UPSEE 2019

Solved Paper

Question 1

Physics

A wire has a mass (0.1 ± 0.001)g, radius (0.5 pm 0.005)mm and length (10 ± 0.1)cm.
The maximum percentage error in the measurement of its density is

Options:

A. 1%

B. 2%

C. 3%

D. 4%

Answer: D

Solution:

Solution:
Here, mass of wire, m = (0.1 ± 0.001)g
Radius, r = (0.5 ± 0.005)mm and length,
L = (10 ± 0.1)cm
As density of a wire,

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{\text{mass}}{\text{area} \times \text{length}}$$

$$\Rightarrow \rho = \frac{m}{\pi R^2 \times L}$$

So relative error in density,

$$\pm\frac{\Delta\rho}{\rho} = \pm\left[\frac{\Delta m}{m} + 2\frac{\Delta R}{R} + \frac{\Delta L}{L}\right]$$

$$\Rightarrow\frac{\Delta\rho}{\rho} = \frac{0.001 \times 10^{-3}}{0.1 \times 10^{-3}} + 2 \times \frac{0.005 \times 10^{-3}}{0.5 \times 10^{-3}} + \frac{0.1 \times 10^{-2}}{10 \times 10^{-2}}$$

$$\Rightarrow\frac{\Delta\rho}{\rho} = 0.01 + 0.02 + 0.01$$

$$\Rightarrow\frac{\Delta\rho}{\rho} = 0.04$$

Maximum percentage error $= \frac{\Delta\rho}{\rho} \times 100 = 0.04 \times 100 = 4\%$
Hence, the maximum percentage error in the measurement of density is 4%.

Question 2

A body slides down a frictionless inclined plane starting from rest. If s_n and s_{n+1} be the distance travelled by the body during n th and (n + 1) th seconds, then the ratio $\frac{s_{n+1}}{s_n}$ is

Options:

A. $\frac{2n-1}{2n+1}$

B. $\frac{2n}{2n+1}$

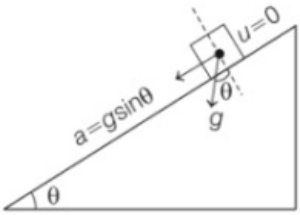
C. $\frac{2n+1}{2n-1}$

D. $\frac{2n}{2n-1}$

Answer: C

Solution:

Solution:
Key Idea For a uniformly accelerated body, the displacement in nth second is given by the expression,
$s_n = u + \frac{1}{2}a(2n-1)$. A body placed on a smooth inclined plane is shown in figure below,



Distance travelled in nth second $s_n = u + \frac{1}{2}(g\sin\theta)(2n-1)$
Here, initial velocity, $u = 0$
$\Rightarrow s_n = \frac{g\sin\theta}{2}(2n-1)$ .....(i)
Similarly, in $(n+1)$th second
$s_{(n+1)} = \frac{g\sin\theta}{2}[2(n+1)-1]$
$\Rightarrow s_{(n+1)} = \frac{g\sin\theta}{2}(2n+1)$ .....(ii)
From Eqs. (i) and (ii), we get the ratio of distances,
$\frac{s_{(n+1)}}{s_n} = \frac{2n+1}{2n-1}$
$\therefore$ The correct option is (c).

Question 3

The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x and y are respectively the horizontal and vertical distances of the projectile from the point of projection. The maximum height attended by projectile is

Options:

A. $\frac{\alpha}{\beta}$

B. $\frac{\alpha^2}{4\beta}$

C. $\frac{\beta}{\alpha}$

D. $\frac{\beta}{2\alpha}$

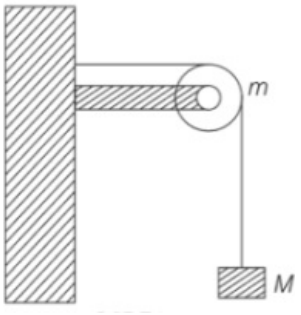
Answer: B

Solution:

Solution:
Here, the trajectory of a projectile motion,
$y = \alpha x - \beta x^2$ .....(i)
As, general equation of trajectory of a projectile motion,
$y = \tan\theta \, . \, x - \frac{g}{2(u\cos\theta)^2} \, . \, x^2$ .....(ii)
Now, comparing Eqs. (i) and (ii), we get
$\tan\theta = \alpha \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = \alpha^2$ .....(iii)
and$\beta = \frac{g}{2u^2\cos^2\theta} \Rightarrow 4\beta = \frac{2g}{u^2\cos^2\theta}$ .....(iv)
Maximum height of projectile,
$H_{max} = \frac{u^2\sin^2\theta}{2g}$
From Eqs. (iii) and (iv), we get
$\frac{\alpha^2}{4\beta} = \frac{u^2\cos^2\theta \times \sin^2\theta}{2g\cos^2\theta} = H_{max}$
Hence, the maximum height is $\frac{\alpha^2}{4\beta}$.

Question 4

A string of negligible mass passing over a clamped pulley of mass m supported a body of mass M as shown in the figure. The force exerted by the clamp on the pulley is



Options:

A. $g\sqrt{(M + m)^2 + M^2}$

B. $\sqrt{(M + m)^2 + m^2}g$

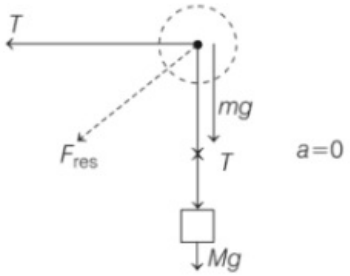
C. $\sqrt{2}Mg$

D. $\sqrt{2}mg$

Answer: A

Solution:

Solution:
A free body diagram of a mass pully system is shown below



Now, tension in the string
$T - Mg = 0$
$\Rightarrow T = Mg$
Force in vertical direction,
$F_1 = T + mg$
$\Rightarrow F_1 = Mg + mg = (M + m)g$
Similarly, force in horizontal direction
$F_2 = T = Mg$
Now, the resultant force
$F_{res} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90°}$
$\Rightarrow F_{res} = \sqrt{F_1^2 + F_2^2} = \sqrt{(M + m)^2g^2 + M^2g^2}$
$\Rightarrow F_{res} = g\sqrt{(M + m)^2 + M^2}$
Hence, the resultant force is $g\sqrt{(M + m)^2 + M^2}$.

Question 5

The linear momentum of a particle moving in X-Y plane under the influence of a force is given as P(t) = $A\left(\hat{i}\cos bt - \hat{j}\sin bt\right)$ where A and b are constants. The angle between the force and momentum is

Options:

A. 0°

B. 45°

C. 60°

D. 90°

Answer: D

Solution:

Solution:
Here, linear momentum of a particle
$p = A\left(\hat{i}\cos bt - \hat{j}\sin bt\right)$ .....(i)
where, A and B are constants.
As we know that,
$F = \frac{dp}{dt}$
From Eq. (i), we get
$\Rightarrow F = \frac{d}{dt}\left(A\hat{i}\cos bt - A\hat{j}\sin bt\right)$
$\Rightarrow F = -A\hat{i}b\sin bt - A\hat{j}b\cos bt$
Now, dot product $= F \cdot p$
$= \left(-A\hat{i}b\sin bt - A\hat{j}b\cos bt\right).\left(A\hat{i}\cos bt - A\hat{j}\sin bt\right)$
$= -Ab\sin bt\cos bt + Ab\sin bt\cos bt$
$= 0$
Since, $F \cdot p = Fp\cos\theta = 0$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°$
Hence, the angle between momentum p and force F is $90°$.

Question 6

According to Keplar's second law, the line joining the planet to sun sweeps out equal area in equal intervals of time. It is a consequence of law of conservation of

Options:

A. linear momentum

B. energy

C. angular momentum

D. All of these

Answer: C

Solution:

Solution:
Earth and all other planets move around the sun under the effect of gravitational force. This force always acts along the line joining the centre of the planet and the sun and is directed towards the sun. In other words, a planet moves around the sun under the effect of a purely radial force. Therefore, the areal velocity of the planet must always remains constant.
$\therefore \frac{\Delta A}{\Delta t} = \frac{L}{2m} =$ constant vector
Therefore, Kepler's 2 nd law is the consequence of the principle of conservation of angular momentum (L).

Question 7

A particle moves in a straight line with its retardation proportional to its displacement. The loss of its kinetic energy for any displacement x is proportional to

Options:

A. $\frac{1}{x}$

B. x

C. x^2

D. e^x

Answer: C

Solution:

Solution:
Given, retardation $a = -kx$

Here, x is the displacement of the particle.

As we know that, $a = \frac{dv}{dt} = \frac{dv}{dx} . \frac{dx}{dt} = -kx$

So, $v . \frac{dv}{dx} = -kx \left[\because \frac{dx}{dt} = v \right]$

Let for any displacement from 0 to x, velocity changes from v_0 to v.

$\Rightarrow \int_{v_0}^{v} v dv = -k \int_{0}^{x} x dx$

$\Rightarrow \left[\frac{v^2}{2} \right]_{v_0}^{v} = -\frac{kx^2}{2}$

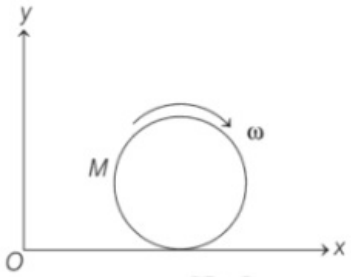
$\Rightarrow \frac{m(v^2 - {v_0}^2)}{2} = -\frac{mkx^2}{2}$

$\Rightarrow \Delta KE \propto x^2$

Hence, the correct option is (c).

Question 8

A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in figure. The magnitude of the angular momentum of the disc about the origin O is



Options:

A. $\frac{1}{2} M R^2 \omega$

B. $\frac{3}{2} M R^2 \omega$

C. $M R^2 \omega$

D. $2M R^2 \omega$

Answer: B

Solution:

Solution:

Here, disc has two types of motion namely translational and rotational. Hence, angular moment of a rigid body in translational cum rotational

= angular momentum of centre of mass about origin (O)+ angular momentum of the body about centre of mass.

$\Rightarrow L = M R^2 \omega + I . \omega$

Here, $I = \frac{M R^2}{2}$ (for disc) and ω = angular velocity of the rigid body

$= M R^2 \omega + \left(\frac{M R^2}{2} \right) \omega$

$= \frac{3}{2} M R^2 \omega$

Question 9

The time period of a simple pendulum is T . If its point of suspension is moved upward according to relation $y = \lambda t^2$ where, λ is a constant, then its new time period T′

Options:

A. is equal to T

B. is greater than T

C. is less than T

D. is infinity

Answer: C

Solution:

Solution:
According to the question, we draw the following situation

As, the point of suspension is moving upward according to the relation
$y = \lambda t^2$
$\frac{dy}{dt} = 2\lambda t$
Further, $\frac{d^2y}{dt^2} = 2\lambda =$ upward acceleration a
Here, 2λ is the acceleration of point of suspension. The bob experiences a downward reaction force.
Hence, the new time period
$T' = 2\pi \sqrt{\frac{L}{g+a}}$ .....(i)
Whereas the time period of simple pendulum,
$T = 2\pi \sqrt{\frac{L}{g}}$ .....(ii)
By comparing Eqs. (i) and (ii), we get
$T' < T$
Hence, the new time period is less than T .

Question 10

Surface tension of a liquid with increase in its temperature

Options:

A. increases

B. decreases

C. remains the same

D. first increases and then decreases

Answer: B

Solution:

Solution:
The surface tension of liquid decreases with rise in temperature and increase with decrease temperature because cohesive force decrease with increase of molecular thermal activity. Hence, the correct option is (b).

Question 11

The dimensional formula of Reynold's number is same as

Options:

A. coefficient of viscosity

B. coefficient of friction

C. universal gravitational constant

D. velocity of light

Answer: B

Solution:

Solution:
As, the formula of Reynold's number
$R_e = \frac{V_c \rho D}{\eta}$
So, the dimension of
$R_e = \frac{[M^0 L T^{-1}][M L^{-3} T^0][M^0 L T^0]}{[M L^{-1} T^{-1}]}$
$\Rightarrow [R_e] = [M^0 L^0 T^0]$
Hence, it is a dimensionless quantity.
Whereas dimension of
(i) Coefficient of viscosity $= [M L^{-1} T^{-1}]$
(ii) Coefficient of friction $= [M^0 L^0 T^0]$
(iii) Universal gravitational constant $= [M^{-1} L^3 T^{-2}]$
(iv) Velocity of light $= [M^0 L T^{-1}]$
Hence, the dimensional formula of Reynold's number is same as coefficient of friction.

Question 12

A ball falling in a lake of depth 200m shows 0.1% decrease in its volume at the bottom. The bulk modulus of the material of the ball is

Options:

A. $1.96 \times 10^9 N / m^2$

B. $1.96 \times 10^{11} N / m^2$

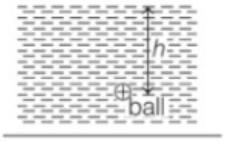
C. $1.96 \times 10^{-9} N / m^2$

D. $1.96 \times 10^{-7} \mathrm{N} / \mathrm{m}^2$

Answer: A

Solution:

Solution:
A ball at a depth of 200m in a tank is shown in figure below



Here, depth h = 200m and percentage decrease in volume = 0.1%
As, pressure at depth h is given by
$p = \rho gh$
where, $g = 9.8 \mathrm{ms}^{-2}$
and $\rho = 10^3 \mathrm{kgm}^{-3}$
So, $p = 10^3 \times 9.8 \times 200$
$\Rightarrow p = 19.6 \times 10^5 \mathrm{N} / \mathrm{m}^2$
Now, the bulk modulus

$$k = \frac{p}{\frac{\Delta V}{V}} \Rightarrow k = \frac{19.6 \times 10^5}{\frac{0.1}{100}}$$

$= 1.96 \times 10^9 \mathrm{N\,m}^{-2}$
Hence, the bulk modulus of the material of ball is $1.96 \times 10^9 \mathrm{N\,m}^{-2}$

Question 13

One mole of a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$. The value of γ for the mixture is

Options:

A. 1.40

B. 1.50

C. 1.53

D. 3.0

Answer: B

Solution:

Solution:
Here, the ratio of specific heat of one mole of monoatomic gas, $\gamma_1 = \frac{5}{3}$ and one mole of diatomic gas, $\gamma_2 = \frac{7}{5}$
As we know, C_V for a mixture

$$C_{V_{mix}} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

and $C_{P_{mix}} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2}$
Now,

$\gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$ .....(i)

As, $\gamma = \frac{C_P}{C_V}$ and $C_P - C_V = R$

So, for $\gamma_1 = \frac{5}{3}$ we get, $C_{P_1} = \frac{5}{2}R$ and

$C_{V_1} = \frac{3}{2}R$

Similarly, for $\gamma_2 = \frac{7}{5}$ we get, $C_{P_2} = \frac{7}{2}R$ and $C_{V_2} = \frac{5}{2}R$

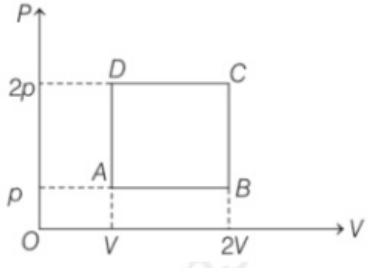
Hence, from Eq. (i), we get

$\lambda_{mix} = \frac{1 \times \frac{5}{2} + 1 \times \frac{7}{2}}{1 \times \frac{3}{2} + 1 \times \frac{5}{2}} = \frac{5+7}{3+5} = \frac{12}{8} = 1.50$

Hence, the ratio of specific heat of mixture is 1.50.

Question 14

An ideal monoatomic gas is taken round the cycle ABCDA as shown in p-V diagram. The work done during the cycle is



Options:

A. pV

B. 2pV

C. $\frac{pV}{2}$

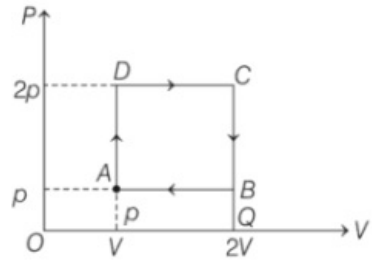
D. 0

Answer: A

Solution:

Solution:

The work done during the process from D to C = area of DCQP, which is in shown figure below



$\Rightarrow$ Area DCQP = $2p \times (2V - V) = 2pV$

Work done during B to A = − area ABQP

Where, negative sign is due to compression along BA.

$\Rightarrow$ area ABQP = $-p \times (2V - V) = -pV$

Hence, the work done in the complete cycle.

$= 2pV - pV = pV$

$\therefore$ Correct option is (a).

Question 15

Two stars emit maximum radiation at wavelengths 4000Å and 6000Å respectively. The ratio of their temperatures is

Options:

A. 1 : 2

B. 2 : 1

C. 2 : 3

D. 3 : 2

Answer: D

Solution:

Solution:
Given, wavelength $\lambda_1 = 4000Å$ and $\lambda_2 = 6000Å$
As, Wien's displacement law, $\lambda_m = \frac{b}{T}$
where, $b =$ Wien's constant and $T =$ temperature
So, $\lambda_m \propto \frac{1}{T}$
Hence, $\frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1}$
So, putting the given values, we get
$\frac{T_1}{T_2} = \frac{\lambda_2}{\lambda_1} = \frac{6000Å}{4000Å} \Rightarrow \frac{T_1}{T_2} = \frac{3}{2}$
Hence, ratio of temperature is 3 : 2.

