Chapter 5 Quadratic Functions

Ex 5.2

Answer 1e.

In order to find the degree, type, and leading coefficient of the polynomial, we have to write it in the standard form by writing the coefficients of $f(x)$ in order of descending exponents.

$f(x) = -5x^4 + 2x^2 + 6$

The coefficient of the variable with highest power is the leading coefficient, which is -5 in the case of $f(x)$. The degree of a polynomial function is the highest power in that function. For $f(x)$, the degree is 4. Since the degree is 4, the function is quadratic.

Answer 1gp.

A polynomial function in standard form is given by

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers. The leading coefficient of the function is a_n, and the degree is n.

The given function is a polynomial function, since the coefficients are real numbers and the exponents are whole numbers. The function can be written in standard form as $f(x) = -2x + 13$.

From the standard form, we get the degree of $f(x)$ as 1, and the leading coefficient as -2. As the degree is 1, the function is linear.

Answer 2e.

The end behavior of a polynomial function's graph is its behavior as x approaches positive infinity $(+\infty)$ or as x approaches negative infinity $(-\infty)$.

For the graph of a polynomial function, the end behavior is determined by the function's degree and the sign of its leading coefficient.

Answer 2gp.

Consider the function $p(x) = 9x^4 - 5x^{-2} + 4$.

Need to determine whether the function is a polynomial function or not.

A polynomial function is a function of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.

Hence the given function $p(x) = 9x^4 - 5x^{-2} + 4$ is not a polynomial function.

Since one of the exponents is -2 which is not a whole number.

Answer 3e.

A polynomial function in standard form is given by $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers. The leading coefficient of the function is a_n, and the degree is n.

The given function is a polynomial function, since the coefficients are real numbers and the exponents are whole numbers. The function can be written in standard form as $f(x) = -x^2 + 8$.

From the standard form, we get the degree of $f(x)$ as 2, and the leading coefficient as -1. As the degree is 2, the function is quadratic.

Answer 3gp.

A polynomial function in standard form is given by $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers. The leading coefficient of the function is a_n, and the degree is n.

The given function is a polynomial function, since the coefficients are real numbers and the exponents are whole numbers. The function can be written in standard form as $h(x) = 6x^2 - 3x + \pi$.

From the standard form, we get the degree of $h(x)$ as 2, and the leading coefficient as 6. As the degree is 2, the function is quadratic.

Answer 4e.

Consider the polynomial function $f(x)=6x+8x^4-3$ $f(x)=6x+8x^4-3$

Need to determine whether the function is a polynomial function or not.

The polynomial function $f(x)=6x+8x^4-3$ can be rewritten as

$$f(x)=8x^4+6x-3$$

A polynomial function is a function of the form $f(x)=a_nx^n+a_{n-1}x^{n-1}+...+a_1x+a_0$ where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.

Hence the given function is a **polynomial function** it is a 4^{th} **degree** (quartic) polynomial with leading coefficient 8.

Answer 4gp.

Consider the polynomial function $f(x)=x^4+2x^3+3x^2-7$; $x=-2$

Need to evaluate the polynomial function for the given value of x by using direct substitution

Now substitute the value of $x=-2$ in $f(x)=x^4+2x^3+3x^2-7$, obtain

$f(x)=x^4+2x^3+3x^2-7$	Write original equation
$f(-2)=(-2)^4+2(-2)^3+3(-2)^2-7$	Substitute -2 for x
$=16-2(8)+3(4)-7$	Evaluate powers
$=16-16+12-7$	Multiply
$=0+5$	
$=\boxed{5}$	Simplify

Answer 5e.

A polynomial function in standard form is given by $f(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$, where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers. The leading coefficient of the function is a_n, and the degree is n.

The given function is a polynomial function, since the coefficients are real numbers and the exponents are whole numbers. The function can be written in standard form as $g(x)=\pi x^4+\sqrt{6}$.

From the standard form, we get the degree of $g(x)$ as 4, and the leading coefficient as π. As the degree is 4, the function is quadratic.

Answer 5gp.

Substitute 4 for x in the given function.
$g(4) = 4^3 - 5(4)^2 + 6(4) + 1$

Evaluate the powers.
$g(4) = 64 - 5(16) + 6(4) + 1$

Multiply.
$64 - 5(16) + 6(4) + 1 = 64 - 80 + 24 + 1$

Simplify.
$64 - 80 + 24 + 1 = 9$

Therefore, the value of $g(x)$ is 9 for the given value of x.

Answer 6e.

Consider the polynomial function $h(x) = x^3\sqrt{10} + 5x^{-2} + 1$
Need to determine whether the function is a polynomial function or not.

A polynomial function is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.
Hence the given function is **not a polynomial** function.
Since one of the exponents is -2 which is not a whole number.

Answer 6gp.

Consider the polynomial function $f(x) = 5x^3 + 3x^2 - x + 7; x = 2$
Need to evaluate the polynomial function for the given value of x by using synthetic substitution.

Step1:

Write the coefficients of $f(x)$ in order of descending exponents. Write the value at which $f(x)$ is being evaluated to the left.

$$\text{x-value} \longrightarrow \begin{array}{r|rrrr} 2 & 5 & 3 & -1 & 7 \\ & & & & \\ \hline \end{array}$$

Step2:

Bring down the leading coefficients. Multiply the leading coefficient by the x-value. Write the product under the second coefficient and then add.

$$\begin{array}{r|rrrr} 2 & 5 & 3 & -1 & 7 \\ & & 10 & & \\ \hline & 5 & 13 & & \end{array}$$

Step3:

Multiply the previous sum by the x-value. Write the product under the third coefficient and then add.
Repeat for all of the remaining coefficients.
The final sum is the value of $f(x)$ at the given x-value.

$$\begin{array}{r|rrrr} 2 & 5 & 3 & -1 & 7 \\ & & 10 & 26 & 50 \\ \hline & 5 & 13 & 25 & \boxed{57} \end{array}$$

Therefore $\boxed{f(2)=57}$.

Answer 7e.

A polynomial function in standard form is given by $f(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$, where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers. The leading coefficient of the function is a_n, and the degree is n.

The given function is a polynomial function, since the coefficients are real numbers and the exponents are whole numbers. The function can be written in standard form as $h(x)=-\frac{5}{2}x^3+3x-10$.

From the standard form, we get the degree of $h(x)$ as 3, and the leading coefficient as $-\frac{5}{2}$. As the degree is 3, the function is cubic.

Answer 7gp.

STEP 1 **Write** the coefficients of g(x) in order of descending exponents. Write the value at which $g(x)$ is being evaluated to the left.

$$x\text{-value} \rightarrow \; -1 \,\big|\; -2 \quad -1 \quad 4 \quad -5 \quad \leftarrow \text{coefficients}$$

STEP 2 **Bring down** the leading coefficient. **Multiply** the leading coefficient by the x-value. Write the product under the second coefficient. **Add**.

The leading coefficient is -2. On multiplying -2 by -1, we get 2. Write 2 below the second coefficient -1. Add them.

$$\begin{array}{r|rrrr} -1 & -2 & -1 & 4 & -5 \\ & & 2 & & \\ \hline & -2 & 1 & & \end{array}$$

STEP 3 **Multiply** the previous sum by the x-value. Write the product under the third coefficient. **Add**. Repeat for all of the remaining coefficients. The final sum is the value of $g(x)$ at the given x-value.

The previous sum is obtained as 1. Multiply 1 by the x-value -1 and write the result under the third coefficient 4. Add the numbers. Repeat this process.

$$\begin{array}{r|rrrr} -1 & -2 & -1 & 4 & -5 \\ & & 2 & 1 & -5 \\ \hline & -2 & 1 & 5 & -10 \end{array}$$

The final sum is -10.

Therefore, the value of g(x) is -10 for the given value of x.

Answer 8e.

Consider the polynomial function $g(x) = 8x^3 - 4x^2 + \frac{2}{x}$.

Need to determine whether the function is a polynomial function or not.

The function $g(x) = 8x^3 - 4x^2 + \frac{2}{x}$ can be rewritten as,

$$g(x) = 8x^3 - 4x^2 + 2x^{-1}$$

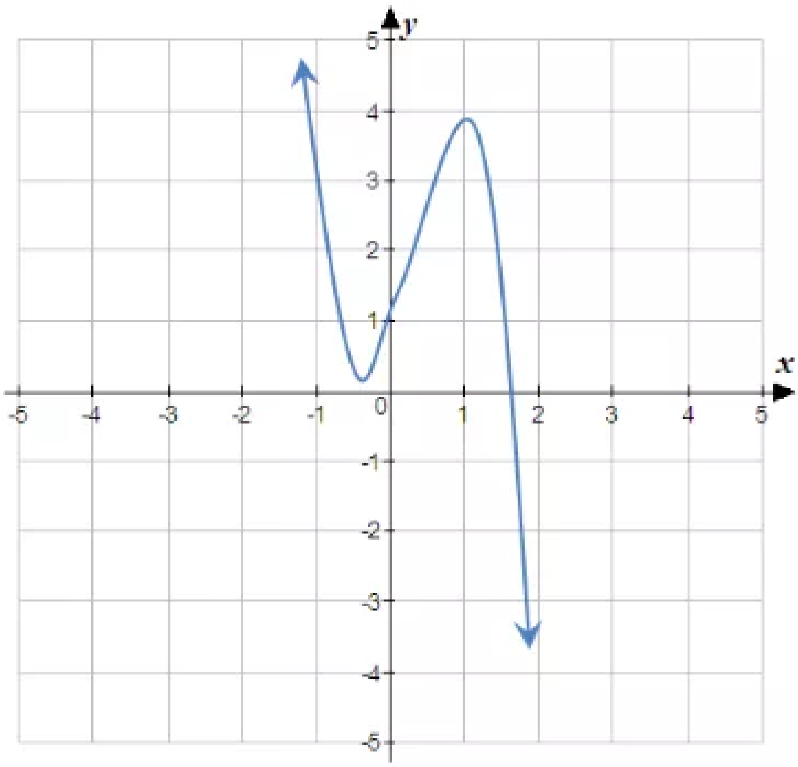
A polynomial function is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.

Hence the given function is **not a polynomial** function.

Since one of the exponents is -1 which is not a whole number.

Answer 8gp.

Consider the following graph:



Need to describe the degree and leading coefficient of the polynomial function for the graph.

Let $f(x)$ be polynomial function of the given graph.

From the given graph, it is clear that

$f(x) \to -\infty$ as $x \to \infty$ and

$f(x) \to \infty$ as $x \to -\infty$

The leading coefficient of $f(x)$ is **negative** and the degree of $f(x)$ is **odd**.

Answer 9e.

Substitute -1 for x in the given function.

$f(-1) = 5(-1)^3 - 2(-1)^2 + 10(-1) - 15$

Evaluate the powers.

$f(-1) = 5(-1) - 2(1) + 10(-1) - 15$

Multiply.

$5(-1) - 2(1) + 10(-1) - 15 = -5 - 2 - 10 - 15$

Simplify.

$-5 - 2 - 10 - 15 = -32$

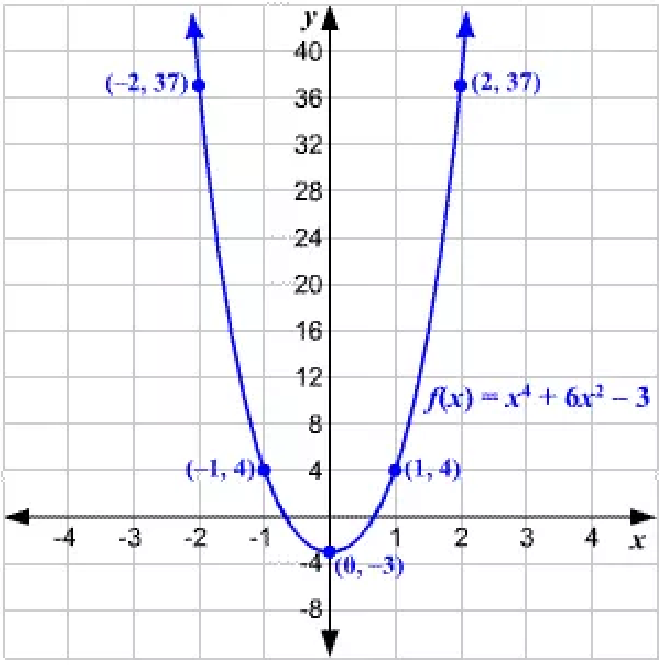
Therefore, the value of $f(x)$ is -32 for the given value of x.

Answer 9gp.

Make a table of values that satisfy the given function. For this, select some values for x and find the corresponding values of $f(x)$ or y.

x	−2	−1	0	1	2
y	37	4	−3	4	37

Plot the points and connect them using a smooth curve.



The degree of the function is even and the leading coefficient is positive. The end behavior of the graph is such that

$f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

Answer 10e.

Consider the polynomial equation $f(x) = 8x + 5x^4 - 3x^2 - x^3$; $x = 2$

Need to use direct substitution be evaluate the polynomial function for the given value of x.

