Chapter 13

Direct and Indirect Proportions

Introduction to Direct and Inverse Proportions

What is Ratio?

The relationship between two quantities of same units expressed as "a to b" or "a : b", sometimes expressed as the quotient of the two quantities that indicates how many

times the first number contains the second.

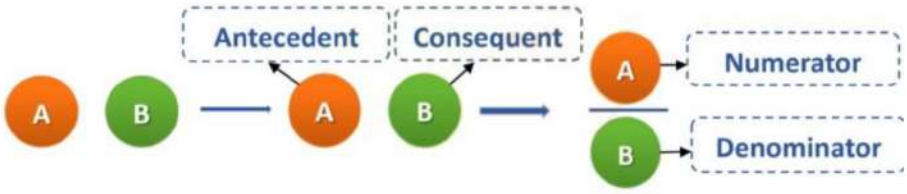
Ratio is represented in the form of a fraction of two quantities, the units of quantities must be the same.

Unit may be Objects, Persons, Students, Quality, Quantity, etc.

For example,

Mohan has 15 pens and 29 pencils, then the ratio of pens to pencils

would be 15:29 and the ratio of pencils to pens would be 29:15.



For example,

a) Find the ratio of 4 km to 200 m.

A= 4 km

B= 200 m

A= 4 km = 4 × 1000 m = 4000 m

Ratio is A: B = 4000 m: 200 m

Ratio is 20: 1

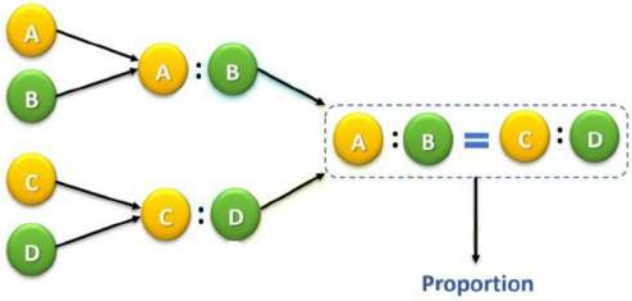
What is Proportion?

When four quantities are so related that the ratio between the first and the second quantities is equal to the ratio between the third and the fourth quantities, then the quantities are said to be in proportion.

The ratios which are equivalent are said to be in proportion.

Four quantities A, B, C, and D are said to be in proportion if A : B = C : D

and A : B :: C : D.



For example,

a) Show the numbers 44, 66, 42 and 63 are in proportion.

A = 44 : 66

$$= \frac{44}{66} = \frac{2}{3}$$ HCF of 44 and 66 is 22

B = 42 : 63

$= \frac{42}{63} = \frac{2}{3}$ HCF of 42 and 63 is 21

$A = B \Rightarrow \frac{2}{3} = \frac{2}{3}$

b) What number must be added to each of the numbers 11, 19, 23, 39

to get the numbers which are in proportion?

Let the number to be added be x. Then,

(11 + x) : (19 + x) :: (23 + x) : (39 + x)

$\Rightarrow \frac{(11+x)}{(19+x)} = \frac{(23+x)}{(39+x)}$

⇒ (11 + x) (39 + x) = (23 + x) (19 + x) [By cross multiplication]
⇒ $429 + 50x + x^2 = 437 + 42x + x^2$

⇒ $50x - 42x + x^2 - x^2 = 437 - 429$

⇒ $8x = 8$

⇒ $x = 8/8$

⇒ $x = 1$

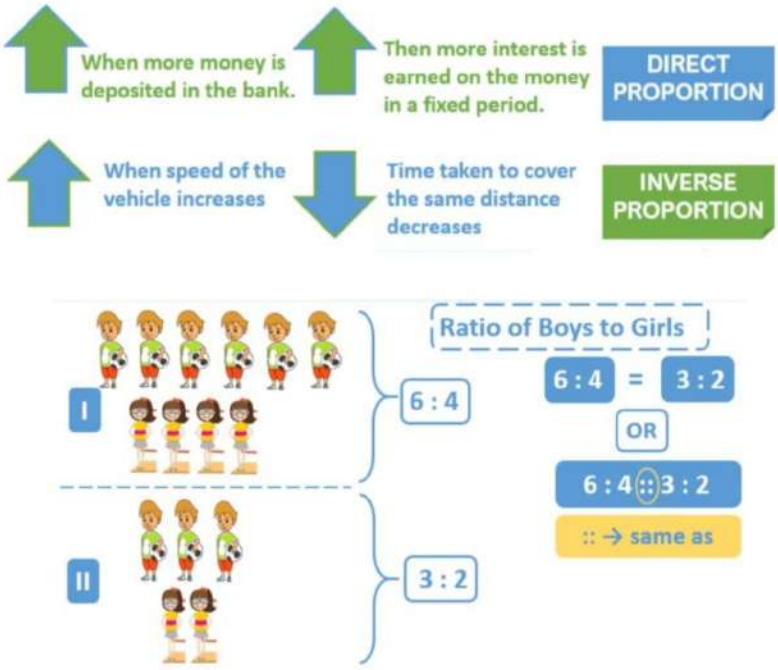
Hence, the required number is 1.

