

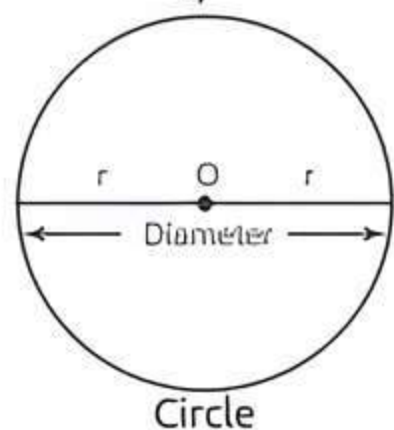
11 Areas Related to Circles

Fastrack Revision

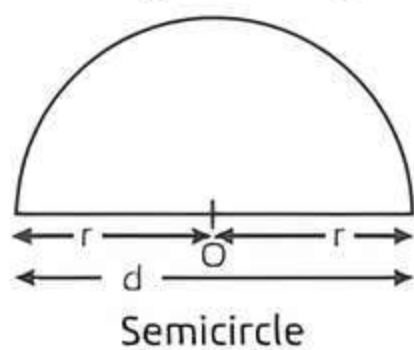
► **Formulae Related to Circle:** If r is the radius of a circle, then:

1. Circumference or perimeter of a circle = $2\pi r$ or πd , where $d = 2r$ is the diameter of the circle.

2. Area of a circle = πr^2 or $\frac{\pi d^2}{4}$



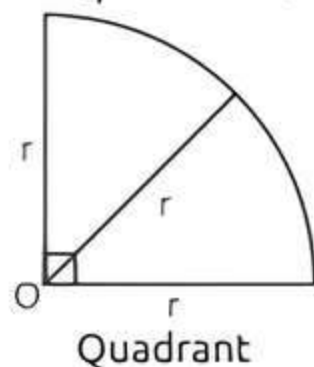
3. Area of a semicircle = $\frac{1}{2}\pi r^2$ or $\frac{\pi d^2}{8}$



4. Circumference of a semicircle = πr or $\frac{\pi d}{2}$

5. Perimeter of a semicircle = $\pi r + 2r$ or $\left(\frac{\pi}{2} + 1\right)d$

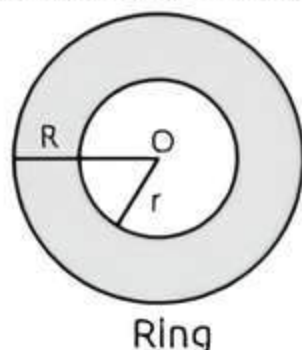
6. Area of a quadrant = $\frac{1}{4}\pi r^2$ or $\frac{\pi d^2}{16}$



7. Circumference of a quadrant = $\frac{\pi r}{2}$ or $\frac{\pi d}{4}$

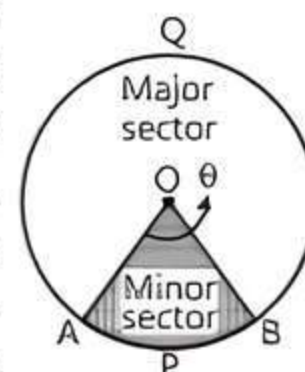
8. Perimeter of a quadrant = $\frac{\pi r}{2} + 2r$ or $\left(\frac{\pi}{4} + 1\right)d$

9. Area of the ring = $\pi (R^2 - r^2) = \pi (R + r)(R - r)$, where R is the outer radius and r is the inner radius.



► **Sector and Segment of Circle:**

► **Sector:** The portion of the circular region enclosed by two radii and the corresponding arc is called the sector of the circle. In the figure, the shaded region OAPB is a sector of the circle with centre O. θ is called the angle of the sector.



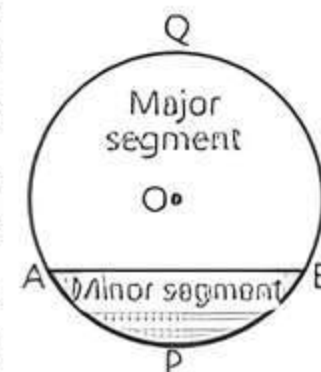
The unshaded region OAQB is also a sector of the circle in the figure. The sector OAPB is called the minor sector of the circle and the sector OAQB is called the major sector of the circle. Therefore, the angle of this major sector will be $(360^\circ - \theta)$.

Knowledge BOOSTER



The angle of minor sector is smaller than 180° and angle of major sector is greater than 180° .

► **Segment:** The portion of the circular region which is bounded between a chord and corresponding arc is called a segment. In the figure, AB is a chord of the circle with centre O. Thus, the shaded region APB is a segment. In the figure, the unshaded region AQB is also a segment formed by chord AB. The segment APB is called the minor segment and the segment AQB is called the major segment of the circle.



► **Formulae Related to Sector and Segment of a Circle:**

1. Length of minor arc (ABC) = $\frac{\theta}{360^\circ} \times 2\pi r$

2. Length of major arc (BDA) = $2\pi r - \frac{\theta}{360^\circ} \times 2\pi r$
 $= \left(1 - \frac{\theta}{360^\circ}\right) 2\pi r$

3. Perimeter of minor sector (OACBO)

= Length of arc ACB + $2r$

= $\frac{\theta}{360^\circ} \times 2\pi r + 2r = \left(1 + \frac{\pi\theta}{360^\circ}\right) \times 2r$

4. Perimeter of major sector (OADBO)

$$= \left(1 - \frac{\theta}{360^\circ}\right) \cdot 2\pi r + 2r$$

$$= \left[1 + \left(1 - \frac{\theta}{360^\circ}\right)\pi\right] \cdot 2r$$

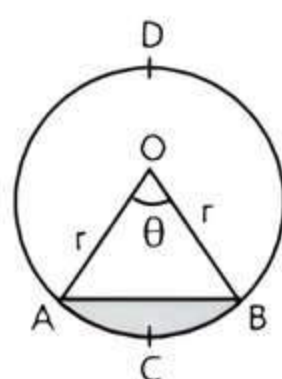
5. Area of minor sector (OACBO) = $\frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{1}{2} \times \text{Length of minor arc ACB} \times r = \frac{1}{2}lr$$

6. Area of major sector = Area of circle

– Area of minor sector

$$= \pi r^2 - \frac{\theta}{360^\circ} \times \pi r^2 = \left(1 - \frac{\theta}{360^\circ}\right) \cdot \pi r^2$$



7. Perimeter of the minor segment of a circle

$$(ACBA) = \frac{\theta}{360^\circ} \times 2\pi r + 2r \sin \frac{\theta}{2} = \left[\frac{\theta \pi}{360^\circ} + \sin \frac{\theta}{2} \right] \times 2r$$

8. Perimeter of the major segment (BDAB)

$$= 2r \sin \frac{\theta}{2} + \left(1 - \frac{\theta}{360^\circ}\right) \cdot 2\pi r$$

$$= \left[\left(1 - \frac{\theta}{360^\circ}\right)\pi + \sin \frac{\theta}{2} \right] \times 2r$$

9. Area of minor segment (ACBA) = $\frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin \theta$

$$= \frac{r^2}{2} \left[\frac{\theta \pi}{180^\circ} - \sin \theta \right]$$

10. Area of major segment (BDAB)

= Area of circle – Area of minor segment

$$= \pi r^2 - \frac{\theta \pi r^2}{360^\circ} + \frac{1}{2} r^2 \sin \theta = \frac{r^2}{2} \left[\frac{(360^\circ - \theta)\pi}{180^\circ} + \sin \theta \right]$$

Knowledge BOOSTER

1. Angle subtended at the circumference by a diameter is always a right angle.

2. Angle made by minute hand in 1 minute

$$= \frac{360^\circ}{60^\circ} = 6^\circ$$

3. Angle made by hour hand in 1 hour = $\frac{360^\circ}{12} = 30^\circ$

4. Angle made by hour hand in 1 minute = $\frac{30^\circ}{60} = 0.5^\circ$



Practice Exercise



Multiple Choice Questions

Q 1. What is the area of a semicircle of diameter 'd'?

[CBSE 2023]

- a. $\frac{1}{16}\pi d^2$ b. $\frac{1}{4}\pi d^2$ c. $\frac{1}{8}\pi d^2$ d. $\frac{1}{2}\pi d^2$

Q 2. The circumferences of two circles are in the ratio 4 : 5. What is the ratio of their radii? [CBSE 2023]

- a. 16 : 25 b. 25 : 16 c. $2 : \sqrt{5}$ d. 4 : 5

Q 3. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is:

[CBSE SQP 2023-24]

- a. 2 units b. π units c. 4 units d. 7 units

Q 4. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park is:

[CBSE SQP 2023-24]

- a. 10 m b. 15 m
c. 20 m d. 24 m

Q 5. What is the length of the arc of the sector of a circle with radius 14 cm and of central angle 90° ?

[CBSE 2023]

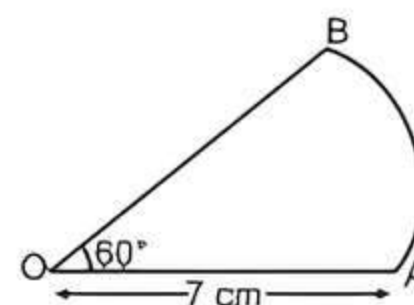
- a. 22 cm b. 44 cm
c. 88 cm d. 11 cm

Q 6. Area of a sector of angle p (in degrees) of a circle with radius (R) is:

[CBSE 2021 Term-I]

- a. $\frac{p}{180^\circ} \times 2\pi R$ b. $\frac{p}{180^\circ} \times \pi R^2$
c. $\frac{p}{360^\circ} \times 2\pi R$ d. $\frac{p}{720^\circ} \times 2\pi R^2$

Q 7. In the given figure, the perimeter of sector OAB will be:



- a. $\frac{64}{3}$ cm b. 26 cm c. $\frac{64}{5}$ cm d. 19 cm

Q 8. A cow is tied to a peg at one corner of a rectangular field of dimensions 10 m \times 8 m by a 3 m long rope. The area of the part of the field in which the cow can graze is:

- a. 14.14 m^2 b. 7.07 m^2
c. 6.07 m^2 d. None of these

Q 9. The area of the sector of a circle of radius 6 cm whose central angle is 30° is: [Take $\pi = 3.14$]

[CBSE 2020]

- a. 9.42 cm^2 b. 10.32 cm^2
c. 10.58 cm^2 d. 11.52 cm^2

Q 10. The area of the circle that can be inscribed in a square of side 6 cm is: [CBSE SQP 2022-23]

- a. $36\pi \text{ cm}^2$ b. $18\pi \text{ cm}^2$
c. $12\pi \text{ cm}^2$ d. $9\pi \text{ cm}^2$

Q 11. The minute hand of a clock is 84 cm long. The distance covered by the tip of minute hand from 10 : 10 am to 10 : 25 am is: [CBSE 2021 Term-I]

- a. 44 cm b. 88 cm
c. 132 cm d. 176 cm

Q 12. The radius of a circle is 5 cm. Find the area of the sector formed by an arc of this circle of length 9 cm.

- a. 45 cm^2 b. 22.5 cm^2
c. 67.5 cm^2 d. 2.25 cm^2

Q 13. Find the area of the major segment of a circle if the area of the minor segment is 25 cm^2 and the area of the circle is 100 cm^2 .

- a. 25 cm^2 b. 100 cm^2
c. 75 cm^2 d. 50 cm^2

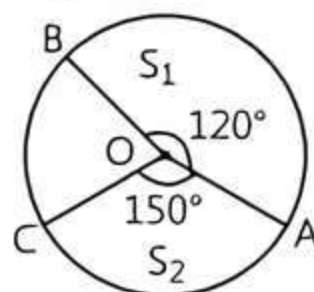
Q 14. A sector is cut from a circular sheet of radius 100 cm, the angle of the sector being 240° . If another circle of the area same as the sector is formed, then radius of the new circle is:

- a. 79.5 cm b. 81.6 cm
c. 83.4 cm d. 88.5 cm

Q 15. Priyanshu has a motor cycle with wheels of diameter 91 cm. There are 22 spokes in the wheel. Find the length of arc between two adjoining spokes.

- a. 26 cm b. 13 cm c. 15 cm d. 18 cm

Q 16. In the given figure, the ratio of the areas of two sectors S_1 and S_2 is:



- a. 5 : 2 b. 3 : 5 c. 5 : 3 d. 4 : 5

Q 17. A car has two wipers which do not overlap. Each wiper has a blade of length 42 cm sweeping through an angle of 120° . Find the total area cleaned at each sweep of the blades.

