MANIPAL
Engineering Entrance Exam Solved Paper 2009

PHYSICS

1. In ruby laser, the stimulated emission is due to transition from
   (a) metastable state to any lower state
   (b) any higher state to lower state
   (c) metastable state to ground state
   (d) any higher state to ground state

2. A direct current I flows along the length of an infinitely long straight thin walled pipe, then the magnetic field is
   (a) uniform throughout the pipe but not zero
   (b) zero only along the axis of the pipe
   (c) zero at any point inside the pipe
   (d) maximum at the centre and minimum at the edge

3. A convex lens made of glass has focal length 0.15 m in air. If the refractive index of glass is $\frac{3}{2}$ and that of water is $\frac{4}{3}$, the focal length of lens when immersed in water is
   (a) 0.45 m (b) 0.15 m
   (c) 0.30 m (d) 0.6 m

4. Two sources are said to be coherent if they produce waves
   (a) having a constant phase difference
   (b) of equal wavelength
   (c) of equal speed
   (d) having same shape of wavefront

5. Three resistors 1 Ω, 2 Ω and 3 Ω are connected to form a triangle. Across 3 Ω resistor a 3 V battery is connected. The current through 3 Ω resistor is
   (a) 0.75 A (b) 1 A
   (c) 2 A (d) 1.5 A

6. In a common emitter amplifier the input signal is applied across
   (a) anywhere (b) emitter-collector
   (c) collector-base (d) base-emitter

7. In a radioactive disintegration, the ratio of initial number of atoms to the number of atoms present at an instant of time equal to its mean life is
   (a) $\frac{1}{e^2}$ (b) $\frac{1}{e}$
   (c) e (d) e^2

8. A ray of light is incident on a surface of glass slab at an angle 45°. If the lateral shift produced per unit thickness is $\frac{1}{\sqrt{3}}$ m, the angle of refraction produced is
   (a) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (b) $\tan^{-1}\left(1-\sqrt{\frac{2}{3}}\right)$
   (c) $\sin^{-1}\left(1-\sqrt{\frac{2}{3}}\right)$ (d) $\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}-1}}\right)$

9. Ferromagnetic materials used in a transformer must have
   (a) low permeability and high hysterisis loss
   (b) high permeability and low hysterisis loss
   (c) high permeability and high hysterisis loss
   (d) low permeability and low hysterisis loss

10. According to Newton's corpuscular theory, the speed of light is
   (a) same in all the media
   (b) lesser in rarer medium
   (c) lesser in denser medium
   (d) independent of the medium

11. For the constructive interference the path difference between the two interfering waves must be equal to
   (a) $(2n + 1)\lambda$ (b) $2n\pi$
   (c) $n\lambda$ (d) $(2n + 1)\frac{\lambda}{2}$

12. The accurate measurement of emf can be obtained using
   (a) multimeter (b) voltmeter
   (c) voltameter (d) potentiometer

13. The kinetic energy of an electron gets tripled, then the de-Broglie wavelength associated with it changes by a factor

(a) $\frac{1}{3}$ (b) $\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) 3

14. Which of the following is not a thermodynamic coordinate?

(a) Gas constant (R) (b) Pressure (p)

(c) Volume (V) (d) Temperature (T)

15. Two solid pieces, one of steel and the other of aluminium when immersed completely in water have equal weights. When the solid pieces are weighed in air

(a) the weight of aluminium is half the weight of steel

(b) steel piece will weigh more

(c) they have the same weight

(d) aluminium piece will weigh more

16. The amount of energy released when one microgram of matter is annihilated is

(a) 25 kWh (b) 9×10^{10} kWh

(c) 3×10^{10} kWh (d) 0.5×10^{5} kWh

17. The number of significant figures in the numbers 4.8000×10^{4} and 48000.50 are respectively

(a) 5 and 6 (b) 5 and 7

(c) 2 and 7 (d) 2 and 6

18. β-decay means emission of electron from

(a) innermost electron orbit

(b) a stable nucleus

(c) outermost electron orbit

(d) radioactive nucleus

19. An electric heater rated 220 V and 550 W is connected to AC mains. The current drawn by it is

(a) 0.8 A (b) 2.5 A

(c) 0.4 A (d) 1.25 A

20. A body of mass m moving along a straight line covers half the distance with a speed of $2\,\text{ms}^{-1}$. The remaining half of the distance is covered in two equal time intervals with a speed of $3\,\text{ms}^{-1}$ and $5\,\text{ms}^{-1}$ respectively. The average speed of the particle for the entire journey is

(a) $\frac{3}{8}\,\text{ms}^{-1}$ (b) $\frac{8}{3}\,\text{ms}^{-1}$

(c) $\frac{4}{3}\,\text{ms}^{-1}$ (d) $\frac{16}{3}\,\text{ms}^{-1}$

21. The moment of inertia of a circular ring of radius r and mass M about diameter is

(a) $\frac{2}{5}\text{Mr}^2$ (b) $\frac{\text{Mr}^2}{4}$

(c) $\frac{\text{Mr}^2}{2}$ (d) $\frac{\text{Mr}^2}{12}$

22. A body of mass 0.05 kg is observed to fall with an acceleration of $9.5\ \text{ms}^{-2}$. The opposing force of air on the body is ($g = 9.8\ \text{ms}^{-2}$)

(a) 0.015 N (b) 0.15 N

(c) 0.030 N (d) zero

23. The colloidal solution in which both the dispersed phase and dispersion medium are liquids are called

(a) emulsions (b) gels

(c) foams (d) liquid crystals

24. In fog, photographs of the objects taken with infrared radiations are more clear than those obtained during visible light because

(a) I-*R* radiation has lesser wavelength than visible radiation

(b) scattering of *I-R* light is more than visible light

(c) the intensity of *I-R* light from the object is less

(d) scattering of *I-R* light is less than visible light

25. Three concurrent co-planar forces 1 N, 2 N and 3 N acting along different directions on a body

(a) can keep the body in equilibrium if 2 N and 3 N act at right angle

(b) can keep the body in equilibrium if 1 N and 2 N act at right angle

(c) cannot keep the body in equilibrium

(d) can keep the body in equilibrium in 1 N and 3 N act at an acute angle

26. Sound waves transfer

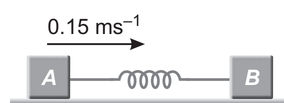
(a) only energy not momentum

(b) energy

(c) momentum

(d) Both (a) and (b)

27. Two rectangular blocks A and B of masses 2 kg and 3 kg respectively are connected by a spring of spring constant $10.8\ \text{Nm}^{-1}$ and are placed on a frictionless horizontal surface. The block A was given an initial velocity of $0.15\ \text{ms}^{-1}$ in the direction shown in the figure. The maximum compression of the spring during the motion is



(a) 0.01 m (b) 0.02 m

(c) 0.05 m (d) 0.03 m

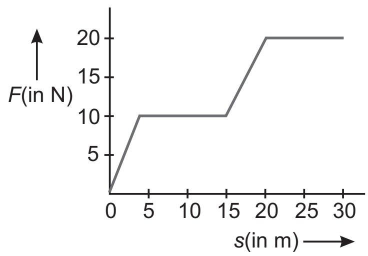
28. G P Thomson experimentally confirmed the existence of matter waves by the phenomena

(a) diffraction (b) refraction
(c) polarisation (d) scattering

29. The resistance of a wire at 300 K is found to be 0.3 Ω. If the temperature coefficient of resistance of wire is $1.5 \times 10^{-3}\ K^{-1}$, the temperature at which the resistance becomes 0.6 Ω is

(a) 720 K (b) 345 K
(c) 993 K (d) 690 K

30. The work done by a force acting on a body is as shown in the graph. The total work done in covering an initial distance of 20 m is



(a) 225 J (b) 200 J
(c) 400 J (d) 175 J

31. Two luminous point sources separated by a certain distance are at 10 km from an observer. If the aperture of his eye is 2.5×10^{-3} m and the wavelength of light used is 500 nm, the distance of separation between the point sources just seen to be resolved is

(a) 12.2 m (b) 24.2 m
(c) 2.44 m (d) 1.22 m

32. A door 1.6 m wide requires a force of 1 N to be applied at the free end to open or close it. The force that is required at a point 0.4 m distance from the hinges for opening or closing the door is

(a) 1.2 N (b) 3.6 N
(c) 2.4 N (d) 4 N

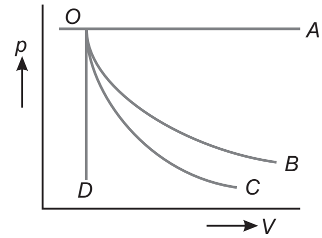
33. 0.1 m^3 of water at 80°C is mixed with 0.3 m^3 of water at 60°C. The final temperature of the mixture is

(a) 65°C (b) 70°C
(c) 60°C (d) 75°C

34. The spectral series of the hydrogen atom that lies in the visible region of the electromagnetic spectrum

(a) Paschen (b) Balmer
(c) Lyman (d) Brackett

35. A graph of pressure *versus* volume for an ideal gas for different processes is as shown. In the graph curve *OC* represents

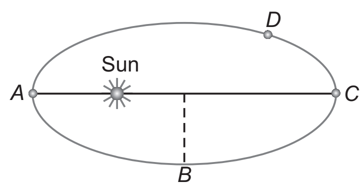


(a) isochoric process
(b) isothermal process
(c) isobaric process
(d) adiabatic process

36. Which of the following statement does not hold good for thermal radiation?

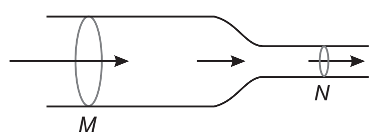
(a) The wavelength changes when it travels from one medium to another
(b) The frequency changes when it travels from one medium to another
(c) The speed changes when it travels from one medium to another
(d) They travel in straight line in a given medium

37. A planet revolves around the sun in an elliptical orbit. The linear speed of the planet will be maximum at



(a) D (b) B
(c) A (d) C

38. Horizontal tube of non-uniform corss-section has radii of 0.1 m and 0.05 m respectively at M and N. For a streamline flow of liquid the rate of liquid flow is



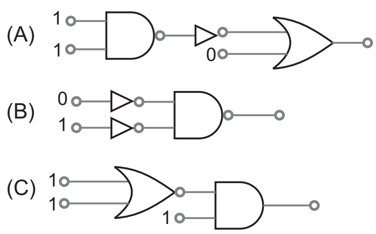
(a) changing continuously with time
(b) greater at *M* than at *N*
(c) greater at *N* than at *M*
(d) same at *M* and *N*

39. A resistor and a capacitor are connected in series with an AC source. If the potential drop across the capacitor is 5 V and that across resistor is 12 V, then applied voltage is
(a) 13 V (b) 17 V
(c) 5 V (d) 12 V

40. The amount of heat energy radiated by a metal at temperature T is E. When the temperature is increased to 3T, energy radiated is
(a) 81E (b) 9E
(c) 3E (d) 27E

41. The angle of minimum deviation for an incident light ray on an equilateral prism is equal to its refracting angle. The refractive index of its material is
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{3}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{3}{2}$

42. In the following combinations of logic gates, the outputs of A, B and C are respectively



(a) 0, 1, 1 (b) 0, 1, 0
(c) 1, 1, 0 (d) 1, 0, 1

43. A stationary point source of sound emits sound uniformly in all directions in a non-absorbing medium. Two points P and Q are at a distance of 4 m and 9 m respectively from the source. The ratio of amplitudes of the waves at P and Q is
(a) $\frac{3}{2}$ (b) $\frac{4}{9}$
(c) $\frac{2}{3}$ (d) $\frac{9}{4}$

44. A galvanometer of resistance 240 Ω allows only 4% of the main current after connecting a shunt resistance. The value of the shunt resistance is
(a) 10 Ω (b) 20 Ω
(c) 8 Ω (d) 5 Ω

45. The phenomena in which proton flips is
(a) nuclear magnetic resonance
(b) lasers
(c) radioactivity
(d) nuclear fusion

46. $y = 3 \sin \pi \left(\frac{t}{2} - \frac{x}{4}\right)$ represents an equation of a progressive wave, where t is in second and x is in metre. The distance travelled by the wave in 5 s is
(a) 8 m (b) 10 m
(c) 5 m (d) 32 m

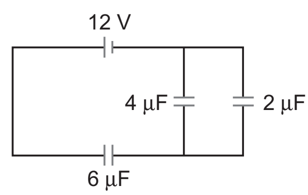
47. According to the quark model, it is possible to build all the hadrons using
(a) 2 quarks and 3 antiquarks
(b) 3 quarks and 2 antiquarks
(c) 3 quarks and 3 antiquarks
(d) 2 quarks and 2 antiquarks

48. An α-particle of mass 6.4×10^{-27} kg and charge 3.2×10^{-19} C is situated in a uniform electric field of 1.6×10^{5} Vm^{-1}. The velocity of the particle at the end of 2×10^{-2} m path when it starts from rest is
(a) $2\sqrt{3} \times 10^{5}$ ms^{-1} (b) 8×10^{5} ms^{-1}
(c) 16×10^{5} ms^{-1} (d) $4\sqrt{2} \times 10^{5}$ ms^{-1}

49. A cylindrical tube open at both the ends has a fundamental frequency of 390 Hz in air. If $\frac{1}{4}$th of the tube is immersed vertically in water the fundamental frequency of air column is
(a) 260 Hz (b) 130 Hz
(c) 390 Hz (d) 520 Hz

50. The surface temperature of the stars is determined using
(a) Planck's law
(b) Wien's displacement law
(c) Rayleigh-Jeans law
(d) Kirchhoff's law

51. The charge deposited on 4 μF capacitor in the circuit is



(a) 6×10^{-6} C (b) 12×10^{-6} C
(c) 24×10^{-6} C (d) 36×10^{-6} C

52. A parallel beam of light is incident on a converging lens parallel to its principal axis. As one moves away from the lens on the other side of the principal axis, the intensity of light

(a) first decreases and then increases
(b) continuously increases
(c) continuously decreases
(d) first increases and then decreases

53. Continuous emission spectrum is produced by
(a) incandescent electric lamp
(b) mercury vapour lamp
(c) sodium vapour lamp
(d) polyatomic substances

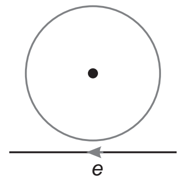
54. A coil of *n* number of turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current of strength *I* is passed through the coil, the magnetic field at its centre is
(a) $\frac{\mu_0 nI}{(b-a)} \log_e \frac{a}{b}$ (b) $\frac{\mu_0 nI}{2(b-a)}$
(c) $\frac{2\mu_0 nI}{b}$ (d) $\frac{\mu_0 nI}{2(b-a)} \log_e \frac{b}{a}$

55. A ray of light is incident on a plane mirror at an angle of 60°. The angle of deviation produced by the mirror is
(a) 120° (b) 30°
(c) 60° (d) 90°

56. The electric potential at any point x, y, z in metres is given by $V = 3x^2$. The electric field at a point (2, 0, 1) is
(a) $12\ Vm^{-1}$ (b) $-6\ Vm^{-1}$
(c) $6\ Vm^{-1}$ (d) $-12\ Vm^{-1}$

57. Young's double slit experiment gives interference fringes of width 0.3 mm. A thin glass plate made of material of refractive index 1.5 is kept in the path of light from one of the slits, then the fringe width becomes
(a) zero (b) 0.3 mm
(c) 0.45 mm (d) 0.15 mm

58. Near a circular loop of conducting wire as shown in the figure an electron moves along a straight line. The direction of the induced current if any in the loop is



(a) variable (b) clockwise
(c) anticlockwise (d) zero

59. Hydrogen atom from excited state comes to the ground stage by emitting a photon of wavelength λ. If R is the Rydberg constant, the principal quantum number *n* of the excited state is
(a) $\sqrt{\frac{\lambda R}{\lambda R - 1}}$ (b) $\sqrt{\frac{\lambda}{\lambda R - 1}}$
(c) $\sqrt{\frac{\lambda R^2}{\lambda R - 1}}$ (d) $\sqrt{\frac{\lambda R}{\lambda - 1}}$

60. The magnetic dipole moment of a current loop is independent of
(a) magnetic field in which it is lying
(b) number of turns
(c) area of the loop
(d) current in the loop

CHEMISTRY

1. The correct statement with regard to H_2^+ and H_2^- is
(a) both H_2^+ and H_2^- are equally stable
(b) both H_2^+ and H_2^- do not exist
(c) H_2^- is more stable than H_2^+
(d) H_2^+ is more stable than H_2^-

2. Arrange the following in the increasing order of their bond order

$$O_2, O_2^+, O_2^- \text{ and } O_2^{2-}$$

(a) $O_2^{2-}, O_2^-, O_2, O_2^+$ (b) $O_2^{2-}, O_2^-, O_2^+, O_2$
(c) $O_2^+, O_2, O_2^-, O_2^{2-}$ (d) $O_2, O_2^+, O_2^-, O_2^{2-}$

3. 2 g of a radioactive sample having half-life of 15 days was synthesised on 1st Jan 2009. The amount of the sample left behind on 1st March, 2009 (including both the days) is
(a) 0 g (b) 0.125 g
(c) 1 g (d) 0.5 g

4. For a chemical reaction $A \rightarrow B$, the rate of the reaction is 2×10^{-3} mol dm^{-3} s^{-1}, when the initial concentration is 0.05 mol dm^{-3}. The rate of the same reaction is 1.6×10^{-2} mol dm^{-3} s^{-1} when the initial concentration is 0.1 mol dm^{-3}. The order of the reaction is
(a) 2 (b) 0
(c) 3 (d) 1

5. For the decomposition of a compound AB at 600 K, the following data were obtained

$[AB]$ mol dm^{-3}	Rate of decomposition of AB in mol dm^{-3} s^{-1}
0.20	2.75×10^{-8}
0.40	11.0×10^{-8}
0.60	24.75×10^{-8}

The order for the decomposition of AB is
(a) 1.5 (b) 0
(c) 1 (d) 2

6. The rate equation for a reaction $A \rightarrow B$ is $r = k[A]^0$. If the initial concentration of the reactant is a mol dm^{-3}, the half-life period of the reaction is
(a) $\frac{a}{2k}$ (b) $\frac{k}{a}$
(c) $\frac{a}{k}$ (d) $\frac{2a}{k}$

7. 30 cc of $\frac{M}{3}$ HCl, 20 cc of $\frac{M}{2}$ HNO_3 and 40 cc of $\frac{M}{4}$ NaOH solutions are mixed and the volume was made up to 1 dm^3. The pH of the resulting solution is
(a) 8 (b) 2
(c) 1 (d) 3

8. An aqueous solution containing 6.5 g of NaCl of 90% purity was subjected to electrolysis. After the complete electrolysis, the solution was evaporated to get solid NaOH. The volume of 1 M acetic acid required to neutralise NaOH obtained above is
(a) 1000 cm^3 (b) 2000 cm^3
(c) 100 cm^3 (d) 200 cm^3

9. The standard electrode potential for the half-cell reactions are

$$Zn^{2+} + 2e^- \longrightarrow Zn;\ E^\circ = -0.76\ V$$
$$Fe^{2+} + 2e^- \longrightarrow Fe;\ E^\circ = -0.44\ V$$

The emf of the cell reaction,

$$Fe^{2+} + Zn \longrightarrow Zn^{2+} + Fe \text{ is}$$

(a) – 0.32 V (b) –1.20 V
(c) +1.20 V (d) +0.32 V

10. 10^{-6} M NaOH is diluted 100 times. The pH of the diluted base is
(a) between 7 and 8
(b) between 5 and 6
(c) between 6 and 7
(d) between 10 and 11

