Engineering Mathematics Test 2

Number of Questions: 25 **Time: 60 min.**

(Ordinary Differential Equations, Calculus (Vector Calculus))

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The directional derivative of $f(x, y) = x^3y^2 + 3xy$ at (1, 2) in the direction of unit vector which makes an angle of $\frac{\pi}{4}$ with the x-axis is ________.

(A) $16\sqrt{2}$ (B) $8\sqrt{2}$

(C) $\frac{15}{\sqrt{2}}$ (D) $\frac{25}{\sqrt{2}}$

2. If the vector $e^{px-y-z}\left(\bar{i}+\bar{j}+\bar{k}\right)$ is solenoidal, then the value of p is ______.

(A) –3 (B) 3

(C) –2 (D) 2

3. Evaluate $\int_c \left(x^2+3yz\right)ds$ where c is the curve defined by $x = 6y$; $z = 4$ from $\left(3, \frac{1}{2}, 4\right)$ to (6, 1, 4).

(A) $15\sqrt{37}$ (B) $2\sqrt{37}$

(C) 199 (D) $37\sqrt{33}$

4. The unit normal vector to the surface $xy^3 + 3yz = 3$ at the point (3, –1, –2) is __________

(A) $\frac{-\bar{i}-3\bar{j}-3\bar{k}}{\sqrt{19}}$ (B) $\frac{-\bar{i}+3\bar{j}-3\bar{k}}{\sqrt{19}}$

(C) $\frac{\bar{i}+3\bar{j}-3\bar{k}}{\sqrt{19}}$ (D) $\frac{\bar{i}-3\bar{j}-3\bar{k}}{\sqrt{19}}$

5. Evaluate the integral $\int_c \left(yz+z+z^2\right)dx + (xz - 1)\,dy + (xy + x + 2xz)\,dz$ from (2, 3, 3) to (3, 4, 5).

(A) 146 (B) 107

(C) 39 (D) 185

6. Evaluate the integral $\oint_c \left(x+2y\right)dx + x^2y\,dy$, C is the triangle with vertices at (0, 0), (3, 0) and (3, 3) taken in that order.

(A) 18 (B) $\frac{45}{4}$

(C) $\frac{37}{3}$ (D) 15

7. Evaluate the surface integral $\iint_S \bar{F}.\bar{n}\,dA$, where $\bar{F} = z^2\,\bar{i} + xy\,\bar{j} + y^2\,\bar{k}$ and S is the portion of the surface of the cylinder $x^2 + y^2 = 49$; $0 \le z \le 5$ included in first octant.

(A) 518 (B) 2590

(C) $\frac{2590}{3}$ (D) 624

8. If $\bar{V} = \left(2x-y\right) - yz^2\,j - y^2z\,\bar{k}$, then evaluate $\oint_c \bar{V}.d\bar{r}$ where C is the curve bounding the projection on the semi sphere $x^2 + y^2 + z^2 = 9$, $z > 0$; in the xy plane.

(A) 18 π (B) 6 π

(C) 8 π (D) 9 π

9. If $\bar{r} = 2x\bar{i} + 3y\,\bar{j} + z\bar{k}$ and V is the volume of the sphere $x^2 + y^2 + z^2 = 16$ then $\iint_S \bar{r}.\bar{n}\,dA =$ _______.

(A) 38 π (B) 512 π

(C) 438 π (D) 348 π

10. The work done by the force $\bar{F} = 2xy\bar{i} + y^2\,\bar{j} + z\bar{k}$ in moving a particle over the circular path $x^2 + y^2 = 9$; $z = 0$ from (3, 0, 0) to (0, 3, 0) is ________.

(A) 17 (B) –12

(C) –9 (D) 6

11. The solution of $3(xdy + ydx) = 2xydy$ when $x = 1$, $y = 1$ is

(A) $3 \log (xy) - 2y + 2 = 0$

(B) $\log(xy) + y - 2 = 0$

(C) $\log x + \log y + 2 = 0$

(D) $\log(xy) - 5y + 2 = 0$

12. Solve $dy = (9x + y - 1)^2\,dx$ when $x = 0$, $y = 1$.

(A) $3 \tan x = 3x + y - 1$

(B) $3 \tan (3x) = 9x + y - 1$

(C) $\tan x = 9x + y - 1$

(D) $3 \tan 3x = 9x - y + 1$

13. The solution of the differential equation $\frac{x\,dy - ydx}{x} = \cos^2\left(\frac{y}{x}\right)dx$ is

(A) $\tan\left(\frac{y}{x}\right) = \log (cx)$ (B) $\tan\left(\frac{y}{x}\right) = x + c$

(C) $\tan^{-1}\left(\frac{y}{x}\right) = \log(cx)$ (D) $\tan\left(\frac{x}{y}\right) = \log x + c$

14. The integrating factor of $\frac{dy}{dx} - y\tan x - \cos x = 0$ is

(A) $\cos x$ (B) $\sin x$
(C) $\sec x$ (D) $\operatorname{cosec} x$

15. Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ with the boundary conditions $x = 1$, $y = 1$.

