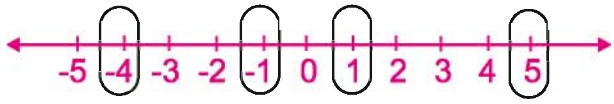
CHAPTER 1

INTEGERS

1.1 We have studied about Whole numbers and Integers in Class VI. We know that the Integers are collection of whole numbers and negative of whole numbers. In this chapter, we will learn the properties and operations of Integers in detail. Following number line displays the integers.



Write the encircled numbers in increasing order.

We know that the numbers increases from left to right along a number line. Hence

$-4 < -1 < 1 < 5.$

We have also studied in Class VI that on a number line when we

1. Add a positive integer then we move towards right.
2. Add a negative integer then we move towards left.
3. Subtract a positive integer then we move towards left.
4. Subtract a negative integer then we move towards right.

Do and learn

1. In which direction one should move on the number line to add -5?
2. In which direction one will move on the number line to subtract -5 from 3 and will reach on what number?

   $3 - (-5) =$ ....
3. In which direction we will move and on which number will we reach by adding 5 to 3?
4. In which direction we will move and on which number will we reach by subtracting + 5 from -3?

Identify true or false statement from the following:

1. Sum of two positive integers is again a positive integer. ( )
2. Sum of two negative integers is again a positive integer. ( )
3. Sum of a positive integer and a negative integer is always a negative integer. ( )
4. Additive inverse of 8 is – 8. ( )
5. $(-7) + 3 = 7 - 3$ ( )
6. $8 + (-7) - (-4) \neq 8 + 7 - 4$ ( )

We check the correctness of above statements as follows:

(1) Statement 1 is true. For example

(i) $7 + 4 = 11$ (ii) $4 + 11 = 15$ (iii) $6 + 7 = 13$ etc.

(2) Statement 2 is false. For example

(i) $(-6) + (-3) = (-9)$

When we add two negative integers, the resulting number is always a negative integer.

(3) Statement 3 is false. For example

$-10 + 15 = 5$, which is not a negative integer.

Therefore the correct statement is that to add a negative and positive integer we take their difference and put the sign of greater integer before it. While choosing the greater integer we neglect the sign.

For example

(i) $(-50)+(70)=20$ (ii) $12+(-20)=-8$

(iii) $16+(-7)=9$ (iv) $(-14)+(10)=-4$

(4) Statement is true because

$-8+8=0=8+(-8)$

Addition of additive inverse to a number gives additive identity "0". Give more examples of it.

So, additive inverse of 'a' is '-a' and that of '-a' is 'a'.

(5) Statement is false because

$(-7)+3=-4$ and $7+(-3)=4$

(6) Statement is true because

$8+(-7)-(-4)=5$ and $8+7-4=11$. Hence $8+(-7)-(-4) \neq 8+7-4$

1.1 Addition and subtraction properties of Integers

1.2.1 Closure property for addition

We have seen that the addition of two whole numbers is always a whole number, hence we can say that whole numbers are closed for addition. Let us check if integers are also closed for addition.

S. No.	Integer 1	Integer 2	Sum	Is the sum Integer?
1.	+ 2	+ 5	+ 7	Yes
2.	- 3	+ 7		
3.	- 4	+ 4		
4.	+ 3	- 5		

Take various integers and check if this is true only for positive integers or it is true for negative integers also. We observe from the table that **all the integers are closed for addition irrespective of being positive or negative.** Can you tell two such integers whose sum is not an integer? For integers **'a'** and **'b'**, (**a**+**b**) is always an integer.

1.2.2 Closure property for subtraction

What happens when we subtract an integer from another integer? Is their difference is also an integer? Complete the following table:

	Statement	Observation
1.	7 – 5 = 2	Result is an integer.
2.	4 – 9 = -5	……………………
3.	(-4) – (-5) = …….	Result is an integer.
4.	(-18) – (-18) = …….	……………………
5.	17 – 0 = …….	……………………

What do you observe? Can we find a pair of integers whose difference is not an integer? Can we say that integers are closed for subtraction? Yes, we can say that **integers are closed for subtraction.**

So, for integers **'a'** and **'b'**, (**a - b**) is always an integer.

Note: It is to be noted that Whole numbers are not closed for subtraction.

