15

Linear Programming

15.01 Introduction

A large number of decision problems faced by a business manager involve allocations to various activities, with the objective of increasing profits or decreasing costs, or both. The manager has to take a decision as to how best to allocate the resources among the various activities. The decision problems can be formulated, and solved, as mathematical programming problems. Mathematical programming involves optimisation of a certain function, called objective function, subject to certain constraints.

Definition : Linear programming deal with the optimization of a linear function of a number of variables subject to a number of conditions on the variables in the form of linear inequation or equations in variables involved.

15.02 Linear programming problem and its mathematical formulation

Let us understand the linear programming and its mathematical formulation with the help of following example:

Example : A developer produced two product P_1 and P_2 with the help of two machines M_1 and M_2. To make a unit of P_1, M_1 takes one hour and M_2 takes 3 hours and to make a unit of P_2, each take two hours. Profit on per unit of P_1 and P_2 be ₹ 60 and ₹ 50 respectively and M_1 and M_2 can work for 40 hrs. and 60 hrs. respectively in a week, then how much unit it can produce for maximum proft. It is clear from this example that,

(i) Developer can produce only P_1 or P_2 or both. Thus he gains maximum profit from different additive incorporate.

(ii) There are certain over riding conditions or constraints like M_1 and M_2 can work only 40 and 60 hrs respectively in a week.

Let developer only wants to produce P_1, then only 10 unit can produce and net profit $=60 \times 20 =$ ₹ 1200

Let developer only wants to produce P_2, then only 20 unit can produce and net profit $=50 \times 20 =$ ₹ 1000

There are too many possibilities. So, we have to know that how developer gain maximum profit from different method. Now, there is a problem, how developer can gain maximum profit from different method of Production. To find the answer, we have to formulate it mathematically.

15.03 Mathematical formulation of the problem

Let, x and y is number of desirable units of product P_1 and P_2 for favourable solution. Now, represent the problem in form of following table :

Machine	**Product**		**Availability**
	P_1	P_2	**(per week)**
M_1	1 hr.	2 hrs.	40 hrs
M_2	3 hrs.	2 hrs.	60 hrs.
Profit (per unit)	₹ 60	₹ 50	

Per unit profit on product P_1 and P_2 are ₹ 60 and ₹ 50 respectively. So, total profit on x unit of P_1 and y unit of P_2.

$$Z = 60x + 50y$$

So, we can relate the total profit with variable x and y linearly. Developer try to maximize that profit.

$$Z = 60x + 50y$$

Constraint for machines M_1 and M_2 : We know that, for production of P_1 and P_2, M_1 occupy for 1 and 2 hours.

So, occupation of M, for production of x unit of P_1 and y unit of P_2 will be $x + 2y$ but availability of M, is 40 hrs. per week then

$$x + 2y \leq 40$$

Similarly for M_2 $\qquad 3x + 2y \leq 60$

Non-negative constraint : Since x and y is number of developing unit whcih never be negative.

So, $\qquad x \geq 0, \quad y \geq 0$

Maximize : $\qquad Z = 60x + 50y$

Constraint : $\qquad x + 2y \leq 40$

$$3x + 2y \leq 60$$

and $\qquad x \geq 0, y \geq 0$

Now, we have define some terms whcih is used in linear programming problems.

Objective Function :

If $c_1, c_2, ..., c_n$ are constants and $x_1, x_2, ..., x_n$ are variables then linear functions $Z = c_1x_1 + c_2x_2 + ... + c_nx_n$ which has to be maximize or minimise, is called objective function.

Constraints : Restriction on the variables of a linear programming problem are called constraints. These are represented in form of linear equation or inequalities.

In above example $x + 2y \leq 40$ and $3x + 2y \leq 60$ are constraints $x \geq 0$ and $y \geq 0$ are non-negative constraints.

Solution : The set of all values which satisfy the constraints of linear programming problems is called 'Solution'.

Feasible solution : Set of values of variables which satisfy the all constraints with non-negative constraints also, called feasible solution.

Optimal solution : Optimal solutions of linear programming problem is a feasible solution for which objective fucntion has maximum or minimum value.

Note : Optimal solution is actual solution of linear programming problem.

15.04 Graphical method to solve linear programming problems :

Graphical method is easiest method to solve linear programming problem. Graphical method is possible only if there is only two variable in linear programming problem.

Corner point method :

This method is based on 'Fundamental Extreme point Theorem', which states that, "If any linear programming problems attains an optimal solution, then one of the corner points (vertices) of the convex polygon at all feasible solution gives the optimal solution",.

Following algorithm can be used to solve a linear programming problem in two variables graphically by using corner-point method:

1. Formulate the given linear programming problem in mathematical form if it is not given in mathematical form.
2. Convert all inequalities (constraints) into equations and drawn their graphs. To draw graph of a linear equation, but $y = 0$ in it and obtain a point on x-axis similarly by putting $x = 0$. Obtain a point on y-axis. Join these points to obtaine graph of the equation.
3. Determine the region represented by each inequation. To determine the region represented by an inequation replace x and y both by zero, if the inequation reduces to as valid statement, then the region containing the origin is the region represented by the given inequation. Otherwise, the region not containing the origin is the region represented by the given inequation.
4. Obtain the region in xy-plane containing all points that simultaneously satisfy all constraints including non-negativity restrictions. The polygonal region so obtained is the feasible region and is known as the convex polygon of the set of all feasible solutions of linear programming problem.
5. Determine the co-ordinates of vertices (corner points) of the convex polygon obtained in setp 2.
6. Obtain the values of the objective function at each vertices of convex polygon. The point where objective function attains its optimum (maximum or minimum) value is the optimal solution of the given linear programming problem.

Now we have to solve the example of 15.03 by graphical method when problem is given following:

Maximize $Z = 60x + 50y$

Constraints $x + 2y \le 40$

$3x + 2y \le 60$

and $x \ge 0,\ y \ge 0$

Firstly we have to convert the constraints into equations;

$$x + 2y = 40 \quad (1)$$

$$3x + 2y = 60 \quad (2)$$

So, there are two points A (40, 0) and B (0, 20). Just like that putting $x = 0$ in equation (2) then $y = 30$ and for $y = 0$, $x = 20$, then we have two point C (0, 30) and D (20, 0). After joining A, B, C and D we have obtained the graph of line (1) and (2).

$x + 2y = 40$

x	40	0
y	0	20

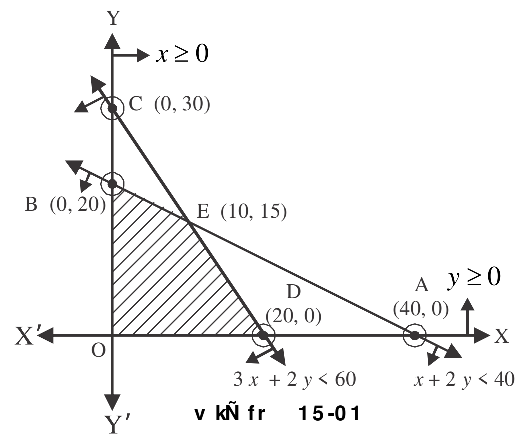
$A(40,\ 0);\ B(0, 20)$

$3x + 2y = 60$

x	0	20
y	30	0

$C(0,\ 30);\ D(20, 0)$

To determine the region of enequality $x+2y \le 40$, we have to put value of x and y equal to zero, inequality $(0)+2(0) \le 40$ satisfied. So, feasible region of inequality is toward origin. Just like that we have investigate the inequality $3x+2y \le 60$ by putting $x = 0$, $y = 0$ which satisfied the inequality. So, feasible region of given inequality is also towards the origin.



Shaded region ODEB is set of all possible values which satisfy the all constraints including non-negative constraints. There is no any solution beyond this region. next step is to find a solution from feasible solution of region ODEB by which we can obtain the optimal solution.

After inspecting the feasible solutions we have find that optimal solution will be on border line of ODEP. Now we have to tabulate the objective function on corner points O, D, E, B of feasible region ODEB.