$f(x) = 8x + 5x^4 - 3x^2 - x^3$	Write original equation
$f(2) = 8(2) + 5(2^4) - 3(2^2) - 2^3$	Substitute 2 for x
$= 16 + 5(8) - 3(4) - 8$	Evaluate powers
$= 16 + 80 - 12 - 8$	Multiply
$= \boxed{76}$	Simplify

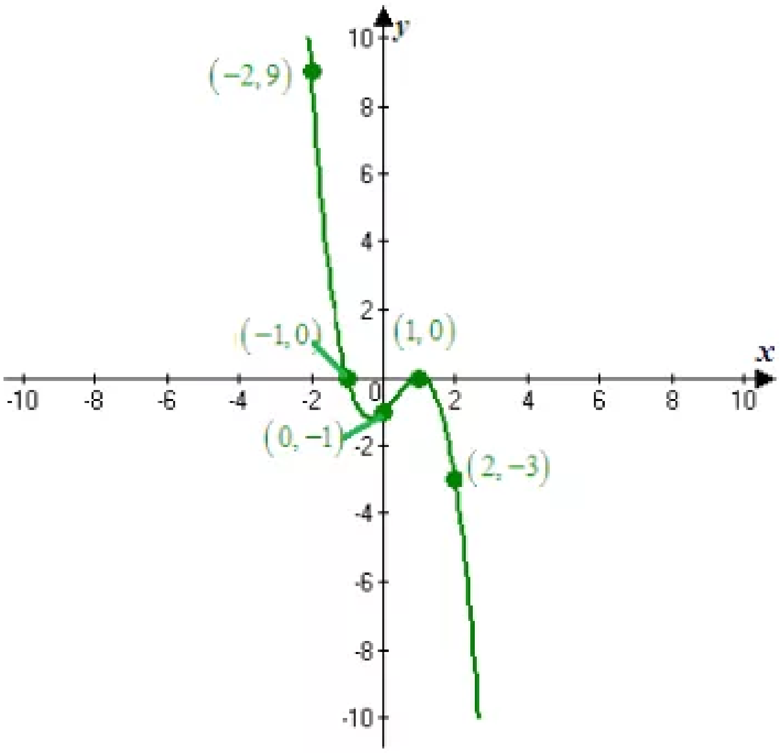
Answer 10gp.

Consider the polynomial function $f(x) = -x^3 + x^2 + x - 1$.

Need to sketch the graph of the function.
To graph the function, make a table of values and plot the corresponding points.
Connect the points with smooth curve and check the end behavior.
The table is as follows:

x	-2	-1	0	1	2
y	9	0	-1	0	-3

The graph of the polynomial function is as follows:



The degree is odd and leading coefficient is negative.

So $f(x) \to -\infty$ as $x \to \infty$ and $f(x) \to \infty$ as $x \to -\infty$

Answer 11e.

Substitute -3 for x in the given function.

$g(-3) = 4(-3)^3 - 2(-3)^5$

Evaluate the powers.

$g(-3) = 4(-27) - 2(-243)$

Multiply.
$4(-27) - 2(-243) = -108 + 486$

Simplify.
$-108 + 486 = 378$

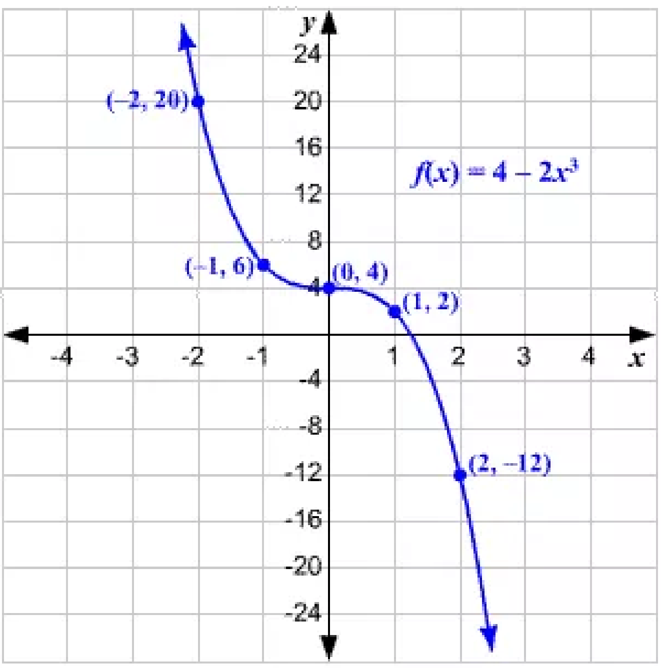
Therefore, the value of $g(x)$ is 378 for the given value of x.

Answer 11gp.

Make a table of values that satisfy the given function. For this, select some values for x and find the corresponding values of $f(x)$ or y.

x	-2	-1	0	1	2
y	20	6	4	2	-12

Plot the points and connect them using a smooth curve.



The degree of the function is odd and the leading coefficient is negative. The end behavior of the graph is such that
$f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$.

Answer 12e.

Consider the polynomial function $h(x)=6x^3-25x+20$; $x=5$.

Need to use direct substitution be evaluate the polynomial function for the given value of x.

$h(x)=6x^3-25x+20$ Write original equation

$h(5)=6(5^3)-25(5)+20$ Substitute 5 for x

$=6(125)-25(5)+20$ Evaluate powers

$=750-125+20$ Multiply

$=\boxed{645}$ Simplify

Answer 12gp.

Consider the energy E (in foot-pounded) in each square root of a wave is given by the model $E=0.0051s^4$

Where s is the wind speed (in miles per hour).

Step1:

Make a table of values.

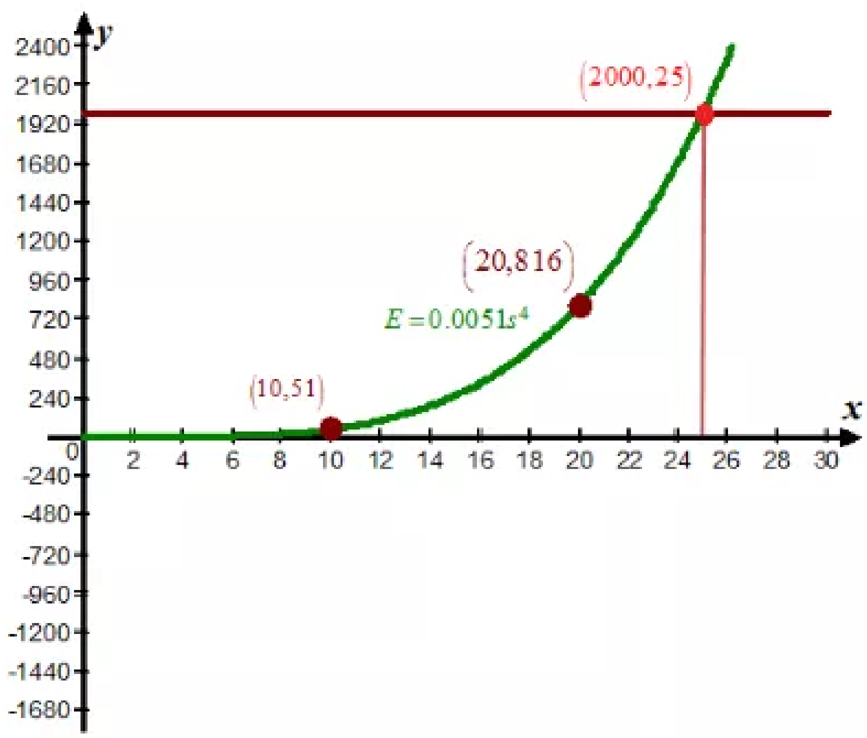
The model only deals with positive values of s.

S	0	10	20	30	40
E	0	51	816	4131	13056

Step2:

Plot the points and connect them with a smooth curve. Because the leading coefficient is positive and the degree is even, the graph rises to the right.

The following diagram contains the graph of the polynomial function $E=0.0051s^4$



Step 3:

Therefore the wind speed that is needed to generate a wave with 2000 foot-pounds of energy per square foot is $\boxed{25}$ miles per hour.

Answer 13e.

Substitute –4 for x in the given function.

$$h(-4) = -4 + \frac{1}{2}(-4)^4 - \frac{3}{4}(-4)^3 + 10$$

Evaluate the powers.

$$h(-4) = -4 + \frac{1}{2}(256) - \frac{3}{4}(-64) + 10$$

Multiply.

$$-4 + \frac{1}{2}(256) - \frac{3}{4}(-64) + 10 = -4 + 128 + 48 + 10$$

Simplify.

$$-4 + 128 + 48 + 10 = 182$$

Therefore, the value of $h(x)$ is 182 for the given value of x.

Answer 14e.

Consider the polynomial function $g(x) = 4x^5 + 6x^3 + x^2 - 10x + 5$; $x = -2$.

Need to use direct substitution be evaluate the polynomial function for the given value of x.

$g(x) = 4x^5 + 6x^3 + x^2 - 10x + 5$	Write original function
$g(-2) = 4(-2)^5 + 6(-2)^3 + (-2)^2 - 10(-2) + 5$	Substitute -2 for x
$= 4(-32) + 6(-8) + 4 + 20 + 5$	Evaluate powers
$= -128 - 48 + 4 + 20 + 5$	Multiply
$= \boxed{-147}$	Simplify

Answer 15e.

STEP 1 **Write** the coefficients of $f(x)$ in order of descending exponents. Write the value at which $f(x)$ is being evaluated to the left.

$$x\text{-value} \rightarrow 3 \;\big|\; 5 \quad -2 \quad -8 \quad 16 \quad \leftarrow \text{coefficients}$$

STEP 2 **Bring down** the leading coefficient. **Multiply** the leading coefficient by the x-value. Write the product under the second coefficient. **Add**.

The leading coefficient is 5. On multiplying 5 by 3, we get 15. Write 15 below the second coefficient −2. Add them.

3	5	−2	−8	16
		15		
	5	13		

STEP 3 **Multiply** the previous sum by the x-value. Write the product under the third coefficient. **Add**. Repeat for all of the remaining coefficients. The final sum is the value of $f(x)$ at the given x-value.

The previous sum is obtained as 13. Multiply 13 by the x-value 3 and write the result under the third coefficient −8. Add the numbers. Repeat this process.

3	5	−2	−8	16
		15	39	93
	5	13	31	109

The final sum is 109.

Therefore, the value of $f(x)$ is 109 for the given value of x.

Answer 16e.

Consider the polynomial function $f(x)=8x^4+12x^3+6x^2-5x+9; x=-2$

Need to evaluate the polynomial function for the given value of x by using synthetic substitution.

Step1:

Write the coefficients of $f(x)$ in order of descending exponents. Write the value at which $f(x)$ is being evaluated to the left.

−2	8	12	6	−5	9

Step2:

Bring down the leading coefficients. Multiply the leading coefficient by the x-value. Write the product under the second coefficient and then add.

$$\begin{array}{r|rrrrr} -2 & 8 & 12 & 6 & -5 & 9 \\ & & -16 & & & \\ \hline & 8 & -4 & & & \end{array}$$

Step3:

Multiply the previous sum by the x-value. Write the product under the third coefficient and then add.
Repeat for all of the remaining coefficients.
The final sum is the value of $f(x)$ at the given x-value.

$$\begin{array}{r|rrrrr} -2 & 8 & 12 & 6 & -5 & 9 \\ & & -16 & 8 & -28 & 66 \\ \hline & 8 & -4 & 14 & -33 & \boxed{75} \end{array}$$

Therefore $\boxed{f(-2)=75}$.

Answer 17e.

STEP 1 **Write** the coefficients of $g(x)$ in order of descending exponents. Write the value at which $g(x)$ is being evaluated to the left.

$$x\text{-value} \rightarrow \begin{array}{r|rrrr} -6 & 1 & 8 & -7 & 35 \\ \hline & & & & \end{array} \leftarrow \text{coefficients}$$

STEP 2 **Bring down** the leading coefficient. **Multiply** the leading coefficient by the x-value. Write the product under the second coefficient. **Add**.

The leading coefficient is 1. On multiplying 1 by -6, we get -6. Write -6 below the second coefficient 8. Add them.

$$\begin{array}{r|rrrr} -6 & 1 & 8 & -7 & 35 \\ & & -6 & & \\ \hline & 1 & 2 & & \end{array}$$

STEP 3 **Multiply** the previous sum by the x-value. Write the product under the third coefficient. **Add**. Repeat for all of the remaining coefficients. The final sum is the value of $g(x)$ at the given x-value.

The previous sum is obtained as 2. Multiply 2 by the x-value -6 and write the result under the third coefficient -7. Add the numbers. Repeat this process.

$$\begin{array}{r|rrrr} -6 & 1 & 8 & -7 & 35 \\ & & -6 & -12 & 114 \\ \hline & 1 & 2 & -19 & 149 \end{array}$$

The final sum is 149.

Therefore, the value of $g(x)$ is 149 for the given value of x.

Answer 18e.

Consider the polynomial function $h(x) = -8x^3 + 14x - 35$; $x = 4$

Need to evaluate the polynomial function for the given value of x by using synthetic substitution.

