Quadrilaterals

Exercise – 5.1

Solution 1:

Let measure of the fourth angle be x°.
By using the angle sum property of quadrilaterals, we get,
$130^\circ + 82^\circ + 40^\circ + x^\circ = 360^\circ$
$\therefore x^\circ = 360^\circ - 252^\circ$
$\therefore x^\circ = 108^\circ$
$\therefore$ The measure of the fourth angle is 108°.

Solution 2:

The angles of the quadrilateral are in the ratio 2:3:5:8.
Let the measures of the angles of the quadrilateral be 2x, 3x, 5x and 8x.
Using the angle sum property of quadrilaterals, we get,
$2x + 3x + 5x + 8x = 360^\circ$
$\therefore 18x = 360^\circ$
$\therefore x = 20^\circ$
$\therefore 2x = 2 \times 20^\circ = 40^\circ$
$\therefore 3x = 3 \times 20^\circ = 60^\circ$
$\therefore 5x = 5 \times 20^\circ = 100^\circ$
$\therefore 8x = 8 \times 20^\circ = 160^\circ$
$\therefore$ The measures of the angles of the quadrilateral are 40°, 60°, 100° and 160°.

Solution 3:

The measures of angles of a $\square$PQRS are 3x, 4x, 5x and 6x respectively.
So $\angle P = 3x$, $\angle Q = 4x$, $\angle R = 5x$ and $\angle S = 6x$.
Using the angle sum property of quadrilaterals, we get,
$m\angle P + m\angle Q + m\angle R + m\angle S = 360^\circ$
$\therefore 3x + 4x + 5x + 6x = 360^\circ$
$\therefore 18x = 360^\circ$
$\therefore x = 20^\circ$
$\therefore m\angle P = 3x = 3 \times 20^\circ = 60^\circ$
$\therefore m\angle Q = 4x = 4 \times 20^\circ = 80^\circ$
$\therefore m\angle R = 5x = 5 \times 20^\circ = 100^\circ$
$\therefore m\angle S = 6x = 6 \times 20^\circ = 120^\circ$
$\therefore$ The measures of the angles of the quadrilateral are 60°, 80°, 100° and 120°.

Solution 4:

Let $\angle B = \angle C = \angle D = x$
Using the angle sum property of quadrilaterals we get,
$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$
$\therefore 120^\circ + x + x + x = 360^\circ$
$\therefore 3x = 240^\circ$
$\therefore x = 80^\circ$
$\therefore m\angle B = x = 80^\circ$
$\therefore m\angle C = x = 80^\circ$
$\therefore m\angle D = x = 80^\circ$
$\therefore$ The measures of angles $\angle B$, $\angle C$ and $\angle D$ are 80°, 80° and 80° respectively.

Solution 5:

Let the measures of all angles of a quadrilateral be x.
By using angle sum property of quadrilaterals, we get,
$x + x + x + x = 360^\circ$
$\therefore 4x = 360^\circ$
$\therefore x = 90^\circ$
$\therefore$ The measure of all the angles of the quadrilateral is 90°.

Solution 6:

The measures of the angles of a quadrilateral are 120°, 90°, 72° and x°.
Using the angle sum property of quadrilaterals, we get,
$120^\circ + 90^\circ + 72^\circ + x = 360^\circ$
$\therefore x = 360^\circ - 282^\circ$
$\therefore x = 78^\circ$
$\therefore$ The measure of x is 78°.

Solution 7:

In □KLMN, $m\angle K = 30^\circ$, $m\angle M = 150^\circ$ and $m\angle N = 110^\circ$.
By using the angle sum property of quadrilaterals we get,
$m\angle K + m\angle M + m\angle N + m\angle KLM = 360^\circ$
$\therefore 30^\circ + 150^\circ + 110^\circ + m\angle KLM = 360^\circ$
$\therefore m\angle KLM = 360^\circ - 290^\circ$
$\therefore m\angle KLM = 70^\circ$
$m\angle KLM + m\angle MLP = 180^\circ$ ….[Linear pair angles]
$\therefore 70^\circ + m\angle MLP = 180^\circ$
$\therefore m\angle MLP = 110^\circ$
$\therefore$ The measures of $\angle KLM$ and $\angle MLP$ are 70° and 110° respectively.

Solution 8:

Let □KLMN be the quadrilateral with $m\angle K = 55^\circ$,
$m\angle L = 55^\circ$, and $m\angle M = 150^\circ$.
Using the angle sum property of quadrilaterals, we get,
$m\angle K + m\angle L + m\angle M + m\angle N = 360^\circ$
$\therefore 55^\circ + 55^\circ + 150^\circ + m\angle N = 360^\circ$
$\therefore m\angle N = 360^\circ - 260^\circ$
$\therefore m\angle N = 100^\circ$
$\therefore$ The measure of fourth angle is 100°.