- a. 4224 cm^2 b. 3696 cm^2
c. 1848 cm^2 d. 5544 cm^2

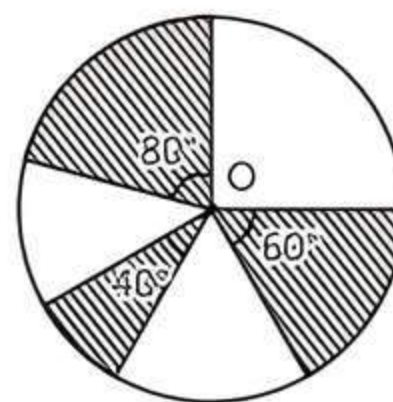
Q 18. A chord of a circle of radius 28 cm subtends an angle of 60° at the centre of the circle. The area of the minor segment is: [Take $\sqrt{3} = 1.732$]

- a. 60.256 cm^2 b. 339.47 cm^2
c. 410.67 cm^2 d. 71.20 cm^2

Q 19. A chord of a circle subtends an angle of 60° at the centre of the circle. If the length of the chord is 10 cm, then the area of the major segment is: [Take $\pi = 3.14$ and $\sqrt{3} = 1.732$]

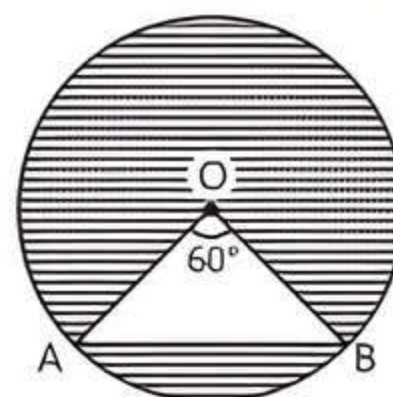
- a. 304.97 cm^2 b. 295 cm^2
c. 310 cm^2 d. 335 cm^2

Q 20. In the given figure, three sectors of a circle of radius 7 cm, making angles of 60° , 80° and 40° at the centre are shaded. The area of the shaded region (in cm^2) is: [Using $\pi = \frac{22}{7}$]



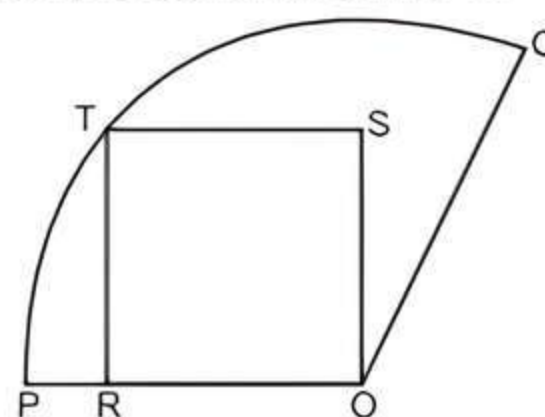
- a. 77 b. 154 c. 44 d. 22

Q 21. O is the centre of a circle of radius 5 cm. The chord AB subtends an angle 60° at the centre. Area of the shaded portion is equal to (approximately):



- a. 50 cm^2 b. 62.78 cm^2
c. 49.88 cm^2 d. 67.74 cm^2

Q 22. In the given figure, a unit square ROST is inscribed in a circular sector with centre O.



Along with the above information, which of these is sufficient to find the area of sector POQ?

- a. Area of the square ROST
b. Radius of sector POQ
c. Arc length PQ
d. Given information is sufficient



Assertion & Reason Type Questions

Directions (Q. Nos. 23-26): In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:

- a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

- Q 23. Assertion (A): In a circle of radius 6 cm, the angle of a sector 60° . Then the area of the sector is $18\frac{6}{7} \text{ cm}^2$.

Reason (R): Area of the circle with radius r is πr^2 .

- Q 24. Assertion (A): The length of the minute hand of a clock is 7 cm. Then the area swept by the minute hand in 5 min is $12\frac{5}{6} \text{ cm}^2$.

Reason (R): The length of an arc of a sector of angle θ and radius r is given by $l = \frac{\theta}{360^\circ} \times 2\pi r$.

- Q 25. Assertion (A): If the perimeter of a sector of a circle of radius 5.6 cm is 27.2 cm, then the area of the sector is 44.8 cm^2 .

Reason (R): The area of a sector of a circle of radius (r) with central angle θ is $\frac{\theta}{360^\circ} \times \pi r$.

- Q 26. Assertion (A): A sector is cut from a circle of radius 42 cm. The central angle of the sector is 150° . The perimeter of the sector is 194 cm.

Reason (R): Perimeter of sector = 2 (radius) + Length of corresponding arc of sector.



Fill in the Blanks Type Questions

- Q 27. The region bounded by chord and the corresponding arc of the circle is a of the circle.
- Q 28. If r is a radius of a circle, then area of major sector is sq. units.
- Q 29. The length of the arc of a sector of angle 30° of a circle of radius 14 cm is



True/False Type Questions

- Q 30. The perimeter of the minor segment of a circle is $\frac{\theta}{360^\circ} \times 2\pi r + 2r \sin \frac{\theta}{2}$.
- Q 31. If the circumference of a circle exceeds its diameter by 180 cm, then its radius is 32 cm.
- Q 32. The region bounded by a chord and the corresponding arc of the circle is called the segment of the circle.
- Q 33. The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.
- Q 34. A chord of a circle subtends an angle of 60° at the centre of the circle, if the length of the chord is 7 cm, then the area of the minor segment is 4.60 cm^2 . [Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.73$]

Solutions

1. (c) Given 'd' be a diameter of circle.
Let 'r' be the radius of circle.

\therefore Diameter (d) = 2 \times radius = 2 \cdot r

$$\Rightarrow r = \frac{d}{2}$$

$$\text{So, area of a semicircle} = \frac{\pi r^2}{2} = \frac{\pi}{2} \left(\frac{d}{2} \right)^2$$

$$= \frac{\pi}{2} \cdot \frac{d^2}{4} = \frac{\pi d^2}{8}$$

2. (d) Let the radius of two circles are r_1 and r_2 respectively.

$$\therefore \frac{\text{Circumference of first circle}}{\text{Circumference of second circle}} = \frac{4}{5}$$

$$\Rightarrow \frac{2\pi r_1}{2\pi r_2} = \frac{4}{5} \Rightarrow \frac{r_1}{r_2} = \frac{4}{5}$$

\therefore Required ratio is 4 : 5.

3. (a) Let the radius of circle be r .

According to condition,

Area of circle = Perimeter of circle

$$\Rightarrow \pi r^2 = 2\pi r$$

$$\Rightarrow 2\pi r - \pi r^2 = 0$$

$$\Rightarrow \pi r(2 - r) = 0$$

$$\Rightarrow r = 0 \text{ or } r = 2$$

But radius cannot be zero.

$$\therefore r = 2.$$

So, radius of circle is 2 units.

4. (a) Let the radius of single circular park is R . Also, the radius of first and second circular park are r_1 and r_2 respectively.

Given, diameter of first circular park (d_1) = 16 m

$$\therefore \text{Its radius } (r_1) = \frac{d_1}{2} = \frac{16}{2} = 8 \text{ m}$$

and diameter of second circular park (d_2) = 12 m

$$\therefore \text{Its radius } (r_2) = \frac{d_2}{2} = \frac{12}{2} = 6 \text{ m}$$

According to question,

Area of single circular park = Area of first circular park + Area of second circular park

$$\Rightarrow \pi R^2 = \pi r_1^2 + \pi r_2^2$$

$$\Rightarrow R^2 = (8)^2 + (6)^2 = 64 + 36 = 100$$

$$\Rightarrow R = 10 \text{ m}$$

So, required radius is 10 m.

5. (a) Given, radius of circle,

$$r = 14 \text{ cm}$$

and central angle ' θ ' = 90° .

$$\therefore \text{Length of the arc of the sector} = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 14$$

$$= \frac{1}{4} \times 2 \times 2 \times 11 \times 2$$

$$= 22 \text{ cm}$$

So, required length is 22 cm.

6. (d) The area of a sector of angle p of a circle with

$$\text{radius } R = \frac{p}{360^\circ} \times \pi R^2 = \frac{p}{360^\circ} \times \pi R^2 \times \frac{2}{2}$$

$$= \frac{p \times 2\pi R^2}{720^\circ}$$

7. (a) Perimeter of sector OAB

$$= OA + OB + \text{arc length of AB}$$

$$= 7 + 7 + \frac{\theta}{360^\circ} \times 2\pi r$$

$$= 7 + 7 + \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 7$$

$$= 14 + \frac{44}{6} = 14 + \frac{22}{3}$$

$$= \frac{42 + 22}{3} = \frac{64}{3} \text{ cm}$$

8. (b) Given, radius of sector (r) = 3m

\therefore Field is rectangular

\therefore Central angle (θ) = 90°

$$\text{So, required area} = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (3)^2$$

$$= \frac{99}{14} = 7.07 \text{ m}^2$$

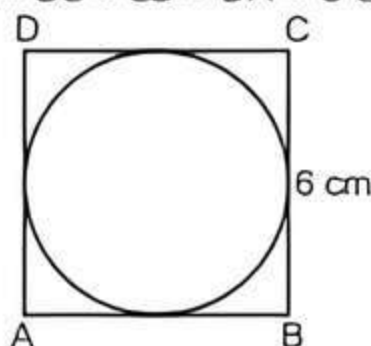
9. (a) Given, radius of the circle (r) = 6 cm and central angle (θ) = 30°

$$\therefore \text{Area of the sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{30^\circ}{360^\circ} \times 3.14 \times (6)^2$$

$$= \frac{1}{12} \times 3.14 \times 36 = 9.42 \text{ cm}^2$$

10. (d) Here $AB = BC = CD = DA = 6 \text{ cm}$



\therefore Diameter of a circle, $d = BC = 6 \text{ cm}$

\therefore Radius of a circle, $r = \frac{6}{2} = 3 \text{ cm}$

\therefore The area of a circle = πr^2
 $= \pi (3)^2 = 9\pi \text{ cm}^2$

11. (c)



Tip

Angle made by minute hand in 1 min = 6°

\therefore Angle made by minute hand from 10 : 10 am to 10 : 25 am is

$$6^\circ \times 15 = 90^\circ$$

\therefore The distance covered by minute hand in 15 minutes

$$= \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 84$$

$$= 132 \text{ cm}$$

12. (b) Let the radius and central angle of the sector be r and θ respectively.

According to the given condition,

$$2\pi r \times \frac{\theta}{360^\circ} = 9$$

$$\Rightarrow 2 \times \pi \times 5 \times \frac{\theta}{360^\circ} = 9$$

$$\Rightarrow \theta = \frac{324}{\pi}$$

$$\therefore \text{Area swept} = \pi r^2 \times \frac{\theta}{360^\circ} \quad [\because r = 5 \text{ cm}]$$

$$= \pi \times 5 \times 5 \times \frac{324}{\pi \times 360^\circ}$$

$$= 22.5 \text{ cm}^2$$

13. (c) Area of the major segment

= Area of the circle – Area of the minor segment

$$= 100 \text{ cm}^2 - 25 \text{ cm}^2 = 75 \text{ cm}^2$$

14. (b) Given, radius of sector (r) = 100 cm and central angle (θ) = 240°

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{240^\circ}{360^\circ} \times \pi (100)^2 = \frac{20000\pi}{3}$$

Let R be the radius of the new circle, then by given condition,

$$\frac{20000\pi}{3} = \pi R^2$$

$$\Rightarrow R = \sqrt{\frac{20000}{3}}$$

$$= \sqrt{6666.66} = 81.6 \text{ cm}$$

15. (b) Radius of wheel (r) = $\frac{91}{2} \text{ cm}$

Angle between two adjoining spokes, $\theta = \frac{360^\circ}{22}$

$$\therefore \text{Length of the arc} = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{360^\circ}{360^\circ \times 22} \times 2 \times \frac{22}{7} \times \frac{91}{2} = 13 \text{ cm}$$

16. (d) Let r be the radius of the circle.