Step1:

Write the coefficients of $f(x)$ in order of descending exponents. Write the value at which $f(x)$ is being evaluated to the left.

$$\begin{array}{r|rrrr} 4 & -8 & 0 & 14 & -35 \\ & & & & \\ \hline \end{array}$$

Step2:

Bring down the leading coefficients. Multiply the leading coefficient by the x-value. Write the product under the second coefficient and then add.

$$\begin{array}{r|rrrr} 4 & -8 & 0 & 14 & -35 \\ & & -32 & & \\ \hline & -8 & -32 & & \end{array}$$

Step3:

Multiply the previous sum by the x-value. Write the product under the third coefficient and then add.
Repeat for all of the remaining coefficients.
The final sum is the value of $f(x)$ at the given x-value.

4	−8	+0	+14	−35
		−32	−128	−456
	−8	−32	−114	$\boxed{-491}$

Therefore $\boxed{h(4) = -491}$.

Answer 19e.

STEP 1 **Write** the coefficients of $f(x)$ in order of descending exponents. Write the value at which $f(x)$ is being evaluated to the left.

In the given function, the x^2 term is missing. Use 0 as the coefficient of the missing term.

x-value →	2	−2	3	0	−8	13	← coefficients

STEP 2 **Bring down** the leading coefficient. **Multiply** the leading coefficient by the x-value. Write the product under the second coefficient. **Add**.

The leading coefficient is −2. On multiplying −2 by 2, we get −4. Write −4 below the second coefficient 3. Add them.

2	−2	3	0	−8	13
		−4			
	−2	−1			

STEP 3 **Multiply** the previous sum by the x-value. Write the product under the third coefficient. **Add**. Repeat for all of the remaining coefficients. The final sum is the value of $f(x)$ at the given x-value.

The previous sum is obtained as −1. Multiply −1 by the x-value 2 and write the result under the third coefficient 0. Add the numbers. Repeat this process.

2	−2	3	0	−8	13
		−4	−2	−4	−24
	−2	−1	−2	−12	−11

The final sum is −11.

Therefore, the value of $f(x)$ is −11 for the given value of x.

Answer 20e.

Consider the polynomial function $g(x)=6x^5+10x^3-27\,;x=-3$

Need to evaluate the polynomial function for the given value of x by using synthetic substitution.

Step1:

Write the coefficients of $f(x)$ in order of descending exponents. Write the value at which $f(x)$ is being evaluated to the left.

$$\begin{array}{r|rrrrrr} -3 & 6 & 0 & 10 & 0 & 0 & -27 \\ & & & & & & \\ \hline \end{array}$$

Step2:

Bring down the leading coefficients. Multiply the leading coefficient by the x-value.
Write the product under the second coefficient and then add.

$$\begin{array}{r|rrrrrr} -3 & 6 & 0 & 10 & 0 & 0 & -27 \\ & & -18 & & & & \\ \hline & 6 & -18 & & & & \end{array}$$

Step3:

Multiply the previous sum by the x-value. Write the product under the third coefficient and then add.
Repeat for all of the remaining coefficients.
The final sum is the value of $f(x)$ at the given x-value.

$$\begin{array}{r|rrrrrr} -3 & +6 & +0 & +10 & +0 & +0 & -27 \\ & & -18 & 54 & -192 & 576 & -1728 \\ \hline & 6 & -18 & 64 & -192 & 576 & \boxed{-1755} \end{array}$$

Therefore $\boxed{g(-3)=-1755}$.

Answer 21e.

STEP 1 Write the coefficients of $h(x)$ in order of descending exponents. Write the value at which $h(x)$ is being evaluated to the left.

In the given function, the constant term is missing. Use 0 as the constant term

$$x\text{-value} \rightarrow \begin{array}{r|rrrr} 3 & -7 & 11 & 4 & 0 \\ & & & & \\ \hline \end{array} \leftarrow \text{coefficients}$$

STEP 2 **Bring down** the leading coefficient. **Multiply** the leading coefficient by the x-value. Write the product under the second coefficient. **Add**.

The leading coefficient is −7. On multiplying −7 by 3, we get −21. Write −21 below the second coefficient 11. Add them.

$$\begin{array}{r|rrrr} 3 & -7 & 11 & 4 & 0 \\ & & -21 & & \\ \hline & -7 & -10 & & \end{array}$$

STEP 3 **Multiply** the previous sum by the x-value. Write the product under the third coefficient. **Add**. Repeat for all of the remaining coefficients. The final sum is the value of $h(x)$ at the given x-value.

The previous sum is obtained as −10. Multiply −10 by the x-value 3 and write the result under the third coefficient 4. Add the numbers. Repeat this process.

$$\begin{array}{r|rrrr} 3 & -7 & 11 & 4 & 0 \\ & & -21 & -30 & -78 \\ \hline & -7 & -10 & -26 & -78 \end{array}$$

The final sum is −78.

Therefore, the value of $h(x)$ is −78 for the given value of x.

Answer 22e.

Consider the polynomial function $f(x) = x^4 + 3x - 20\,;\, x = 4$

Need to evaluate the polynomial function for the given value of x by using synthetic substitution.

Step1:

Write the coefficients of $f(x)$ in order of descending exponents. Write the value at which $f(x)$ is being evaluated to the left.

$$\begin{array}{r|rrrrr} 4 & 1 & 0 & 0 & 3 & -20 \\ & & & & & \\ \hline \end{array}$$

Step2:

Bring down the leading coefficients. Multiply the leading coefficient by the x-value. Write the product under the second coefficient and then add.

4	1	0	0	3	−20
		16			
	4	16			

Step3:

Multiply the previous sum by the x-value. Write the product under the third coefficient and then add.
Repeat for all of the remaining coefficients.
The final sum is the value of $f(x)$ at the given x-value.

4	1	0	0	3	−20
		4	16	64	268
	1	4	16	67	$\boxed{248}$

Therefore $\boxed{f(4)=248}$.

Answer 23e.

Observe the given function. We can see that the coefficient of x^3 is missing. The synthetic substitution has been done without considering the x^3 term, which has led to the wrong evaluation.

In synthetic division method, the missing terms must also be included using 0 as the coefficient. The given function thus becomes $f(x) = -4x^4 + 0x^3 + 9x^2 - 21x + 7$.

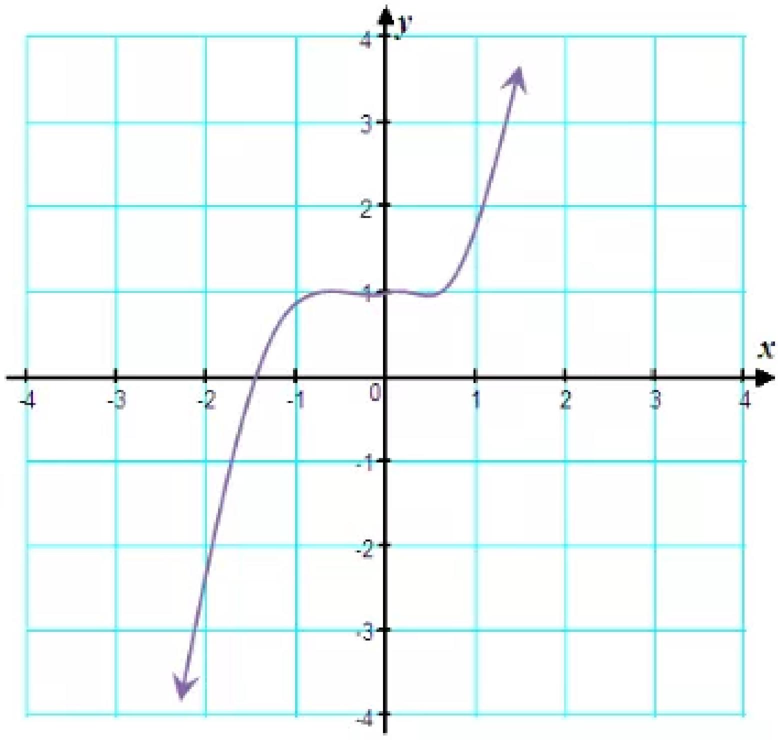
Now, evaluate the function at $x = -2$ using the synthetic substitution method.

−2	−4	0	9	−21	7
		8	−16	14	14
	−4	8	−7	−7	21

The value of $f(x)$ at $x = -2$ is 21.

Answer 24e.

Consider the following graph:



Let $f(x)$ be the polynomial function of the given graph.

From the graph, it is clear that

$f(x) \to \infty$ as $x \to \infty$ and

$f(x) \to -\infty$ as $x \to -\infty$

The leading coefficient of $f(x)$ is positive and the degree of $f(x)$ is odd.

Therefore the answer is matched with the option $\boxed{(\mathbf{A})}$.

Answer 25e.

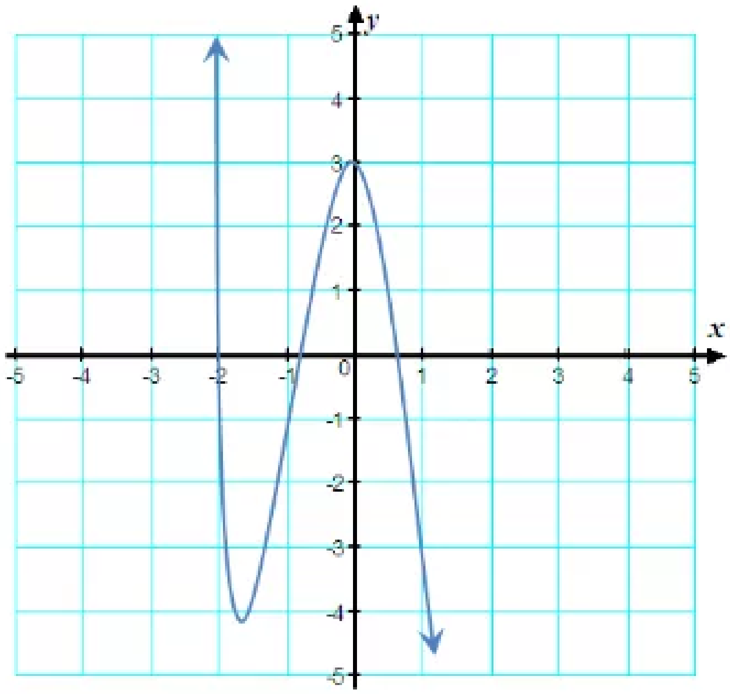
Let $f(x)$ be the function represented by the graph given.

From the graph, it is clear that $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

This implies that the degree of the function is even and the leading coefficient is positive.

Answer 26e.

Consider the following graph.



Let $f(x)$ be the polynomial function of the given graph.

From the graph, it is clear that

$f(x) \to -\infty$ as $x \to \infty$ and

$f(x) \to \infty$ as $x \to -\infty$

The leading coefficient of $f(x)$ is **negative** and the degree of $f(x)$ is **odd.**

Answer 27e.

Let $f(x)$ be the function represented by the graph given.

From the graph, it is clear that $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to -\infty$.

This implies that the degree of the function is even and the leading coefficient is negative.

Answer 28e.

Consider the function $f(x) = 10x^4$

Need to describe the end behavior of the graph of the polynomial function $f(x) = 10x^4$

A polynomial function is a function of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$

where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.

For this function a_n is the leading coefficient, n is the degree.

The function $f(x) = 10x^4$ is a polynomial function of even degree 10 with positive leading coefficients 4.

Therefore, $f(x) \to \boxed{\infty}$ as $x \to \infty$ and $f(x) \to \boxed{\infty}$ as $x \to -\infty$.

Answer 29e.

The degree of the given function is 6, which is even. The leading coefficient is -1, which is negative.

For a function with even degree and negative leading coefficient, the end behavior o the graph is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$.

Thus, for the given function, the statement can be completed as
$f(x) \to \underline{-\infty}$ as $x \to -\infty$ and $f(x) \to \underline{-\infty}$ as $x \to +\infty$.

Answer 30e.

Consider the polynomial function $f(x) = -2x^3 + 7x - 4$.

Need to describe the end behavior of the graph of the polynomial function $f(x) = -2x^3 + 7x - 4$.

A polynomial function is a function of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.

For this function a_n is the leading coefficient, n is the degree.

The function $f(x) = -2x^3 + 7x - 4$ is a polynomial function of odd degree 3 with negative leading coefficient -2.

Therefore, $f(x) \to \boxed{-\infty}$ as $x \to \infty$ and $f(x) \to \boxed{\infty}$ as $x \to -\infty$.

Answer 31e.

The degree of the given function is 7, which is odd. The leading coefficient is 1, which is positive.

For a function with odd degree and positive leading coefficient, the end behavior o the graph is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

Thus, for the given function, the statement can be completed as
$f(x) \to \underline{-\infty}$ as $x \to -\infty$ and $f(x) \to \underline{+\infty}$ as $x \to +\infty$.

Answer 32e.

Consider the polynomial function $f(x)=3x^{10}-16x$

Need to describe the end behavior of the graph of the polynomial function $f(x)=3x^{10}-16x$

A polynomial function is a function of the form $f(x)=a_nx^n+a_{n-1}x^{n-1}+...+a_1x+a_0$ where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.

For this function a_n is the leading coefficient, n is the degree.

The function $f(x)=3x^{10}-16x$ is a polynomial function of even degree 10 with positive leading coefficient 3.

Therefore, $f(x)\to\boxed{\infty}$ as $x\to\infty$ and $f(x)\to\boxed{\infty}$ as $x\to-\infty$.

Answer 33e.