11. In the electrolysis of acidulated water, it is desired to obtain 1.12 cc of hydrogen per second under STP condition. The current to be passed is
(a) 1.93 A (b) 9.65 A
(c) 19.3 A (d) 0.965 A

12. The one which decreases with dilution is
(a) molar conductance
(b) conductance
(c) specific conductance
(d) equivalent conductance

13. Vapour pressure of pure 'A' is 70 mm of Hg at 25°C. It forms an ideal solution with 'B' in which mole fraction of A is 0.8. If the vapour pressure of the solution is 84 mm of Hg at 25°C, the vapour pressure of pure 'B' at 25°C is
(a) 28 mm (b) 56 mm
(c) 70 mm (d) 140 mm

14. A 6% solution of urea is isotonic with
(a) 1 M solution of glucose
(b) 0.05 M solution of glucose
(c) 6% solution of glucose
(d) 25% solution of glucose

15. In countries nearer to polar region, the roads are sprinkled with $CaCl_2$. This is
(a) to minimise the wear and tear of the roads
(b) to minimise the snow fall
(c) to minimise pollution
(d) to minimise the accumulation of dust on the road

16. For the reaction $H_2O(l) \rightleftharpoons H_2O(g)$ at 373 K and 1 atm pressure
(a) $\Delta H = 0$ (b) $\Delta E = 0$
(c) $\Delta H = T\Delta S$ (d) $\Delta H = \Delta E$

17. A compound of 'A' and 'B' crystallises in a cubic lattice in which 'A' atoms occupy the lattice points at the corners of the cube. The 'B' atoms occupy the centre of each face of the cube. The probable empirical formula of the compound is
(a) AB_2 (b) A_3B
(c) AB (d) AB_3

18. In electrophilic aromatic substitution reaction, the nitro group is *meta* directing because it
(a) decreases electron density at *ortho* and *para* positions
(b) decreases electron density at *meta* position
(c) increases electron density at *meta* position
(d) increases electron density at *ortho* and *para* positions

19. $CH_3COOH \xrightarrow{LiAlH_4} X \xrightarrow[300^\circ C]{Cu} Y \xrightarrow[NaOH]{Dilute} Z$

In the above reaction Z is

(a) butanol (b) aldol
(c) ketol (d) acetal

20. The best method for the conversion of an alcohol into an alkyl chloride is by treating the alcohol with
(a) PCl_3
(b) PCl_5
(c) $SOCl_2$ in presence of pyridine
(d) dry HCl in the presence of anhydrous $ZnCl_2$

21. The electrophile involved in the sulphonation of benzene is
(a) SO_3^+ (b) SO_3^{2-}
(c) H_3^+O (d) SO_3

22. The carbon-carbon bond length in benzene is
(a) in between C_2H_6 and C_2H_4
(b) same as in C_2H_4
(c) in between C_2H_6 and C_2H_2
(d) in between C_2H_4 and C_2H_2

23. The compound which is not formed during the dry distillation of a mixture of calcium formate and calcium acetate is
(a) methanal (b) propanal
(c) propanone (d) ethanal

24. An organic compound *X* is oxidised by using acidified $K_2Cr_2O_7$. The product obtained reacts with phenyl hydrazine but does not answer silver mirror test. The possible structure of *X* is
(a) CH_3CH_2OH (b) $CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$
(c) $(CH_3)_2CHOH$ (d) CH_3CHO

25. The reaction involved in the oil of winter green test is salicylic acid $\xrightarrow[\text{Conc. } H_2SO_4]{\Delta}$ product. The product is treated with Na_2CO_3 solution. The missing reagent in the above reaction is
(a) phenol (b) NaOH
(c) ethanol (d) methanol

26. The compound which forms acetaldehyde when heated with dilute NaOH, is
(a) 1, 1-dichloroethane
(b) 1, 1, 1-trichloroethane
(c) 1-chloroethane
(d) 1, 2-dichloroethane

27. Arrange the following in the increasing order of their basic strengths CH_3NH_2, $(CH_3)_2NH$, $(CH_3)_3N$, NH_3
(a) $NH_3 < (CH_3)_3N < (CH_3)_2NH < CH_3NH_2$
(b) $NH_3 < (CH_3)_3N < CH_3NH_2 < (CH_3)_2NH$
(c) $(CH_3)_3N < NH_3 < CH_3NH_2 < (CH_3)_2NH$
(d) $CH_3NH_2 < (CH_3)_2NH < (CH_3)_3N < NH_3$

28. The one which has least iodine value is
(a) sunflower oil (b) ginger oil
(c) ghee (d) groundnut oil

29. A diabetic person carries a packet of glucose with him always, because
(a) glucose reduces the blood sugar level slowly
(b) glucose increases the blood sugar level slowly
(c) glucose reduces the blood sugar level
(d) glucose increases the blood sugar level almost instantaneously

30. There are 20 naturally occurring amino acids. The maximum number of tripeptides that can be obtained is
(a) 8000 (b) 6470
(c) 7465 (d) 5360

31. Cooking is fast in a pressure cooker, because
(a) food particles are effectively smashed
(b) water boils at higher temperature inside the pressure cooker
(c) food is cooked at constant volume
(d) loss of heat due to radiation is minimum

32. The ore that is concentrated by froth floatation process is
(a) zincite (b) cinnabar
(c) bauxite (d) malachite

33. The correct set of four quantum numbers for outermost electron of potassium ($Z = 19$) is
(a) $4, 1, 0, \frac{1}{2}$ (b) $3, 1, 0, \frac{1}{2}$
(c) $4, 0, 0, \frac{1}{2}$ (d) $3, 0, 0, \frac{1}{2}$

34. A body of mass *x* kg is moving with a velocity of 100 ms^{-1}. Its de-Broglie wavelength is 6.62×10^{-35} m. Hence, x is ($h = 6.62 \times 10^{-34}$ Js)
(a) 0.1 kg (b) 0.25 kg
(c) 0.15 kg (d) 0.2 kg

35. The correct order of ionisation energy of C, N, O, F is
(a) $F < O < N < C$ (b) $F < N < C < O$
(c) $C < N < O < F$ (d) $C < O < N < F$

36. The oxide of an element whose electronic configuration is $1s^2, 2s^2, 2p^6, 3s^1$ is
(a) neutral (b) amphoteric
(c) basic (d) acidic

37. The characteristic not related to alkali metal is
(a) high ionisation energy
(b) their ions are isoelectronic with noble gases
(c) low melting point
(d) low electronegativity

38. Among the following, the compound that contains ionic, covalent and coordinate linkage is
(a) NH_3 (b) NH_4Cl
(c) NaCl (d) CaO

39. A covalent molecule AB_3 has pyramidal structure. The number of lone pair and bond pair of electrons in the molecule are respectively
(a) 2 and 2 (b) 0 and 4
(c) 3 and 1 (d) 1 and 3

40. Excess of carbon dioxide is passed through 50 mL of 0.5 M calcium hydroxide solution. After the completion of the reaction, the solution was evaporated to dryness. The solid calcium carbonate was completely neutralised with 0.1 N hydrochloric acid. The volume of hydrochloric acid required is (Atomic mass of calcium = 40)
(a) 300 cm^3 (b) 200 cm^3
(c) 500 cm^3 (d) 400 cm^3

41. A bivalent metal has an equivalent mass of 32. The molecular mass of the metal nitrate is
(a) 182 (b) 168
(c) 192 (d) 188

42. The rms velocity of molecules of a gas of density 4 kg m^{-3} and pressure 1.2×10^5 Nm^{-2} is
(a) 300 ms^{-1} (b) 900 ms^{-1}
(c) 120 ms^{-1} (d) 600 ms^{-1}

43. 0.5 mole of each of H_2, SO_2 and CH_4 are kept in a container. A hole was made in the container. After 3 h, the order of partial pressures in the container will be
(a) $pSO_2 > pH_2 > pCH_4$
(b) $pSO_2 > pCH_4 > pH_2$
(c) $pH_2 > pSO_2 > pCH_4$
(d) $pH_2 > pCH_4 > pSO_2$

44. The enthalpy of formation of NH_3 is – 46 kJ mol^{-1}. The enthalpy change for the reaction

$$2NH_3(g) \longrightarrow N_2(g) + 3H_2(g) \text{ is}$$

(a) +184 kJ (b) +23 kJ
(c) +92 kJ (d) +46 kJ

45. 5 moles of SO_2 and 5 moles of O_2 are allowed to react. At equilibrium, it was found that 60% of SO_2 is used up. If the partial pressure of the equilibrium mixture is one atmosphere, the partial pressure of O_2 is
(a) 0.82 atm (b) 0.52 atm
(c) 0.21 atm (d) 0.41 atm

46. $2HI(g) \rightleftharpoons H_2(g) + I_2(g)$
The equilibrium constant of the above reaction is 6.4 at 300 K. If 0.25 mole each of H_2 and I_2 are added to the system, the equilibrium constant will be
(a) 6.4 (b) 0.8
(c) 3.2 (d) 1.6

47. Rate of physical adsorption increases with
(a) decrease in surface area
(b) decrease in temperature
(c) decrease in pressure
(d) increase in temperature

48. IUPAC name of $(CH_3)_3CCl$ is
(a) *n*-butyl chloride
(b) 3-chloro butane
(c) 2-chloro 2-methyl propane
(d) *t*-butyl chloride

49. Lucas test is associated with
(a) aldehydes (b) phenols
(c) carboxylic acids (d) alcohols

50. An organic compound on heating with CuO produces CO_2 but no water. The organic compound may be
(a) carbon tetrachloride
(b) chloroform
(c) methane
(d) ethyl iodide

51. The condensation polymer among the following is
(a) rubber (b) protein
(c) PVC (d) polyethene

52. The order of stability of metal oxides is
(a) $Al_2O_3 < MgO < Fe_2O_3 < Cr_2O_3$
(b) $Cr_2O_3 < MgO < Al_2O_3 < Fe_2O_3$
(c) $Fe_2O_3 < Cr_2O_3 < Al_2O_3 < MgO$
(d) $Fe_2O_3 < Al_2O_3 < Cr_2O_3 < MgO$

53. The temperature of the slag zone in the metallurgy of iron using blast furnace is
(a) 1200–1500°C (b) 1500–1600°C
(c) 400–700°C (d) 800–1000°C

54. The function of $Fe(OH)_3$ in the contact process is
(a) to remove arsenic impurity
(b) to detect colloidal impurity
(c) to remove moisture
(d) to remove dust particles

55. In which of the following, NH_3 is not used?
(a) Tollen's reagent
(b) Nessler's reagent
(c) Group reagent for the analysis of IV group basic radicals
(d) Group reagent for the analysis of III group basic radicals

56. Argon is used
(a) in filling airships
(b) to obtain low temperature
(c) in high temperature welding
(d) in radiotherapy for treatment of cancer

57. The incorrect statement in respect of chromyl chloride test is
(a) formation of red vapours
(b) formation of lead chromate
(c) formation of chromyl chloride
(d) liberation of chlorine

58. The magnetic moment of a transition metal ion is $\sqrt{15}$ BM. Therefore, the number of unpaired electrons present in it, is
(a) 3 (b) 4
(c) 1 (d) 2

59. The IUPAC name of $[Co(NH_3)_5ONO]^{2+}$ ion is
(a) penta ammine nitrito cobalt (IV) ion
(b) penta ammine nitrito cobalt (III) ion
(c) penta ammine nitro cobalt (III) ion
(d) penta ammine nitro cobalt (IV) ion

60. The oxidation state of Fe in the brown ring complex: $[Fe(H_2O)_5NO]SO_4$ is
(a) +3 (b) 0
(c) +2 (d) +1

MATHEMATICS

1. If $f : [2, 3] \to R$ is defined by $f(x) = x^3 + 3x - 2$, then the range $f(x)$ is contained in the interval
(a) [1, 12] (b) [12, 34]
(c) [35, 50] (d) [−12, 12]

2. $\left\{x \in R : \dfrac{2x-1}{x^3+4x^2+3x} \in R\right\}$ equals
(a) $R - \{0\}$
(b) $R - \{0, 1, 3\}$
(c) $R - \{0, -1, -3\}$
(d) $R - \left\{0, -1, -3, +\dfrac{1}{2}\right\}$

3. Using mathematical induction, the numbers a_n's are defined by,
$$a_0 = 1, a_{n+1} = 3n^2 + n + a_n, (n \geq 0).$$
Then, a_n is equal to
(a) $n^3 + n^2 + 1$ (b) $n^3 - n^2 + 1$
(c) $n^3 - n^2$ (d) $n^3 + n^2$

4. The number of subsets of {1, 2, 3, ..., 9} containing at least one odd number is
(a) 324 (b) 396
(c) 496 (d) 512

5. *p* points are chosen on each of the three coplanar lines. The maximum number of triangles formed with vertices at these points is
(a) $p^3 + 3p^2$ (b) $\dfrac{1}{2}(p^3 + p)$
(c) $\dfrac{p^2}{2}(5p - 3)$ (d) $p^2(4p - 3)$

6. A binary sequence is an array of 0's and 1's. The number of *n*-digit binary sequences which contain even number of 0's is
(a) 2^{n-1} (b) $2^n - 1$
(c) $2^{n-1} - 1$ (d) 2^n

7. The coefficient of x^{24} in the expansion of $(1 + x^2)^{12}(1 + x^{12})(1 + x^{24})$ is
(a) $^{12}C_6$ (b) $^{12}C_6 + 2$
(c) $^{12}C_6 + 4$ (d) $^{12}C_6 + 6$

8. If x is numerically so small so that x^2 and higher powers of x can be neglected, then $\left(1 + \dfrac{2x}{3}\right)^{3/2} \cdot (32 + 5x)^{-1/5}$ is approximately equal to
(a) $\dfrac{32 + 31x}{64}$ (b) $\dfrac{31 + 32x}{64}$
(c) $\dfrac{31 - 32x}{64}$ (d) $\dfrac{1 - 2x}{64}$

9. For $|x| < 1$, the constant term in the expansion of $\dfrac{1}{(x-1)^2(x-2)}$ is
(a) 2 (b) 1
(c) 0 (d) $-\dfrac{1}{2}$

10. $\dfrac{1}{e^{3x}}(e^x + e^{5x}) = a_0 + a_1x + a_2x^2 + \ldots$
$\Rightarrow 2a_1 + 2^3a_3 + 2^5a_5 + \ldots$ is equal to
(a) e (b) e^{-1}
(c) 1 (d) 0

11. The roots of $(x-a)(x-a-1)+(x-a-1)(x-a-2)+(x-a)(x-a-2)=0$, $a \in R$ are always

(a) equal (b) imaginary

(c) real and distinct (d) rational and equal

12. Let $f(x)=x^2+ax+b$, where $a, b \in R$. If $f(x)=0$ has all its roots imaginary, then the roots of $f(x)+f'(x)+f''(x)=0$ are

(a) real and distinct (b) imaginary

(c) equal (d) rational and equal

13. If α, β, γ are the roots of $x^3+4x+1=0$, then the equation whose roots are $\frac{\alpha^2}{\beta+\gamma}, \frac{\beta^2}{\gamma+\alpha}, \frac{\gamma^2}{\alpha+\beta}$ is

(a) $x^3-4x-1=0$ (b) $x^3-4x+1=0$

(c) $x^3+4x-1=0$ (d) $x^3+4x+1=0$

14. If $f(x)=2x^4-13x^2+ax+b$ is divisible by x^2-3x+2, then (a, b) is equal to

(a) $(-9, -2)$ (b) $(6, 4)$

(c) $(9, 2)$ (d) $(2, 9)$

15. Let *A* and *B* be two symmetric matrices of same order. Then, the matrix $AB-BA$ is

(a) a symmetric matrix

(b) a skew-symmetric matrix

(c) a null matrix

(d) the identity matrix

16. If one of the roots of $\begin{vmatrix} 3 & 5 & x \\ 7 & x & 7 \\ x & 5 & 3 \end{vmatrix}=0$ is -10, then the other roots are

(a) 3, 7 (b) 4, 7

(c) 3, 9 (d) 3, 4

17. If *x, y, z* are all positive and are the *p*th, *q*th and *r*th terms of a geometric progression respectively, then the value of the determinant

$$\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} \text{ equals}$$

(a) $\log xyz$

(b) $(p-1)(q-1)(r-1)$

(c) pqr

(d) 0

18. If $\begin{bmatrix} 1 & -1 & x \\ 1 & x & 1 \\ x & -1 & 1 \end{bmatrix}$ has no inverse, then the real value of *x* is

(a) 2 (b) 3

(c) 0 (d) 1

19. If α and β are the roots of $x^2-2x+4=0$, then the value of $\alpha^6+\beta^6$ is

(a) 32 (b) 64

(c) 128 (d) 256

20. The locus of *z* satisfying the inequality $\left|\frac{z+2i}{2z+i}\right|<1$, where $z=x+iy$, is

(a) $x^2+y^2<1$ (b) $x^2-y^2<1$

(c) $x^2+y^2>1$ (d) $2x^2+3y^2<1$

21. If *n* is an integer which leaves remainder one when divided by three, then $(1+\sqrt{3}i)^n+(1-\sqrt{3}i)^n$ equals

(a) -2^{n+1} (b) 2^{n+1}

(c) $-(-2)^n$ (d) -2^n

22. The period of $\sin^4 x+\cos^4 x$ is

(a) $\frac{\pi^4}{2}$ (b) $\frac{\pi^2}{2}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

23. $\frac{\cos x}{\cos(x-2y)}=\lambda \Rightarrow \tan(x-y)\tan y$ is equal to

(a) $\frac{1+\lambda}{1-\lambda}$ (b) $\frac{1-\lambda}{1+\lambda}$

(c) $\frac{\lambda}{1+\lambda}$ (d) $\frac{\lambda}{1-\lambda}$

24. $\cos A \cos 2A \cos 4A \dots \cos 2^{n-1}A$ equals

(a) $\frac{\sin 2^n A}{2^n \sin A}$ (b) $\frac{2^n \sin 2^n A}{\sin A}$

(c) $\frac{2^n \sin A}{\sin 2^n A}$ (d) $\frac{\sin A}{2^n \sin 2^n A}$

25. If $3\cos x \neq 2\sin x$, then the general solution of $\sin^2 x-\cos 2x=2-\sin 2x$ is

(a) $n\pi+(-1)^n\frac{\pi}{2}, n \in Z$

(b) $\frac{n\pi}{2}, n \in Z$

(c) $(4n \pm 1)\frac{\pi}{2}, n \in Z$

(d) $(2n-1)\pi, n \in Z$

26. $\cos^{-1}\left(\frac{-1}{2}\right) - 2\sin^{-1}\left(\frac{1}{2}\right) + 3\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) - 4\tan^{-1}(-1)$ equals

(a) $\frac{19\pi}{12}$ (b) $\frac{35\pi}{12}$

(c) $\frac{47\pi}{12}$ (d) $\frac{43\pi}{12}$

27. $\sinh^{-1} 2 + \sinh^{-1} 3 = x \Rightarrow \cosh x$ is equal to

(a) $\frac{1}{2}(3\sqrt{5} + 2\sqrt{10})$ (b) $\frac{1}{2}(3\sqrt{5} - 2\sqrt{10})$

(c) $\frac{1}{2}(12 + 2\sqrt{50})$ (d) $\frac{1}{2}(12 - 2\sqrt{50})$

28. In any Δ ABC, $a(b\cos C - c\cos B)$ equals

(a) $b^2 + c^2$ (b) $b^2 - c^2$

(c) $\frac{1}{b} + \frac{1}{c}$ (d) $\frac{1}{b^2} - \frac{1}{c^2}$

29. In a Δ ABC

$$\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$$

equals

(a) $\cos^2 A$ (b) $\cos^2 B$

(c) $\sin^2 A$ (d) $\sin^2 B$

30. *P* is a point on the segment joining the feet of two vertical poles of heights *a* and *b*. The angles of elevation of the tops of the poles from *P* are 45° each. Then, the square of the distance between the tops of the poles is