Corner points	***x*-coordinate**	***y*-coordinate**	**Objective function** $Z = 60x + 50y$
O	0	0	$Z_0 = 0$
D	20	0	$Z_D = 1200$
E	10	15	$Z_E = 1350$
B	0	20	$Z_B = 1000$

From the above table, it is clear that objective function has its maximum value at E(10, 15), so, solution given by E is optional solution.

Note :

(1) If feasible solution of any linear programming problem gives a convex polygon then any corner point of polygon attain maximum value of objective fucntion and any other corner point attain minimum value of objective function.

(2) Sometimes the feasible region of linear programming problem is not a bounded convex polygon. That is, it extends indefenetely in any direction. In such case, we say that the feasible region is unbounded. Above algorithm is applicable when the feasible region is bounded. If the feasible region is unbounded, then we find the values of the objective function $Z = ax + by = M$ by at each corner point of the feasible region. Let M and m respectively denote the largest and smallest values of Z at there points. In order to check whether Z has maximum and minimum values at M and m respectively, we proceed as follows:

(i) Draw the line $ax + by > M$ and find the open half plane $ax + by > M$. If the open half plane represented by $ax + by > M$ has no point common with the unbounded feasible region, then M is the maximum value of Z otherwise Z has no maximum value.

(ii) Draw the line $Z = ax + by = m$ and find the half plane $ax + by < m$. If the half plane $ax + by < m$ has no point common with the unbounded feasible region, then m is the minimum value of z. Otherwise, Z has no minimum value.

Illustrative Examples

Example 1. Solve the following LPP graphically

Maximize $Z = 5x + 3y$

Constraints $3x + 5y \le 15$

$5x + 2y \le 10$

and $x \ge 0,\ y \ge 0$

Solution : Converting the given inequalities into equations, we obtain the following equation :

$$3x + 5y = 15 \quad (1)$$

$$5x + 2y = 10 \quad (2)$$

Region represented by $3x + 5y \le 15$: The line $3x + 5y = 15$ meets the coordinate axes at A (5, 0) and B (0, 3). Join these points to obtain the line $3x + 5y = 15$. Clearly (0, 0) satisfies the inequality $3x + 5y \le 15$. So the region containing the origin represents the solution set of inequation $3x + 5y \le 15$.

3x + 5y = 15

x	5	0
y	0	3
	A(5, 0),	B (0, 3)

Region represented by $5x + 2y \le 10$. The line $5x + 2y = 10$ meets the coordinate axes at C (2,0) and D (0,5) respectively.

$5x + 2y = 10$

x	2	0
y	0	5

Join these points to obtain line $5x + 2y \le 10$. Clearly (0, 0) satisfies the inequation 5 (0) + 2 (0) = 0 $\le$ 10. So, the region containing the origin represents the solution set of this inequation.

The shaded region OCEB in figure represents the common region of the inequations. This region is feasible region of given LPP.

The coordinates of the vertices (conrer points) of the shaded feasible region are 0(0, 0), C(2, 0), $E(20/19,\ 45/19)$ and B(0, 3). These points have been obtained by solving the equations at the corresponding intersecting lines, simultaneously.

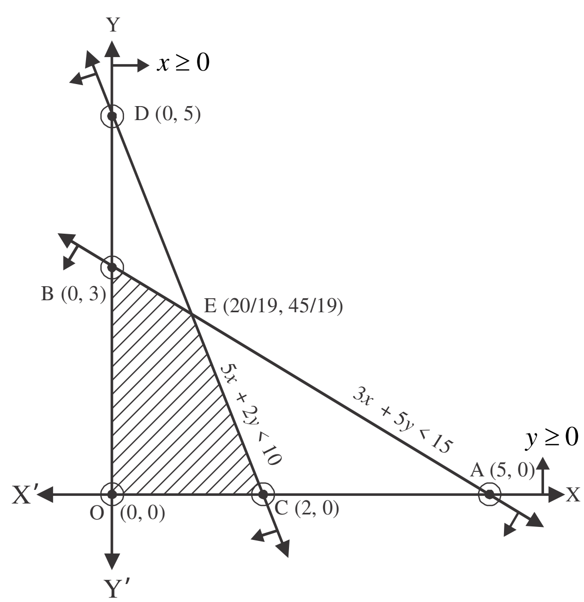


Fig. 15.02

The values of objective function at these points are given in the following table.

Points	***x*-co-ordiante**	***y*-co-ordinate**	**Objective function Z = 5*x* + 3*y***
O	0	0	$Z_0 = 5(0) + 3(0) = 0$
C	2	0	$Z_C = 5(2) + 3(0) = 10$
E	20 / 19	45 / 19	$Z_E = 5(20/19) + 3(45/19) = 235/19$
B	0	3	$Z_B = 5(0) + 3(3) = 9$

Clearly Z is maximum at $E(20/19, 45/19)$. Hence $x = 20/19, y = 45/19$ is the optimal solution of the given LPP. The optimal value of Z is 235 / 19.

Example 2. Solve the following linear programming problem graphically.

Minimize $Z = 200x + 500y$

Subject to the constraints $x + 2y \geq 10$

$3x + 4y \leq 24$

and $x \geq 0, \ y \geq 0$

Solution : The inequalities in the form of equations are

$$x + 2y = 10 \quad (1)$$

$$3x + 4y = 24 \quad (2)$$

Area shown by the inequality $x + 2y \geq 10$

Line $x + 2y = 10$ meets the coordinate axes at points A (10, 0) and B (0,5).

x + 2y = 10

x	10	0
y	0	5

A (10, 0) ; B (0, 5)

Area shown by the inequality $3x + 4y \leq 24$

Line $3x + 4y = 24$ meets the coordinate axes at points C(8, 0) and D (0, 6).

3x + 4y = 2y

x	8	0
y	0	6

C (8, 0) ; D (0, 6)

The shaded region in figure is the feasible region determined by the system of constraints. We observe that the feasible region BED is bounded. So, we now use corner points method to determine the maximum value of Z.

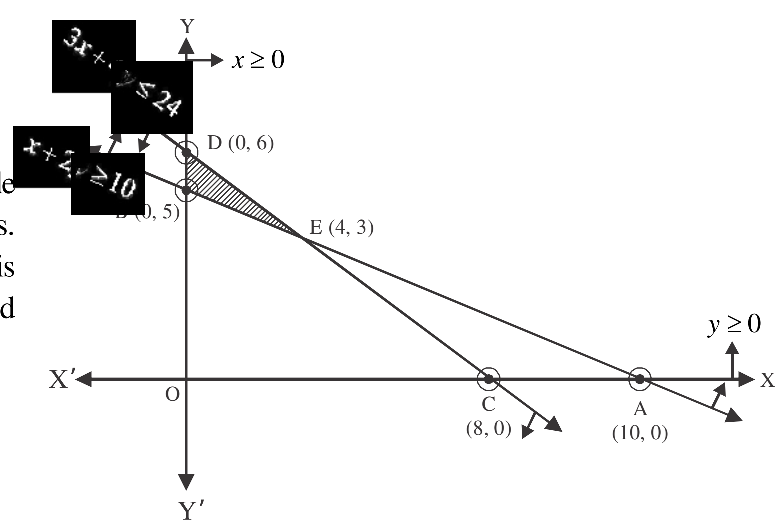


Fig. 15.03

Now we evaluate Z at each corner point.

Points	*x*-co-ordinate	*y* co-ordinate	Objective function Z = 200*x* + 500*y*
B	0	5	$Z_B = 200(0) + 500\ (5) = 2500$
E	4	3	$Z_E = 200\ (4) + 500\ (3) = 2300$
D	0	6	$Z_D = 200\ (0) + 500\ (6) = 3000$

Hence the minimum value at point E (4, 3) is 2300.

Example 3. Solve the following linear programming problem graphically.

Maximize $$Z = y + \frac{3}{4}x$$

subject to the constraints $$x - y \geq 0$$

$$-x/2 + y \leq 1$$

and $$x \geq 0,\ y \geq 0$$

Solution : The inequalities in the form of equations are

$$x - y = 0 \qquad (1)$$

$$-x/2 + y = 1 \qquad (2)$$

Area shown by the inequality $x - y \geq 0$

Line $x - y = 0 \Rightarrow x = y$ meets at points O (0, 0) ; A (1, 1).

x = y

x	0	1
y	0	1

Area shown by the inequality $-x/2 + y \leq 1$

Line $-x/2 + y = 1$ meets the coordinate axes at points B(–2,0) and C(0,1).