(A) $x^2 + y^2 = 3$ (B) $x^2 - y^2 = 0$
(C) $x^2 - y^2 = 2$ (D) $x^2 + y^2 = 2$

16. The solution of the differential equation $ydx = (x + 3y^3)\,dy$ when $x = 1$, $y = 1$ is

(A) $x = 3y^2 - 1$ (B) $3x = y(2y^2 - 1)$
(C) $2x = y(3y^2 + 1)$ (D) $2x = y(3y^2 - 1)$

17. The particular integral solution of the differential equation

$\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 3y = e^{4x}\sinh x$ is

(A) $\frac{1}{8}\left[e^{5x} - 2e^{3x}\right]$

(B) $\frac{1}{16}\left[e^{3x} - xe^{-5x}\right]$

(C) $\frac{1}{64}\left[e^{5x} - 8x\,e^{3x}\right]$

(D) $\frac{1}{4}\left[x\,e^{5x} - 2e^{3x}\right]$

18. The solution of the differential equation $(D^3 + 5D^2)y = 4$ is

(A) $y = C_1 + C_2e^{-5x} + \frac{1}{5}x^2$

(B) $y = C_1 + C_2x + C_3e^{-5x} + \frac{2}{5}x^2$

(C) $y = (C_1 + C_2x)\,e^{5x} + \frac{2}{5}x^2$

(D) $y = (C_1 + C_2x)\,e^{x} + \frac{2}{5}x^2 + e^{5x}$

19. The particular integral of the differential equation given by $(D^2 - 2D + 4)y = x^2 e^x$ is

(A) $\frac{1}{9}e^x(3x^2 - 2)$ (B) $\frac{1}{6}e^x(2x^2 - 3)$

(C) $\frac{1}{8}e^x(3x^2 - 1)$ (D) $\frac{1}{3}e^x(2x^2 - 3)$

20. The solution of the differential equation $(D^4 + D^2 + 36D + 52)y = 0$ is

(A) $y = (C_1 + C_2x)e^{-2x} + (C_3 + C_4x)e^{2x}$
(B) $y = (C_1 + C_2x + C_3\cos 3x + C_4\sin 3x)e^{2x}$
(C) $y = (C_1 + C_2x + (C_3\cos 3x + C_4\sin 3x)e^{-2x}$
(D) $y = (C_1 + C_2x)e^{-2x} + (C_3\cos 3x + C_4\sin 3x)e^{2x}$

21. The Laplace Transform of the function $f(t) = t^2\sin 3t$, $t > 0$ is ______.

(A) $\frac{(s^2 - 2s + 9)}{(s^2 + 9)^3}$ (B) $\frac{18(s^2 - 3)}{(s^2 + 9)^3}$

(C) $\frac{-(s^2 - 2s + 9)}{(s^2 + 9)^3}$ (D) $\frac{6(s^2 - 2s + 9)}{(s^2 + 9)^3}$

22. If $u(t - a)$ denotes the unit step function, then the Laplace Transform of $(t^2 + 3)\,u(t - 2)$ is ________.

(A) $\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{7}{s}\right)e^{-2s}$ (B) $\left(\frac{2}{s^3} + \frac{3}{s}\right)e^{-2s}$

(C) $\left(\frac{2}{s^2} + 3\right)e^{-2s}$ (D) $\left(\frac{1}{s^3} + \frac{4}{s^2} + \frac{7}{s}\right)e^{-2s}$

23. The Laplace Transform of solution of the initial value problem $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0$, $y(0) = 1$ and $y^1(0) = -2$ will be ____