(a) $\frac{a^2 + b^2}{2}$ (b) $a^2 + b^2$

(c) $2(a^2 + b^2)$ (d) $4(a^2 + b^2)$

31. In a quadrilateral *ABCD,* the point *P* divides *DC* in the ratio 1 : 2 and *Q* is the mid point of *AC*. If $\overrightarrow{\mathbf{AB}} + 2\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{BC}} - 2\overrightarrow{\mathbf{DC}} = k\,\overrightarrow{\mathbf{PQ}}$, then k is equal to

(a) –6 (b) –4

(c) 6 (d) 4

32. The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$, $l^2 + m^2 - n^2 = 0$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

33. If $\vec{\mathbf{a}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, then the angle between $2\vec{\mathbf{a}} - \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

34. If m_1, m_2, m_3 and m_4 are respectively the magnitudes of the vectors

$\vec{\mathbf{a}}_1 = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{a}}_2 = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$,

$\vec{\mathbf{a}}_3 = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{a}}_4 = -\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$,

then the correct order of m_1, m_2, m_3 and m_4 is

(a) $m_3 < m_1 < m_4 < m_2$

(b) $m_3 < m_1 < m_2 < m_4$

(c) $m_3 < m_4 < m_1 < m_2$

(d) $m_3 < m_4 < m_2 < m_1$

35. Suppose $\vec{\mathbf{a}} = \lambda\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\lambda\hat{\mathbf{k}}$. If the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is greater than 90°, then λ satisfies the inequality

(a) $-7 < \lambda < 1$ (b) $\lambda > 1$

(c) $1 < \lambda < 7$ (d) $-5 < \lambda < 1$

36. The volume of the tetrahedron having the edges $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$ as coterminous, is $\frac{2}{3}$ cubic unit. Then λ equals

(a) 1 (b) 2

(c) 3 (d) 4

37. If *A* and *B* are events of a random experiment such that $P(A \cup B) = \frac{4}{5}$, $P(\bar{A} \cup \bar{B}) = \frac{7}{10}$ and $P(B) = \frac{2}{5}$, then $P(A)$ equals

(a) $\frac{9}{10}$ (b) $\frac{8}{10}$

(c) $\frac{7}{10}$ (d) $\frac{3}{5}$

38. The probability of choosing randomly a number *c* from the set {1, 2, 3, ..., 9} such that the quadratic equation $x^2 + 4x + c = 0$ has real roots is

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$

(c) $\frac{3}{9}$ (d) $\frac{4}{9}$

39. Suppose that E_1 and E_2 are two events of a random experiment such that $P(E_1) = \frac{1}{4}$, $P(E_2/E_1) = \frac{1}{2}$ and $P(E_1/E_2) = \frac{1}{4}$, observe the lists given below

List I	**List II**
(A) $P(E_2)$	(i) 1/4
(B) $P(E_1 \cup E_2)$	(ii) 5/8
(C) $P(\bar{E}_1/\bar{E}_2)$	(iii) 1/8
(D) $P(E_1/\bar{E}_2)$	(iv) 1/2
	(v) 3/8
	(vi) 3/4

The correct matching of the List I from the List II is

	(A)	(B)	(C)	(D)
(a)	(ii)	(iii)	(vi)	(i)
(b)	(iv)	(v)	(vi)	(i)
(c)	(iv)	(ii)	(vi)	(i)
(d)	(i)	(ii)	(iii)	(iv)

40. If *m* and σ^2 are the mean and variance of the random variable *X*, whose distribution is given by

$X = x$	0	1	2	3
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{2}$	0	$\frac{1}{6}$

Then
(a) $m = \sigma^2 = 2$ (b) $m = 1, \sigma^2 = 2$
(c) $m = \sigma^2 = 1$ (d) $m = 2, \sigma^2 = 1$

41. If *X* is a binomial variate with the range $\{0, 1, 2, 3, 4, 5, 6\}$ and $P(X = 2) = 4P(X = 4)$, then the parameter *p* of *X* is
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

42. The transformed equation of $x^2 + y^2 = r^2$ when the axes are rotated through an angle 36° is
(a) $\sqrt{5}X^2 - 4XY + Y^2 = r^2$
(b) $X^2 + 2XY - \sqrt{5}Y^2 = r^2$
(c) $X^2 - Y^2 = r^2$
(d) $X^2 + Y^2 = r^2$

43. The area (in square unit) of the circle which touches the lines $4x + 3y = 15$ and $4x + 3y = 5$ is
(a) 4π (b) 3π
(c) 2π (d) π

44. The point on the line $3x + 4y = 5$ which is equidistant from (1, 2) and (3, 4) is
(a) (7, –4) (b) (15, –10)
(c) (1/7, 8/7) (d) (0, 5/4)

45. The equation of the straight line perpendicular to the straight line $3x + 2y = 0$ and passing through the point of intersection of the lines $x + 3y - 1 = 0$ and $x - 2y + 4 = 0$ is
(a) $2x - 3y + 1 = 0$ (b) $2x - 3y + 3 = 0$
(c) $2x - 3y + 5 = 0$ (d) $2x - 3y + 7 = 0$

46. The value of λ with $|\lambda| < 16$ such that $2x^2 - 10xy + 12y^2 + 5x + \lambda y - 3 = 0$ represents a pair of straight lines, is
(a) –10 (b) –9
(c) 10 (d) 9

47. The area (in square unit) of the triangle formed by $x + y + 1 = 0$ and the pair of straight lines $x^2 - 3xy + 2y^2 = 0$ is
(a) $\frac{7}{12}$ (b) $\frac{5}{12}$
(c) $\frac{1}{12}$ (d) $\frac{1}{6}$

48. The pairs of straight lines $x^2 - 3xy + 2y^2 = 0$ and $x^2 - 3xy + 2y^2 + x - 2 = 0$ form a
(a) square but not rhombus
(b) rhombus
(c) parallelogram
(d) rectangle but not a square

49. The equations of the circle which pass through the origin and makes intercepts of lengths 4 and 8 on the *x* and *y*-axes respectively are
(a) $x^2 + y^2 \pm 4x \pm 8y = 0$
(b) $x^2 + y^2 \pm 2x \pm 4y = 0$
(c) $x^2 + y^2 \pm 8x \pm 16y = 0$
(d) $x^2 + y^2 \pm x \pm y = 0$

50. The locus of centre of a circle which passes through the origin and cuts off a length of 4 unit from the line $x = 3$ is
(a) $y^2 + 6x = 0$
(b) $y^2 + 6x = 13$
(c) $y^2 + 6x = 10$
(d) $x^2 + 6y = 13$

51. The diameters of a circle are along $2x + y - 7 = 0$ and $x + 3y - 11 = 0$. Then, the equation of this circle, which also passes through (5, 7), is

(a) $x^2 + y^2 - 4x - 6y - 16 = 0$
(b) $x^2 + y^2 - 4x - 6y - 20 = 0$
(c) $x^2 + y^2 - 4x - 6y - 12 = 0$
(d) $x^2 + y^2 + 4x + 6y - 12 = 0$

52. The point $(3, -4)$ lies on both the circles $x^2 + y^2 - 2x + 8y + 13 = 0$ and $x^2 + y^2 - 4x + 6y + 11 = 0$. Then, the angle between the circles is
(a) $60°$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$
(c) $\tan^{-1}\left(\frac{3}{5}\right)$ (d) $135°$

53. The equation of the circle which passes through the origin and cuts orthogonally each of the circles $x^2 + y^2 - 6x + 8 = 0$ and $x^2 + y^2 - 2x - 2y = 7$ is
(a) $3x^2 + 3y^2 - 8x - 13y = 0$
(b) $3x^2 + 3y^2 - 8x + 29y = 0$
(c) $3x^2 + 3y^2 + 8x + 29y = 0$
(d) $3x^2 + 3y^2 - 8x - 29y = 0$

54. The number of normals drawn to the parabola $y^2 = 4x$ from the point $(1, 0)$ is
(a) 0 (b) 1
(c) 2 (d) 3

55. If the distance between the foci of an ellipse is 6 and the length of the minor axis is 8, then the eccentricity is
(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{1}{2}$
(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

56. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points (x_i, y_i), for $i = 1, 2, 3$ and 4, then $y_1 + y_2 + y_3 + y_4$ equals
(a) 0 (b) c
(c) a (d) c^4

57. The mid point of the chord $4x - 3y = 5$ of the hyperbola $2x^2 - 3y^2 = 12$ is
(a) $\left(0, -\frac{5}{3}\right)$ (b) $(2, 1)$
(c) $\left(\frac{5}{4}, 0\right)$ (d) $\left(\frac{11}{4}, 2\right)$

58. The eccentricity of the conic
$$\frac{5}{r} = 2 + 3\cos\theta + 4\sin\theta \text{ is}$$
(a) $\frac{1}{2}$ (b) 1
(c) $\frac{3}{2}$ (d) $\frac{5}{2}$

59. The perimeter of the triangle with vertices at $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ is
(a) 3 (b) 2
(c) $2\sqrt{2}$ (d) $3\sqrt{2}$

60. If a line in the space makes angle α, β and γ with the coordinate axes, then
$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma + \sin^2\alpha + \sin^2\beta + \sin^2\gamma \text{ equals}$$
(a) –1 (b) 0
(c) 1 (d) 2

61. The image of the point $(3, 2, 1)$ in the plane $2x - y + 3z = 7$ is
(a) $(1, 2, 3)$ (b) $(2, 3, 1)$
(c) $(3, 2, 1)$ (d) $(2, 1, 3)$

62. The radius of the sphere $x^2 + y^2 + z^2 = 12x + 4y + 3z$ is
(a) $\frac{13}{2}$ (b) 13
(c) 26 (d) 52

63. $\lim_{x \to \infty}\left(\frac{x+5}{x+2}\right)^{x+3}$ equals
(a) e (b) e^2
(c) e^3 (d) e^5

64. If $f : R \to R$ is defined by
$$f(x) = \begin{cases} \dfrac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases},$$
then the value of a so that f is continuous at 0 is
(a) 2 (b) 1
(c) –1 (d) 0

65. $x = \frac{1-\sqrt{y}}{1+\sqrt{y}} \Rightarrow \frac{dy}{dx}$ is equal to
(a) $\frac{4}{(x+1)^2}$ (b) $\frac{4(x-1)}{(1+x)^3}$
(c) $\frac{x-1}{(1+x)^3}$ (d) $\frac{4}{(x+1)^3}$

66. $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right), y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right) \Rightarrow \frac{dy}{dx}$ is equal to

(a) 0 (b) tan t

(c) 1 (d) sin t cos t

67. $\frac{d}{dx}\left[a \tan^{-1} x + b \log\left(\frac{x-1}{x+1}\right)\right] = \frac{1}{x^4 - 1}$

$\Rightarrow$ a – 2b is equal to

(a) 1 (b) –1

(c) 0 (d) 2

68. $y = e^{a \sin^{-1} x} \Rightarrow (1 - x^2) y_{n+2} - (2n+1) x y_{n+1}$ is equal to

(a) $-(n^2 + a^2) y_n$

(b) $(n^2 - a^2) y_n$

(c) $(n^2 + a^2) y_n$

(d) $-(n^2 - a^2) y_n$

69. There is an error of ± 0.04 cm in the measurement of the diameter of a sphere. When the radius is 10 cm, the percentage error in the volume of the sphere is

(a) ± 1.2 (b) ± 1.0

(c) ± 0.8 (d) ± 0.6

70. The function $f(x) = x^3 + ax^2 + bx + c,\ a^2 \le 3b$ has

(a) one maximum value

(b) one minimum value

(c) no extreme value

(d) one maximum and one minimum value

71. The maximum value of $\frac{\log x}{x}$, $0 < x < \infty$ is

(a) ∞ (b) *e*

(c) 1 (d) e^{-1}

72. $z = \tan(y + ax) + \sqrt{y - ax} \Rightarrow z_{xx} - a^2 z_{yy}$ is equal to

(a) 0 (b) 2

(c) $z_x + z_y$ (d) $z_x z_y$

73. $\int \frac{dx}{(x+1)\sqrt{4x+3}}$ is equal to

(a) $\tan^{-1}\sqrt{4x+3} + c$

(b) $3\tan^{-1}\sqrt{4x+3} + c$

(c) $2\tan^{-1}\sqrt{4x+3} + c$

(d) $4\tan^{-1}\sqrt{4x+3} + c$

74. $\int \left(\frac{2 - \sin 2x}{1 - \cos 2x}\right) e^x\, dx$ is equal to

(a) $-e^x \cot x + c$ (b) $e^x \cot x + c$

(c) $2e^x \cot x + c$ (d) $-2e^x \cot x + c$

75. If $I_n = \int \sin^n x\, dx$, then $nI_n - (n-1)I_{n-2}$ equals

(a) $\sin^{n-1} x \cos x$ (b) $\cos^{n-1} x \sin x$

(c) $-\sin^{n-1} x \cos x$ (d) $-\cos^{n-1} x \sin x$

76. $\int_0^{\pi} \frac{1}{1 + \sin x}\, dx$ is equal to

(a) 1 (b) 2

(c) –1 (d) –2

77. The line $x = \frac{\pi}{4}$ divides the area of the region bounded by $y = \sin x$, $y = \cos x$ and *x*-axis $\left(0 \le x \le \frac{\pi}{2}\right)$ into two regions of areas A_1 and A_2. Then A_1, A_2 equals

(a) 4 : 1 (b) 3 : 1

(c) 2 : 1 (d) 1 : 1

78. The velocity of a particle which starts from rest is given by the following table.

t (in second)	0	2	4	6	8	10
v (in m/s)	0	12	16	20	35	60

The total distance travelled (in metre) by the particles in 10 s, using Trapezoidal rule is given by

(a) 113 (b) 226

(c) 143 (d) 246

79. The solution of the differential equation $\frac{dy}{dx} = \sin(x + y)\tan(x + y) - 1$ is

(a) $\operatorname{cosec}(x + y) + \tan(x + y) = x + c$

(b) $x + \operatorname{cosec}(x + y) = c$

(c) $x + \tan(x + y) = c$

(d) $x + \sec(x + y) = c$

80. The differential equation of the family $y = ae^x + bx\,e^x + cx^2 e^x$ of curves, where *a, b, c* are arbitrary constants, is

(a) $y''' + 3y'' + 3y' + y = 0$

(b) $y''' + 3y'' - 3y' - y = 0$

(c) $y''' - 3y'' - 3y' + y = 0$

(d) $y''' - 3y'' + 3y' - y = 0$

GENERAL ENGLISH AND APTITUDE

◆ *Directions* (Q. 1-4) : In each of the following questions, choose the alternative which best expresses the meaning of the word given in capital letters.

1. INTREPID
(a) Coward (b) Fearless
(c) Selfish (d) Ugly

2. LAMENTABLE
(a) Deplorable (b) Contemptible
(c) Remorseful (d) Scornful

3. DEFT
(a) Skilful (b) Vigorous
(c) Swift (d) Deceitful

4. HOARD
(a) Destroy (b) Hide
(c) Store (d) Divide

◆ ***Directions* (Q. 5-8)** : In each of the following questions, choose the most suitable alternative to fill in the blank.

5. He is too dull …… the problem.
(a) solves (b) to solve
(c) solving (d) to solving

6. The speaker drew the attention of the audience …… the burning issue.
(a) to (b) towards
(c) on (d) into

7. It's nine o'clock …… and I'm still at breakfast.
(a) till (b) yet
(c) so (d) already

8. Although he is blind, he is very fast …… calculations.
(a) in (b) with
(c) at (d) about

◆ ***Directions* (Q. 9-12) :** In each of the following questions, choose the alternative which is opposite in meaning to the word given in capital letters.

9. VIRTUE
(a) Vice (b) Fraud
(c) Wickedness (d) Crime

10. ZEAL
(a) Hostility (b) Diffidence
(c) Apathy (d) Contempt

11. AGONY
(a) Enmity (b) Cruelty
(c) Abhorrence (d) Ecstasy

12. ERUDITE
(a) Professional (b) Immature
(c) Unimaginative (d) Ignorant

◆ ***Directions* (Q. 13-16) :** In each of the following questions, choose the alternative which can be substituted for the given words/sentence.

13. A person who speaks for or supports an idea
(a) Pioneer (b) Adviser
(c) Advocate (d) Ideologist

14. A man of odd habits
(a) Eccentric (b) Cynical
(c) Introvert (d) Moody

15. A thing or person behind time
(a) Lazy (b) Sluggish
(c) Indolent (d) Antiquated

16. One whose attitude is : 'eat, drink and be merry'
(a) Epicurean (b) Cynic
(c) Materialistic (d) Stoic

◆ ***Directions* (Q. 17-20) :** In each of the following questions, choose the alternative which can best improve the given sentence by substituting the italicised portion. If the sentence is correct as it is, your answer is (d).

17. Older people *often* stay at home and watch TV because it is cold and dark in winter.
(a) seldom (b) frequently
(c) sometimes (d) No improvement

18. You must find *someone* to accompany you to Mumbai
(a) no one (b) everyone
(c) anyone (d) No improvement

19. No sooner *he reached* home than all the villagers gathered at his home to listen to his story.
(a) would he reach (b) did he reach
(c) have he reached (d) No improvement

20. I wish I *was* with him.
(a) have been (b) were
(c) am (d) No improvement

◆ ***Directions* (Q. 21-25) :** In the following questions, choose the option which shows common feature in the relationship given in each question.

21. Sandstone : Limestone : Coal
(a) They are formed by metamorphic rocks.
(b) They are chemical minerals.
(c) They are found in river beds.
(d) They are formed in sedimentary rocks.

22. Sweep : Scrub : Wipe
(a) These are terms connected with rubbing.
(b) These are games of cards.
(c) These are terms used by motor mechanics.
(d) These are terms connected with cleaning.

23. Delhi : Agra : Mathura
(a) They have been capitals of the country
(b) They have exquisite temples.
(c) They have religious background
(d) They are situated on the bank of river Yamuna.

24. Press : Television : Cinema
(a) They are means of entertainment.
(b) They are means of mass media.
(c) They give world wide news.
(d) All are public undertakings.

25. Comets : Stars : Satellites
(a) They are shining masses.
(b) They give out light.
(c) They are rotating from left to right
(d) They are heavenly bodies.

◆ ***Directions* (*Q.* 26-30) :** In the questions, choose the group of words that shows the same relationship as given at the top of every question.

26. Magazine : Story : Article
(a) Tea : Milk : Sugar
(b) Television : Newspaper : Entertainment
(c) Bed : Quilt : Pillow
(d) Novel : Drama : Literature

27. Juice : Orange : Banana
(a) Table : Chair : Wood
(b) Fish : Shork : Water
(c) Cow : Milk : Curd
(d) Ink : Pen : Pencil

28. Carnivorous : Tiger : Wolf
(a) Mango : Banana : Fruit
(b) Worker : Master : Manager
(c) Cat : Cow : Milk
(d) Student : Boy : Girl

29. Rain : Cloud : Evaporation
(a) Pain : Injury : Accident
(b) Cold : Cough : Sneezing
(c) Purse : Leather : Tanning
(d) Fragrance : Flower : Bud

30. Dog : Squirrel : Tail
(a) Cottage : Hut : Palace
(b) Fish : Crocodile : Water
(c) Horse : OX : Horn
(d) Truck : Scooter : Gear

31. The five intertwined rings or circles on the olympic flag made of white [From left to right] are
(a) Blue, yellow, black, green and red.
(b) Yellow, red, green, black and blue.
(c) Red, green, black, yellow and blue.
(d) Yellow, green black, blue and red.