$-x/2 + y = 1$

x	–2	0
y	0	1

B(–2, 0) ; C(0, 1)

We draw the graph of the equations. The shaded region in fig 15.04 is the feasible region determined by the system of constraints. We observe that the feasible region is unbounded. So, We can see that there is no point satisfying all the constraints simultaneosuly. Thus the problem is having no feasible region and hence no feasible solution.

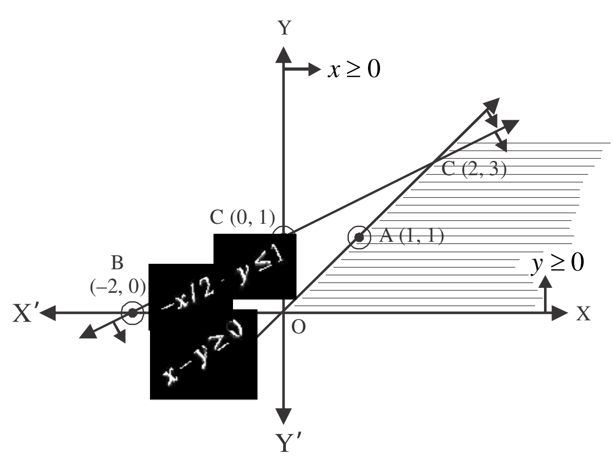


Fig. 15.04

Example 4. Solve the following linear programming problem graphically:

Maximize $Z = 3x + 4y$

Subject to constraint $x + y \leq 3$

$2x + 2y \leq 12$

and $x \geq 0, \; y \geq 0$

Solution : The inequalities in the form of equations are

$$x + y = 3 \qquad (1)$$

$$2x + 2y = 12 \qquad (2)$$

Area shown by the inequality $x + y \leq 3$:

Line $x + y = 3$ meets the coordinate axes at points A (3, 0) and B (0, 3).

x + y = 3

x	3	0
y	0	3

A (3, 0) ; B (0, 3)

Area shown by the inequality $2x + 2y \geq 12$:

Line $2x + 2y = 12$ meets the coordinate axes at points C(6, 0) and D(0, 6)

2x + 2y = 12

x	6	0
y	0	6

C (6, 0) ; D (0, 6)

We draw the graph of the equations. The shaded region in fig. 15.05 is the feasible region determined by the system of constraints. We observe that the feasible region is unbounded. So, we can see that there is no point satisfying all the constraints simultanesouly. Thus, the problem is having no feasible region and hence no feasible solution.

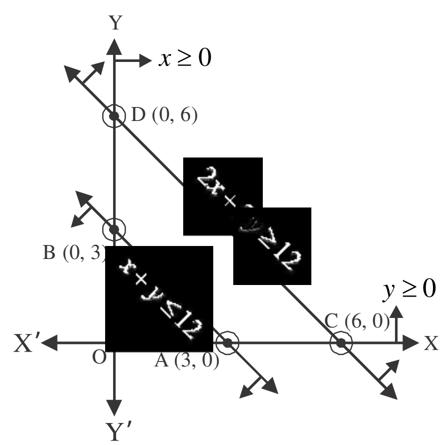


Fig. 15.05

Example 5. Solve the following linear programming problem graphically:

Maximize $Z = 2x + 3y$

Subject to constraints $4x + 6y \leq 60$

$2x + y \leq 20$

and $x \geq 0, \; y \geq 0$

Solution : The inequalities in the form of equations are

$$4x + 6y = 60 \qquad (1)$$

$$2x + y = 20 \qquad (2)$$

Area shown by the inequality $4x + 6y \leq 60$:

Line $4x+6y=60$ meets the coordinate axes at points A (15, 0) and B(0,10).

4x + 6y = 60

x	15	0
y	0	10

A(15, 0) ; B (0, 10)

Area shown by the inequality $2x + y \leq 20$:

Line $2x+y=20$ meets the coordiante axes at points C(10, 0) and D(0, 20).

2x + y = 20

x	10	0
y	0	20

C (10, 0) ; D (0, 20)

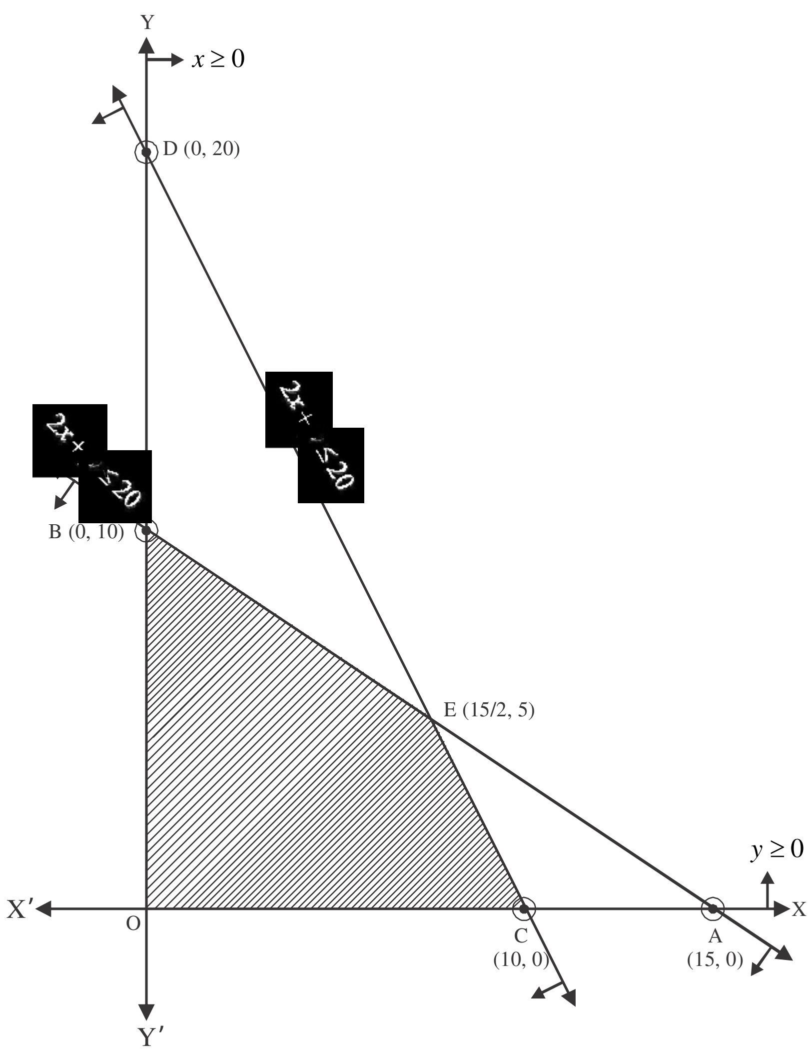


Fig. 15.06

The shaded region in Fig. 15.06 is the feasible region determined by the system of constraints. We observe that the feasible region OCEB is bounded. So, we now use corner point method to determine the maximum value of Z. The coordinates of the corner points O, C, E and B are O(0, 0), C (10, 0), $E(15/2, 5)$ and B (0, 10). Now we evaluate Z at each corner point.

Points	***x*-coordinate**	***y*-coordiante**	**Objective function $Z = 2x + 3y$**
O	0	0	$Z_O = 2(0) + 3(0) = 0$
C	10	0	$Z_C = 2(10) + 3(0) = 20$
E	15 / 2	5	$Z_E = 2(15/2) + 3(5) = 30$
B	0	10	$Z_B = 2\ (0) + 3(10) = 30$

Hence the maximum value at point $E(15/2, 5)$ and $B(0, 10)$ is the maximum value is obtained at points E and B.

Note: The reason for the inifinte solution is the objective function $Z = 2x + 3y$ which is parallel to the line $4x + 6y = 60$.