1.2.3 Commutative Property

We know that 2 + 4 = 4 + 2 = 6, i.e., change of order in the addition of whole numbers does not alter the result. Hence, commutative property is followed. Similarly do integers also follow commutative property? Let us check. Are the following same?

(-8) + (-4) and (-4) + (-8);

(-2) + 5 and 5 + (-2);

12 + 0 and 0 + 12.

Add other integers and check if there exist any pair of integers where the result is unchanged by changing the order of addition.

We have seen that if order of addition is changed, the result does not change i.e., **integers follow commutative property under addition operation**. In general, for two integers **'a'** and **'b'** we can say that

a+b = b + a

We know that the whole numbers do not follow commutative property for subtraction. Does the commutative property applies for subtraction of integers? Consider two integers (-6) and (+4).

Are (-6) – (+4) and (+4) - (-6) same?

No, because

(-6) – (+4) = -10 and (+4) - (-6) = 10

and -10, +10 are not equal.

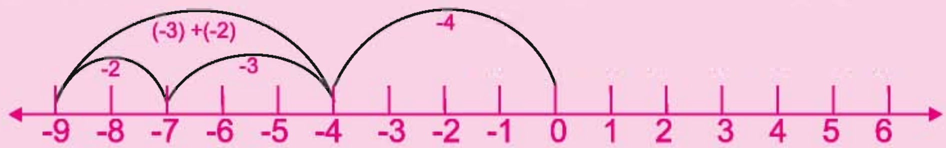
Thus, we conclude that **subtraction is not commutative for integers.**

1.2.4 Associative Property

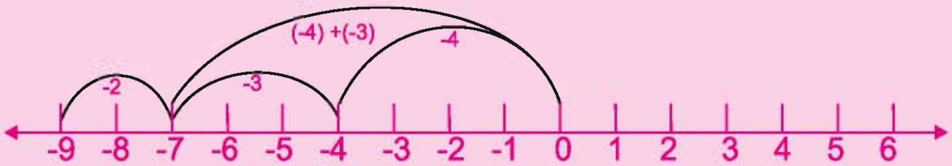
Test for associative property for integers -4, -3 and -2. Calculate

-4 + [(-3) + (-2)] and [(-4) + (-3)]+(-2).

-4 + [(-3) + (-2)] means we first add (-3) and (-2) and then add (-4) to the result.



[(-4) + (-3)] + (-2) means we first add (-4) and (-3) and then add (-2) to the result.



The result is (-9) in both the cases. Give three more such examples. You won't find any example for which the results are different. This shows that **the addition of integers follow the associativity property**, i.e.,

$$\mathbf{a + (b + c) = (a + b) + c.}$$

1.2.5 Additive Identity

Observe the following and fill in the blanks:

(i) (-4) + 0 = -4 (ii) 7 + 0 = 7 (iii) 0 + (-14) = …….

(iv) -8 + …... = -8 (v) …... + 0 = 15 (vi) -23 + ……. = -23

It is clear from the above examples that the same integers is obtained when we add 0 to it. Hence, **'0' is the additive identity for integers**. Justify this by taking some more examples.

Exercise 1.1

1. The temperature in Churu is measured in °C at different time and represented on number line

(i) The temperature of Churu on following date from above number line
   (a) 26 January ……………..   (b) 25 December ……………
   (c) 25 February ……………..   (d) 25 March ………………..
(ii) What is the difference in temperature between the hottest and the coldest day?
(iii) The temperature of 26 January is how much less than the temperature of 25 February?
(iv) Can we say that the sum of temperatures on 25 December and 25 February is higher than the temperature on 26 January?

2. Sheela deposits Rs. 5000 in post office and withdraws Rs. 3700 after one month. If the amount withdrawn is represented in the form of negative number then how will we represent the deposited amount? What is the amount left in the account after withdrawal?