32. The national remote sensing agency is located at
(a) Delhi (b) Hyderabad
(c) Bangalore (d) Lucknow

33. The Nobel Prize are given to Dec, 10 on the death anniversary of
(a) Linus Pauling (b) Frederic Sanger
(c) Alfred Bernard (d) John Bardeen

34. The term Gambit is used in
(a) Chess (b) Boning
(c) Baseball (d) Polo

35. Word's fastest missileship 'INS Prahar' was commissioned in
(a) 1996 (b) 1997
(c) 1998 (d) 1999

36. Nati is the classical dances of
(a) Tamil Nadu (b) Assam
(c) Himachal Pradesh (d) Andhra Pradesh

37. The headquarter of International Labour Organisation is located at
(a) Geneva (b) Paris
(c) New York (d) Rome

38. The first Indian woman to receive Bharat Ratna was
(a) Mother Teresa (b) Mrs. Indira Gandhi
(c) Ashapurana devi (d) Aarti Saha

39. 'Kip' is the currency of
(a) Kuwait (b) Lebanon
(c) Laos (d) Malaysia

40. The charter of the United Nations was signed on June 26, 1945 in
(a) San Francisco (b) Washington D.C.
(c) Landon (d) Trygue Le

Answers

❐ PHYSICS

1. (c)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (b)	**10.** (b)
11. (c)	**12.** (d)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (d)	**19.** (b)	**20.** (b)
21. (c)	**22.** (a)	**23.** (a)	**24.** (d)	**25.** (c)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (b)
31. (c)	**32.** (d)	**33.** (a)	**34.** (b)	**35.** (d)	**36.** (b)	**37.** (c)	**38.** (c)	**39.** (a)	**40.** (a)
41. (b)	**42.** (c)	**43.** (d)	**44.** (a)	**45.** (a)	**46.** (b)	**47.** (c)	**48.** (d)	**49.** (a)	**50.** (b)
51. (c)	**52.** (d)	**53.** (a)	**54.** (d)	**55.** (c)	**56.** (d)	**57.** (b)	**58.** (a)	**59.** (a)	**60.** (a)

❐ CHEMISTRY

1. (d)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (a)
11. (b)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (b)	**20.** (c)
21. (d)	**22.** (a)	**23.** (b)	**24.** (c)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (c)	**29.** (d)	**30.** (a)
31. (b)	**32.** (b)	**33.** (c)	**34.** (a)	**35.** (d)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (d)	**40.** (c)
41. (d)	**42.** (a)	**43.** (b)	**44.** (c)	**45.** (d)	**46.** (a)	**47.** (b)	**48.** (c)	**49.** (d)	**50.** (a)
51. (b)	**52.** (c)	**53.** (d)	**54.** (a)	**55.** (b)	**56.** (c)	**57.** (d)	**58.** (a)	**59.** (b)	**60.** (d)

❐ MATHEMATICS

1. (b)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (d)
11. (c)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (d)	**18.** (d)	**19.** (c)	**20.** (c)
21. (c)	**22.** (d)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (d)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (c)
31. (a)	**32.** (c)	**33.** (b)	**34.** (a)	**35.** (a)	**36.** (a)	**37.** (c)	**38.** (d)	**39.** (c)	**40.** (c)
41. (a)	**42.** (d)	**43.** (d)	**44.** (b)	**45.** (d)	**46.** (b)	**47.** (c)	**48.** (c)	**49.** (a)	**50.** (b)
51. (c)	**52.** (d)	**53.** (b)	**54.** (b)	**55.** (c)	**56.** (a)	**57.** (b)	**58.** (d)	**59.** (d)	**60.** (c)
61. (c)	**62.** (a)	**63.** (c)	**64.** (d)	**65.** (b)	**66.** (c)	**67.** (b)	**68.** (c)	**69.** (d)	**70.** (c)
71. (d)	**72.** (a)	**73.** (c)	**74.** (a)	**75.** (c)	**76.** (b)	**77.** (d)	**78.** (b)	**79.** (b)	**80.** (d)

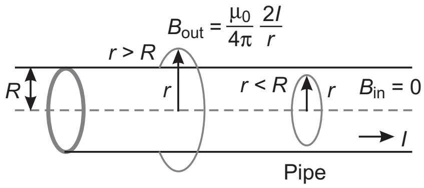
❐ GENERAL ENGLISH AND APTITUDE

1. (b)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (c)
11. (d)	**12.** (d)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (d)	**18.** (d)	**19.** (b)	**20.** (b)
21. (d)	**22.** (d)	**23.** (d)	**24.** (b)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (d)	**29.** (a)	**30.** (d)
31. (a)	**32.** (b)	**33.** (c)	**34.** (a)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (c)	**40.** (a)

Solutions

PHYSICS

1. For a ruby laser, a crystal of ruby is formed into a cylinder. A fully reflecting mirror is placed on one end and a partially reflecting mirror on the other. A high-intensity lamp is spiraled around the ruby cylinder to provide a flash of white light that triggers the laser action. The green and blue wavelengths in the flash excite electrons in the chromium atoms to a higher energy level. Upon returning to their normal state, the electrons emit their characteristic ruby-red light. The mirrors reflect some of this light back and forth inside the ruby crystal, stimulating other excited chromium atoms to produce more red light, until the light pulse builds up to high power and drains the energy stored in the crystal. In ruby laser stimulated emission is due to transition from metastable state to ground state.
2. Required arrangement is shown in figure. According to Ampere's circuital law.



For an internal point, r < R

$$B_{internal} = \frac{\mu_0(0)}{2\pi r} = 0$$

For a point on the pipe, r = R

$$B = \frac{\mu_0 I}{2\pi r}$$

For an external point, r < R

$$B_{external} = \frac{\mu_0 I}{2\pi r}$$

Therefore, option (c) is correct.

3. Given, $f_a = 0.15$ m, $\mu_g = \frac{3}{2}, \mu_w = \frac{4}{3}$

According to Lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{where } \mu = \frac{\mu_L}{\mu_M}$$

$$\frac{1}{f_a} = \left(\frac{\mu_g}{\mu_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{(3/2)}{1} - 1\right)C \quad \text{where } C = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$\frac{1}{f_a} = \frac{C}{2} \qquad \ldots\text{(i)}$$

Also, $$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{(3/2)}{(4/3)} - 1\right)C$$

or $$\frac{1}{f_w} = \frac{C}{8} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

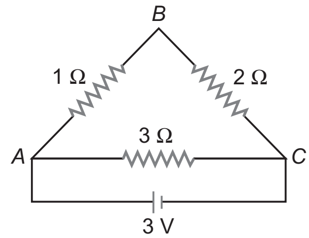
$$\frac{f_w}{f_a} = \frac{C}{2} \times \frac{8}{C} = 4$$

or $$f_w = 4f_a$$
$$= 4 \times 0.15 = 0.6 \text{ m}$$

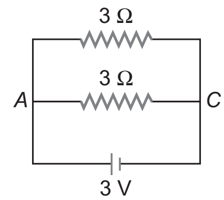
4. Phase difference $= \frac{2\pi}{\lambda} \times$ (path difference)

The phase difference is independent of time and depends only on the path difference $(x_2 - x_1)$. This holds only if the two sources are 'coherent', *ie*, they have a constant fixed phase difference between them.

5. Required arrangement is shown in figure.

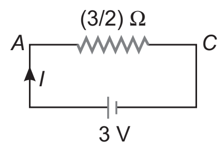


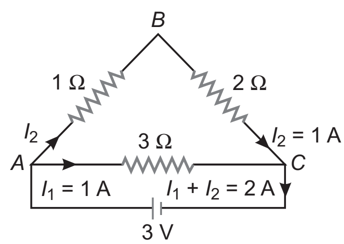
The equivalent circuit will look like (since the two resistances of 1 Ω and 2 Ω are in series, which form 3 Ω which is in parallel with 3 Ω resistance).



Therefore, the effective resistance is

$$\frac{(1+2)\times 3}{(1+2)+3} = \frac{3}{2}\,\Omega$$



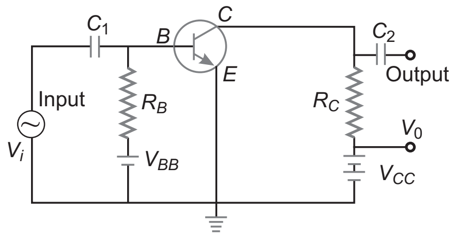


∴ Current in the circuit,

$$I = \frac{3}{(3/2)} = 2 \text{ A}$$

∴ Current in 3 Ω resistor $= \frac{I}{2} = 1$ A

6. In CE amplifier, the input signal is applied across base-emitter junction as shown in the figure below.



7. Let the initial number of atoms at time t = 0 be N_0.

Let N be the number of atoms at any instant t.

Mean life $\tau = \frac{1}{\lambda}$, where λ is disintegration constant.

Given, t = τ

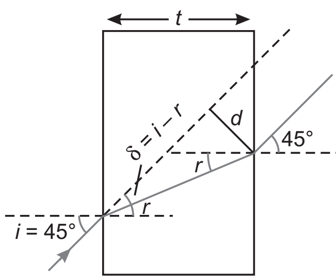
According to radioactive disintegration law,

$$N = N_0 e^{-\lambda t}$$

or $$N = N_0 e^{-\lambda \times \frac{1}{\lambda}} = \frac{N_0}{e}$$

or $$\frac{N_0}{N} = e$$

8. Here, angle of incidence i = 45°



$$\frac{\text{Lateral shift (d)}}{\text{Thickness of glass slab (t)}} = \frac{1}{\sqrt{3}}$$

$$\text{Lateral shift d} = \frac{t \sin \delta}{\cos r} = \frac{t \sin (i - r)}{\cos r}$$

$\Rightarrow$ $$\frac{d}{t} = \frac{\sin (i - r)}{\cos r}$$

or $$\frac{d}{t} = \frac{\sin i \cos r - \cos i \sin r}{\cos r}$$

or $$\frac{d}{t} = \frac{\sin 45° \cos r - \cos 45° \sin r}{\cos r}$$

$$= \frac{\cos r - \sin r}{\sqrt{2} \cos r}$$

or $$\frac{d}{t} = \frac{1}{\sqrt{2}}(1 - \tan r)$$

or $$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{2}}(1 - \tan r)$$

or $$\tan r = 1 - \frac{\sqrt{2}}{\sqrt{3}}$$

or $$r = \tan^{-1}\left(1 - \frac{\sqrt{2}}{\sqrt{3}}\right)$$

9. Ferromagnetic materials used in a transformer must have high permeability and low hysterisis loss.

10. According to Newton's corpuscular theory, speed of light in a rarer medium (like air) is lesser than that in a denser medium (like water, glass).

11. Phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

In a constructive interference,

$$\Delta\phi = 2n\pi \quad \text{where } n = 0, 1, 2, 3, \ldots$$

$\therefore$ $$2n\pi = \frac{2\pi}{\lambda}\Delta x$$

or $$\Delta x = n\lambda$$

12. In a potentiometer there is no current drawn from the cell whose emf is to be measured whereas a voltmeter always draws some current from the cell. Hence, the emf of a cell can be measured accurately using a potentiometer.

13. de-Broglie wavelength of an electron is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

or $$\lambda \propto \frac{1}{\sqrt{K}}$$

$\therefore$ $$\frac{\lambda'}{\lambda} = \frac{1}{\sqrt{3K}} \frac{\sqrt{K}}{1} = \frac{1}{\sqrt{3}}$$

or $$\lambda' = \frac{\lambda}{\sqrt{3}}$$

Hence, de-Broglie wavelength will change by factor $\frac{1}{\sqrt{3}}$.

14. Pressure (p), volume (V) and temperature (T) are the thermodynamic coordinates whereas R is a universal gas constant valued at 8.314 J mol^{-1} K^{-1}.

15. In air, force of gravity acts on metals. Thus, these have their actual weight. Atomic weight of steel, *ie*, iron is 56 and that of aluminium is 27. Hence, it can be said that in air the weight of aluminium is half the weight of steel.

16. According to Einstein's mass energy equivalence relation

$$\begin{aligned} E &= mc^2 \\ &= 10^{-6} \times 10^{-3} \times (3 \times 10^8)^2 \text{ J} \\ &= 9 \times 10^7 \text{ J} \\ &= \frac{9 \times 10^7}{3.6 \times 10^6} \text{ kWh} \\ & \qquad (\because 1 \text{ kWh} = 3.6 \times 10^6 \text{ J}) \\ &= 25 \text{ kWh} \end{aligned}$$

17. The number of significant figures in 4.8000×10^4 is 5 (zeros on right after decimal are counted while zeros in powers of 10 are not counted).

The number of significant figures in 48000.50 is 7 (all the zeros between two non-zero digits are significant).

18. β-emission takes place from a radioactive nucleus as

$$^{32}_{15}P \xrightarrow{-\beta} {}^{32}_{16}S + {}_{-1}e^0 + \bar{\nu}$$

where $\bar{\nu}$ is the anti-neutrino.

In β^+ decay a positron is emitted as

$$^{22}_{11}Na \longrightarrow {}^{22}_{10}Ne + {}_{+1}e^0 + \nu$$

where ν is the neutrino.

19. $I = \dfrac{P}{V} = \dfrac{550}{220} = 2.5 \text{ A}$

20. Let the total distance travelled by the body is $2S$.

If t_1 is the time taken by the body to travel first half of the distance, then

$$t_1 = \frac{S}{2}$$

Let t_2 be the time taken by the body for each time interval for the remaining half journey.

$\therefore \qquad S = 3t_2 + 5t_2 = 8t_2$

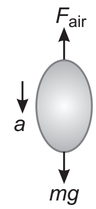
So, average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{2S}{t_1 + 2t_2} = \frac{2S}{\frac{S}{2} + \frac{S}{4}} = \frac{8}{3} \text{ ms}^{-1}$$

21. Moment of inertia of a circular ring about a diameter

$$I = \frac{1}{2} Mr^2$$

22. From, Newton's laws of motion,



$$mg - F_{air} = ma$$

or $\qquad F_{air} = m(g - a)$

$$= 0.05\,(9.8 - 9.5) = 0.015 \text{ N}$$

23. Emulsion is a colloidal solution in which both the dispersed phase and dispersion medium are liquids. In most common emulsions, water is used as dispersed phase and oil is used as dispersion medium.

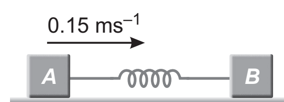
24. In fog, visible light is scattered more according to Rayleigh scattering, but scattering of infrared radiations is less due to high wavelengths, hence in fog, photographs of the objects taken with infrared radiations are more clear.

25. If we keep 1 N and 2 N forces act in same direction then these are balanced by 3 N force, but this is against statement of question.

Hence, options (c) is correct.

26. Sound waves transfer both energy and momentum.

27. As the block A moves with velocity 0.15 ms^{-1}, it compresses the spring which pushes B towards right. A goes on compressing the spring till the velocity acquired by B becomes equal to the velocity of A, *ie*, 0.15 ms^{-1}. Let this velocity be v. Now, spring is in a state of maximum compression. Let x be the maximum compression at this stage.



According to the law of conservation of linear momentum, we get

$$m_A u = (m_A + m_B) v$$

or $\qquad v = \dfrac{m_A u}{m_A + m_B}$

$$= \frac{2 \times 0.15}{2+3} = 0.06 \text{ ms}^{-1}$$

According to the law of conservation of energy.

$$\frac{1}{2} m_A u^2 = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2} kx^2$$

$$\frac{1}{2} m_A u^2 - \frac{1}{2}(m_A + m_B)v^2 = \frac{1}{2} kx^2$$

$$\frac{1}{2} \times 2 \times (0.15)^2 - \frac{1}{2}(2+3)(0.06)^2 = \frac{1}{2} kx^2$$

$$0.0225 - 0.009 = \frac{1}{2} kx^2 \text{ or } 0.0135 = \frac{1}{2} kx^2$$

or $\quad x = \sqrt{\frac{0.027}{k}} = \sqrt{\frac{0.027}{10.8}} = 0.05 \text{ m}$

28. G P Thomson experimentally confirmed the existence of matter waves (de-Broglie's hypothesis) by demonstrating that electron beams are diffracted when they are scattered by the regular atomic arrays of crystals.

29. Given, $R_{300} = 0.3\,\Omega$, $R_t = 0.6\,\Omega$,

$$T = 300 \text{ K} = 27°\text{C}$$

Temperature coefficient of resistance,

$$\alpha = 1.5 \times 10^{-3} \text{ K}^{-1}$$

$\therefore \qquad R_{300} = R_0(1 + \alpha \times 27)$

$$0.3 = R_0(1 + 1.5 \times 10^{-3} \times 27) \qquad \ldots\text{(i)}$$

Again, $\qquad R_t = R_0(1 + \alpha t)$

$$0.6 = R_0(1 + 1.5 \times 10^{-3} \times t) \qquad \ldots\text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{0.6}{0.3} = \frac{1 + 1.5 \times 10^{-3}\, t}{1 + 1.5 \times 10^{-3} \times 27}$$

$\Rightarrow \; 2(1 + 1.5 \times 10^{-3} \times 27) = 1 + 1.5 \times 10^{-3} t$

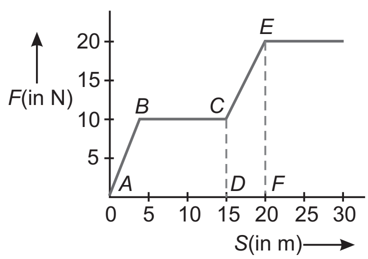
$\Rightarrow \quad 2 + 81 \times 10^{-3} = 1 + 1.5 \times 10^{-3} t$

$\Rightarrow \qquad 2 + 0.081 = 1 + 1.5 \times 10^{-3} t$

$\Rightarrow \quad t = \frac{1.081}{1.5 \times 10^{-3}} = 720°\text{C} = 993 \text{ K}$

30. Word done W = Area ABCEFDA

= Area ABCD + Area CEFD



$$= \frac{1}{2} \times (15 + 10) \times 10 + \frac{1}{2} \times (10 + 20) \times 5$$

$$= 125 + 75 = 200 \text{ J}$$

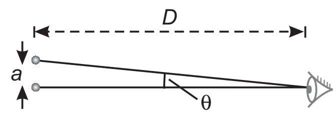
31. According to Rayleigh's criterion,

$$\theta = \frac{1.22\lambda}{d_e}$$

where λ = wavelength of light,

d_e = diameter of the pupil of the eye

$$\therefore \; \theta = \frac{1.22 \times 500 \times 10^{-9}}{2.5 \times 10^{-3}} = 2.44 \times 10^{-4} \text{ radian}$$



But $\qquad \theta = \frac{a}{D}$

$\therefore$ Distance of separation,

$$a = D \times \theta = 10 \times 10^3 \times 2.44 \times 10^{-4}$$

$$= 2.44 \text{ m}$$

32. Here, torque $\tau = 1.6 \times 1 = 1.6$ N-m

So, when d = 0.4 m,

$$F = \frac{\tau}{d} = \frac{1.6}{0.4} = 4 \text{ N}$$

33. Let the final temperature of the mixture be *t*

Heat lost by water at 80°C

$= ms\Delta t$

$= 0.1 \times 10^3 \times s_{water} \times (80° - t)$

$(\because m = V \times d = 0.1 \times 10^3 \text{ kg})$

Heat gained by water at 60°C

$= 0.3 \times 10^3 \times s_{water} \times (t - 60°)$

According to principle of calorimetry

Heat lost = Heat gained

$\therefore \qquad 0.1 \times 10^3 \times s_{water} \times (80° - t)$

$= 0.3 \times 10^3 \times s_{water} \times (t - 60°)$

or $\qquad (80° - t) = 3 \times (t - 60°)$

or $\qquad 4t = 260°$

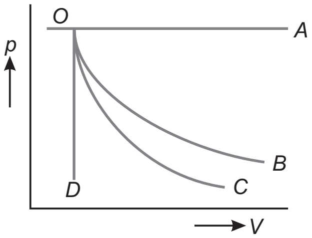
or $\qquad t = 65°\text{C}$

34.