Exercise 15.1

Solve the following linear Programming problems graphically:-

1. Minimize $Z = -3x + 4y$

   Subject to the constraints $x + 2y \le 8$

   $3x + 2y \le 12$

   and $x \ge 0,\ \ y \ge 0$

2. Maximize $Z = 3x + 4y$

   Subject to the constraints $x + y \le 4$

   and $x \ge 0,\ \ y \ge 0$

3. Minimize $Z = -50x + 20y$

   Subject to the constraints $2x - y \ge -5$

   $3x + y \ge 3$

   $2x - 3y \le 12$

   and $x \ge 0,\ \ y \ge 0$

4. Minimize $Z = 3x + 5y$

   Subject to the constraints $x + 3y \ge 3$

   $x + y \ge 2$

   and $x \ge 0,\ \ y \ge 0$

5. Find the maximum and minimum value of $Z = 3x + 9y$

Subject to the constraints $x + 3y \le 60$

$x + y \ge 10$

and $x \ge 0, \quad y \ge 0$

6. Minimize $Z = x + 2y$

Subject to the constraints $2x + y \ge 3$

$x + 2y \ge 6$

and $x \ge 0, \quad y \ge 0$

7. Find the maximum and minimum value of $Z = 5x + 10y$

Subject to the constraints $x + 2y \le 120$

$x + y \ge 60$

$x - 2y \ge 0$

and $x \ge 0, \quad y \ge 0$

8. Maximize $Z = x + y$

Subject to the constraints $x - y \le -1$

$-x + y \le 0$

and $x \ge 0, \quad y \ge 0$

9. Maximize $Z = 3x + 2y$

Subject to the constraints $x + y \ge 8$

$3x + 5y \le 15$

and $x \ge 0, \quad y \ge 0$

10. Maximize $Z = -x + 2y$

Subject to the constraints $x \ge 3$

$x + y \ge 5$

$x + 2y \ge 6$

and $x \ge 0, \quad y \ge 0$

15.05 Different types of linear programming problems

In this section, we will discuss about some important linear programming problem like diet related problem, manufacturing related problem and transportation related problem.

Diet related problem:

In these problems, we determine the amount of different kind of constituents / nutrients which should be included in a diet so as to minimize the cost of the desired diet such that it contains a certain minimum amount of each constituent / nutrients.

Illustrative Examples

Example 6. A human requires definite amount of two type of vitamin (Vitamin A and Vitamin B) for balanced food. These vitamins find in two different food product (F_1 and F_2). Vitamin contained in one unit of each food product, minimum requirement for balanced food and prices of per unit food product is given in table.

Vitamin	Food product		Daily requirement
	F_1	F_2	
A	2	4	40
B	3	2	50
Price per unit (in ₹)	3	2.5	

How much unit of both produce its used os that the minimum requirement for balanced food is fullfilled?

Solution : Let x unit of F_1 and y unit of F_2 is required for minimum necesstiy. Then price of x unit of F_1 will be $3x$ and y unit of F_2 will be 2.54. Total price will be $3x + 2.5\,y$, we have to calculate minimum value.

Objective function is $Z = 3x + 2.5y$

Subject to constraint for vitamin A :

$$2x + 4y \geq 40$$

Subject to constraint for vitamin B :

$$3x + 2y \geq 50$$

Since units of required food product may not be negative, so, non-negative constraint

$$x \geq 0, \quad y \geq 0$$

So, mathematical formulation of given LPP

Minimize $Z = 3x + 2.5y$

Constraint $2x + 4y \geq 40$

$3x + 2y \geq 50$

and $x \geq 0, \quad y \geq 0$

Region represented by indequation $2x + 4y \geq 40$:

Line $2x + 4y = 40$ meets the coordiante axes at A(20, 0) and B(0, 10) respectively.

2x + 4y = 40

x	20	0
y	0	10

Join these points to obtain line $2x \;\; 4y = 40$, But (0, 0) doesn't satisfy the inequation $2(0) + 4(0) = 0 \geq 40$, So, the region opposite to the origin represents the solution set of this inequation.

Region represented by $3x + 2y \geq 50$

Line $3x + 2y \geq 50$ meets the coordiante axes at point $C(50/3, 0)$ and D (0, 25).

3x + 2y = 50

x	50/3	0
y	0	25

Join these points to obtain line $3x + 2y = 50$. But (0, 0) doesn't satisfy the in equation $3x + 2y \geq 50$, so, the region opposite the origin represents the solution set of inequations.

Region represented by $x \geq 0$ and $y \geq 0$.

Since every point in first quadrant satisfies, the both inequation. So, the region represented by inequations $x \geq 0$ and $y \geq 0$ in first quadrant.

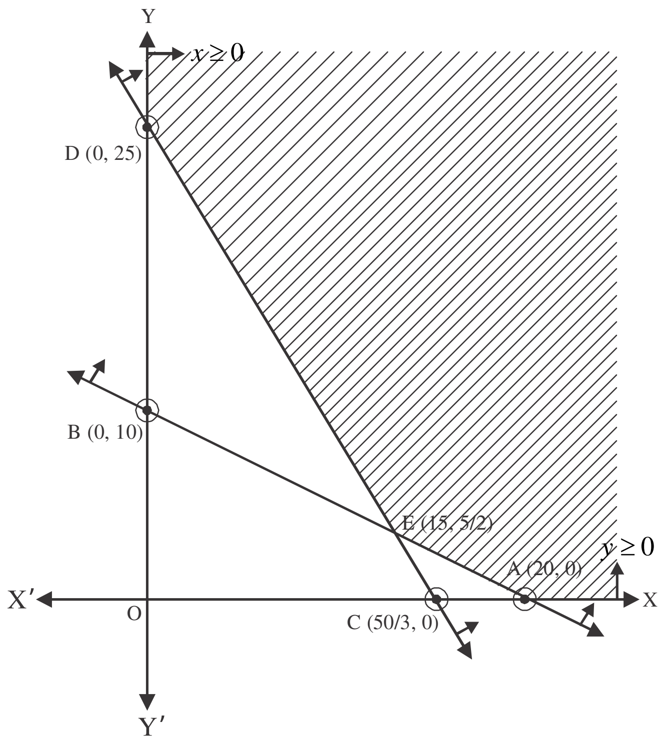


Fig. 15.07

The coordinates of vertices (corner points) of shaded region are A (20, 0) ; $E(15, 5/2)$ and D (0, 25). Where E is intersection point of line $2x + 4y = 40$ and $3x + 2y = 50$.

The values of objective function at these points are given in following table:

Points	*x* co-ordinate	*y* co-ordinate	Objective function Z = 3*x* + 2.5*y*
A	20	0	$Z_A = 3(20) + 2.5(0) = 60$
E	15	5 / 2	$Z_E = 3(15) + 2.5\ (5/2) = 51.25$
D	0	25	$Z_D = 3(0) + 2.5\ (25) = 62.5$

Clearly Z is minimum at point $E(15, 5/2)$. Since feasible region is unbounded. So, we have to draw graph of $3x + 2.5y < 51.25$. Resultant open half plane represented by in equation $3x + 2.5y < 51.25$ doesn't have any common point with feasible region. So, minimum value of LPP is 51.25, Rs. 50, for optimal solution we have 15 unit of F_1 and 5/2 unit of F_2.

Manufacturing problems:

In these prloblems, we determine the number of units of different products which should be produced and sold by a firm when each product requires a fixed manpower, maching hours, labour hour per unit of product, warehouse space per unit of output etc., in order to make maximum profit.

Illustrative Examples

Example 7. A firm manufacturing two types of electric items, A and B. Can make a profit of ₹ 20 per unit of A and ₹ 30 per unit of B. Each unit of A requires 3 motors and 2 transformers and each unit of B requires 12 motors and 4 transformers. The total supply of these per month is restricted to 210 motors and 300 transformers. Type B is an expert model requiring a voltage stabilzer which has a supply restricted to 65 units per month. Formulate the LPP for maximum profit and solve it graphically.

Solution : Left firm manufactures x and y unit respectively of A and B to get maximum profit. Profit per unit of A and B is ₹ 20 and ₹ 30 respectively. So, profit from x and y unit of A and B is,

Z is objective function. $Z = 20x + 30y$

Constraint for motor

For manufacturing x unit of A and y unit of B we have need of $3x$ and $2y$ motors and total supply of motor per month is 210 only. So,

$$3x + 2y \leq 210$$

Constraint for transformer.

