Introduction to Trigonometry

NOTES

As we know, the trigonometry is the branch of Mathematics in which we study about the relationship between angles and its sides. In this chapter, we will discuss about trigonometric ratios which are defined in a right-angled triangle.

Trigonometrical Ratios

In the given triangle ABC, $\angle B = 90^{\circ}$ and let angle C is θ .



Then the trigonometrical ratios are defined as follows:

$$\sin \theta = \frac{Perpendicular}{Hypotenuse} = \frac{AB}{AC}$$

$$\cos \theta = \frac{Base}{Hypotenuse} = \frac{BC}{AC}$$

$$\tan \theta = \frac{Perpendicular}{Hypotenuse} = \frac{AB}{BC}$$

$$\cot \theta = \frac{Base}{Perpendicular} = \frac{BC}{AB}$$

$$\sec \theta = \frac{Hypotenuse}{Base} = \frac{AC}{BC}$$

$$\csc \theta = \frac{Hypotenuse}{Perpendicular} = \frac{AC}{BC}$$

Relationship between T-ratios

$$\sin \theta = \frac{1}{\cos ec \ \theta}, \ \cos \ \theta = \frac{1}{\sec \ \theta} \ \text{and} \ \cot \ \theta = \frac{1}{\tan \ \theta}$$

From the above, we conclude that sine of an angle is reciprocal to the cosec of that angle and so on.

Trigonometric Ratios of Complementary Angles

- \succ sin $(90^\circ \theta) = \cos \theta$
- $\succ \cos (90^\circ \theta) = \sin \theta$
- \succ tan $(90^\circ \theta) = \cot \theta$
- \succ cot $(90^{\circ} \theta) = \tan \theta$

- \succ sec $(90^{\circ} \theta)$ = cosec θ
- \succ cosec $(90^\circ \theta) = \sec \theta$

Trigonometric Identities

The trigonometric ratios of an angle θ is said to be a trigonometric identity if it is satisfied for all values of θ . In this chapter we will learn about following identities.

- $\Rightarrow \sin^2\theta + \cos^2\theta = 1$
- \succ sec² $\theta = 1 + tan^2 \theta$
- \succ cosec² $\theta = 1 + \cot^2 \theta$

Angle→ Ratios ¬	0 °	30 °	45 °	60 °	90 °
$\sin heta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cos \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

T-ratios of Some standard Angles

> Example:

If $\csc\theta - \cot\theta = \mathbf{a}$, then the value of $\cot \theta$ is:

(a)
$$\frac{a-1}{2a}$$
 (b) $\frac{1-a^2}{2a}$
(c) $a^2 - 1$ (d) $\frac{a^2 - 1}{a^2 + 1}$

(e) None of these

Ans. (b)

Explanation: Here it is given that $cosec\theta - cot\theta = a$

We know that, $\csc^2\theta - \cot^2\theta = 1$

$$\Rightarrow \csc \theta + \cot \theta = \frac{1}{\csc \theta - \cot \theta}$$
$$\Rightarrow (\csc \theta + \cot \theta) = \frac{1}{a}$$
$$\Rightarrow \cot \theta = \frac{1 - a^2}{2a}$$

> Example:

The value of $\frac{\sin^2 60^\circ + \cos^2 30^\circ}{\sin^2 45^\circ}$ (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 3
(c) 3
(d) 1

(e) None of these

Ans. (c)

Explanation:
$$\frac{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}{\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{\frac{6}{4}}{\frac{1}{2}} = \frac{12}{4} = 3$$

> Example:

The value of $2(\cot^2 45^\circ + \sec^2 30^\circ) - 6(\tan^2 45^\circ - \csc^2 60^\circ)$ is:

(a)
$$\frac{4}{7}$$
 (b) $\frac{20}{3}$
(c) $\frac{3}{20}$ (d) $\frac{1}{3}$

(e) None of these.

Ans. (b)

Explanation: Given trigonometric expression is: $2(\cot^2 45^\circ + \sec^2 30^\circ) - 6(\tan^2 45^\circ - \cos ec^2 60^\circ)$

$$= 2\left[1 + \left(\frac{2}{\sqrt{3}}\right)^{2}\right] - 6\left[1 - \left(\frac{2}{\sqrt{3}}\right)^{2}\right] = 2\left[1 + \frac{4}{3}\right] - 6\left[1 - \frac{4}{3}\right] = \frac{20}{3}$$