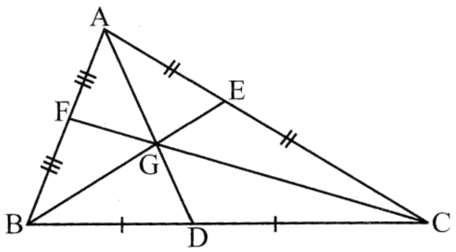
Properties of Triangle

NOTES



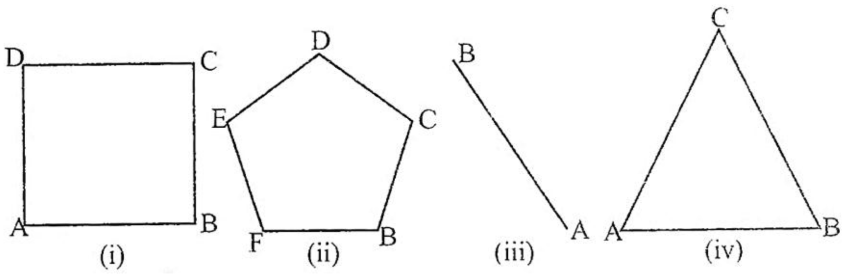
$$\frac{AG}{GD}=\frac{2}{1}$$

FUNDAMENTALS

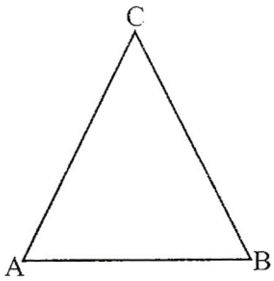
- A triangle (denoted as A delta) is a closed figure bounded by three line segments, it has three vertices, three sides and three angles. The three sides and three angles of a triangle are called its six elements.

Elementary Question -1

Identify triangle among following figures and also identify its six elements and vertices.



The figure (iv) is a triangle



Its sides are AB, BC, CA and angles are $\angle A, \angle B, \angle C$ (also written as $\angle BAC, \angle CBA$ and $\angle ACB$). These are six elements and its vertices are points A, B, C.

- A triangle is said to be

  (a) An acute angled triangle, if each one of its angles measures less than $90°$.

  (b) A right angled triangle, if any one of its angles measures $90°$.

  (c) An obtuse angled triangle, if any one of its angles measures more than $90°$.

Note: A triangle cannot have more than one right angle.

A triangle cannot have more than one obtuse angle.

In a right triangle, the sum of the acute angles is $90°$.

- **Angle sum property:** The sum of the angle of a triangle is $180°$.

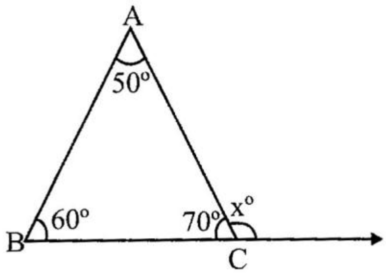
  This is such an important property that it will be used right from class VII to graduation level and even higher (post - graduation level). Hence it is very important, commit to memory, and apply wherever required.

- **Properties of sides:**

  (a) The sum of any two sides of a triangle is greater than the third side.

  (b) The difference of any two sides is less than the third side.

- **Property of exterior angles:** If a side of a triangle is produced, the exterior angle so formed is equal to the sum of interior opposite angles.



  e.g., Exterior angle,

  $x° = \angle A + \angle B = 50° + 60° = 110°$

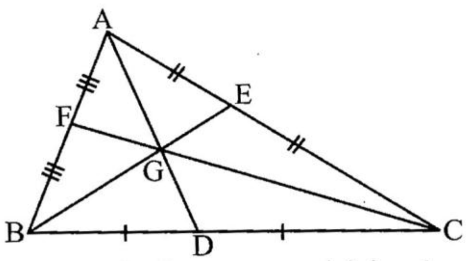
- A triangle is said to be

  (a) An equilateral triangle, If all of its sides are equal.

  (b) An isosceles triangle, if any two of its sides are equal.

  (c) A scalene triangle, if all of its sides are of different lengths.