For manufacturing of x unit of A and y unit of B we have need of $2x$ and $4y$ transformers and total supply at transformer per month is 300 only, So,

$$2x + 4y \leq 300$$

Voltage stabilizer is used in only B and its supply per month is only 65, 50

$$y \leq 65$$

Manufactured unit may not be negative. So,

$$x \geq 0, \quad y \geq 0$$

So, mathematical formulation of LPP is given below,

Maximize $Z = 20x + 30y$

constraint $3x + 2y \leq 210$

$2x + 4y \leq 300$

$y \leq 65$

and $x \geq 0, \quad y \geq 0$

Convert all the inequations into equation,

$$3x + 2y = 210 \quad (1)$$

$$2x + 4y = 300 \quad (2)$$

$$y = 65 \quad (3)$$

Region represented by $3x + 2y \leq 210$:

Line $3x + 2y = 210$ meets the coordinate axes at point A(70, 0) and B(0, 105).

3x + 2y = 210

x	70	0
y	0	105

Join A and B to obtain the line $3(0) + 2(0) = 0 \leq 210$. (0, 0) satisifes the inequation, So the region containing the origin represents the solution set of inequation.

Region represented by 2x + 4y $\leq$ 300:

Line $2x+4y=300$ meets the co-ordinate axes at C(150, 0) and D(0, 75) respectively.

2x + 4y = 300

x	150	0
y	0	75

Join C and D to obtain the line $2(0)+4(0)=0\leq 300$. (0, 0) satisifes the inequation, so, region containing the origin represents the solution set of inequation.

Region represented y $\leq$ 65 :

Line $0x+y=65$ meets at point E (5, 65) an F(10, 65).

0x + y = 65

x	5	10
y	65	65

Join E and F to obtain line 0x + y = 65. (0, 0) satisifes the inequation, so, region containing the origin represent the solution set of inequation.

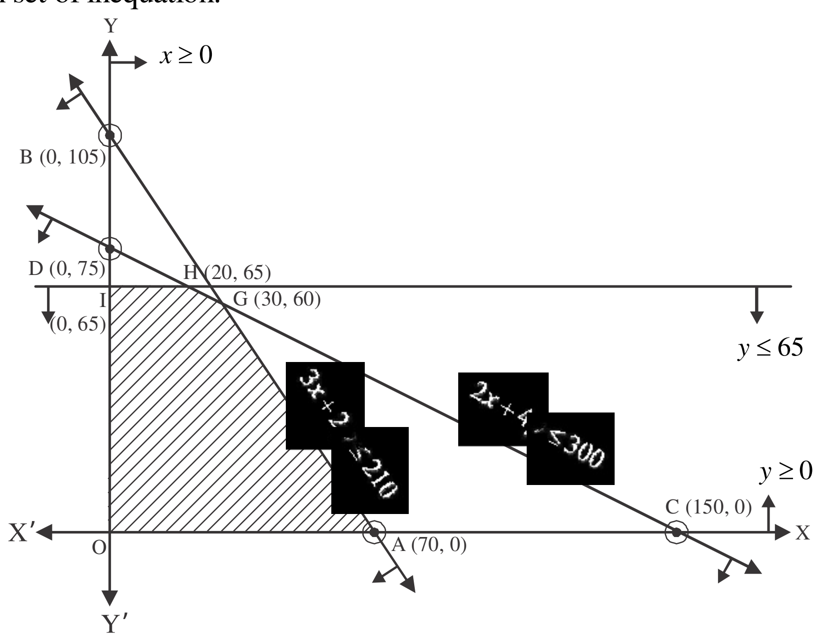


Fig. 15.08

Region represented by x $\geq$ 0 and y $\geq$ 0 :

Since points on first quadrant satisfy the both inequation. So, region represented by $x \geq 0$ and $y \geq 0$ is first quadrant.

Shaded region OAGHI represents the common region of above inequations. This region is feasible region of given LPP. Vertices of shaded feasible region are O(0,0), A (70,0), G(30, 60), H (20, 65) and I(0, 65). Where G and H are intersection points of $2x+4y=300$ and $3x+2y=210$ and $y=65$. Values of objective function is given in following table at these points.

Points	x-co-ordinate	y-co-ordinate	Objective function $Z = 20x + 30y$
O	0	0	$Z_O = 20(0) + 30(0) = 0$
A	70	0	$Z_A = 20(70) + 30(0) = 1400$
G	30	60	$Z_G = 20(30) + 30(60) = 2400$
H	20	65	$Z_H = 20(20) + 30(65) = 2350$
I	0	65	$Z_I = 20(0) + 30(65) = 1950$

Clearly, it is clear from table that objective function has its maximum value at point G(30, 60). So, for maximum profit, firm will manufacture 30 unit of A and 60 unit of B from which it gain maximum profit of ₹2400.

Transportation problems :

In this type of problems, we have transport diffrent objects from diefferent factories and different-different places according to demands on market. This type of transport according to supply from factories to the market so that cost of transportation is minimum.

Illustrative Examples

Example 8. There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

From / To	Cost (In ₹)		
	A	**B**	**C**
P	16	10	15
Q	10	12	10

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. What will be the minimum transportation cost?

Solution : The problem can be explained diagrammatically as follows : Let x units and y units of the commodity be transported from the factory at P to the depots at A and B respectively. Then $(8 - x - y)$ units will be transported to depot at C.

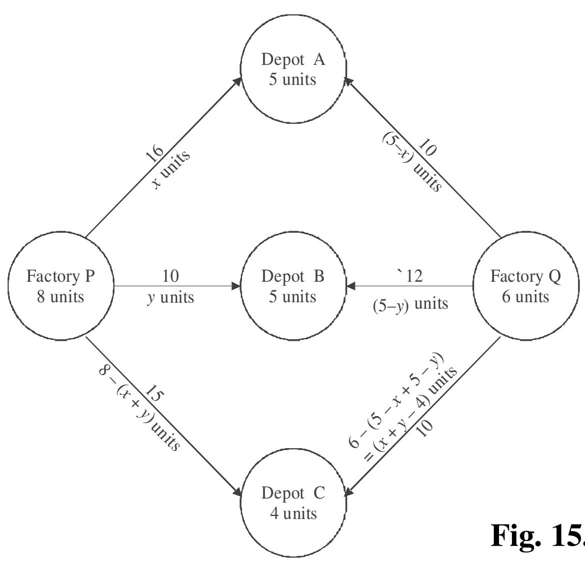


Fig. 15.09

Hence, we have $x \geq 0, \quad y \geq 0$ and $\quad 8 - x - y \geq 0$

$\Rightarrow \quad x \geq 0, \quad y \geq 0$ and $\quad x + y \leq 8$

Now, the weekly requirement of the depot at A is 5 units of the commodity. Since x units are transported from the factory at P, the remaining $(5 - x)$ units need to be transported from the factory at Q. Obviously $x \leq 5$

Similarly, $\quad y \leq 5$ and $x + y \geq 4$

Total transportation cost Z is given by

$$Z = 16x + 10y + 15(8 - x - y) + 10(5 - x) + 12(5 - y) + 10(x + y - 4)$$

$$Z = x - 7y + 190$$

Therefore, the problem reduces to

Minimize $\quad Z = (x - 7y + 190)$

subject to the constraints $\quad x \geq 0, \; y \geq 0$

$$y \leq 5$$

$$x \leq 5$$

$$x + y \geq 4$$

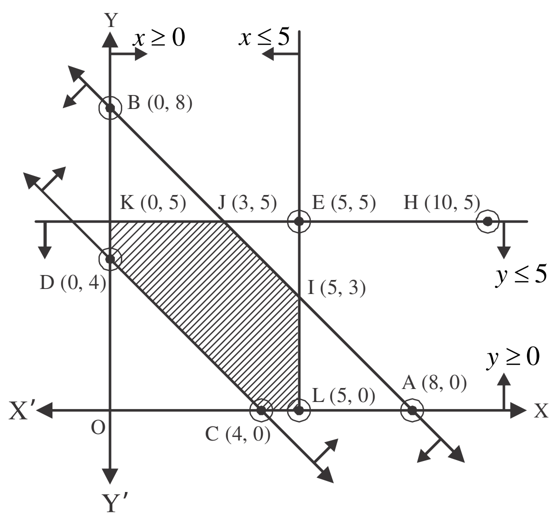


Fig. 15.10

The shaded region CLIJKD represented by the constraints above

Observe that the feasible region is bounded. The coordinates of the corner points of the feasible region are (0, 4), (0, 5), (3, 5), (5, 3), (5, 0) and (4, 0). Let us evaluate Z at these points.