Question 16

A carnot engine takes 300 calories of heat at 500K and rejects 150 calories of heat to the sink. The temperature of the sink is

Options:

A. 1000K

B. 750K

C. 500K

D. 250K

Answer: D

Solution:

Solution:
Key-Idea The efficiency of a carnot engine is given as

$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{Q_2}{Q_1}$
Where, T_1 = source temperature, T_2 = sink temperature Q_1 = heat extracted from source and Q_2 = heat given to sink.
Here, $T_1 = 500K$, $Q_1 = 300cal$ and $Q_2 = 150cal$
Now putting these values in above equation,
$1 - \frac{T_2}{500} = 1 - \frac{150}{300}$
$\Rightarrow T_2 = 500 \times \frac{150}{300} = 250K$
Hence, the temperature of sink is $250K$.

Question 17

The rate of heat transfer is maximum in

Options:

A. conduction

B. convection

C. radiation

D. None of these

Answer: C

Solution:

Solution:
Radiation is considered the fastest mode of the heat transfer as it travels as fast as light.
Whereas conduction is the slowest mode of heat transfer as it moves from particle to particle.
Hence, the correct option is (c).

Question 18

The latent heat of ice is 80cal / g. The change in entropy when 10g of ice at 0°C is converted into water of same temperature is

Options:

A. 0.293cal / K

B. 2.93cal / K

C. 80cal / K

D. 8cal / K

Answer: B

Solution:

Solution:

Key-Idea The change in entropy of a system is given as $\Delta S = \frac{\Delta Q}{T}$
where, T = temperature and ΔQ = heat absorbed by the system.
Here, latent heat of ice, $L = 80\text{cal / g}$, mass of water, $m = 10\text{g}$ and temperature,
$T = 0°C = 273K$
Heat given to 10g ice,
$E = mL = 10 \times 80$
$\Rightarrow E = 800\text{cal}$
So, the entropy change
$\Delta s = \frac{E}{T} = \frac{800}{273}$
$\Rightarrow \Delta s = 2.93\text{cal / K}$
Hence, the correct option is (b).

Question 19

A police car with a siren of frequency 8kHz is moving with uniform velocity of 20m / s towards a tall building which reflects the sound waves. If speed of sound in air be 320m / s, the frequency of siren heard by car driver is

Options:

A. 7.1kHz

B. 8.5kHz

C. 9.1kHz

D. 10.1kHz

Answer: C

Solution:

Solution:
Key-Idea If source and observer are moving towards a wall, then apparent frequency after the reflection from wall is same as the apparent frequency by the source and observer moving towards each other.
Here, speed of sound, $v_s = 320\text{m / s}$, actual frequency, $v = 8000\text{Hz}$ and speed of source $v_s = 20\text{m / s}$
Since, both the observer and source are moving with same speed.
$\Rightarrow v_s = 20\text{m / s}$
Apparent frequency,
$v' = \left(\frac{v + v_0}{v - v_s}\right) v$
$\Rightarrow v' = \left(\frac{320 + 20}{320 - 20}\right) \times 8000$
$= 9066.66 \approx 9.1\text{kHz}$
Hence, the frequency of siren heard by car driver is 9.1kHz

Question 20

The phase difference between two waves $x_1 = A\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x_2 = A\cos\omega t$ is

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. π

Answer: B

Solution:

Solution:
Here, the equation of two waves
$x_1 = A\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x_2 = A\cos(\omega t)$
$\because \sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$
Hence, $x_2 = A\sin\left(\omega t + \frac{\pi}{2}\right)$
Now, the phase difference
$\phi = \left(\omega t + \frac{\pi}{2}\right) - \left(\omega t + \frac{\pi}{6}\right)$
$\Rightarrow \phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}\,\text{rad}$
Hence, the phase difference between the waves is $\frac{\pi}{3}\,\text{rad}$.

Question 21

The velocity of sound waves in a medium does not depend on

Options:

A. temperature

B. pressure

C. humidity

D. direction of air

Answer: B

Solution:

Solution:
Speed of sound does not depend on pressure, since at the constant temperature, $= \frac{\text{pressure}}{\text{density of medium}} = \text{constant}$
Whereas speed of sound depends on the variation to temperature, humadity and flow of the medium.

Question 22

The number of beats heard per second, by three sound sources of equal

intensities and frequencies of 300,301 and 302H z, is

Options:

A. 4

B. 3

C. 2

D. 1

Answer: C

Solution:

Solution:
Here, $f_1 = 300\text{H z}$, $f_2 = 301\text{H z}$ and $f_3 = 302\text{H z}$
Now, beats heard in 1s,
$B_{12} = f_2 - f_1 = 301 - 300 = 1$
$B_{31} = f_3 - f_1 = 302 - 300 = 2$ and
$B_{23} = f_3 - f_2 = 302 - 301 = 1$
Hence, the maximum beats heard in 1s is 2 .

Question 23

A steady current flows through a metallic conductor of non-uniform area of cross-section. Along the length of conductor

Options:

A. only current is constant

B. only drift speed is constant

C. both current and drift speed are constant

D. neither current nor drift speed is constant

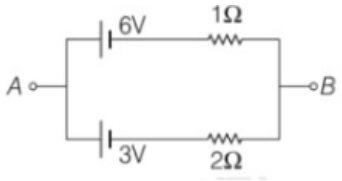
Answer: A

Solution:

Solution:
As we know that the current entering in a conductor is equal to the current leaving the conductor. Which is based on the conservation of the charge.
As, drift velocity of electrons,
$v_d = \frac{eE\tau}{m} \Rightarrow v_d \propto E$
Where, $e =$ charge of electron, $\tau =$ relaxation time and $m =$ mass of electron and $E =$ electric field inside a conductor.
Since, drift velocity depends on the electric field inside a conductor, so it varies with change in cross-section of conductor.
Hence, only current is constant in such conductor.

Question 24

Two batteries of emf 6V and 3V with internal resistances 1Ω and 2Ω respectively are connected as shown in figure. The potential difference across A and B is



Options:

A. 9V

B. 5V

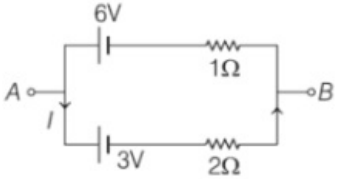
C. 3V

D. 1V

Answer: B

Solution:

Solution:
According to the question, we draw the circuit as below



Let the current in the circuit is I ,
Now applying KVL rule,
$6 - 1I - 2I - 3 = 0$
$\Rightarrow 6 - 3 = 3I$
$\Rightarrow I = \frac{3}{3} = 1A$
As V_{AB} is the potential across battery 6V and resistance of 1Ω.
So, $V_{AB} = 6 - 1I$
$\Rightarrow V_{AB} = 6 - 1 = 5V$
Hence, the potential difference across A and B is 5 V.

Question 25

A parallel plate capacitor is charged and the charging battery is disconnected. A dielectric slab is now introduced between the plates to the capacitor. Which of the following is correct?

Options:

A. Potential difference across the capacitor remains constant

B. Capacitance of the capacitor remains constant

C. Energy associated with the capacitor increases

D. Energy associated with the capacitor decreases

Answer: C

Solution:

Solution:
Initially, a capacitor of capacitance C is connected to a battery of potential V, then the charge stored
$Q = \frac{C}{V}$
If battery is removed then stored charge remains constant. i.e.
$Q =$ constant ($\because$ conservation of charge)
As, new capacitance of the capacitor
$C' = \varepsilon_r \left(\frac{\varepsilon_0 A}{d} \right)$
$\Rightarrow C' = \varepsilon_r C \left[\because C = \frac{\varepsilon_0 A}{d} \right]$
Which shows that capacitance is increased.
Potential across new capacitor,
$V' = \frac{C'}{Q} = \varepsilon_r \frac{C}{Q} = \varepsilon_r V \left[\because V = \frac{C}{Q} \right]$
So, potential difference across capacitor increased.
Energy stored in new capacitor and old capacitor.
$E = \frac{1}{2} C V^2$ and
$E' = \frac{1}{2} C' V'^2 = \frac{1}{2} (\varepsilon_r C)(\varepsilon_r V)^2$
$\Rightarrow E' = \frac{1}{2} \varepsilon_r^3 C V^2$
Hence, the energy associated with capacitor is increased.

Question 26

An electric bulb is rated as $200V - 1000W$. The powers consumed by the bulb when operated at $100V$ is

Options:

A. 25W

B. 50W

C. 75W

D. 100W

Answer: A

Solution:

Solution:
Key-Idea Power consumed by a electric device is given as
$P = \frac{V^2}{R}$
where, $V =$ potential across device and
$R =$ resistance of device
Here, $V_1 = 200V$
$P_1 = 100W$
and $V_2 = 100V$
Resistance of bulb, $R_1 = \frac{V_1^2}{P_1}$
$\Rightarrow R_1 = \frac{(200)^2}{100} = 400\Omega$
Hence, the power consumed by bulb

$P_2 = \frac{V_2^2}{R_2} [\because R_1 = R_2]$

$\Rightarrow P_2 = \frac{(100)^2}{400} = 25W$

Question 27

Displacement current is caused due to

Options:

A. a time varying electric field

B. a constant electric field

C. free electrons flow

D. All of the above

Answer: A

Solution:

Solution:
As, displacement current,
$I_d = \varepsilon_0 \frac{d\phi}{dt}$ .....(i)
Where, ϕ = electric flux
$\because \phi = E \cdot A$
For area, A = constant
$\therefore d\phi = A dE$ .....(ii)
From Eqs. (i) and (ii), we get $I_d = \varepsilon_0 A \frac{dE}{dt}$
So, time varying electric field generates the displacement current.
∴ Option (a) is correct.

Question 28

The dipole moment of a dipole formed by a proton and electron at a distance of 1nm is

Options:

A. $1.6 \times 10^{-19} Cm$

B. $1.6 \times 10^{-25} Cm$

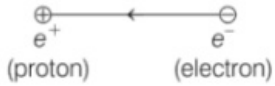
C. $1.6 \times 10^{-28} Cm$

D. $1.6 \times 10^{-29} Cm$

Answer: C

Solution:

Solution:
According to the question, we draw the following situation



Hence, the dipole moment $p = q \cdot d$
Here, $q = 1.6 \times 10^{-19}$ and $d = 10^{-9}$m
$\Rightarrow p = 1.6 \times 10^{-19} \times 10^{-9}$
$\Rightarrow p = 1.6 \times 10^{-28}$Cm

Question 29

The resistance of a platinum wire is 100Ω at 0°C. If its temperature coefficient of resistance is 0.0045 / °C, then its resistance at 60°C temperature will be

Options:

A. 127Ω

B. 73Ω

C. 370Ω

D. 2800Ω

Answer: A

Solution:

Solution:
Given, $R_0 = 100\Omega$, $T_1 = 0°C$, $T_2 = 60°C$ and temperature coefficient of resistance, $\alpha = 0.0045°C^{-1}$
As, resistance of a metal at the temperature T is given by
$R_t = R_0[1 + \alpha(T_2 - T_1)]$
$\Rightarrow R_t = 100[1 + 0.0045(60° - 0°)]$
$\Rightarrow R_t = 100[1 + 0.27] \Rightarrow R_t = 127\Omega$
So, the resistance of platinum wire at 60°C is 127Ω.

Question 30

The temperature, above which a ferromagnetic material becomes paramagnetic, is called

Options:

A. critical temperature

B. neutral temperature

C. temperature of inversion

D. Curie temperature

Answer: D

Solution:

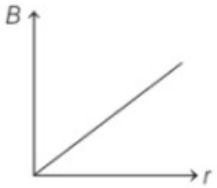
Solution:
The Curie temperature of a ferromagnetic material is the temperature above which it losses its characteristic ferromagnetic ability to possess a net magnetisation in the absence of an external magnetic field. Hence, above Curie temperature material is purely paramagnetic.

Question 31

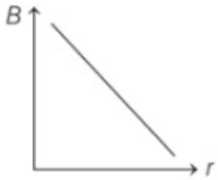
Which of the following graphs shows that the variation of magnetic induction B with distance r from a current carrying long wire?

Options:

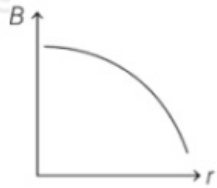
A.



B.



C.



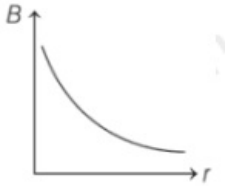
D.

Answer: D

Solution:

Solution:

As magnetic field due to a current carrying straight wire of infinite length $B = \frac{\mu_0 i}{2\pi}$, $\frac{1}{r} \Rightarrow B \propto \frac{1}{r}$So, the graph between magnetic induction B and distance r from the conductor is rectangular hyperbola.



∴ Hence, the correct option is (d).

Question 32

An electron having charge e is moving with a constant speed v along a circle of radius r. Its magnetic moment will be

Options:

A. evr

B. $\frac{evr}{2}$

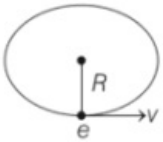
C. 2π rev

D. zero

Answer: B

Solution:

Solution:
An electron moving in a circular motion is shown in figure below



Current due to circular motion

$i = \frac{e}{t}$, where t = time period

So, the magnetic moment of electron

$m = i \cdot A = \frac{e}{T} \cdot \pi r^2$

$\Rightarrow m = er^2\frac{2\pi}{2T} = \frac{e\omega r^2}{2}\left[\because \frac{2\pi}{t} = \omega\right]$

$\Rightarrow m = \frac{evr}{2}\left[\because \omega = \frac{v}{r}\right]$

Hence, the correct option is (b).

Question 33

You are given an ammeter, a galvanometer and a voltmeter. From these, the device having maximum resistance is

Options:

A. ammeter

B. galvanometer

C. voltmeter

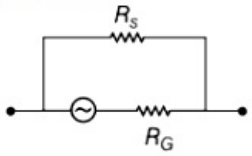
D. all will have the same resistance

Answer: C

Solution:

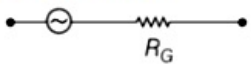
Solution:
Electric circuit diagram of ammeter, galvanometer and voltmeter is shown below,

(i) Ammeter



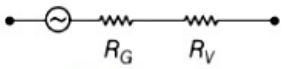
$$\Rightarrow R_{eq(A)} = \frac{R_s R_G}{R_G + R_s}$$

(ii) Galvanometer



$$\Rightarrow R_{eq(G)} = R_G$$

(iii) Voltmeter



$$\Rightarrow R_{eq(V)} = R_G + R_V$$

Hence, the order of resistances

$$R_{eq(V)} > R_{req(G)} > R_{eq(A)}$$

So, the maximum resistance device is voltmeter.

Question 34

The unit of self inductance is

Options:

A. Joule/Ampere

B. Volt/Ampere

C. Volt-Ampere/Second

D. Volt-second/Ampere

Answer: D

Solution:

Solution:
As we know, EMF induced in a coil of self-inductance L is given as

$$E = -L\frac{dI}{dt} \Rightarrow L = -E\frac{dt}{dI}$$

So, SI unit of $L = \frac{\text{Volt second}}{\text{Ampere}}$

Hence, the unit of self induction is Volt-second/Ampere.

Question 35

Eddy currents are produced when

Options:

A. a metal is kept in varying magnetic field

B. a metal is kept in steady magnetic field

C. a circular coil is placed is a magnetic field

D. a current is passed through a circular coil

Answer: A

Solution:

Solution:
Eddy currents are loops of electrical current induced within the conductors by changing magnetic field in the conductor. It is according to Faraday's law of induction. Hence, the correct option is (a).

Question 36

Lenz's law is consequence of the law of conservation of

Options:

A. charge

B. momentum

C. mass

D. energy

Answer: D

Solution:

Solution:
Lenz's law of electromagnetic induction corresponding to the law of conservation of energy.

Question 37

Two coherent monochromatic light beams of intensities ratio 1 : 4 are superposed. The ratio of maximum and minimum intensities in the resulting beam will be

Options:

A. 9 : 1

B. 5 : 3

C. 25 : 9

D. 9 : 25

Answer: A

Solution:

Solution:
Given, $I_1 : I_2 = 1 : 4$
As, we know
$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$
and $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$
Hence, the ratio

$$\frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right]^2 = \left[\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right]^2$$

$$\Rightarrow \frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{\frac{1}{4}} + 1}{\sqrt{\frac{1}{4}} - 1}\right] = \left[\frac{\frac{1+2}{2}}{\frac{1-2}{2}}\right]^2 = [3-1]^2 = 9 : 1$$

$$\Rightarrow \frac{I_{max}}{I_{min}} = \frac{9}{1}$$

Hence, the ratio of maximum and minimum intensities in the resulting beam is 9 : 1.