Solution 9:

In □PQRS,
Using the angle sum property of quadrilaterals, we get,

$m\angle P + m\angle Q + m\angle R + m\angle S = 360^\circ$

$\therefore m\angle P + m\angle Q + 110^\circ + 50^\circ = 360^\circ$

$\therefore m\angle P + m\angle Q = 360^\circ - 160^\circ$

$\therefore m\angle P + m\angle Q = 200^\circ$

Now, $\frac{1}{2}m\angle P + \frac{1}{2}m\angle Q = \frac{1}{2} \times 200^\circ$

i.e. $m\angle APQ + m\angle AQP = 100^\circ$........$(i)$

In ΔAPQ,

$m\angle APQ + m\angle AQP + m\angle PAQ = 180^\circ$

$\therefore 100^\circ + m\angle PAQ = 180^\circ$ [from (i)]

$\therefore m\angle PAQ = 80^\circ$

$\therefore$ The measure of $\angle PAQ$ is 80°.

Solution 10:

In figure (a),

Using the angle sum property of quadrilaterals, we get,

$P + 130° + 80° + 70° = 360°$

$\therefore P + 280° = 360°$

$\therefore P = 360° - 280°$

$\therefore P = 80°$

In figure (b),

Using the angle sum property of quadrilaterals, we get,

$90° + 150° + 2y + y = 360°$

$\therefore 240° + 3y = 360°$

$\therefore 3y = 120°$

$\therefore y = 40°$

$\therefore$ The value of P is 80° and y is 40°.

Solution 11:

Given that $\angle DAE = n$, $\angle CDA = p$, $\angle BCF = m$ and $\angle ABC = q$ ...(1)

$m\angle DAE + m\angle DAB = 180°$ [Linear pair angles]

$\therefore n + m\angle DAB = 180°$

$\therefore m\angle DAB = 180° - n$ ...(2)

Simillarly, $m\angle DCB = 180° - m$ ...(3)

In ▫ABCD

Using the angle sum property of quadrilaterals, we get,

$m\angle DAB + m\angle ABC + m\angle DCB + m\angle CDA = 360°$

$(180° - n) + q + (180° - m) + p = 360°$ ...$\begin{bmatrix}\text{from equations}\\ \text{(1), (2) and (3)}\end{bmatrix}$

$\therefore p + q - m - n + 360° = 360°$

$\therefore p + q - m - n = 360° - 360°$

$\therefore p + q - m - n = 0$

$\therefore p + q = m + n$

Exercise – 5.2

Solution 1:

The opposite angles of a parallelogram are congruent.
$\therefore \angle A = \angle C = x^\circ$
$\therefore \angle B = \angle D = 3x^\circ + 20^\circ$
In parallelogram ABCD
Using the angle sum property of quadrilaterals, we get,
$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$
$\therefore x^\circ + (3x^\circ + 20^\circ) + x^\circ + (3x^\circ + 20^\circ) = 360^\circ$
$\therefore 8x^\circ + 40^\circ = 360^\circ$
$\therefore 8x^\circ = 320^\circ$
$\therefore x^\circ = 40^\circ$
$\therefore m\angle C = x^\circ = 40^\circ$
$\therefore m\angle D = 3x^\circ + 20^\circ = 120^\circ + 20^\circ = 140^\circ$

Solution 2:

The ratio of two sides of a parallelogram is 3:5.
Suppose the lengths of the sides of the parallelogram are 3x, 5x, 3x and 5x.
Perimeter of the parallelogram is 48cm.
$\therefore 3x + 5x + 3x + 5x = 48$
$\therefore 16x = 48$
$\therefore x = 3$ cm
$\therefore 3x = 3 \times 3 = 9$ cm
$\therefore 5x = 5 \times 3 = 15$cm
$\therefore$The lengths of the sides of the parallelogram are 9cm, 15cm, 9cm and 15cm.

Solution 3:

Let the length of the first side of the parallelogram be x cm.
The other side of the parallelogram is greater than the first by 25cm.
So the length of other side is (25 + x) cm.
Perimeter of the parallelogram is 150cm.
$\therefore x + (25 + x) + x + (25 + x) = 150$
$\therefore 4x + 50 = 150$
$\therefore x = 25$ cm
The length of the other side
$= (25 + x)$ cm $= (25 + 25)$ cm $= 50$ cm
$\therefore$The lengths of the sides of the parallelogram are 25cm, 50cm, 25cm and 50cm.

Solution 4:

Adjacent angles of a parallelogram are in the ratio 1:2.
Let the measures of the adjacent angles of the parallelogram be x, 2x.
We know that the opposite angles of a parallelogram are congruent.
So there are two angles having measure x and two angles having measure 2x.
By angle sum property of quadrilaterals we get,
$x + 2x + x + 2x = 360^\circ$
$\therefore 6x = 360^\circ$
$\therefore x = 60^\circ$
$\therefore 2x = 120^\circ$
$\therefore$ The measures of all the angles of the parallelogram are 60°, 120°, $60^\circ$and 120°.