The degree of the given function is 5, which is odd. The leading coefficient is –6, which is negative.

For a function with odd degree and negative leading coefficient, the end behavior of the graph is $f(x)\to+\infty$ as $x\to-\infty$ and $f(x)\to-\infty$ as $x\to+\infty$.

Thus, for the given function, the statement can be completed as $f(x)\to\underline{+\infty}$ as $x\to-\infty$ and $f(x)\to\underline{-\infty}$ as $x\to+\infty$.

Answer 34e.

Consider the polynomial function $f(x)=0.2x^3-x+45$.

Need to describe the end behavior of the graph of the polynomial function $f(x)=0.2x^3-x+45$.

A polynomial function is a function of the form $f(x)=a_nx^n+a_{n-1}x^{n-1}+...+a_1x+a_0$ where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.

For this function a_n is the leading coefficient, n is the degree.

The function $f(x)=0.2x^3-x+45$ is a polynomial function of odd degree 3 with positive leading coefficients 0.2

Therefore, $f(x)\to\boxed{\infty}$ as $x\to\infty$ and $f(x)\to-\infty$ as $x\to\boxed{-\infty}$.

Answer 35e.

The degree of the given function is 8, which is even. The leading coefficient is 5, which is positive.

For a function with even degree and positive leading coefficient, the end behavior o the graph is $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

Thus, for the given function, the statement can be completed as
$f(x) \to \underline{+\infty}$ as $x \to -\infty$ and $f(x) \to \underline{+\infty}$ as $x \to +\infty$.

Answer 36e.

Consider the polynomial function $f(x) = -x^{273} + 500x^{271}$.
Need to describe the end behavior of the graph of the polynomial function $f(x) = -x^{273} + 500x^{271}$

A polynomial function is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.
For this function a_n is the leading coefficient, n is the degree.

The function $f(x) = -x^{273} + 500x^{271}$ is a polynomial function of odd degree 273 with negative leading coefficients -1.
Therefore, $f(x) \to \boxed{-\infty}$ as $x \to \infty$ and $f(x) \to \boxed{\infty}$ as $x \to -\infty$.

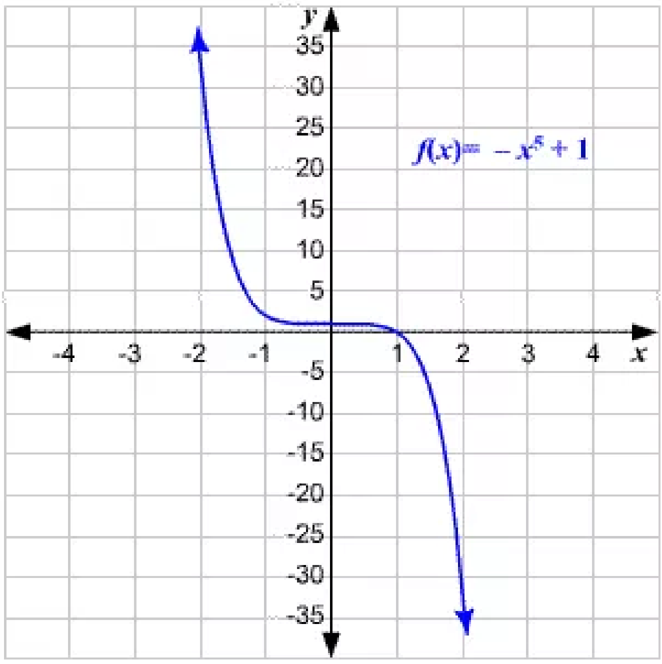
Answer 37e.

The end behavior of a function $f(x)$ is given as
$f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$. Using the end behavior, we can say that the degree of the function is odd, and the leading coefficient is negative.

We seek a function of degree 5. The leading coefficient must be negative for the occurrence of the given end behavior. Such a function can be written as $f(x) = -x^5 + 1$.

Other functions are also possible.

Graph the function to verify the end behavior.



In the graph, we can see that the y or $f(x)$ tends to infinity when x tends to negative infinity and vice versa.

Answer 38e.

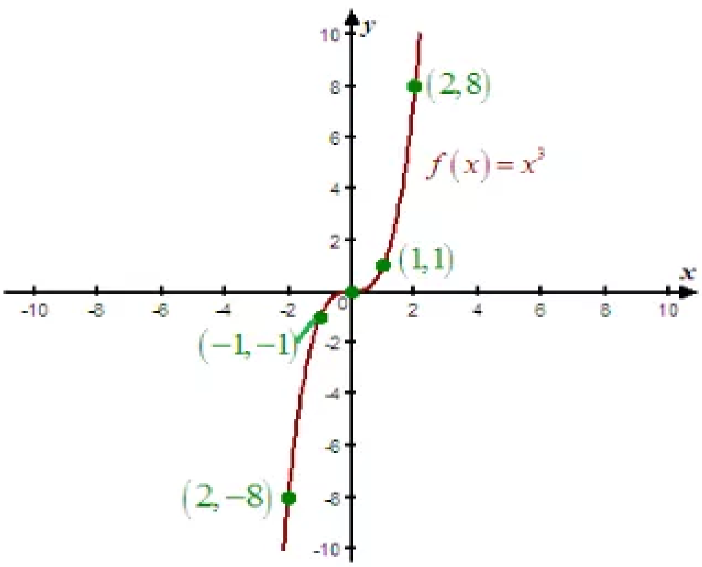
Consider the polynomial function $f(x) = x^3$.

Need to sketch the graph of the function $f(x) = x^3$.

To graph the function, make a table of values and plot the corresponding points. Connect the points with smooth curve and check the end behavior.
The table is as follows:

x	-2	-1	0	1	2
y	-8	-1	0	1	8

The graph of the polynomial function $f(x) = x^3$ with points is as follows:



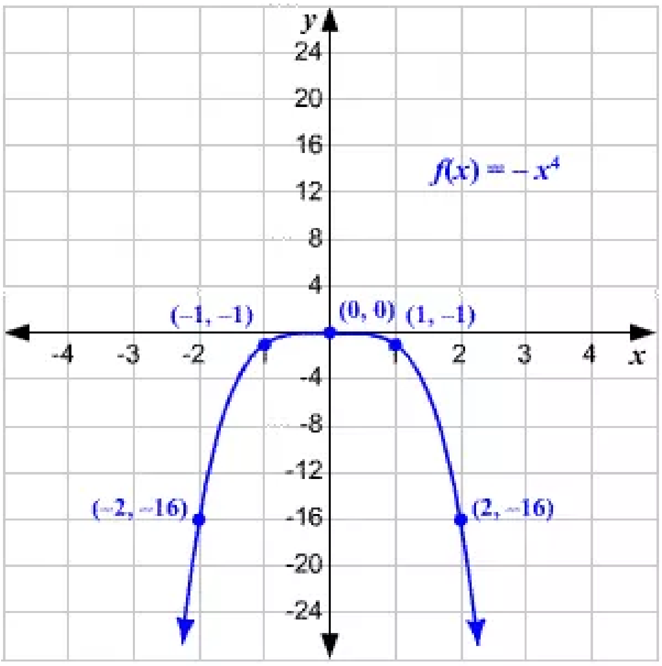
For the polynomial function $f(x) = x^3$ the degree is odd and leading coefficient is positive.
So $f(x) \to \infty$ as $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$

Answer 39e.

Make a table of values that satisfy the given function. For this, select some values for x and find the corresponding values of $f(x)$ or y.

x	-2	-1	0	1	2
y	-16	-1	0	-1	-16

Plot the points and connect them using a smooth curve.



The degree of the function is even and the leading coefficient is positive. The end behavior of the graph is such that

$f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$.

Answer 40e.

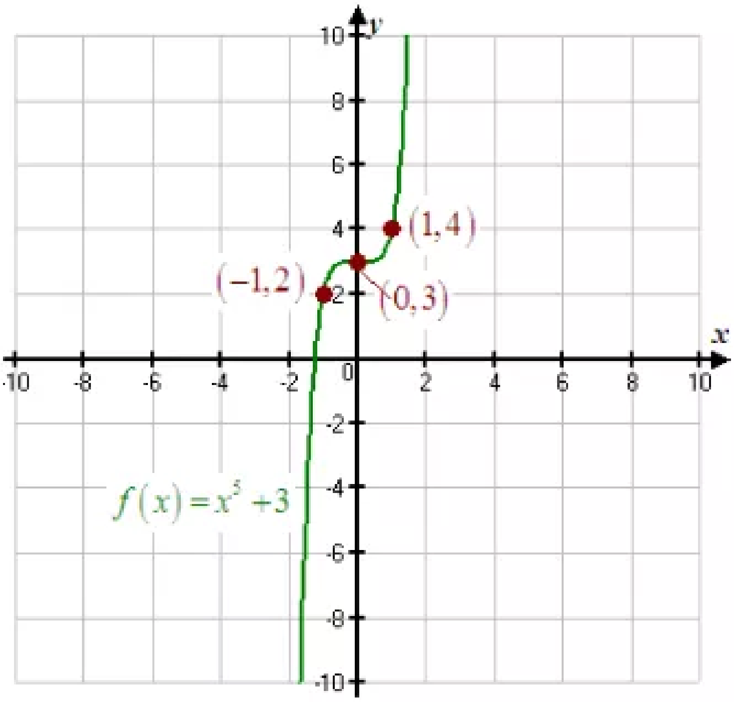
Consider the polynomial function $f(x) = x^5 + 3$.

Need to sketch the graph of the function $f(x) = x^5 + 3$.

To graph the function, make a table of values and plot the corresponding points. Connect the points with smooth curve and check the end behavior. The table is as follows:

x	-2	-1	0	1	2
y	-29	2	3	4	35

The graph of the polynomial function $f(x)=x^5+3$ with points is as follows:



For the polynomial function $f(x)=x^5+3$ the degree is odd and leading coefficient is positive.

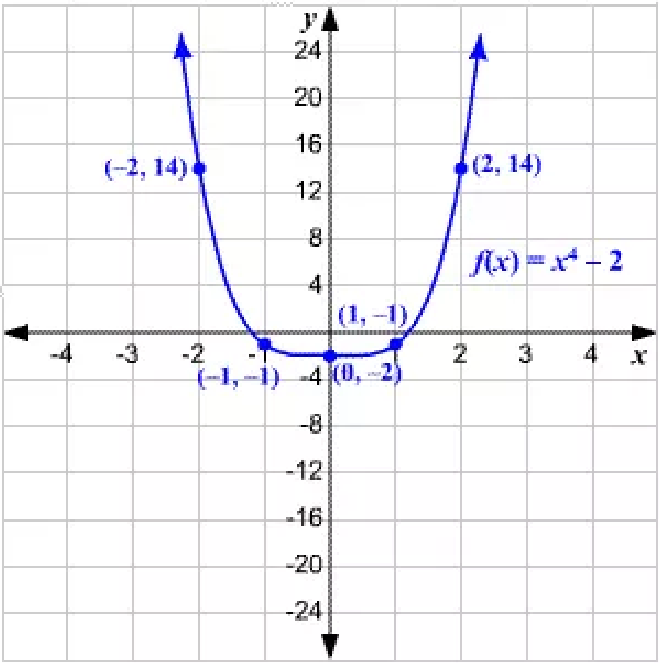
So $f(x)\to\infty$ as $x\to\infty$ and $f(x)\to-\infty$ as $x\to-\infty$

Answer 41e.

Make a table of values that satisfy the given function. For this, select some values for x and find the corresponding values of $f(x)$ or y.

x	−2	−1	0	1	2
y	14	−1	−2	−1	14

Plot the points and connect them using a smooth curve.



The degree of the function is even and the leading coefficient is positive. The end behavior of the graph is such that

$f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

Answer 42e.

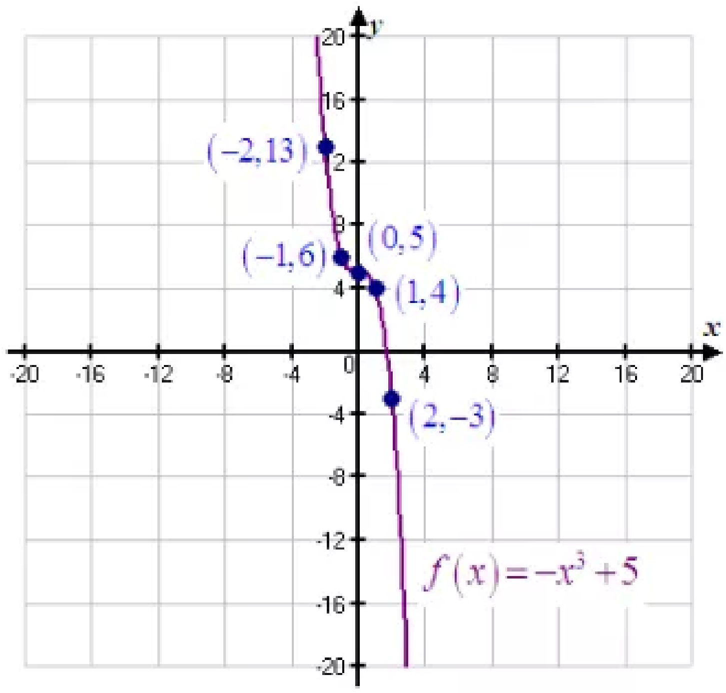
Consider the polynomial function $f(x) = -x^3 + 5$.