Question 38

A convex lens of focal length 10cm and refractive index 1.5 is dipped in a liquid of refractive index 1.75. It will behaves as

Options:

A. a convex lens of focal length 10cm

B. a convex lens of focal length 35cm

C. a concave lens of focal length 10cm

D. a concave lens of focal length 35cm

Answer: D

Solution:

Solution:
Given, focal length of convex lens, $f = 10cm$,
$\mu_{lens} = 1.5$ and $\mu_{liquid} = 1.75$
As, lens maker's formula

$$\frac{1}{f} = (\mu_{lens} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$\because$ For a equiconvex lens $R_1 = R$ and $R_2 = -R$

$\frac{1}{f} = (\mu_{lens})\left[\frac{1}{R} - \frac{1}{(-R)}\right]$
$\Rightarrow \frac{1}{f} = (\mu_{lens} - 1)\frac{2}{R}$
$\Rightarrow \frac{1}{10} = (1.5 - 1)\frac{2}{R}$
$\Rightarrow R = 10cm$
If lens is dropped in liquid, then
$\frac{1}{f'} = \left[\frac{\mu_{lens}}{\mu_{liq}} - 1\right]\frac{2}{R}$
$\Rightarrow \frac{1}{f'} = \left[\frac{1.5}{1.75} - 1\right]\frac{2}{10}$
$\Rightarrow f' = -35.71cm$
Hence, the lens behaves like a concave (negative focal length) lens of focal length $35cm$.

Question 39

Two beams of red and violet colour are made to pass separately through a prism with angle of prism 60°. In the position of minimum deviation, the angle of refraction will be

Options:

A. 60° for both colours

B. 30° for both colours

C. greater for violet colour

D. greater for red colour

Answer: D

Solution:

Solution:
As, we know, from Cauchy's formula,
Wavelength $\propto \frac{1}{\mu}$
Since, $\lambda_{voilet} < \lambda_{red}$
Hence from the above equation,
$\mu_{violet} > \mu_{red}$
For a prism,
$\mu = \frac{\sin i}{\sin r}$
As, it is given that angle of incidence is same for both the beams
So, $\sin r \propto \frac{1}{\mu}$
$\because \mu_{violet} > \mu_{red}$
$\therefore \sin r_{violet} < \sin r_{red}$
$\angle r_{violet} < \angle r_{red}$
Hence, the angle of refraction will be greater for red colour.

Question 40

The resolving power of a telescope can be increased by

Options:

A. increasing the diameter of objective

B. increasing the wavelength of light used

C. decreasing the diameter of objective

D. decreasing the frequency of light used

Answer: A

Solution:

Solution:
As we know,
the resolving power of a telescope $= \frac{d}{1.22\lambda}$
where, d is the diameter of objective. So if the diameter of objective lens of telescope is increased then the resolving power will be increase.

Question 41

'Lumen' is the unit of

Options:

A. luminous flux

B. luminous intensity

C. illuminance

D. light frequency

Answer: A

Solution:

Solution:
The lumen is the SI derived unit of luminous flux, a measured of the total quantity of visible light emitted by a source per unit time.
where,
1 lumen (lm) = 1Cd / Sr
$\because$Cd = candela and Sr = steradians
$\therefore$ The correct option is (a).

Question 42

When an unpolarised beam of light of intensity I_0 is incident on a polaroid, the intensity of transmitted light is

Options:

A. 0

B. I_0

C. $\frac{I_0}{2}$

D. $\frac{I_0}{4}$

Answer: C

Solution:

Solution:
An unpolarised beam of light of intensity I_0 is incident on a polaroid, the intensity coming out is half of the incident intensity $\frac{I_0}{2}$.

Question 43

An achromatic combination of lens is formed by joining

Options:

A. two convex lenses

B. two concave lenses

C. one convex lens and one concave lens

D. one convex lens and one plane mirror

Answer: C

Solution:

Solution:
For an achromatic combination of lenses, there should be
$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \Rightarrow \frac{\omega_1}{f_1} = -\frac{\omega_2}{f_2}$
Here, one focal length is positive and another one is negative i.e., one lens is convex and another is concave lens.

Question 44

A metal surface of work function 3eV is illuminated by photons of energy 2eV . The kinetic energy of emitted photo-electrons will be

Options:

A. 1eV

B. 2eV

C. 3eV

D. 0

Answer: D

Solution:

Solution:
Here, work function $\omega_0 = 3\text{eV}$ and photon energy $E_0 = 2\text{eV}$
As, Einstein's photoelectric equation $E_0 = W_0 + KE_0$
So, $KE_0 = E_0 - W_0 \Rightarrow KE_0 = 2 - 3 = -1\text{eV}$
Since, kinetic energy is a scalar quantity i.e., $KE > 0$, positive.
So, the photoelectron does not ejected and $KE = 0$
∴ Correct option is (d).

Question 45

The potential difference applied to an X-ray tube is 5kV and the current through it is 3.2mA. Then the number of electrons striking the target per second are

Options:

A. 5×10^6

B. 2×10^{16}

C. 1×10^{17}

D. 4×10^{18}

Answer: B

Solution:

Solution:
Given, potential difference, $V = 5\text{kV}$ and discharge current, $I = 3.2\text{mA} = 3.2 \times 10.3\text{A}$
As, we know that current, $I = \frac{ne}{t}$
where, $n =$ number of electron, $e =$ charge on electron and $t =$ time
$\Rightarrow n = \frac{It}{e}$
Putting $t = 1\text{s}$ and $e = 1.6 \times 10^{-19}\text{C}$
$\Rightarrow n = \frac{3.2 \times 10^{-3} \times 1}{1.6 \times 10^{-19}} = 2 \times 10^{16}$
Hence, 2×10^{16} electrons are striking the target per second.

Question 46

The mass density of a nucleus varies with mass number A as

Options:

A. A^2

B. A

C. A^0

D. A^{-1}

Answer: C

Solution:

Solution:
The nuclear radius r varies with mass number A according to the relation, $r = r_0 A^{1/3}$
$\Rightarrow r \propto A^{1/3} \Rightarrow A \propto r^3$
Now, density $= \frac{\text{mass}}{\text{volume}}$
Further, mass $\propto A$ and volume $\propto r^3$
$\therefore$ Density $= \frac{\text{Mass}}{\text{Volume}} = \frac{A}{r^3} = \frac{A}{A} = A^0 =$ Constant
Hence, the mass density of nucleus varies with mass number A as A^0.

Question 47

The half-life time of a radioactive sample is 5min. The amount of substance decayed in 20 min will be

Options:

A. 93.75%

B. 75%

C. 25%

D. 6.25%

Answer: A

Solution:

Solution:
Given, half-life $T_{1/2} = 5\text{min}$ and timet = 20min
As, fraction remains after n th half-life,
$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{20}{5}} = \left(\frac{1}{2}\right)^4$
$\Rightarrow \frac{N}{N_0} = \frac{1}{16}$
Now, \% amount of substance decayed in 20min,
$X = \frac{N_0 - N}{N_0} \times 100 = \frac{(16-1)}{16} \times 100 = \frac{15}{16} \times 100$
$X = 93.75\%$

Question 48

The depletion layer of an unbiased p − n junction consists of

Options:

A. only electrons

B. only holes

C. both electrons and holes

D. neither electrons nor holes

Answer: D

Solution:

Solution:
In the vicinity of the $p-n$ -junction, a region is created, which is devoid of the free charge carriers and has immobile ions.
Hence, the depletion layer consists of neither electrons nor holes.

Question 49

Which of the following is not the property of laser beams?

Options:

A. Highly intense

B. Monochromatic

C. Directional

D. Incoherent

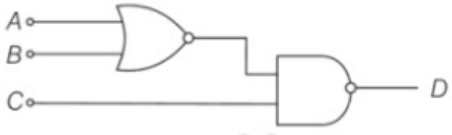
Answer: D

Solution:

Solution:
(d) A laser beam has following properties, like
(i) highly intense
(ii) monochromatic
(iii) unidirectional and
(iv) coherent
∴ Correct option is (d).

Question 50

For the given combination of gates in figure, the logic states of inputs are A = B = 1 and C = 0, then the logic state of output D is



Options:

A. 0

B. 1

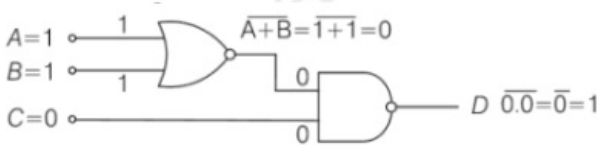
C. 2

D. 3

Answer: B

Solution:

Solution:
For a logical circuit given below, the output can be explained as



Hence, the output D = 1.

Question 51

Chemistry
The reaction
$P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + PH_3$
is an example of

Options:

A. disproportionation reaction

B. neutralisation reaction

C. double decomposition reaction

D. pyrolytic reaction

Answer: A

Solution:

Solution:
Given reaction,
$P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + PH_3$
Oxidation state of P in $P_4 = 0$
Oxidation state of P in $PH_3 = -3$
Oxidation state of P in $NaH_2PO_2 = +1$
So, $P_4^{0} \rightarrow \overset{-3}{PH}_3 + H_2\overset{+1}{P}O_2^{-}$
Hence, this reaction is an example of disproportionation reaction. It is a chemical reaction, typically a redox reaction, where a molecule is transformed into two or more dissimilar products.

Question 52

For the following three reactions (i), (ii) and(iii), equilibrium constants are given
(i) $CO_{(g)} + H_2O_{(g)} \rightleftharpoons CO_{2(g)} + H_2(g)$; K_1

(ii) $CH_{4(g)} + H_2O_{(g)} \rightleftharpoons CO(g) + 3H_{2(g)}$; K_2
(iii) $CH_{4(g)} + 2H_2O(g) \rightleftharpoons CO_{2(g)} + 4H_{2(g)}$; K_3
Which of the following relation is correct?

Options:

A. $K_3 . K_2^3 = K_1^2$

B. $K_1\sqrt{K_2} = K_3$

C. $K_2K_3 = K_1$

D. $K_3 = K_1K_2$

Answer: D

Solution:

Solution:
Given reaction:
(i) $CO_{(g)} + H_2O_{(g)} \rightleftharpoons CO_{2(g)} + H_2(g)$; K_1
Equilibrium constant (K_1) = $\frac{[CO_2][H_2]}{[CO][H_2O]}$
(ii) $CH_{4(g)} + H_2O_{(g)} \rightleftharpoons CO(g) + 3H_{2(g)}$; K_2
Equilibrium constant (K_2) = $\frac{[CO][H_2]^3}{[CH_4][H_2O]}$
(iii) $CH_{4(g)} + 2H_2O(g) \rightleftharpoons CO_{2(g)} + 4H_{2(g)}$; K_3
Equilibrium constant (K_3) = $\frac{[CO_2][H_2]^4}{[CH_4][H_2O]^2}$
So, $K_3 = K_1 . K_2$

Question 53

Three electrolytic cells A, B, C containing solutions of $ZnSO_4$, $AgNO_3$ and $CuSO_4$ respectively are connected in series. A steady current of 1.5 amperes was passed through them until 1.45g of silver deposited at the cathode of cell B. What mass of Cu and Zn were deposited?

Options:

A. Zn = 0.44g; Cu = 63.5g

B. Zn = 65.4g; Cu = 63.5g

C. Zn = 0.44g; Cu = 0.427g

D. Zn = 1.45g; Cu = 1.45g

Answer: C

Solution:

Solution:

Atomic mass of $Zn = 65.4$
Atomic mass of $Ag = 108$
Atomic mass of $Cu = 63.5$

$$\underset{1F}{Ag^{+} + e^{-}} \rightarrow \underset{1mol}{Ag}$$

$\because 108g$ of Ag is deposited by $= 96500C$
$1.45g$ of Ag will be deposited by

$$= \frac{96500 \times 1.45}{108} C = 1295.6C$$

Since, the reaction related to deposition of Cu is

$$\underset{2F}{Cu^{2+} + 2e^{-}} \rightarrow \underset{63.5g}{Cu}$$

$\because 2 \times 96500C$ electricity deposits $63.5g$ of Cu
$\therefore 1295.6C$ electricity will departs of

$$Cu = \frac{63.5 \times 1295.6}{2 \times 96500} = 0.42g$$

Since, the reaction related to deposition of zinc is

$$\underset{2F}{Zn^{2+} + 2e^{-}} \rightarrow \underset{65.4g}{Zn}$$

$\because 2 \times 96500C$ electricity deposits $65.4g$ of Zn.
$\therefore 1295.6$ C electricity will deposit of

$$Zn = \frac{65.4 \times 1295.6}{2 \times 96500} = 0.44g$$

So, option (c) $Zn = 0.44g$ and $Cu = 0.42g$ is correct.

Question 54

Which of the following does not have linear shape?

Options:

A. Cl_2^-

B. CH_3^+

C. XeF_2

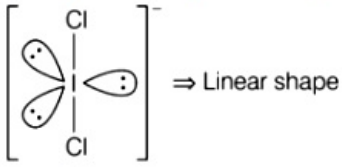
D. ClO^-

Answer: B

Solution:

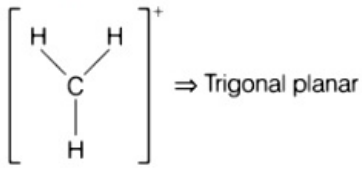
Solution:

(a) ICl_2^- ⇒ 2 bond pair +3 lone pair



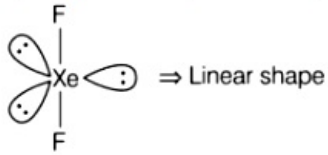
⇒ Linear shape

(b) CH_3^+ ⇒ 3 bond pair +0 lone pair



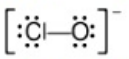
⇒ Trigonal planar

(c) XeF_2 ⇒ 2 bond pair +3 lone pair



⇒ Linear shape

(d) ClO^- ⇒ 1 bond pair +0 lone pair


⇒ Linear shape

Hence, the option (b) is correct, it does not have linear shape.

Question 55

The correct decreasing order of the boiling points of the compounds H_2O, HF and NH_3 is

Options:

A. $HF > H_2O > NH_3$

B. $H_2O > HF > NH_3$

C. $NH_3 > HF > H_2O$

D. $NH_3 > H_2O > HF$

Answer: B

Solution:

Solution:
Decreasing order of the boiling points of compounds will be, $H_2O > HF > NH_3$; due to the strength of hydrogen bonding present in given compound. So, more the hydrogen bonding, higher will be boiling point. Each water molecule can potentially form four hydrogen bonds with surrounding water molecules. Due to higher partial charge on H and F in HF, a hydrogen bond between HF molecules is stronger than that of NH_3 molecules.
Hence, the option (b) $H_2O > HF > NH_3$ is correct.

Question 56

What is the maximum volume of water required to dissolve 2g of

calcium sulphate at 298K ?(K_{sp} for $CaSO_4$ is 9.0×10^{-6})

Options:

A. 2.45L

B. 4.08L

C. 4.90L

D. 3.00L

Answer: C

Solution:

Solution:
Given,
Solubility product for $CaSO_4(K_{sp}) = 9.0 \times 10^{-6}$
Gram value of $CaSO_4$ = 2g
Suppose solubility of $CaSO_4$ in water is 'S'.
$CaSO_4 \rightleftharpoons Ca^{2+}(aq) + SO_4^{2-}(aq)$
Here, $K_{sp}(CaSO_4) = S^2$
$S^2 = 9.0 \times 10^{-6}$
$S = 3 \times 10^{-3}$
Molar mass of $CaSO_4$ = 136
Solubility of $CaSO_4 = 3 \times 10^{-3}$
When, 2g of $CaSO_4$ dissolve in water.
$\because$ Solubility $= \frac{2}{136 \times \text{ Volume}}$
$\therefore 3 \times 10^{-3} = \frac{2}{136 \times \text{ Volume}}$
Volume $= \frac{2}{136 \times 3 \times 10^{-3}} = \frac{2 \times 1000}{136 \times 3} = 4.90L$

Question 57

What is the correct volume of equilibrium constant for the following reaction at 400K , if the values of $\Delta H°$ is $77.5 kJ\,mol^{-1}$ and $\Delta S° = 135 J\,K^{-1} mol^{-1}$
$2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g)$

Options:

A. 8.545×10^{-4}

B. 8.545×10^{-2}

C. 8.314

D. 135

Answer: A

Solution:

Solution:
Given reaction,
$2N\,OCl\,(g) \rightleftharpoons 2N\,O(g) + Cl_2(g)$
Enthalpy $(\Delta H\,°) = 77.5 kJ\,mol^{-1} = 77500 J\,mol^{-1}$
Entropy $(\Delta S°) = 135 J\,K^{-1} mol^{-1}$
Temperature $(T) = 400K$
We know that, Gibbs free energy;
$\Delta G° = \Delta H\,° - T\,\Delta S°$
$\Delta G° = 77500 J\,mol^{-1} - 400 \times 135$
$\Delta G = 23500 J\,mol^{-1}$
Further, $\Delta G° = -2.303 RT\,\log K$
$23500 = -2.303 \times 8.314 \times 400 \log K$
$\log K = -\frac{23500}{2.303 \times 8.314 \times 400}$
$\log K = -3.06$
$K = \text{Antilog}\,(-3.06)$
$K = 8.545 \times 10^{-4}$

Question 58

Acetic acid dissociates 1.3%. What will be the pH of N / 10 solution of the acid?