Corner Point	**Z = 10 (*x* – 7*y* + 190)**
(4, 0)	162
(5, 0)	155
(5, 3)	158
(3, 5)	174
(0, 5)	195
(0, 4)	194

From the table, we see that the minimum vlaue of Z is 155 at the point (5, 0)

Hence, the optimal transportation strategy will be to deliver 0, 5 and 3 units from the factory at P and 5, 0 and 1 units from the factory at Q to the depots at A, B and C respectively. Corresponding to this strategy, the transportation cost would be minimum, i.e., ₹ 155.

Exercise 15.2

1. A dietician wishes to mix two type sof foods in such a way that the vitamin content of the mixture contains atleast 8 units of vitamin A and 10 units of vitamin C. Food 1 contains 2 units / kg of vitamin A and 1 unit / kg of vitamin C. Food II contains 1 unit / kg of vitamin A and 2 units / kg of vitamin C. It costs Rs. 50 / kg to purchase Food I and Rs. 70 / kg to purchase Food II. Formulate a linear programming problem to minimise the cost of the mixture.
2. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture cotnains atleast 10 uns if vitamin A, 12 units of vitamin B and 8 units of vitamin C the vitmain contents of one kg food is given below:

	Vitamin A	**Vitamin B**	**Vitamin C**
Food *X*	1	2	3
Food *Y*	2	2	1

   One kg of food X costs ₹ 16 and one kg of food Y costs ₹ 20. Find the least cost of the mixture which will produce the required diet?
3. One kind of cake requires 300 grams of flour and 15 grams of fat and another kind of cake requires 150 grams of flour and 30 grams of fat. Find the maximum number of cake which can be made from 7.5 kg of flour and 600 grams of fat assuming that their is no shortage of other ingredients in making the cake.
4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 17.50 per package nuts and ₹ 7 pr package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day?
5. A furniture dealer deals in tables and chairs. He has ₹ 5760 to invest and has a storage space of atmost 20 pieces. A table cost ₹ 360 and a chair cost ₹ 240. He estimates that from the sale of one table he can make a profit of ₹ 22 and by selling one chair. He makes a profit of ₹ 18. He wants to know how many tables and chairs he should buy from the available money, so as to maximize his profit, assuming that he can sell all the items which he buys. Solve the following optimising problem graphically.
6. A factory manufactures two types of screws A and B. Each type of screw requires the use of two machines automatic and a hand operated. It takes 4 minutes on automatic and 6 minutes on hand operated machines to manufacture a package of screws A while it takes 6 minutes on automatic and 3 minutes on hand operated machine to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufactuer can sell a package of screws A at a profit to 70 paise and screw B at a profit of ₹ 1. Assuming that he sells all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.
7. A company manufactures two types of novelty sourvenirs made of plywood. Sourvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes for assembling. There are 3 hours and 20 minutes available for cuttign and 4 hours for assembling. The profit is Rs 5 each for type A and ₹ 6 each for type B sourvenirs. How many sourvenirs of each type should the company manufacture in order to maximize the profit?

8. There are two types of fertilisers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for his corporation. If F_1 costs ₹ 6 / kg and F_2 costs ₹ 5 / kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

9. A merchant plans to sell two types of personal computers - desktop model and a portable model that will cost ₹ 25,000 and ₹ 40,000 respectively he estimates that total monthly demand of computer will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than ₹ 70 lakhs.

10. Two godowns A and B have grain capacity of 100 quintals and 50 quintals resp. They supply to three ration shops D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportantion per quintal from the godwon to the shops are given in the following table:-

Transportation cost per quintal (in ₹)

From / To	**A**	**B**
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

Miscellaneous Examples

Example 9. A company produces two types of leather belts, say type A and B. Belt A is a superior quality and belt B is of lower quality. Profits on each type of belt are ₹ 2 and ₹ 1.50 per belt respectively. Each belt of type A requires twice as much time as required by a blet of type B. If all belts were of type B, the company could produce 1000 belts per day,. But the supply of leather is sufficient only for 800 belts per day (Both A and B comined). Belt A requires a fancy buckle and only 400 fancy buckles are available for this per day. For belt B, only 700 buckles are available per day.

Solution : Let company produces x unit of A and y unit of B. Profit from A and B are ₹2 and ₹ 1.50 respectively. So, objective function is.

Maximize $$Z = 2x + 1.50y$$

If all belts be of type B then company produces 1000 belts per day. Time taken to produce y unit of B type belt $= \frac{y}{1000}$

Since, time taken to produce A-type belt is twice with respect to B. So, time taken to produce A type belt $= \frac{x}{500}$

$$\frac{x}{500} + \frac{y}{1000} \le 1$$

$$\Rightarrow \quad 2x + y \le 1000$$

Supply of leather is limited produce only 800 belt. So,

$$x + y \le 800$$

Since 400 buckles are available for A-type belt and 700 buckle for B-type belt.

$$x \le 400, \qquad y \le 700$$

No. of belt never be negative. So,

$$x \ge 0, \qquad y \ge 0$$

Mathematical formulation of given LPP is

Maximize $\quad Z = 2x + 1.50y$

Constraint $\quad 2x + y \le 1000$

$$x + y \le 800$$

$$x \le 400$$

$$y \le 700$$

and $\quad x, \ y \ge 0$

Convert the inequations in equations,

$$2x + y = 1000 \quad (1)$$

$$x + y = 800 \quad (2)$$

$$x = 400 \quad (3)$$

$$y = 700 \quad (4)$$

Region represented by $2x + y \le 1000$:

Line $2x + y = 1000$ meets the co-ordinate axes at A (500, 0) and B(0, 1000).

2x + y = 1000

x	500	0
y	0	1000

Join A and B to obtain line $2(0) + (0) = 0 \le 1000$. (0, 0) satisfies the inequation. So, region containing the origin represents the solution set of inequation.

Region represented by $x + y \le 800$

Line $x + y = 800$ meets the coordinate axes at point C(800, 0) and D (0, 800).

x + y = 800

x	800	0
y	0	800

Join C and D to obtain line $x + y = 800$. (0, 0) satisfies the inequlaity. So, region containg the origin represents the solution set of inequation.

Region represented by $x \le 400$:

Line $x + 0y = 400$ meets at the point E(400, 10) and F(400, 20).

x + 0y = 400

x	400	400
y	10	20

Join E and F to obtain the line $x + 0y = 400$. (0, 0) satisfies the inequation $x \leq 400$. So, region containing the origin represents the solution set of inequation.

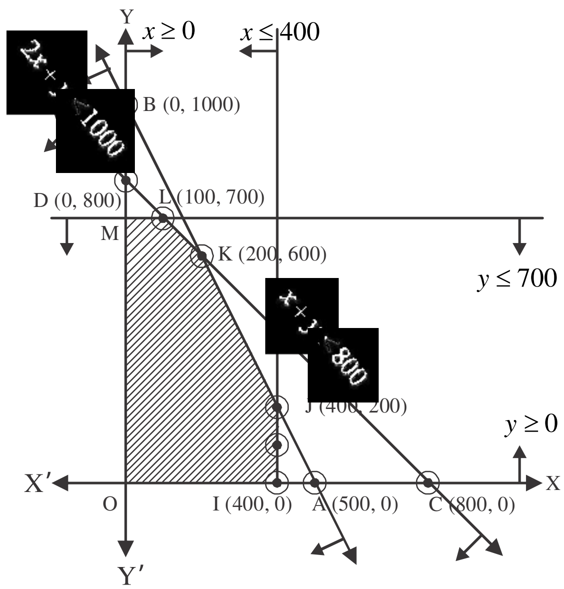


Fig. 15.11

Region represented by $x \geq 0$ and $y \geq 0$

Since, every point in first quadrant satisfy the both inequalities, So, region represented by $x \geq 0$, $y \geq 0$ is in first quadrant.

