CBSE Class 09

Mathematics

Sample Paper 9 (2019-20)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. If $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5} - \frac{19}{11}b$, then the value of 'b' is

   a. 3

   b. 1

   c. -1

   d. 2

2. $\sqrt{2}$ is a polynomial of degree

   a. 0

   b. 1

c. $\sqrt{2}$

d. 2

3. If two lines intersect each other then

    a. Corresponding angles are equal

    b. Alternate interior angles are equal

    c. Co-interior angles are equal

    d. Vertically opposite angles are equal

4. The orthocenter of an obtuse triangle lies ___________.

    a. outside the triangle

    b. on the greatest side of the triangle

    c. inside the triangle

    d. on the smallest side of the triangle

5. If $x+\frac{1}{x}=3$, then the value of $x^2+\frac{1}{x^2}$ is

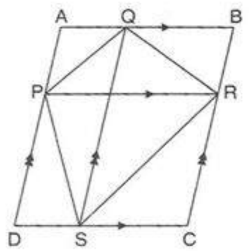
    a. 0

    b. 7

    c. 1

    d. 9

6. ABCD is a parallelogram formed by drawing lines parallel to diagonals of quadrilateral PQRS through its vertices. If $ar\ (quad\ PQRS)=15\ cm^2,$ then $ar\ (\|\ ABCD)$ is



a. $25\ cm^2$.

b. $30\ cm^2$.

c. $40\,cm^2$.

d. $40\ cm^2$.

7. If $a+b+c=0$, then $\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right)=$

a. 3

b. 2

c. 1

d. 0

8. Semiperimeter of scalene triangle of side k, 2k and 3k is

a. 3k

b. 4k

c. 2k

d. k

9. A hemispherical bowl is made of steel 0.25 cm thick. If the inner radius of the bowl is 3.25 cm, then the outer curved surface area of the bowl is

a. 77 cm^2.

b. 115.5 cm^2.

c. 154 cm^2.

d. 38.5 cm^2.

10. The probability of an event (other than sure and impossible event) lies between

a. - 1 and 1

b. $\frac{1}{2}$ *and* 1

c. 0 and 1

d. $0\ and \frac{1}{2}$

11. Fill in the blanks:

The exponent form of $\sqrt[3]{7}$ is ________.

12. Fill in the blanks:

If x = 1, y = 1 and 5x + 2ky = 3k, then the value of k is ________.

OR

Fill in the blanks:

Any point on the X-axis is of the form of ________.

13. Fill in the blanks:

Points (0, 4) and (3, 0) lies on ________ , ________ axis respectively.

14. Fill in the blanks:

The longest chord of a circle is ________ of the circle.

15. Fill in the blanks:

The total surface area of cylinder is equal to _______sq units.

16. Is $(1 + \sqrt{5}) - (4 + \sqrt{5})$ a rational number?

17. Evaluate the following: $(9.9)^3$

18. Find the length of the longest rod that can be placed in a room having dimensions 12 m × 9 m × 8 m.

OR

How many 3 metre cubes can be cut from a cuboid measuring $18m \times 12m \times 9m$?

19. In a parallelogram PQRS, if $\angle P = (3x - 5)^o$ and $\angle Q = (2x + 15)^o$. Find the value of x.

20. Express the given equation as linear equation in two variables in standard form: $\sqrt{3}y = 2x$.

21. Write a following in decimal form and state its kind of decimal expansion. $4\frac{1}{48}$

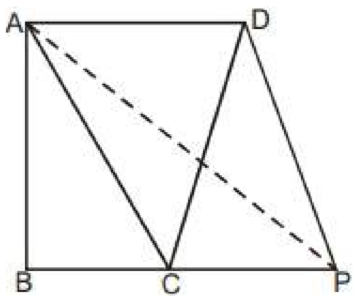
22. If the point $(2k - 3, k + 2)$ lies on the graph of the equation $2x + 3y + 15 = 0$, find the value of k.

23. Factorise: $125x^3 - 343y^3$.

OR

Using suitable identity Factorise : $9x^2 + 6xy + y^2$

24. There is a plot in a village in the shape of a quadrilateral ABCD. Sarpanch wants to get floor cemented so as to use it for social gatherings and Panchayat meetings. Later due to construction of park in the neighbourhood for children, they decided to change the shape to triangle ABP.
If $AC || DP$, prove that ar(quad. ABCD) = ar($\triangle$ABP)



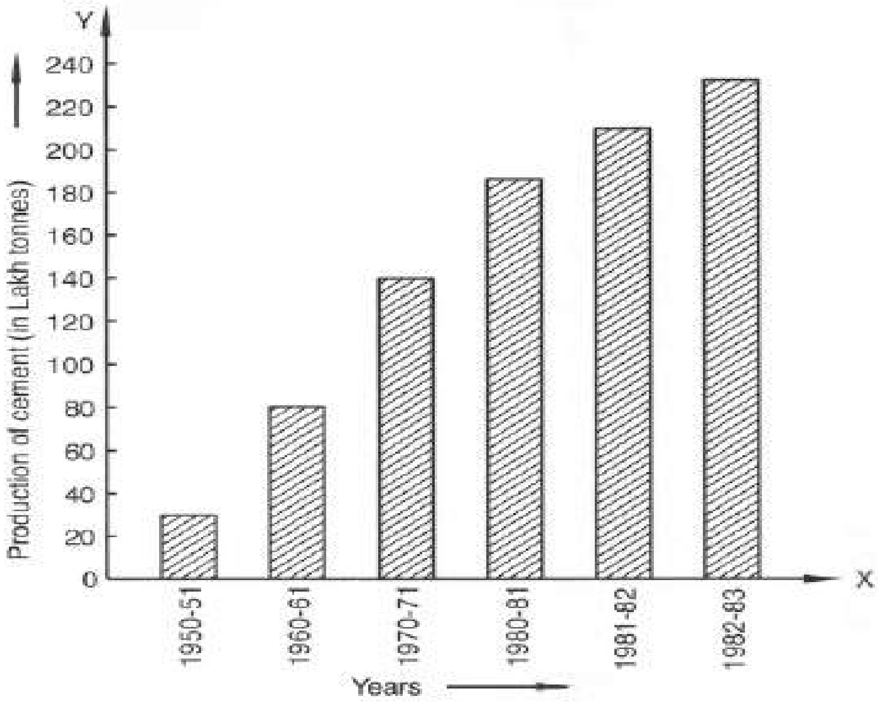
25. Find the mean of the following distribution:

x	10	12	20	25	35
f	3	10	15	7	5

OR

Read the following bar graph in the figure and answer the following questions:

i. What information is given by the bar graph?
ii. What was the production of cement in the year 1980-81?
iii. What is the minimum and maximum productions of cement and corresponding years?