\therefore Area of the sector is given by $\frac{\theta}{360^\circ} \times \pi r^2$

$$\text{Area of sector } S_1 = \frac{120^\circ}{360^\circ} (\pi r^2) = \frac{\pi r^2}{3}$$

$$\text{Area of sector } S_2 = \frac{150^\circ}{360^\circ}(\pi r^2) = \frac{5\pi r^2}{12}$$

$$\text{Required ratio} = \frac{\pi r^2}{3} : \frac{5\pi r^2}{12} = 4 : 5$$

17. (b) Clearly, each wiper sweeps a sector of a circle of radius 42 cm and sector angle 120° .

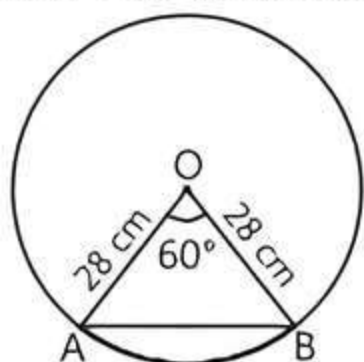
i.e., $r = 42$ cm and $\theta = 120^\circ$

$$\therefore \text{Total area cleaned at each sweep} = 2 \times \frac{\theta}{360^\circ} \times \pi r^2$$

$$= 2 \times \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 42 \times 42 \text{ cm}^2$$

$$= 3696 \text{ cm}^2$$

18. (d) Here, radius $r = 28$ cm and central angle $\theta = 60^\circ$



Since, $OA = OB$ (radii)
 $\Rightarrow \angle A = \angle B = 60^\circ$ (angles opposite to equal sides are also equal)

$\Rightarrow \triangle AOB$ is an equilateral triangle.

\therefore Area of $\triangle AOB$

$$= \frac{\sqrt{3}}{4} r^2 = \frac{1.732}{4} \times 28 \times 28 = 339.472 \text{ cm}^2$$

Area of minor segment

$$= \frac{\theta}{360^\circ} \pi r^2 - \text{Area of } \triangle AOB$$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 28 \times 28 - 339.472$$

$$= 410.67 - 339.47$$

$$= 71.20 \text{ cm}^2$$

19. (a) We have, $OA = OB$ (radii of circle)

$\Rightarrow \angle A = \angle B = 60^\circ$

($\because \angle AOB = 60^\circ$)

$\Rightarrow \triangle AOB$ is equilateral triangle.

$\therefore OA = OB = AB = 10$ cm

and central angle, $\theta = 60^\circ$

Area of the minor segment

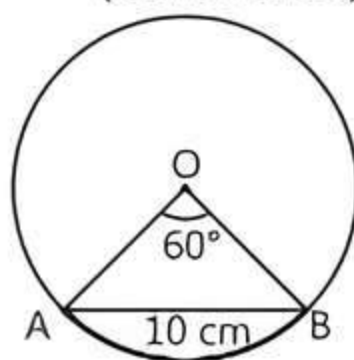
$= \text{Area of minor sector} - \text{Area of } \triangle AOB$

$$= \left(\frac{\pi r^2 \theta}{360^\circ} - \frac{\sqrt{3}}{4} r^2 \right)$$

$$= \left\{ \left(3.14 \times 10 \times 10 \times \frac{60^\circ}{360^\circ} \right) - \left(\frac{\sqrt{3}}{4} \times 10 \times 10 \right) \right\}$$

$$= \left(\frac{157}{3} - 25 \times \sqrt{3} \right) = \left(\frac{157}{3} - \frac{433}{10} \right)$$

$$= \frac{(1570 - 1299)}{30} = \frac{271}{30} = 9.03 \text{ cm}^2$$



\therefore Area of major segment

$$= \pi r^2 - \text{Area of minor segment}$$

$$= [(\pi \times 10 \times 10) - 9.03] \text{ cm}^2$$

$$= [(3.14 \times 100) - 9.03] \text{ cm}^2$$

$$= [(314 - 9.03)] \text{ cm}^2 = 304.97 \text{ cm}^2$$

20. (a) Area of shaded region = Area of sector with angles $(60^\circ + 80^\circ + 40^\circ)$

$$= \frac{180^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2$$

21. (d) $\angle AOB = 60^\circ$ and $OA = OB$ (radii)

$$\Rightarrow \angle OAB = \angle OBA = 60^\circ$$

$\therefore \triangle AOB$ is an equilateral triangle.

$$\text{Area of triangle} = \frac{\sqrt{3}}{4} \times 5^2 = \frac{25\sqrt{3}}{4} \text{ cm}^2$$

\therefore Area of shaded region = Area of circle - Area of triangle

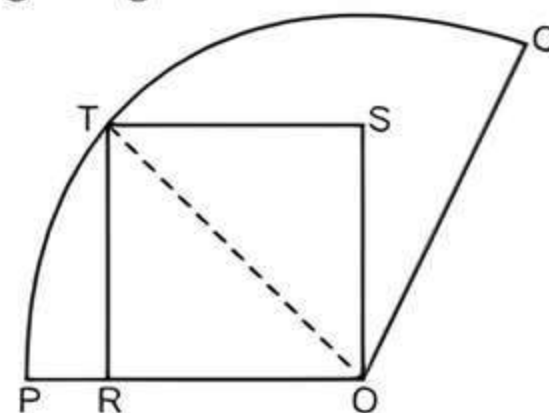
$$= \left(\frac{22}{7} \times 5 \times 5 - \frac{25\sqrt{3}}{4} \right) \text{ cm}^2$$

$$= \frac{550}{7} - \frac{433}{4} \quad (\text{use } \sqrt{3} = 1.732)$$

$$= 78.57 - 10.83$$

$$= 67.74 \text{ cm}^2$$

22. (c) In right-angled $\triangle ORT$.



$$OT = \sqrt{TR^2 + OR^2}$$

(by Pythagoras theorem)

$$= \sqrt{1^2 + 1^2} \quad (\text{given } TR = OR = 1)$$

$$= \sqrt{2}$$

We know that area of sector is $\frac{1}{2} lr$.

Here we have $r = OT = \sqrt{2}$

So, we need to require the arc length PQ to find the area of sector.

23. (b) **Assertion (A):** Given, radius (r) = 6 cm and central angle (θ) = 60°

Area of the sector

$$= \frac{\theta}{360^\circ} \pi r^2 = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{132}{7} = 18\frac{6}{7} \text{ cm}^2$$

So, Assertion (A) is true.

Reason (R): It is also true, but it is not the correct explanation of Assertion (A).

24. (b) **Assertion (A):** Area swept by minute hand in 5 min

$$= \frac{\theta}{360} \times \pi r^2 = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7$$

(\because radius (r) = 7 cm and angle made by minute hand in 5 min is 30°)

$$= \frac{77}{6} = 12\frac{5}{6} \text{ cm}^2$$

So, Assertion (A) is true.

Reason (R): It is also a true statement.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

25. (c) **Assertion (A):** Radius of circle (r) = 5.6 cm
Let θ be the central angle of sector.

$$\therefore \text{Perimeter of sector} = r + r + \frac{\theta}{360^\circ} \times 2\pi r$$

$$\Rightarrow 27.2 = 5.6 + 5.6 + \frac{\theta}{360^\circ} \times 2 \times \frac{22}{7} \times 5.6$$

$$\Rightarrow 27.2 = 11.2 + 35.2 \times \frac{\theta}{360^\circ}$$

$$\Rightarrow 16 = 35.2 \times \frac{\theta}{360^\circ}$$

$$\Rightarrow \frac{\theta}{360^\circ} = \frac{16}{35.2} = \frac{16 \times 10}{352} = \frac{5}{11} \quad \dots(1)$$

$$\begin{aligned} \text{Now, area of sector} &= \frac{\theta}{360^\circ} \times \pi r^2 \\ &= \frac{5}{11} \times \frac{22}{7} \times 5.6 \times 5.6 \quad (\text{using eq. (1)}) \\ &= 44.8 \text{ cm}^2 \end{aligned}$$

So, Assertion (A) is true.

Reason (R): It is false, because area of sector is

$$\frac{\theta}{360^\circ} \times \pi r^2$$

Hence, Assertion (A) is true but Reason (R) is false.

26. (a) **Assertion (A):** We have, radius of circle,
 $r = 42$ cm

Central angle, $\theta = 150^\circ$

$$\begin{aligned} \therefore \text{Perimeter of sector} &= 2r + \frac{\theta}{360^\circ} \times 2\pi r \\ &= 2 \times 42 + \frac{150^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 42 \\ &= 84 + 110 = 194 \text{ cm} \end{aligned}$$

So, Assertion (A) is true.

Reason (R): It is also true statement.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

27. segment

$$28. \pi r^2 \left(1 - \frac{\theta}{360^\circ}\right)$$

29. Given, central angle $\theta = 30^\circ$ and radius $r = 14$ cm

TR!CK

The length of an arc of a sector is $\frac{\theta}{360^\circ} \times 2\pi r$.

\therefore The length of an arc of a sector

$$\begin{aligned} &= \frac{30^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 14 \\ &= 7.33 \text{ cm} \end{aligned}$$

30. True

31. Let radius of a circle be r cm. Then according to the given condition,

$$2\pi r = 2r + 180$$

$$\Rightarrow 2 \times \frac{22}{7} r - 2r = 180$$

$$\Rightarrow \frac{44r - 14r}{7} = 180$$

$$\begin{aligned} \Rightarrow 30r &= 180 \times 7 \\ \Rightarrow r &= 42 \text{ cm} \end{aligned}$$

Hence, given statement is false.

32. True

33. True

34. We have,

$$OA = OB.$$

(radii)

$$\Rightarrow \angle A = \angle B = 60^\circ$$

$$(\because \angle AOB = 60^\circ)$$

$\Rightarrow \triangle AOB$ is an equilateral triangle.

$$\therefore OA = OB = AB = 7 \text{ cm}$$

and central angle, $\theta = 60^\circ$

\therefore Area of minor segment

$$= \text{Area of minor sector} - \text{Area of } \triangle AOB$$

$$= \frac{\pi r^2 \theta}{360^\circ} - \frac{\sqrt{3}}{4} r^2$$

$$= \frac{22}{7} \times (7)^2 \times \frac{60^\circ}{360^\circ} - \frac{\sqrt{3}}{4} \times 7 \times 7$$

$$= 25.67 - 1.73 \times 12.25$$

$$= 25.67 - 21.19$$

$$= 4.48 \text{ cm}^2$$

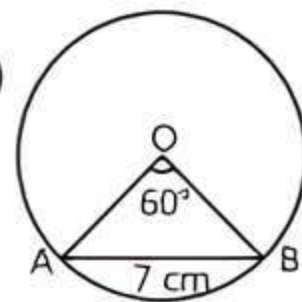
Hence, given statement is false.



Case Study Based Questions

Case Study 1

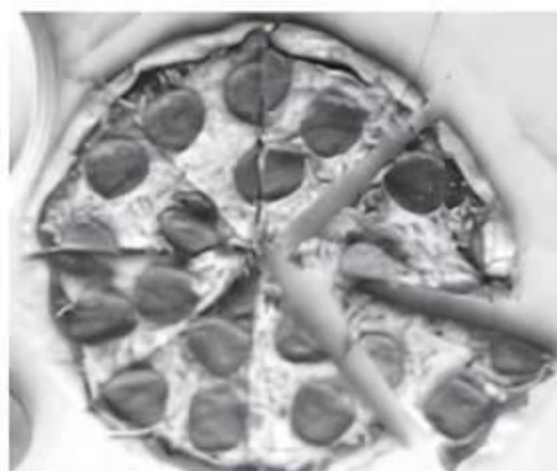
We all love to eat pizzas, especially kids. And a variety of pizzas are available in India which have been modified according to Indian taste and menu. From the Greeks to the Egyptians, from the Persians to the Indians, there have been incarnations of pizza served throughout history, Flatbreads, naan, and plakountos are all early preparations that could be considered cousins to the modern pizza, but there isn't a consensus as to which is first and whether these could even be considered precursors to pizza at all.



Consider two pizzas, both of equal diameter, namely, 12 inches. The first pizza marked (I) has been cut into six equal slices, whereas the second pizza, marked (II) has been cut into eight equal slices.