Need to sketch the graph of the function $f(x) = -x^3 + 5$.

To graph the function, make a table of values and plot the corresponding points.
Connect the points with smooth curve and check the end behavior.
The table is as follows:

x	−2	−1	0	1	2
y	13	6	5	4	−3

The graph of the polynomial function $f(x)=-x^3+5$ with points is as follows:



For the polynomial function $f(x)=-x^3+5$ the degree is odd and leading coefficient is negative.

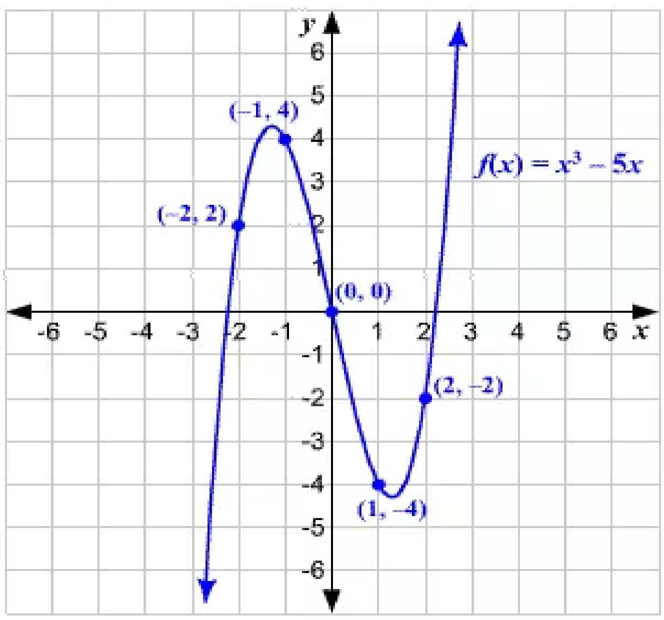
So $f(x)\to\infty$ as $x\to-\infty$ and $f(x)\to-\infty$ as $x\to\infty$

Answer 43e.

Make a table of values that satisfy the given function. For this, select some values for x and find the corresponding values of $f(x)$ or y.

x	−2	−1	0	1	2
y	2	4	0	−4	−2

Plot the points and connect them using a smooth curve.



The degree of the function is odd and the leading coefficient is positive. The end behavior of the graph is such that $f(x)\to-\infty$ as $x\to-\infty$ and $f(x)\to+\infty$ as $x\to+\infty$.

Answer 44e.

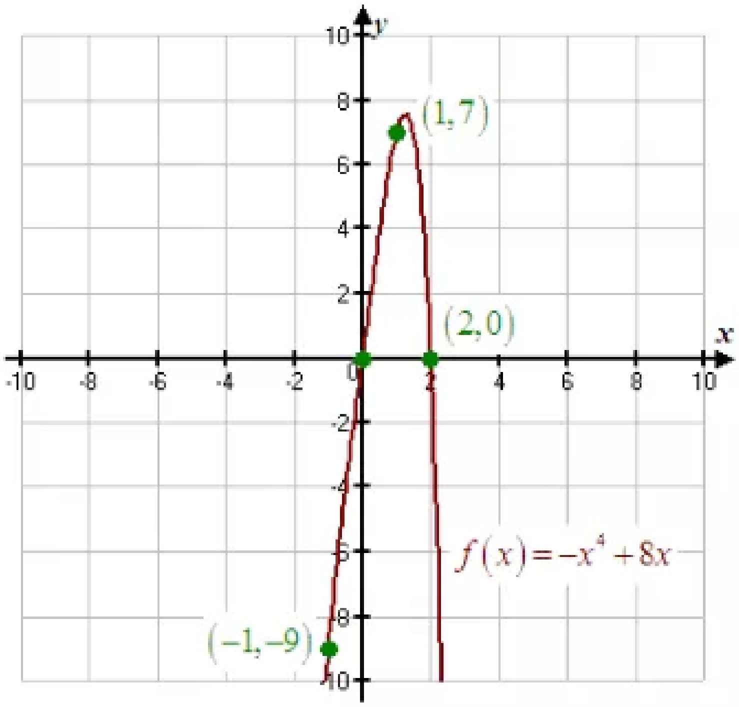
Consider the polynomial function $f(x)=-x^4+8x$.

Need to sketch the graph of the function $f(x)=-x^4+8x$.

To graph the function, make a table of values and plot the corresponding points.
Connect the points with smooth curve and check the end behavior.
The table is as follows:

x	-2	-1	0	1	2
y	-32	-9	8	7	0

The graph of the polynomial function $f(x)=-x^4+8x$ with points is as follows:

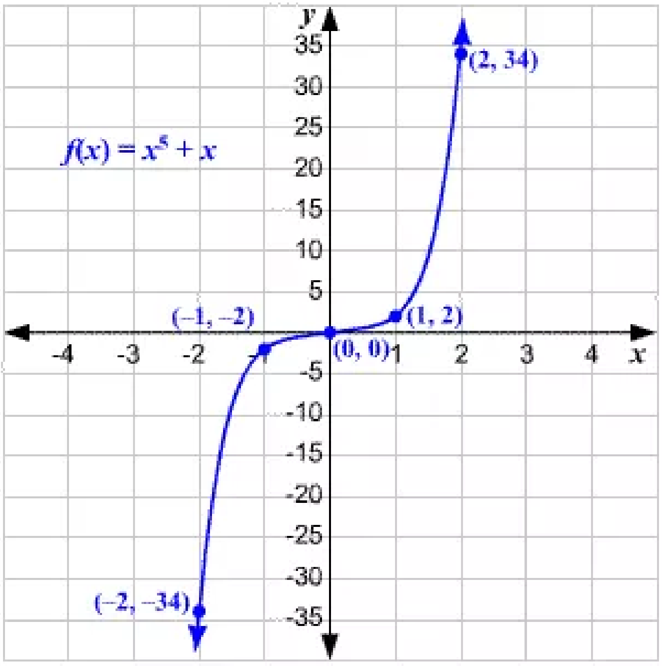


For the polynomial function $f(x)=-x^4+8x$ the degree is even and leading coefficient is negative.

So $f(x)\to-\infty$ as $x\to-\infty$ and $f(x)\to-\infty$ as $x\to\infty$

Answer 45e.

Plot the points and connect them using a smooth curve.



The degree of the function is odd and the leading coefficient is positive. The end behavior of the graph is such that $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

Answer 46e.

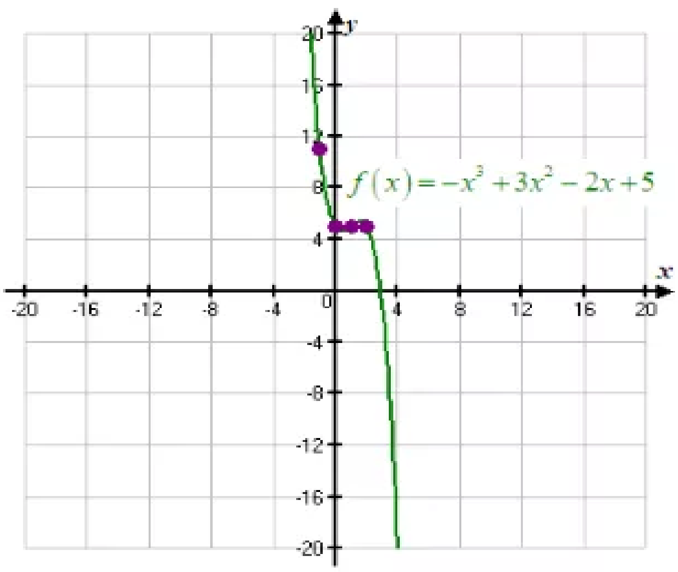
Consider the polynomial function $f(x) = -x^3 + 3x^2 - 2x + 5$

Need to sketch the graph of the function $f(x) = -x^3 + 3x^2 - 2x + 5$.

To graph the function, make a table of values and plot the corresponding points.
Connect the points with smooth curve and check the end behavior.
The table is as follows:

x	-2	-1	0	1	2
y	29	11	5	5	5

The graph of the polynomial function $f(x) = -x^3 + 3x^2 - 2x + 5$ with points is as follows:



For the polynomial function $f(x) = -x^3 + 3x^2 - 2x + 5$ the degree is odd and leading coefficient is negative.

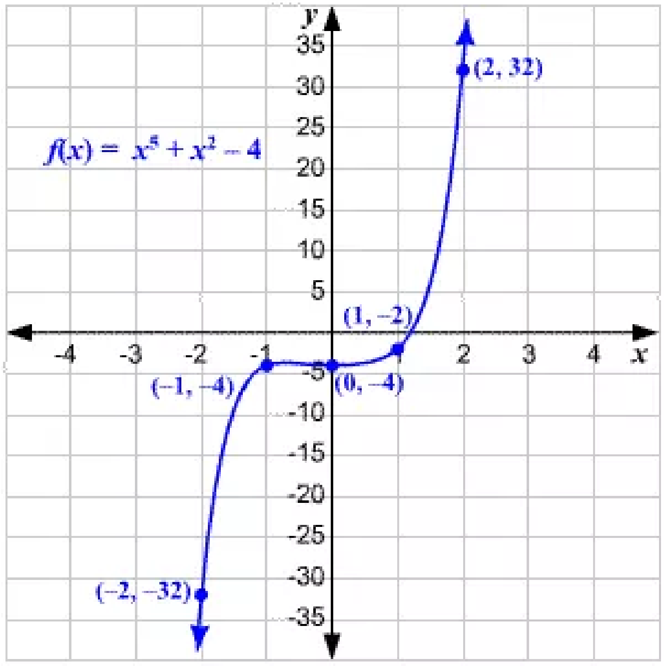
So $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$.

Answer 47e.

Make a table of values that satisfy the given function. For this, select some values for x and find the corresponding values of $f(x)$ or y.

x	−2	−1	0	1	2
y	−32	−4	−4	−2	32

Plot the points and connect them using a smooth curve.



The degree of the function is odd and the leading coefficient is positive. The end behavior of the graph is such that $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

Answer 48e.

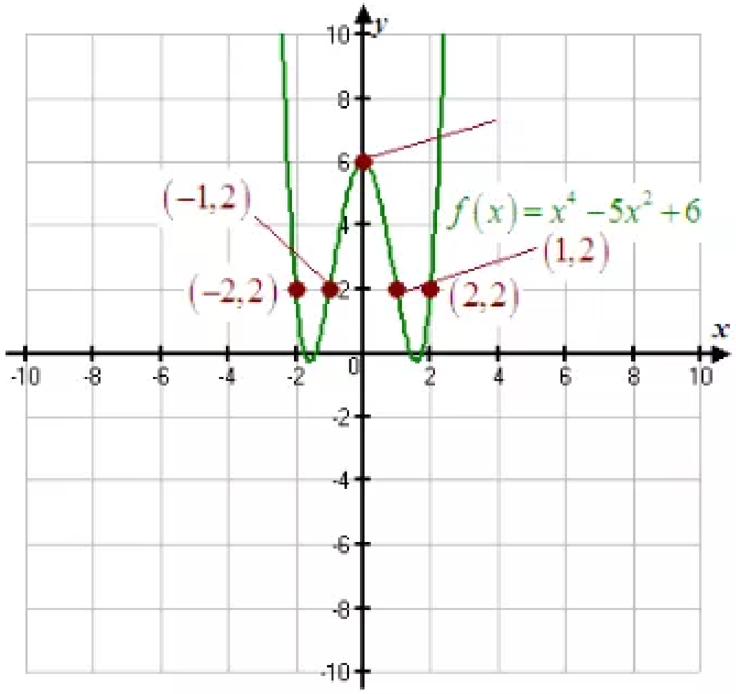
Consider the polynomial function $f(x) = x^4 - 5x^2 + 6$.

Need to sketch the graph of the function $f(x) = x^4 - 5x^2 + 6$.

To graph the function, make a table of values and plot the corresponding points. Connect the points with smooth curve and check the end behavior. The table is as follows:

x	−2	−1	0	1	2
y	2	2	6	2	2

The graph of the polynomial function $f(x) = -x^3 + 3x^2 - 2x + 5$ with points is as follows:



For the polynomial function $f(x) = -x^3 + 3x^2 - 2x + 5$ the degree is odd and leading coefficient is negative.

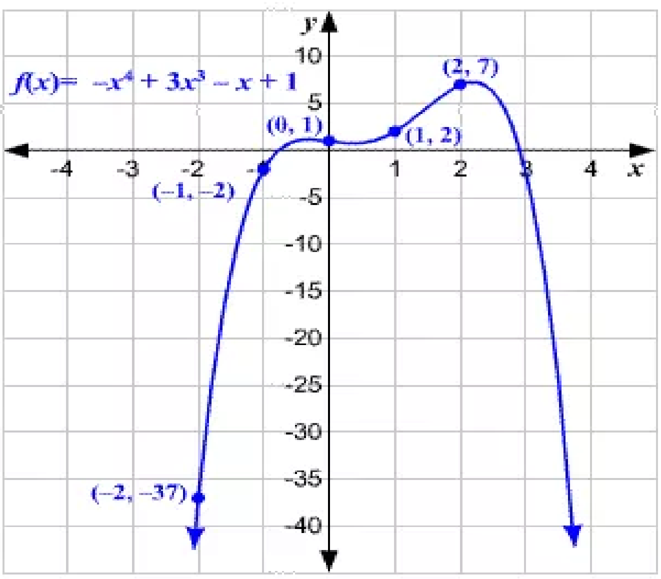
So $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$.