Solution 5:

□PQRS is parallelogram.
The opposite angles of a parallelogram are congruent.
So $\angle P \cong \angle R = 110^\circ$ and $\angle S \cong \angle Q$

□ABCR is parallelogram.
The opposite angles of a parallelogram are congruent.
So $\angle B \cong \angle R = 110^\circ$ and $\angle A \cong \angle C$

By angle sum property of quadrilaterals, we get,
$m\angle A + m\angle B + m\angle C + m\angle R = 360^\circ$
$\therefore m\angle A + 110^\circ + m\angle A + 110^\circ = 360^\circ$
$\therefore 2m\angle A + 220^\circ = 360^\circ$
$\therefore 2m\angle A = 140^\circ$
$\therefore m\angle A = 70^\circ$
$\therefore \angle C \cong \angle A = 70^\circ$

The measures of all the angles of the parallelogram ABCR are,
$m\angle A = 70^\circ, m\angle B = 110^\circ, m\angle C = 70^\circ$ and $m\angle R = 110^\circ$.

Solution 6:

□ABCD is parallelogram.

side AB || side CD and BD is a transversal

$\therefore \angle ADB = \angle CBD$ and $\angle ABD = \angle CDB$

$\therefore m\angle CBD = 40^\circ$ and $m\angle CDB = 30^\circ$

$\angle ADC = \angle ADB + \angle CDB$

$\therefore m\angle ADC = 40^\circ + 30^\circ = 70^\circ$

□ABCD is parallelogram.

$\therefore \angle ABC = \angle ADC$ and $\angle BAD = \angle DCB$

$\therefore m\angle ABC = 70^\circ$ and $\angle BAD = \angle DCB$

By angle sum property of the quadrilateral we get,

$m\angle BAD + m\angle DCB + m\angle ADC + m\angle ABC = 360^\circ$

$\therefore m\angle BAD + m\angle BAD + 70^\circ + 70^\circ = 360^\circ$

$\therefore 2\angle BAD = 360^\circ - 140^\circ$

$\therefore m\angle BAD = 110^\circ$

$\therefore m\angle BAD = m\angle DCB = 110^\circ$

The measures of the angles of parallelogram ABCD are,

$m\angle BAD = 110^\circ, m\angle DCB = 110^\circ, m\angle ADC = 70^\circ$ and $m\angle ABC = 70^\circ$.

Solution 7:

□KLMN is a parallelogram.

The opposite sides of a parallelogram are congruent.
$\therefore$ side LM = side KN = 8 units

The diagonala of a parallelogram bisect each other.
$\therefore$ KP = PM = 6 units and NP = PL = 4 units

Perimeter of ΔMPL
= LM + PM + PL
= 8 units + 6 units + 4 units
= 18 units

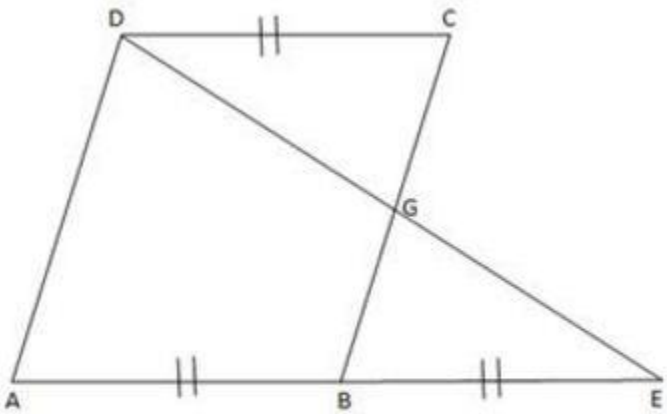
Perimeter of ΔMPL is 18 units.

Solution 8:

□WXYZ is parallelogram.
side XY || side WZ and XZ is a transversal
$\therefore \angle WXZ = \angle XZY$ and $\angle ZXY = \angle WZX$
$\therefore 10x = 60^\circ$ and $4y = 28^\circ$
$\therefore x = 6^\circ$ and $y = 7^\circ$

The values of x and y are 6° and 7° reapectively.

Solution 9:



Let G be the point of intersection of seg BC and seg DE.

Opposite sides of a parallelogram are congruent.

$\therefore$ side AB $\cong$ side DC

also side AB $\cong$ side BE

$\therefore$ side DC $\cong$ side BE ...(1)

side AB || side DC

i.e. AE || DC and seg BC is the transversal

$\therefore$ $\angle DCB = \angle CBE$

$\therefore$ $\angle DCG = \angle GBE$ ...(2)

In ΔGCD and ΔGBE

$\angle GCD = \angle GBE$ ...[from (2)]

$\angle DGC = \angle EGB$ ...[vertically opposite angles are equal]

side DC $\cong$ side BE ...[from (1)]

$\therefore$ $\Delta GCD \cong \Delta GBE$ ...(A A S test)

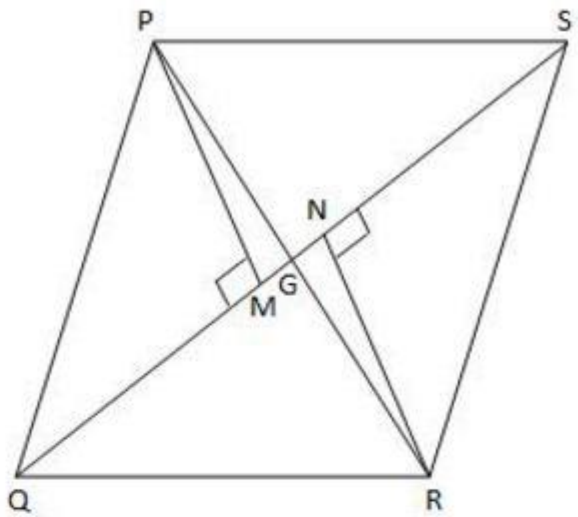
$\therefore$ seg CG $\cong$ seg BG ...(c.s.c.t.)