3. Solve the following:
   (i) (-4) + (-3)   (ii) 15 – 8 + (-9)
   (iii) 400 + (-1000) + (-500)   (iv) 23 – 41 – 11
   (v) -27 + (-3) +30

4. Put the appropriate sign (<, >, =) for the following statements:

(i)	-14 + 11 + 5	( )	14 – 11 – 5
(ii)	30 + (-5) + (-8)	( )	(-5) + (-8) + 30
(iii)	7 + 11 + (-5)	( )	(-7) – 11 + 5
(iv)	(-14) + 11 + (-12)	( )	14 + 11 + 12
(v)	6 + 7 – 13	( )	6 + 7 + (-13)

5. Write two such integers whose
   (i) sum is (-7)   (ii) difference is 4   (iii) sum is 0   (iv) difference is -2.

6. Fill in the blanks.
   (i) (-3) + 5 = 5 + ……….
   (ii) 17 + ……… = 17
   (iii) ………. + (-5) = 0
   (iv) -11 + [(-12) + 4] = [(-11) + (-12)] + ……..

7. Examples and some properties of integers are given below. Match the correct property and its example.

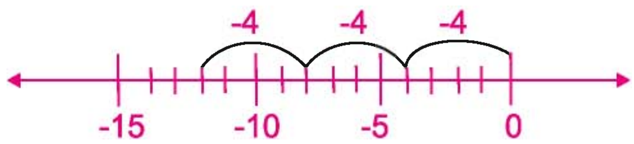
	Example		**Property**
(i)	(a + b) + c = a + (b + c)	(a)	Identity
(ii)	3 + 4 = 4 + 3	(b)	Associativity
(iii)	(-4) + 0 = (-4)	(c)	Commutativity

1.3 Multiplication of Integers

1.3.1 Multiplication of Positive Integers with Negative Integers

$3 \times 4 = 4 + 4 + 4 = 12$

$3 \times (-4) = (-4) + (-4) + (-4) = -12$



Similarly, $5 \times (-3) = (-3) + (-3) + (-3) + (-3) + (-3) = -15$

Do and learn:

Solve:

(i) $4 \times (-8) =$ .... = ... (ii) $3 \times (-3) =$ .... = ...

(iii) $5 \times (-9) =$ .... = ...

By using this method we find that the product of a positive integer with a negative integer is a negative integer. But what happens when we multiply a negative integer with a positive integer?

Observe the following pattern:

$3 \times 4 = 12$

$2 \times 4 = 8 = 12 - 4$

$1 \times 4 = 4 = 8 - 4$

$0 \times 4 = 0 = 4 - 4$

$-1 \times 4 = -4 = 0 - 4$

$-2 \times 4 = -8 = -4 - 4$

$-3 \times 4 = -12 = -8 - 4$

We have already obtained that $3 \times (-4) = -12$.

Thus, we find that $(-3) \times 4 = -12 = 3 \times (-4)$.

Similarly, we can also obtain $(-5) \times 3 = -15 = 3 \times (-5)$.

Do and learn :

Find -

(i) 15 × (-5) (ii) 27 × (-10)

(iii) -12 × 12 (iv) -7 × 4

1.3.2 Product of Two Negative Integers

Observe the following:

-3 × 4 = -12

-3 × 3 = -9 = -12 – (-3)

-3 × 2 = -6 = -9 – (-3)

-3 × 1 = -3 = -6 – (-3)

-3 × 0 = 0 = -3 – (-3)

-3 × -1 = 3 = 0 – (-3)

-3 × -2 = 6 = 3 – (-3)

Similarly, complete the following:

(i) -3 × -3 = ……………. (ii) -3 × -4 = …………….

Fill in the blanks in similar manner

-5 × 3 = -15

-5 × 2 = -10 = -15 – (-5)

-5 × 1 = -5 = -10 – (-5)

-5 × 0 = 0 = …………..

-5 × -1 = ………… = …………

-5 × -2 = ………… = …………

-5 × -3 = ………… = …………

Observing this pattern we can say that the product of two negative integers is a positive integer. We multiply two negative integers by considering them whole numbers and put (+) sign before the product value.

For example: (-10) × (-14) = 140, (-5) × (-6) = 30

In general, for two positive integers **'a'** and **'b'**

$$(-a) \times (-b) = a \times b$$

Do and learn :

Find the following products:

(i) (-12) × (-15) (ii) (-25) × (-4) (iii) (-17) × (-11)

1.3.3 Product of Three or More Negative Integers

We have seen that the product of two negative integers is a positive integer. What will be the product of three or more negative integers? Let us see following examples:

(i) $(-2) \times (-3) = 6$

(ii) $(-2) \times (-3) \times (-4) = [(-2) \times (-3)] \times (-4) = (6) \times (-4) = -24$

(iii) $(-2) \times (-3) \times (-4) \times (-5) = [(-2) \times (-3)] \times [(-4) \times (-5)] = 6 \times 20 = 120$

(iv) $(-2) \times (-3) \times (-4) \times (-5) \times (-6) = [(-2) \times (-3)] \times [(-4) \times (-5)] \times (-6)$
$= 6 \times 20 \times (-6) \quad = 120 \times (-6) = -720$

We conclude from the above examples that

(i) The product of two negative integers is a positive integer.