(I)



(II)

Based on the above information, solve the following questions:

- Q 1. The area of one slice in pizza, marked (I) is:**
 a. 6π sq. inches b. 8π sq. inches
 c. 10π sq. inches d. None of these
- Q 2. The perimeter of the pizza slice shown in (I) is:**
 a. $(\pi + 12)$ inch b. $(\pi + 10)$ inch
 c. $(2\pi + 10)$ inch d. $(2\pi + 12)$ inch
- Q 3. The ratio of area of slice to the area of remaining pizza in (I) is:**
 a. 5 : 1 b. 1 : 5 c. 2 : 5 d. 5 : 3
- Q 4. The ratio of areas of each slice of pizza (I) and (II) is:**
 a. 3 : 4 b. 5 : 3 c. 4 : 3 d. 2 : 5
- Q 5. The relation between area of a sector (A), length of the arc (l), angle (θ) subtended by the arc at the centre and radius of circle is:**
 a. $\frac{1}{2}lr$ b. lr
 c. $\frac{1}{3}lr$ d. $\frac{1}{2}lr^2$

Solutions

1. We have.

$$\text{Area of sector of a circle} = \frac{\theta}{360^\circ} \times \pi r^2$$

where, θ is the angle subtended by the sector at the centre. The radius of pizza (l) = 6 inches and as the pizza has been cut into six equal slices, the angle (θ) subtended at the centre is equal $= \frac{360^\circ}{6}$, i.e., to 60° .

$$\therefore \text{Required area} = \frac{60^\circ}{360^\circ} \times \pi \times 6^2 = 6\pi \text{ sq. inches.}$$

So, option (a) is correct.

2. Perimeter of a sector $= l + 2r$, where l is the length of the arc given by $\frac{\theta}{360^\circ} \times 2\pi r$

Here, $\theta = 60^\circ$ and $r = 6$ inch

$$l = \frac{60^\circ}{360^\circ} \times 2 \times \pi \times 6 = 2\pi$$

and

$$2r = 2 \times 6 = 12$$

$$\therefore \text{Perimeter of pizza} = (2\pi + 12) \text{ inch}$$

So, option (d) is correct.

3. The angle subtended by a slice of pizza (I) at the centre is 60° .

Therefore, angle subtended by the remaining pizza at the centre is 300° .

\therefore Ratio of areas of minor sector and major sector

$$= \frac{60^\circ}{360^\circ} \times \pi \times (6)^2 : \frac{300^\circ}{360^\circ} \times \pi \times (6)^2$$

$$= 60^\circ : 300^\circ = 1 : 5$$

So, option (b) is correct.

4. The angle subtended by a slice in pizza (I) is 60° , and the angle subtended by a slice in pizza (II) is 45°

$$\text{i.e., } \frac{360^\circ}{8}$$

Therefore, ratio of area of slice in the two cases

$$= \frac{60^\circ}{360^\circ} \times \pi \times (6)^2 : \frac{45^\circ}{360^\circ} \times \pi \times (6)^2$$

$$= 60^\circ : 45^\circ = 4 : 3$$

So, option (c) is correct.

5. The area of sector is given by $A = \frac{\theta}{360^\circ} \times \pi r^2$ and

$$\text{length of the arc is given by } l = \frac{\theta}{360^\circ} \times 2\pi r$$

Multiplying and dividing A by 2, we get

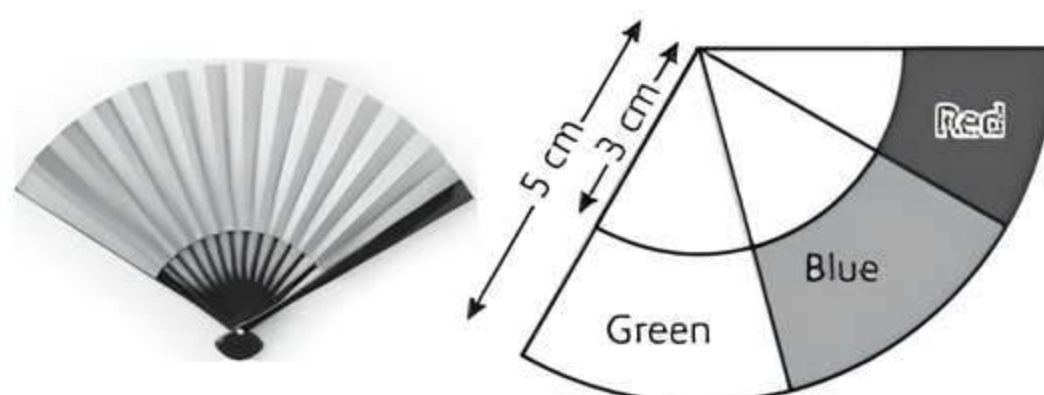
$$A = \frac{1}{2} \times \frac{\theta}{360^\circ} \times 2 \times \pi r^2$$

$$= \frac{1}{2} \times \left(\frac{\theta}{360^\circ} \times 2\pi r \right) \times r = \frac{1}{2} \times l \times r = \frac{1}{2}lr$$

So, option (a) is correct.

Case Study 2

Sara hold a Japanese folding fan in her hand as shown in the figure. It is shaped like a sector of a circle and made of a thin material such as paper or feather. The inner and outer radii are 3 cm and 5 cm respectively. The fan has three colours, i.e., red, blue and green.



Based on the given information, solve the following questions:

- Q 1.** If the region containing blue colour makes an angle of 80° at the centre, the area of the region having blue colour is:
 a. 9.17 cm^2 b. 10.1 cm^2
 c. 11.17 cm^2 d. 13.17 cm^2
- Q 2.** If the region containing green colour makes an angle of 60° at the centre, the area of the region having green colour is:
 a. 6.2 cm^2 b. 8.38 cm^2
 c. 9.9 cm^2 d. 11.12 cm^2
- Q 3.** If the region containing red colour makes an angle of 20° at the centre, the perimeter of the region containing red colour is:
 a. 2.9 cm b. 4.2 cm c. 5.4 cm d. 6.79 cm
- Q 4.** The area of the region having radius 3 cm is:
 a. 12.57 cm^2 b. 14.8 cm^2
 c. 20 cm^2 d. 26.57 cm^2
- Q 5.** The region given in the figure represents:
 a. minor sector b. major sector
 c. minor segment d. major segment

Solutions

1. Area of the region containing blue colour

$$= \frac{22}{7} \times 5 \times 5 \times \frac{80^\circ}{360^\circ} - \frac{22}{7} \times 3 \times 3 \times \frac{80^\circ}{360^\circ}$$

$$= \frac{22}{7} \times \frac{2}{9} \times [25 - 9] = \frac{44}{63} (16) = 11.17 \text{ cm}^2$$

So, option (c) is correct.

2. Area of the region containing green colour

$$= \frac{22}{7} \times \frac{60^\circ}{360^\circ} (5 \times 5 - 3 \times 3) = \frac{22}{7} \times \frac{1}{6} \times 16 = 8.38 \text{ cm}^2$$

So, option (b) is correct.

3. Perimeter of the region containing red colour
 $\Rightarrow (5 - 3) + (5 - 3) + \text{length of arc of sector having radius } 3 \text{ cm} + \text{length of arc of sector having radius } 5 \text{ cm}$

$$= 2 + 2 + 2 \times \frac{22}{7} \times 3 \times \frac{20^\circ}{360^\circ} + 2 \times \frac{22}{7} \times 5 \times \frac{20^\circ}{360^\circ}$$

$$\Rightarrow 4 + \frac{44}{7} \times \frac{1}{18} (3 + 5)$$

$$= 4 + \frac{44}{7} \times \frac{1}{18} \times 8 = 4 + \frac{176}{63} = 4 + 2.79 = 6.79 \text{ cm}$$

So, option (d) is correct.

4. Required area $= \frac{22}{7} \times (3)^2 \times (80^\circ + 60^\circ + 20^\circ)$

$$= \frac{22}{7} \times 3 \times 3 \times \frac{160^\circ}{360^\circ} = \frac{88}{7} = 12.57 \text{ cm}^2$$

So, option (d) is correct.

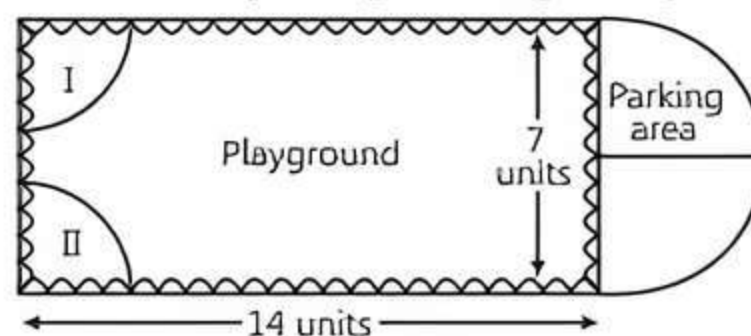
5. (a) Angle of given sector $= 80^\circ + 60^\circ + 20^\circ = 160^\circ$

Thus, the given region represents minor sector of a circle.

So, option (a) is correct.

Case Study 3

Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking.



After survey, it was decided to build rectangular playground, with a semicircular area allotted for parking at one end of the playground. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats.

Based on the above information, solve the following questions: [CBSE 2023]

- Q 1.** What is the total perimeter of the parking area?
Q 2. What is the total area of parking and the two quadrants?

Or

What is the ratio of area of playground to the area of parking area?

- Q 3.** Find the cost of fencing the playground and parking area at the rate of ₹ 2 per unit.

Solutions

1. Given, diameter of semicircular parking area.

$$d = 7 \text{ units}$$

$$\therefore \text{Its radius } (r) = \frac{d}{2} = \frac{7}{2} \text{ units}$$

Now, the total perimeter of the parking area

$$\Rightarrow 2r + \pi r$$

$$= 2 \times \frac{7}{2} + \frac{22}{7} \times \frac{7}{2} = 7 + 11 = 18 \text{ units}$$

2. We have, radius of parking area $(r) = \frac{7}{2}$ units.

$$\text{Now, area of both quadrant (I and II)} \Rightarrow 2 \times \frac{\pi R^2}{4}$$

$$\Rightarrow \frac{1}{2} \times \frac{22}{7} \times (2)^2 = \frac{44}{7} \text{ sq. units}$$

$$\text{and area of semicircular parking area} \Rightarrow \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{77}{4} \text{ sq. units}$$

So, total area of parking and the two quadrants

$$= \frac{44}{7} + \frac{77}{4} = \frac{176 + 539}{28} = \frac{715}{28}$$

$$\approx 25.54 \text{ sq. units}$$

Or
Given, length of the rectangular playground.
 $l = 14$ units
and breadth of the rectangular playground.
 $b = 7$ units

$$\therefore \text{Area of playground} = l \times b = 14 \times 7 \\ = 98 \text{ sq. units}$$

$$\text{From above part, area of parking area} \\ = \frac{77}{4} \text{ sq. units}$$

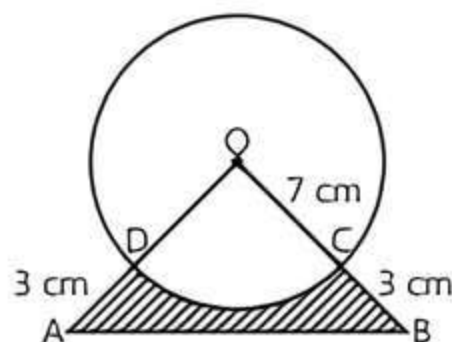
$$\therefore \text{Required ratio} = 98 : \frac{77}{4} = 56 : 11$$

3. Perimeter of playground and parking area
 $= 2l + b + \pi r$
 $= 2 \times 14 + 7 + \frac{22}{7} \times \frac{7}{2}$
 $= 28 + 7 + 11 = 46$ units

$$\text{So, cost of fencing} = 46 \times 2 = ₹ 92$$

Case Study 4

In an annual day function of a school, the organisers wanted to give a cash prize along with a memento to their best students. Each memento is made as shown in the figure and its base ABCD is shown from the front side. The rate of silver plating is ₹ 20 per cm^2 .



Based on the above information, solve the following questions: [CBSE 2023]

- Q 1. What is the area of the quadrant ODCO?
Q 2. Find the area of $\triangle AOB$.
Q 3. What is the total cost of silver plating the shaded part ABCD?

Or
What is the length of arc CD?