Options:

A. 2.886

B. 2.066

C. 1.300

D. 2.086

Answer: A

Solution:

Solution:

CH_3COOH	$\rightleftharpoons$ CH_3COO^- +	H^+	(weak acid)
C	0	0	(initially)
$C(1-\alpha)$	$C\alpha$	$C\alpha$	at equilibrium

Given, concentration $(C) = \frac{N}{10}$
degree of dissociation $(\alpha) = \frac{1.3}{100}$
$[H^+] =$ Concentration $(C) \times$ degree of dissociation (α)
$[H^+] = \frac{1.3}{100} \times \frac{1}{10} = 1.3 \times 10^{-3}$
Thus, $pH = -\log[H^+]$
$= -\log(1.3 \times 10^{-3})$
$= -\log(0.0013)$
$= -(-2.88) = 2.88$

Question 59

In which compound does H show oxidation number of −1?

Options:

A. SiH_4

B. A_8H_3

C. N_3H

D. CaH_2

Answer: D

Solution:

Solution:
Oxidation number of CaH_2 :
$2 + 2x = 0$
$2x = -2$
$x = \frac{-2}{2} = -1$
Thus, the option (d) is correct.

Question 60

Which of the following oxides would be reduced by C? Al_2O_3, MgO, ZnO, CaO, Fe_2O_3, PbO

Options:

A. Al_2O_3, PbO and CaO

B. ZnO, Fe_2O_3 and PbO

C. Fe_2O_3, MgO and Al_2O_3

D. MgO, CaO and Al_2O_3

Answer: B

Solution:

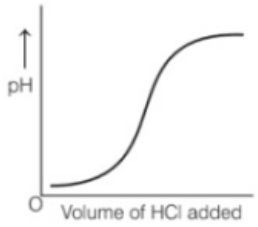
Solution:
Carbon is used to reduce the elements which are placed lower in the reactivity series. Metals which are highly reactive can not be reduced by carbon.
Hence, ZnO, Fe_2O_3, and PbO can be reduced by carbon.

Question 61

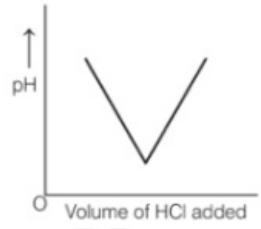
Titration curve if a strong base is titrated with strong acid is

Options:

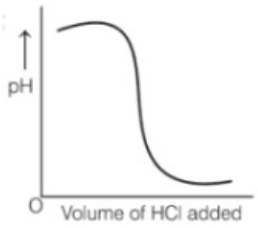
A.



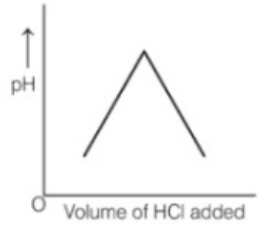
B.



C.



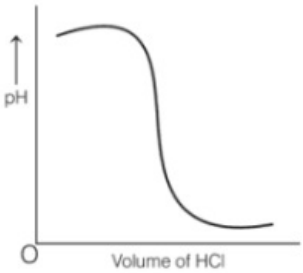
D.



Answer: C

Solution:

Solution:
In a strong acid and strong base titration, the acid and base react to form a neutral solution. At the equivalence point of the reaction hydronium ion $[H^+]$ and hydroxide ions $[OH^-]$ with react to form water, leading to pH of 7 .
$HCl + H_2O \rightarrow H_3O^+ + Cl^-$
$pH = -\log[H_3O^+]$



Question 62

Kelvin equation is related to

Options:

A. vapour pressure of droplets of liquids

B. temperature of a liquid

C. adsorption of liquid on solids

D. None of the above

Answer: A

Solution:

Solution:
The Kelvin equation describes the change in vapour pressure due to a curved liquid vapour interface, such as the surface of a droplet. The vapour pressure at a convex curved surface is higher than that at a flat surface.The Kelvin equation is dependent upon thermodynamic principles.

Question 63

Nylon-66 is formed by

Options:

A. free radical addition polymer

B. ionic addition polymerisation

C. condensation polymerisation

D. All of the above

Answer: C

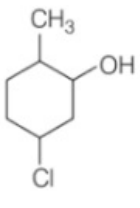
Solution:

Solution:
Nylon-6, 6 is formed by condensation polymerisation. In this mode of polymerisation, there is combination of molecules of same or different monomers. e.g.

$$\underset{\text{Adipic acid}}{HOOC(CH_2)_4COOH} + \underset{\text{Hexamethylene diamine}}{H_2N-(CH_2)_6-NH_2} \xrightarrow{-H_2O} -\left[\underset{\text{Nylon-66}}{NH(CH_2)_6NHCO(CH_2)_4CO}\right]_n$$

Question 64

Which of the following is the correct IUPAC name of the following structure



Options:

A. 3-chloro-2-methyl cyclohexanol

B. 2-methyl-5-chloro cyclohexanol

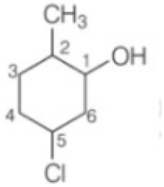
C. 1-chloro-4-methyl cyclohexanol

D. 5-chloro-2- methyl cyclohexanol

Answer: D

Solution:

Solution:
IUPAC name of the structure:



5-chloro-2- methyl cyclohexanol

Question 65

Average atomic weight of an element M is 51.7. If two isotopes of M , M^{50} and M^{52} are present, then percentage of occurrence of M^{50} in nature will be

Options:

A. 85%

B. 15%

C. 50%

D. 100%

Answer: B

Solution:

Solution:
Given, Average atomic weight of element (M) = 51.7
Let's assume M^{50} occurs with x%.
Hence, M^{52} = (100 − x)%
Average atomic weight

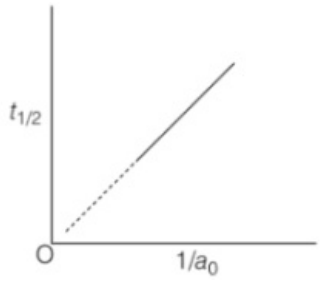
$$51.7 = \frac{x \times M^{50} + (100 - x) \times M^{52}}{100}$$

$$51.7 = \frac{x \times 50 + (100 - x)52}{100}$$

$5170 = 5200 - 2x$
$2x = 5200 - 5170$
$x = 15$
Hence, % of M^{50} = 15%

Question 66

The following graph shows how ($t_{1/2}$) (half-life) of a reactant R changes with the initial reactant concentration a_0. The order of the reaction will be



Options:

A. 0

B. 1

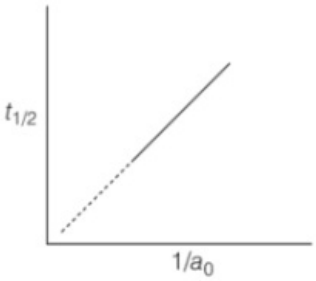
C. 2

D. 3

Answer: C

Solution:

Solution:
For second order reaction,



It can be represented as: $-\frac{d\,x}{d\,t} = K\,x^2$

$\frac{1}{[a]} = \frac{1}{[a_0']} + K\,t$; $K\,t_{1/2} = \frac{1}{[a_0']}$

half-life ($t_{1/2}$) = $\frac{1}{K\,[a_0']}$

So, graph of $t_{1/2}$ and $\frac{1}{a_0'}$ is linear.

Question 67

Which of the following is not permissible arrangement of electrons is an atom?

Options:

A. $n = 3, l = 2, m = -2, s = -1/2$

B. $n = 4, l = 0, m = 0, s = -1/2$

C. $n = 5, l = 3, m = 0, s = +1/2$

D. $n = 3, l = 2, m = -3, s = -1/2$

Answer: D

Solution:

Solution:
$n = 3, l = 2, m = -3, s = -1/2$
For a given value of n, 1 can be 0 to $(n - 1)$
Hence, for $n = 3$, value of 1 can be 2 for a given value of 1, m can be from -1 to $+1$.
Thus, m can be -2 to $+2$,
but given $m = 3$, which is not possible.

Question 68

The molecules of which of the following has the highest speed?

Options:

A. O_2 at 0°C

B. N_2 at 1000°C

C. CH_4 at 298K

D. H_2 at -50°C

Answer: D

Solution:

Solution:
(a) Velocity of particle (v_{rms}) = $\sqrt{\frac{3RT}{M}}$
O_2 at 0°C = $\sqrt{\frac{3 \times R \times 273}{32}} = \sqrt{25.5R}$
(b) N_2 at 1000°C = $\sqrt{\frac{3 \times R \times 1273}{28}} = \sqrt{136.3R}$
(c) CH_4 at 298K = $\sqrt{\frac{3 \times R \times 298}{16}} = \sqrt{55.8R}$
(d) H_2 at -50°C = $\sqrt{\frac{3 \times R \times 223}{2}}$
$= \sqrt{334.5R}$
Hence, H_2 has the highest speed.

Question 69

The vapour density of a mixture containing NO_2 and N_2O_4 is 38.3 at 27°C. What will be the mole of NO_2 in 100 mole mixture?

Options:

A. 76.6mol

B. 33.48mol

C. 50mol

D. 46mol

Answer: B

Solution:

Solution:
Given,
Vapour density of mixture NO_2 and N_2O_4 = 38.3
Temperature = 27°C
Moles of NO_2 + $2NO_2$ mixture = 100
$3(NO_2) = 100$
$NO_2 = \frac{100}{3} = 33.4mol$

Question 70

The enthalpy change of a reaction does not depend on

Options:

A. state of reactants and products

B. nature of reactants and products

C. different intermediate reactions

D. initial and final enthalpy of a reaction.

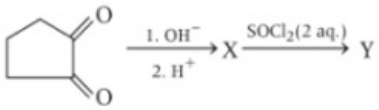
Answer: C

Solution:

Solution:
The change in enthalpy of reaction depends upon the quantity of reactants used. When the number of moles of reactants are doubled, the enthalpy change also becomes double. For elements existing in different allotropic modification, the heat of reaction is different if different allotropic form is involved in reaction. While the enthalpy change of a reaction does not depend on different intermediate reactions.

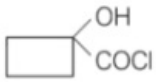
Question 71

In the following sequence of reactions, the product Y will be

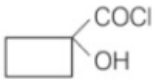


Options:

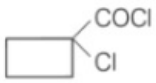
A.



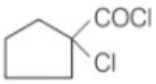
B.



C.



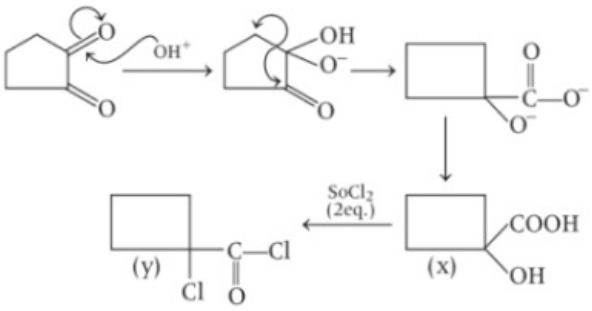
E.



Answer: C

Solution:

Solution:



Question 72

The reaction of P_4 with X leads selectively to P_4O_6. The X is

Options:

A. dry O_2

B. a mixture of O_2 and N_2

C. moist O_2

D. O_2 in the presence of aqueous NaOH

Answer: B

Solution:

Solution:
When phosphorous is burnt in limited supply of air, formation of P_4O_6 occur.
$\because P_4 +$ air $\rightarrow P_4O_6$
$\therefore P_4 + 3O_2 + N_2 \rightarrow P_4O_6 + N_2$
Nitrogen is used here to retard the further oxidation.

Question 73

Bleaching powder contains a salt of an oxoacid as one of its components. The anhydride of that oxoacid is

Options:

A. Cl_2O

B. Cl_2O_7

C. ClO_2

D. Cl_2O_6

Answer: A

Solution:

Solution:
Chemical formula of bleaching powder $= Ca(OCl)Cl$
Salt of oxoacid is $= OCl^-$
Here, oxoacid is $= HOCl$
Anhydride of $HOCl$ is Cl_2O.

Question 74

Which is the following is correct statement?

Options:

A. Starch is polymer of 2 -glucose

B. Amylose is a component of cellulose

C. Proteins are composed of only one type of amino acid.

D. In cyclic structure of fructose, there are four carbons and one oxygen atom.

Answer: A

Solution:

Solution:
(a) From these given statement, the option (a) is correct and statements (b), (c) and (d) are as following.
(b) Amylose is a component of starch.
(c) Proteins are composed of one or more type of amino acids.
(d) In cyclic structure of fructose, there are six carbons and six oxygen atom.

Question 75

The correct composition of malachite is

Options:

A. $CuF\ eS_2$

B. $CuCO_3$

C. $CuCO_3\ .\ Cu(OH)_2$

D. $Cu(OH)_2$

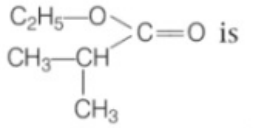
Answer: C

Solution:

Solution:
The malachite is an ore of copper. The composition of malachite is $CuCO_3\ .\ Cu(OH)_2$.

Question 76

IUPAC name of the compound



Options:

A. ethoxy-methanone

B. ethyl-2-methyl propanoate

C. ethoxy-propanone

D. 2-methyl-ethoxy propanone

Answer: B

Solution:

Solution:
The given compound has ester group as main functional group along with ethyl group, which contain one methyl group at number 2 -position.
Thus, IUPAC name of the given compound is ethyl-2-methyl propanoate.

Question 77

Which shows highest magnetic moment?

Options:

A. Cr^{2+}

B. Mn^{2+}

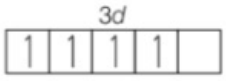
C. Cu^{2+}

D. CO^{2+}

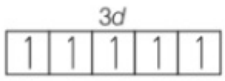
Answer: B

Solution:

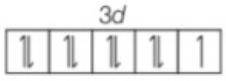
Solution:
Magnetic moment (μ) is calculated as follows:
$\mu = \sqrt{n(n+2)}$. BM
where, n = number of unpaired electrons.

(a) Cr^{2+} = 

It has 4-unpaired electron
$\therefore \mu = \sqrt{4(4+2)} = \sqrt{24}$ BM

(b) Mn^{2+} = 

It has 5-unpaired electrons.
$\therefore \quad \mu = \sqrt{5(5+2)} = \sqrt{35}$BM

(c) Cu^{2+} = 

It has 1-unpaired electron.
$\therefore \quad \mu = \sqrt{1(1+2)} = \sqrt{3}$ BM

(d) CO^{2+} = 6 + 8 − 2 = 12 electrons.
Thus, has 4 electrons in 2p-subshell with 2-unpaired electron.
$\therefore \mu = \sqrt{2(2+2)} = \sqrt{8}$
Hence, Mn^{2+} has highest magnetic moment and option (b) is the correct answer.

Question 78

In the Sandmeyer's reaction, $-N = N - X$ group of diazonium salt is replaced by

Options:

A. halide group

B. nitro group

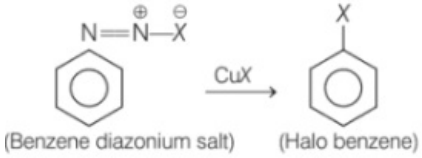
C. - OH group

D. $-NHNH_2$ group

Answer: A

Solution:

Solution:
The Sandmeyer's reaction occurs as follows



where, [$X = Cl$ or Br]
Hence, in Sandmeyer reaction,
$-N = N - X$ group is replaced by halide group and option (a) is the correct answer.

Question 79

Which of the following carbohydrates are branched polymer of glucose?

Options:

A. Amylase

B. Amylopectin

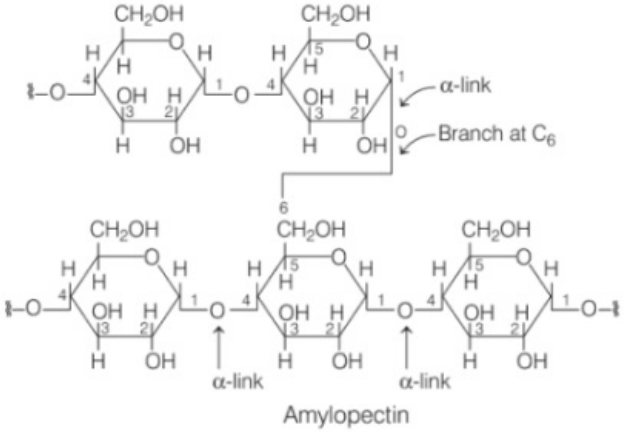
C. Cellulose

D. Glycogen

Answer: B

Solution:

Solution:
Amylopectin is a component of starch composed by branched polymer of glucose. Its structure is as follows:



Hence, (b) is the correct option.

Question 80

Which of the following acid has the lowest pK_a value?