(A) $\frac{1}{(s-4)(s+2)}$ (B) $\frac{1}{s-4}$

(C) $\frac{-1}{(s-4)(s+2)}$ (D) $\frac{1}{s+2}$

24. The inverse Laplace Transform of $\frac{3}{2}\left[\frac{1}{\sqrt{s^5}} - \frac{1}{\sqrt{s^3}}\right]$ is ______.

(A) $\sqrt{\frac{t}{\pi}}\,(2t - 3)$ (B) $\sqrt{\frac{t}{\pi}}\,(4t - 3)$

(C) $\sqrt{\frac{\pi}{t}}\,(2t + 3)$ (D) $\sqrt{\frac{\pi}{t}}\,(4t + 3)$

25. The inverses Laplace Tranform of $\frac{4(2s+3)}{(s^2 + 4s + 20)}$ is

(A) $e^{2t}[2\cos 4t - \sin 4t]$
(B) $e^{2t}\{2\sin 4t - \cos 4t]$
(C) $e^{-2t}[2\cos 4t - \sin 4t]$
(D) $e^{-2t}[2\sin 4t - \cos 4t]$

Answer Keys

1. D	**2.** D	**3.** A	**4.** B	**5.** B	**6.** B	**7.** C	**8.** D	**9.** B	**10.** C
11. A	**12.** B	**13.** A	**14.** A	**15.** B	**16.** D	**17.** C	**18.** B	**19.** A	**20.** D
21. B	**22.** A	**23.** D	**24.** A	**25.** C					

Hints and Explanations

1. Given $f(x, y, z) = x^3 y^2 + 3xy$

$$\nabla f = \frac{\partial f}{\partial x}\bar{i} + \frac{\partial f}{\partial y}\bar{j} = (3x^2 y^2 + 3y)\bar{i} + (2x^3 y + 3x)\bar{j}$$

$$(\nabla f)_{(1,2)} = (12 + 6)\bar{i} + (4 + 3)\bar{j} = 18\bar{i} + 7\bar{j}$$

Given that unit vector makes an angle $\frac{\pi}{4}$ with x-axis

$\therefore$ unit vector must be $\bar{b} = \cos\frac{\pi}{4}\bar{i} + \sin\frac{\pi}{4}\bar{j} = \frac{\bar{i}+\bar{j}}{\sqrt{2}}$

The directional derivative of f in the direction $\bar{b}$ is $\nabla f.\bar{b}$

$$(18\bar{i}+7\bar{j}).\frac{(\bar{i}+\bar{j})}{\sqrt{2}} = \frac{18+7}{\sqrt{2}} = \frac{25}{\sqrt{2}}$$

Choice (D)

2. Given $\bar{V} = e^{px-y-z}(\bar{i}+\bar{j}+\bar{k})$ is solenoidal

We know that if $\bar{V}$ is solenoidal, div $\bar{V} = 0$

$$\frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z} = 0$$

$$= e^{px-y-z}.p + e^{px-y-z}.(-1) + e^{px-y-z}(-1) = 0$$

$\therefore p - 1 - 1 = 0 \Rightarrow p = 2$

Choice (D)

3. Let $x = t$; $y = \frac{t}{6}$; $z = 4$ and $3 \le t \le 6$.

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,.dt = \sqrt{1+\frac{1}{36}}\,dt = \frac{\sqrt{37}}{6}dt$$

$$\int_C (x^2+3yz)\,ds = \int_3^6 \left(t^2+3\frac{t}{6}.4\right)\frac{\sqrt{37}}{6}dt = d\left(\tan\left(\frac{y}{x}\right)\right)$$

$$= \tan\left(\frac{y}{x}\right) = \tan\left(\frac{y}{x}\right)$$

Choice (A)

4. Let $f(x, y, z) = xy^3 + 3yz - 3$

The normal vector to the surface $f(x, y, z)$ is

$$\frac{dy}{dx} - y\tan x - \cos x$$

$$\nabla f = y^3\,\bar{i} + (3xy^2+3z)\bar{j} + 3y\bar{k}$$

The normal vector at (3, –1, –2)

$$(\nabla f)_{(3,-1,-2)} = -\bar{i} + (9-6)\bar{j} - 3\bar{k} = -\bar{i} + 3\bar{j} - 3\bar{k}$$

$\therefore$ The unit normal vector to the surface f is $\frac{\nabla f}{|\nabla f|}$

$$= \frac{-\bar{i}+3\bar{j}-3\bar{k}}{\sqrt{19}}$$

Choice (B)

5. $\int (yz+z+z^2)dx + (xz-1)dy + (xy+x+2xz)dz$

Let $f(x, y, z) = yz + z + z^2$

$g(x, y, z) = xz - 1$

$h(x, y, z) = xy + x + 2xz$

$$\frac{\partial f}{\partial y} = z = \frac{\partial g}{\partial x}$$

$$\frac{\partial f}{\partial z} = y + 1 + 2z = \frac{\partial h}{\partial x}$$

$$\frac{\partial g}{\partial z} = x = \frac{\partial h}{\partial y}$$

The integral is independent of the path C.