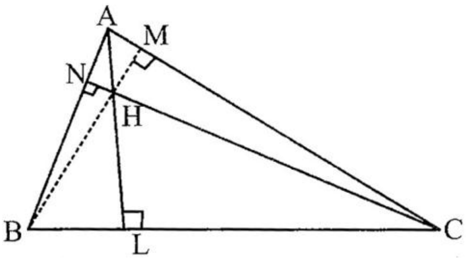
Important terms; Medians & centroid, altitudes & orthocenter:



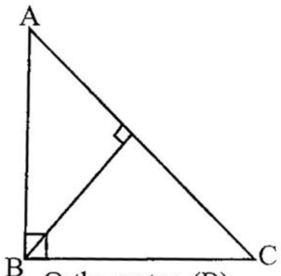
- The medians of a triangle are the line segments joining the vertices of the triangle to the midpoints of the opposite sides.
- Here AD, BE and CF are medians of ΔABC
- The medians of a triangle are concurrent.
- The centroid of a triangle is the point of concurrence of its medians.
- The centroid is denoted by G.
- The centroid of a triangle divides the medians in the ratio $2:1$.

- The centroid of a triangle always lies in the interior of the triangle.
- The medians of an equilateral triangle are equal.
- The medians to the equal sides of an isosceles triangle are equal.
- Altitudes of triangle are the perpendiculars drawn from the vertices of a triangle to the opposite sides.

  Here AL, BM and CN are the altitudes of ΔABC.



- The altitudes of a triangle are concurrent, they meet at the same point and point of meeting is called orthocenter.
- Thus, orthocenter is the point of concurrence of the altitudes of a triangle. Orthocenter is denoted by H.
- The orthocenter of an acute angled triangle lies in the interior of the triangle.
- The orthocenter of a right angled triangle is the vertex containing the right angle.



Orthocenter (B)

- The orthocenter of an obtuse angled triangle lies in the exterior of the triangle.

Properties:

(a) The altitudes drawn on equal sides of an isosceles triangle are equal.

(b) The altitude bisects the base of an isosceles triangle.

(c) The altitudes of an equilateral triangle are equal.

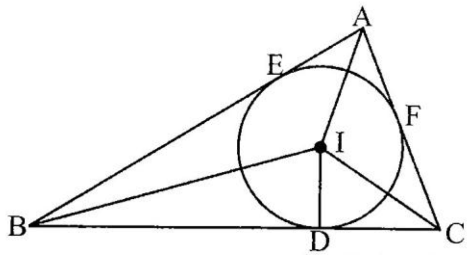
(d) The centroid of an equilateral triangle coincides with its orthocenter.

Exercise for the students

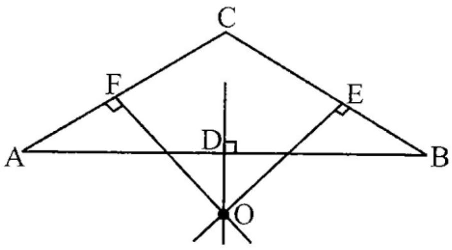
You should practice each of these properties by drawing roughly appropriate Ale and drawing altitudes in them.

In center: Draw angle bisectors of a triangle as shown they meet at point 'I', called in centre.

From I, draw a perpendicular to line BC so that $ID \perp BC$. Taking ID as radius we can draw an in circle DEF. Hence, it is called in centre and $ID = IE = IF$ (where $IE \perp AB,\ IF \perp AC$) which are called radii's of in circle.

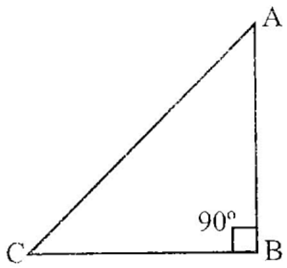


Circumcentre: Draw perpendicular from midpoints D, E, F lying on sides AB, BC and CA respectively. Let them meet at 0, which is called circumcentre. This ensures that $OA = OB = OE$ If we draw a circle with radius $OA = OB = OC,$ then we get a circle touching vertices A, B and C of triangle. Hence, O is given the name -circum centre of ΔABC because it circumscribes ΔABC .



Properties of Right - angled

- In a right angled triangle, the side opposite to the right angle is called the hypotenuse and the other two sides are known as it legs.
- '**Pythagoras**' **Theorem:** In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the remaining two sides.
  In the right angled triangle
  $ABC,\ AC^2 = AB^2 + BC^2.$



- In a right angled triangle, the hypotenuse is the longest side.