Options:

A. $CH_3 - \overset{Cl}{\overset{|}{C}}H - COOH$

B. $Cl - CH_2 - CH_2 - COOH$

C. CCl_3COOH

D. $CHCl_2COOH$

Answer: C

Solution:

Solution:
Key Point -I groups; if more in number and more close to - COOH group, more is the value of K_a while + I group if are more in number and close to - COOH group, decreases the value of K_a

$pK_a \propto \frac{1}{K_a}$

where, K_a = acidic ionisation constant.
(a) It has one-(I) group (i.e. Cl) at $C-2$ position.
(b) It has also one (-)I group (i.e. Cl) at $C-3$ position.
(c) It has three (−) I groups (i.e. Cl)at $C-2$ position.
(d) It has one (-)I group (i.e. Cl) at $C-2$ position.
Thus, option (c) is the correct answer.

Question 81

Which of the following alcohol is dehydrated most easily by concentrated H_2SO_4?

Options:

A. $p-O_2NC_6H_4CH(OH)CH_3$

B. $p-ClC_6H_4CH(OH)CH_3$

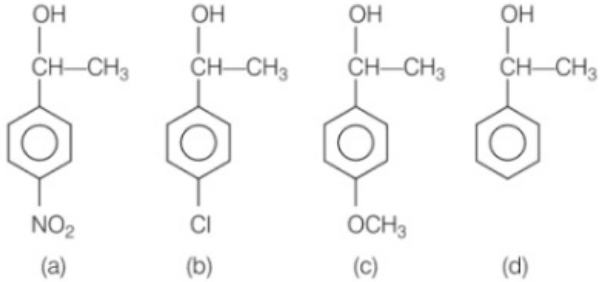
C. $p-CH_3OC_6H_4CH(OH)CH_3$

D. $C_6H_5CH(OH)CH_3$

Answer: C

Solution:

Solution:



The case of dehydration of alcohol takes place most easily in case (c).
In all cases during dehydration carbocation is formed.
In case (a), carbocation destabilise by $NO_2(-I)$ group hence, dehydration reaction proceed with difficulty.
In case (b) dehydration reaction takes place less difficulty than case (a) because Cl group show (-I) and (+ R) effect.
In case (c) dehydration reaction proceed easily because carbocation is stabilised by $OCH_3(+R)$ group.

Question 82

Which of the carbonyl compound will be most polar?

Options:

A. $C_2H_5-\overset{\overset{O}{\|}}{C}-C_2H_5$

B. $CH_3-\overset{\overset{O}{\|}}{C}-CH_3$

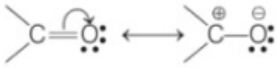
C. $CH_3-\overset{\overset{O}{\|}}{C}-H$

D. $H-\overset{\overset{O}{\|}}{C}-H$

Answer: D

Solution:

Solution:
In carbonyl compound the (C = O) bond is polarised due to high electronegativity of O atom relative to C atom, as shown below:



If any electron withdrawing group attached with carbon atom of carbonyl group than polarity increases.
If any electron releasing group attached with carbon atom of carbonyl group than polarity decreases.
Among the given case HCHO doesn't have strong electron releasing group hence it will be most polar. Because in the absence of strong electron releasing group the bond will more polar than other.

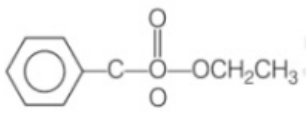
Question 83

The product (C) obtained from sequence of reaction will be

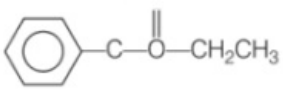
$$CH_3CH_2COOH \xrightarrow{PCl_3} (A) \xrightarrow[Al\,Cl_3]{C_6H_6} (B) \xrightarrow[Base,\,\Delta]{NH_2NH_2} (C)$$

Options:

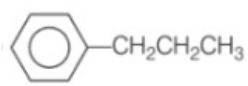
A.



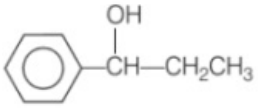
B.



C.



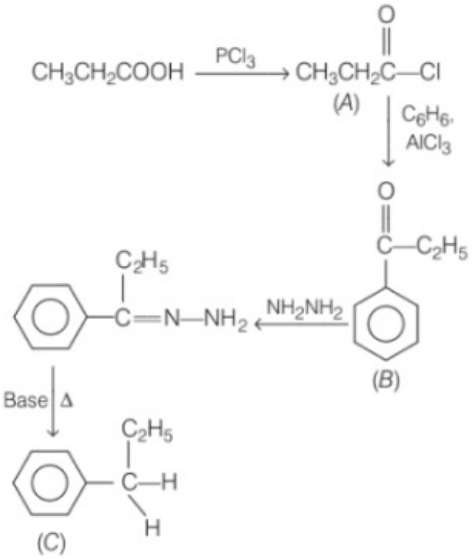
D.



Answer: C

Solution:

Solution:



Question 84

Which one of the following is most stable?

Options:

A. $CH \equiv \overset{\ominus}{C}$

B. $CH_2 = \overset{\ominus}{C}H_2$

C. $CH_3 - \overset{\ominus}{C}H_2$

D. $CH_3 - \overset{\ominus}{O}$

Answer: D

Solution:

Solution:
In $CH_3 - O^{\ominus}$, electronegative atom oxygen$O$) is more capable of bearing negative charge than carbon hence is more stable than the other given option. In general carbon ions increase in stability with increase in the amount of s-character at the carbonion ion. Higher the s-character, closer the electrons are to the nucleus and lower the energy or greater the stability.
Hence, the stability order is
$CH_3 - O^{\ominus} > \underset{50\%(sp)}{HC \equiv C^{\ominus}} > \underset{33\%(sp^2)}{H_2C = C^{\ominus}H} > \underset{25\%(sp^3)}{H_3C - C^{\ominus}H_2}$

Question 85

Liquid hydrocarbons can be converted to a mixture of gaseous hydrocarbons by

Options:

A. oxidation

B. distillation under reduced pressure

C. cracking

D. vaporisation

Answer: C

Solution:

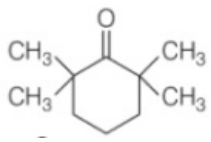
Solution:
Lower hydrocarbon exists in gaseous state while higher ones are in liquid state or solid state.
On cracking or pyrolysis, the hydrocarbon with higher molecular mass gives a mixture of hydrocarbons having lower molecular mass. Hence, cracking a liquid hydrocarbon can be converted into a mixture of gaseous hydrocarbons.

Question 86

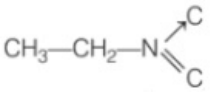
Tautomerism is exhibited by

Options:

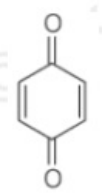
A.



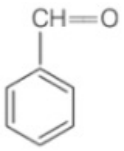
B.



C.



D.

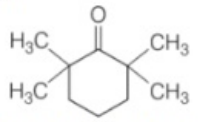


Answer: B

Solution:

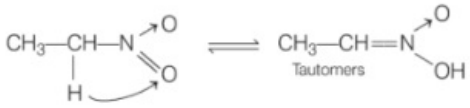
Solution:

(a)



does not have α -hydrogen

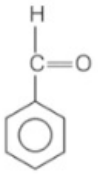
(b)



(c)

does not have H at ortho and para position so doesn't show tautomerism

(d)



no α hydrogen so doesn't show tautomerism.

Question 87

Most hazardous metal pollutants of automobile exhaust is

Options:

A. H g

B. Cd

C. Pb

D. Cu

Answer: C

Solution:

Solution:
Lead (Pb) is most hazardous metal pollutants of automobile exhaust.
Hence, option (c) is correct.

Question 88

The wavelength associated with a golf ball weighing 200g and moving with a speed of 5m / h is of the order

Options:

A. 10^{-10}m

B. 10^{-20}m

C. 10^{-30}m

D. 10^{-40}m

Answer: C

Solution:

Solution:
Given value,
Weight of a golf ball $= 200$g
Moving with speed $= 5\text{m / h} = \frac{5}{60 \times 60} = \frac{5}{3600}\text{m / sec}$
From de-Broglie equation; $\lambda = \frac{h}{mv}$
Here, h is proportionality constant of plank constant $= 6.626 \times 10^{-34}\text{kgm}^2\text{s}^{-1}$
Wavelength,$\lambda = \frac{6.626 \times 10^{-34}}{200 \times 10^{-3} \times \frac{5}{3600}\text{m / sec}}$
$= \frac{6.62 \times 3600 \times 10^{-34}}{1000 \times 10^{-3}}$
Order of wavelength $= 10^{-30}$m
Hence, option (c) is correct.

Question 89

Silicones are

Options:

A. organometallic compound

B. compounds obtained from silica

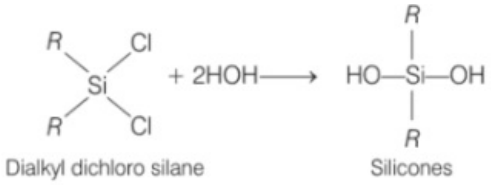
C. compounds obtained by hydrolysis of organo chloro silance

D. macromolecules prepared from silicates

Answer: C

Solution:

Solution:
Silicones are polymers that include any synthetic compound made up of repeating units of siloxane. Silicones compound obtained by hydrolysis of organo chloro silane.



Dialkyl dichloro silane Silicones

Question 90

10% solution of urea is isotonic with 6% solution of nonvolatile solute ' x′. What will be the atomic mass of solute ' x′ ?

Options:

A. $6g\ mol^{-1}$

B. $60g\ mol^{-1}$

C. $36g\ mol^{-1}$

D. $32g\ mol^{-1}$

Answer: C

Solution:

Solution:
Given value,
Solution of urea = 10%
Solution of non-volatile solute ' x′ = 6%
∵ Isotonic solution has the same osmotic pressure and hence the same concentration in the solution. We know that, osmotic pressure
$(\pi) = CRT$
here, R and T are constant
$\because \pi_1 = \pi_2$
$\therefore C_1RT_1 = C_2RT_2$
10% solution of urea = 6% of solution of ' x′
$\frac{10}{100 \times 60} = \frac{6}{100 \times x}$ (Molecular weight of urea = 60)
$x = 6 \times 6 = 36gmol^{-1}$
Hence, the atomic mass of solute ' x′ is $36gmol^{-1}$.

Question 91

Which is the correct order of size of O, O^{2-}, F^- and F ?

Options:

A. $O^{2-} > O > F^- > F$

B. $O > O^{2-} > F > F^-$

C. $O^{2-} > F^- > F > O$

D. $O^{2-} > F^- > O > F$

Answer: D

Solution:

Solution:
Radii of anion is more than neutral element. Here, O^{-2} has $10e^-$ and F has $10e^-$ for isoelectronic species radius increases as $\frac{e^-}{p}$ ratio increases.
$\because O^{-2}\frac{e}{p} = \frac{10}{8}$
$\therefore F^{-}\frac{e}{p} = \frac{10}{9}$
So, radius order is $O^{2-} > F^-$.
On moving left to right in a period, effective nuclear charge increases as radius decreases. So, order of size is $O^{2-} > F^- > O > F$

Question 92

Covalent molecules are usually held in a crystal structure by

Options:

A. dipole-dipole attraction

B. electrostatic attraction

C. van der Waals' attraction

D. hydrogen bond

Answer: C

Solution:

Solution:
Covalent molecules are usually held in a crystal structure by van der Waals' attraction.
(London forces, dipole-dipole interactions, dispersion).

Question 93

The normal dehydrating agent, used in a laboratory is

Options:

A. $MgCO_3$

B. CaF_2

C. NaCl

D. $CaCl_2$

Answer: D

Solution:

Solution:
The most desirable dehydrating agents that can be used for commercial dehydration purposes. The normal dehydrating agent, used in a laboratory is $CaCl_2$.

Question 94

B_2H_6 reacts with $(CH_3)_3N$ to produce

Options:

A. $BH_3{}^+N^-(CH_3)_3$

B. $B_2H_6{}^+N^-(CH_3)_2CH_3 . BH_3$

C. $(CH_3)_3N^+BH_3$

D. $BH_3N^+(CH_3)_2CH_3BH_3$

Answer: C

Solution:

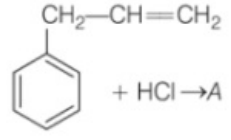
Solution:
When B_2H_6 reacts with $(CH_3)_3N$;

$$\underset{\text{Trimethyl amine}}{2(CH_3)_3N} + \underset{\text{Diborane}}{B_2H_6} \rightarrow \underset{\text{Complex}}{2(CH_3)_3\overset{+}{N}BH_3}$$

in this reaction trimethyl amine, reacts readily with diborane and form a $2(CH_3)_3\overset{+}{N}BH_3$ complex.
Hence, option (c) is correct.

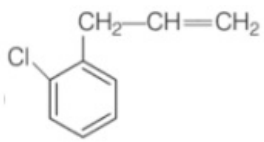
Question 95

What is 'A' in the following reaction

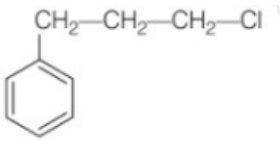


Options:

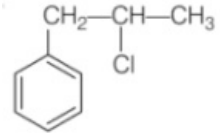
A.



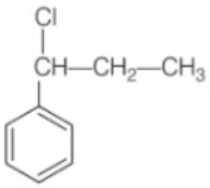
B.



C.



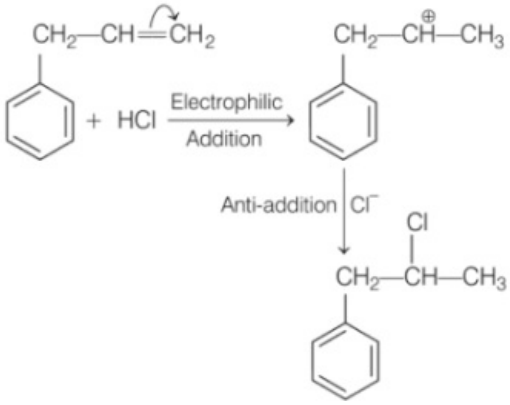
D.



Answer: C

Solution:

Solution:



Hence, option (c) is correct.

Question 96

The number of structural isomers for C_6H_{14} is

Options:

A. 3

B. 4

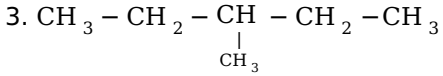
C. 5

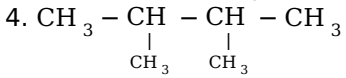
D. 6

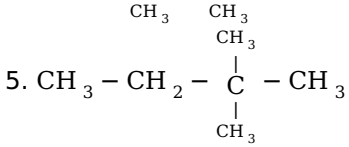
Answer: C

Solution:

Solution:
Structural isomers of C_6H_{14}
1. $CH_3-CH_2-CH_2-CH_2-CH_2-CH_3$
2. $CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2-CH_3$

3. 

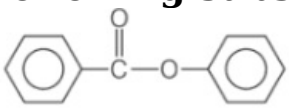
4. 

5. 

Hence, option (c) is correct.

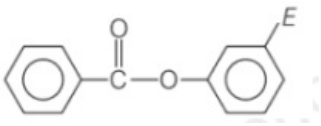
Question 97

Find the major product (Considering E as the electrophile) when the following substrate is subjected to electrophilic aromatic substitution.

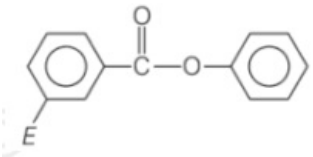


Options:

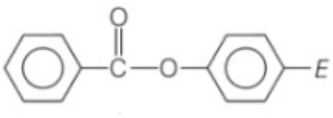
A.



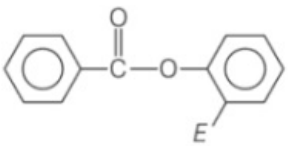
B.



C.



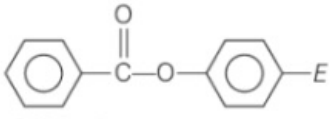
D.



Answer: C

Solution:

Solution:
Aromatic compounds and it's derivatives slow electrophilic substitution reaction and electrophile attack of para group due to less hindrance



Question 98

The second order Bragg-diffraction of X-rays with wavelength of 2.00Å from a set of parallel planes in a crystal occurs at 60°. The distance between the scattering planes in the crystal is

Options:

A. 5.75Å

B. 2.00Å

C. 4.00Å

D. 2.30Å

Answer: D

Solution:

Solution:
Given,
Wavelength (λ) = 2Å
Angle ($\sin\theta$) = $60° \Rightarrow \sin 60° = \frac{\sqrt{3}}{2}$
From Bragg equation;
$\lambda = 2d \sin\theta$
distance, $d = \frac{2Å}{2 \times \sqrt{3}/2}$
$d = \frac{2}{\sqrt{3}} = 1.154Å$
distance between the scattering planes in a crystal $= 2 = 2 \times 1.154Å$
$= 2.30Å$

Question 99

Gold sol is not

Options:

A. a macromolecular colloid

B. a lyophobic colloid

C. a multimolecular colloid

D. negatively charged colloid

Answer: C

Solution:

Solution:
Gold sol or colloidal gold is a sol or colloidal suspension of nanoparticles of gold in a fluid, usually water.
These are macromolecular and lyophobic colloid. Gold sol is a negatively charged colloid and nanomolecular colloid.
Hence, option (c) is correct.

