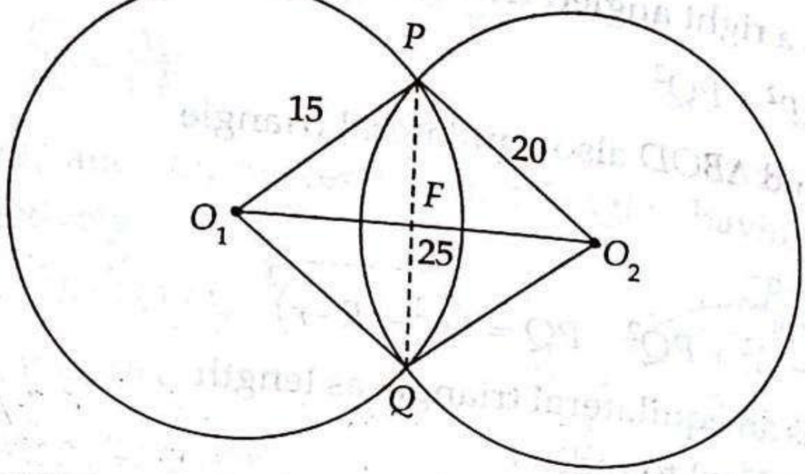
Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 2x \times x = x^2$

($\because$ greatest base $= AB =$ greatest height $OP = x$)

26. (a) $\because\ 15^2 + 20^2 = 25^2$

$\therefore\ \angle O_1PO_2 = 90° = \angle O_1QO_2$



Hence ΔO_1PO_2 is a right angled triangle

$\therefore$ Area of $O_1PO_2 = \frac{1}{2} \times OP_1 \times OP_2 = \frac{1}{2} O_1O_2 \times PF$

$\Rightarrow \frac{1}{2} \times 15 \times 20 = \frac{1}{2} \times 25 \times PF$

$\Rightarrow PF = \frac{15 \times 20}{25} = \frac{15 \times 4}{5} = 12$ cm

$\therefore\ PQ = 12 \times 2 = 24$ cm

27. (c) Common tangent $= \sqrt{d^2 - (R - r)^2}$

$= \sqrt{13^2 - (8 - 3)^2} = \sqrt{169 - 25} = \sqrt{144} = 12$

★★★