26. A team of 10 interns and 1 professor from zoological department visited a forest, where they set up a conical tent for their accommodation. There they perform activities like planting saplings, yoga, cleaning lakes, testing the water for contaminants and pollutant levels and desilt the lake bed and also using the silt to strengthen bunds.

    Find the radius and height of the tent if the base area of tent is 154 cm^2 and curved surface area of the tent is 396 cm^2.

27. If $9^{x+2} = 720 + 9^x$, find the value of $(4x)^{\frac{1}{x}}$.

OR

Simplify: $\frac{\sqrt{25}}{\sqrt[3]{64}} + \left(\frac{256}{625}\right)^{-1/4} + \frac{1}{\left(\frac{64}{125}\right)^{2/3}}$

28. Take a rectangle ABCD with A (-6, 4), B (-6, 2), C (-2, 2) and D (-2, 4). Find its mirror image with respect to x- axis.

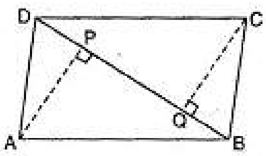
29. Draw the graph of the following equation. Read a few solutions from the graph and verify the same by actual substitution and find the points where the line meets the two axes. 2(x – 1) + 3y = 4

OR

Draw the graph of the following linear equation in two variables: 2y = -x + 1.

30. Construct an angle of 30° whose initial point is given ray.

31. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively.



Show that :

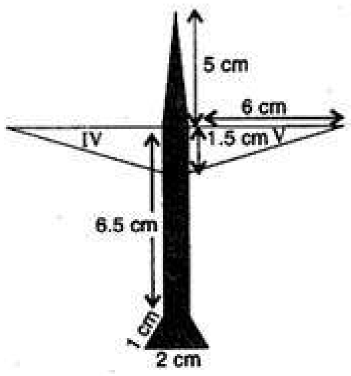
i. $\triangle$APB $\cong$ $\triangle$CQD
ii. AP = CQ.

32. AD and BE are respectively altitudes of a triangle ABC such that AE = BD. Prove that AD = BE.

OR

S is any point on side QR of a $\triangle$PQR. Show that: PQ + QR + RP > 2PS.

33. Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used.

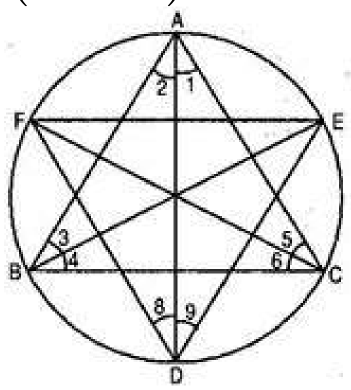


34.

Marks (out of 100)	Number of Students
0-20	7
20-30	10
30-40	10
40-50	20
50-60	20
60-70	15
70-above	8
Total	90

i. Find the probability that a student obtained less than 20% in the mathematics test.
ii. Find the probability that a student obtained marks 60 or above.

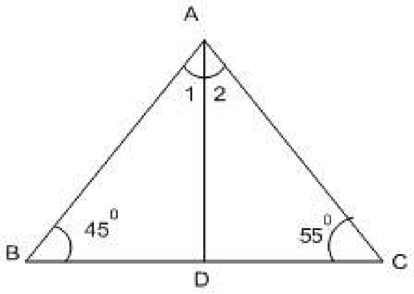
35. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that angles of the triangle are $\left(90^\circ - \frac{A}{2}\right)$, $\left(90^\circ - \frac{B}{2}\right)$ and $\left(90^\circ - \frac{C}{2}\right)$ respectively.



OR

Prove that angle bisector of any angle of a triangle and perpendicular bisector of the opposite side if intersect, they will intersect on the circumcircle of the triangle.

36. In $\triangle ABC$ $\angle B = 45°$, $\angle C = 55°$ and bisector $\angle A$ meets BC at a point D. Find $\angle ADB$ and $\angle ADC$.



37. Using factor theorem, factorize the polynomial $x^4 + 2x^3 - 13x^2 - 14x + 24$

OR

If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a.

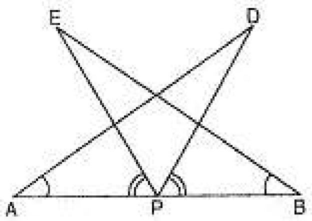
38. A wooden toy is in the form of a cone surmounted on a hemisphere. The diameter of the base of the cone is 16 cm and its height is 15 cm. Find the cost of painting the toy at Rs. 7 per 100 cm^2.

OR

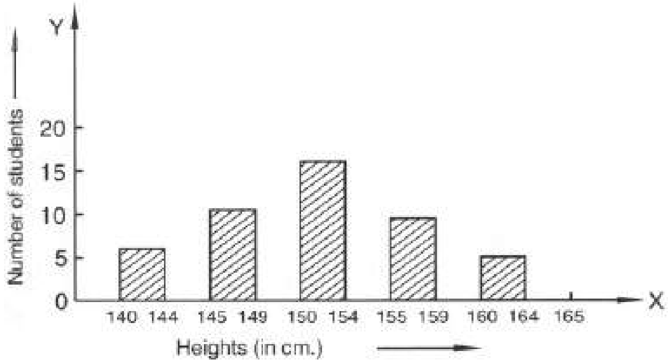
An agricultural field is in the form of a rectangle of length 20 m and width 14 m. A pit 6 m long, 3 m wide and 2.5 m deep is dug in a corner of the field and the earth taken out of the pit is spread uniformly over the remaining area of the field. Find the extent to which the level of the field has been raised.

39. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$.
Show that:

i. DDAP $\cong$ DEBP
ii. AD = BE



40. The following bar graph in the figure represents the heights (in cm) of 50 students of Class XI of a particular school. Study the graph and answer the following questions:



i. What percentage of the total number of students have their heights more than 149 cm?
ii. How many students in the class are in the range of maximum height of the class?
iii. The school wants to provide a particular type of tonic to each student below the height of 150 cm to improve his height. If the cost of the tonic for each student comes out to be 55, how much amount of money is required?
iv. How many students are in the range of the shortest height of the class?

CBSE Class 09

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Solution

Section A

1. (b) 1

   Explanation: $\frac{3-\sqrt{5}}{3+2\sqrt{5}}=a\sqrt{5}-\frac{19}{11}b$

   taking LHS, $\Rightarrow\frac{3-\sqrt{5}}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

   $\Rightarrow\frac{3(3-2\sqrt{5})-\sqrt{5}(3-2\sqrt{5})}{9-20}$
   $\Rightarrow\frac{9-6\sqrt{5}-3\sqrt{5}+10}{-11}$
   $\Rightarrow\frac{19-9\sqrt{5}}{-11}$
   $\Rightarrow\frac{-19}{11}+\frac{9\sqrt{5}}{11}$
   equating this with RHS,

   we get,
   $\frac{-19}{11}b=-\frac{19}{11}$
   $\Rightarrow b=1$

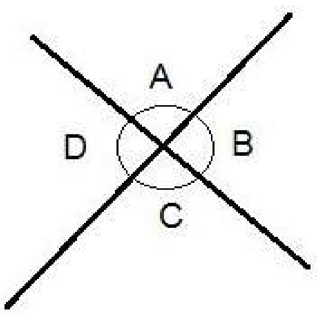
2. (a) 0

   Explanation:

   $\sqrt{2}$ is a constant term. Therefore, the degree of $\sqrt{2}$ is 0.

3. (d) Vertically opposite angles are equal

   Explanation:

   $\angle$A + $\angle$B = 180 (Linear Pair)

   $\angle$B + $\angle$C = 180 (Linear Pair)

On equating ablove equations, we get

$\angle$A + $\angle$B = $\angle$B + $\angle$C

$\angle$A = $\angle$C

Similarly, $\angle$B = $\angle$D

4. (a) outside the triangle

   Explanation: This is so because orthocentre of a triangle is the point of intersection of the three altitudes of a triangle and as we know, the three altitudes of an obtuse triangle meet outside the triangle .

   Thus, the orthocentre of an obtuse triangle is outside the triangle.

5. (b) 7

   Explanation:

   $x+\frac{1}{x}=3$

   Squaring both sides, we get

   $x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}=9$

   $\Rightarrow x^2+\frac{1}{x^2}+2=9$

   $\Rightarrow\ x^2+\frac{1}{x^2}=7$

6. (b) $30\ cm^2$.

   Explanation: Since triangle PQR and parallelogram PRBA are on the same base PR and between the same parallels, then

   $\text{area}\,(\triangle\text{PQR})=\frac{1}{2}\times\text{area}\,(\|gm\text{PRBA})$ ....(i)

   Similarly, $\text{area}\,(\triangle\text{PRS})=\frac{1}{2}\times\text{area}\,(\|gm\text{PDCR})$....(ii)

   Adding eq.(i) and (ii), we have

   $\text{area}\,(\triangle\text{PQR})+\text{area}\,(\triangle\text{PRS})=\frac{1}{2}\times\text{area}\,(\|gm\text{PRBA})\,\frac{1}{2}\times\text{area}\,(\|gm\text{PDCR})$

   $\Rightarrow\text{area}\,(\text{PQRS})=\frac{1}{2}\times\text{area}\,(\|gm\text{ABCD})$

   $\Rightarrow\frac{1}{2}\times\text{area}\,(\|gm\text{ABCD})=15\text{ sq. cm}$

   $\Rightarrow\text{area}\,(\|gm\text{ABCD})=30\text{ cm}^2$

7. (a) 3

    Explanation:

    $\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right)$

    = $\frac{a^3+b^3+c^3}{abc}$

    Since $a+b+c=0$, then $a^3+b^3+c^3=3abc$

    Therefore,

    = $\frac{3abc}{abc}$

    = 3

8. (a) 3k

    Explanation:

    Semiperimeter of scalene triangle of side k, 2k and 3k = $\frac{k+2k+3k}{2}=3k$

9. (a) 77 cm^2.

    Explanation:

    Here,

    Outer curved surface area is asked,

    so, thickness would be added to radius

    Thus, r=3.25+0.25= 3.5 cm

    Surface area of bowl = $2\pi r^2$

    =2× $\frac{22}{7}$ × $(3.5)^2$

    = $\frac{2\times22\times3.5\times3.5}{7}$

    = 77 cm^2

10. (c) 0 and 1

Explanation: The probability of an event (other than sure and impossible event) lies between 0 and 1 . The probability of any event can never be less than 0 and more than 1.

11. $7^{1/3}$

12. 5

OR

(x, 0)

13. y-axis, x-axis

14. diameter

15. $2\pi r(h+r)$

16. $(1+\sqrt{5})-(4+\sqrt{5})=-3$, which is a rational number.