The integral is exact differential

So there exists a function Φ

Such that $\frac{\partial\phi}{\partial x} = yz + z + z^2 \quad \rightarrow (1)$

$\frac{\partial\phi}{\partial y} = xz - 1 \quad \rightarrow (2)$

$\frac{x}{y} = \int 3y^2 . \frac{1}{y}dy + c = xy + x + 2xz \quad \rightarrow (3)$

$$\frac{\partial\phi}{\partial x} = yz + z + z^2$$

Integrate wrt x.

$\phi = xyz + xz + xz^2 + Q(y, z) \quad \rightarrow (4)$

Diff wrt y

$\frac{\partial\phi}{\partial y} = xz + \phi^1(y, z) \quad \rightarrow (5)$

Comparing (2) and (5)

$\phi^1(y, z) = -1 \Rightarrow \phi(y, z) = -y + R(z)$

$\therefore \phi = xyz + xz + xz^2 - y + R(z) \quad \rightarrow (6)$

$$\frac{ydx - xdy}{y^2} = 3y\,dy = xy + x + 2xz + R^1(z)$$

Comparing (3) and (6)

$R^1(z) = 0$

$\Rightarrow R(z) = k$

$\therefore \phi = xyz + xz + xz^2 - y + k$

$$\therefore \int_{(3,4,5)} (yz+z+z^2)dx + (xz-1)dy + (xy+x+2xz)dz$$

$$= \int_{(2,3,3)} d(xyz + xz + xz^2 - y)$$

$$= xyz + xz + xz^2 - y\Big]_{(2,3,3)}^{(3,4,5)}$$

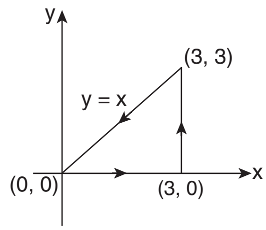
$= 146 - 39 = 107$

Choice (B)

6. $\oint_C (x+2y)dx + x^2 y\,dy$

$f(x, y) = x + 2y \qquad g(x, y) = x^2 y$

$$\frac{\partial f}{\partial y} = 2 \text{ and } \frac{\partial g}{\partial x} = 2xy$$



By green's theorem

$$\oint_C (x+2y)dx + x^2 y dy = \iint_R (2xy-2)dxdy$$

$$= \int_0^3 \int_0^x (2xy-2)dydx$$

$$= \int_0^3 \left. xy^2 - 2y \right]_0^x dx$$

$$= \int_0^3 (x^3 - 2x)dx$$

$$= \left. \frac{x^4}{4} - x^2 \right]_0^3 = \frac{81}{4} - 9 = \frac{45}{4}$$

Choice (B)

7. Let $f(x, y, z) = x^2 + y^2 - 49$
Surface then

grad $f = 2x\bar{i} + 2y\,j$

$$\overset{\Delta}{n} = \frac{\text{grad } f}{|\text{grad } f|} = \frac{2x\bar{i} + 2y\,j}{2\sqrt{x^2+y^2}} = \frac{1}{7}(x\bar{i} + y\bar{j})$$

Consider the projection of S on the yz plane. It is a rectangle with sides 7 and 5.

$$dA = \frac{dy\,dz}{\bar{n}.\bar{i}} = \frac{dy\,dz}{\frac{x}{7}}$$

$$\bar{F}.\overset{\Delta}{n} = (z^2\,\bar{i} + xy\,\bar{j} + y^2\,\bar{k}).\left(\frac{x\bar{i} + y\bar{j}}{7}\right) = \frac{xz^2 + xy^2}{7}$$

$$\therefore \iint_S \bar{F}.\overset{\Delta}{n}\,dA = \iint_s \frac{xz^2 + xy^2}{7} \frac{dy\,dz}{\frac{x}{7}}$$

$$= \int_{z=0}^{5} \int_{y=0}^{7} (z^2 + y^2)\,dy\,dz = \int_{z=0}^{5} \left. z^2 y + \frac{y^3}{3} \right]_0^7 dz$$

$$= \int_{z=0}^{5} (7z^2 + \frac{343}{3})dz$$

$$\left. \frac{7z^3}{3} + \frac{343}{3} z \right]_0^5 = \frac{875}{3} + \frac{1715}{3} = \frac{2590}{3}$$

Choice (C)