Solutions

1. Given, radius of the circle (r) = 7 cm
and central angle of quadrant (ODCO).
 $\theta = 90^\circ$

$$\therefore \text{Area of the quadrant ODCO} = \frac{1}{4} \pi r^2 \\ = \frac{1}{4} \times \frac{22}{7} \times (7)^2 \\ = \frac{77}{2} = 38.5 \text{ cm}^2$$

2. Since, ODCO is a quadrant so, $\triangle AOB$ is right angle at A.
 $\therefore OD = OC$ (radii)
 $\therefore OD = 7$ cm
 $\Rightarrow OB = OC + BC = 7 + 3 = 10$ cm

and $OA = OD + AD = 7 + 3 = 10$ cm
So, area of right-angled isosceles $\triangle AOB$

$$= \frac{1}{2} \times OA \times OB \\ = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$$

3. Area of shaded region ABCD = Area of $\triangle AOB$ – Area of quadrant ODCO $= 50 - 38.5 = 11.5 \text{ cm}^2$.
Since, the rate of silver plating is ₹ 20 per cm^2 .
So, total cost of silver plating the shaded part ABCD
 $= ₹ 20 \times 11.5$
 $= ₹ 230$

Or
Given, radius of sector ODCO.
 $r = 7$ cm
and central angle (θ) = 90°

$$\therefore \text{The length of arc CD} = \frac{\theta}{360^\circ} \times 2\pi r \\ = \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 7 \\ = 11 \text{ cm}$$

Case Study 5

Kritika bought a pendulum clock for her living room. The clock contains a small pendulum of length 15 cm. The minute hand and hour hand of the clock are 9 cm and 6 cm long respectively.



Based on the above information, solve the following questions:

- Q 1. Find the area swept by the minute hand in 10 minutes.
Q 2. If the pendulum covers a distance of 22 cm in the complete oscillation, then find the angles described by pendulum at the centre.
Q 3. Find the area swept by the hour hand in 1 hour.
Q 4. Find the area swept by the hour hand between 11 am and 5 pm.

Solutions

1. Angles made by minutes hand in 60 minutes $= 360^\circ$
 \therefore Angle made by minute hand in 10 minutes
 $= \frac{360^\circ}{60} \times 10 = 60^\circ$
Length of minute hand (r) = 9 cm

$$\begin{aligned}
 \therefore \text{Area swept by minute hand in 10 minutes} \\
 &= \text{Area of sector having central angles } 60^\circ \\
 &= \pi r^2 \left(\frac{60^\circ}{360^\circ} \right) = \frac{22}{7} \times 9 \times 9 \times \frac{1}{6} = \frac{297}{7} \\
 &= 42.42 \text{ cm}^2
 \end{aligned}$$

2. We have, $R = 15 \text{ cm}$ and arc length $l = \frac{1}{2} \times \text{complete oscillation} = \frac{1}{2}(22) = 11 \text{ cm}$

We know that, $l = 2\pi R \left(\frac{\theta}{360^\circ} \right)$

$$\Rightarrow \theta = \frac{11 \times 360^\circ}{2 \times \frac{22}{7} \times 15} = \frac{90^\circ \times 7}{15} = 6^\circ \times 7 = 42^\circ$$

COMMON ERROR

Some of the students make a mistake of considering the length of arc as 22 cm instead of taken $\frac{22}{2} \text{ cm}$, which leads to the wrong solutions.

3. Angle made by hour hand in 1 hour $= \frac{360^\circ}{12} = 30^\circ$

Also, $r_1 = 6 \text{ cm}$

$$\begin{aligned}
 \therefore \text{Area swept by hour hand in 1 hour} \\
 &= \text{Area of sector having central angle } 30^\circ \\
 &= \pi r_1^2 \times \left(\frac{30^\circ}{360^\circ} \right) = \frac{22}{7} \times 6 \times 6 \times \frac{1}{12} = \frac{66}{7} \\
 &= 9.428 \text{ cm}^2
 \end{aligned}$$

4. Number of hours from 11 am to 5 pm = 6
 Area swept by hour hand in 1 hour $= 9.428 \text{ cm}^2$
 \therefore Area swept by hour hand in 6 hours $= 9.428 \times 6$
 $= 56.568 \text{ cm}^2$

Very Short Answer Type Questions

Q 1. Find the area of the circle inscribed in a square of side 14 cm. [U. Imp.]

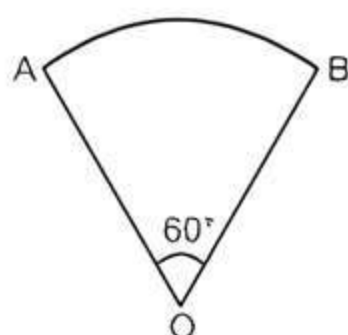
Q 2. Find the area of the sector of a circle of radius 6 cm whose central angle is 30° . [Take $\pi = 3.14$]

[CBSE 2020]

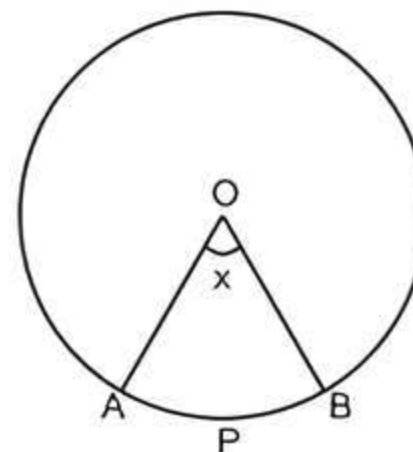
Q 3. In the given figure, OAB is a sector of circle of radius 10.5 cm. Find the perimeter of the sector.

[Take $\pi = \frac{22}{7}$]

[CBSE 2020]



Q 4. In the given figure, O is the centre of the circle. The area of the sector OAPB is $\frac{5}{12}$ part of the area of the circle. Find the value of x .



Short Answer Type-I Questions

Q 1. A piece of wire 20 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle.

[NCERT EXEMPLAR]

Q 2. Find the area of the sector of a circle with radius 4 cm and of angle 30° . Also find the area of the corresponding major sector. [Use $\pi = 3.14$]

[NCERT SOLVED EXAMPLE]

Q 3. Find the radius of a circle if an arc of angle 40° has length of $4\pi \text{ cm}$. Hence, find the area of the sector formed by this arc.

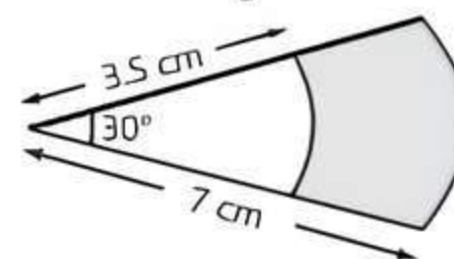
Q 4. Area of a sector of a circle of radius 36 cm is $54\pi \text{ cm}^2$. Find the length of the corresponding arc of the sector.

[NCERT EXEMPLAR]

Q 5. The length of the minute hand of a clock is 6 cm. Find the area swept by it when it moves from 7:05 pm to 7:40 pm. [Use $\pi = \frac{22}{7}$]

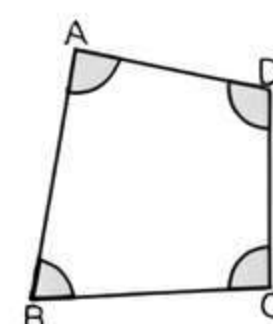
[CBSE SQP 2022-23]

Q 6. In the given figure, sectors of two concentric circles of radii 7 cm and 3.5 cm are shown, then find the area of the shaded region.



Q 7. In the given figure, arcs have been drawn of radius 7 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region. [Use $\pi = \frac{22}{7}$]

[CBSE SQP 2022-23]



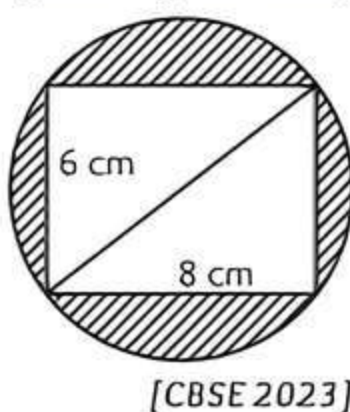
- Q 8. An Umbrella has 8 ribs which are equally spaced (see the figure). Assuming umbrella to be a flat circle of radius 45 cm, then find the area between the two consecutive ribs of the umbrella.



- Q 9. With vertices A, B and C of $\triangle ABC$ as centres, arcs are drawn with radii 14 cm and the three portions of the triangle so obtained are removed. Find the total area removed from the triangle.

[CBSE SQP 2023-24]

- Q 10. Reeti prepares a Rakhi for her brother Ronit. The Rakhi consists of a rectangle of length 8 cm and breadth 6 cm inscribed in circle as shown in the figure. Find the area of the shaded region. [Use $\pi = 3.14$]

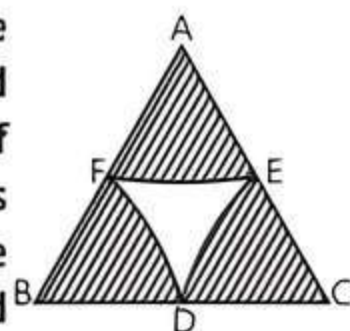


[CBSE 2023]



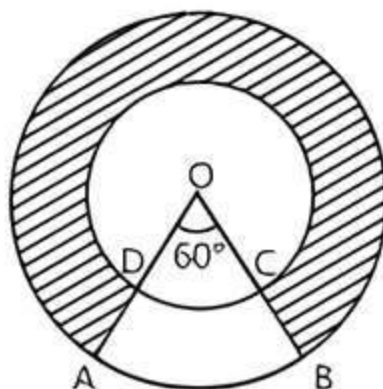
Short Answer Type-II Questions

- Q 1. In the given figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm, to intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region. [Use $\pi = 3.14$]



[NCERT EXEMPLAR]

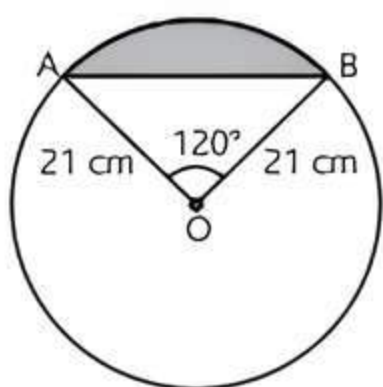
- Q 2. In the given figure, two concentric circles with centre O, have radii 21 cm and 42 cm. If $\angle AOB = 60^\circ$, find the area of the shaded region. [CBSE 2019, 17]



- Q 3. Find the area of the segment shown in figure, if radius of the circle is 21 cm and $\angle AOB = 120^\circ$.

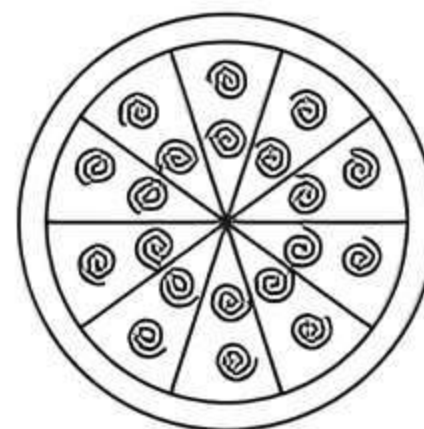
[Use $\pi = \frac{22}{7}$]

[NCERT EXERCISE; CBSE 2019]

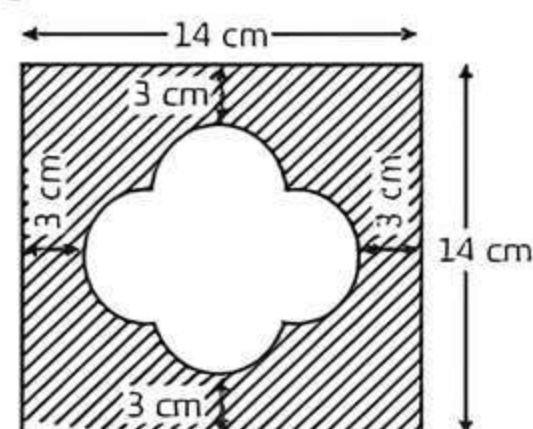


- Q 4. A chord AB of a circle of radius 15 cm makes an angles of 60° at the centre of the circle. Find the area of the major and minor segment. [Take $\pi = 3.14, \sqrt{3} = 1.73$]

- Q 5. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Figure. Find:
(i) the total length of the silver wire required
(ii) the area of each sector of the brooch.



- Q 6. Find the area of the unshaded region shown in the given figure. [CBSE SQP 2023-24]



Long Answer Type Questions

- Q 1. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
(i) length of the arc
(ii) area of the sector formed by the arc
(iii) area of the segment formed by the corresponding chord of the arc.