Question 100

Carbon monoxide forms volatile compound with

Options:

A. Ni

B. Cu

C. Al

D. Si

Answer: A

Solution:

Solution:
Nickel (Ni) on heating in a stream of carbon monoxide forms a volatile complex, that is nickel tetracarbonyl.

$$\underset{\text{Nickel}}{Ni} + \underset{\text{Carbon monoxide}}{HCO} \xrightarrow{330-350K} \underset{\text{Nickel tetracarbonyl (volatilecompound)}}{[Ni(CO)_4]}$$

Question 101

Mathematics
If the parabola $y = -x^2 - 2x + k$, touches the parabola $y = -\frac{1}{2}x^2 - 4x + 3$, then the value of k is

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Solution:
Let the parabola $y = -x^2 - 2x + k$ and the parabola $y = -\frac{1}{2}x^2 - 4x + 3$ touches the point $P(x_1, y_1)$

Now, $y = -x^2 - 2x + k$

$\frac{dy}{dx})_{(x_1, y_1)} = -2x_1 - 2$ .....(i)

and $y = -\frac{1}{2}x^2 - 4x + 3$

$\left(\frac{dy}{dx}\right) = -x - 4$

$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = x_1 - 4$ .....(ii)
Since, parabola touches of (x_1, y_1).
$\therefore$ Slope of their tangents are equal
$-2x_1 - 2 = -x_1 - 4$
$\therefore x_1 = 2$
put the value of x_1 in $y_1 = -\frac{1}{2}x_1^2 - 4x_1 + 3$
we get $y_1 = -\frac{1}{2}(2)^2 - 4(2) + 3 = -7$
$\therefore y_1 = -x_1^2 - 2x_1 + k$
$-7 = -4 - 4 + k$
$\therefore k = 1$

Question 102

If f (x) = $\frac{1}{x-1}$ and g(x) = $\frac{x-1}{x+1}$, then the domain of (fog) (x) is

Options:

A. $\{x \in R \mid x \neq -1\}$

B. $\{x \in R \mid x \neq 1\}$

C. $\{x \in R \mid x \neq 1, -1\}$

D. $\{x \in R \mid x \neq 0, -1, 1\}$

Answer: A

Solution:

Solution:
We have,
$f(x) = \frac{1}{x-1}$ and $g(x) = \frac{x-1}{x+1}$
$(\text{ fog })(x) = f(g(x)) = \frac{1}{g(x) - 1}$
$f(g(x)) = \frac{1}{\frac{x-1}{x+1} - 1}$, $x \neq -1$
$f(g(x)) = \frac{x+1}{-2}$, $x \neq -1$
$\therefore$ Domain of $(\text{ fog })(x) = R - \{-1\}$ i.e.
$\{x \in R : x \neq -1\}$

Question 103

If x = $3 - 2\sqrt{2}$, then the value of $\sqrt{x} - \frac{1}{\sqrt{x}}$ is

Options:

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. 1

D. -2

Answer: D

Solution:

Solution:
We have,
$x = 3 - 2\sqrt{2}$
$x = (\sqrt{2} - 1)^2$
$\sqrt{x} = \sqrt{2} - 1$
$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$
$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = (\sqrt{2} - 1) - (\sqrt{2} + 1) = -2$

Question 104

Let z = x+ iy be a complex number satisfying the following equation $|z - (2 + i)| = |\text{Re}(z) - 4|$
Which of the following options describes the above equation?

Options:

A. $y = 1 \pm 2\sqrt{3 - x}$

B. $y = 2 \pm \sqrt{3 - x}$

C. $y = 1 \pm 3\sqrt{2 - x}$

D. $y = 3 \pm \sqrt{2 - x}$

Answer: A

Solution:

Solution:
We have,
$z = x + iy$
and $|z - (2 + i)| = |\text{Re}(z) - 4|$
$\Rightarrow |x + iy - 2 - i| = |x - 4|$
$\Rightarrow |(x - 2) + (y - 1)i| = |x - 4|$
$\Rightarrow (x - 2)^2 + (y - 1)^2 = (x - 4)^2$
$\Rightarrow x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 - 8x + 16$
$\Rightarrow y^2 - 2y + 1 = 12 - 4x$
$\Rightarrow (y - 1)^2 = 12 - 4x$
$\Rightarrow y - 1 = \pm\sqrt{12 - 4x}$
$\Rightarrow y = 1 \pm 2\sqrt{3 - x}$

Question 105

Let z = x+ iy be a complex number. The equation $\arg\left(\frac{z + 1}{z}\right) = \frac{\pi}{4}$ represents

Options:

A. $x^2 + x + y + y^2 = 0$

B. $x^2 - x + y + y^2 = 0$

C. $x^2 + x - y + y^2 = 0$

D. $x^2 + x + y - y^2 = 0$

Answer: A

Solution:

Solution:
We have,
$z = x + iy$ and $\arg\left(\frac{z+1}{z}\right) = \frac{\pi}{4}$
$\because \frac{z+1}{z} = \frac{x+iy+1}{x+iy} = \frac{((x+1)+iy)(x-iy)}{x^2+y^2}$
$\Rightarrow \frac{z+1}{z} = \frac{x^2+x+y^2+(xy-xy-y)i}{x^2+y^2}$
$= \frac{x^2+y^2+x-yi}{x^2+y^2}$
$\therefore \arg\left(\frac{z+1}{z}\right) = \tan^{-1}\left(\frac{-y}{x^2+y^2+x}\right) = \frac{\pi}{4}$
$\frac{-y}{x^2+y^2+x} = \tan\frac{\pi}{4} = 1$
$\Rightarrow -y = x^2 + y^2 + x$
$\Rightarrow x^2 + y^2 + x + y = 0$

Question 106

The imaginary part of $\left(\frac{1}{2} + \frac{1}{2}i\right)^{10}$ is

Options:

A. 0

B. $\frac{1}{30}$

C. $\frac{1}{31}$

D. $\frac{1}{32}$

Answer: D

Solution:

Solution:
We have,
$= \left(\frac{1}{2} + \frac{1}{2}i\right)^{10} = \frac{1}{2^{10}}(1+i)^{10}$
$= \frac{1}{2^{10}}[(1+i)^2]^5 = \frac{1}{2^{10}}(2i)^5$

$= \frac{1}{2^{10}} \times 2^5 i = \frac{1}{32} i$

$\therefore$ Imaginary part $= \frac{1}{32}$

Question 107

Let x be an n × 1 matrix. Let O and I be the zero, and identity matrices of order n, respectively. Define $P = -\frac{xx^T}{x^T x}$ is the transpose of x. Then, which of the following options is always correct?

Options:

A. $P^2 - P = O$

B. $P^2 - P = I$

C. $P^2 + P = O$

D. $P^2 + P = I$

Answer: C

Solution:

Solution:
Since it is given that
$P = -\frac{xx^T}{x^T x}$
$\Rightarrow x^T \times P = -xx^T$
On applying transpose both sides, we get
$P^T x^T x = -x^T x$
$\Rightarrow (P^T + I)x^T x = O$
$\Rightarrow P^T + I = O$
$\Rightarrow P^T = -I \Rightarrow P = -I \{\because I^T = I\}$
so $P^2 + P = I - I = O$
$\Rightarrow P^2 + P = O$

Question 108

Let P be a 2 × 2 matrix such that $[10]P = -\frac{1}{\sqrt{2}}[11]$ and $[01]P = \frac{1}{\sqrt{2}}[-11]$If O and I denote the zero and identity matrices of order 2 respectively, then which of the following options is correct?

Options:

A. $P^8 - P^6 + P^4 + P^2 = 0$

B. $P^8 + P^6 - P^4 + P^2 = I$

C. $P^8 + P^6 + P^4 - P^2 = 2I$

D. $P^8 - P^6 - P^4 - P^2 = 0$

Answer: C

Solution:

Solution:

Let $P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given,

$\therefore [10]P = -\frac{1}{\sqrt{2}}[11]$

$[10]\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$

$[ab] = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$

and $[01]P = \frac{1}{\sqrt{2}}[-11]$

$[01]\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

$[cd] = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

$\therefore a = \frac{-1}{\sqrt{2}}, b = \frac{-1}{\sqrt{2}}, c = \frac{-1}{\sqrt{2}}, d = \frac{1}{\sqrt{2}}$

$P = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$

$P^2 = \frac{1}{2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$P^2 = I$

$P^4 = I = P^6 = P^8$

$P^8 + P^6 + P^4 - P^2 = I + I + I - I = 2I$

Question 109

If det $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 9 \\ t & t^2 & 1+t^3 \end{bmatrix}$ = 0, then the values of t are

Options:

A. $1, 2, \frac{1}{2}$

B. $-1, 2, \frac{1}{2}$

C. $1, -2, \frac{1}{2}$

D. $1, 2, -\frac{1}{2}$

Answer: D

Solution:

Solution:
We have,

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 9 \\ t & t^2 & 1+t^3 \end{bmatrix} = 0$$

$\Rightarrow 1(4 + 4t^3 - 9t^2) - 1(2 + 2t^3 - 9t) + 2(2t^2 - 4t) = 0$
$\Rightarrow 4 + 4t^3 - 9t^2 - 2 - 2t^3 + 9t + 4t^2 - 8t = 0$
$\Rightarrow 2t^3 - 5t^2 + t + 2 = 0$
$\Rightarrow (t-1)(t-2)(2t+1) = 0$
$\Rightarrow t = 1, 2, -\frac{1}{2}$

Question 110

Let $x^2 + \alpha x + \beta = 0$ be the equation whose roots are the negatives of the roots of $x^2 + 7x - 2 = 0$, then the value of $\alpha + \beta$ is

Options:

A. 5

B. -5

C. 9

D. -9

Answer: D

Solution:

Solution:
We have,
$x^2 + \alpha x + \beta = 0$ be the equation whose roots are the negative of the roots of $x^2 + 7x - 2 = 0$
$\therefore \alpha = -7$
and $\beta = -2$
$\alpha + \beta = (-7) + (-2) = -9$

Question 111

If the roots of $4k^2 - (5k + 1)x + 5k = 0$ differ by unity, then the sum of all possible values of k is

Options:

A. $\frac{11}{5}$

B. $\frac{12}{5}$

C. $\frac{13}{5}$

D. $\frac{14}{5}$

Answer: D

Solution:

Solution:
Some error in question$4k^2 - (5k + 1)x + 5k = 0$

let α, β be roots
then α-β = 1

we know In a standard Quadratic Equation $ax^2 + bx + c = 0$
then sum of roots $= -\frac{b}{a}$

products of roots $= \frac{c}{a}$

Similarly
$\alpha + \beta = \frac{5k + 1}{4}$

$\alpha\beta = \frac{5k}{4}$

$\Rightarrow |\ \alpha = \beta\ | = 1$
$\Rightarrow (\alpha - \beta)^2 = 1$
$\Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 1$
$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1$
$\left(\frac{5k + 1}{4}\right)^2 - 4 \times \frac{5k}{4} = 1$
$\frac{(5k + 1)^2}{16} - \frac{20k}{4} = 1$
$25k^2 + 1 + 10k - 80k - 16 = 0$
$25k^2 - 70k - 15 = 0$
$(5k + 1)(k - 3) = 0$
Possible values of $k = 3, k = -\frac{1}{5}$

sum $= 3 + \left(-\frac{1}{5}\right) = \frac{14}{5}$

Question 112

Let a_1, a_2, a_3, ... be an arithmetic progression with non-zero common difference. It is given that $\sum_{i=4}^{12} a_i = 63$ and $a_k = 7$ for some k. Then the value of k is

Options:

A. 6

B. 7

C. 8

D. 9

Answer: C

Solution:

Solution:
Given, a_1, a_2, a_3, ... be an arithmetic progression

$a_1 + a_2 + a_3 + \ldots\ldots . a_n = \frac{n}{2}(a_1 + a_n)$

$\sum_{i=4}^{12} a_i = 63$

$\Rightarrow a_4 + a_5 + a_6 + \ldots. + a_{12} = 63$

$\Rightarrow \frac{9}{2}(a_4 + a_{12}) = 63$

$\Rightarrow a_4 + a_{12} = 14$

$\Rightarrow a + 3d + a + 11d = 14$

$\Rightarrow 2a + 14d = 14$

$\Rightarrow a + 7d = 7$

$\Rightarrow a_8 = 7 \Rightarrow k = 8$

Question 113

Let S be the set of all right angled triangles with integer sides forming consecutive terms of an arithmetic progression. The number of triangles in S with perimeter less than 30 is

Options:

A. 0

B. 1

C. 2

D. 3

Answer: C

Solution:

Solution:
Let the side of right angled triangles are
$x, x + 1, x + 2$
$\therefore (x + 2)^2 = (x + 1)^2 + (x)^2$
$\Rightarrow x^2 + 4x + 4 = x^2 + 2x + 1 + x^2$
$\Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow (x - 3)(x + 1) = 0$
$\Rightarrow x = 3, x \neq -1$
Perimeter of triangle $= 3x + 3$
$3x + 3 < 30$
$x + 1 < 10$
$x < 9$
The possible triangle whose sides are
(3, 4, 5), (6, 8, 10)
$\therefore$ Two triangles are possible.

Question 114

It is given that $\lim_{x \to 0} \frac{e^{ax} - bx - 1}{x^2} = 2$. Thenthe value of |a|+ | b| is

Options:

A. 1

B. 2

C. 3

D. 4

Answer: D

Solution:

Solution:
We have,
$\lim_{x \to 0} \frac{e^{ax} - bx - 1}{x^2} = 2$
Apply L, Hospital's rule, we get
$\lim_{x \to 0} \frac{ae^{ax} - b}{2x}$
$\therefore a - b = 0$
Again apply L, Hospital's rule, we get
$\lim_{x \to 0} \frac{a^2 e^{ax}}{2} = 2$
$\Rightarrow a^2 = 4$
$\Rightarrow a = \pm 2$
$\therefore b = \pm 2$
$|a| + |b| = 2 + 2 = 4$

Question 115

The sum of intercepts on the axes of the tangent to the curve $\sqrt{x} + \sqrt{y} = 3$ at (4, 1) is

Options:

A. 3

B. 5

C. 7

D. 9

Answer: D

Solution:

Solution:
Given, curve,
$\sqrt{x} + \sqrt{y} = 3$
On differentiating with respect to x, we get
$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$
$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$
$\Rightarrow \left(\frac{dy}{dx}\right)_{(4, 1)} = -\frac{\sqrt{1}}{\sqrt{4}} = \frac{-1}{2}$ $[\because x_1 = 4, y_1 = 1]$
Equation of tangent of curve at (4, 1)
$y - 1 = \frac{-1}{2}(x - 4)$

$\Rightarrow x + 2y = 6 \Rightarrow \frac{x}{6} + \frac{y}{3} = 1$
Sum of intercept $6 + 3 = 9$

Question 116

If $f(x) = (x-1)^2(x+1)^3$, then the function f has

Options:

A. a local maximum at $x = \frac{1}{5}$

B. a local minimum at $x = \frac{1}{5}$

C. a local minimum at $x = -1$

D. a local maximum at $x = -1$

Answer: A

Solution:

Solution:
We have,
$f(x) = (x-1)^2(x+1)^3$
$f'(x) = 2(x-1)(x+1)^3 + 3(x-1)^2(x+1)^2$
$\Rightarrow f'(x) = (x-1)(x+1)^2[2x + 2 + 3x - 3]$
$\Rightarrow f'(x) = (x-1)(x+1)^2(5x-1)$
For maxima or minima put $f'(x) = 0$
$\therefore (x-1)(x+1)^2(5x-1) = 0$
$\Rightarrow x = -1, \frac{1}{5}, 1$
$f''(x) = (x+1)^2(5x-1) + 2(x-1)$
$(x+1)(5x-1) + 5(x-1)(x+1)^2$
$f''(-1) = 0$
$f''(1) = 16 > 0$
$f''\left(\frac{1}{5}\right) = \frac{-144}{25} < 0$
$\therefore f(x)$ has local maxima at $x = \frac{1}{5}$

Question 117

If the function

$$f(x) = \begin{cases} x + 2; & \text{if } x < 2 \\ ax^2 + bx + 3; & \text{if } 2 \le x < 3 \\ 2x + a + b; & \text{if } x \ge 3 \end{cases}$$

is continuous, then the value of $(a^2 + b^2)$ is

Options:

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. $\frac{1}{9}$

Answer: A

Solution:

Solution:
We have,

$$f(x) = \begin{cases} x + 2; & \text{if } x < 2 \\ ax^2 + bx + 3; & \text{if } 2 \le x < 3 \\ 2x + a + b; & \text{if } x \ge 3 \end{cases}$$

is continuous.
$\therefore \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$
$\Rightarrow 2 + 2 = 4a + 2b + 3 \Rightarrow 4a + 2b = 1$ .....(i)
and $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$
$\Rightarrow 9a + 3b + 3 = 6 + a + b$
$\Rightarrow 8a + 2b = 3$ .....(ii)
From Eqs, (i) and (ii), we get
$a = \frac{1}{2}, b = -\frac{1}{2}$
$\therefore a^2 + b^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Question 118

Suppose f is differentiable function such that $f(g(x)) = x^2$ and $f'(x) = 1 + (f(x))^2$, then the value of $g'(2)$ is

Options:

