Sets

Concept of sets

In our daily lives, we talk about different collections such as collection of maths books in a cupboard, collection of toys in a shop, collection of shirts in a shop, students in a school, collection of all natural numbers, etc.

These collections are said to be **sets.**

> **A set is a well-defined collection of objects.**

Sets are usually represented by capital letters *A*, *B*, *C*, *D*, *X*, *Y*, *Z*, etc. The objects inside a set are called **elements** or members of a set. They are denoted by small letters *a*, *b*, *c*, *d*, *x*, *y*, *z*, etc.

Now, let us consider the set of natural numbers. We know that 4 is a natural number. However, −1 is not a natural number. We denote it as $4 \in \mathbf{N}$ and $-1 \notin \mathbf{N}$.

If *a* is an element of a set *A* then we say that "*a* belongs to *A*" and mathematically we write it as "*a*∈*A*"; if *b* is not an element of *A* then we say that "*b* does not belong to *A*" and represent it as"*b*∉*A*".

There are three different ways of representing a set:

1. Description method
2. Roster method or listing method or tabular form
3. Set-builder form or rule method

Let us study about them one by one.

Description method: In this method, a description about the set is made and it is enclosed in curly brackets { }.

For example: The set of composite numbers less than 30 is written as

{Composite numbers less than 30}

Roster method or listing method or tabular form : In the roster form, all the elements of a set are listed in such a manner that different elements are separated by commas and enclosed within the curly brackets { }. The roster form enables us to see all the members of a set at a glance.

For example: A set of all integers greater than 5 and less than 9 will be represented in roster form as {6, 7, 8}. However, it must be noted that in roster form, the order in which the elements are listed is immaterial. Hence, the set {6, 7, 8} can also be written as {7, 6, 8}.

Set-builder form or rule method: In set-builder representation of a set, all the elements of the set have a single common property that is exclusive to the elements of the set i.e., no other element outside the set has that property.

We have learnt how to write a set of all integers greater than 5 and less than 9 in roster form. Now, let us understand how we write the same set in set-builder form. Let us denote this set by L.

$L = \{x : x$ is an integer greater than 5 and less than 9$\}$

Hence, in set-builder form, we describe an element of a set by a symbol x (though we may use any other small letter), which is followed by a colon (:). After the colon, we describe the characteristic property possessed by all the elements of that set.

Note:

1. The order of listing the elements in a set can be changed.
2. If one or more elements in a set are repeated, then the set remains the same.
3. Each element of the set is listed once and only once.

Now, consider the following three sets.

$A = \{x: x \in \mathbf{Z}, -18 < x \leq 5\}$

$B = \{x: x \in \mathbf{W}\}$

$C = \{x: x \in \mathbf{N}, -7 < x < -1\}$

Did you observe anything about the number of elements of these sets?

Observe that if we count the elements of set A, then we find that the number of elements is limited in this set. However, the number of elements in set B is not limited and we cannot count the number of elements of this set. Also, observe that set C does not contain any element as there does not exist any natural number lying between −7 and −1.

Therefore, on this basis i.e., on the basis of number of elements, the sets are classified into following categories:

(a) Finite set

(b) Infinite set

(c) Empty set

(d) Singleton set

Let us now study about them one by one.

(a) Finite set – A set that contains limited (countable) number of different elements is called a finite set.

(b) Infinite set – A set that contains unlimited (uncountable) number of different elements is called an infinite set.

(c) Empty set – A set that contains no element is called an empty set. It is also called null (or void) set. An empty set is denoted by Φ or {}. Also, since an empty set has no element, it is regarded as a finite set.

(d) Singleton set – A set having exactly one element is known as singleton set.

Therefore, we can now classify the above discussed sets as follows:

A = {−17, −16, −15, …, 0, 1, 2, 3, 4, 5} → Finite set

B = {0, 1, 2, 3, 4, 5 …} → Infinite set

C = Φ or {} → Empty set

D = {4} → Singleton set

Now, again consider set A. We have seen that it is a finite set.

Can you find the number of elements in this set?

We have A = {−17, −16, −15, −14, −13, −12, −11, −10, −9, −8, −7, −6, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5}

We see that the number of elements in set A is 23. This number 23 is known as the **cardinal number** of set A.

Cardinal number of a set is defined as:

> **The number of distinct elements in a finite set _A_ is called its cardinal number. It is denoted by _n_ (_A_).**

Note: The cardinal number of an infinite set is not defined.

Now, can you find what the cardinal number of an empty set is?

As the empty set has no elements, therefore, its cardinal number is 0 i.e., $n(\Phi) = 0$

We have learnt different ways of representing a set such as description method, roster method, and set-builder method. However, there is one more way of representing a given set and that is through Venn diagrams.