(ii) The product of three negative integers is a negative integer.

(iii) The product of four negative integers is a positive integer.

(iv) What is the product of five negative integers?

(v) In continuation, what is the product of six negative integers?

From above examples, we conclude that if the number of negative integers is even (2,4,6,…) then their product is positive integer and if the number of negative integers is odd (1,3,5,…) then the result is a negative integer.

Do and learn

Find the following products:

(i) $(-1) \times (-1) \times (-1) =$ ………..(ii) $(-1) \times (-1) \times (-1) \times (-1) =$ ………..

1.3.4 Multiplication by Zero

Observe the pattern given below and fill in the blanks:

$-4 \times 3 = -12$ $\quad$ $-4 \times 2 = -8 = -12 - (-4)$

$-4 \times 1 = -4 = -8 - (-4)$, $\quad$ $-4 \times 0 = 0 = -4 - (-4)$

We find that $-4 \times 0 = 0$. Make the pattern in similar manner and check.

Again $\quad 3 \times (-5) = -15$

$2 \times (-5) = -10 = -15 - (-5)$

$1 \times (-5) = -5 = -10 - (-5)$

$0 \times (-5) = 0 = -5 - (-5)$

From above pattern, we can say that the product of any integer by zero gives zero.

In general, we can say that for any integer **a**, $\boxed{\mathbf{a \times 0 = 0 = 0 \times a}}$

1.3.5 Division of Integers

We know that the division is the inverse of multiplication. For example 4 × 5 = 20, 20 ÷ 4 = 5 or 20 ÷ 5 = 4. Therefore we can say that there exists a division statement for every multiplication statement.

Multiplication Statement	Corresponding Division Statement
3 × (-5) = (-15)	(-15) ÷ (3) = -5, (-15) ÷ (-5) = 3
(-3) × 4 = (-12)	(-12) ÷ (-3) = 4, (-12) ÷ 4 = -3
(-2) × (-7) = 14	14 ÷ (-7) = -2, ...
(-4) × 5 = (-20)	(-20) (-4) = 5, ...
5 × (-9) = -45	....
(-6) × 5 = ...	....
(+5) × (+2) = ...	....

Observe the division statements in the table and accordingly check the following statements and put the sign [√ or ×]:

(i) Negative Integer ÷ Positive Integer = Negative Integer ( )
(ii) Positive Integer ÷ Negative Integer = Negative Integer ( )
(iii) Positive Integer ÷ Positive Integer = Positive Integer ( )
(iv) Negative Integer ÷ Negative Integer = Positive Integer ( )

Division of integers is carried out in the same manner as in the case of whole numbers. The only point we need to take care whether the result is positive or negative.

In general, **a ÷ (-b) = (-a) ÷ (b)** (where **b** and -**b** are not zero).

Exercise 1.2

1. Find the product of following:
   (i) (-3) × 4 (ii) (-1) × 24
   (iii) (-30) × (-24) (iv) (-214) × 0
   (v) (-15) × (-7) × 6 (vi) (-5) × (-7) × (-4)
   (vii) (-3) × (-2) × (-1) × (-5)
2. Start with (-1) × 5 and make pattern to show that (-1) × (-1) = +1.
3. The rate of decrease in temperature in a fridge is 3°C per minute. A thing whose temperature is 25°C is placed in the fridge. After how much time the temperature of the thing will be -2°C.
4. In a game if a blue card is chosen then 2 balls are to be given and if a red card is chosen then 3 balls are given. Sheetal has 27 balls. 9 blue cards are chosen in a row in the game. Show that how many balls are with her.