8. Given $\bar{V} = (2x-y)\bar{i} - yz^2\,\bar{j} - y^2\,z\,\bar{k}$

$$\oint \bar{V}.d\bar{r} = \iint_S \nabla \times V.ndA$$

(By stoke's theorem)

$$\nabla \times V = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x-y & -yz^2 & -y^2 z \end{vmatrix}$$

$$= i(-2yz + 2yz) - j(0) + \bar{k}\,(0 - (-1)) = \bar{k}$$

$f(x, y, z) = x^2 + y^2 + z^2 - 9$ be the surface

grad $f = 2x\bar{i} + 2y\bar{j} + 2z\bar{k}$

$$\hat{n} = \frac{\text{grad } f}{|\text{grad } f|} = \frac{2x\bar{i} + 2y\bar{j} + 2z\bar{k}}{2\sqrt{x^2+y^2+z^2}}$$

$$\hat{n} = \frac{x\bar{i} + y\bar{j} + z\bar{k}}{3}$$

$$\therefore \quad \nabla \times \bar{V}.\overset{\Delta}{n} = \bar{k}.\frac{(x\bar{i} + y\bar{j} + z\bar{k})}{3} = \frac{z}{3}$$

$$\iint_S \nabla \times V.\overset{\Delta}{n}\,dA = \iint_R \frac{z}{3} \frac{dx\,dy}{\bar{n}.\bar{k}}$$

$$= \iint_R \frac{z}{3} \frac{dx\,dy}{\frac{z}{3}} = \iint_R dx\,dy$$

Area of circular region in x-y plane = 9π Choice (D)

9. By using divergence theorem

$$\iint_S \bar{r}.\bar{n}\,dA = \iiint_V div\,\bar{r}\,dV$$

$$\bar{r} = 2x\,\bar{i} + 3y\,\bar{j} + z\bar{k}$$

$$\text{Div } \bar{r} = 2 + 3 + 1 = 6 = \iiint_V div\,\bar{r}\,dv = \iiint_V 6dV = 6V$$

V is the volume of the sphere

$= 6 \times \frac{4}{3} \times \pi(4)^3 = 512\,\pi$ Choice (B)

10. Given $\bar{F} = 2xy\bar{i} + y^2\,\bar{j} + z\bar{k}$

Work done by the force is $\int_C F.d\bar{r}$

$$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k} \Rightarrow d\bar{r} = dx\bar{i} + dy\,\bar{j} + dz\bar{k}$$

$$\bar{F}.d\bar{r} = (2xy\bar{i} + y^2\,\bar{j} + z\bar{k}).(dx\bar{i} + dy\,\bar{j} + dz\bar{k})$$

$$\bar{F}.d\bar{r} = 2xy\,dx + y^2\,dy + z\,dz$$

$$\int_C F.d\bar{r} = \int (2xy\,dx + y^2\,dy + z\,dz$$

Convert x, y, z in parametric form
$x = 3\cos t, y = 3\sin t, z = 0$
$dx = -3\sin t\, dt, dy = 3\cos t\, dt$
The limit of t is 0 to $\pi/2$

$$\int_C F.d\bar{r} = \int_0^{\frac{\pi}{2}} 2.3\cos t.3\sin t.(-3\sin t)\,dt + 9\sin^2 t\, 3\cos t\, dt$$

$$= -\int_0^{\frac{\pi}{2}} 27\sin^2 t\cos t\, dt$$

$$= -27\left.\frac{\text{Sin}^3 t}{3}\right]_0^{\frac{\pi}{2}} = \frac{-27}{3} = -9$$

Choice (C)

11. $3(x\,dy + y\,dx) = 2xy\,dy$
$x(3-2y)\,dy + 3y\,dx = 0$

$$\frac{(3-2y)dy}{y} + \frac{3dx}{x} = 0$$

$$\int\left(\frac{3}{y} - 2\right)dy + 3\int\frac{dx}{x} = C_1$$

$3\log y - 2y + 3\log x = \log C$
$3\log(xy) - 2y = \log C$
Given when $x = 1, y = 1$
$\Rightarrow\ -2 = \log C$
$\therefore$ The required solution is $3\log(xy) - 2y + 2 = 0$.

Choice (A)

12. $dy = (9x + y - 1)^2 dx$ or $\frac{dy}{dx} = (9x + y - 1)^2$ ……. (1)

Let $9x + y - 1 = u$

$$9 + \frac{dy}{dx} = \frac{du}{dx}$$

$$\Rightarrow \frac{du}{dx} - 9 = u^2 \text{ (from 1)}$$

$$\frac{du}{dx} = u^2 + 9 \text{ or } \frac{du}{u^2+9} = dx$$

$$\int\frac{du}{u^2+9} = \int dx + c \Rightarrow \frac{1}{3}\text{Tan}^{-1}\left(\frac{u}{3}\right) = x + c$$

i.e., $\frac{1}{3}\tan^{-1}\left(\frac{9x+y-1}{3}\right) = x + c$ or

$3\tan(3x + 3c) = 9x + y - 1$, when $x = 0, y = 1$

$$\frac{1}{3}\tan^{-1}(0) = C \Rightarrow C = 0$$

$\therefore$ The required solution is $3\tan(3x) = 9x + y - 1$.