$\therefore$ CG = BG

$\therefore$ Point G is the midpoint of side BC.

Hence ED bisects BC.

Solution 10:



Let ▫PQRS be a parallelogram such that,
seg PM ⊥ diagonal QS and seg RN ⊥ diagonal QS
Join diagonal PR intersecting diagonal QS at point G.
G is the point of intersection of the diagonals of the parallelogram PQRS.
∴ seg PG ≅ seg RG

In ΔPGM and ΔRGN
∠PGM ≅ ∠RGN
∠PMG ≅ ∠RNG
seg PG ≅ seg RG
∴ ΔPGM ≅ ΔRGN (A A S test)
∴ seg PM ≅ seg RN (c.s.c.t.)
i.e. the opposite vertices of a parallelogram are equidistant from the diagonal not containing these vertices.

Solution 11:

Let ▱PQRS be a parallelogram such that,

side PQ = 4.8 cm and QR = $\frac{3}{2}$PQ

$\therefore$ QR = $\frac{3}{2}PQ = \frac{3}{2} \times 4.8 = 7.2$ cm

▱PQRS is a parallelogram

$\therefore$ PQ = RS and QR = PS

$\therefore$ RS = 4.8 cm and PS = 7.2 cm

Perimetre of parallelogram PQRS

= PQ + QR + RS+ PS

= 4.8cm + 7.2cm + 4.8cm + 7.2cm

= 24 units

Solution 12:

The ratio of two sides of a parallelogram is 3 : 4.

Let the two sides be 3x and 4x respectively.

The opposite sides of a parallelogram are congruent.

$\therefore$ The sides of the parallelogram are 3x, 4x, 3x and 4x.

The perimeter of the parallelogram = 112

$\therefore 3x + 4x + 3x + 4x = 112$

$\therefore 14x = 112$

$\therefore x = 8$

$\therefore 3x = 3 \times 8 = 24$

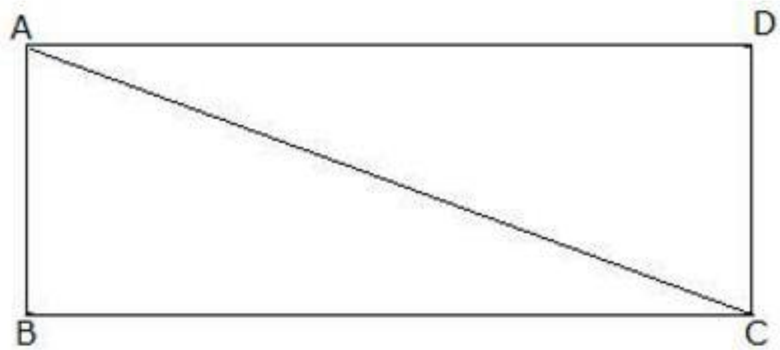
$\therefore 4x = 4 \times 8 = 32$

The lengths of the sides of the given parallelogram are

24 cm, 32 cm, 24 cm and 32 cm.

Exercise – 5.3

Solution 1:



Let ▭ABCD be a rectangle with side AB = 7 cm and BC = 24 cm.

In ΔABC, $m\angle ABC = 90^\circ$ (Angle of a rectangle)

$\therefore$ By Pythagoras' Theorem

$AC^2 = AB^2 + BC^2$

$\therefore AC^2 = (7)^2 + (24)^2$

$\therefore AC^2 = 49 + 576$

$\therefore AC^2 = 625$

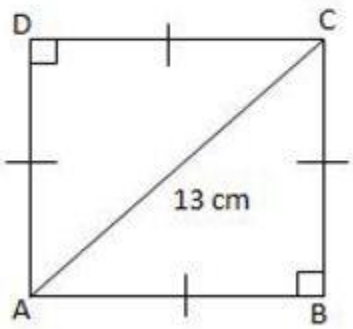
$\therefore AC = 25 cm$

Diagonals of the rectangle are congruent

$\therefore AC = BD = 25$ cm

$\therefore$ The length of the diagonals is 25 cm.

Solution 2:



Let ▫ABCD be a square with diagonal AC of length 13 cm.