5. Solve the following division problems:

(i) (-35) ÷ 7 (ii) 15 ÷ (-3)
(iii) -25 ÷ (-25) (iv) 25 ÷ (-1)
(v) 0 ÷ (-3) (vi) 15 ÷ [(-2) + 1]
(vii) [(-6) + 3] ÷ [(-2) + 1]

6. A shopkeeper earns a profit of Rs.1 by selling a pen and loses 50 Paisa by selling a pencil. Represent the profit and loss in terms of integers.

(i) There is a loss of Rs. 5 in a month. If he had sold 45 pen then find the number of pencils sold in that month.

(ii) There is no profit and no loss in the second month. If he had sold 70 pen the find the number of pencils sold.

7. Fill in the following table by multiplying the integers:

x	2	3	- 4	- 2	1
3					
-2					
-1					
4					
2					

8. If going up a 60 feet high multi-story building by a lift is represented by positive integers then

(i) How will we show the height of flat at 60 feet?

(ii) Represent the parking at 15 feet below in integers.

(iii) If lift goes up at a speed of 5 feet per second then it is represented by +5 and if travels in a opposite direction then what is the integer representing downward direction?

1.4 Properties of Multiplication of Integers

1.4.1 Closure under Multiplication

Integer -1	Integer -2	Product	Product an integer Yes/No
2	-3	-6	Integer
-3	4	-12	Integer
-2	-3		
5	4		
-5	3		

What do you see? Can you find any two integers whose product is not an integer?

No. Hence we can say that the product of two integers is again an integer, i.e., **Integers follow closure property under multiplication.**

1.4.2 Commutativity

We know that the product of whole numbers is commutative. Is the multiplication of integers also commutative?

Observe the following table and complete it:

Integer Pair	Product	Changing the order	Conclusion
5, -4	5 × (-4) = -20	-4 × 5 = -20	5 × -4 = -4 × 5
-10, 12	(-10) × 12 = ...	12 × (-10) = ...	
-3, -4	(-3) × (-4) = ...		
-5, -7		(-7) × (-5) = ...	
+8, -3	(+8) × (-3) = ...		

What do you see? You will find that product of integers does not depend on their order. Hence, **multiplication of integers is commutative.** In general, for any two integers

a × b = b × a

1.4.3 Multiplicative Identity

We know that the multiplicative identity for whole numbers is 1. Check for the integers.

(-3) × 1 = -3 1 × 5 = 5
(-4) × 1 = 1 × 8 =
1 × (-5) = 3 × 1 =
1 × (-6) = 7 × 1 =

This shows that 1 is multiplicative identity for integers. In general, for any integer

a × 1 = a= 1 × a

What happens if an integer is multiplied by -1? -3 × (-1) = 3
3 × -1 = -3
-6 × -1 = 6
-1 × 13 = -13
Is -1 multiplicative identity for integers?

1.4.4 Associative property for Multiplication

Take 3, -4, -2.

[3 × (-4)] × (-2)
First multiply 3 and -4 and then multiply the product by (-2).
= (-12) × (-2) = 24

Consider 3 × [(-4) × (-2)]
First multiply (-4) and (-2) and then multiply the product by 3.
= 3 × (+8) = 24

Hence, [3 × (-4)] × (-2) = 3 × [(-4) × (-2)].

Take similar sets of three integers and repeat the above activity. Is the product affected by different sets of integers? In general, for any three integers a, b and c

(a × b) × c = a × (b × c)

1.4.5 Distributive Property

We have seen distributive property for whole numbers

$$\mathbf{a \times (b + c) = a \times b + a \times c}$$

Let us check if it is true for integers.

(i) $(-7) \times [2 + (-5)]$ $= (-7) \times (-3) = +21$	$(-7) \times 2 + (-7) \times (-5)$ $= -14 + 35 = +21$
(ii) $(-4) \times [(-3) + (-7)]$ $= (-4) \times (-10) = 40$	$(-4) \times (-3) + (-4) \times (-7)$ $= 12 + 28 = 40$
(iii) $(-8) \times [(-2) + (-1)]$ $= (-8) \times (-3) = 24$	$(-8) \times (-2) + (-8) \times (-1)$ $= +16 + 8 = 24$

Can we say that the distributive property (distribution of multiplication over addition) is also true for integers? Yes. In general,

$$a \times (b + c) = a \times b + a \times c$$

1.4.6 Division Property of Integers

Complete the following table:

Statement	Conclusion	Statement	Conclusion
$(-8) \div (-2) = 4$	Result is an Integer.	$(-8) \div 4 =$	…………………
$(-2) \div (-8) = \frac{-2}{-8}$	Result is not an Integer.	$3 \div (-8) = \frac{3}{-8}$	…………………

What do you see? We see that the **integers are not closed under division operation.** It is not necessary that division of two integers result in an integer.