Ultraviolet region	Lyman series
Visible region	Balmer series
Infrared region	Paschen series, Brackett sereis, Pfund series

From the above chart it is clear that Balmer series lies in the visible region of the electromagnetic spectrum.

35.



(i) Curve OA represents isobaric process (since pressure is constant)

Since, the slope of adiabatic process is more steeper than the isothermal process.

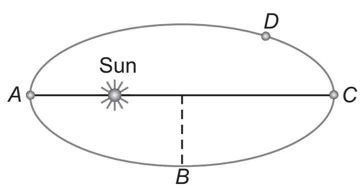
(ii) Curve OB represents isothermal process.

(iii) Curve OC represents adiabatic process.

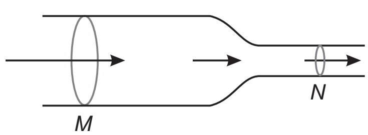
(iv) Curve OD represents isochoric process (since volume is constant).

36. Thermal radiation obeys the law of refraction and it travels along straight line like light. Hence, frequency does not change when travelling from one medium to another.

37. From Kepler's second law of planetary motion, the linear speed of a planet is maximum, when its distance from the sun is least, *ie*, at point *A*.



38. The velocity of flow will increase if cross-section decreases and *vice-versa*.

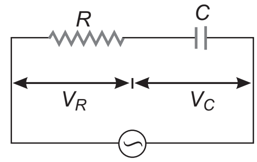


ie, $A_1v_1 = A_2v_2$

or $Av = \text{constant}$

Therefore, the rate of liquid flow will be greater at *N* than at *M*.

39. Let the applied voltage be *V* volt.



Here, $V_R = 12\text{ V}, V_C = 5\text{ V}$

$$\therefore \quad V = \sqrt{V_R^2 + V_C^2} = \sqrt{(12)^2 + (5)^2}$$

$$= \sqrt{144 + 25} = \sqrt{169} = 13\text{ V}$$

40. According to Stefan's law

$$E \propto T^4$$

$$\therefore \quad \frac{E'}{E} = \left(\frac{3T}{T}\right)^4 \text{ or } E' = 81E$$

41. For an equilateral prism,

angle of prism of refracting angle A = 60°

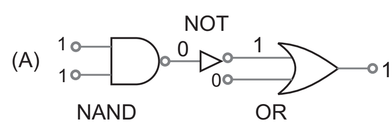
Here, $\delta_m = A = 60°$

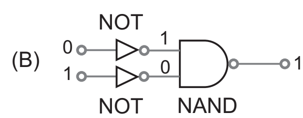
$\therefore$ Refractive index,

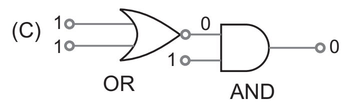
$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{60° + 60°}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$= \frac{\sin 60°}{\sin 30°} = \frac{\sin 60°}{\cos 60°} = \tan 60° = \sqrt{3}$$

42. The output gate circuit will be as shown below.

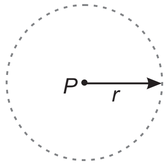






Hence, outputs of A, B and C are 1, 1 and 0 respectively.

43. For an isotropic point source of power P, intensity I at a distance r from it is



$$I = \frac{P}{4\pi r^2}$$

Since power P remains the same,

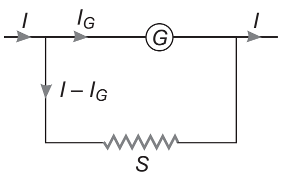
$$\therefore \quad \frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{9}{4}\right)^2$$

$$\because \quad I \propto A^2$$

where A is the amplitude of a wave

$$\therefore \quad \frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = \frac{9}{4}$$

44. Given, galvanometer resistance $G = 240\ \Omega$



Shunt resistance $S = ?$

$$I_G = \frac{4}{100} I$$

From figure voltage through the circuit.

$$(I - I_G)S = I_G G$$

or $$\left(1 - \frac{4I}{100}\right) S = \frac{4I}{100} \times 240$$

or $$S = \frac{4 \times 240}{96} = 10\ \Omega$$

45. The phenomena in which proton flips is nuclear magnetic resonance.

46. The given equation of a progressive wave is

$$y = 3 \sin \pi \left(\frac{t}{2} - \frac{x}{4}\right) = 3 \sin 2\pi \left(\frac{t}{4} - \frac{x}{8}\right)$$

The standard equation of a progressive wave is

$$y = y_0 \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Comparing these two equations, we get

$$T = 4\ \text{s}, \quad \lambda = 8\ \text{m}$$

$\therefore$ Velocity of wave,

$$v = \frac{\lambda}{T} = \frac{8}{4} = 2\ \text{ms}^{-1}$$

Distance travelled by wave in time t is

$$s = vt$$

or $$s = 2 \times 5 = 10\ \text{m}$$

47. Mesons and baryons are collectively known as hadrons. According to the quark model, it is possible to build all the hadrons using 3 quarks and 3 antiquarks.

48. Given, $m_\alpha = 6.4 \times 10^{-27}$ kg,

$q_\alpha = 3.2 \times 10^{-19}$ C, $E = 1.6 \times 10^5\ \text{Vm}^{-1}$

Force on α-particle

$$F = q_\alpha E = 3.2 \times 10^{-19} \times 1.6 \times 10^5$$
$$= 51.2 \times 10^{-15}\ \text{N}$$

Now, acceleration of the particle

$$\alpha = \frac{F}{m_\alpha} = \frac{51.2 \times 10^{-15}}{6.4 \times 10^{-27}}$$
$$= 0.8 \times 10^{13}\ \text{ms}^{-2}$$

$\because$ Initial velocity, $u = 0$

$$\therefore \quad v^2 = 2\alpha S$$
$$= 2 \times 8 \times 10^{12} \times 2 \times 10^{-2}$$
$$= 32 \times 10^{10}$$

or $$v = 4\sqrt{2} \times 10^5\ \text{ms}^{-1}$$

49. Fundamental frequency of cylindrical open tube

$$n = \frac{v}{2L} = 390\ \text{Hz}$$

When it is immersed in water it becomes a closed tube of length $\frac{3}{4}$th of the initial length.

Therefore, its fundamental frequency is

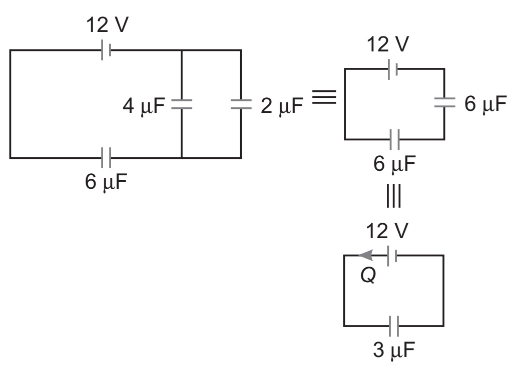
$$n' = \frac{v}{4\left(\frac{3}{4}L\right)} = \frac{v}{3L} = \frac{2}{3}\left(\frac{v}{2L}\right)$$
$$= \frac{2}{3} \times 390\ \text{Hz} = 260\ \text{Hz}$$

50. The surface temperature of the stars is determined using Wien's displacement law.

According to this law, $\lambda_m T = b$

where b is Wien's constant whose value is 2.898×10^{-3} mK.

51.

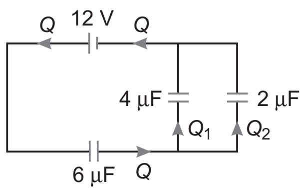


As the capacitors $4\,\mu F$ and $2\,\mu F$ are connected in parallel and are in series with $6\,\mu F$ capacitor, their equivalent capacitance is

$$\frac{(2+4)\times 6}{2+4+6} = 3\,\mu F$$

Charge in the circuit,

$$Q = 3\,\mu F \times 12\,V = 36\,\mu C$$



Since, the capacitors $4\,\mu F$ and $2\,\mu F$ are connected in parallel, therefore potential difference across them is same.

$$\Rightarrow \quad \frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{4}{2} \text{ or } Q_1 = 2Q_2$$

Also, $\quad Q = Q_1 + Q_2$

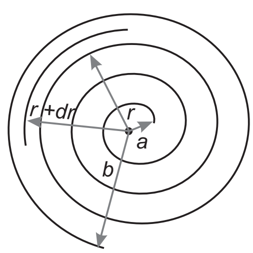
$$\therefore\ 36\mu C = 2Q_2 + Q_2 \quad \text{or} \quad Q_2 = \frac{36\,\mu C}{3} = 12\,\mu C$$

$$Q_1 = Q - Q_2 = 36\mu C - 12\mu C$$
$$= 24\,\mu C = 24\times 10^{-6}\,C$$

52. Beam first converges and then diverges *ie* intensity of light first increases and then decreases.

53. Incandescent electric lamp produces continuous emission spectrum whereas mercury and sodium vapour give line emission spectrum. Polyatomic substances such as H_2, CO_2 and $KMnO_4$ produces band absorption spectrum.

54. Consider an element of thickness dr at a distance r from the centre of spiral coil.



Number of turns in coil = n

Number of turns per unit length

$$= \frac{n}{b-a}$$

Number of turns in element $dr = dn$

Number of turns per unit length in element dr

$$= \frac{ndr}{b-a}$$

ie, $$dn = \frac{ndr}{b-a}$$

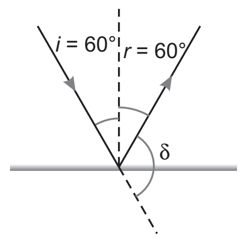
Magnetic field at its centre due to element dr is

$$dB = \frac{\mu_0 Idn}{2r} = \frac{\mu_0 I}{2}\,\frac{n}{(b-a)}\,\frac{dr}{r}$$

$$\therefore \quad B = \int_a^b \frac{\mu_0 Indr}{2(b-a)r} = \frac{\mu_0 In}{2(b-a)} \int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 In}{2(b-a)} \log_e\left(\frac{b}{a}\right)$$

55. Here, angle of incidence, $i = 60°$



$\therefore$ Angle of deviation

$$\delta = 180° - (i + r)$$
$$= 180° - 2i \qquad \text{(As } i = r\text{)}$$
$$= 180° - 2\times 60° = 60°$$

56. Here, electric potential

$$V = 3x^2$$

Electric field,

$$E = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(3x^2) = -6x$$

$$\therefore \qquad E_{(2,\,0,\,1)} = -12\,Vm^{-1}$$

57. When a this glass plate of thickness t and refractive index μ is introduced in one of the paths of the interfering waves then the path increases by $(\mu - 1)t$ and the whole pattern shifts by

$$y_0 = \frac{D}{d}(\mu - 1)t$$

Shifting is towards the side in which the plate is introduced without any change in fringe width. Therefore, when a glass plate of refractive index

1.5 is kept in the path of light from one of the slits, only the fringes get shifted but the fringe width remains unchanged.

58. Since, electron is moving from left to right, the flux linked with loop will first increase and then decrease as the electron passes by. Therefore, induced current *I* in the loop will be first clockwise and then will move in anticlockwise direction as the electron passes by.

59. Here, $n_f = 1, \; n_i = n$

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow \frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right) \quad \ldots(i)$$

or $\quad \frac{1}{\lambda R} = 1 - \frac{1}{n^2} \quad$ or $\quad \frac{1}{n^2} = 1 - \frac{1}{\lambda R}$

or $\quad n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$

60. Magnetic dipole moment of a current is given by

$$M = NIA$$

where N = number of turns

I = current in a loop

A = area of the loop

From the above relation it is clear that magnetic dipole moment of a current loop is independent of the magnetic field in which it is lying.

CHEMISTRY

1. $H_2^+ : \sigma 1s^1, \overset{*}{\sigma} 1s^0$

$$\text{Bond order} = \frac{1}{2}$$

$H_2^- : \sigma 1s^2, \overset{*}{\sigma} 1s^1$

$$\text{Bond order} = \frac{2-1}{2} = \frac{1}{2}$$

The bond order of H_2^+ and H_2^- are same but H_2^+ is more stable than H_2^-. It is due to the presence of one electron in the antibonding molecular orbital in H_2^-.

2. Species : O_2^{2-} O_2^- O_2 O_2^+

Bond order : 1 1.5 2 2.5

3. $N = N_0 \left(\frac{1}{2}\right)^n$

Given, $N_0 = 2\text{ g}$

$t_{1/2} = 15\text{ days}$

$T = 60\text{ days}$

$\Rightarrow \quad n = \frac{60}{15} = 4$

$\therefore \quad N = 2\left(\frac{1}{2}\right)^4$

or $\quad N = 0.125\text{ g}$

4. $A \longrightarrow B$

$$\text{Rate} = k[A]^n$$

$(\text{Rate})_1 = k(0.05)^n = 2 \times 10^{-3} \quad \ldots(i)$

$(\text{Rate})_2 = k(0.1)^n = 1.6 \times 10^{-2} \quad \ldots(ii)$

Dividing the Eq. (ii) by Eq. (i).

$$\frac{(\text{Rate})_2}{(\text{Rate})_1} = \frac{k(0.1)^n}{k(0.05)^n} = \frac{1.6 \times 10^{-2}}{2 \times 10^{-3}}$$

$$(2)^n = 8 \quad \text{or} \quad (2)^n = 2^3$$

$\therefore \quad n = 3$

5. $AB \longrightarrow \text{Product}$

$$\text{Rate} = k[AB]^n$$

$(\text{Rate})_1 = k[0.20]^n = 2.75 \times 10^{-8} \quad \ldots(i)$

$(\text{Rate})_2 = k[0.40]^n = 11.0 \times 10^{-8} \quad \ldots(ii)$

Dividing Eq. (ii) by Eq. (i),

$$\frac{(\text{Rate})_2}{(\text{Rate})_1} = \frac{k[0.40]^n}{k[0.20]^n} = \frac{11.0 \times 10^{-8}}{2.75 \times 10^{-8}}$$

$$2^n = 4$$

or $\quad 2^n = 2^2$

Hence, $\quad n = 2$

6. $A \longrightarrow B$

$\because \quad r = k[A]^0$

or $\quad r = k$

This is a zero order reaction.

$\therefore \quad t_{1/2} = \frac{a}{2k}$

7. Total milliequivalents of H^+

$$= 30 \times \frac{1}{3} + 20 \times \frac{1}{2} = 20$$

Total milliequivalents of OH^-

$$= 40 \times \frac{1}{4} = 10$$

Milli equivalence of H^+ left

$$= 20 - 10 = 10$$

$$\therefore \quad [H^+] = \frac{10}{1000} \text{ g ions/dm}^3 = 10^{-2}$$

$$\therefore \quad pH = 2$$

8. Weight of pure NaCl $= 6.5 \times 0.9 = 5.85$ g

No. of equivalence of NaCl $= \frac{5.85}{58.5} = 0.1$

No. of equivalence of NaOH obtained $= 0.1$

Volume of 1 M acetic acid required for the neutralisation of NaOH $= \frac{0.1 \times 1000}{1}$

$$= 100 \text{ cm}^3$$

9. $Zn^{2+} + 2e^- \longrightarrow Zn; \ E^\circ = -0.76$ V

$Fe^{2+} + 2e^- \longrightarrow Fe; E^\circ = -0.44$ V

Cell reaction is

$$Fe^{2+} + Zn \longrightarrow Zn^{2+} + Fe$$

$$\begin{aligned} E_{cell} &= E_{cathode} - E_{anode} \\ &= -0.44 - (-0.76) \\ &= -0.44 + 0.76 \\ &= 0.32 \text{ V} \end{aligned}$$

10. $[OH^-]$ in the diluted base $= \frac{10^{-6}}{10^2} = 10^{-8}$

$$\begin{aligned} \text{Total } [OH^-] &= 10^{-8} + [OH^-] \text{ of water} \\ &= (10^{-8} + 10^{-7}) \text{ M} \\ &= 10^{-8}[1 + 10] \text{ M} \\ &= 11 \times 10^{-8} \text{ M} \end{aligned}$$

$$\begin{aligned} pOH &= -\log 11 \times 10^{-8} \\ &= -\log 11 + 8 \log 10 \\ &= 6.9586 \\ pH &= 14 - 6.9586 \\ &= 7.0414 \end{aligned}$$

11. No. of moles of $H_2 = \frac{1.12}{22400}$

No. of equivalence of hydrogen

$$= \frac{1.12 \times 2}{22400} = 10^{-4}$$

No. of Faradays required $= 10^{-4}$

$\therefore$ Current to be passed in one second

$$\begin{aligned} &= 96500 \times 10^{-4} \\ &= 9.65 \text{ A} \end{aligned}$$

12. The number of ions per cc decreases with dilution and therefore, specific conductance decreases with dilution.

13. $p = p_A^\circ x_A + p_B^\circ x_B$

$$\Rightarrow \quad 84 = 70 \times 0.8 + p_B^\circ \times 0.2$$

$$84 = 56 + p_B^\circ \times 0.2$$

$$p_B^\circ = \frac{28}{0.2} = 140 \text{ mm}$$

14. Molarity of urea $= \dfrac{\frac{6}{60}}{\frac{100}{1000}} = 1$ M

Hence, 1 M solution of glucose is isotonic with 6% urea solution.

15. In countries nearer to polar region, the roads are sprinkled with $CaCl_2$ because $CaCl_2$ decreases the freezing point of ice and therefore, minimise the wear and tear of the roads.

16. $\Delta G = \Delta H - T \cdot \Delta S$

For the reaction,

$$H_2O(l) \rightleftharpoons H_2O(g)$$

$$\Delta G = 0 \qquad \text{(at equilibrium)}$$

$$\therefore \quad \Delta H = T \cdot \Delta S$$

17. *A* occupies corners, thus number of *A* atoms per unit cell $= 8 \times \frac{1}{8} = 1$

B occupies face centres, thus number of *B* atoms per unit cell $= 6 \times \frac{1}{2} = 3$

$\therefore$ The empirical formula of the compound is AB_3.

18. When nitro group is present in the benzene nucleus, it withdraws electrons from *o* and *p*-positions. Thus, the electron density at the *o* and *p*-positions decreases. *m*-positions become positions of comparatively higher electron density and therefore, electrophilic attack occurs at *m*-positions.

19. $CH_3COOH \xrightarrow{LiAlH_4} CH_3CH_2OH \xrightarrow[300°C]{Cu} CH_3CHO$

acetic acid (X) (Y)

$\xrightarrow[\text{(aldol condensation)}]{\text{dil. NaOH}}$ $CH_3-\underset{\underset{OH}{|}}{CH}-CH_2CHO$

(Z) aldol

20. The best method for the conversion of an alcohol into an alkyl chloride is by treating the alcohol with $SOCl_2$ in the presence of pyridine.

$$ROH + SOCl_2 \longrightarrow RCl + HCl + SO_2$$

The other products being gases escape leaving behind pure alkyl halide.

21. The electrophile involved in the sulphonation of benzene is SO_3.

$$2H_2SO_4 \longrightarrow SO_3 + H_3O^+ + HSO_4^-$$

22. The carbon-carbon bond length in benzene (1.39 Å) is between that of C—C (1.54 Å) and C=C (1.34 Å) *ie*, in between that of C_2H_6 and C_2H_4.