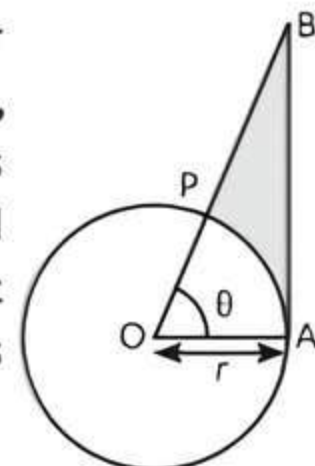
[Use $\pi = \frac{22}{7}, \sqrt{3} = 1.732$]

Or

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of the sector formed by the arc. Also, find the length of the arc.

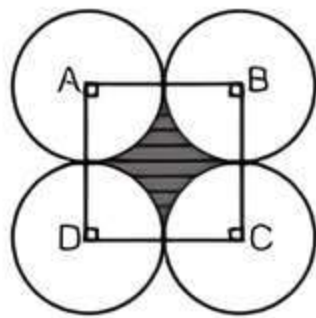
[CBSE 2023]

- Q 2. In the adjoining figure, a sector OAP of a circle with centre O, containing $\angle \theta$ is shown AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r \left(\tan \theta + \sec \theta + \frac{\pi \theta}{180^\circ} - 1 \right)$.

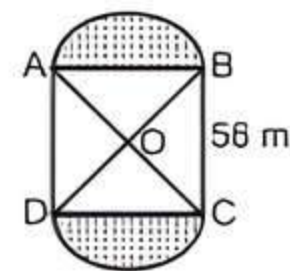


[CBSE 2016]

- Q 3. Four circular cardboard pieces of radii 7 cm are placed on a paper in such a way that each piece touches other two pieces. Find the area of the unshaded region within quadrilateral ABCD.

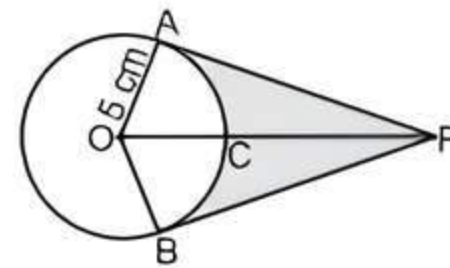


- Q 4. In the given figure, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the total area of the flower beds. Use $\pi = \frac{22}{7}$



- Q 5. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also, find the shaded area. [Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]

[CBSE 2016]



Solutions

Very Short Answer Type Questions

1. Let 'r' be the radius of the circle.

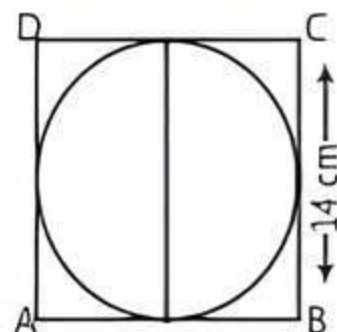


TIP

If circle is inscribed in a square, then diameter of a circle is equal to the side of a square.

Diameter of circle
= Side of square
= 14 cm

$$\therefore \text{Radius of circle } (r) = \frac{14}{2} = 7 \text{ cm}$$



$$\begin{aligned} \text{Area of circle} &= \pi r^2 = \frac{22}{7} \times 7^2 \\ &= \frac{22}{7} \times 49 = 154 \text{ cm}^2 \end{aligned}$$

Hence, area of circle is 154 cm^2 .

2. Given, radius of the circle (r) = 6 cm
and central angle (θ) = 30°

$$\begin{aligned} \therefore \text{Area of the sector} &= \frac{\theta}{360^\circ} \times \pi r^2 \\ &= \frac{30^\circ}{360^\circ} \times 3.14 \times (6)^2 \\ &= \frac{1}{12} \times 3.14 \times 36 = 9.42 \text{ cm}^2 \end{aligned}$$

3. Given, radius of circle (r) = 10.5 cm
and central angle (θ) = 60°

TRICK

$$\text{Length of the arc} = \frac{\theta}{360^\circ} \times 2\pi r$$

$$\therefore \text{Perimeter of sector} = OA + OB + \widehat{AB}$$

$$\begin{aligned} &= r + r + \text{length of arc AB} \\ &= 10.5 + 10.5 + \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 10.5 \\ &= 21 + \frac{22 \times 15}{3} = 21 + 11 = 32 \text{ cm} \end{aligned}$$

COMMON ERROR

Sometimes students take the perimeter of sector as a length of the sector, but it is wrong. Here add twice of radius in length of the sector to get the perimeter of the sector.

4. Given, area of sector OAPB = $\frac{5}{12}$ × Area of circle

$$\Rightarrow \frac{x}{360^\circ} \times \pi r^2 = \frac{5}{12} \times \pi r^2 \Rightarrow x = 150^\circ$$

Short Answer Type-I Questions

1. Given, length of arc of circle = length of wire = 20 cm
Here, central angle $\theta = 60^\circ$

$$\therefore \text{Length of arc} = \frac{\theta}{360^\circ} \times 2\pi r$$

$$\Rightarrow 20 = \frac{60^\circ}{360^\circ} \times 2\pi r$$

$$\Rightarrow 20 = \frac{\pi r}{3}$$

$$\Rightarrow r = \frac{60}{\pi} \text{ cm}$$

Therefore, required radius of circle is $\frac{60}{\pi} \text{ cm}$.

2. Given, radius of circle (r) = 4 cm
and angle of sector (θ) = 30°

$$\begin{aligned}\text{Then, area of sector} &= \frac{\theta}{360^\circ} \times \pi r^2 \\ &= \frac{30^\circ}{360^\circ} \times 3.14 \times 4 \times 4 \\ &= \frac{12.56}{3} = 4.19 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{and area of corresponding major sector} &= \text{area of circle} - \text{area of sector} \\ &= \pi r^2 - 4.19 = 3.14 \times 4 \times 4 - 4.19 \\ &= 50.24 - 4.19 \\ &= 46.05 \text{ or } 46.1 \text{ cm}^2\end{aligned}$$

3. Let ' r ' be the radius of the circle.

$$\begin{aligned}\therefore \text{Length of arc} &= \frac{\theta}{360^\circ} \times 2\pi r \\ 4\pi &= \frac{40^\circ}{360^\circ} \times 2\pi r = \frac{2\pi r}{9} \\ (\because \theta &= 40^\circ \text{ and length of arc} = 4\pi) \\ \Rightarrow r &= 18 \text{ cm} \\ \therefore \text{Area of sector} &= \frac{1}{2}lr \quad (\because l = \text{length of arc}) \\ &= \frac{1}{2} \times 4\pi \times 18 = 36\pi \text{ cm}^2\end{aligned}$$

4. Given, radius (r) of circle = 36 cm
 \therefore Area of sector = $54\pi \text{ cm}^2$

$$\begin{aligned}\frac{\theta}{360^\circ} \times \pi r^2 &= 54\pi \\ \therefore \frac{\theta}{360^\circ} \times (36)^2 &= 54 \\ \Rightarrow \theta &= \frac{54 \times 360^\circ}{36 \times 36} = 15^\circ \\ \text{So, length of corresponding arc} &= \frac{\theta}{360^\circ} \times 2\pi r = \frac{15^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 36 \\ &= \frac{3}{2} \times 2 \times \frac{22}{7} = \frac{66}{7} = 9.43 \text{ cm}\end{aligned}$$

5. We know that, in 60 min, the tip of minute hand moves 360° .

$$\text{In 1 minute, it will move} = \frac{360^\circ}{60} = 6^\circ$$

\therefore From 7:05 pm to 7:40 pm i.e., 35 min, It will move through = $35 \times 6^\circ = 210^\circ$

TR!CK

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

\therefore Area swept by the minute hand in 35 min = Area of sector with sectorial angle 210° and radius of 6 cm.

$$\begin{aligned}&= \frac{210^\circ}{360^\circ} \times \pi \times 6^2 = \frac{7}{12} \times \frac{22}{7} \times 6 \times 6 \\ &= 66 \text{ cm}^2\end{aligned}$$

6. Area of shaded region
= (Area of sector having $r = 7 \text{ cm}$, $\theta = 30^\circ$)
– (Area of sector having $r = 3.5 = \frac{7}{2} \text{ cm}$, $\theta = 30^\circ$)
$$= \left[\frac{22}{7} \times (7)^2 \times \frac{30^\circ}{360^\circ} \right] - \left[\frac{22}{7} \times \left(\frac{7}{2} \right)^2 \times \frac{30^\circ}{360^\circ} \right]$$

$$\left(\because \text{Area of a sector} = \frac{\theta}{360^\circ} \times \pi r^2 \right)$$

$$= \left(\frac{77}{6} - \frac{77}{24} \right) = \frac{308 - 77}{24} = \frac{231}{24} = \frac{77}{8} = 9.625 \text{ cm}^2$$

7. Let the measure of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ be θ_1 , θ_2 , θ_3 and θ_4 respectively.

Required area = Area of sector with centre A + Area of sector with Centre B + Area of sector with centre C + area of sector with centre D

$$\begin{aligned}&= \frac{\theta_1}{360^\circ} \times \pi 7^2 + \frac{\theta_2}{360^\circ} \times \pi 7^2 + \frac{\theta_3}{360^\circ} \times \pi 7^2 + \frac{\theta_4}{360^\circ} \times \pi 7^2 \\ &= \frac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{360^\circ} \times \pi 7^2\end{aligned}$$

TR!CK

The sum of all interior angles of a quadrilateral is 360° .

$$\begin{aligned}&= \frac{(360^\circ)}{360^\circ} \times \frac{22}{7} \times 7 \times 7 \\ &= 154 \text{ cm}^2\end{aligned}$$

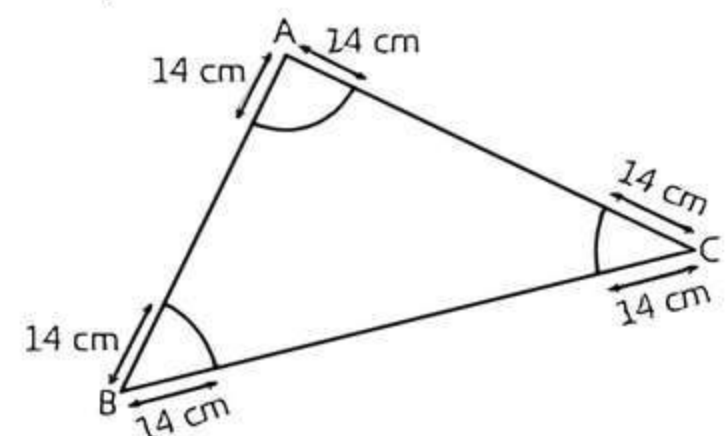
8. Given, umbrella to be a flat circle.
So, the central angle of an umbrella is 360° .
Since, umbrella has 8 ribs.

$$\therefore \text{Angle between two ribs} = \frac{360^\circ}{8} = 45^\circ$$

Area between two ribs = Area of one sector of the umbrella

$$\begin{aligned}&= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times (45)^2 \quad (\because r = 45 \text{ given}) \\ &= \frac{22}{7 \times 8} \times 45 \times 45 = \frac{22275}{28} \text{ cm}^2\end{aligned}$$

9. Let the measure of $\angle A$, $\angle B$ and $\angle C$ be θ_1 , θ_2 and θ_3 respectively.



\therefore Required area = Area of sector with centre A
+ Area of sector with centre B
+ Area of sector with centre C.

$$= \frac{\theta_1}{360^\circ} \times \pi(14)^2 + \frac{\theta_2}{360^\circ} \times \pi(14)^2 + \frac{\theta_3}{360^\circ} \times \pi(14)^2$$

$$= \frac{(\theta_1 + \theta_2 + \theta_3)}{360^\circ} \times \pi \times 14 \times 14$$

TR!CK

The sum of all interior angles of a triangle is 180° .

$$= \frac{180^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 = 11 \times 2 \times 14 = 308 \text{ cm}^2.$$

10. Given, length of rectangle (l) = 8 cm
and breadth of rectangle (b) = 6 cm
So, area of rectangle $= l \times b = 8 \times 6 = 48 \text{ cm}^2$
In rectangle, each consecutive sides are perpendicular to each other.