Choice (B)

13. $\frac{x\,dy - y\,dx}{x} = \cos^2\left(\frac{y}{x}\right)dx$

Dividing both sides by $x\cos^2\left(\frac{y}{x}\right)$, we have

$$\left(\frac{x\,dy - y\,dx}{x^2}\right)\sec^2\left(\frac{y}{x}\right) = \frac{dx}{x}$$

$$d\left(\tan\left(\frac{y}{x}\right)\right) = d(\log x)$$

Integrating on both sides, we have

$$\tan\left(\frac{y}{x}\right) = \log x + \log c$$

$$\tan\left(\frac{y}{x}\right) = \log(cx).$$

Choice (A)

14. $\frac{dy}{dx} - y\tan x - \cos x = 0$

$$\Rightarrow \frac{dy}{dx} - y\tan x = \cos x$$

which is in the form of $\frac{dy}{dx} + Py = Q$

The integrating factor of above equation $e^{\int p dx} = e^{-\int \tan x\, dx}$
$= e^{\log\cos x} = \cos x$.

Choice (A)

15. $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ --------- (1)

$(x^2 + y^2)dx - 2xy\,dy = 0$
which is in the form of $Mdx + Ndy = 0$
Here $M = x^2 + y^2$, $N = -2xy$

$$\frac{\partial M}{\partial y} = 2y, \frac{\partial N}{\partial x} = -2y$$

$$\Rightarrow \text{Here } \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{-1}{2xy}(2y + 2y) = \frac{-2}{x}$$

which is a function of x alone say $f(x)$ then the Integrating factor ($I.F$) is

$$e^{\int f(x)dx} = e^{\int\frac{-2}{x}dx} = e^{-2\log x} = e^{\log x^{-2}} = x^{-2}$$

Multiplying (1) by $I.F$, we have

$\left(1 + \frac{y^2}{x^2}\right)dx - \frac{2y}{x}dy = 0$ which is in the form of $M\,dx + N\,dy = 0$

$\therefore$ Solution is

$$\int m\,dx(\text{taking } y \text{ constant}) + \int(\text{terms of } N \text{ not containing } x)\,dy = C$$

$$\Rightarrow \int\left(1 + \frac{y^2}{x^2}\right)dx + \int 0\,dy$$

$$x - \frac{y^2}{x} = C$$

Given when $x = 1, y = 1 \Rightarrow C = 0$

$\therefore$ The required solution is $x - \frac{y^2}{x} = 0$ or $x^2 - y^2 = 0$.

Choice (B)

16. $y\,dx = (x + 3y^3)\,dy$

or $y\frac{dx}{dy} = x + 3y^3$

or $\frac{dx}{dy} - \frac{x}{y} = 3y^2$, which is a linear equation in y of the form $\frac{dx}{dy} + px = Q$ here $P = -\frac{1}{y}$ and $Q = 3y^2$

$\therefore\ I.F = e^{\int p dy} = e^{-\int \frac{1}{y} dy} = e^{\log y^{-1}} = \frac{1}{y}$

$\therefore$ The solution is $x\,.\,e^{I.F} = \int Q.e.^{I.F}.dy$

$\Rightarrow\ \frac{x}{y} = \int 3y^2.\frac{1}{y}dy + c$

$\frac{x}{y} = \frac{3y^2}{2} + c$

Given when $x = 1, y = 1$

$\Rightarrow\ 1 = \frac{3}{2} + c$ or $c = \frac{-1}{2}$

$\therefore$ The required solutions is $\frac{x}{y} = \frac{3y^2}{2} - \frac{1}{2}$

i.e., $2x = y(3y^2 - 1)$

Alternate solution:

Given

$ydx = (x + 3y^3)dy$

$ydx - xdy = 3y^3\,dy$

$\frac{ydx - xdy}{y^2} = 3y\,dy$

$d\left(\frac{x}{y}\right) = 3y\,dy$

Integrating on both sides

$\left(\frac{x}{y}\right) = \left(\frac{3y^2}{2}\right) + C$

$2x = y(3y^2 + C_1)$

Given $x = 1; y = 1$

$2 = 3 + C_1 \Rightarrow C_1 = -1$

$\therefore$ required solution is $2x = y\,(3y^2 - 1)$. Choice (D)