23. Propanal is not formed during the dry distillation of a mixture of calcium formate and calcium acetate.

$$(HCOO)_2Ca \xrightarrow{\text{Dry distillation}} HCHO + CaCO_3$$

$$(CH_3COO)_2Ca \xrightarrow{\Delta} CH_3COCH_3 + CaCO_3$$

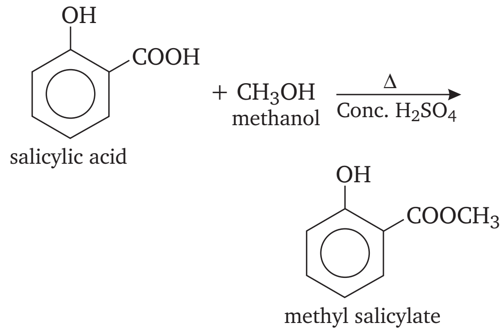
$$(HCOO)_2Ca + (CH_3COO)_2Ca \xrightarrow{\Delta} 2CH_3CHO + 2CaCO_3$$

24. $(H_3C)_2CHOH + [O] \xrightarrow[K_2Cr_2O_7]{\text{Acidified}} (H_3C)_2C{=}O$

secondary alcohol (X) acetone

Ketone (*ie*, acetone) reacts with phenyl hydrazine but does not give silver mirror test.

25. Methanol reacts with salicylic acid in the presence of a few drops of conc H_2SO_4 to give methyl salicylate having the smell of oil of winter green.



26. $CH_3-\overset{\overset{Cl}{|}}{\underset{\underset{Cl}{|}}{C}}-H \xrightarrow{\text{Dil. NaOH}} CH_3-C(H)(OH)(OH)$

1, 1 - dichloroethane unstable

$\downarrow$

$CH_3CHO + H_2O$

acetaldehyde

27. Aliphatic amines are more basic than NH_3 due to $+I$ effect of alkyl groups. In aqueous medium tertiary amine is less basic than secondary amine because the cation formed by protonation of tertiary amine is less solvated as compared to the cation formed by protonation of secondary amine. Hence, the increasing order of their basic strength is as:

$$NH_3 < (CH_3)_3N < CH_3NH_2 < (CH_3)_2NH$$

28. Ghee has least iodine value among the given options because it is the least unsaturated.

29. Sometimes the blood sugar level of diabetic patients decreases suddenly. So, diabetic patients generally carry a packet of glucose which can increase the blood sugar level almost instantaneously.

30. Naturally occurring amino acids are 20.

Hence, number of possible tripeptides

$$= 20^3 = 8000$$

31. Water boils at higher temperature inside the pressure cooker because pressure is high in the pressure cooker and therefore, cooling becomes fast.

32. Cinnabar (HgS) is a sulphide ore, hence it is concentrated by froth floatation process.

33. K(19) : $1s^2, 2s^2 2p^6, 3s^2 3p^6, 4s^1$

$4s^1$ is the valence electron in potassium, hence the correct set of four quantum numbers for outermost electron is 4, 0, 0, $\frac{1}{2}$.

34. $\lambda = \frac{h}{mv}$

$= \frac{6.62 \times 10^{-34}}{6.62 \times 10^{-35} \times 100}$

$= 0.1$ kg

35. Ionisation energy generally increases from left to right in a period but ionisation energy of nitrogen is greater than oxygen due to stable p^3 configuration. Hence, the order is as

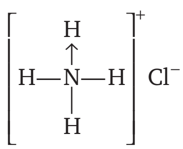
$$C < O < N < F$$

36. Na(11) : $1s^2, 2s^2 2p^6, 3s^1$

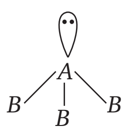
It is an alkali metal. Alkali metal oxides are basic in nature.

37. Alkali metals have low ionisation energy. They posses minimum value of ionisation energy in their period.

38. NH_4Cl contains ionic, covalent and coordinate linkage.



39. The pyramidal structure of covalent molecule AB_3 is as:



No. of lone pair = 1

No. of bond pair = 3

40. No. of millimoles of $Ca(OH)_2 = 50 \times 0.5 = 25$

No. of millimoles of $CaCO_3$ = 25

No. of milliequivalents of $CaCO_3$ = 50

$\therefore$ Volume of 0.1 N HCl $= \frac{50}{0.1} = 500\ cm^3$

41. Atomic mass of the metal $= 32 \times 2 = 64$

Formula of metal nitrate $= M(NO_3)_2$

$\therefore$ Molecular mass = 64 + 28 + 96 = 188

42. $v_{rms} = \sqrt{\frac{3p}{d}}$

$= \sqrt{\frac{3 \times 1.2 \times 10^5}{4}} = 300\ ms^{-1}$

43. Rate of diffusion $\propto \frac{1}{\sqrt{\text{molecular mass}}}$

$\therefore$ Order of diffusion : $H_2 > CH_4 > SO_2$

and amount left is in the order $SO_2 > CH_4 > H_2$

Hence, order of partial pressure is

$$pSO_2 > pCH_4 > pH_2$$

44. $2NH_3(g) \longrightarrow N_2(g) + 3H_2(g)$

$\Delta H_r = -(2 \times \text{enthalpy of formation of } NH_3)$

$= -(2 \times -46) = 92$ kJ

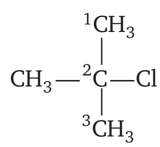
45. $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

	$2SO_2(g)$	$O_2(g)$	$2SO_3(g)$
Initial	5	5	0
At equilibrium	(5 – 3) = 2	(5 – 1.5) = 3.5	3

$\therefore \quad pO_2 = \frac{3.5 \times 1}{8.5} = 0.41$ atm

46. The value of equilibrium constant remains constant for a given reaction at constant temperature.

47. Physical adsorption decreases with increasing temperature or rate of physical adsorption increases with decreasing temperature.

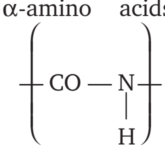
48. 

2-chloro-2-methylpropane

49. Lucas test is used to distinguish primary, secondary and tertiary alcohols.

50. Since, the compound on heating with CuO produced CO_2, it contains carbon. Again, it does not produce water, hence it does not contain hydrogen. So, the organic compound is carbon tetrachloride (CCl_4).

51. Proteins are the condensation polymers of α-amino acids. Proteins contain peptide

 linkages.

52. The order of stability of metal oxides is as:

$$Fe_2O_3 < Cr_2O_3 < Al_2O_3 < MgO$$

53. The temperature of the slag zone in the metallurgy of iron using blast furnace is 800–1000°C.

54. The function of $Fe(OH)_3$ in the contact process is to remove arsenic impurity. $Fe(OH)_3$ is a positive sol, hence it removes arsenic impurity which is a negative sol.

55. Potassium tetraiodo mercurate (II) *ie*, $K_2[HgI_4]$ dissolves in KOH solution to give Nessler's reagent. Nessler's reagent is used to test NH_4^+ ions.

56. Argon is used in high temperature welding and other operations which require a non-oxidising atmosphere and the absence of nitrogen.

57. Chromyl chloride test is used for Cl^- ions. Chlorine is not liberated in this test.

$$4NaCl + K_2Cr_2O_7 + 3H_2SO_4 \longrightarrow K_2SO_4 + 2Na_2SO_4 + \underset{\substack{\text{chromyl chloride} \\ \text{(reddish brown vapour)}}}{2CrO_2Cl_2\uparrow} + 3H_2O$$

$$CrO_2Cl_2 + 4NaOH \longrightarrow 2NaCl + Na_2CrO_4 + 2H_2O$$

$$Na_2CrO_4 + (CH_3COO)_2Pb \longrightarrow 2CH_3COONa + \underset{\text{lead chromate}}{PbCrO_4}$$

58. $\mu = \sqrt{n(n+2)}$

$\Rightarrow \quad \sqrt{15} = \sqrt{n(n+2)}$

$\therefore \quad n = 3$

59. $[Co(NH_3)_5ONO]^{2+}$

Penta ammine nitrito cobalt (III) ion.

60. Let the oxidation state of Fe in $[Fe(H_2O)_5NO]SO_4$ is x.

$$[Fe(H_2O)_5NO]^{2+}$$

$\Rightarrow \quad x + 0 + 1 = 2$

$\therefore \quad x = +1$

Here NO exists as nitrosyl ion (NO^+).

MATHEMATICS

1. Given, $f(x) = x^3 + 3x - 2$

On differentiating w.r.t. x, we get

$$f'(x) = 3x^2 + 3$$

Put $\quad f'(x) = 0 \quad \Rightarrow \quad 3x^2 + 3 = 0$

$\Rightarrow \quad x^2 = -1$

$\therefore$ $f(x)$ is either increasing or decreasing.

At $x = 2$, $f(2) = 2^3 + 3(2) - 2 = 12$

At $x = 3$, $f(3) = 3^3 + 3(3) - 2 = 34$

$\therefore f(x) \in [12, 34]$.

2. Let $A = \left\{x \in R : \dfrac{2x-1}{x^3 + 4x^2 + 3x}\right\}$

Now, $x^3 + 4x^2 + 3x = x(x^2 + 4x + 3)$

$= x(x+3)(x+1)$

$\therefore \quad A = R - \{0, -1, -3\}$

3. Given, $a_0 = 1$, $a_{n+1} = 3n^2 + n + a_n$

$\Rightarrow \quad a_1 = 3(0) + 0 + a_0 = 1$

and $a_2 = 3(1)^2 + 1 + a_1 = 3 + 1 + 1 = 5$

From option (b),

Let $\quad P(n) = n^3 - n^2 + 1$

$\therefore \quad P(0) = 0 - 0 + 1 = 1 = a_0$

$P(1) = 1^3 - 1^2 + 1 = 1 = a_1$

and $\quad P(2) = (2)^3 - (2)^2 + 1 = 5 = a_2$

Hence, option (b) is correct.

4. The total number of subsets of given set is $2^9 = 512$

Case I When selecting only one even number $\{2, 4, 6, 8\}$
Number of ways $= {}^4C_1 = 4$

Case II When selecting only two even numbers $= {}^4C_2 = 6$

Case III When selecting only three even numbers $= {}^4C_3 = 4$

Case IV When selecting only four even numbers $= {}^4C_4 = 1$

$\therefore$ Required number of ways

$$= 512 - (4 + 6 + 4 + 1) - 1$$
$$= 496$$

[Here, we subtract 1 for due to the null set]

5. Total number of points in a plane is 3p.

$\therefore$ Maximum number of triangles

$$= {}^{3p}C_3 - 3\cdot {}^{p}C_3$$

[Here, we subtract those triangles which points are in a line]

$$= \frac{(3p)!}{(3p-3)!\,3!} - 3\cdot\frac{p!}{(p-3)!\,3!}$$

$$= \frac{3p(3p-1)(3p-2)}{3\times 2} - \frac{3\times p(p-1)(p-2)}{3\times 2}$$

$$= \frac{p}{2}[9p^2 - 9p + 2 - (p^2 - 3p + 2)]$$

$$= p^2(4p-3)$$

6. The required number of ways = The even number of 0's *ie*, {0, 2, 4, 6, ...}

$$= \frac{n!}{n!} + \frac{n!}{2!(n-2)!} + \frac{n!}{4!(n-4)!}$$

$$= {}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = 2^{n-1}$$

7. Now, $(1+x^2)^{12}(1 + x^{12} + x^{24} + x^{36})$

$$= [1 + {}^{12}C_1(x^2) + {}^{12}C_2(x^2)^2 + {}^{12}C_3(x^2)^3$$
$$+ {}^{12}C_4(x^2)^4 + {}^{12}C_5(x^2)^5 + {}^{12}C_6(x^2)^6$$
$$+ \ldots + {}^{12}C_{12}(x^2)^{12}] \times (1 + x^{12} + x^{24} + x^{36})$$

Coefficient of $x^{24} = {}^{12}C_6 + {}^{12}C_{12} + 1$

$$= {}^{12}C_6 + 2$$

8. $\left(1 + \frac{2x}{3}\right)^{3/2}(32 + 5x)^{-1/5}$

$$= \left[1 + \frac{3}{2}\left(\frac{2x}{3}\right)\right](32)^{-1/5}\left(1 + \frac{5}{32}x\right)^{-1/5}$$

(Neglect higher powers of x)

$$= [1 + x]2^{-1}\left[1 - \frac{1}{5}\left(\frac{5}{32}\right)x\right]$$

(Neglect higher powers of x)

$$= \frac{1}{2}(1 + x)\left(1 - \frac{x}{32}\right)$$

$$= \frac{(1+x)(32-x)}{64} = \frac{32 + 31x}{64}$$

(Neglect x^2 term)

9. $\frac{1}{(x-1)^2(x-2)} = \frac{1}{-2(1-x)^2\left(1 - \frac{x}{2}\right)}$

$$= -\frac{1}{2}\left[(1-x)^{-2}\left(1 - \frac{x}{2}\right)^{-1}\right]$$

$$= -\frac{1}{2}\left[(1 + 2x + \ldots)\left(1 + \frac{x}{2} + \ldots\right)\right]$$

$\therefore$ Coefficient of constant term is $-\frac{1}{2}$.

10. Given, $\frac{1}{e^{3x}}(e^x + e^{5x}) = a_0 + a_1x + a_2x^2 + \ldots$

$\Rightarrow$ $(e^{-2x} + e^{2x}) = a_0 + a_1x + a_2x^2 + \ldots$

$\Rightarrow$ $2\left[1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \ldots\right]$

$$= a_0 + a_1x + a_2x^2 + \ldots$$

$\Rightarrow$ $a_1 = a_3 = a_5 = \ldots = 0$

$\therefore$ $2a_1 + 2^3a_3 + 2^5a_5 + \ldots = 0$

11. Given,

$(x-a)(x-a-1) + (x-a-1)(x-a-2)$
$+ (x-a)(x-a-2) = 0$

Let $x - a = t$, then

$t(t-1) + (t-1)(t-2) + t(t-2) = 0$

$\Rightarrow$ $t^2 - t + t^2 - 3t + 2 + t^2 - 2t = 0$

$\Rightarrow$ $3t^2 - 6t + 2 = 0$

$\Rightarrow$ $t = \frac{6 \pm \sqrt{36 - 24}}{2(3)} = \frac{6 \pm 2\sqrt{3}}{2(3)}$

$\Rightarrow$ $x - a = \frac{3 \pm \sqrt{3}}{3}$

$\Rightarrow$ $x = a + \frac{3 \pm \sqrt{3}}{3}$

Hence, x is real and distinct.

12. Given, $f(x) = x^2 + ax + b$ has imaginary roots.

$\therefore$ Discriminant, $D < 0 \Rightarrow a^2 - 4b < 0$

Now, $f'(x) = 2x + a$

$f''(x) = 2$

Also, $f(x) + f'(x) + f''(x) = 0$ ...(i)

$\Rightarrow$ $x^2 + ax + b + 2x + a + 2 = 0$

$\Rightarrow$ $x^2 + (a+2)x + b + a + 2 = 0$

$\therefore$ $x = \frac{-(a+2) \pm \sqrt{(a+2)^2 - 4(a+b+2)}}{2}$

$$= \frac{-(a+2) \pm \sqrt{a^2 - 4b - 4}}{2}$$

Since, $a^2 - 4b < 0$

$\therefore \quad a^2 - 4b - 4 < 0$

Hence, Eq. (i) has imaginary roots.

13. Given, α, β and γ are the roots of $x^3 + 4x + 1 = 0$.

$\therefore \quad \alpha + \beta + \gamma = 0, \alpha\beta + \beta\gamma + \gamma\alpha = 4, \alpha\beta\gamma = -1$

Now, $\dfrac{\alpha^2}{\beta+\gamma} + \dfrac{\beta^2}{\gamma+\alpha} + \dfrac{\gamma^2}{\alpha+\beta} = \dfrac{\alpha^2}{-\alpha} + \dfrac{\beta^2}{-\beta} + \dfrac{\gamma^2}{-\gamma}$

$= -(\alpha + \beta + \gamma) = 0$

$\dfrac{\alpha^2\beta^2}{(\beta+\gamma)(\gamma+\alpha)} + \dfrac{\beta^2\gamma^2}{(\gamma+\alpha)(\alpha+\beta)} + \dfrac{\gamma^2\alpha^2}{(\beta+\gamma)(\alpha+\beta)}$

$= \alpha\beta + \beta\gamma + \gamma\alpha = 4$

and $\dfrac{\alpha^2\beta^2\gamma^2}{(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)} = -\alpha\beta\gamma = 1$

$(\because \ \alpha + \beta + \gamma = 0)$

$\therefore$ Required equation is

$$x^3 + 4x - 1 = 0$$

14. Given, $f(x) = 2x^4 - 13x^2 + ax + b$ is divisible by $(x-2)(x-1)$.

$\therefore \quad f(2) = 2(2)^4 - 13(2)^2 + a(2) + b = 0$

$\Rightarrow \quad 2a + b = 20 \quad \ldots(\text{i})$

and $\quad f(1) = 2(1)^4 - 13(1)^2 + a + b = 0$

$\Rightarrow \quad a + b = 11 \quad \ldots(\text{ii})$

On solving Eqs. (i) and (ii), we get

$$a = 9, b = 2$$

15. Given, $A = A'$, $B = B'$

Now, $(AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B'$

$= BA - AB$

$= -(AB - BA)$

$\therefore$ $AB - BA$ is a skew-symmetric matrix.

16. Given, $\begin{vmatrix} 3 & 5 & x \\ 7 & x & 7 \\ x & 5 & 3 \end{vmatrix} = 0$

$\Rightarrow \quad 3(3x - 35) - 5(21 - 7x) + x(35 - x^2) = 0$

$\Rightarrow 9x - 105 - 105 + 35x + 35x - x^3 = 0$

$\Rightarrow \quad x^3 - 79x + 210 = 0$

$\Rightarrow \quad (x + 10)(x - 3)(x - 7) = 0$

$\Rightarrow \quad x = -10, 3, 7$

17. Let a and R be the first term and common ratio of a GP.

$\therefore \quad T_p = aR^{p-1} = x$

$T_q = aR^{q-1} = y$

and $\quad T_r = aR^{r-1} = z$

$\Rightarrow \quad \log x = \log a + (p-1)\log R$

$\log y = \log a + (q-1)\log R$

and $\quad \log z = \log a + (r-1)\log R$

$\therefore \begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = \begin{vmatrix} \log a + (p-1)\log R & p & 1 \\ \log a + (q-1)\log R & q & 1 \\ \log a + (r-1)\log R & r & 1 \end{vmatrix}$

$= \begin{vmatrix} \log a & p & 1 \\ \log a & q & 1 \\ \log a & r & 1 \end{vmatrix} + \begin{vmatrix} (p-1)\log R & p & 1 \\ (q-1)\log R & q & 1 \\ (r-1)\log R & r & 1 \end{vmatrix}$

$= \log a \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p-1 & p-1 & 1 \\ q-1 & q-1 & 1 \\ r-1 & r-1 & 1 \end{vmatrix}$

$(C_2 \to C_2 - C_3)$

$= 0 + 0 = 0$ ($\because$ two columns are identical)

18. If matrix has no inverse it means the value of determinant should be zero.

$\therefore \quad \begin{vmatrix} 1 & -1 & x \\ 1 & x & 1 \\ x & -1 & 1 \end{vmatrix} = 0$

If we put $x = 1$, then column Ist and IIIrd are identical.

Hence, option (d) is correct.