▫ABCD is a square

$\therefore$ AB = BC = CD = AD

In right angle ABC,

by Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$\therefore (13)^2 = AB^2 + AB^2$

$\therefore 2AB^2 = 169$

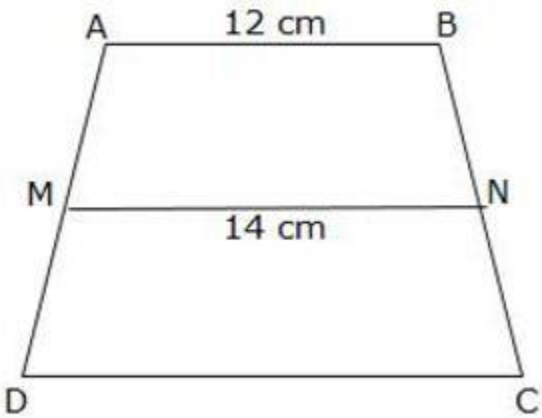
$\therefore AB = \frac{13}{\sqrt{2}}$ cm

$\therefore AB = \frac{13}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ cm

$\therefore AB = \frac{13\sqrt{2}}{2}$ cm

$\therefore$ The length of each side of the square is $\frac{13\sqrt{2}}{2}$ cm.

Solution 3:



In a trapezium, the line segment joining mid-points of the non-parallel sides is half the sum of the lengths of its parallel sides.

$\therefore MN = \frac{1}{2}(AB + CD)$

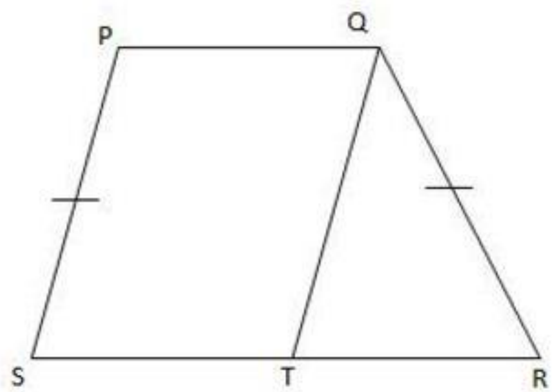
$\therefore 14 = \frac{1}{2}(12 + CD)$ [Given: MN = 14 and AB = 12]

$\therefore 28 = 12 + CD$

$\therefore CD = 16$ cm

$\therefore$ The length of CD is 16 cm.

Solution 4:



Let □PQRS be an isosceles trapezium having side PQ $\parallel$ side SR and side PS $\cong$ side QR

Construct seg QT $\parallel$ side PS

$\therefore \square$PQTS is a parallelogram.

The opposite sides and opposite angles of a parallelogram are congruent.

$\therefore$ side PS $\cong$ side QT and $\angle S \cong \angle PQT$ ...(i)

side PS $\cong$ side QR ...(Given)

$\therefore$ side QT $\cong$ side QR

$\therefore \angle QTR \cong \angle R$ (Isosceles Triangle Theorem) ...(ii)

side PQ $\cong$ side SR

side QT is a transversal

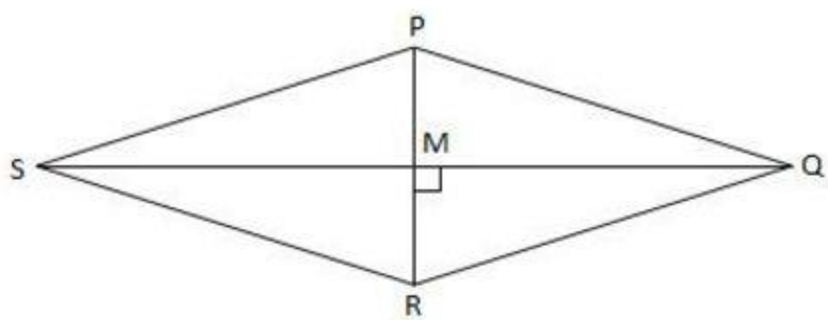
$\therefore \angle PQT \cong \angle QTR$ (Alternate angles) ...(iii)

from (i), (ii) and (iii) we get,

$\angle S \cong \angle R$

i.e. $\angle PSR \cong \angle QRS$

Solution 5:



$\square$PQRS is a rhombus having diagonals PR and QS of length 20 cm and 48 cm respectively.

Let diagonals PR and QS intersect at point M.

Diagonals of a rhombus are perpendicular bisector of each other.

$\therefore$ seg PM = seg RM = $\frac{1}{2}$PM = $\frac{1}{2} \times 20 = 10$ cm

and seg SM = seg QM = $\frac{1}{2}$SQ = $\frac{1}{2} \times 48 = 24$cm

Consider ΔPMS,

by Pythagoras' Theorem,

$PS^2 = PM^2 + SM^2$

$\therefore PS^2 = (10)^2 + (24)^2$

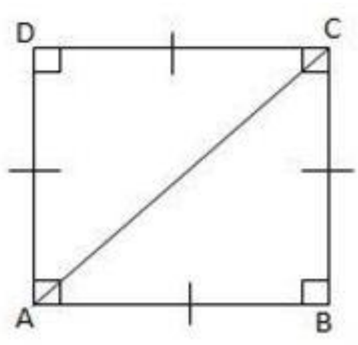
$\therefore PS^2 = 676$

$\therefore$ PS = 26cm

All the sides of the rhombus are congruent.

$\therefore$ The length of each side of the rhombus is 26 cm.

Solution 6:



□ABCD is a square.