17. $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 3y = e^{4x}\sin hx$

i.e., $(D^3 - 5D^2 + 7D - 3)y$

$= e^{4x}\left[\frac{e^x - e^{-x}}{2}\right] = \frac{e^{5x} - e^{3x}}{2}$

i.e., $(D - 1)^2\,(D - 3)y = \frac{e^{5x} - e^{3x}}{2}$

Particular integral is $\frac{e^{5x} - e^{3x}}{(D-1)^2(D-3)}.\frac{1}{2}$

$\frac{1}{2}\frac{e^{5x}}{(D-1)^2(D-3)} - \frac{1}{2(D-1)^2(D-3)}.e^{3x}$

$= \frac{1}{2}.\frac{e^{5x}}{(5-1)^2(5-3)} - \frac{1}{2}\frac{1}{(3-1)^2}.\frac{e^{3x}}{(D-3)}$

$= \frac{e^{5x}}{64} - .\frac{1}{8}.x.e^{3x} = \frac{1}{64}\left[e^{5x} - 8x\,e^{3x}\right]$ Choice (C)

18. $(D^3 + 5D^2)\,y = 4$

Auxiliary equation $m^3 + 5m^2 = 0$

$m^2(m + 5) = 0$

$\Rightarrow\ m = 0, 0, -5$

$\therefore\ C.\,F$ is $(C_1 + C_2x)e^{0.x} + C_3e^{-5x} = (C_1 + C_2x) + C_3e^{-5x}$

$P.\,I = \frac{1}{D^2(D+5)}4.e^{0.x}$

$= \frac{1}{5}.\frac{1}{D^2}4 = \frac{1}{5}\left[\frac{1}{D^2}.4\right] = \frac{2x^2}{5}$.

Complementary Solution $y = C.F + P.I$

$y = C_1 + C_2x + C_3\,e^{-5x} + \frac{2x^2}{5}$ Choice (B)

19. $(D^2 - 2D + 4)\,y = x^2\,e^x$

$P.\,I = \frac{e^x\,x^2}{(D^2 - 2D + 4)} = e^x\frac{1}{(D+1)^2 - 2(D+1) + 4}.x^2$

$= e^x\frac{1}{D^2 + 3}x^2$

$\Rightarrow\ e^x\frac{1}{3\left(1 + \frac{D^2}{3}\right)}x^2 = e^x\frac{1}{3}\left(1 + \frac{D^2}{3}\right)^{-1}x^2$

$= e^x\frac{1}{3}\left(1 - \frac{D^2}{3}\right)x^2$

$= e^x\frac{1}{3}\left(x^2 - \frac{2}{3}\right) = e^x\frac{1}{9}\left(3x^2 - 2\right)$ Choice (A)

20. $(D^4 + D^2 + 36D + 52)\,y = 0$

Auxiliary equation of the above is $m^4 + m^2 + 36m + 52 = 0$

By trail and error we notice m = –2, –2, are the roots of the above

$\therefore\ (m + 2)^2\,(m^2 - 4m + 13) = 0$

The roots are $m = -2, -2$, and $2 \pm 3i$

$\therefore$ The solution is

$y = (C_1 + C_2x)e^{-2x} + e^{2x}\,(C_3\cos 3x + C_4\sin 3x)$

Choice (D)

21. Let $g(t) = \sin 3t \Rightarrow f(t) = t^2 g(t)$

Now $L[g(t)] = L:[\sin 3t] = \dfrac{3}{s^2+9}$

$$L[f(t)] = L[t^2 g(t)] = \frac{d^2}{ds^2}\ (L[g(t)])$$

$$= \frac{d}{ds}\left(\frac{d}{ds}\left(\frac{3}{s^2+9}\right)\right)$$

$$= \frac{d}{ds}\left(\frac{-6s}{\left(s^2+9\right)^2}\right)$$

$$= \frac{-6\left[\left(s^2+9\right)^2.1 - s.2\left(s^2+9\right).2s\right]}{\left(s^2+9\right)^4}$$

$$= \frac{-6\left[s^2+9-4s^2\right]}{\left(s^2+9\right)^3}$$

$$= \frac{18(s^2-3)}{\left(s^2+9\right)^3}$$

Choice (B)