A. $\frac{1}{17}$

B. $\frac{2}{17}$

C. $\frac{3}{17}$

D. $\frac{4}{17}$

Answer: D

Solution:

Solution:
We have, $f(g(x) = x^{2.}$
$\therefore f'(g(x) \cdot g'(x) = 2x$
$g'(x) = \frac{2x}{f'(g(x))}; g'(x) = \frac{2x}{1 + (f(gx))^2}$ $[\because f'(x) = 1 + (f(x))^2]$

$g'(x) = \frac{2x}{1+x^4}$

$g'(2) = \frac{4}{1+16} = \frac{4}{17}$

Question 119

The area bounded by the graphs of functions $f(x) = x^4 - 2x^2$ and $g(x) = 2x^2$ is

Options:

A. $\frac{121}{15}$

B. $\frac{124}{15}$

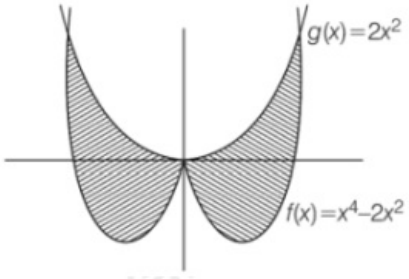
C. $\frac{128}{15}$

D. $\frac{131}{15}$

Answer: C

Solution:

Solution:
We have,
$f(x) = x^4 - 2x^2$ .....(i)
$g(x) = 2x^2$ .....(ii)
The intersection point of $f(x)$ and $g(x)$ are $(0, 0)$ $(2, 8)$ and $(-2, 8)$



Area of shaded region

$= 2\int_0^2 (g(x) - f(x))dx$

$= 2\int_0^2 (2x^2 - x^4 + 2x^2)dx$

$= 2\int_0^2 (4x^2 - x^4)dx$

$= 2\left[\frac{4x^3}{3} - \frac{x^5}{5}\right]_0^2 = 2\left(\frac{32}{3} - \frac{32}{5}\right)$

$= \frac{128}{15}$

Question 120

If the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 9$ into regions o. equal area, then the value of b is

Options:

A. $\frac{3}{\sqrt[3]{4}}$

B. $\frac{5}{\sqrt[3]{4}}$

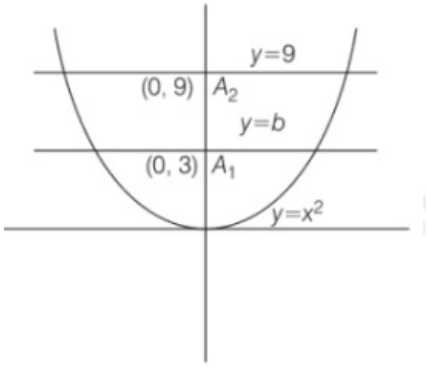
C. $\frac{7}{\sqrt[3]{4}}$

D. $\frac{9}{\sqrt[3]{4}}$

Answer: D

Solution:

Solution:
Given curve,



$y = x^2$, $y = b$, $y = 9$
$A_1 = A_2$
$\therefore \int_0^b \sqrt{y}\,dy = \int_b^9 \sqrt{y}\,dy$
$\Rightarrow \frac{2}{3}[y^{3/2}]_0^b = \frac{2}{3}[y^{3/2}]_0^9$
$\Rightarrow b^{3/2} = 9^{3/2} - (b)^{3/2} \Rightarrow 2b^{3/2} = 27$
$\Rightarrow b = \left(\frac{27}{2}\right)^{2/3} = \frac{9}{\sqrt[3]{4}}$

Question 121

If [y] denotes the greatest integer less than or equal to y for all y ∈ R, then the value of the integral $\int_{1/2}^{13/2} [\sqrt{x}]\,dx$ is

Options:

A. 8

B. 9

C. 10

D. 11

Answer: A

Solution:

Solution:

Let $I = \int_{1/2}^{13/2} [\sqrt{x}]dx$

$\Rightarrow I = \int_{1/2}^{1} 0dx + \int_{1}^{4} dx + \int_{4}^{13/2} 2dx$

$\because |\sqrt{x}| = \begin{cases} 0, & 0 < x < 1 \\ 1, & 1 < x < 4 \\ 2, & 4 < x < 9 \end{cases}$

$\Rightarrow I = 0 + (4 - 1) + 2\left(\frac{13}{2} - 4\right)$

$\Rightarrow I = 3 + 5 = 8$

Question 122

If $J = \int_{\pi/6}^{\pi/3} \frac{1}{\sqrt{\cos x} + \sqrt{\sin x}} dx$, then $\int_{\pi/6}^{\pi/3} \frac{x}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ equals

Options:

A. $\frac{\pi}{6} J$

B. $\frac{\pi}{3} J$

C. $\frac{\pi}{4} J$

D. πJ

Answer: C

Solution:

Solution:

We have,

$J = \int_{\pi/6}^{\pi/3} \frac{1}{\sqrt{\cos x} + \sqrt{\sin x}} dx$

Let $I = \int_{\pi/6}^{\pi/3} \frac{x}{\sqrt{\cos x} + \sqrt{\sin x}} dx$

On applying $\int_a^b f(x) = \int_a^b f(a + b - x)dx$

$I = \int_{\pi/6}^{\pi/3} \frac{\frac{\pi}{3} + \frac{\pi}{6} - x}{\sqrt{\cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)} + \sqrt{\sin\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}} dx$

$I = \int_{\pi/6}^{\pi/3} \frac{\pi/2 - x}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

$2I = \int_{\pi/6}^{\pi/3} \frac{\pi/2}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

$2I = \frac{\pi}{2} J$

$I = \frac{\pi}{4} J$

Question 123

The differential equations representing a family of circles having centre on the Y-axis and radius 4 is

Options:

A. $\left(\frac{dy}{dx}\right)^2 + \frac{x^2}{(x^2-2)} = 0$

B. $\left(\frac{dy}{dx}\right)^2 + \frac{x^2}{(x^2-4)} = 0$

C. $x^2\left(\frac{dy}{dx}\right)^2 + (x^2-4) = 0$

D. $\left(\frac{dy}{dx}\right)^2 + \frac{y^2}{(y^2-4)} = 0$

E. None of Above

Answer: E

Solution:

Solution:
Let Center of Circle is $C : (0, K)$, Any point on it $P(X, Y)$ and Given Radius of Circle $R = 4$
Then Equation of circle becomes after applying distance formula
$(X-0)^2 + (Y-K)^2 = 16$…… (i)
Now d.w.r.t X of equation (i)
$2X + 2(Y-K)Y = 0$
$(Y-K) = \frac{(-X)}{(Y)}$…… (ii)
From Eq. (i) and (ii) we have get,
$X^2 + \left[\frac{(-X)}{(Y)}\right]^2 = 16$
$(XY)^2 + X^2 = (4Y)^2$: Represents the family of circles whose center $(0, K)$ lies on Y-axis and Radius is 4.

Question 124

If y(x) satisfies equations $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ and y(0) = 0, then y(1) is

Options:

A. $\frac{2}{3}$

B. 1

C. $\frac{4}{3}$

D. $\frac{5}{3}$

Answer: A

Solution:

Solution:
We have,
$(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$
$\Rightarrow \frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{4x^2}{1+x^2}$
This is linear differential equation
$\therefore I.F=e^{\int \frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$
Solution of given differential equation is
$y(1+x^2)=\int 4x^2 dx$
$y(1+x^2)=\frac{4x^3}{3}+C$
put $x=0, y=0$, we get
$C=0$
$\therefore y(1+x^2)=\frac{4x^3}{3}$
put $x=1, y=\frac{2}{3}$
$\therefore y(1)=\frac{2}{3}$

Question 125

If y(x) satisfies equations $(1+e^x)\frac{dy}{dx}+ye^x=1$ and $y(1)=\frac{3}{1+e}$, then maximum value of y(x) is

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Solution:
Given,
$(1+e^x)\frac{dy}{dx}+ye^x=1$
$\frac{dy}{dx}+\frac{ye^x}{1+e^x}=\frac{1}{1+e^x}$
This is a linear differential equation is
$\therefore IF=e^{\int \frac{e^x}{1+e^x}dx}=e^{\log(1+e^x)}$
$=1+e^x$
Solution of given differential equation is
$y(1+e^x)=\int dx$
$y(1+e^x)=x+C$
putting $x=1, y=\frac{3}{1+e}$, we get
$C=2$
$\therefore y=\frac{x+2}{1+e^x}$
Maximum value of $y=1$ at $x=0$

Question 126

The equation $6x^2 + xy - 12y^2 - 13x + 6y + 6 = 0$ represents

Options:

A. a pair of straight lines through the origin

B. a pair of perpendicular straight lines

C. a pair of parallel straight lines

D. a pair of straight lines not passing through the origin, neither parallel nor perpendicular

Answer: D

Solution:

Solution:
We have,
$6x^2 + xy - 12y^2 - 13x + 6y + 6 = 0$
Here, $a = 6, h = \frac{1}{2}, b = -12$
$g = -\frac{13}{2}$
$f = 3, c = 6$
$\because a + b = 6 - 12 = -6 \neq 0$
$\therefore$ A pair of line is not perpendicular and $\because h^2 \neq ab$
$\therefore$ Pair of line is not parallel
Hence, pair of straight line is not passing through origin, neither parallel nor perpendicular.

Question 127

The normal at the point (2, 3) to the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ intersects the circle $x^2 + y^2 = 1$ at points P and Q. The area of the circle with PQ as diameter is

Options:

A. $\frac{\pi}{2}$

B. π

C. 2π

D. $\frac{3\pi}{2}$

Answer: A

Solution:

Solution:
Given circle,
$x^2 + y^2 - 2x - 4y + 3 = 0$ having

Centre (1, 2)
Equation of normal of circle at (2, 3) is
$y - 3 = \frac{3-2}{2-1}(x - 2)$
$y - 3 = x - 2$
$\Rightarrow x - y + 1 = 0$ .....(i)
Normal of circle intersect the circle
$x^2 + y^2 = 1$ .....(ii)
From Eqs. (i) and (ii)
$x^2 + (x + 1)^2 = 1$
$x^2 + x^2 + 2x + 1 = 1$
$2x^2 + 2x = 0$
$2x(x + 1) = 0$
$x = 0, -1$
On putting the value of x in Eq. (i), we get
$y = 1, 0$
$P(0, 1), Q(-1, 0)$
$PQ = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$
Radius of circle $= \frac{\sqrt{2}}{2}$
Area of circle $= \pi\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{\pi}{2}$

Question 128

The circles $x^2 + y^2 + kx + 4y = 20$ and $x^2 + y^2 + 6x - 8y + 10 = 0$ intersect orthogonally. Also circles $x^2 + y^2 - p(x - y) + 1 = 0$ andp$(x^2 + y^2) + x - y = l$ intersect orthogonally. Then, k / p equals

Options:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 2

D. 4

Answer: C

Solution:

Solution:
Given, circle $x^2 + y^2 + kx + 4y = 20$ and $x^2 + y^2 + 6x - 8y + 10 = 0$ intersect orthogonally
$\therefore 2(3)\frac{(k)}{2} + 2(-4)(2) = -20 + 10$
$\Rightarrow 3k - 16 = -10$
$\Rightarrow k = 2$
Also, $x^2 + y^2 - px + py + 1 = 0$ and $p(x^2 + y^2) + x - y = 1$ intersect orthogonally
$\therefore 2\left(\frac{-p}{2}\right)\left(\frac{1}{2p}\right) + 2\left(\frac{p}{2}\right)\left(-\frac{1}{2p}\right) = 1 - \frac{1}{p}$
$\Rightarrow -1 = 1 - \frac{1}{p} \Rightarrow p = \frac{1}{2}$
$\therefore k / p = \frac{2}{\frac{1}{2}} = 4$

Question 129

The common tangent to the parabolas $y^2 = 32x$ and $x^2 = 108y$ intersects the coordinate axes at points P and Q, respectively. Then, length of PQ is

Options:

A. $2\sqrt{13}$

B. $3\sqrt{13}$

C. $5\sqrt{13}$

D. $6\sqrt{13}$

Answer: D

Solution:

Solution:
Equation of tangent of parabola
$y^2 = 32x$ having slope m is $y = mx + \frac{8}{m}$ .....(i)
Equation of tangent of parabola
$x^2 = 108y$ having slope m is $y = mx - 27m^2$ .....(ii)
Eqs. (i) and (ii) are identical
$\therefore \frac{8}{m} = -27m^2 \Rightarrow m = -\frac{2}{3}$
Equation of common tangent of parabola is
$y = \frac{-2}{3}x - 12$
$\Rightarrow 2x + 3y + 36 = 0$
$\therefore P(-18, 0)$ and $Q(0, -12)$
$PQ = \sqrt{(18)^2 + (12)^2} = \sqrt{468} = 6\sqrt{13}$

Question 130

Tangent to the ellipse $\frac{x^2}{4} + y^2 = 1$ at the point $P\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$ touches a circle $x^2 + y^2 = r^2$ at the point Q. Then the length of PQ is

Options:

A. $\frac{1}{\sqrt{10}}$

B. $\frac{3}{\sqrt{10}}$

C. $\frac{7}{\sqrt{10}}$

D. $\frac{11}{\sqrt{10}}$

Answer: B

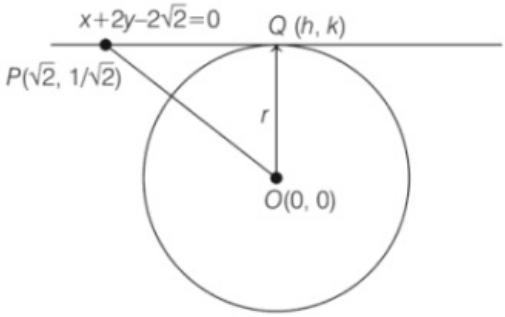
Solution:

Solution:

Equation of tangent of ellipse $\frac{x^2}{4} + y^2 = 1$ at $P\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$ is

$\frac{\sqrt{2}x}{4} + \frac{y}{\sqrt{2}} = 1 \Rightarrow x + 2y - 2\sqrt{2} = 0$

This is also tangent of circle $x^2 + y^2 = r^2$



$\therefore r = \frac{|-2\sqrt{2}|}{\sqrt{1+4}} = \frac{2\sqrt{2}}{\sqrt{5}}$

$PQ = \sqrt{OP^2 - OQ^2} = \sqrt{OP^2 - r^2}$

$PQ = \sqrt{\left(2 + \frac{1}{2}\right) - \left(\frac{2\sqrt{2}}{\sqrt{5}}\right)^2}$

$PQ = \sqrt{\frac{5}{2} - \frac{8}{5}} = \frac{3}{\sqrt{10}}$

Question 131

If the line $3x + 4y = 7$ is a normal at a point $P = (x_1, y_1)$ of the hyperbola $3x^2 - 4y^2 = 1$,then the distance of P from the origin is

Options:

A. $\frac{\sqrt{319}}{12}$

B. $\frac{\sqrt{337}}{12}$

C. $\frac{\sqrt{423}}{12}$

D. $\frac{\sqrt{527}}{12}$

Answer: B

Solution:

Solution:

Equation of normal of the hyperbola

$3x^2 - 4y^2 = 1$ at

$\frac{4x}{x_1} + \frac{3y}{y_1} = 7$ .....(i)

Also, normal at $P(x_1, y_1)$ is $3x + 4y = 7$ .....(ii)

Eqs. (i) and (ii) are identical

$\because \frac{4}{3x_1} = \frac{3}{4y_1} = \frac{7}{7}$

$\Rightarrow x_1 = \frac{4}{3}, y_1 = \frac{3}{4}$

$\therefore P\left(\frac{4}{3}, \frac{3}{4}\right)$

Distance from P to origin

$= \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{3}{4}\right)^2}$

$= \sqrt{\frac{16}{9} + \frac{9}{16}} = \frac{\sqrt{337}}{12}$

Question 132

Let p, q, r be all distinct real numbers and the vectors $p\hat{i} + p^2\hat{j} + (1 + p^3)\hat{k}$, $q\hat{i} + q^2\hat{j} + (1 + q^3)\hat{k}$ and $r\hat{i} + r^2\hat{j} + (1 + 3)\hat{k}$ are coplanar. Then, pqr equals

Options:

A. 2

B. 1

C. - 1

D. -2

Answer: C

Solution:

Solution:

We have,

$p\hat{i} + p^2\hat{j} + (1 + p^3)\hat{k}$, $q\hat{i} + q^2\hat{j} + (1 + q^3)\hat{k}$ and $r\hat{i} + r^2\hat{j} + (1 + 3)\hat{k}$ are coplanar.

$$\therefore \begin{vmatrix} p & p^2 & 1+p^3 \\ q & q^2 & 1+q^3 \\ r & r^2 & 1+r^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p & p^2 & 1 \\ q & q^2 & 1 \\ r & r^2 & 1 \end{vmatrix} + \begin{vmatrix} p & p^2 & p^3 \\ q & q^2 & q^3 \\ r & r^2 & r^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p & p^2 & 1 \\ q & q^2 & 1 \\ r & r^2 & 1 \end{vmatrix} + pqr \begin{vmatrix} 1 & p & p^2 \\ 1 & q & q^2 \\ 1 & r & r^2 \end{vmatrix} = 0$$

$$\Rightarrow (1 + pqr) \begin{vmatrix} p & p^2 & 1 \\ q & q^2 & 1 \\ r & r^2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1 + pqr = 0 \Rightarrow pqr = -1$

Question 133

Let a, b, c be three unit vectors such that$b \times (c \times a) = \frac{c}{2}$. If a and c are non-parallel, then the angles which b makes with c and a are respectively

Options:

A. 30° and 60°

B. 60° and 45°

C. 30° and 90°

D. 90° and 60°

Answer: D

Solution:

Solution:
We have,
$b \times (c \times a) = \frac{c}{2}$
and $|a| = |b| = |c| = 1$
Now, $b \times (c \times a) = \frac{c}{2}$
$\Rightarrow (b \cdot a)c - (b \cdot c)a = \frac{c}{2}$
$\Rightarrow b \cdot a = \frac{1}{2}$ and $b \cdot c = 0$
$\therefore$ Angle between b and c = 90° and angle between b and a is 60°.