Shaded region of IKLM is common region of all inequation. That is feasible region of given LPP. Vertices of this region are O (0, 0), I (400, 0) J (400, 200), K (200, 600), L (100, 700), M (0, 700). Where J, K, L are intersection points of lines $x = 400$ and $2x + y = 1000$; $2x + y = 1000$ and $x + y = 800$; $y = 700$ and $x + y = 800$.

Values of objective function at these points are–

Points	x Co-ordinate	y Co-ordinate	Objective functions Z = 2x + 1.50y
O	0	0	$Z_O = (2)(0) + (1.50)(0) = 0$
I	400	0	$Z_I = (2)(400) + (1.50)(0) = 800$
J	400	200	$Z_J = (2)(400) + (1.50)(200) = 1100$
K	200	600	$Z_K = (2)(200) + (1.50)(600) = 1300$
L	100	700	$Z_L = 2(100) + (1.50)(700) = 1250$
M	0	700	$Z_M = (2)(0) + (1.50)(700) = 1050$

It is clear from table, objective function is maximum at K(200, 600). So, company produces 200 unit of A and 600 unit of B for maximum profit.

Example 10. The old hen can be purchased at ₹ 2 per hen whereas the price of new hen is 5 Rs. per hen. Old hense give 3 eggs and new hens give 5 eggs per week. Price of one egg is 30 paise. Investment on food of a hen per week is ₹ 1. How many hens of both type a man buy if he has only ₹ 80 and he earned profit more than ₹ 6. If than person can not keep more than 20 hens the solve the LPP by graphical method.

Solution: Let he purchases x new hens and y old hens. Since, a new hen gives 5 eggs per week, So, he earns ₹ 1.50 earn per week. After deducting food investment, gross profit is 50 paise.

Similarly, profit from old hen = ₹ (0.30 × 3 – 1) = ₹ (–0.10). So, objective function is

$Z = (.50)x - (.10)y$. Price of old hen is ₹ 2 per hen and price of new hen is ₹ 5 per hen. Also, the person has only ₹ 80. So, $5x + 2y \le 80$.

Again, that person can not keep more 20 hens in his house.

So, $x + y \le 20$. Person wants to get profit more than ₹ 6, $0.5x - 0.1y \ge 6$.

Purchased hens never be negative.

So, $x \ge 0,\ y \ge 0$

Mathematical formulation of given LPP is,

Maximize $Z = (\cdot 50)x - (\cdot 10)y$

Constraints $5x + 2y \le 80$

$x + y \le 20$

$0\cdot 5x - 0\cdot 1y \ge 6$

and $x \ge 0,\ y \ge 0$

Since the person wants to get profit more than ₹6. Therefore, it is not necessary to consider $0.5x - 0.1y \ge 6$.

$\therefore$ The LPP is maximize $Z = (\cdot 50)x - (\cdot 10)y$

Such that $5x + 2y \le 80$, $x + y \le 20$ and $x \ge 0,\ y \ge 0$

On converting the inequation into the equation, we get

$$5x + 2y = 80 \qquad (1)$$

$$x + y = 20 \qquad (2)$$

Region represented by 5*x* + 2*y* ≤ 80:

Line $5x + 2y = 80$ meets the coordinate axes at A (16, 0) and B (0, 40).

5x + 2y = 80

x	16	0
y	0	40

Join A and B to obtain line $5(0) + 2(0) = 0 \le 80$. (0, 0). Satisfy the inequation. So, region containing the origin gives the solution set of inequation.

Region represented by *x* + *y* ≤ 20:

Line $x + y = 20$ meets the coordinate axes at C(20, 0) and D(0, 20).

x + y = 20

x	20	0
y	0	20

Join C and D to obtain line $x + y = 20$, (0, 0) satisfy the inequation. So, region containing the origin represent the solution set of inequation.

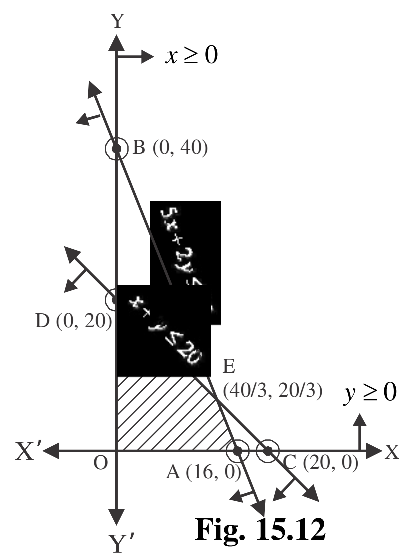


Fig. 15.12

Region represented by $x \geq 0$ and $y \geq 0$:

Since every point of first quadrant satisfy both the inequation. So, region represented by $x \geq 0$ and $y \geq 0$ is in first quadrant.

Shaded region OAED represents the common region of inequations. That is the feasible region. Vertices (corner points) of this region are 0(0, 0), A (16, 0), $E(40/3,\ 20/3)$ and D (0, 20) and E is intersection point of $x+y=20$ and $5x+2y=80$.

So, value of objective function on these point given in table.

Points	*x*-co-ordinate	*y*-co-ordinate	Objective function Z = (.50) *x* – (.10) *y*
0	0	0	Z_O = (·50) (0) – (·10) (0) = 0
A	16	0	Z_A (·50) (16) – (·10) (0) = 8
E	40 / 3	20 / 3	Z_E = (·50) (40 / 3) – (·10) (20 / 3) = 6
D	0	20	Z_D = (·50) (0) = (·10) (20) = –2

It is clear from above table that objective function is maximum at corner point (16, 0). So, for maximum profit the purchase 16 new hens to get profit of ₹ 8.

Miscellaneous Exercise – 15

Solve the following Linear Programming Problems graphically:

1. Maximize $Z=4x+y$

   constraints $x+y\leq 50$

   $3x+y\leq 90$

   and $x\geq 0,\ y\geq 0$

2. Maximize $Z=3x+2y$

   constraints $x+y\geq 8$

   $3x+5y\leq 15$

   and $x\geq 0,\ y\leq 15$

3. Maximize and Minimize $Z=x+2y$

   constraints $x+2y\geq 100$

   $2x-y\leq 0$

   $2x+y\leq 200$

   and $x\geq 0,\ y\geq 0$

4. Maximize $Z=3x+2y$

   constraints $x+2y\leq 10$

   $3x+y\leq 15$

   and $x\geq 0,\ y\geq 0$

5. Food for pateint must include a mixture of atleast 4000 units of vitamin 50 units mineral and 1400 units

calories. Two food products A and B are available at the cost of ₹ 3 and ₹ 4 per unit. Food product A constains 200 units of vitmain, 1 unit of mineral and 40 calories and food B contains 100 units of vitamin, 2 units mineral and 40 calories. What should be the mixture of food so that the cost is minimum.

6. A diet is to contain atleast 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs ₹ 4 per unit food and F_2 costs ₹ 6 per unit. One unit of food F_1 contains 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consist of mixture of these two foods and also meets the minimal nutritional requriements.

7. A furniture manufacturer makes table and chairs. These are made on two machines A and B. Machine A takes 2 hours and machine B takes 6 hours to make a chair, whereas machine A taskes 4 hours and machine B takes 2 hours to make a table. Machine A and B are used for 16 hours and 30 hours respectively. The manufacturer earns a profit of ₹ 3 and ₹ 5 on selling one chair and one table. Find the number of chairs and tables to be manufactured per day so as to get the maximum profit.

8. A firm manufactures two types of pills for headache size A and size B. Size A pill contains 2 grams aspirin, 5 grams bicorbonate and 1 gram sulphur whereas size B pill contains 1 gram aspirin, 8 grams bicorbonate and 66 grams sulphur. It is been found that for quick relief atleast 12 grams aspirin, 7.4 grams bicorbonate and 24 grams sulphur is required. For quick relief from pain what should be the minimum number of pill a patient should take.