22. Let $f(t) = (t^2 + 3)\, u(t - 2)$

$L[f(t)] = L[(t^2 + 3)\, u(t - 2)]$

$L[((t - 2) + 2)^2 + 3)\, u(t - 2)]$

$\therefore\quad L[f(t)] = L[(t-2)^2 + 4(t - 2) + 7)\, u(t - 2)]$ ----- (1)

Let $g(t) = t^2 + 4t + 7$

$\therefore\quad L[g(t)] = L[t^2 + 4t + 7] = L[t^2] + 4L[t] + 7L[1]$

$\therefore\quad L[g(t)] = \dfrac{2}{s^3} + \dfrac{4}{s^2} + \dfrac{7}{s}$

Now from (1)

$L[f(t)] = L[(t - 2)^2 + 4(t - 2) + 7)\, u(t - 2)]$

$= L[g(t - 2).\ U(t - 2)$

$= L[g(t)]\, e^{-2s}$ (By second shifting theorem)

$= \left(\dfrac{2}{s^3} + \dfrac{4}{s^2} + \dfrac{7}{s}\right)e^{-2s}.$

Choice (A)

23. Given initial value problem is

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0 \qquad \text{----------- (1)}$$

Where $y(0) = 1$ and $y^1(0) = -2$

Applying laplace transform on both sides of (1)

$$L\left[\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y\right] = L[0]$$

$$\Rightarrow\ L\left[\frac{d^2y}{dt^2}\right] - 2L\left[\frac{dy}{dt}\right] - 8L[y] = 0$$

$$\Rightarrow\ \left(s^2\bar{y} - sy(0) - y^1(0)\right) - 2\left(s\bar{y} - y(0)\right) - 8\bar{y} = 0$$

Where $\bar{y} = L[y]$

$$\Rightarrow\ s^2\bar{y} - s \times 1 - (-2) - 2s\bar{y} + 2 \times 1 - 8\bar{y} = 0$$

$$\Rightarrow\ \left(s^2 - 2s - 8\right)\bar{y} - s + 4 = 0$$

$$\Rightarrow\ \left(s^2 - 2s - 8\right)\bar{y} = s - 4$$

$$\Rightarrow\ \bar{y} = \frac{(s-4)}{\left(s^2-2s-8\right)} = \frac{(s-4)}{(s-4)(s+2)}$$

$$\bar{y} = \frac{1}{s+2}$$

The laplace transform of the solution of (1) is

$\bar{y} = L[y] = \dfrac{1}{s+2}.$

Choice (D)

24. We have to find $L^{-1}\left[\dfrac{3}{2}\left(\dfrac{1}{\sqrt{s^5}} - \dfrac{1}{\sqrt{s^3}}\right)\right]$

$$= \frac{3}{2}\left(L^{-1}\left[\frac{1}{s^{\frac{5}{2}}}\right] - L^{-1}\left[\frac{1}{s^{\frac{3}{2}}}\right]\right)$$

$$= \frac{3}{2}\left[\frac{t^{\frac{3}{2}}}{\Gamma\left(\frac{5}{2}\right)} - \frac{t^{\frac{1}{2}}}{\Gamma\left(\frac{3}{2}\right)}\right]$$

$$= \frac{3}{2}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}\times\frac{1}{2}\left(\sqrt{\pi}\right)} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}\left(\sqrt{\pi}\right)}\right]$$

$$= \frac{3}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{4}\sqrt{\pi}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}\sqrt{\pi}}\right) = - \times \frac{\sqrt{t}}{\sqrt{\pi}}\left[\frac{4t}{3} - 2\right]$$

$$= \sqrt{\frac{t}{\pi}} \times 3\left[\frac{2t}{3} - 1\right]$$

$$= \sqrt{\frac{t}{\pi}}\ (2t - 3).$$

Choice (A)

25. We have $L^{-1}\left[\dfrac{4(2s+3)}{s^2+4s+20}\right]$

$$= 4\,L^{-1}\left[\frac{2s+3}{\left(s^2+4s+4\right)+16}\right]$$

$$= 4L^{-1}\left[\frac{2(s+2-2)+3}{(s+2)^2+4^2}\right] = 4L^{-1}\left[\frac{2(s+2)-1}{(s+2)^2+4^2}\right]$$

$$= 4L^{-1}\left[\frac{2(s+2)}{(s+2)^2+4^2}\right] - 4L^{-1}\left[\frac{1}{(s+2)^2+4^2}\right]$$

$$= 8 \times \frac{1}{4}e^{-2t}\cos 4t - 4 \times \frac{1}{4}e^{-2t}\sin 4t$$

$$= e^{-2t}\,[2\cos 4t - \sin 4t].$$

Choice (C)