Venn diagrams are closed figures such as square, rectangle, circle, etc. inside which some points are marked. The closed figure represents a set and the points marked inside it represent the elements of the set.

For example, consider the set of all letters in the word AMERICA. This set consists of the letters A, M, E, R, I, and C.

This set can be represented by a Venn diagram as follows:
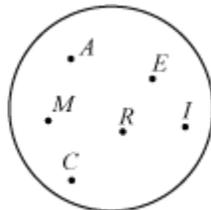
Sometimes, in Venn diagrams, points are not marked, only the elements are written inside the closed figure. For example, the set of letters in the word AMERICA can also be shown as follows:
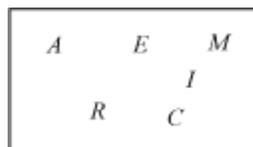
Now, consider the set of all natural numbers. **How will we represent this set by a Venn diagram?**

In such cases, when the number of elements in a set is large, the description of the set is written in the closed figure.

Therefore, the set of all natural numbers can be shown by a Venn diagram as follows:
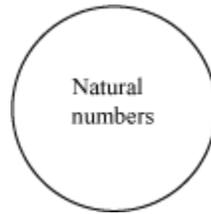
Let us now look at some examples to understand the above discussed concepts better.

Example 1:

Which of the following collection are sets?

1. **Collection of rivers in India**
2. **Collection of good dancers in a locality**
3. **Collection of integers which are less than 21**
4. **Collection of best runners**
5. **Collection of all states of America**
6. **Collection of all vowels**

Solution:

1. The collection of rivers is a set because every river of India will be included in it.
2. The collection of good dancers in a locality is not a set because some dancers of the locality may be good from the point of view of one person, but the same may not be good from the point of view of another person.
3. The collection of integers which are less than 21 is a set as the range of integers in the collection is defined.
4. The collection of best runners is not a set because some runners may be good from the point of view of one person, but they may not be good from the point of view of another person.
5. The collection of states of America is a set because all the states of America will be included in it.
6. The collection of all vowels is a set because all the five vowels will be included in it.

Example 2:

Write the roster form for the set A = {x : x is a letter in the word AEROPLANE which has vowels just before and after it}.

Solution:

In the word AEROPLANE, the vowels are A, E, and O.

Now, the third letter (i.e., R) has a vowel (i.e., E) just before it and a vowel (i.e., O) just after it. Hence, it satisfies the given condition.

Now, look at letter N, which has vowel (i.e., A) just before it and a vowel (i.e., E) just after it. Hence, this letter also satisfies the given condition.

Thus, the set can be written in roster form as

A = {R, N}

Example 3:
State whether each of the following sets is finite or infinite:
1. **Set of multiples of 7**
2. **Set of lines passing through the point (1,1) as well as the origin**

Solution:

1. The multiples of 7 are 7, 14, 21, 28, 35 …
   Hence, the number of elements in set *A* = {7, 14, 21, 28, 35…} is not definite. Hence, it is an infinite set.
2. The two given points are (1,1) and (0,0) and we know that there is one and only one line passing through two fixed points. Hence, there will be only one line that passes through the given points.
   Thus, the set contains only one element. Hence, it is a finite set.
3.

Example 4:

Write the following sets in set builder form.

1. **{1, 8, 27, 64, 125}**
2. **{0, 1, 2, 3, 4, 5, 6 ,7}**
3. **{w, x, y, z}**
4. **{3, 6, 9, 12, 15, 18, 21}**
5. **{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}**

Solution:

1. {*x* : *x* is the cube of first five natural numbers}
2. {*x* : *x* ∈ **W**, *x* < 8}
3. {*x* : *x* is a letter amongst the last four letters of the English alphabet}
4. {*x* : *x* is a multiple of 3, *x* ≤ 21}
5. {*x* : *x* is a day of a week}

Example 5:

Write the following sets in roster and descriptive forms:

1. **{x : x is a letter in the word MATHEMATICS}**
2. **{y : y≤ 23 and it is odd}**

Solution:

1. Roster form: {M, A, T, H, E, I, C, S}

   Descriptive form: {letters of the word MATHEMATICS}

2. Roster form: {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23}

   Descriptive form: {Odd numbers less than and equal to 23}

Example 6:

Classify the following sets into finite set, infinite set, and empty sets. Also, find the cardinal number in case of finite sets.

(a) A = {x: x is a letter in the word ENGINEER}

(b) B = {x: x is a multiple of 9}

(c) C = {x: x is a factor of 48}

(d) D = {x: x is a vowel in the word RHYTHM}

(e) E = {x: x is a vowel in the word SKY}

(f) F = {x: x∈ Z}

(g) G = {all stars in universe}

(h) H = {x : x > 2, where x is an even prime number}

Solution:

(a) A = {E, N, G, I, R}

It is a finite set and $n(A) = 5$

(b) $B = \{9, 18, 27, 36, 45, ...\}$

It is an infinite set.