9. A brick manufacturer has tow depots A and B with a storage capacity of 30,000 and 20,000 bricks. He takes the order from three builder P, Q and R of 15,000, 20,000 and 15,000 number of bricks. The cost of transportation to deliver 1000 bricks is given below in the table.

From/To	**P**	**Q**	**R**
A	40	20	30
B	20	60	40

keeping the transportation cost minimum how would the manufacturer send the bricks.

10. Constraints $x + y \leq 3$

$y \leq 6$

and $x,\ y \leq 0$

The area bounded by the above inequalities

(A) unbounded in first quadrant
(B) unbounded in first and second quadrant
(C) bounded in first quadrant
(D) None of these

IMPORTANT POINTS

1. Linear programming is mathematical method which is used to distribute the limited resources in optimized manner in competitive activities, while all used variables have linear relationship.
2. Set of values of variable which satisfied the all constraint of LPP is called a solution LPP.
3. Solution of LPP which satisifed the non-negative constraint is feasible solution and set of all feasible solution is called feasible region.
4. A feasible solution which gives optimal solution of LPP is called optimal solution.
5. Graphical method is applicable in LPP when there is only two variable in problem.
6. Graphical method maninly depends upont he extreme point theorem which states that 'An optimal solution

of a LPP, if it exists, occurs at one of the extrem (corner) points of the convex polygon of the set of all feasible solutions'.

7. Following algorithm can be used to solve a LPP in two variables graphically by using corner point method:
   (i) Formulate the given LPP in mathematical form it is not given in mathematical form.
   (ii) Convert all inequations into equations and draw their graphs. To draw the graph of a linear equation, puty y =0 and obtain the point on x-axis. Simiarly by putting $x = 0$ obtain a point on y-axis. Join these points to obtain the graph of the equation.
   (iii) Determine the region represented by each inequation. To determine the region represented by an inequation replace x and y both by zero, if the inequation reduces to a valid statement, then the region containing the origin is the region represented by the given inequation. Otherwise, the region not containing the origin is the region represented by the given inequation.
   (iv) Obtain the region in xy- plane containing all points that simultaneosuly satisfy all constraint including non-negative restrictions. The polygonal region is so obtained is the feasible region and is known as the convex polygon of the set of all feasible solutions of LPP.
   (v) Determine the coordinates of the vertices (corner points) of the convex polygon obtained in step II. There vertices are known as extreme points of the set of all feasible solutions of LPP.
   (vi) Obtain the values of the objective functions to each of vertices of the convex polygon. The point where the objective function attains its optimum (maximum or minimum) value is the optimal solutions of given LPP.
8. If the feasible region of LPP is bounded, i.e., it is a convex polygon. Then, the objective function $Z = ax + by$ has both maximum value M and minimum value m and each of these values is the optimal solution of given LPP.
9. Sometimes the feasible region of a LPP is not a bounded convex polygon. That is, it extends indefinitely in any direction. In such cases, we say that the feasible region is unbounded. The above algorithm is applicable when the feasible region is bounded. If the feasible region is unbounded, then we find values of objective function $Z = ax + by$ at each corner points of feasible region. Let M and m respectively largest and smallest values of Z at these points. In order to check whether Z has maximum and minimum values at M and m respectively.

We proced as follows:

(i) Draw the line $ax + by > M$ and find the open half plane $ax + by > M$. if the open half plane represented by $ax + by > M$ has no point common with the unbounded feasible region, then M is maximum value of Z has no maximum value.

(ii) Draw the line $ax + by < m$ and find the open half plane represented by $ax + by < m$. If the half-plane $ax + by < m$ has no point common with the unbounded feasible region, then m is the minimum value of Z, otherwise Z has no minimum value.

Answers

Exericse 15.1

1. point (4, 0), minimum $Z = -12$
2. point (0, 4), maximum $Z = 16$
3. For the given constraints no minimum value exists.
4. point (3 / 2, 1 / 2), minimum $Z = 7$
5. point (5, 5) minimum $Z = 60$ and points (15, 15) and (0, 20), maximum z = 120
6. points (6, 0) and (0, 3), minimum $Z = 6$
7. point (60, 0), minimum $Z = 300$ points (120, 0) and (60, 30) maximum $Z = 600$

8. For the given constraints no maximum valeu exists.

9. For the given constraints no feasible solution exists.

10. For the given constraints no maximum value of objective function exists.

Exercise 15.2

1. Minimum $Z = 5x + 7y$

constraints $2x + y \geq 8$

$x + 2y \geq 10$

$x \geq 0,\ y \geq 0$

For food I, 2 kg and for food II. 4 kg mixture is requried whose minimum value is ₹ 38

2. Minimum $Z = 6x + 10y$

constraints $x + 2y \geq 10$

$2x + 2y \geq 12$

$3x + y \geq 8$

$x \geq 0,\ y \geq 0$

For food I, 2 kg and for food II, 4 kg mixture is requried whose minimum valeu is ₹ 52.

3. 20, 10

4. Maximize $Z = 2.50x + y$

constraints $x + 3y \leq 12$

$3x + y \leq 12$

$x \geq 0,\ y \geq 0$

3 and 4 packets of nuts and bolts everday with a profit of ₹ 10.50

5. Maximize $Z = 22x + 18y$

constraints $x + y \leq 20$

$360x + 240y \leq 5760$

$x \geq 0,\ y \geq 0$

the dealer would buy 8 fans and 12 sweing macnine sto get the profit of ₹ 392

6. Maximize $Z = 0.7x + y$

constraints $4x + 6y \leq 240$

$6x + 3y \leq 240$

and $x \geq 0,\ y \geq 0$

the dealer would make 30 packets of bolts A and 20 packets of bolt B to get the maximum profit of ₹ 41.

7- Maximize $Z = 5x + 6y$

constraints $5x + 8y \leq 200$

$10x + 8y \leq 240$

and $x \geq 0,\ y \geq 0$

Firm should make 8 mementos of type A and s 20 memntos of type B to get the maximum profit of ₹ 160

8. Minimize $Z = (.60)x + (.40)y$

constraints $\frac{10x}{100} + \frac{5y}{100} \le 14$

$\frac{6x}{100} + \frac{10y}{100} \le 14$

and $x \ge 0,\ y \ge 0$

1000 kg of fertiliser F_1 and 80 kg of fertiliser F_2 should be used

minimum vlaue = ₹ 92

9. Maximize $Z = 4500x + 5000y$

Constraints $25000x + 40000y \le 7000000$

$x + y \le 250$

and $x \ge 0,\ y \ge 0$

The dealer will store 200 desktop and 50 portable computers so that to get the maximum profit of ₹ 1150000

10. Let depot A, provide x and y quintals of food to D and E

Minimize $Z = 6x + 3y + 5/2(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60)$

Constraints $x + y \le 100$

$x \le 60$

$y \le 50$

$x + y \ge 60$

and $x \ge 0,\ y \ge 0$

10, 50 and 40 Quintals for depot A to D, E and F

50, 0 and 0 Quintals for depot B to D, E and F

Miscellaneous Exercise - 15

1. At point (30, 0) Maximum is $Z = 120$

2. There is no feasible solution

3. At points (0, 50) and (20, 40) Minimum $Z = 100$ at (0, 200) and maximum Z = 400

4. At point (4, 3), Maximum $Z = 18$

5. 5 units of food material A and 30 units of food material B

6. Let x and y denotes the units of food material F_1 and F_2

Minimize $Z = 4x + 6y$

constraints $3x + 6y \ge 80$

$4x + 3y \ge 100$

and $x \ge 0,\ y \ge 0$

Minimum value = ₹ 104

7. 22 / 5 Chairs and 9 / 5 tables, maximum profit = ₹ 22.2

8. 2 tablest of size A and 8 tablets of size B

9. From depot A, the builders P, Q and R should be supplied 0, 20000, 10000 number of bricks and from depot B, the builders P, Q and R should be supplied, 15000, 0, 5000 number of bricks

10- (C)