17. We have,
$(9.9)^3 = (10 - 0.1)^3$
$= (10)^3 - (0.1)^3 - 3 \times 10 \times 0.1(10 - 0.1)$
$= 1000 - 0.001 - 3 \times 9.9$
$= 999.999 - 29.7$
$= 970.299$

18. For room : l = 12 m, b = 9 m, h = 8 m.
∴ Length of the longest rod that can be placed in the room
= Length of the diagonal
$= \sqrt{l^2+b^2+h2} = \sqrt{(12)^2+(9)^2+(8)^2}$
$= \sqrt{144+81+64} = \sqrt{289} = 17$ m.

OR

We have,
Edge of each cube = 3m
Volume of each cube = $(\text{edge})^3 = (3)^3\ m^3 = 27\ m^3$

Volume of the cuboid = $(18 \times 12 \times 9)m^3$ = 1944 m^3

$\therefore$ Number of cubes = $\frac{Volume\ of\ the\ cuboid}{Volume\ of\ each\ cube}$ = $\frac{1944}{27}$ = 72

19. $\angle P + \angle Q = 180^\circ$ (Angles on the same side of a transversal are supplementary)

$\Rightarrow\ 3x - 5 + 2x + 15 = 180^\circ$

5x + 10 = 180^o

$\Rightarrow$ 5x + 170^o $\Rightarrow$ x = 34^o

20. The standard form of linear equation in two variable is $ax + by + c = 0$

2x - $\sqrt{3}$y + 0 = 0

21. $4\frac{1}{8} = \frac{4\times8+1}{8} = \frac{32+1}{8} = \frac{33}{8}$

```
8) 33.000 (4.125
   32
   ----
    10
     8
    ----
     20
     16
    ----
      40
      40
    ----
       0
```

$\therefore 4\frac{1}{8}$ = 4.125

The decimal expansion is terminating.

22. According to the question, given equation is $2x + 3y + 15 = 0$.

So, putting x = 2k - 3 and y = k + 2 in equation, we get

$\Rightarrow 2(2k - 3) + 3(k + 2) + 15 = 0$

$\Rightarrow 4k - 6 + 3k + 6 + 15 = 0$

$\Rightarrow 7k + 15 = 0$

$\Rightarrow 7k = -15$

$\Rightarrow\ k = -\frac{15}{7}$

23. $125x^3 - 343y^3$

$= 5^3x^3 - 7^3y^3 = (5x)^3 - (7y)^3$

$= (5x - 7y)[(5x)^2 + 5x \times 7y + (7y)^2]$

$= (5x - 7y)(25x^2 + 35xy + 49y^2)$

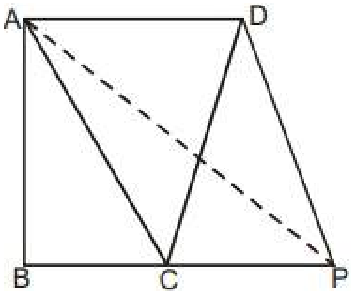
OR

$9x^2 + 6xy + y^2$

$= (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x + y)^2 = (3x + y)(3x + y)$

{Using Identity $(a+ b)^2= a^2+ 2ab+ b^2$}

24. 

Since AC || DP,

ar($\triangle$ADC) = ar($\triangle$APC) ...(i) ($\because$ triangles on the same base AC and between same parallels AC & DP are equal in area)

ar($\triangle$ADC) + ar($\triangle$ABC) = ar($\triangle$APC) + ar($\triangle$ABC)

$\Rightarrow$ ar(quad. ABCD) = ar($\triangle$ABP)

Hence proved.

25.

x_i	f_i	f_ix_i
10	3	30
12	10	120
20	15	300
25	7	175
35	5	175
	N = 40	$\sum f_ix_i$ = 800

Mean($\overline{x}$) = $\frac{\sum f_ix_i}{N}$ = $\frac{800}{40}$ = 20

OR

i. It gives information regarding the industrial production of cement in different years in India.
ii. The production of cement in the year 1980-81 = 186 lakh tonnes.
iii. The minimum production is 30 lakh tonnes in 1950-51 and maximum production is 232 lakh tonnes in 1982-83

26. A tent is of conical shape. Thus,

Base area = πr^2 = 154 cm^2

So, radius r = 7 cm

Curved surface area = πrl = 396 cm^2

396 = 3.14 $\times$ 7 $\times$ l

$\Rightarrow$ l = 18 cm

Now, height $h = \sqrt{l^2 - r^2}$ = $\sqrt{18^2 - 7^2}$ = 16.5 cm

27. First, let's find the value of x, by considering

9^{x+2} = 720 + 9^x

$\Rightarrow 9^x \times 9^2$ = 720 + 9^x

$\Rightarrow 9^x \times 9^2 - 9^x$ = 720

$\Rightarrow 9^x(9^2 - 1)$ = 720

$\Rightarrow 9^x \times 80$ = 720

$\Rightarrow 9^x = \frac{720}{80}$ = 9

$\Rightarrow 9^x = 9^1$

$\Rightarrow$ x = 1

$\therefore (4x)^{\frac{1}{x}}$ = $(4 \times 1)^1$ = 4.

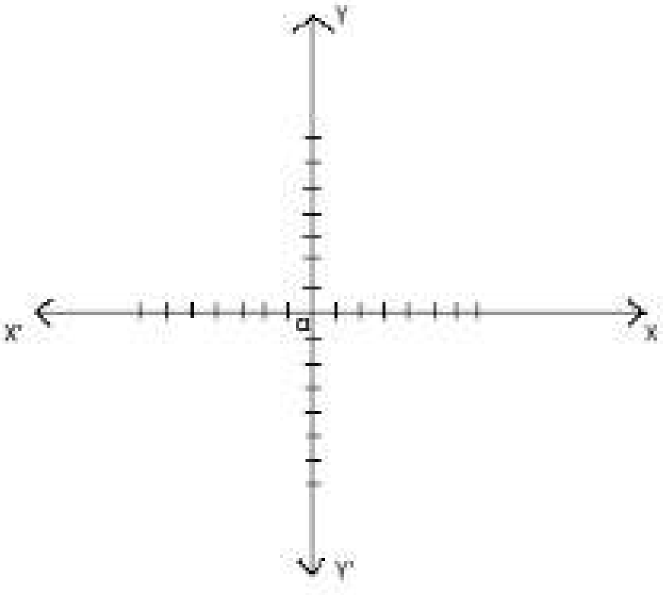
OR

Given, $\frac{\sqrt{25}}{\sqrt[3]{64}} + \left(\frac{256}{625}\right)^{-1/4} + \frac{1}{\left(\frac{64}{125}\right)^{2/3}}$