19. Given, α, β are the roots of $x^2 - 2x + 4 = 0$

$\therefore \quad \alpha + \beta = 2 \quad \ldots(\text{i})$

and $\quad \alpha\beta = 4 \quad \ldots(\text{ii})$

Now, $\quad \alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{4 - 4 \times 4} = \sqrt{-12}$

$\Rightarrow \quad \alpha - \beta = 2\sqrt{3}\,i \quad \ldots(\text{iii})$

On solving Eqs. (i) and (ii), we get

$\alpha = \dfrac{2 + 2\sqrt{3}\,i}{2} = -2\left(\dfrac{-1 - \sqrt{3}\,i}{2}\right) = -2\omega^2$

and $\beta = \dfrac{2 - 2\sqrt{3}\,i}{2} = -2\left(\dfrac{-1 + \sqrt{3}\,i}{2}\right) = -2\omega$

Now, $\quad \alpha^6 + \beta^6 = (-2\omega^2)^6 + (-2\omega)^6$

$= 64(\omega^3)^4 + 64(\omega^3)^2$

$= 128 \quad [\because \ \omega^3 = 1]$

20. Let $z = x + iy$

Given, $\quad \left|\dfrac{z + 2i}{2z + i}\right| < 1$

$$\Rightarrow \quad \frac{\sqrt{(x)^2+(y+2)^2}}{\sqrt{(2x)^2+(2y+1)^2}} < 1$$

$$\Rightarrow x^2+y^2+4+4y < 4x^2+4y^2+1+4y$$

$$\Rightarrow \quad 3x^2+3y^2 > 3 \quad \Rightarrow \quad x^2+y^2 > 1$$

21. Now, $(1+\sqrt{3}\,i)^n + (1-\sqrt{3}\,i)^n$

$$= \left[2\left(\frac{1+\sqrt{3}\,i}{2}\right)\right]^n + \left[2\left(\frac{1-\sqrt{3}\,i}{2}\right)\right]^n$$

$$= (-2\omega^2)^n + (-2\omega)^n$$

$$= (-2)^n\,[(\omega^2)^{3r+1} + (\omega)^{3r+1}]$$

[$\because$ $n = 3r+1$, where r is an integer]

$$= (-2)^n\,(\omega^2+\omega)$$

$$= -(-2)^n$$

22. Let $f(x) = \sin^4 x + \cos^4 x$

$$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$$

$$= 1 - \frac{1}{4}\cdot 2(\sin 2x)^2$$

$$= 1 - \frac{1}{4}(1-\cos 4x)$$

$$= \frac{3}{4} + \frac{\cos 4x}{4}$$

$\therefore$ Period of $f(x) = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

23. Now, $\tan(x-y)\tan y$

$$= \frac{\sin(x-y)\sin y}{\cos(x-y)\cos y} \times \frac{2}{2}$$

$$= \frac{\cos(x-2y) - \cos(x)}{\cos(x-2y) + \cos(x)}$$

$$= \frac{1 - \dfrac{\cos x}{\cos(x-2y)}}{1 + \dfrac{\cos(x)}{\cos(x-2y)}}$$

$$= \frac{1-\lambda}{1+\lambda}$$

$$\left(\text{Given, } \lambda = \frac{\cos x}{\cos(x-2y)}\right)$$

24. It is a standard result.

$$\cos A \cos 2A \cos 2^2 A \ldots \cos 2^{n-1} A$$

$$= \frac{\sin 2^n A}{2^n \sin A}$$

25. $\sin^2 x - \cos 2x = 2 - \sin 2x$

$$\Rightarrow 1 - \cos^2 x - (2\cos^2 x - 1)$$

$$= 2 - 2\sin x \cos x$$

$$\Rightarrow \quad -3\cos^2 x + 2\sin x \cos x = 0$$

$$\Rightarrow \quad \cos x\,(2\sin x - 3\cos x) = 0$$

$\Rightarrow \quad \cos x = 0, \quad (\because$
$2\sin x - 3\cos x \neq 0)$

$$\Rightarrow \quad x = 2n\pi \pm \frac{\pi}{2}$$

$$\Rightarrow \quad x = (4n \pm 1)\frac{\pi}{2}$$

26. $\cos^{-1}\left(-\dfrac{1}{2}\right) - 2\sin^{-1}\left(\dfrac{1}{2}\right) + 3\cos^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)$
$- 4\tan^{-1}(-1)$

$$= \pi - \cos^{-1}\left(\frac{1}{2}\right) - 2\left(\frac{\pi}{6}\right) + 3\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) + 4\tan^{-1}(1)$$

$$= \pi - \frac{\pi}{3} - \frac{\pi}{3} + 3\left(\pi - \frac{\pi}{4}\right) + 4\cdot\frac{\pi}{4}$$

$$= \frac{\pi}{3} + 3\cdot\frac{3\pi}{4} + \pi$$

$$= \frac{43\pi}{12}$$

27. Given, $\sinh^{-1} 2 + \sinh^{-1} 3 = x$

$$\Rightarrow \cosh(\sinh^{-1} 2 + \sinh^{-1} 3) = \cosh x$$

$$\Rightarrow \quad \cosh(\sinh^{-1} 2)\cosh(\sinh^{-1} 3) + \sinh(\sinh^{-1} 2)\sinh(\sinh^{-1} 3)$$

$$= \cosh x$$

$$\Rightarrow \quad \cosh x = \cosh\left(\cosh^{-1}\sqrt{1+2^2}\right) \times \cosh\left(\cosh^{-1}\sqrt{1+3^2}\right) + 2\times 3$$

$$\Rightarrow \quad \cosh x = (\sqrt{5}\,\sqrt{10} + 6) \times \frac{2}{2}$$

$$= \frac{1}{2}(12 + 2\sqrt{50})$$

28. $a(b\cos C - c\cos B)$

$$= ab\cos C - ac\cos B$$

$$= \frac{a^2+b^2-c^2}{2} - \frac{a^2+c^2-b^2}{2}$$

$$= b^2 - c^2$$

29. We know that, $2s = a+b+c$

$$\therefore \frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$$

$$= \frac{2s(2s-2a)(2s-2b)(2s-2c)}{4b^2c^2}$$

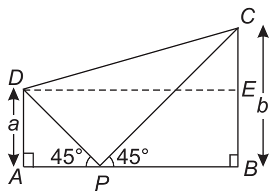
$$= 4\,\frac{s(s-a)}{bc} \times \frac{(s-b)(s-c)}{bc}$$

$$= 4\cos^2\frac{A}{2} \times \sin^2\frac{A}{2}$$

$$= \sin^2 A$$

30. In Δ APD,

$$\tan 45° = \frac{a}{AP} \Rightarrow AP = a$$



and in Δ BPC,

$$\tan 45° = \frac{b}{PB} \Rightarrow PB = b$$

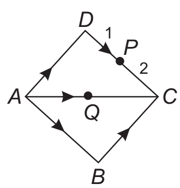
$\therefore$ DE = a + b and CE = b − a

In Δ DEC,

$$DC^2 = DE^2 + EC^2$$
$$= (a+b)^2 + (b-a)^2$$
$$= 2(a^2 + b^2)$$

31. Now, $\overrightarrow{\mathbf{AB}} + 2\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{BC}} - 2\overrightarrow{\mathbf{DC}}$

$$= \overrightarrow{\mathbf{AC}} + 2\overrightarrow{\mathbf{AD}} - 2\overrightarrow{\mathbf{DC}}$$
$$= \overrightarrow{\mathbf{AC}} + 2(\overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{CD}}) - 2\overrightarrow{\mathbf{DC}}$$



$$= 3\overrightarrow{\mathbf{AC}} - 4\overrightarrow{\mathbf{DC}}$$
$$= 3(2\overrightarrow{\mathbf{QC}}) - 4\left(\frac{3}{2}\overrightarrow{\mathbf{PC}}\right)$$
$$= 6\overrightarrow{\mathbf{QC}} - 6\overrightarrow{\mathbf{PC}} = 6(\overrightarrow{\mathbf{QC}} + \overrightarrow{\mathbf{CP}})$$

$\Rightarrow$ $k\overrightarrow{\mathbf{PQ}} = 6\overrightarrow{\mathbf{QP}} = -6\overrightarrow{\mathbf{PQ}}$ (given)

$\Rightarrow$ $k = -6$

32. Given, $l + m + n = 0$, $\Rightarrow$ $l = -m - n$

and $l^2 + m^2 - n^2 = 0$

$\therefore$ $(-m-n)^2 + m^2 - n^2 = 0$

$\Rightarrow$ $2m^2 + 2mn = 0$

$\Rightarrow$ $2m(m+n) = 0$

$\Rightarrow$ $m = 0$ or $m + n = 0$

If $m = 0$, then $l = -n$

$$\therefore \quad \frac{l_1}{-1} = \frac{m_1}{0} = \frac{n}{1}$$

and if $m + n = 0 \Rightarrow m = -n$, then $l = 0$

$$\therefore \quad \frac{l_2}{0} = \frac{m_2}{-1} = \frac{n_2}{1}$$

ie, $(l_1, m_1, n_1) = (-1, 0, 1)$

and $(l_2, m_2, n_2) = (0, -1, 1)$

$$\therefore \quad \cos\theta = \frac{0+0+1}{\sqrt{1+0+1}\,\sqrt{0+1+1}} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

33. Now, $2\vec{\mathbf{a}} - \vec{\mathbf{c}} = 2(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) - (-2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$$= \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

and $\vec{\mathbf{a}} + \vec{\mathbf{b}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} + 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$$= \hat{\mathbf{i}} + \hat{\mathbf{k}}$$

Let θ be the angle between $2\vec{\mathbf{a}} - \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} + \vec{\mathbf{b}}$.

$$\therefore \quad \cos\theta = \frac{(\hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{k}})}{\sqrt{1^2+1^2}\,\sqrt{1^2+1^2}}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

34. Given, $m_1 = |\vec{\mathbf{a}}_1| = \sqrt{2^2 + (-1)^2 + (1)^2} = \sqrt{6}$

$$m_2 = |\vec{\mathbf{a}}_2| = \sqrt{3^2 + (-4)^2 + (-4)^2} = \sqrt{41}$$

$$m_3 = |\vec{\mathbf{a}}_3| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$$

and $m_4 = |\vec{\mathbf{a}}_4| = \sqrt{(-1)^2 + (3)^2 + (1)^2} = \sqrt{11}$

$$\therefore \quad m_3 < m_1 < m_4 < m_2$$

35. Given, $\vec{\mathbf{a}} = \lambda\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\lambda\hat{\mathbf{k}}$

$$\therefore \quad \cos\theta = \frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|}$$

$$= \frac{\lambda^2 - 7 + 6\lambda}{\sqrt{\lambda^2 + 49 + 9}\,\sqrt{\lambda^2 + 1 + 4\lambda^2}} < 0$$

$\Rightarrow \quad (\lambda + 7)(\lambda - 1) < 0$

$\Rightarrow \quad -7 < \lambda < 1$

36. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

and $\vec{c} = \hat{i} - \hat{j} + \lambda\hat{k}$

Since, volume of tetrahedron $= \frac{1}{6}[\vec{a}\,\vec{b}\,\vec{c}]$

$$\Rightarrow \frac{2}{3} = \frac{1}{6}\begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & \lambda \end{vmatrix}$$

$$\Rightarrow \frac{2}{3} = \frac{1}{6}[1(\lambda + 1) - 2(\lambda - 1) - 1(-1 - 1)]$$

$\Rightarrow \quad 4 = [-\lambda + 5]$

$\Rightarrow \quad \lambda = 1$

37. Given, $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = \frac{7}{10}$

Since, $P(A \cap B) + P(\overline{A \cap B}) = 1$

$$\Rightarrow P(A \cap B) = 1 - \frac{7}{10} = \frac{3}{10}$$

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{4}{5} = P(A) + \frac{2}{5} - \frac{3}{10}$$

$$\Rightarrow P(A) = \frac{4}{5} - \frac{2}{5} + \frac{3}{10}$$

$$= \frac{2}{5} + \frac{3}{10} = \frac{7}{10}$$

38. Given, $x^2 + 4x + c = 0$

For real roots,

$$D = b^2 - 4ac \geq 0$$

$$= 16 - 4c \geq 0$$

$\Rightarrow$ $c = 1, 2, 3, 4$ will satisfy the above inequality.

$\therefore$ Required probability $= \frac{4}{9}$

39. (A) Given, $P(E_1) = \frac{1}{4}$, $P\left(\frac{E_1}{E_2}\right) = \frac{1}{4}$

and $P\left(\frac{E_2}{E_1}\right) = \frac{1}{2}$

$$\Rightarrow \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{1}{2}$$

$$\Rightarrow P(E_2 \cap E_1) = \frac{1}{8}$$

Also, $P\left(\frac{E_1}{E_2}\right) = \frac{1}{4}$

$$\Rightarrow \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1}{4}$$

$$\Rightarrow \frac{1}{8\,P(E_2)} = \frac{1}{4} \Rightarrow P(E_2) = \frac{1}{2}$$

(B) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$$

(C) $P\left(\frac{\bar{E}_1}{\bar{E}_2}\right) = \frac{P(\bar{E}_1 \cap \bar{E}_2)}{P(\bar{E}_2)}$

$$= \frac{1 - P(E_1 \cup E_2)}{1 - P(E_2)} = \frac{1 - \frac{5}{8}}{1 - \frac{1}{2}}$$

$$= \frac{3}{4}$$

(D) $P\left(\frac{E_1}{\bar{E}_2}\right) = \frac{P(E_1 \cap \bar{E}_2)}{P(\bar{E}_2)}$

$$= \frac{P(E_1) - P(E_1 \cap E_2)}{1 - P(E_2)}$$

$$= \frac{\frac{1}{4} - \frac{1}{8}}{1 - \frac{1}{2}} = \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4}$$

40. Given, distribution is

$X = x$	0	1	2	3
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{2}$	0	$\frac{1}{6}$

$\therefore$ Mean, $m = \sum_{i=1}^{4} p_i x_i$

$$= 0 \times \frac{1}{3} + 1 \times \frac{1}{2} + 2 \times 0 + 3 \times \frac{1}{6}$$

$$= 0 + \frac{1}{2} + 0 + \frac{1}{2} = 1$$

Variance, $\sigma^2 = \sum_{i=1}^{4} p_i (x_i - m)^2$

$$= \frac{1}{3}(0 - 1)^2 + \frac{1}{2}(1 - 1)^2 + 0(2 - 1)^2 + \frac{1}{6}(3 - 1)^2$$

$$= \frac{1}{3} + 0 + 0 + \frac{2}{3} = 1$$

$\therefore \quad m = \sigma^2 = 1$

41. Here, $n = 6$

According to the question

$$^6C_2 p^2 q^4 = 4 \cdot {}^6C_4 p^4 q^2$$

$\Rightarrow$ $q^2 = 4p^2$

$\Rightarrow$ $(1-p)^2 = 4p^2$

$\Rightarrow$ $3p^2 + 2p - 1 = 0$

$\Rightarrow$ $(p+1)(3p-1) = 0$

$\Rightarrow$ $p = \dfrac{1}{3}$

($\because$ p cannot be negative)

42. Given equation is $x^2 + y^2 = r^2$.

After rotation

$x = X\cos 36^\circ - Y\sin 36^\circ$

and $y = X\sin 36^\circ + Y\cos 36^\circ$

$\therefore$ $X^2(\cos^2 36^\circ + \sin^2 36^\circ)$

$+ Y^2(\sin^2 36^\circ + \cos^2 36^\circ) = r^2$

$\Rightarrow$ $X^2 + Y^2 = r^2$

43. Since, given lines are parallel.

$\therefore$ $d = \dfrac{15-5}{\sqrt{4^2+3^2}} = \dfrac{10}{5}$

$\Rightarrow$ $d = 2 =$ diameter of the circle

$\therefore$ Radius of circle $= 1$

$\therefore$ Area of circle $= \pi r^2 = \pi$ sq unit

44. Let point (x_1, y_1) be on the line $3x + 4y = 5$.

$\therefore$ $3x_1 + 4y_1 = 5$ ...(i)

Also, $(x_1 - 1)^2 + (y_1 - 2)^2$

$= (x_1 - 3)^2 + (y_1 - 4)^2$

$\Rightarrow$ $x_1^2 + y_1^2 - 2x_1 - 4y_1 + 5 = x_1^2 + y_1^2 - 6x_1$

$- 8y_1 + 25$

$\Rightarrow$ $4x_1 + 4y_1 = 20$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x_1 = 15, y_1 = -10$

45. The point of intersection of lines $x + 3y - 1 = 0$ and $x - 2y + 4 = 0$ is $(-2, 1)$.

Let equation of line perpendicular to the given line is $2x - 3y + \lambda = 0$.

Since, it passes through $(-2, 1)$.

$\therefore$ $2(-2) - 3(1) + \lambda = 0$

$\Rightarrow$ $\lambda = 7$

$\therefore$ Required line is $2x - 3y + 7 = 0$

46. Given equation is

$$2x^2 - 10xy + 12y^2 + 5x + \lambda y - 3 = 0$$

Here, $a = 2, h = -5, b = 12, g = \dfrac{5}{2}, f = \dfrac{\lambda}{2}, c = -3$

For pair of lines $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

$\Rightarrow$ $\begin{vmatrix} 2 & -5 & 5/2 \\ -5 & 12 & \lambda/2 \\ 5/2 & \lambda/2 & -3 \end{vmatrix} = 0$

$\Rightarrow$ $2\left(-36 - \dfrac{\lambda^2}{4}\right) + 5\left(15 - \dfrac{5\lambda}{4}\right)$

$+ \dfrac{5}{2}\left(\dfrac{-5\lambda}{2} - 30\right) = 0$

$\Rightarrow -72 - \dfrac{\lambda^2}{2} + 75 - \dfrac{25\lambda}{4} - \dfrac{25\lambda}{4} - 75 = 0$

$\Rightarrow$ $\lambda^2 + 25\lambda + 144 = 0$

$\Rightarrow$ $(\lambda + 9)(\lambda + 16) = 0$

$\Rightarrow$ $\lambda = -9$ $(\because |\lambda| < 16)$

47. Given, $x^2 - 2xy - xy + 2y^2 = 0$

$\Rightarrow$ $(x - 2y)(x - y) = 0$

$\Rightarrow$ $x = 2y, \quad x = y$ ...(i)

Also, $x + y + 1 = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$A\left(-\dfrac{2}{3}, -\dfrac{1}{3}\right), B\left(-\dfrac{1}{2}, -\dfrac{1}{2}\right), C(0, 0)$

$\therefore$ Area of $\Delta ABC = \dfrac{1}{2}\begin{vmatrix} -\dfrac{2}{3} & -\dfrac{1}{3} & 1 \\ -\dfrac{1}{2} & -\dfrac{1}{2} & 1 \\ 0 & 0 & 1 \end{vmatrix}$

$= \dfrac{1}{2}\left[\dfrac{1}{3} - \dfrac{1}{6}\right] = \dfrac{1}{2}\left[\dfrac{1}{6}\right] = \dfrac{1}{12}$

48. Given pair of lines are $x^2 - 3xy + 2y^2 = 0$and $x^2 - 3xy + 2y^2 + x - 2 = 0$.

$\therefore$ $(x - 2y)(x - y) = 0$

and $(x - 2y + 2)(x - y - 1) = 0$

$\Rightarrow$ $x - 2y = 0, \quad x - y = 0$ and $x - 2y + 2 = 0,$

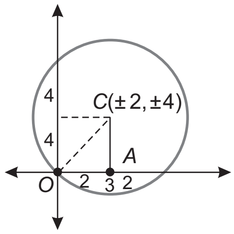
$x - y - 1 = 0$

Since, the lines $x - 2y = 0$, $x - 2y + 2 = 0$ and $x - y = 0$, $x - y - 1 = 0$ are parallel.

Also, angle between $x - 2y = 0$ and $x - y = 0$ is not 90°.

$\therefore$ It is a parallelogram.