Commutativity : We know that the division of whole numbers is not commutative. Let us check for integers. From above table, we can observe.

$$(-8) \div (-2) \neq (-2) \div (-8)$$

Is $[(-6) \div 2]$ and $[2 \div (-6)]$ same?

Hence, we can say that **division is not commutative for integers**.

Exercise 1.3

1. Following are the properties of multiplication of integers and opposite to them are the examples. Match the correct pair.

(i)	$(-4) \times 5 = 5 \times (-4)$	(a) Associativity
(ii)	$(-4) \times [(-3) + (-2)] = (-4) \times (-3) + (-4) \times (-2)$	(b) Commutativity
(iii)	-4 (an integer), +7 (another integer), product $(-4) \times (+7) = (-28)$ (an integer)	(c) Distributive
(iv)	$(-4) \times [(-7) \times (5)] = [(-4) \times (-7)] \times (5)$	(d) Closure

2. Fill in the blanks keeping in view the properties of multiplication of integers:

(i) $26 \times (-48) = (-48) \times$ ……… Commutative

(ii) $(-6) \times [(-2) + (-1)] = (-6) \times (-2) + (-6)$ ……… Distributive property

(iii) $100 \times [(-4) \times (-52)] = [100 \times$ …….$] \times (-52)$ Associativity

3. Find the product by using appropriate property.

(i) $26 \times (-48) + (-48) \times (-56)$ (ii) $8 \times 78 \times (-125)$

(iii) $9 \times (50 - 2)$ (iv) 999×45

4. Identify True/False. Correct the false statements and write.

(i) Multiplication of integers is closed.

(ii) Division of integers is closed.

(iii) Division of integers is not commutative but multiplication is commutative.

(iv) Multiplication of integer is distributive over addition.

(v) Division of integers is distributive over subtraction.

We Learnt

1. Integers can be considered as huge collection of numbers which contains whole numbers and their negatives.

2. Sum of two positive integers is again a positive integer and sum of two negative integers is again a negative integer.

3. We have studied the properties satisfied by sum and difference.

(i) Addition and subtraction of integers is closed, i.e., **a + b** and **a – b** both are integers, where **'a'** and **'b'** are any integers.

(ii) Addition is commutative for integers, i.e., for all integers **'a'** and **'b'**, **a + b = b + a.**

(iii) Addition is associative for integers, i.e., for all integers **'a', 'b'**and **'c', (a + b) + c = (a + b) + c.**

(iv) Zero is the additive identity for integers. In addition of two integers of opposite sign we subtract their absolute values. Result will be positive if the absolute value of positive integer is more and result will be negative if the absolute value of negative integer is more.

4. We have also learnt the product of integers. We have seen that the product of a positive integer and a negative integer is a negative integer and the product of two negative integers is a positive integer.

5. Following are the properties of integer multiplication:

(I) Integers are closed under multiplication. If 'a' and 'b' are integers then $a \times b$ is also an integer.

(ii) Multiplication of integers is commutative. If 'a' and 'b' are integers then $a \times b = b \times a$ hold true.

(iii) Integer 1 is multiplicative identity, i.e., for any integer 'a', $a \times 1 = 1 \times a$

(iv) Multiplication of integers is associative, i.e., for any three integers $(a \times b) \times c = a \times (b \times c)$ holds true.

6. Integers exhibit distributive property for addition and multiplication, i.e., for any three integers a, b and c, $[a \times (b + c) = a \times b + a \times c]$ holds true.

7. Commutativity, associativity and distributive properties under addition and multiplication make our calculations easy.

8. We have also learnt the division of integers. We found that (a) result obtained by dividing a negative integer by a positive integer or a positive integer by a negative integer is negative. (b) result obtained by dividing a negative integer by a negative integer is positive.

9. For any integer 'a', we find that (i) $a \div 0$ is undefined and (ii) $a \div 1 = a$.