$\therefore$ AD = DC (sides of the square) ...(i)

$m\angle ADC = 90^\circ$ (an angle of a square)

In ΔADC,

side AD = side DC [from (i)]

$\therefore \angle DAC = \angle DCA$ (angles opposite to equal sides) ...(ii)

But $m\angle DAC + m\angle DCA + m\angle ADC = 180^\circ$

(sum of the angles of a triangle)

$\therefore m\angle DCA + m\angle DCA + m\angle ADC = 180^\circ$ ...[from (ii)]

$\therefore 2m\angle DCA + 90^\circ = 180^\circ$

$\therefore 2m\angle DCA = 180^\circ - 90^\circ$

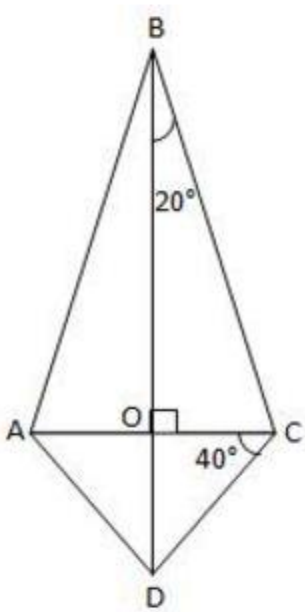
$\therefore 2m\angle DCA = 90^\circ$

$\therefore m\angle DCA = \dfrac{90^\circ}{2}$

$\therefore \angle DCA = 45^\circ$

The measure of $\angle DCA$ is 45°.

Solution 7:



▫ABCD is a kite.

$\therefore$ Diagonal AC is the perpendicular bisector of diagonal BD.

$\therefore$ OA = OC and $m\angle BOA = \angle BOC = 90^\circ$

In ΔBOA and ΔBOC

OA = OC

$m\angle BOA = m\angle BOC = 90^\circ$

BO = BO

$\therefore \Delta BOA \cong \Delta BOC$ (SAS test)

$\therefore \angle ABO = \angle CBO$ (c.a.c.t.)......(i)

$\therefore m\angle ABO = 20^\circ$

$\angle ABC = \angle ABO + \angle CBO$

$\therefore m\angle ABC = 20^\circ + 20^\circ$

$\therefore \angle ABC = 40^\circ$

As per (i) it can be proved that

$\Delta AOD \cong \Delta COD$

$\therefore \angle DAO = \angle DCO$ (c.a.c.t.)

$\therefore m\angle DAO = 40^\circ$

In ΔDAC,

$m\angle ADC + m\angle DAC + m\angle DCA = 180^\circ$

$\therefore m\angle ADC + 40^\circ + 40^\circ = 180^\circ$

$\therefore m\angle ADC = 100^\circ$

In ΔAOB,

$m\angle AOB = 90^\circ$ and $m\angle ABO = 20^\circ$

$\therefore m\angle BAO = 180^\circ - \left(90^\circ + 20^\circ\right)$

$\therefore m\angle BAO = 70^\circ$

$\angle BAD = \angle BAO + \angle DAO$

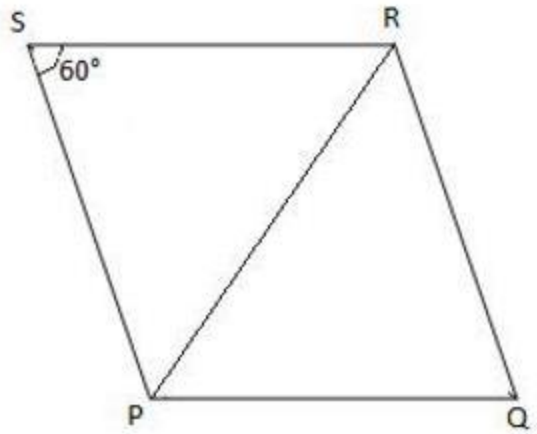
$\therefore m\angle BAD = 70^\circ + 40^\circ$

$\therefore m\angle BAD = 110^\circ$

The measures of angles are

$m\angle ABC = 40^\circ$, $m\angle ADC = 100^\circ$ and $m\angle BAD = 110^\circ$

Solution 8:

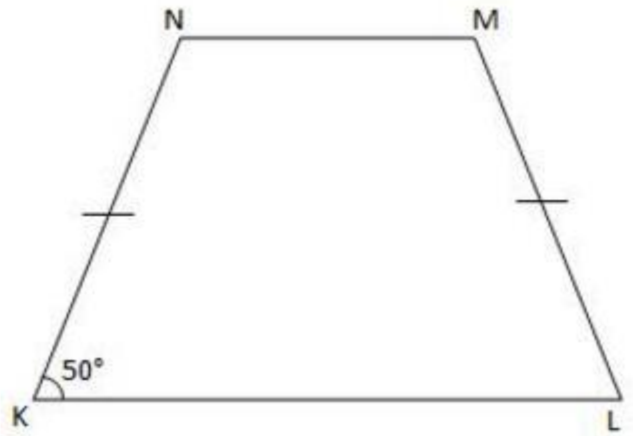


Let ▫PQRS be a rhombus with $\angle PSR = 60°$.
In ΔPSR,
$PS = SR$ .....(Sides of rhombus)
$\angle SPR = \angle SRP$....(Angles opposite to equal sides)....(i)