$= \frac{\sqrt{5\times5}}{\sqrt[3]{4\times4\times4}} + \left(\frac{625}{256}\right)^{1/4} + \left(\frac{125}{64}\right)^{2/3}$

$$= \frac{5}{4} + \left(\frac{5^4}{4^4}\right)^{1/4} + \left(\frac{5^3}{4^3}\right)^{2/3}$$
$$= \frac{5}{4} + \left(\frac{5}{4}\right)^{4\times\frac{1}{4}} + \left(\frac{5}{4}\right)^{3\times\frac{2}{3}}$$
$$= \frac{5}{4} + \frac{5}{4} + \left(\frac{5}{4}\right)^{2} = \frac{5}{4} + \frac{5}{4} + \frac{25}{16}$$
$$= \frac{20+20+25}{16} = \frac{65}{16}$$

28. The mirror image of A(-6, 4) is A'(-6, -4) and B(-6, 2) is B'(-6, -2), C(-2, 2) is C'(-2, -2) and D(-2, 4) is D'(-2, -4).



29. 2(x- 1) + 3y = 4

$\Rightarrow$ 2x- 2 + 3y = 4

$\Rightarrow$ 2x + 3y = 6

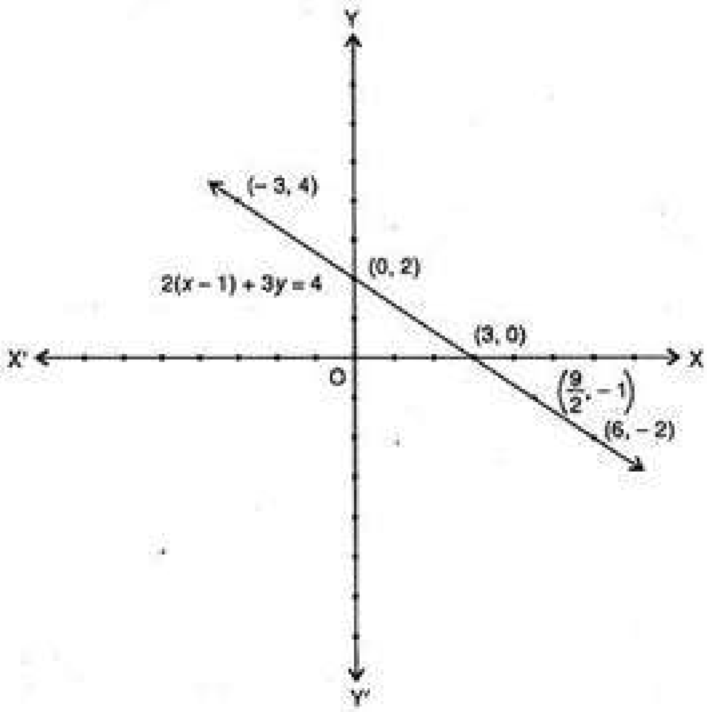
$\Rightarrow$ 3y = 6- 2x

$\Rightarrow y = \frac{6-2x}{3}$

x	0	3
y	2	0

We plot the points (0, 2) and (3, 0) on the graph paper and join the same by a ruler to get the line which is the graph of the equation 2(x- 1) + 3y = 4.

Few solutions read from the graph are
$(-3, 4), (\frac{9}{2}, -1)$ and (6, -2).
For (-3, 4)
L.H.S. = 2(-3- 1) + 3(4) = -8 + 12 = 4 = R.H.S.
$\Rightarrow$ The solution (- 3, 4) is verified.
For $(\frac{9}{2}, -1)$
L.H.S. = $2(\frac{9}{2} - 1)$ + 3(-1) = 7- 3 = 4 = R.H.S.
$\therefore$ The solution $(\frac{9}{2}, -1)$ is verified.
For (6, -2)
L.H.S. = 2(6- 1) + 3(-2) = 10- 6 = 4 = R.H.S.
$\therefore$ The solution (6, -2) is verified.
The points where the given line meets the x-axis and y-axis are respectively (3, 0) and (0, 2).



OR

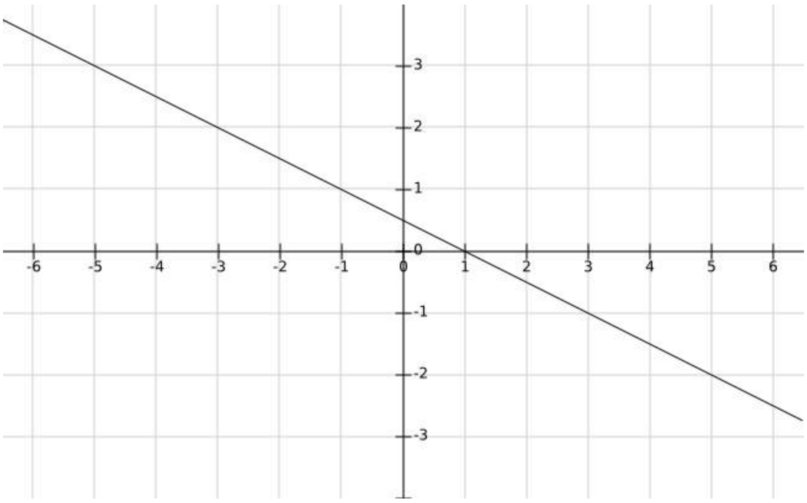
We have,
2y = -x + 1
$\Rightarrow$ y = $\frac{1-x}{2}$ ......(i)
Putting x = 1 in eq. (i), we get y = $\frac{1-1}{2}$ = 0
Putting x = -1in eq. (i), we get y = $\frac{1-(-1)}{2}$ = $\frac{1+1}{2}$ = 1