Question 134

Let $a = -\hat{i} + \hat{j}$ and $b = \hat{i} + 3\hat{j}$. Then, angle between the vectors $4a + b$ and $\frac{1}{4}(7b - a)$ is

Options:

A. 30°

B. 45°

C. 60°

D. 90°

Answer: B

Solution:

Solution:
We have
$a = -\hat{i} + \hat{j}$ and $b = \hat{i} + 3\hat{j}$
$4a + b = 4\left(-\hat{i} + \hat{j}\right) + \left(\hat{i} + 3\hat{j}\right) = -3\hat{i} + 7\hat{j}$
$\frac{1}{4}(7b - a) = \frac{1}{4}\left(7\hat{i} + 21\hat{j} + \hat{i} - \hat{j}\right) = 2\hat{i} + 5\hat{j}$

Angle between $4a + b$ and $\frac{1}{4}(7b - a)$

i.e, $\cos\theta = \frac{\left(-3\hat{i} + 7\hat{j}\right).\left(2\hat{i} + 5\hat{j}\right)}{\sqrt{9+49}\sqrt{4+25}}$

$\cos\theta = \frac{-6+35}{\sqrt{58}\sqrt{29}} = \frac{29}{29\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\theta = 45°$

Question 135

Let a and b be two vectors with |a| = 13, | b | = 19 and |a − b| = 22. The value of |a + b| is

Options:

A. 16

B. 18

C. 20

D. 24

Answer: D

Solution:

Solution:

Given,

$|a| = 13, | b | = 19$ and $|a - b| = 22$

$|a + b|^2 + | a - b |^2 = 2[\,|a|^2 + | b|^2\,]$

$|a + b|^2 + (22)^2 = 2(13^2 + 19^2)$

$|a + b|^2 = 2(169 + 361) - 484 \Rightarrow | a + b |^2 = 576$

$|a + b| = \sqrt{576} = 24$

Question 136

The value of $20! + \frac{21!}{1!} + \frac{22!}{2!} + ... + \frac{60!}{40!}$ is

Options:

A. $20!^{61}C_{20}$

B. $21!^{60}C_{20}$

C. $20!^{61}C_{21}$

D. $21!^{61}C_{19}$

Answer: C

Solution:

Solution:
We have,
$20! + \frac{21!}{1!} + \frac{22!}{2!} + ... + \frac{60!}{40!}$
$= 20!\left[1 + \frac{21!}{20!1!} + \frac{22!}{20!2!} + ... + \frac{60!}{40!20!}\right]$
$= 20![^{21}C_0 + {}^{21}C_1 + {}^{22}C_2 + ... + {}^{60}C_{40}]$
$= 20![^{22}C_1 + {}^{22}C_2 + ... + {}^{60}C_{40}]$ $[\because {}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r]$
$= 20![^{61}C_{40}] = 20! \cdot {}^{61}C_{21}$

Question 137

The ratio of coefficients of 9 th and 7 th terms in the expansion of $(1 + x)^n$ is 9 : 7. Then, the coefficient of 4 th term is

Options:

A. 395

B. 455

C. 530

D. 645

Answer: B

Solution:

Solution:
We have, $(1 + x)^n$
$T_9 = {}^{n}C_8 x^8$
$T_7 = {}^{n}C_6 x^6$
Given, $^{n}C_8 : {}^{n}C_6 = 9 : 7 \Rightarrow \frac{{}^{n}C_8}{{}^{n}C_6} = \frac{9}{7}$
$\Rightarrow \frac{n!6!(n-6)!}{8!(n-8)!n!} = \frac{9}{7}$
$\frac{(n-6)(n-7)}{8 \times 7} = \frac{9}{7}$
$\Rightarrow (n-6)(n-7) = 9 \times 8$
$\therefore n = 15$
Coefficient of 4 th term of $(1 + x)^{15}$
$^{15}C_3 = \frac{15!}{3!12!} = 455$

Question 138

Six balls are placed randomly into six cells. Then the probability that exactly one cell remains empty is

Options:

A. $\frac{29}{216}$

B. $\frac{35}{216}$

C. $\frac{25}{108}$

D. $\frac{31}{108}$

Answer: C

Solution:

Solution:
Six balls are placed randomly into six cells Total number of outcomes $= 6^6$
Number of ways exactly one cells remain empty is
${}^6C_1 \times {}^5C_4 \times \frac{6!}{2!} = 6 \times 5 \times 3600 = 10800$
Required probability $= \frac{10800}{6^6} = \frac{25}{108}$

Question 139

A pair of fair dice is tossed repeatedly until a sum of four or an odd sum appears. Then, the probability that a sum of four appears first is

Options:

A. $\frac{1}{3}$

B. $\frac{2}{5}$

C. $\frac{3}{8}$

D. $\frac{1}{7}$

Answer: D

Solution:

Solution:
The number of result of pair of dice which sum of 4 is 3 . The number of result of a pair of dice which odd sum is 18 .
The probability the sum 4 appear before the odd sum is $= \frac{3}{3+18} = \frac{3}{21} = \frac{1}{7}$

Question 140

Let A and B be two events with $P(A^c) = 0.55, P(B) = 0.36, (A \cup B) = 0.60$. Then $P(A \mid B^c)$ is

Options:

A. $\frac{1}{4}$

B. $\frac{3}{8}$

C. $\frac{1}{2}$

D. $\frac{5}{8}$

Answer: B

Solution:

Solution:
Given,
$P(A') = 0.55, P(B) = 0.36, P(A \cup B) = 0.60$
$P(A \cap B') = P(A) - P(A \cap B)$
$= P(A) - [P(A) + P(B) + P(A \cup B)]$
$= P(A \cup B) - P(B)$
$= 0.60 - 0.36 = 0.24$
$P(A / B') = \frac{P(A \cap B')}{P(B')} = \frac{0.24}{1 - 0.36} = \frac{0.24}{0.64} = \frac{3}{8}$

Question 141

Rural and urban students are equally likely to get admission in a college. If 100 students get admission, then the probability that more rural students get admission than urban students, is

Options:

A. $\left(\frac{1}{2}\right)^{100} {}^{100}P_{50}$

B. $\left(\frac{1}{2}\right)^{100} {}^{100}C_{50}$

C. $1 - \left(\frac{1}{2}\right)^{100} {}^{100}C_{50}$

D. $\frac{1}{2}\left(1 - \left(\frac{1}{2}\right)^{100} {}^{100}C_{50}\right)$

Answer: D

Solution:

Solution:
We have,
$n = 100, p = \frac{1}{2}, q = \frac{1}{2}$
Required probability
$p(r > 50) = {}^{100}C_{51}\left(\frac{1}{2}\right)^{100} + {}^{100}C_{52}\left(\frac{1}{2}\right)^{100} + \ldots\ldots {}^{100}C_{100}\left(\frac{1}{2}\right)^{100}$
$= \left(\frac{1}{2}\right)^{100}({}^{100}C_{51} + {}^{100}C_{52} + \ldots\ldots {}^{100}C_{100})$
$= \left(\frac{1}{2}\right)^{100}\left(\frac{1}{2}(2^{100} - {}^{100}C_{50})\right)$
$= \frac{1}{2}\left[1 - \left(\frac{1}{2}\right)^{100} {}^{100}C_{50}\right]$

Question 142

The value of $\sin 20° \sin 40° \sin 80°$ equals

Options:

A. $\frac{\sqrt{3}}{2}$

B. $\frac{\sqrt{3}}{4}$

C. $\frac{\sqrt{3}}{8}$

D. $\frac{1}{8}$

Answer: C

Solution:

Solution:
We have, $\sin 20° \sin 40° \sin 80°$
$= \sin 20° \sin(60° - 20°) \sin(60° + 20°)$
$= \frac{1}{4} \sin 3(20°) = \frac{\sin 60°}{4} = \frac{\sqrt{3}}{8}$

Question 143

The sum of all solutions of the equation $\cos 3\theta = \sin 2\theta$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is

Options:

A. $-\frac{\pi}{5}$

B. $-\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{3\pi}{10}$

Answer: A

Solution:

Solution:
We have, $\cos 3\theta = \sin 2\theta$
$\Rightarrow 4\cos^3\theta - 3\cos\theta = 2\sin\theta\cos\theta$
$\Rightarrow 4\cos^2\theta - 3 = 2\sin\theta$ or $\cos\theta = 0$
$\Rightarrow 4(1 - \sin^2\theta) - 3 = 2\sin\theta$

or $\theta = \frac{\pi}{2}, -\frac{\pi}{2}$

Now, $4\sin^2\theta + 2\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{-2 \pm \sqrt{20}}{8}$

$\Rightarrow \sin\theta = \frac{-1 \pm \sqrt{5}}{4}$

$\Rightarrow \sin\theta = \frac{\sqrt{5}-1}{4}$ and $\sin\theta = \frac{-(\sqrt{5}+1)}{4}$

$\theta_1 = \frac{\pi}{10}$ and $\theta_2 = \frac{-3\pi}{10}$

$\Rightarrow \theta_1 + \theta_2 = \frac{\pi}{10} - \frac{3\pi}{10} = -\frac{\pi}{5}$

Question 144

In ΔABC, ∠B = 90° and perpendicular from B on AC intersects it at D. If AC = 4BD, then the smallest angle of ΔABC is

Options:

A. $\frac{\pi}{12}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{6}$

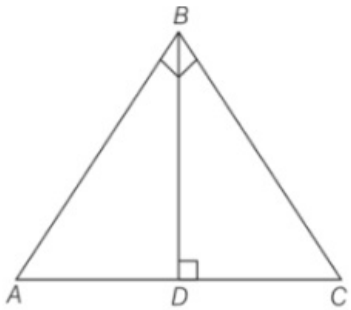
D. $\frac{\pi}{5}$

Answer: A

Solution:

Solution:

Given, In ΔABC



∠B = 90°

BD is perpendicular on AC

AC = 4BD

In ΔABD

$\Rightarrow \tan A = \frac{BD}{AD}$.....(i)

$AD = BD\cot A$

In ΔBDC

$\Rightarrow \tan C = \frac{BD}{CD}$

$CD = BD\cot C$ .....(ii)

From Eqs. (i) and (ii)

$AD + CD = BD(\cot A + \cot C)$

$AD + CD = BD(\cot A + \cot C)$

$AC = BD\left(\cot A + \cot\left(\frac{\pi}{2} - A\right)\right)\ \left[\because A + C = \frac{\pi}{2}\right]$

$4BD = BD(\cot A + \tan A)$

$4 = \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$

$\Rightarrow 4 = \frac{1}{\sin A \cos A}$

$\Rightarrow 2 = \frac{1}{2\sin A \cos A}$

$\Rightarrow \sin 2A = \frac{1}{2}$

$\Rightarrow 2A = 30°, 150°$

$\Rightarrow A = 15°, 75°$

$\Rightarrow$ Hence, $C = 75°, 15°$

$\therefore$ Smallest angle is 15° or, $\frac{\pi}{12}$

Question 145

A solution of the equation $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$ is

Options:

A. 1

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{6}$

Answer: D

Solution:

Solution:
We have,

$\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{2x + 3x}{1 - 6x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{5x}{1 - 6x^2} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{5x}{1 - 6x^2} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow (6x - 1)(x + 1) = 0$

$\Rightarrow x = \frac{1}{6}$

[x must be positive]

Question 146

The angle of elevation from a window to the top of a flag is 60° and the angle of depression to the base of the flag is 30°. The horizontal distance of the window from the flag is 6m. Then, the height of the flag

is

Options:

A. $2\sqrt{3}$m

B. $4\sqrt{3}$m

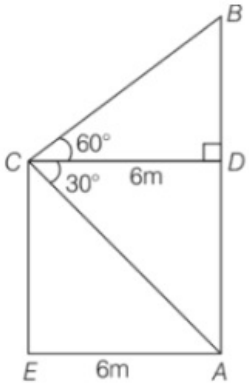
C. $8\sqrt{3}$m

D. $16\sqrt{3}$m

Answer: C

Solution:

Solution:
Let AB = height of flag



In ΔBDC

$\tan 60° = \frac{BD}{CD}$

$BD = 6\sqrt{3}$

In ΔACD,

$\tan 30° = \frac{AD}{CD} \Rightarrow AD = \frac{6}{\sqrt{3}}$

Height of flag $= BD + AD = 6\sqrt{3} + \frac{6}{\sqrt{3}}$

$= 6\sqrt{3} + 2\sqrt{3} = 8\sqrt{3}$m

Question 147

A car travels 200km is 2h and travels 240km in next 3h. If the acceleration is constant, then the distance it will travel in the next one hour, is

Options:

A. 48km

B. 64km

C. 72km

D. 84km

Answer: D

Solution:

Solution:
Let u and a are initial velocity and acceleration of cars, respectively.
$\therefore s = ut + \frac{1}{2}at^2$
$s = 200, t = 2$
$200 = 2u + 2a \Rightarrow u + a = 100$ .....(i)
$s_1 + s_2 = u(t_1 + t_2) + \frac{1}{2}a(t_1 + t_2)^2$
$s_1 = 200, s_2 = 240, t_1 = 2, t_2 = 3$
$440 = 5u + \frac{25a}{2} \Rightarrow 10u + 25a = 880$ .....(ii)
From Eqs. (i) and (ii), we get $u = 108, a = -8$
$s_1 + s_2 + s_3 = u(t_1 + t_2 + t_3) + \frac{1}{2}a(t_1 + t_2 + t_3)^2$
$t_1 = 2, t_2 = 3, t_3 = 1, s_1 = 200, s_2 = 240, s_3 = ?$
$440 + s_3 = 108(6) - \frac{8}{2}(6)^2$
$s_3 = 648 - 144 - 440 = 84km$

Question 148

A ball is thrown vertically upwards. It is at the same height h after 10s and 20s. Then the value of h is (assume g = 9.8m / s^2)

Options:

A. 270m

B. 360m

C. 490m

D. 980m

Answer: D

Solution:

Solution:
$h = ut + \frac{1}{2}gt^2$
$t = 10, u = u, g = -9.8$
$h = 10u - \frac{9.8(100)}{2}$
$t = 20, u = u, g = -9.8$
$h = 20u - \frac{9.8(400)}{2}$
Since height are equal
$\therefore 10u - \frac{9.8}{2}(100) = 20u - \frac{9.8}{2}(400)$
$u = 147$
$h = 10(147) - \frac{9.8}{2}(100) = 980m$

Question 149

Two forces P and 2P are inclined at 120° with each other. If their resultant makes an angle α with their bisector, then the value of α (in degree) is

Options:

A. 15°

B. 30°

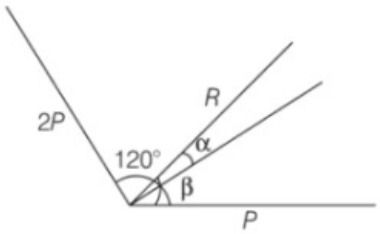
C. 45°

D. 60°

Answer: B

Solution:

Solution:
Two force P and 2P



$\tan\theta = \frac{2P\sin 120°}{P + 2P\cos 120°}$

$\tan\theta = \frac{\sqrt{3}P}{0} = \infty$

$\theta = 90°$

$\therefore \alpha = 90° - 60° = 30°$

Question 150

Two forces, of equal magnitude P, are inclined at angle α with each other. If the resultant is also P, then the angle α (in degree) is

Options:

A. 30°

B. 45°

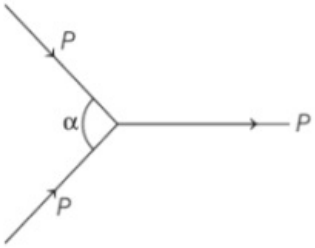
C. 60°

D. 120°

Answer: D

Solution:

Solution:
Two force of equal magnitude P



$P = \sqrt{P^2 + P^2 + 2P^2 \cos\alpha}$

$\Rightarrow P = \sqrt{2P^2 + 2P^2 \cos\alpha}$

$\Rightarrow P^2 = 2P^2(1 + \cos\alpha)$

$\Rightarrow 1 + \cos\alpha = \frac{1}{2}$

$\Rightarrow \cos\alpha = -\frac{1}{2}$

$\Rightarrow \alpha = 120°$