$$\therefore \text{Length of the diagonal} = \sqrt{l^2 + b^2}$$

$$= \sqrt{(8)^2 + (6)^2} = \sqrt{64 + 36}$$

$$= \sqrt{100} = 10 \text{ cm}$$

$$\therefore \text{Diameter of the circle} = \text{length of the diagonal} = 10 \text{ cm}$$

$$\Rightarrow \text{Radius of the circle} = \frac{10}{2} = 5 \text{ cm}$$

$$\therefore \text{Area of circle} = \pi r^2 = 3.14 \times (5)^2$$

$$= 3.14 \times 25 = 78.5 \text{ cm}^2$$

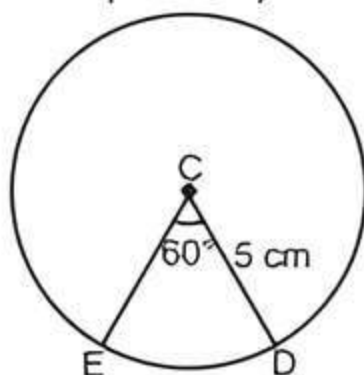
$$\text{So, required area of shaded region}$$

$$= \text{Area of circle} - \text{Area of rectangle}$$

$$= 78.5 - 48 = 30.5 \text{ cm}^2.$$

Short Answer Type-II Questions

1. $\triangle ABC$ is an equilateral triangle.
 $\therefore \angle A = \angle B = \angle C = 60^\circ$
and $AB = BC = CA = 10 \text{ cm}$ (given)
Given that D, E and F are the mid-points of the sides BC, CA and AB respectively of equilateral $\triangle ABC$.



$$\therefore \text{Radius} = AE = EC = CD = BD = BF = FA$$

$$= \frac{10}{2} = 5 \text{ cm}$$

$$\text{Now, area of sector CDE} = \frac{\theta \pi r^2}{360^\circ} \quad (\text{where, } r = 5 \text{ cm})$$

$$= \frac{60^\circ \times 3.14}{360^\circ} \times (5)^2$$

$$= \frac{3.14 \times 25}{6} = \frac{78.5}{6}$$

$$= 13.0833 \text{ cm}^2$$

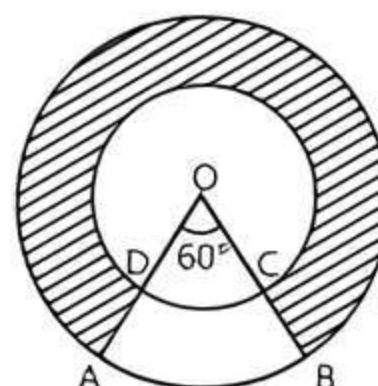
$$\text{Therefore, area of shaded region}$$

$$= 3 \times \text{area of sector CDE}$$

$$= 3 \times 13.0833$$

$$= 39.25 \text{ cm}^2$$

2. Let the sector AOB meet the inner circle at C and D.



Given, radius of inner circle (r_2) = 21 cm,
Radius of external circle (r_1) = 42 cm and central angle $\angle AOB$, $\theta = 60^\circ$.



TiP

For finding the area of shaded region we will subtract area of major sector COD from major sector AOB.

$$\text{Area of region ABCD} = \pi \times \frac{\theta}{360^\circ} (r_1^2 - r_2^2)$$

$$= \pi \times \frac{60^\circ}{360^\circ} (42^2 - 21^2)$$

$$= \frac{22}{7} \times \frac{1}{6} \times 63 \times 21$$

$$(\because a^2 - b^2 = (a + b)(a - b))$$

$$= 693 \text{ cm}^2$$

TR!CKS

- Area of sector $= \frac{\theta}{360^\circ} \cdot \pi r^2$
- Area of ring $= \pi(r_1^2 - r_2^2)$

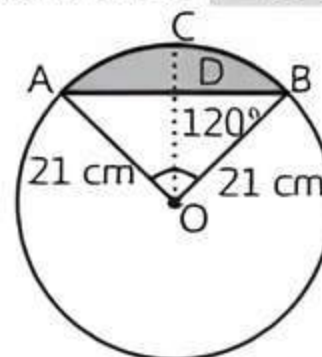
$$\therefore \text{Area of shaded region} = \pi (42^2 - 21^2)$$

$$- \text{Area of region ABCD}$$

$$= \frac{22}{7} \times 63 \times 21 - 693$$

$$= 4158 - 693 = 3465 \text{ cm}^2$$

3. Given, radius of a circle, $r = 21 \text{ cm}$ and $\angle AOB = 120^\circ$



$$\text{Now, area of the sector OACB} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times (21)^2$$

$$= \frac{1}{3} \times 22 \times 3 \times 21 = 462 \text{ cm}^2$$



TiP

A perpendicular drawn from centre to chord bisects it.

Now, we draw a perpendicular line OD to the chord AB.

Also, $\triangle AOD \cong \triangle BOD$ (by RHS congruency)

$\therefore \angle AOD = \angle BOD = 60^\circ$ (by CPCT)

In right-angled $\triangle ADO$,

$$\sin 60^\circ = \frac{AD}{OA}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AD}{21} \Rightarrow AD = \frac{21}{2}\sqrt{3} \text{ cm}$$

$$\text{and } \cos 60^\circ = \frac{OD}{OA} \Rightarrow \frac{1}{2} = \frac{OD}{21}$$

$$\Rightarrow OD = \frac{21}{2} \text{ cm}$$

Now, $AB = 2 AD$

$$= 2 \times \frac{21}{2}\sqrt{3} = 21\sqrt{3} \text{ cm}$$

Now, $\text{area of } \triangle AOB = \frac{1}{2} \times AB \times OD$

$$= \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$$

$$= \frac{441}{4}\sqrt{3} \text{ cm}^2$$

\therefore Required area of segment = Area of sector OACB
– Area of $\triangle AOB$

$$= \left(462 - \frac{441}{4}\sqrt{3}\right)$$

$$= \left(462 - \frac{441}{4} \times 1.73\right)$$

$$= 462 - 190.73 = 271.27 \text{ cm}^2$$

4. The area of a minor segment of angle θ (in degrees) in a circle of radius r is given by

$$A = \left\{ \frac{\pi\theta}{360^\circ} - \frac{1}{2}\sin\theta \right\} r^2$$

$$\Rightarrow A = \left\{ \frac{3.14 \times 60^\circ}{360^\circ} - \frac{1}{2}\sin 60^\circ \right\} (15)^2 \text{ cm}^2$$

$$\Rightarrow A = \left\{ \frac{3.14}{6} - \frac{\sqrt{3}}{4} \right\} \times 225 \text{ cm}^2$$

$$= \left\{ \frac{3.14}{6} - \frac{1.73}{4} \right\} \times 225 \text{ cm}^2$$

$$\Rightarrow A = (0.5233 - 0.4325)225 \text{ cm}^2$$

$$= 225 \times 0.0908 \text{ cm}^2 = 20.43 \text{ cm}^2$$

Area of the major segment

= Area of the circle – Area of the minor segment

$$= [3.14 \times (15)^2 - 20.43] \text{ cm}^2 = [706.5 - 20.43] \text{ cm}^2$$

$$= 686.07 \text{ cm}^2$$

5. (i) Let l be the total length of the silver wire. Then,

l = Circumference of the circle of radius $\frac{35}{2}$ mm

+ Length of five diameters

$$\Rightarrow l = 2\pi \times \frac{35}{2} + 5 \times 35 \text{ mm}$$

$$= \left(2 \times \frac{22}{7} \times \frac{35}{2} + 175 \right) \text{ mm} = 285 \text{ mm}$$

- (ii) The circle is divided into 10 equal sectors.

$$\therefore \text{Central angle of each sector } (\theta) = \frac{360}{10} = 36^\circ$$

$$\text{and radius } (r) = \frac{35}{2} \text{ mm}$$

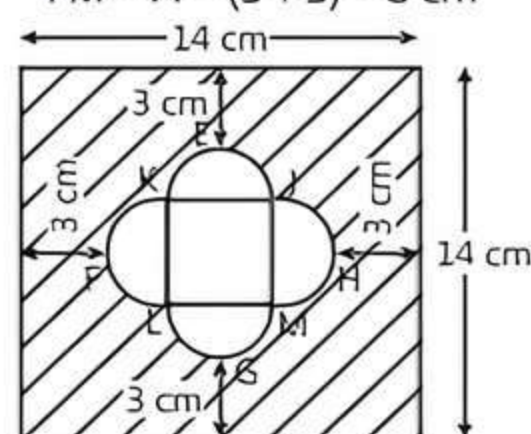
So, area A of each sector of the brooch is given by

$$A = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{36^\circ}{360^\circ} \times \frac{22}{7} \times \left(\frac{35}{2}\right)^2$$

$$= \frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} = \frac{385}{4} \text{ mm}^2$$

6. Join JK, KL, LM and MJ. Here four equal semicircles are obtained and LMKJ represents a square.

$$FM = 14 - (3 + 3) = 8 \text{ cm}$$



So, the side of the square must be 4 cm and the radius of each semicircle at both ends must be 2 cm.

\therefore Area of square JKLM = (side of square)²

$$= (4)^2 = 16 \text{ cm}^2$$

$$\text{and area of semicircle} = \frac{\pi r^2}{2} = \frac{\pi}{2} (2)^2 = 2\pi \text{ cm}^2$$

\therefore Area of four semicircles = $4 \times 2\pi = 8\pi$

$$= 8 \times 3.14 = 25.12 \text{ cm}^2$$

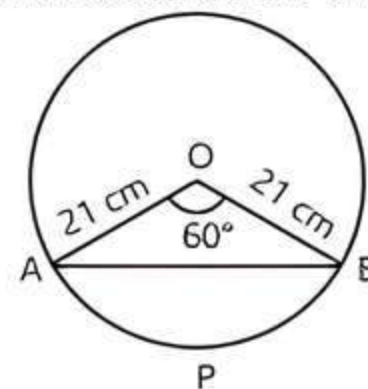
\therefore Area of unshaded region = Area of square JKLM

+ Area of four semicircles

$$= 16 + 25.12 = 41.12 \text{ cm}^2$$

Long Answer Type Questions

1. Let O be the centre of the circle of radius 21 cm such that an arc APB subtends 60° angle at the centre O.



- (i) Length of the arc APB

$$= \frac{\theta}{360^\circ} \times 2\pi r = \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21 \text{ cm} = 22 \text{ cm}$$

- (ii) Area of sector OAPB

$$\frac{\theta}{360^\circ} \times \pi r^2 = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 231 \text{ cm}^2$$

- (iii) Area of the segment APB = $\left\{ \frac{\pi\theta}{360^\circ} - \frac{1}{2}\sin\theta \right\} r^2$

$$= \left\{ \frac{22}{7} \times \frac{60^\circ}{360^\circ} - \frac{1}{2}\sin 60^\circ \right\} \times 21 \times 21 \text{ cm}^2$$

$$= \left\{ \frac{11}{21} - \frac{1}{2} \times \frac{\sqrt{3}}{2} \right\} \times 21 \times 21 \text{ cm}^2$$

$$= \left\{ 11 \times 21 - \frac{\sqrt{3}}{4} \times 21 \times 21 \right\} \text{ cm}^2$$

$$= \left\{ 231 - \frac{441 \times 1.732}{4} \right\} \text{ cm}^2$$

$$= (231 - 190.95) \text{ cm}^2 = 40.05 \text{ cm}^2$$

2. Given, radius of circle, $OA = OP = r$.

\therefore Tangent is perpendicular to the radius through the point of contact.

$$\therefore \angle OAB = 90^\circ$$

Now, in right-angled $\triangle OAB$,

$$\tan \theta = \frac{AB}{OA} = \frac{AB}{r}$$

$$\Rightarrow AB = r \tan \theta \quad \dots(1)$$

Again, in right-angled $\triangle OAB$,

$$\cos \theta = \frac{OA}{OB} = \frac{r}{OB}$$

$$\Rightarrow OB = r \sec \theta \quad \dots(2)$$

$$\text{Then, } PB = OB - OP = r \sec \theta - r \quad \dots(3)$$

Now, length of arc \widehat{AP} (minor)

$$= \frac{\theta}{360^\circ} \times 2\pi r = \frac{\theta}{180^\circ} \times \pi r \quad \dots(4)$$

\therefore Perimeter of shaded region

$$= \widehat{AP} + AB + PB$$

$$= \frac{\theta}{180^\circ} \times \pi r + r \tan \theta + r \sec \theta - r$$

$$= r \left(\tan \theta + \sec \theta + \frac{\pi \theta}{180^\circ} - 1 \right)$$

Hence proved.