Putting x = 3 in eq. (i), we get y = $\frac{1-(3)}{2}$ = $\frac{-2}{2}$ = -1

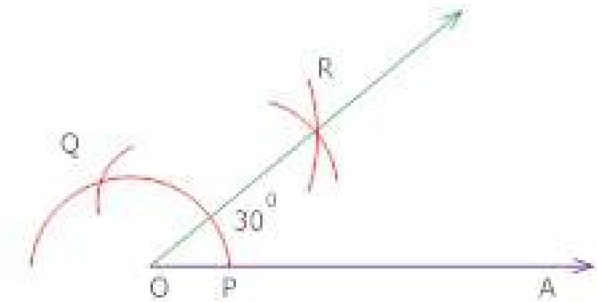
Thus, we have the following table represent the equation 2y = -x + 1.

x	1	-1	3
y	0	1	-1

Graph of the equation 2y = -x + 1:



30. Steps of construction



i. Draw a ray OA
ii. Taking O as a centre draw an arc of any radius which intersects OA at point P.
iii. Taking P as a centre draw an arc of same radius which intersect previous arc at point Q
iv. Bisect $\angle POQ$
v. $\angle AOR$ is bisector of $\angle AOQ$
vi. $\angle AOR = 30^\circ$

31. Given: ABCD is a parallelogram and AP and CQ are perpendicular from vertices A and

C on diagonal BD respectively.

To Prove :

i. $\triangle APB \cong \triangle CQD$
ii. AP = CQ.

Proof :

i. In $\triangle APB$ and $\triangle CQD$

   AB = CD . . . [Opp. sides of || gm ABCD]

   $\angle ABP = \angle CDQ$ . . .[Alternate interior angles for AB||CD]

   $\therefore \angle APB = \angle CQD$ . . .[Each 90^o]

   $\triangle APB \cong \triangle CQD$ . . . [By AAS rule]

ii. As $\triangle APB \cong \triangle CQD$ . . .[As proved above]

   $\therefore$ AP = CQ . . .[c.p.c.t.]

32. In DPDB and DPEA,

$\angle PDB = \angle PEA$ . . . . [Each 90^o]

$\angle BPD = \angle APE$ . . . [Vertically opposite angles]

AE = BD . . . . [Given]

$\therefore$ DPDB $\cong$ DPEA . . . .[By AAS property]

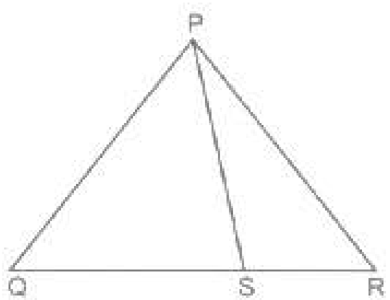
$\therefore$ PA = PB . . . [c.p.c.t.] . . . . (1)

PD = PE . . . [c.p.c.t.] . . . . (2)

PA + PD = PB + PE $\Rightarrow$ AD = BE . . . [By adding (1) and (2)]

OR

Given: A Point S on side QR of $\triangle$PQR.



To prove: PQ + QR + RP > 2PS

Proof: In ΔPQS, we have

PQ + QS > PS...(1)

[∵ Sum of the length of any two sides of a triangle must be greater than the third side]

Now, in $\triangle$PSR, we have

RS + RP > PS...(2)

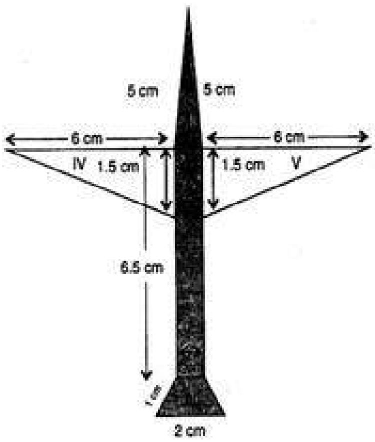
[∵ Sum of the length of any two sides of triangle must be greater than the third side]

Adding (1) and (2), we get

PQ + QS + RS + RP > 2PS

$\Rightarrow$ PQ + QR + RP > 2PS

Hence, proved.

33. 

For triangular area I

a = 5 cm, b = 5 cm, c = 1 cm

$s = \frac{a+b+c}{2}$

$\therefore s = \frac{5+5+1}{2} = \frac{11}{2} = 5.5cm$

$\therefore$ Area I $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$

$= \sqrt{5.5(.5)(.5)(4.5)}$

$= (.5)\sqrt{(5.5)(4.5)}$

$= (.5)\sqrt{(.5)(11)(.5)(9)}$

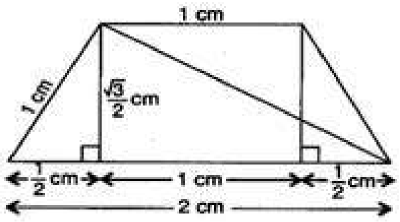
$= (.5)(.5)(3)\sqrt{11}$

$= 0.75\sqrt{11}$ = 0.75 (3.3)

= 2.5 cm^2

Area II = 6.5 $\times$ 1 = 6.5 cm^2

For Area III



Area III

$= \frac{1}{2} \times 2 \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times 1 \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}$

$= \frac{3\sqrt{3}}{4} = \frac{3 \times 1.732}{4}$

$= \frac{5.196}{4} = 1.3 cm^2$

Area IV $= \frac{6 \times 1.5}{2} = 4.5\, cm^2$

Area V $= \frac{6 \times 1.5}{2} = 4.5\, cm^2$

$\therefore$ Total area of the paper used

= Area I + Area II + Area III + Area IV + Area V

= 2.5 cm^2 + 6.5 cm^2 + 1.3 cm^2 + 4.5 cm^2 + 4.5 cm^2

= 19.3 cm^2

34. i. Number of students obtaining less than 20%

in the mathematics test = 7

$\therefore$ Probability that a student obtained less than 20%

in the mathematics $= \frac{7}{90}$

ii. Number of students obtaining marks 60 or above = 15 + 8 = 23

$\therefore$ Probability that a student obtained marks 60 or above $= \frac{23}{90}$.