(c) $C = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

It is a finite set and $n(C) = 10$

(d) $D = \Phi$

It is an empty set and $n(D) = 0$

(e) $E = \Phi$

It is an empty set and $n(E) = 0$

(f) $F = \{..., -2, -1, 0, 1, 2, ...\}$

It is an infinite set.

(g) $G = \{$all stars in universe$\}$

The number of elements in set G is not defined and hence it is an infinite set.

(h) $H = \{x : x > 2,$ where x is an even prime number$\}$

We can see that no value of x will satisfy the given property as 2 is the only even prime number and no even number greater than 2 will be a prime number. Hence, A will be an empty set as it has no elements.

$n(H) = 0$

Equal Sets and Equivalent Sets

Consider two sets:

$A = \{-9, -3, 0, 5, 12\}$

$B = \{-2, 1, 2, 4, 7\}$

Did you notice any relation between the sets A and B?

Let us see.

We have,

A = {−9, −3, 0, 5, 12}

B = {−2, 1, 2, 4, 7}

Therefore, we have $n(A)$ = 5 and $n(B)$ = 5

Observe that both the sets A and B have same number of elements. Therefore, in this case, we say that the sets A and B are **equivalent sets** and it can be defined as:

> **Two finite sets are called equivalent, if they have the same number of elements.**

Thus, two finite sets X and Y are equivalent, if $n(X) = n(Y)$. We write it as $X \leftrightarrow Y$ (read as "X is equivalent to Y")

Now, consider the two sets:

X = {all letters in the word STONE}

Y = {all letters in the word NOTES}

Did you notice any relation between the sets X and Y?

Let us see.

We have,

X = {S, T, O, N, E} and Y = {N, O, T, E, S}

Observe that both the sets X and Y have same elements. Therefore, in this case, we say that the sets X and Y are **equal sets**.

> **Two sets are called equal, if they have same elements.**

When two sets X and Y are equal, we denote it as $X = Y$; and if they are not equal, then we write it as $X \neq Y$

Also, note that $n(X) = 5$ and $n(Y) = 5$

Therefore, we can conclude that:

If *A* and *B* are finite sets and *A* = *B*, then *n* (*A*) = *n* (*B*) i.e., *A* and *B* are equivalent. However, the converse of the above statement may not be true.

For example, if $A = \{2, 4, 6\}$ and $B = \{1, 3, 5\}$, then $n(A) = n(B) = 3$; however, $A \neq B$

In this example we see that no element is common between the two sets A and B. So, we can say that the given sets are **disjoint sets**.

Two sets are called disjoint, if they have no element in common.

For example, if A = Set of students of class III and B = Set of students of class VI
No students can be common to the two classes, hence set A and set B are disjoint.

Let us now look at some more examples to understand the above discussed concepts better.

Example 1:

Which of the following sets are equal?

(a) *X* = {*x*: *x* is a letter in the word REFRESH},

Y = {A letter in the word FRESHER}

(b) *X* = {4}, *Y* = {*x*: *x*∈ N, *x* − 4 = 0}

(c) *X* = {*x/x* is a vowel letter in the word WEIGHT},

Y = {*x/x* is a vowel letter in the word HEIGHT}

(d) *X* = {*x*: *x*∈ N, 0 < *x* < 4}, *Y* = {*x*: *x*∈ W}

Solution:

(a) $X = \{R, E, F, S, H\}$, $Y = \{F, R, E, S, H\}$

∴ *X* and *Y* are equal sets i.e., *X* = *Y*

(b) X = {4}, Y = {4}

∴ X and Y are equal sets i.e., $X = Y$

(c) X = {E, I}, Y = {E, I}

∴ X and Y are equal sets i.e., $X = Y$

(d) X = {1, 2, 3}, Y = {0, 1, 2, 3, 4, 5 ...}

∴ X and Y are not equal sets i.e., $X \neq Y$

Example 2:

Which of the following sets are equivalent?

(a) X = {x: x is a vowel in the word MATRIX}

Y = {A vowel in the word SHIVANI}

(b) X = {5, 0, 1, 0, 2, 1}, Y = {3, 2, 8, 3, 3, 2}

Solution:

(a) X = {A, I}, Y = {A, I}

∴ n (X) = 2 and n (Y) = 2

This means n (X) = n (Y)

Therefore, X and Y are equivalent sets.

(b) X = {5, 0, 1, 0, 2, 1} = {0, 1, 2, 5}, Y = {3, 2, 8, 3, 3, 2} = {2, 3, 8}

∴ n (X) = 4 and n (Y) = 3

This means n (X) $\neq$ n (Y)

Therefore, X and Y are not equivalent sets.