49. In ΔOAC, $OC^2 = 2^2 + 4^2 = 20$



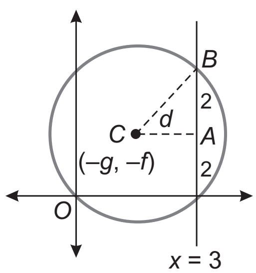
$\therefore$ Required equation of circle is

$$(x \pm 2)^2 + (y \pm 4)^2 = 20$$

$$\Rightarrow \quad x^2 + y^2 \pm 4x \pm 8y = 0$$

50. Let centre of circle be $C(-g, -f)$, then equation of circle passing through origin be

$$x^2 + y^2 + 2gx + 2fy = 0$$



$\therefore$ Distance, $d = |-g - 3| = g + 3$

In ΔABC, $(BC)^2 = AC^2 + BA^2$

$$\Rightarrow \quad g^2 + f^2 = (g+3)^2 + 2^2$$

$$\Rightarrow \quad g^2 + f^2 = g^2 + 6g + 9 + 4$$

$$\Rightarrow \quad f^2 = 6g + 13$$

Hence, required locus is $y^2 + 6x = 13$

51. The intersection point of diameter lines is (2, 3) which is the centre of circle.

Now, radius $= \sqrt{(5-2)^2 + (7-3)^2}$

$$= \sqrt{9 + 16} = 5$$

$\therefore$ Required equation of circle is

$$(x-2)^2 + (y-3)^2 = 5^2$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 6y - 12 = 0$$

52. Given circles are $x^2 + y^2 - 2x + 8y + 13 = 0$ and $x^2 + y^2 - 4x + 6y + 11 = 0$.

Here, $C_1 = (1, -4), C_2 = (2, -3)$,

$$\Rightarrow \quad r_1 = \sqrt{1 + 16 - 13} = 2$$

and $\quad r_2 = \sqrt{4 + 9 - 11} = \sqrt{2}$

Now, $d = C_1C_2 = \sqrt{(2-1)^2 + (-3+4)^2} = \sqrt{2}$

$$\therefore \cos\theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1r_2} = \frac{2 - 4 - 2}{2 \times 2 \times \sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = 135°$$

53. Let the required equation of circle be $x^2 + y^2 + 2gx + 2fy = 0$. Since, the above circle cuts the given circles orthogonally.

$$\therefore \quad 2(-3g) + 2f(0) = 8 \quad \Rightarrow \quad 2g = -\frac{8}{3}$$

and $\quad -2g - 2f = -7$

$$\Rightarrow \quad 2f = +7 + \frac{8}{3} = \frac{29}{3}$$

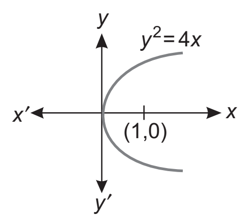
$\therefore$ Required equation of circle is

$$x^2 + y^2 - \frac{8}{3}x + \frac{29}{3}y = 0$$

or $\quad 3x^2 + 3y^2 - 8x + 29y = 0$

54. Given curve is $y^2 = 4x$.

Also, point (1, 0) is the focus of the parabola. It is clear from the graph that only one normal is possible.



55. Here, $2ae = 6$ and $2b = 8 \Rightarrow b = 4$

$$\Rightarrow \quad b^2 = 16 \quad \Rightarrow \quad a^2(1 - e^2) = 16$$

$$\Rightarrow \frac{9}{e^2}(1 - e^2) = 16 \quad \Rightarrow \quad 9 - 9e^2 = 16e^2$$

$$\Rightarrow \quad 25e^2 = 9 \quad \Rightarrow \quad e = \frac{3}{5}$$

56. Given, $\quad x^2y^2 = c^4$

$$\Rightarrow \quad y^2(a^2 - y^2) = c^4$$

$$\Rightarrow \quad y^4 - a^2y^2 + c^4 = 0$$

Let y_1, y_2, y_3 and y_4 are the roots.

$$\therefore \quad y_1 + y_2 + y_3 + y_4 = 0$$

57. Given, $4x - 3y = 5$ and $2x^2 - 3y^2 = 12$

$$\therefore \quad 2\left(\frac{5 + 3y}{4}\right)^2 - 3y^2 = 12$$

$$\Rightarrow \frac{(25 + 9y^2 + 30y)}{8} - 3y^2 = 12$$

$\Rightarrow \quad 15y^2 - 30y + 71 = 0$

$$\Rightarrow \quad y = \frac{30 \pm \sqrt{900 - 4260}}{30}$$

$$= 1 \pm \frac{\sqrt{-3360}}{30}$$

Also, $\quad 2x^2 - 3\left(\frac{4x-5}{3}\right)^2 = 12$

$\Rightarrow \quad 10x^2 - 40x + 61 = 0$

$$\Rightarrow \quad x = \frac{40 \pm \sqrt{1600 - 4 \times 10 \times 61}}{2 \times 10}$$

$$= \frac{40 \pm \sqrt{-840}}{20} = 2 \pm \frac{\sqrt{-840}}{20}$$

$\therefore$ Points are $A\left(2 + \frac{\sqrt{-840}}{20}, 1 + \frac{\sqrt{-3360}}{30}\right)$ and $B\left(2 - \frac{\sqrt{-840}}{20}, 1 - \frac{\sqrt{-3360}}{30}\right)$.

$\therefore$ Mid point of *AB* is (2, 1).

58. Given, $\frac{5}{r} = 2 + 3\cos\theta + 4\sin\theta$

$$\Rightarrow \quad \frac{5}{r} = 2 + 5\left(\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta\right)$$

$$\Rightarrow \frac{5/2}{r} = 1 + \frac{5}{2}(\cos\phi\cos\theta + \sin\phi\sin\theta)$$

$$\left(\text{put } \cos\phi = \frac{3}{5}, \text{ then } \sin\phi = \frac{4}{5}\right)$$

$$\Rightarrow \quad \frac{5/2}{r} = 1 + \frac{5}{2}\cos(\theta - \phi)$$

It is of the form $\frac{l}{r} = 1 + e\cos\theta$

$$\therefore \quad e = \frac{5}{2}$$

59. Let $A = (1, 0, 0)$, $B = (0, 1, 0)$ and $C = (0, 0, 1)$

Now, $AB = \sqrt{(0-1)^2 + (1-0)^2 + 0^2} = \sqrt{2}$

$BC = \sqrt{0^2 + (0-1)^2 + (1-0)^2} = \sqrt{2}$

and $CA = \sqrt{(1-0)^2 + 0^2 + (0-1)^2} = \sqrt{2}$

$\therefore$ Perimeter of triangle $= AB + BC + CA$

$= \sqrt{2} + \sqrt{2} + \sqrt{2}$

$= 3\sqrt{2}$

60. $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + \sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= (\cos^2\alpha - \sin^2\alpha) + (\cos^2\beta - \sin^2\beta) + (\cos^2\gamma - \sin^2\gamma) + \sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= \cos^2\alpha + \cos^2\beta + \cos^2\gamma$

$= 1$

61. We know that image (x, y, z) of a point (x_1, y_1, z_1) in a plane $ax + by + cz + d = 0$ is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

Here, point is (3, 2, 1) and plane is $2x - y + 3z = 7$.

$$\therefore \quad \frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{3} = \frac{-2[2(3) - (2) + 3(1) - 7]}{2^2 + 1^2 + 3^2}$$

$$\Rightarrow \quad \frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{3} = -2(0)$$

$\Rightarrow \quad x = 3, \; y = 2, \; z = 1$

62. Given equation of sphere is

$$x^2 + y^2 + z^2 - 12x - 4y - 3z = 0$$

$\therefore$ Centre of sphere is $\left(6, 2, \frac{3}{2}\right)$.

$\therefore$ Radius of sphere $= \sqrt{(6)^2 + (2)^2 + \left(\frac{3}{2}\right)^2}$

$$= \sqrt{36 + 4 + \frac{9}{4}} = \sqrt{\frac{169}{4}}$$

$$= \frac{13}{2}$$

63. $\lim_{x \to \infty}\left(\frac{x+5}{x+2}\right)^{x+3} = \lim_{x \to \infty}\left(1 + \frac{3}{x+2}\right)^{x+3}$

$$= \lim_{x \to \infty}\left[\left(1 + \frac{3}{x+2}\right)^{\frac{x+2}{3}}\right]^{\frac{3(x+3)}{x+2}}$$

$$= e^{\lim_{x \to \infty} 3\left(\frac{1 + \frac{3}{x}}{1 + \frac{2}{x}}\right)} = e^3$$

64. Given, $f(x) = \begin{cases} \dfrac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases}$

$$\text{Now, } \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{2\sin x - \sin 2x}{2x\cos x} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 0} \frac{2\cos x - 2\cos 2x}{2(\cos x - x\sin x)}$$

$$= \lim_{x \to 0} \frac{2-2}{2(1-0)} = 0$$

Since, $f(x)$ is continuous at $x = 0$

$$f(0) = \lim_{x \to 0} f(x) \quad \Rightarrow \quad a = 0$$

65. Given, $\frac{x}{1} = \frac{1-\sqrt{y}}{1+\sqrt{y}}$

Applying componendo and dividendo, we get

$$\frac{1+x}{1-x} = \frac{(1+\sqrt{y})+(1-\sqrt{y})}{(1+\sqrt{y})-(1-\sqrt{y})}$$

$$\Rightarrow \quad \frac{1+x}{1-x} = \frac{2}{2\sqrt{y}}$$

$$\Rightarrow \quad y = \left(\frac{1-x}{1+x}\right)^2$$

On differentiating w.r.t. *x*, we get

$$\frac{dy}{dx} = \frac{-2(1+x)^2(1-x)-(1-x)^2 \cdot 2(1+x)}{(1+x)^4}$$

$$= \frac{(1-x)(1+x)(-2-2x-2+2x)}{(1+x)^4}$$

$$= \frac{4(x-1)}{(x+1)^3}$$

66. Given, $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

and $\quad y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

$\Rightarrow \quad x = \tan^{-1} t,$

and $\quad y = \tan^{-1} t$

$\therefore \quad y = x \quad \Rightarrow \quad \frac{dy}{dx} = 1$

67. Given, $\frac{d}{dx}\left[a\tan^{-1}x + b\log\left(\frac{x-1}{x+1}\right)\right] = \frac{1}{x^4-1}$

On integrating both sides, we get

$$a\tan^{-1}x + b\log\left(\frac{x-1}{x+1}\right) = \frac{1}{2}\int\left[\frac{1}{x^2-1} - \frac{1}{x^2+1}\right]dx$$

$$\Rightarrow \quad a\tan^{-1}x + b\log\left(\frac{x-1}{x+1}\right) = \frac{1}{4}\log\left(\frac{x-1}{x+1}\right) - \frac{1}{2}\tan^{-1}x$$

$$\Rightarrow \quad a = -\frac{1}{2}, \quad b = \frac{1}{4}$$

$$\therefore \quad a - 2b = -\frac{1}{2} - 2\left(\frac{1}{4}\right) = -1$$

68. Given, $y = e^{a\sin^{-1}x}$

On differentiating w.r.t. *x*, we get

$$y_1 = e^{a\sin^{-1}x}\, a \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad y_1\sqrt{1-x^2} = ay$$

$$\Rightarrow \quad (1-x^2)y_1^2 = a^2y^2$$

Again, differentiating w.r.t. *x*, we get

$$(1-x^2)2y_1y_2 - 2xy_1^2 = a^2 2yy_1$$

$$\Rightarrow \quad (1-x^2)y_2 - xy_1 - a^2y = 0$$

Using Leibnitz's rule,

$$(1-x^2)y_{n+2} + {}^nC_1y_{n+1}(-2x) + {}^nC_2y_n(-2)$$

$$-xy_{n+1} - {}^nC_1y_n - a^2y_n = 0$$

$$\Rightarrow \quad (1-x^2)y_{n+2} + xy_{n+1}(-2n-1)$$

$$+ y_n[-n(n-1) - n - a^2] = 0$$

$$\Rightarrow \quad (1-x^2)y_{n+2} - (2n+1)xy_{n+1}$$

$$= (n^2 + a^2)y_n$$

69. Given, error in diameter $= \pm\, 0.04$

$\therefore$ Error in radius, $dr = \pm\, 0.02$

$\therefore$ Per cent error in the volume of sphere

$$= \frac{dV}{V} \times 100 = \frac{d\left(\frac{4}{3}\pi r^3\right)}{\frac{4}{3}\pi r^3} \times 100$$

$$= \frac{3dr}{r} \times 100$$

$$= \frac{3 \times (\pm\, 0.02)}{10} \times 100 = \pm\, 0.6$$

70. Given, $f(x) = x^3 + ax^2 + bx + c$, $a^2 \le 3b$.

On differentiating w.r.t. *x*, we get

$$f'(x) = 3x^2 + 2ax + b$$

Put $f'(x) = 0$

$\Rightarrow \quad 3x^2 + 2ax + b = 0$

$$\Rightarrow \quad x = \frac{-2a \pm \sqrt{4a^2 - 12b}}{2 \times 3}$$

$$= \frac{-2a \pm 2\sqrt{a^2 - 3b}}{3}$$

Since, $a^2 \leq 3b$,

$\therefore$ x has an imaginary value.

Hence, no extreme value of x exist.

71. Let $y = \dfrac{\log x}{x}$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \log x}{x^2} = \frac{1 - \log x}{x^2}$$

Put $\dfrac{dy}{dx} = 0 \quad \Rightarrow \quad \log x = 1$

$\Rightarrow \quad x = e$

$$\text{Now, } \frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)\,2x}{(x^2)^2}$$

$$= -\frac{(3 - 2\log x)}{x^3}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{(x=e)} = \frac{-(3-2)}{e^3} = -\frac{1}{e^3} < 0, \text{ maxima}$$

Hence, maximum value at $x = e$ is $\dfrac{1}{e}$.

72. Given, $z = \tan(y + ax) + \sqrt{y - ax}$

$$\Rightarrow z_x = \sec^2(y + ax)\,a + \frac{1}{2\sqrt{y - ax}}(-a)$$

$$\Rightarrow z_{xx} = 2\sec^2(y + ax)\tan(y + ax)a^2 + \frac{1(-a^2)}{4(y - ax)^{3/2}}$$

and $z_y = \sec^2(y + ax) + \dfrac{1}{2\sqrt{y - ax}}$

$$\Rightarrow \quad z_{yy} = 2\sec^2(y + ax)\tan(y + ax) - \frac{1}{4(y - ax)^{3/2}}$$

$$\therefore \quad z_{xx} - a^2 z_{yy} = 0$$

73. Let $I = \displaystyle\int \frac{dx}{(x+1)\sqrt{4x+3}}$

Put $4x + 3 = t^2 \Rightarrow 4\,dx = 2t\,dt$

$$\therefore \quad I = \frac{1}{2}\int \frac{t\,dt}{\left(\frac{t^2 - 3}{4} + 1\right)t} = 2\int \frac{dt}{1 + t^2}$$

$$= 2\tan^{-1} t + c = 2\tan^{-1}\sqrt{4x + 3} + c$$

74. Let $I = \displaystyle\int \left(\frac{2 - \sin 2x}{1 - \cos 2x}\right) e^x\,dx$

$$= \int \left(\frac{2 - 2\sin x\cos x}{2\sin^2 x}\right) e^x\,dx$$

$$= \int \underset{\text{II}}{\operatorname{cosec}^2 x}\; \underset{\text{I}}{e^x}\,dx - \int \cot x\, e^x\,dx$$

$$= -\cot x\, e^x - \int (-\cot x)\, e^x\,dx - \int \cot x\, e^x\,dx + c$$

$$= -\cot x\, e^x + c$$

75. We know that, if

$$I_n = \int \sin^n x\,dx, \text{ then}$$

$$I_n = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n} I_{n-2}$$

where n is a positive integer.

$\Rightarrow \quad nI_n - (n-1)I_{n-2} = -\sin^{n-1} x\cos x$

76. Let $I = \displaystyle\int_0^{\pi} \frac{1}{1 + \sin x}\,dx$

$$= \int_0^{\pi} \frac{1}{1 + \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}}\,dx$$

$$= \int_0^{\pi} \frac{\sec^2\frac{x}{2}}{\left(1 + \tan\frac{x}{2}\right)^2}\,dx$$

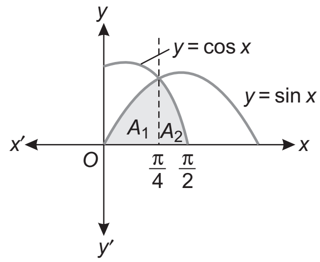
Put $\tan\dfrac{x}{2} = t \quad \Rightarrow \quad \dfrac{1}{2}\sec^2\dfrac{x}{2}\,dx = dt$

$$\therefore \quad I = \int_0^{\infty} \frac{2\,dt}{(1+t)^2} = \left[-\frac{2}{1+t}\right]_0^{\infty} = 2$$

77. Area, $A_1 = \displaystyle\int_0^{\pi/4} \sin x\,dx$

$$= -[\cos x]_0^{\pi/4} = 1 - \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2} - 1}{\sqrt{2}}$$



and area, $A_2 = \int_{\pi/4}^{\pi/2} \cos x \, dx$

$$= [\sin x]_{\pi/4}^{\pi/2} = \left[1 - \frac{1}{\sqrt{2}}\right]$$

$$= \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\therefore \quad A_1 : A_2 = \frac{\sqrt{2}-1}{\sqrt{2}} : \frac{\sqrt{2}-1}{\sqrt{2}} = 1 : 1$$

78. Given table is

t	0	2	4	6	8	10
v	0	12	16	20	35	60

Here, $h = \frac{10-0}{5} = 2$

$\therefore$ Total distance

$$= \frac{h}{2}[f(x_0) + 2\{f(x_1) + f(x_2) + f(x_3) + f(x_4)\} + f(x_5)]$$

$$= \frac{2}{2}[0 + 2(12 + 16 + 20 + 35) + 60]$$

$$= 166 + 60 = 226$$

79. Given, $\frac{dy}{dx} = \sin(x+y)\tan(x+y) - 1$

Put $x + y = z \quad \Rightarrow \quad 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$$\therefore \quad \frac{dz}{dx} - 1 = \sin z \tan z - 1$$

$$\Rightarrow \quad \int \frac{\cos z}{\sin^2 z} dz = \int dx$$

Put $\sin z = t$

$\Rightarrow \quad \cos z \, dz = dt$

$$\therefore \quad \int \frac{1}{t^2} dt = x - c \quad \Rightarrow \quad -\frac{1}{t} = x - c$$

$\Rightarrow \quad -\operatorname{cosec} z = x - c$

$\Rightarrow \quad x + \operatorname{cosec}(x+y) = c$

80. Given, $y = ae^x + bx e^x + cx^2 e^x$ ...(i)

On differentiating w.r.t. *x*, we get

$$y' = ae^x + b(xe^x + e^x) + c(x^2e^x + 2xe^x)$$

$\Rightarrow \quad y' = ae^x + bxe^x + cx^2e^x + be^x + 2cxe^x$

$\Rightarrow \quad y' = y + be^x + 2cxe^x$ ...(ii)

Again differentiating w.r.t. *x*, we get

$$y'' = y' + be^x + 2c(xe^x + e^x)$$

$\Rightarrow \quad y'' = y' + be^x + 2cxe^x + 2ce^x$

$\Rightarrow \quad y'' = 2y' - y + 2ce^x$ ...(iii)

[from Eq. (ii)]

Again differentiating w.r.t. *x*, we get

$$y''' = 2y'' - y' + 2ce^x$$

$\Rightarrow \quad y''' = 2y'' - y' + (y'' - 2y' + y)$

[from Eq. (iii)]

$\Rightarrow \quad y''' - 3y'' + 3y' - y = 0$