35. According to question, AD is bisector of $\angle$A.

$\therefore \angle 1 = \angle 2 = \frac{A}{2}$

And BE is the bisector of $\angle$B.

$\therefore \angle 3 = \angle 4 = \frac{B}{2}$

Also CF is the bisector of $\angle$C.

$\therefore \angle 5 = \angle 6 = \frac{C}{2}$

Since the angles in the same segment of a circle are equal.

$\therefore \angle 9 = \angle 3$ [angles subtended by $\overset{\frown}{AE}$] ...(i)

And $\angle 8 = \angle 5$ [angles subtended by $\overset{\frown}{FA}$ ...(ii)

Adding both equations,

$\angle 9 + \angle 8 = \angle 3 + \angle 5$

$\Rightarrow \angle D = \frac{B}{2} + \frac{C}{2}$

Similarly, $\angle E = \frac{A}{2} + \frac{C}{2}$ and $\angle F = \frac{A}{2} + \frac{B}{2}$

In triangle DEF,

$\angle D + \angle E + \angle F = 180^\circ$

$\Rightarrow \angle D = 180^\circ - (\angle E + \angle F)$

$\Rightarrow \angle D = 180^\circ - \left(\frac{A}{2} + \frac{C}{2} + \frac{A}{2} + \frac{B}{2}\right)$

$\Rightarrow \angle D = 180^\circ - \left(\frac{A}{2} + \frac{B}{2} + \frac{C}{2}\right) - \frac{A}{2}$

$\Rightarrow \angle D = 180^\circ - 90^\circ - \frac{A}{2}$ [$\because \angle A + \angle B + \angle C = 180^\circ$]

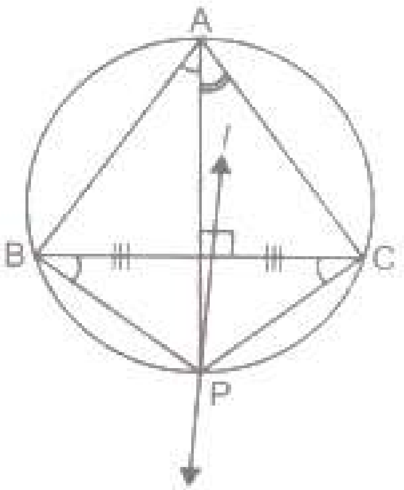
$\Rightarrow \angle D = 90^\circ - \frac{A}{2}$

Similarly, we can prove that

$\angle E = 90^\circ - \frac{B}{2}$ and $\angle F = 90^\circ - \frac{C}{2}$

OR

Here we have a ΔABC and L is perpendicular bisector of side BC.



To prove: Angles bisector of $\angle A$ and perpendicular bisector of BC intersect on the circumcircle of ΔABC.

Proof: Let the angle bisector of $\angle A$ intersect circumcircle of ΔABC at P.

Construction: Join BP and CP.

$\Rightarrow \angle BAP = \angle BCP$

[Angles in the same segment are equal]

$\Rightarrow \angle BAP = \angle BCP = \frac{1}{2}\angle A$ ....(1) [AP is bisector of $\angle A$]

Similarly, we have

$\angle PAC = \angle PBC = \frac{1}{2}\angle A$..................(2)

From equation (1) and (2), we obtain

$\angle BCP = \angle PBC$

Since we know that if the angles subtended by two Chords of a circle at the centre are equal, the chords are equal.

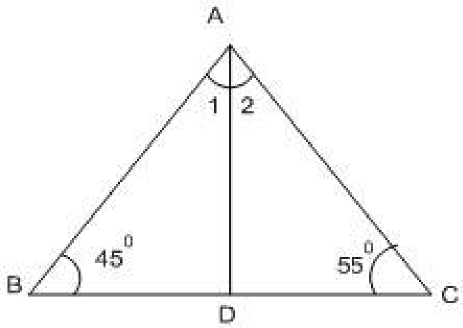
Therefore, BP = CP

$\Rightarrow$ P is on perpendicular bisector of BC.

Hence, angle bisector of $\angle A$ and perpendicular bisector of BC intersect on the circumcircle of ΔABC .

36. In $\triangle$ABC

$\angle \mathrm{A} + \angle \mathrm{B} + \angle \mathrm{C} = 180^0$ [Sum of three angle of a $\triangle$ is 180°]



$\Rightarrow \angle \mathrm{A} + 45^0 + 55^{\circ} = 180^0$

$\angle \mathrm{A} = 180^0 - 100^0 = 80^0$

Now as AD bisects $\angle$A, therefore $\angle BAD = \angle CAD = \frac{80^\circ}{2} = 40^\circ$

Now in $\triangle$ADB, We have

$\angle 1 + \angle \mathrm{B} + \angle \mathrm{ADB} = 180^0$

$\Rightarrow\ 40^0 + 45^0 + \angle ADB = 180^0$

$\Rightarrow\ \angle \mathrm{ADB} = 180^0 - 85^0 = 95^0$

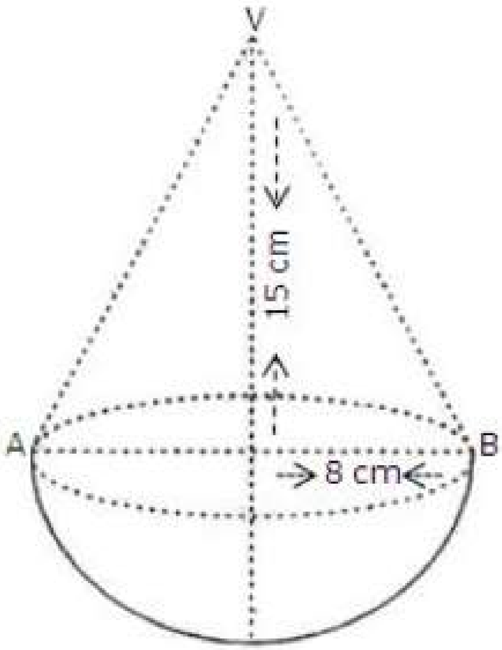
$\angle \mathrm{ADB} + \angle \mathrm{ADC} = 180^0$ [Linear Pair]