In ΔPSR,
$m\angle PSR + m\angle SPR + m\angle SRP = 180°$
$\therefore 2m\angle SPR + 60° = 180°$ [from (i)]
$\therefore m\angle SPR = 60° = m\angle SRP$
$\therefore \Delta PSR$ is an equilateral triangle.
Since in a parallelogram, the opposite angles are equal, we have $m\angle PQR = 60°$.
In ΔPQR,
$PQ = QR$ .....(Sides of rhombus)
$\angle QPR = \angle QRP$....(Angles opposite to equal sides)....(ii)

In ΔPQR,
$m\angle PQR + m\angle QPR + m\angle QRP = 180°$
$\therefore 2m\angle QPR + 60° = 180°$ [from (ii)]
$\therefore m\angle QPR = 60° = m\angle QRP$
$\therefore \Delta PQR$ is an equilateral triangle.

Solution 9:



□KLMN is an isosceles trapezium in which side KL $\parallel$ side NM.

The base angles of an isosceles trapezium are congruent.

$\therefore \angle K = \angle L$

$\therefore m\angle L = 50^\circ$

side NM $\parallel$ side KL and side NK is the transversal.

$\therefore m\angle K + m\angle N = 180^\circ$

$\therefore 50^\circ + m\angle N = 180^\circ$

$\therefore m\angle N = 130^\circ$

In a quadrilateral, sum of all the angles is 360°.

$\therefore m\angle K + m\angle L + m\angle M + m\angle N = 360^\circ$

$\therefore 50^\circ + 50^\circ + m\angle M + 130^\circ = 360^\circ$

$\therefore m\angle M + 230^\circ = 360^\circ$

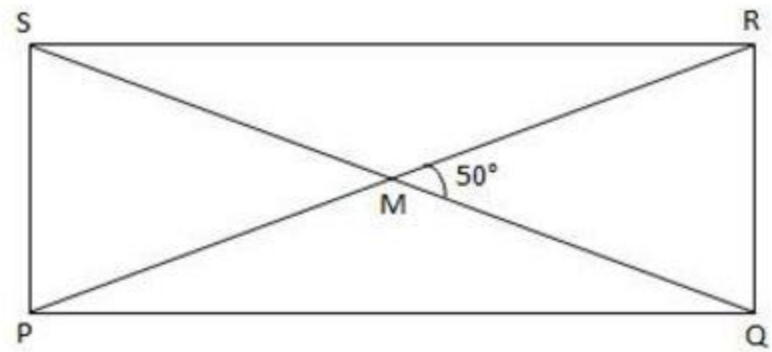
$\therefore m\angle M = 360^\circ - 230^\circ$

$\therefore m\angle M = 130^\circ$

$\therefore$ The measures of the remaining angles is,

$m\angle L = 50^\circ, m\angle N = 130^\circ$ and $m\angle M = 130^\circ$.

Solution 10:



$\angle PMS = \angle QMR$ (opposite angle)

$m\angle QMR = 50^\circ$

$\therefore m\angle PMS = 50^\circ$

The diagonals of a rectangle are congruent and bisect each other.

$\therefore MS = MP$

$\therefore \angle MPS = \angle MSP$ (angle opposite to equal sides)

In ΔMSP,

$m\angle PMS + m\angle MPS + m\angle MSP = 180^\circ$

$\therefore 50^\circ + 2m\angle MPS = 180^\circ$

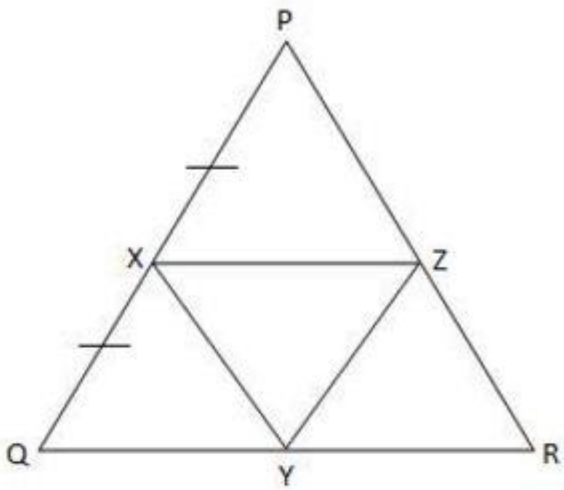
$\therefore 2m\angle MPS = 130^\circ$

$\therefore m\angle MPS = 65^\circ$

$\therefore$ The measure of $\angle MPS$ is 65°.

Exercise – 5.4

Solution 1:



In ΔPQR,

Point X is the midpoint of side PQ and seg XZ $\parallel$ side QR.

$\therefore$ By the converse of the midpoint theorem,

point Z is the midpoint of side PR .....(i)

Similarly,

Point X is the midpoint of side PQ and seg XY $\parallel$ side PR.

$\therefore$ Point Y is the midpoint of side QR. .....(ii)

From (i) and (ii), points Z and Y are the midpoints of sides PR and QR of ΔPQR respectively.