Types of Proportion

(a) Direct Proportion

(a) Inverse Proportion

We come across many situations in daily life where we see that variation in one quantity brings a variation in other quantity.



Direct Proportions

Direct Proportions

An increase in one quantity causes an increase in other quantity or a decrease in one quantity causes a decrease in other quantity, then we say that they are related directly.

Two quantities x and y are said to be in direct proportion if whenever the value of x increases (or decreases), then the value of y increases

(or decreases) in such a way that the ratio $\frac{x}{y}$ remains constant.

No. of Pencils (x)	Cost of Pencils (in ₹) (y)
5	20
10	40
15	60

Increasing

Increasing

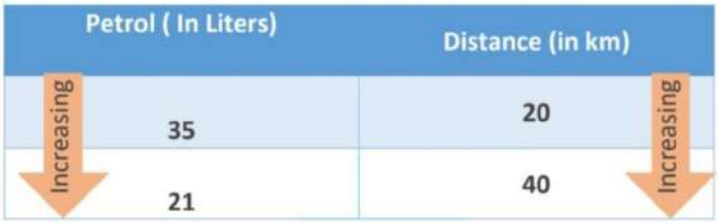
x and y are in direct proportion, if $\frac{x}{y}$ = k, where k is constant.

$$\frac{x}{y} = \frac{5}{20} = \frac{10}{40} = \frac{15}{60}$$

$$\frac{x}{y} = \frac{1}{4} = \text{K}$$

A car covers 430 km in 35 litres of petrol. How much distance would it cover in 21 litres of petrol? (Assuming distance covered and quantity of petrol are in direct proportion)

Let the required distance be x km.



Lesser the quantity of petrol consumed
The distance covered will be less.
As they are in Direct Proportion.

$$\frac{35}{430} = \frac{21}{x}$$

$$\frac{7}{86} = \frac{21}{x}$$

$7 \times x = 21 \times 86$

$$x = \frac{21 \times 86}{7}$$

$x = 3 \times 86 = 258$

$x = 258$

An electric pole, 12 meters high, casts a shadow of 10 meters. Find the height of a tree that casts a shadow of 15 meters under similar conditions?

Let the height of the tree be x meters when the length of the shadow is 15 m.

Height of the object (in meters)	Length of the shadow (in meters)
12	10
x	15

Increasing (Height of the object)

Increasing (Length of the shadow)

Longer the length of the shadow, higher is the height of the object. Therefore, they are in Direct Proportion.

$$\frac{12}{10} = \frac{x}{15}$$

$$\frac{6}{5} = \frac{x}{15}$$

5 X x = 15 X 6

$$x = \frac{15 \times 6}{5}$$

x = 3 X 6 = 18

x = 18

Inverse Proportions

Inverse Proportion

An increase in one quantity causes a decrease in other quantity or decrease in one quantity causes an increase in other quantity and vice versa, then we say that both quantities are related inversely.

If two quantities x and y are in inverse proportion, then their product will be constant, i.e. xy = c, where c is a constant.

When the number of men increases, time taken to complete a given work decreases

No. of Men (x)	No of hours (y)
10	12
20	6
30	4

Increasing

Decreasing

The total quantity of work

$10 \times 12 = 120$ hours

$20 \times 6 = 120$ hours

$30 \times 4 = 120$ hours

If 50 men can do a piece of work in 36 days, in how many days 25 men will do it?

Let the required number of days be x

No of men (x)	No of days (y)
50	36
25	x

Increasing

Decreasing

As the number of men is decreased, the time taken to complete the work is increased in the same ratio.

$\therefore x_1y_1 = x_2y_2$

$50 \times 36 = 25 \times x$

$$x = \frac{36 \times 50}{25}$$

$x = 36 \times 2 = 72$

$x = 72$

6 pipes are required to fill a tank in 1 hour 10 minutes. How long will it take if only 5 pipes of the same type are used?

Let the required distance be y km.

No of pipes (x)	Time in minutes (y)
6	70
5	y

Increasing ↓ (No of pipes)

Decreasing ↓ (Time in minutes)

1 hour = 60 minutes

1 hour 10 minutes = (60 +10) minutes = 70 minutes

As the number of pipes are decreased, the time taken to fill the tank is increased in the same ratio.

$\therefore x_1y_1 = x_2y_2$

$6 \times 70 = 5 \times y$

$$y = \frac{6 \times 70}{5}$$

$y = 6 \times 14 = 84$

$y = 84$