$\Rightarrow 95^0 + \angle ADC = 180^0$
$\angle \mathrm{ADC} = 180^0 - 95^0 = 85^0$
$\angle \mathrm{ADB} = 95^0 and \angle \mathrm{ADC} = 85^0$

37. Let f(x) = $x^4 + 2x^3 -13x^2 - 14x + 24$ be the given polynomial. The factors of the constant term 24 in this polynomial are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12$ and ±24
We have,
f(1) = 1 + 2 - 13 - 14 + 24 = 0
$\Rightarrow$ (x - 1) is a factor of f(x).
Similarly, it can be verified that (x - 3), (x + 2) and (x + 4) are factors of f(x).
Since f(x) is a polynomial of degree 4. So, it cannot have more than 4 linear factors.
Hence, f(x) = k(x - 1)(x + 2)(x - 3)(x - 4), where k is constant.
$\Rightarrow x^4 + 2x^3 - 13x^2 - 14x + 24$ = k(x - 1)(x + 2)(x - 3)(x + 4)
Putting x = 0 on both sides, we get,
24 = k(-1)(2)(-3)(4) $\Rightarrow$ 24 = 24k $\Rightarrow$ k = 1
Hence, $x^4 + 2x^3 - 13x^2 - 14x + 24$ = (x - 1)(x + 2)(x - 3)(x + 4)

OR

Let p(z) = $az^3 + 4z^2 + 3z - 4$
And q(z) = $z^3 - 4z + a$
As these two polynomials leave the same remainder, when divided by z - 3, then p(3) = q(3).
$\therefore$ p(3) = $a(3)^3 + 4(3)^2 + 3(3) - 4$
= 27a + 36 + 9 - 4
Or p(3) = 27a + 41
And q(3) = $(3)^3 - 4(3) + a$
= 27 - 12 + a = 15 + a
Now, p(3) = q(3)
$\Rightarrow$ 27a + 41 = 15 + a
$\Rightarrow$ 26a = -26; a = -1
Hence, the required value of a = -1.

38. 

Diameter of cone = 16 cm

$\therefore$ Radius of cone = 8 cm

Height of cone = 15 cm

Slant height of cone = $\sqrt{8^2 + 15^2}$

= $\sqrt{64 + 225} = \sqrt{289}$

= 17 cm

$\therefore$ Total curved surface area of toy

= $\pi r l$ + $2\pi r^2$

= $\frac{22}{7} \times 8 \times 17 + 2 \times \frac{22}{7} \times 8^2$

= $\frac{5808}{7}$ cm^2

Now, Cost of 100 cm^2 = Rs. 7

$\therefore$ 1 cm^2 = Rs. $\frac{7}{100}$

Hence, Cost of $\frac{5808}{7}$ cm^2 = Rs. $\left(\frac{5808}{7} \times \frac{7}{100}\right)$ = Rs. 58.08

OR

Let ABCD be the field and let $AB_1C_1D_1$ be the part of the field where a pit is to be dug.

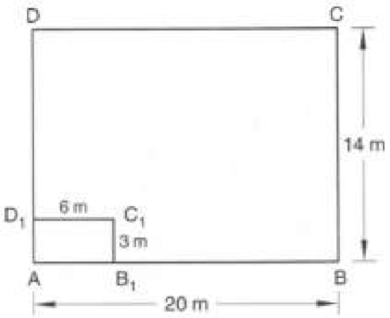
Volume of the earth dug out = $(6 \times 3 \times 2.5)$ m^3 = 45 m^3 ...(i)

Area of the remaining part of the field = Area of the field - Area of pit

= $(20 \times 14 - 6 \times 3)$m^2 = 262m^2

The earth taken out of the pit is spread uniformly over the remaining area of the field. Let h metres be the level raised over the field uniformly. Clearly, the earth taken out forms a cuboid of base area 262 m^2 and height h.

Volume of the earth dug out = $(262 \times h)\ m^3$ ......(ii)



From (i) and (ii), we have

$262h = 45 \Rightarrow h = \frac{45}{262} = 0.1718m = 17.18cm$

Hence, the level is raised by 17.18 cm

39. Given: AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$.

To prove:

i. DDAP $\cong$ DEBP
ii. AD = BE

Proof :(ii)

$\angle EPA = \angle DPB$ ...[Given]

$\angle EPA + \angle EPD = \angle EPD + \angle DPB$ ...[Adding $\angle EPD$ to both sides]

$\angle APD = \angle BPE$ ...(1)

In DDAP and DEBP

$\angle DAP = \angle EBP$ ...[Given]

AP = BP ...[As P is the mid-point of the line AB]

$\angle APD = \angle BPE$ ...[From (1)]

$\therefore$ DDAP $\cong$ DEBP proved ...[ASA property] ...(2)

(i) As DDAP $\cong$ DEBP ...[From (2)]

$\therefore$ AD = BE ...[c.p.c.t.]

40. i. Total number of students have their heights more than 149 cm = 16 + 10 + 5 = 31.
The percentage of the total number of students have their heights more than 149

cm = $\frac{31}{50} \times 100$

$= 31 \times 2$

= 62 %

ii. The number of students in the range of maximum height of the class is 5.

iii. Total number of students below the height of 150 cm = 7 + 12 = 19.

The coast of the tonic for each student = Rs. 55

The cost of the tonic for 19 students = 19×55

= Rs 1045

iv. The number of students in the range of the shortest height of the class = 7