Answer 49e.

Make a table of values that satisfy the given function. For this, select some values for x and find the corresponding values of $f(x)$ or y.

x	−2	−1	0	1	2
y	−37	−2	1	2	7

Plot the points and connect them using a smooth curve.

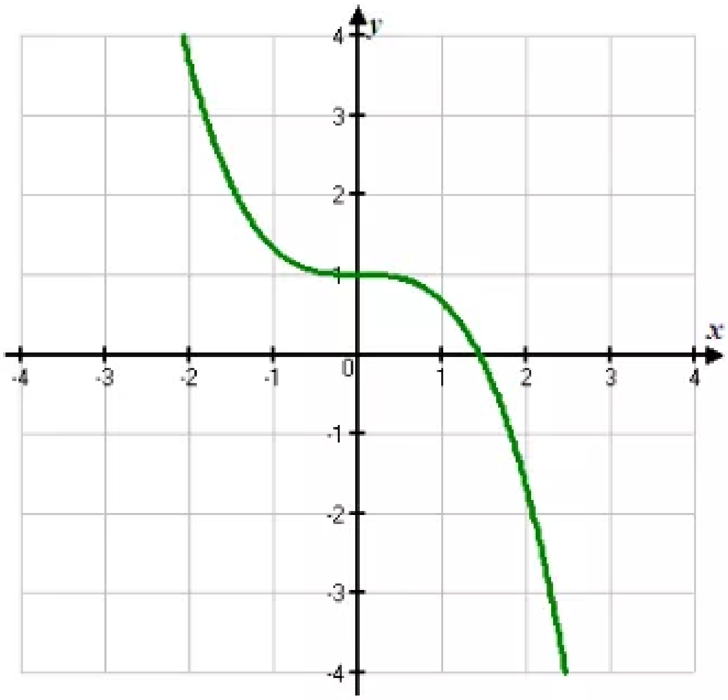


The degree of the function is even and the leading coefficient is negative. The end behavior of the graph is such that

$f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$.

Answer 50e.

Consider the following graph,



Let $f(x)$ be the polynomial corresponding to the given graph.

Then,

$f(x) \to -\infty$ as $x \to \infty$ and

$f(x) \to \infty$ as $x \to -\infty$

The polynomial $f(x)$ is a polynomial of odd degree with negative leading coefficient.

Therefore the graph is matched with the option $\boxed{\mathbf{B}}$ $f(x) = -\frac{1}{3}x^3 + 1$. Because the graph passes through the point $(0,1)$.

Answer 51e.

The given end behavior is of a function $f(x)$ having odd degree and negative leading coefficient. When $-f(x)$ is taken, the negative sign of the leading coefficient in $f(x)$ will change to positive. Since $g(x) = -f(x)$ and the degree of the two functions being the same, we can say that the leading coefficient of $g(x)$ will be positive.

For a function with odd degree and positive leading coefficient, the end behavior of the graph is $g(x) \to -\infty$ as $x \to -\infty$ and $g(x) \to +\infty$ as $x \to +\infty$.

Answer 52e.

Consider a cubic polynomial f with leading coefficient 2 and constant term -5 such that $f(1)=0$ and $f(2)=3$.

Need to find $f(-5)$.

A polynomial function is a function of the form $f(x)=a_nx^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0$ where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.

For this function a_n is the leading coefficient, n is the degree.

Now substitute the leading coefficient and constant term into the polynomial function, obtain

$$f(x)=2x^3+px^2+qx-5$$

For $x=1$,

$$f(1)=2+p+q-5$$

It is given that $f(1)=0$ then $2+p+q-5=0$.

Also for $x=2$,

$$f(2)=16+4p+2q-5$$

It is given that $f(2)=3$ then $16+4p+2q-5=3$.

$$\begin{cases} f(1)=2+p+q-5=0 \\ f(2)=16+4p+2q-5=3 \end{cases}$$

$$\begin{cases} p+q=3 \\ 4p+2q=-8 \end{cases}$$

On solving,

$$(p+q=3)2$$
$$(4p+2q=-8)(-1)$$

$$2p+\cancel{2q}=6$$
$$-4p-\cancel{2q}=+8$$

$$-2p=14$$

$$p=\frac{-14}{2}$$
$$=-7$$

Substitute the value of $p=-7$ in $p+q=3$, obtain

$$p+q=3$$
$$-7+q=3$$
$$q=3+7$$
$$q=10$$

Thus,

$$(p,q)=(-7,10)$$

Now substitute these values in $f(x)=2x^3+px^2+qx-5$, obtain

$$f(x)=2x^3-7x^2+10x-5$$

Now need to find $f(-5)$.

Substitute -5 for x in $f(x)=2x^3-7x^2+10x-5$, obtain

$$\begin{aligned} f(-5) &= 2(-125)-7(25)+10(-5)-5 \\ &= -250-175-50-5 \\ &= -480 \end{aligned}$$

Therefore $f(-5)=\boxed{-480}$.

Answer 53e.

a. The table can be completed by finding the values of $f(x)$, $g(x)$, and $\frac{f(x)}{g(x)}$ for each of the given values of x.

Let us first find $f(10)$ by substituting 10 for x in the expression for $f(x)$.

$$\begin{aligned} f(10) &= 10^3 \\ &= 1000 \end{aligned}$$

Now, find g(10).

$$\begin{aligned} g(10) &= 10^3 - 2(10)^2 + 4(10) \\ &= 1000 - 200 + 40 \\ &= 840 \end{aligned}$$

Divide 1000 by 840 to find $\frac{f(x)}{g(x)}$.

$$\begin{aligned} \frac{f(x)}{g(x)} &= \frac{1000}{840} \\ &\approx 1.19 \end{aligned}$$

Similarly, proceed for all values of x and complete the table.

x	$f(x)$	$g(x)$	$\frac{f(x)}{g(x)}$
10	1000	840	1.19
20	8000	7280	1.09
50	125,000	120,200	1.04
100	1,000,000	980,400	1.02
200	8,000,000	7,920,800	1.01

b. In the table, we can see that as the value of x is increasing, the value of $\frac{f(x)}{g(x)}$ is decreasing and approaching 1.

For any higher value of x, we can say that $\frac{f(x)}{g(x)}$ approximately evaluates to 1.

Thus, the statement can be completed as as $x \to \infty$, $\frac{f(x)}{g(x)} \to \underline{1}$.

c. We have obtained the result as $x \to \infty$, $\frac{f(x)}{g(x)} \to 1$. For the ratio of $f(x)$ and $g(x)$ to be 1, their values must always be positive. This means that as x approaches infinity, $f(x)$ and $g(x)$ also approaches higher positive values.

Therefore, it is clear that the functions f and g have the same end behavior as $x \to \infty$.

Answer 54e.

Consider the weight of an ideal round-cut diamond is modeled by

$$w = 0.0071d^3 - 0.090d^2 + 0.48d$$

Where w is the diamond's weight (in carats) and d is its diameter (in millimeters).
Need to find w when $d = 15$

$w = 0.0071d^3 - 0.090d^2 + 0.48d$ Write the original equation

$w = 0.0071(15)^3 - 0.090(15)^2 + 0.48(15)$ Substitute 15 for d

$= 0.0071(3375) - 0.090(225) + 0.48(15)$ Evaluate powers

$= 23.9625 - 20.25 + 2.7$ Multiply

$= 6.4125$ carats Simplify

Therefore the weight of a diamond is $\boxed{w = 6.4125 \text{ carats}}$.

Answer 55e.

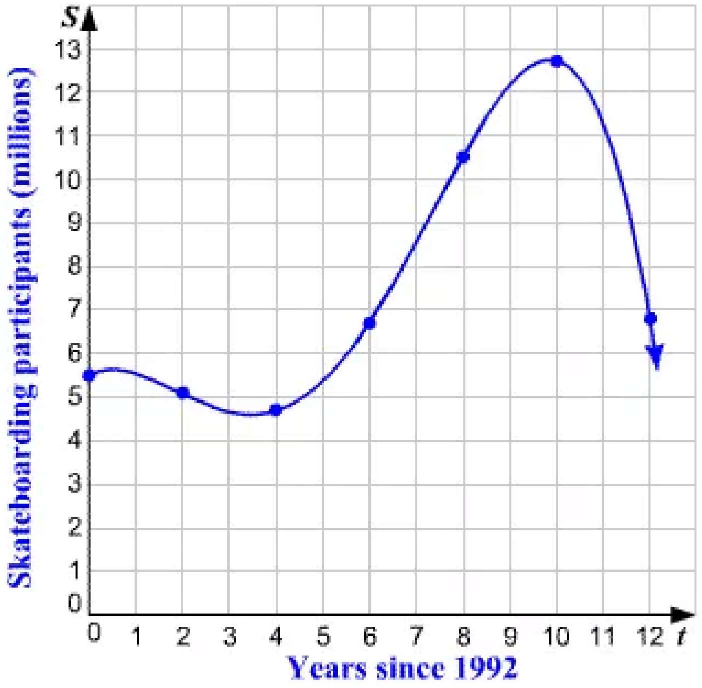
STEP 1 Make a table of values.

Since t represents the number of years, the model deals with only positive values of t. Also, as t is the number of years since 1992, $t = 0$ corresponds to the year 1992.

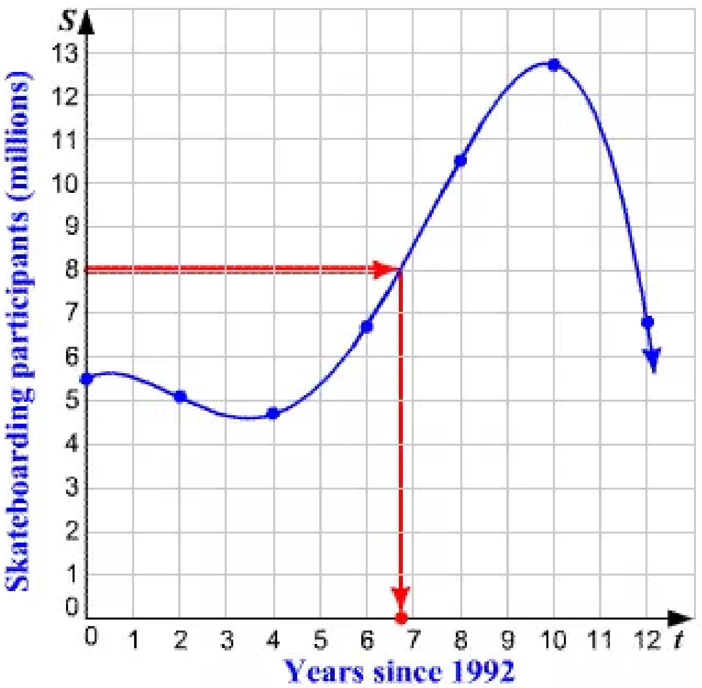
t	0	2	4	6	8	10	12
S	5.5	5.1	4.7	6.7	10.5	12.7	6.8

STEP 2 **Plot** the points and connect them with a smooth curve.

Since the degree is even and the leading coefficient is negative, the graph tends to fall to the right.



STEP 3 Examine the graph to find the first year that the number of skateboarding participants was greater than 8 million.



From the graph, it is clear that the number of participants was greater than 8 million at some value after $t = 6$. This value corresponds to some months in the year 1998.

Thus, the year in which the number of skateboarding participants was greater than 8 million was 1998.

Answer 56e.

Consider, from 1987 to 2003, the number of indoor movie screens M in the country can be modeled by $M = -11.0t^3 + 267t^2 - 592t + 21600$
Where t is the number of years since 1987.

(a)

Need to state the degree and type of the function.
A polynomial function is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.
Hence the function $M = -11.0t^3 + 267t^2 - 592t + 21600$ is a **polynomial function** it is a 3^{rd} **degree** (cubic) polynomial function with leading coefficient -11.

(b)

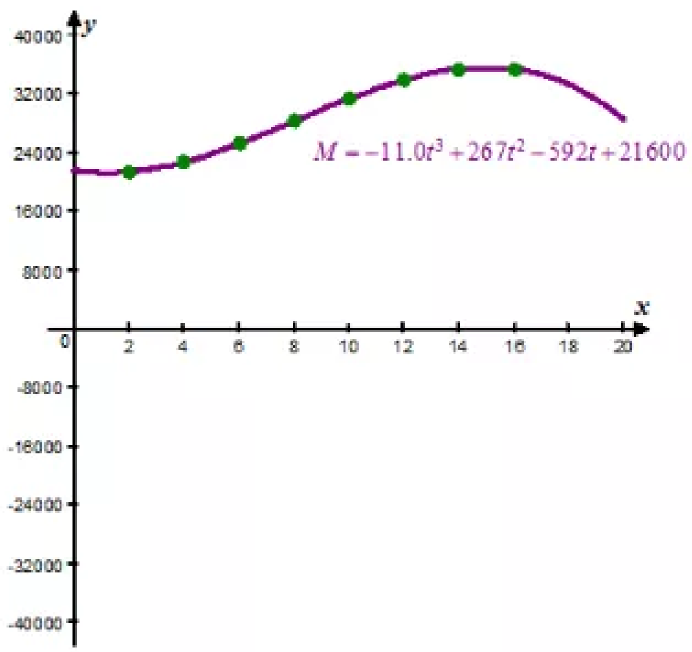
Need to make a table of values for the function $M = -11.0t^3 + 267t^2 - 592t + 21600$.