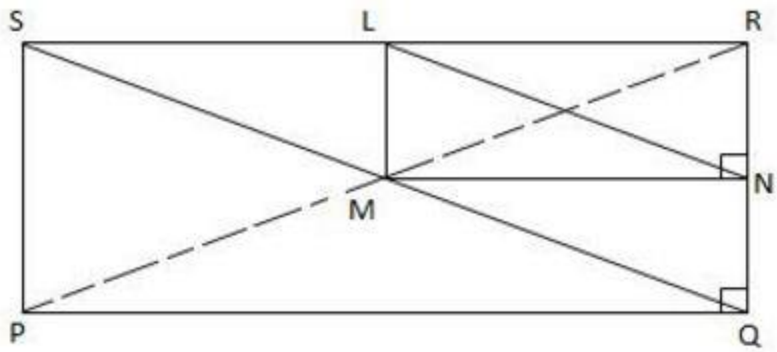
$\therefore$ By the converse of the midpoint theorem,

$ZY = \frac{1}{2} \times PQ$

$\therefore ZY = \frac{1}{2} \times 20$

$\therefore ZY = 10$ cm

Solution 2:



□LMNR is a rectangle.

$\therefore$ LM $\parallel$ RN....(Opposite sides of a rectangle)

i.e. LM $\parallel$ RQ

but RQ $\parallel$ SP....(Opposite sides of a rectangle)

$\therefore$ LM $\parallel$ RQ $\parallel$ SP

In ΔPSR, M is the midpoint of PR and LM $\parallel$ SP

$\therefore$ by the converse of the midpoint theorem,

point L is the midpoint of side SR.

$\therefore$ SL = LR

The diagonals of a rectangle are congruent.

$\therefore$ In rectangle LMNR,

seg LN $\cong$ seg MR

$\therefore$ LN = MR

Point M is the midpoint of PR

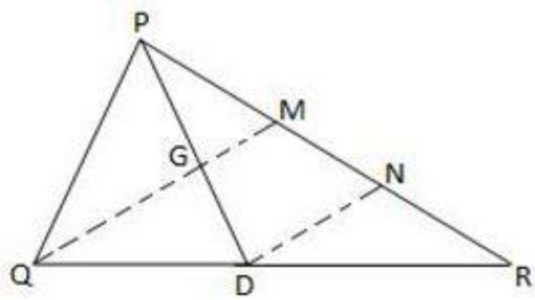
$\therefore$ RM = $\frac{1}{2}$PR

In rectangle PQRS, seg PR $\cong$ seg SQ

$\therefore$ PR = SQ

$\therefore$ LN = $\frac{1}{2}$SQ

Solution 3:



Draw seg DN $\parallel$ seg QM

In ΔPDN,

G is the midpoint of seg PD

Seg GM $\parallel$ seg DN

$\therefore$ by the converse of the midpoint theorem,

M is the midpoint of seg PN.

$\therefore$ PM = MN......(i)

In ΔQRM,

D is the midpoint of seg QR

Seg QM $\parallel$ seg DN

$\therefore$ by the converse of midpoint theorem,

N is the midpoint of seg MR.

$\therefore$ MN = NR.......(ii)

From (i) and (ii)

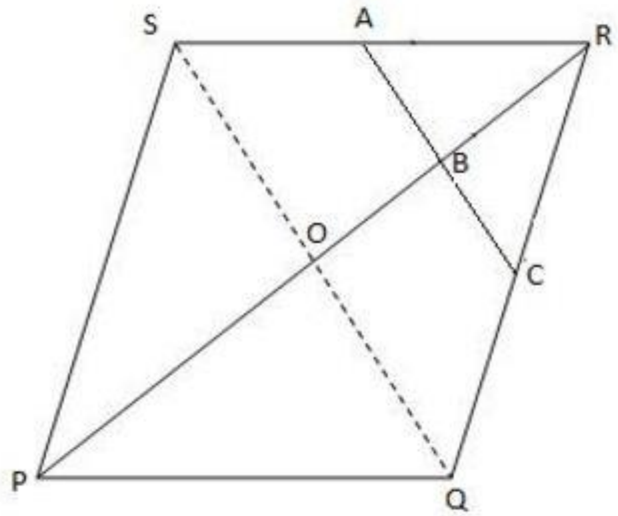
PM = MN = NR

PR = PM + MN + NR

$\therefore$ PR = 3PM

$$\therefore \frac{PM}{PR} = \frac{1}{3}$$

Solution 4:



The diagonals of a parallelogram bisect each other.

$\therefore$ RO = PO

$\therefore$ PR = 2RO.....(i)

$RB = \frac{1}{4}PR$

$\therefore$ 4RB = PR

$\therefore$ 4RB = 2RO......[from(i)]

$\therefore$ 2RB = RO

$\therefore$ point B is the midpoint of RO.......(ii)

Point A is the midpoint of side SR

By the midpoint theorem,

seg AB $\parallel$ seg SQ

In ΔRSQ,

A is the midpoint of side SR and

seg AC $\parallel$ seg SQ

$\therefore$ by the converse of the midpoint theorem,

point C is the midpoint of side QR.