3. Four circular card board pieces are placed on a paper in such a way that each piece touches other two pieces.

Now, the centres of four circles are joined to one-another by a line segment. Since, the radius of each circle is 7 cm.

$$\therefore AB = 2 \times \text{radius of circle} \\ = 2 \times 7 = 14 \text{ cm}$$

$$\Rightarrow AB = BC = CD = AD = 14 \text{ cm}$$

which shows that the quadrilateral ABCD is a square, each of whose side is 14 cm.

We know that the angle between two adjacent sides of a square is 90° .

$$\therefore \text{Area of sector with central angle } A = \frac{\angle A}{360^\circ} \times \pi r^2$$

$$(\because \angle A = \angle B = \angle C = \angle D = 90^\circ)$$

$$= \frac{90^\circ}{360^\circ} \times \pi \times (7)^2 = \frac{1}{4} \times \pi \times 49$$

$$= \frac{49}{4} \times \frac{22}{7} = \frac{154}{4} = \frac{77}{2} = 38.5 \text{ cm}^2$$

\therefore Area of unshaded region with ABCD

= Area of four sector

= $4 \times$ area of sector with central angle A

$$= 4 \times 38.5 = 154 \text{ cm}^2$$

4. Given, length of the side of square lawn ABCD = 56 m

$$\therefore \text{Area of square lawn ABCD} = (\text{side})^2 = 56 \times 56 \text{ cm}^2$$

Let $OA = OB = x$ m

(since, ABCD is a square and diagonals of a square bisect each other at right angle)

\therefore In right-angled $\triangle AOB$,

$$AB^2 = OA^2 + OB^2$$

(from Pythagoras theorem)

$$\Rightarrow (56)^2 = x^2 + x^2$$

$$\Rightarrow 56 \times 56 = 2x^2 \Rightarrow x^2 = 28 \times 56 \quad \dots(1)$$

$$\text{Now, area of sector AOB} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times x^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 28 \times 56$$

(from eq. (1))

$$\text{and area of } \triangle AOB = \frac{1}{2} \times OA \times OB$$

$$= \frac{1}{2} \times x \times x \quad (\because \angle AOB = 90^\circ)$$

$$= \frac{1}{2} x^2 = \frac{1}{2} \times 28 \times 56$$

(from eq. (1))

\therefore Area of flower bed AB = area of sector AOB

– area of $\triangle AOB$

$$= \frac{1}{4} \times \frac{22}{7} \times 28 \times 56 - \frac{1}{2} \times 28 \times 56$$

$$= \frac{1}{4} \times 28 \times 56 \times \left(\frac{22}{7} - 2 \right)$$

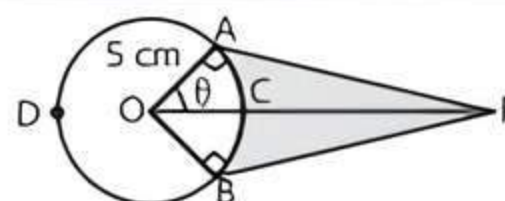
$$= \frac{1}{4} \times 28 \times 56 \times \frac{8}{7}$$

\therefore Area of both flower beds

= $2 \times$ Area of flower bed AB

$$= 2 \times \frac{1}{4} \times 28 \times 56 \times \frac{8}{7} = 896 \text{ m}^2$$

5. Given, radius of circle = $OA = OB = 5$ cm, $OP = 10$ cm



\therefore Tangent is perpendicular to the radius through the point of contact.

$$\therefore \angle OAP = 90^\circ$$

Now, in right $\triangle OAP$,

$$\cos \theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2} \quad (\because OP = 10 \text{ cm, given})$$

$$\Rightarrow \cos \theta = \cos 60^\circ$$

$$\Rightarrow \theta = 60^\circ$$

\therefore If two tangents are drawn from an external point to a circle, then both tangents subtend equal angles at the centre of the circle.

$\therefore \angle AOB = 60^\circ + 60^\circ = 120^\circ$ --(1)
 Reflex $\angle AOB = 360^\circ - \angle AOB = 360^\circ - 120^\circ = 240^\circ$
 \therefore Length of the belt that is still in
 Contact with the pulley $= ADB =$ Length of major arc

$$\begin{aligned}
 &= \left(\frac{\text{Reflex } \angle AOB}{360^\circ} \right) 2\pi r \\
 &= \frac{240^\circ}{360^\circ} \times 2 \times 3.14 \times 5 = \frac{62.8}{3} = 20.93 \text{ cm}
 \end{aligned}$$

Again, in right-angled $\triangle OAP$,

$$\sin 60^\circ = \frac{AP}{OP} = \frac{AP}{10}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AP}{10} \Rightarrow AP = \frac{\sqrt{3} \times 10}{2} = 5\sqrt{3} \text{ cm}$$

Now, area of $\triangle OAP$

$$= \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times AP \times OA$$

$$= \frac{1}{2} \times 5\sqrt{3} \times 5 = \frac{25}{2}\sqrt{3} \text{ cm}^2$$

$$\therefore \text{ar}(\triangle OAP) = \text{ar}(\triangle OBP) = \frac{25}{2}\sqrt{3} \text{ cm}^2$$

Now, area of minor sector OACB

$$\begin{aligned}
 &= \frac{\theta}{360^\circ} \times \pi r^2 \\
 &= \frac{120^\circ}{360^\circ} \times 3.14 \times (5)^2 \quad [\text{from eq.(1)}] \\
 &= \frac{78.5}{3} = 26.17 \text{ cm}^2
 \end{aligned}$$

$$\therefore \text{Shaded area} = \text{ar}(\triangle OAP) + \text{ar}(\triangle OBP) - \text{area of minor sector OACB}$$

$$\begin{aligned}
 &= \frac{25}{2}\sqrt{3} + \frac{25}{2}\sqrt{3} - 26.17 = 25\sqrt{3} - 26.17 \\
 &= 25(1.73) - 26.17 = 43.25 - 26.17 = 17.08 \text{ cm}^2
 \end{aligned}$$

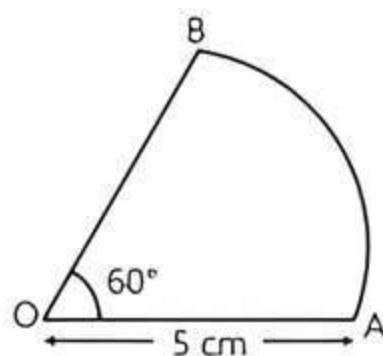
Hence, the length of the belt that is still in contact with the pulley, is 20.93 cm and the shaded area is 17.08 cm^2 .



Chapter Test

Multiple Choice Questions

Q 1. The perimeter of the sector OAB shown in figure is:



- a. $\frac{61}{3}$ cm b. $\frac{320}{21}$ cm
 c. 60 cm d. $\frac{54}{7}$ cm

Q 2. If AB is a chord of length $5\sqrt{3}$ cm of a circle with centre O and radius 5 cm, then area of sector OAB is:

- a. $\frac{21\pi}{5} \text{ cm}^2$ b. $\frac{25}{3} \text{ cm}^2$
 c. $25\pi \text{ cm}^2$ d. $\frac{25\pi}{3} \text{ cm}^2$

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:

- a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
 b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
 c. Assertion (A) is true but Reason (R) is false
 d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): The area of the sector of a circle of radius 5 cm is 9.75 cm^2 , if the corresponding arc length is 3.5 cm.

Reason (R): Area of a sector of a circle of radius r and central angle θ is $\frac{\theta}{360^\circ} \pi r^2$.

Q 4. Assertion (A): A sector is cut from a circle of radius 35 cm. The central angle of the sector is 120° . The perimeter of the sector is $\frac{430}{3}$ cm.

Reason (R): Perimeter of sector = $2r$ – length of corresponding arc.

Fill in the Blanks

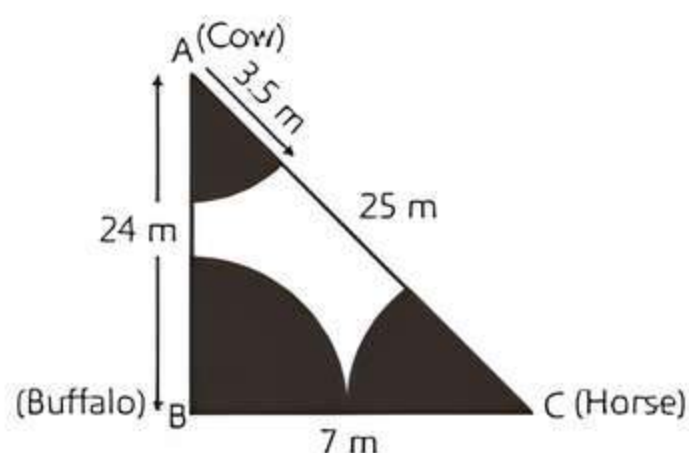
- Q 5. If l is the arc length of a sector of a circle of a radius r and A is the area of the sector. Then, $A : l = \dots\dots\dots$
 Q 6. The angle subtended at the centre of a circle of radius 6 cm by an arc of length 3π cm is $\dots\dots\dots$

True/False

- Q 7. An arc subtends an angle of 90° at the centre of the circle of radius 14 cm. The area of minor sector thus formed in terms of π is $49\pi \text{ cm}^2$.
 Q 8. If a sector of a circle of radius 4 cm contains an angle of 30° , then area of the sector is $\frac{2\pi}{3} \text{ cm}^2$.

Case Study Based Question

Q 9. Gayatri have a triangle shaped grass field. At the three corners of the field, a cow, a buffalo and a horse are tied separately to the pegs through ropes of 3.5 m each to graze in the field, as shown in the figure. Sides of the triangle field are 25 m, 24 m and 7 m.



Based on the given information, solve the following questions:

- (i) Find the area of triangular field.
- (ii) Find the total area grazed by a cow, a buffalo and a horse.

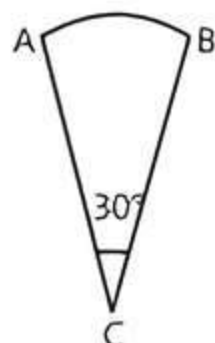
Or

Find the area of the field that cannot be grazed.

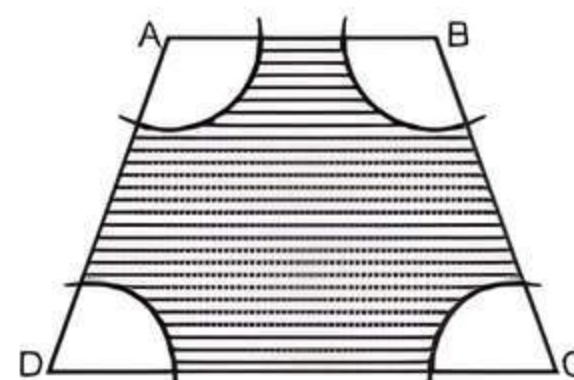
- (iii) Find the sum of the field arc lengths grazed by a buffalo, a cow and a horse.

Very Short Answer type Questions

- Q 10. If the figure is a sector of a circle of radius 7 cm, what is the perimeter of the sector?



- Q 11. In the figure, ABCD is a trapezium with $AB \parallel DC$, $AB = 18$ cm, $DC = 32$ cm and distance between AB and DC is 14 cm. If arcs of equal radii 7 cm with centres A, B, C and D have been drawn, then find the area of the non-shaded region of the figure.



Short Answer Type-I Questions

- Q 12. The length of the minute hand of a clock is 21.5 cm. Find the area swept by the minute hand in 5 minutes.
- Q 13. A sector of 56° cut out from a circle contains area 4.4 cm^2 . Find the radius of the circle.

Short Answer Type-II Questions

- Q 14. If a chord of a circle of radius 28 cm makes an angle of 90° at the centre, then find the area of the major segment.
- Q 15. From a circular piece of cardboard of radius 3 cm two sectors of 90° have been cut off. Find the perimeter of the remaining portion nearest hundredth centimetres. $\left[\text{Use } \pi = \frac{22}{7} \right]$

Long Answer Type Question

- Q 16. In the adjoining figure, three circles of radius 2 cm touch one another externally. These circles are circumscribed by a circle of radius R cm. Find the value of R and the area of the non-shaded region in $\triangle ABC$.