Consider the value of $t = 2, 4, 6, 8, 10, 12, 14, 16$ and substitute these values into the function, obtain the values and make the table as follows:

t	2	4	6	8	10	12	14	16
M	21396	22800	25284	28320	31380	33936	35460	35424

(c)

Need to sketch the graph of the function by using the table in part (b).
The following figure represents the graph of the given function.



Answer 57e.

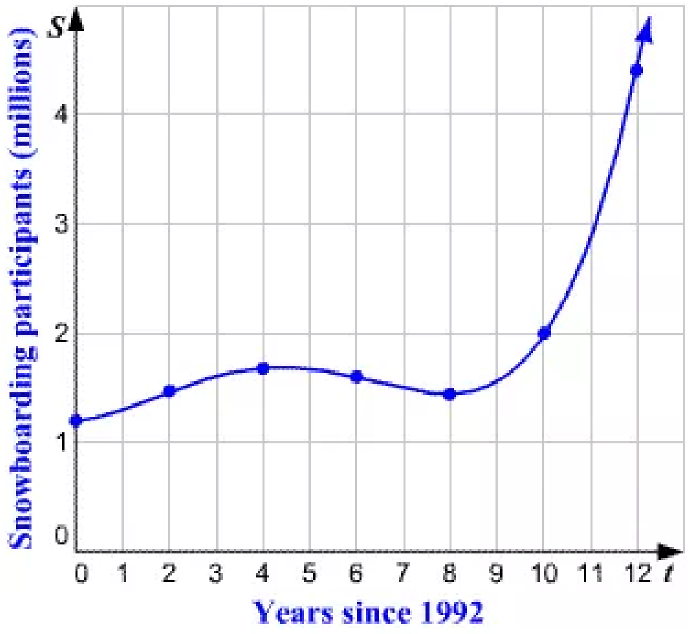
STEP 1 **Make** a table of values.

Since t represents the number of years, the model deals with only positive values of t. Also, as t is the number of years since 1992, $t = 0$ corresponds to the year 1992.

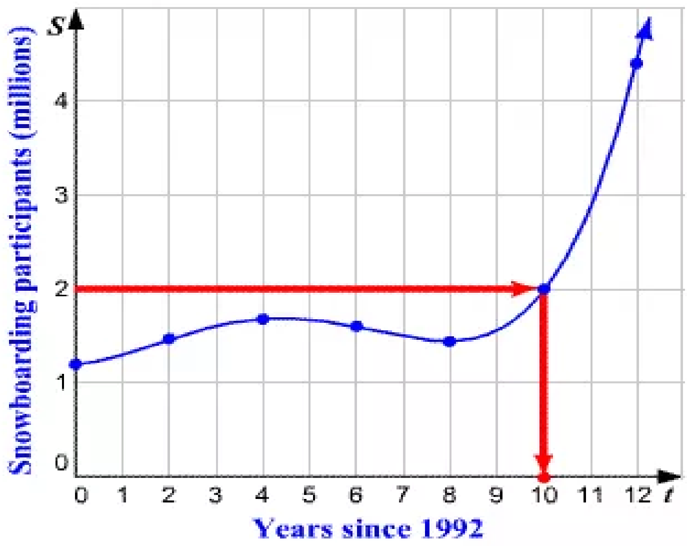
t	0	2	4	6	8	10	12
S	1.2	1.5	1.7	1.6	1.4	2	4.4

STEP 2 **Plot** the points and connect them with a smooth curve.

Since the degree is even and the leading coefficient is positive, the graph tends to rise to the right.



STEP 3 Examine the graph to find the first year that the number of snowboarding participants was greater than 2 million.



From the graph, it is clear that the number of participants was greater than 2 million in and after $t = 10$. This value corresponds to the year 2002.

Thus, the number of snowboarding participants was greater than 2 million in the year 2002.

Answer 58e.

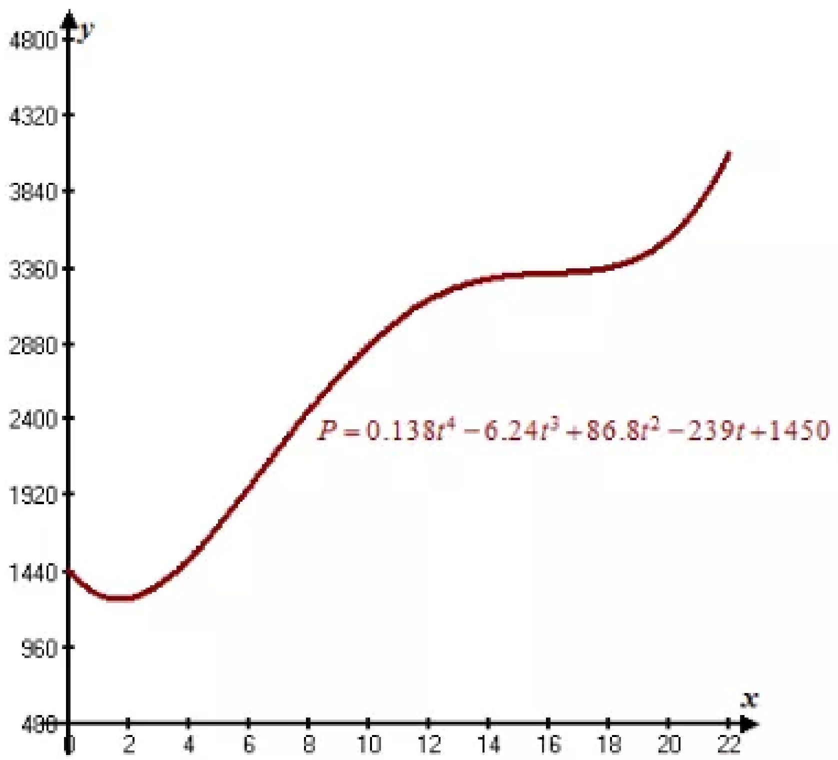
Consider, from 1980 to 2002, the number of quarterly periodicals P published in the country can be modeled by $P = 0.138t^4 - 6.24t^3 + 86.8t^2 - 239t + 1450$
Where t is the number of years since 1980.

(a)

Need to describe the end behavior of the graph of the model.
A polynomial function is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ where $a_n \neq 0$, the exponents are all whole numbers and the coefficients are all real numbers.
Hence the function $P = 0.138t^4 - 6.24t^3 + 86.8t^2 - 239t + 1450$ is a polynomial function of degree 4 or a quartic polynomial function and it has a positive leading coefficient 0.138.
Thus the end behavior of the function is $\boxed{P(t) \to \infty \text{ as } t \to \infty}$.

(b)

Need to sketch the function on the domain $0 \le t \le 22$
The following diagram contains the graph of the given function P over the domain $0 \le t \le 22$



(c)

If we use the given function to predict the number of quarterly periodicals in 2010.
We get,

$$P(30) = 0.138(30^4) - 6.24(30^3) + 86.4(30^2) - 239(30) + 1450$$
$$= 0.138(810000) - 6.24(27000) + 86.4(900) - 239(30) + 1450$$
$$= 111780 - 168480 + 77760 - 7170 + 1450$$
$$= 15340$$

But, it is not appropriate to use the model to make the prediction about 2010 as it is clearly mentioned that the given is for the years 1980 to 2002.

Answer 59e.

a. Replace t with 5 in the expression for S and evaluate to find the weight of the 5-day-old Sarus crane chick.

$$\begin{aligned} S &= -0.122(5)^3 + 3.49(5)^2 - 14.6(5) + 136 \\ &= -15.25 + 87.25 - 73 + 136 \\ &= 135 \end{aligned}$$

A 5-day-old Sarus chick weighs 135 grams.

Substitute 5 for t in the expression for H to find the weight of a 5-day-old hooded chick.

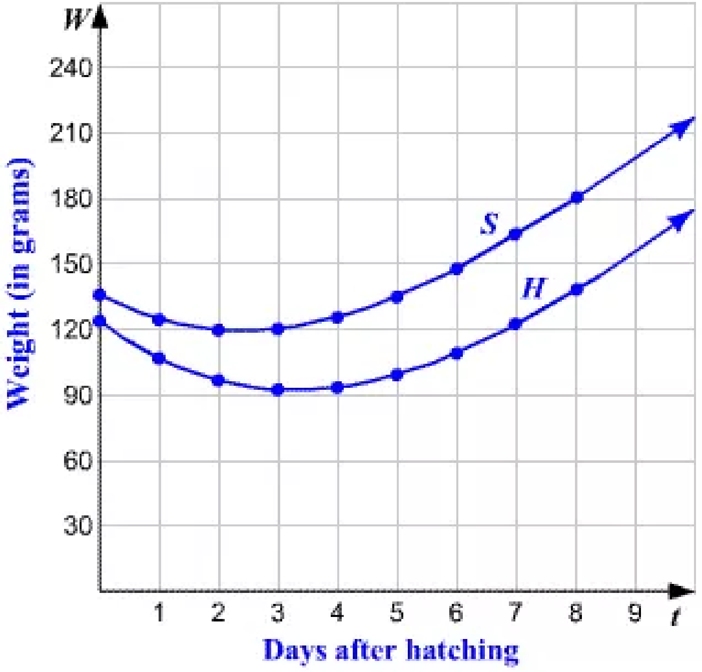
$$\begin{aligned} H &= -0.115(5)^3 + 3.71(5)^2 - 20.6(5) + 124 \\ &= -14.375 + 92.75 - 103 + 124 \\ &= 99.375 \end{aligned}$$

A 5-day-old hooded chick weighs 99.375 grams.
The difference in the weights of the two chicks is 135 – 99.375, or 35.625 grams.

b. Make a table of values of the two functions. Since t represents the number of days after hatching, it can take only positive values.

t	0	1	2	3	4	5	6	7	8
S	136	124.7	119.7	120.3	125.6	135	147.7	162.9	180
H	124	106.9	96.7	92.5	93.6	99.4	109.1	122.1	137.8

Plot the points and connect them with a smooth curve.



c. Let us find the weight of each kind of chick 3 days after hatching. For this, substitute 3 for t in both functions and evaluate.

$$\begin{aligned} S &= -0.122(3)^3 + 3.49(3)^2 - 14.6(3) + 136 \\ &= 120.3 \end{aligned}$$

$$\begin{aligned} H &= -0.115(3)^3 + 3.71(3)^2 - 20.6(3) + 124 \\ &= 92.5 \end{aligned}$$

The weight of a 3-day-old Sarus chick is closer to 130 grams that that of a hooded chick.
Thus, the crane chick is likely to be of Sarus species.

Answer 60e.

Consider, the weight y (in ponds) of a rainbow trout can be modeled by $y = 0.000304x^3$ where x is the length of the trout (in inches).

(a)

Need to write a function that relates the weight y (in kilograms) and the length x (in centimeters).

Since 1 kilogram ≈ 2.20 pounds and
1 Centimeter ≈ 0.394 inch

$$\frac{y}{2.20} = 0.000304\left(\frac{x}{0.394}\right)^3$$

$$y = 0.000304\left(\frac{2.2}{(0.394)^3}\right)x^3 \quad \text{Multiply by 2.2 on each side}$$

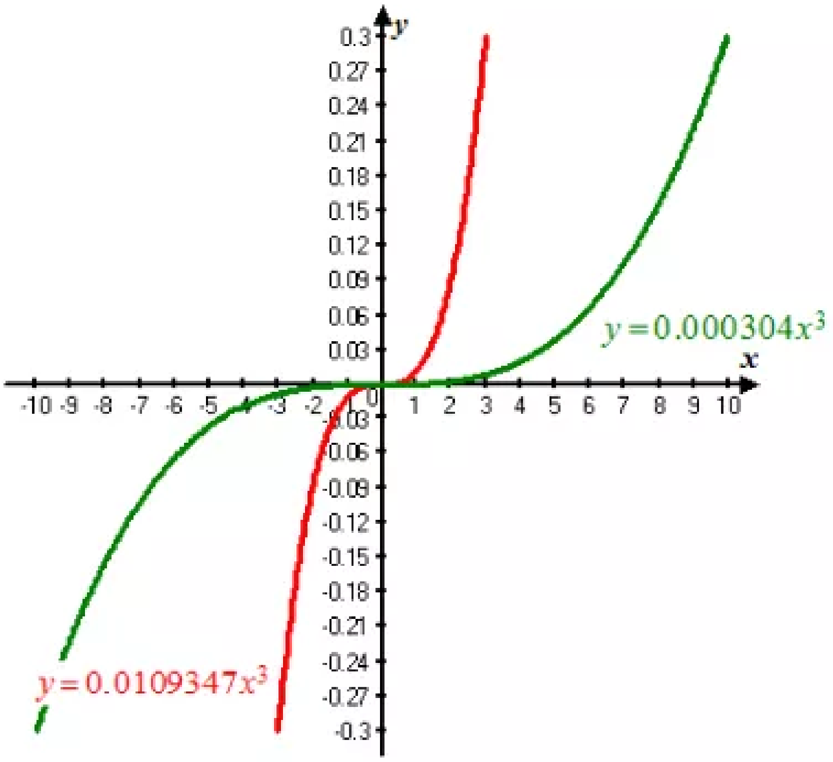
$$= 0.000304\left(\frac{2.2}{0.061162984}\right)x^3$$

$$= 0.000304(35.9694)x^3$$

$$y = \boxed{0.0109347x^3}$$

(b)

The following diagram contains the graph of the function $y = 0.000304x^3$ (green curve) and $y = 0.0109347x^3$ (red curve).



Clearly, the graph of the function $y = 0.000304x^3$ can be stretched vertically upwards to obtain the graph of the function $y = 0.0109347x^3$.

Answer 61e.

Add $6b$ to each side of the given equation.

$$2b + 11 + 6b = 15 - 6b + 6b$$
$$8b + 11 = 15$$

Subtract 11 from each side.

$$8b + 11 - 11 = 15 - 11$$
$$8b = 4$$

Divide each side by 8.

$$\frac{8b}{8} = \frac{4}{8}$$

$$b = \frac{1}{2}$$

The solution appears to be $\frac{1}{2}$.

CHECK

Substitute $\frac{1}{2}$ for b in the original equation.

$$2b + 11 = 15 - 6b$$

$$2\left(\frac{1}{2}\right) + 11 \stackrel{?}{=} 15 - 6\left(\frac{1}{2}\right)$$

$$1 + 11 \stackrel{?}{=} 15 - 3$$

$$12 = 12 \checkmark$$

The solution checks.

Therefore, the solution to the given equation is $\frac{1}{2}$.

Answer 62e.

Consider the equation $2.7n + 4.3 = 12.94$

Need to solve the equation.

$$2.7n + 4.3 = 12.94$$

$$2.7n = 12.94 - 4.3 \qquad \text{Subtract 4.3 from each side}$$

$$2.7n = 8.64$$

$$n = \frac{8.64}{2.7} \qquad \text{Divide by 2.7 on each side}$$

$$n = 3.2 \qquad \text{Simplify}$$

Therefore $\boxed{n = 3.2}$ is the solution of the equation $2.7n + 4.3 = 12.94$

Answer 63e.

The solution of an "and" compound inequality includes the solutions that are common to both inequalities.

Add 1 to each part.

$$-7 + 1 < 6y - 1 + 1 < 5 + 1$$

$$-6 < 6y < 6$$

Divide each part by 6.

$$\frac{-6}{6} < \frac{6y}{6} < \frac{6}{6}$$

$$-1 < y < 1$$

Thus, the solutions for x are all real numbers greater than -1 and less than 1.

Answer 64e.

Consider the equation $x^2 - 14x + 48 = 0$

Need to solve the equation.

$$x^2 - 14x + 48 = 0$$

$$x^2 - 6x - 8x + 48 = 0 \quad \text{Factorize}$$

$$x(x-6) - 8(x-6) = 0$$

$$(x-6)(x-8) = 0 \quad \text{Simplify}$$

$$x - 6 = 0 \quad \text{Or} \quad x - 8 = 0 \quad \text{Use zero product property}$$

$$x = 6 \quad \text{Or} \quad x = 8 \quad \text{Simplify}$$

$$x = 6, 8$$

Therefore $\boxed{x = 6, 8}$ are the solutions of the equation $x^2 - 14x + 48 = 0$.

Answer 65e.

The first step in solving an equation is to get 0 on one side of that equation.

Subtract 21 from both sides of the given equation.

$$-24q^2 - 90q - 21 = 21 - 21$$

$$-24q^2 - 90q - 21 = 0$$

The equation can be rewritten in intercept form by factoring the expression on the right side.

The first step in factoring an expression is to check whether there is any common monomial, other than 1. Since -3 is common to all terms, take out this monomial.

$$-3(8q^2 + 30q + 7) = 0$$

The trinomial is of the form $aq^2 + bq + c$, where $a = 8$, $b = 30$, and $c = 7$. We need to find k and l such that their product is 8, and m and n such that their product is 7. Since the middle term is positive, we consider only the positive factors of 7.

k, l	8, 1	8, 1	2, 4	2, 4
m, n	7, 1	1, 7	7, 1	1, 7
$(kq + m)(lq + n)$	$(8q + 7)(q + 1)$	$(8q + 1)(q + 7)$	$(2q + 7)(4q + 1)$	$(2q + 1)(4q + 7)$
$aq^2 + bq + c$	$8q^2 + 15q + 7$	$8q^2 + 57q + 7$	$8q^2 + 30q + 7$	$8q^2 + 18q + 7$

The correct factorization is obtained as $(2q + 7)(4q + 1)$. We thus have the equation

$$-3(2q + 7)(4q + 1) = 0$$

Apply the Zero product property.
$2q + 7 = 0$ or $4q + 1 = 0$

Solve the equations.

$$\begin{aligned} 2q + 7 &= 0 \quad \text{or} \quad & 4q + 1 &= 0 \\ 2q &= -7 & 4q &= -1 \\ q &= -\frac{7}{2} & q &= -\frac{1}{4} \end{aligned}$$

Therefore, the solutions of the given equation are $-\frac{7}{2}$ and $-\frac{1}{4}$.

Consider the inequality $z^2 + 5z < 36$
Need to solve the inequality $z^2 + 5z < 36$

$$\begin{aligned} z^2 + 5z &< 36 \\ z^2 + 5z - 36 &< 0 && \text{Subtract 36 from each side} \\ z^2 + 9z - 4z - 36 &< 0 && \text{Factorize} \\ z(z+9) - 4(z+9) &< 0 \\ (z+9)(z-4) &< 0 && \text{Simplify} \\ z + 9 > 0 \text{ Or } z - 4 &< 0 && \text{Use zero product property} \\ z = -9 \text{ or } z &= 4 \\ -9 < z &< 4 \\ z &\in (-9, 4) \end{aligned}$$

Therefore the solution set of the inequality $z^2 + 5z < 36$ is $\boxed{(-9, 4)}$.

Answer 67e.

The direct variation equation for the variables x and y is $y = ax$.

Substitute 4 for x, and 12 for y in the equation to find the value of a.
$12 = a(4)$

Divide each term by 4 to solve for a.

$$\begin{aligned} \frac{12}{4} &= \frac{a(4)}{4} \\ 3 &= a \end{aligned}$$

Replace a with 3 in $y = ax$.
$y = 3x$

Thus, the direct variation equation that relates x and y is $y = 3x$.

Substitute -3 for y in the direct variation equation obtained and find the value of x.

$$\begin{aligned} -3 &= 3x \\ -1 &= x \end{aligned}$$

The value of x is -1, when y is -3.

Answer 68e.

Consider $x = 3, y = -21$

Need to write an equation that relates x and y such that they vary directly and also find the value of x when $y = -3$

Let $y = x - 24$

$\Rightarrow \quad x = y + 24$

When $y = -3$

$x = y + 24$

$x = -3 + 24$ Substitute -3 for y

$\boxed{x = 21}$ Simplify

Answer 69e.

The direct variation equation for the variables x and y is $y = ax$.

Substitute 10 for x, and -4 for y in the equation to find the value of a.

$-4 = a(10)$

Divide each term by 10 to solve for a.

$$\frac{-4}{10} = \frac{a(10)}{10}$$

$$-\frac{2}{5} = a$$

Replace a with $-\frac{2}{5}$ in $y = ax$.

$$y = -\frac{2}{5}x$$

Thus, the direct variation equation that relates x and y is $y = -\frac{2}{5}x$.

Substitute -3 for y in the direct variation equation obtained and find the value of x.

$$-3 = -\frac{2}{5}x$$

$$\frac{15}{2} = x$$

The value of x is $\frac{15}{2}$, when y is -3.

Answer 70e.

Consider $x = 0.8, y = 0.2$

Need to write an equation that relates x and y such that they vary directly and also find the value of x when $y = -3$.

Let $x = 4y$

When $y = -3$,

Substitute the value of -3 for y in $x = 4y$, obtain

$$x = 4(-3)$$

$$\boxed{x = -12}$$

Answer 71e.

The direct variation equation for the variables x and y is $y = ax$.

Substitute -0.45 for x, and -0.35 for y in the equation to find the value of a.

$-0.45 = a(-0.35)$

Divide each term by -0.35 to solve for a.

$$\frac{-0.45}{-0.35} = \frac{a(-0.35)}{-0.35}$$

$$\frac{9}{7} = a$$

Replace a with $\frac{9}{7}$ in $y = ax$.

$$y = \frac{9}{7}x$$

Thus, the direct variation equation that relates x and y is $y = \frac{9}{7}x$.

Substitute -3 for y in the direct variation equation obtained and find the value of x.

$$-3 = \frac{9}{7}x$$

$$-\frac{7}{3} = x$$

The value of x is $-\frac{7}{3}$, when y is -3.

Answer 72e.

Consider $x = -6.5, y = 3.9$

Need to write an equation that relates x and y such that they vary directly and also find the value of x when $y = -3$

Let $x = y - 10.4$

When $y = -3$,

Substitute the value of -3 for y in $x = y - 10.4$, obtain

$$x = -3 - 10.4$$

$$\boxed{x = -13.4}$$

Answer 73e.

Apply the FOIL method and multiply.

$$\begin{aligned} y &= x(x) + x(-7) + 3(x) + 3(-7) \\ &= x^2 - 7x + 3x - 21 \end{aligned}$$

Combine the like terms.

$$\begin{aligned} x^2 - 7x + 3x - 21 &= x^2 + (-7+3)x - 21 \\ &= x^2 + (-4)x - 21 \\ &= x^2 - 4x - 21 \end{aligned}$$

Thus, the given function can be written in standard form as $y = x^2 - 4x - 21$.

Answer 74e.

Consider the quadratic function $y = 8(x-4)(x+2)$

Need to write the quadratic function $y = 8(x-4)(x+2)$ in standard form.

$y = 8(x-4)(x+2)$

$y = 8(x^2 + 2x - 4x - 8)$ Apply FOIL method

$y = 8(x^2 - 2x - 8)$ Combine like terms

$y = 8x^2 - 16x - 64$ Multiply

This is the required quadratic equation.

Answer 75e.

Rewrite $(x-5)^2$.

$y = -3(x-5)(x-5) - 25$

Apply the FOIL method and multiply.

$$\begin{aligned} -3(x-5)(x-5) - 25 &= -3[x(x) + x(-5) + (-5)(x) + (-5)(-5)] - 25 \\ &= -3[x^2 - 5x - 5x + 25] - 25 \end{aligned}$$

Combine the like terms within the brackets.
$-3[x^2 - 5x - 5x + 25] - 25 = -3[x^2 - 10x + 25] - 25$

Remove the brackets using the distributive property.
$-3[x^2 - 10x + 25] - 25 = -3x^2 + 30x - 75 - 25$

Combine the like terms again.
$-3x^2 + 30x - 75 - 25 = -3x^2 + 30x - 100$

Thus, the given function can be written in standard form as $y = -3x^2 + 30x - 100$.

Answer 76e.

Consider the quadratic function $y = 2.5(x-6)^2 + 9.3$.

Need to write the quadratic function $y = 2.5(x-6)^2 + 9.3$ in standard form.

$y = 2.5(x-6)^2 + 9.3$ Write the original function

$y = 2.5(x^2 - 12x + 36) + 9.3$ Simplify

$y = (2.5x^2 - 30x + 90) + 9.3$ Multiply

$y = 2.5x^2 - 30x + 99.3$ Combine like terms

This the required quadratic equation.

Answer 77e.

Rewrite $(x-4)^2$.

$$y = \frac{1}{2}(x-4)(x-4)$$

Apply the FOIL method and multiply.

$$\frac{1}{2}(x-4)(x-4) = \frac{1}{2}\left[x(x) + x(-4) + (-4)(x) + (-4)(-4)\right]$$
$$= \frac{1}{2}\left[x^2 - 4x - 4x + 16\right]$$

Combine the like terms within the brackets.

$$\frac{1}{2}\left[x^2 - 4x - 4x + 16\right] = \frac{1}{2}\left[x^2 - 8x + 16\right]$$

Remove the brackets using the distributive property.

$$\frac{1}{2}\left[x^2 - 8x + 16\right] = \frac{x^2}{2} - 4x + 8$$

Thus, the given function can be written in standard form as $y = \frac{x^2}{2} - 4x + 8$.

Answer 78e.

Consider the quadratic function $y=-\frac{5}{3}(x+4)(x+9)$.

Need to write the quadratic function $y=-\frac{5}{3}(x+4)(x+9)$ in standard form.

$y=-\frac{5}{3}(x+4)(x+9)$ Write the original equation

$y=-\frac{5}{3}(x^2+4x+9x+36)$ Apply FOIL method

$y=-\frac{5}{3}(x^2+13x+36)$ Combine like terms

$y=-\frac{5}{3}x^2-\frac{65}{3}x-60$ Multiply

This is the required quadratic equation.