15. PROBABILITY

1. Classical (A priori) Definition of Probability :

If an experiment results in a total of (m + n) outcomes which are equally likely and mutually exclusive with one another and if 'm' outcomes are favorable to an event 'A' while 'n' are unfavorable, then the probability of occurrence of the event 'A' = $P(A) = \frac{m}{m+n} = \frac{n(A)}{n(S)}$.

We say that odds in favour of 'A' are m : n, while odds against 'A' are n : m.

$$P(\bar{A}) = \frac{n}{m+n} = 1 - P(A)$$

2. Addition theorem of probability : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

De Morgan's Laws : (a) $(A \cup B)^c = A^c \cap B^c$ (b) $(A \cap B)^c = A^c \cup B^c$

Distributive Laws : (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(i) $P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(ii) P (at least two of A, B, C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 2P(A \cap B \cap C)$

(iii) P(exactly two of A, B, C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 3P(A \cap B \cap C)$

(iv) P(exactly one of A, B, C occur) =
$P(A) + P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

3. Conditional Probability : $P(A/B) = \frac{P(A \cap B)}{P(B)}$.

4. Binomial Probability Theorem

If an experiment is such that the probability of success or failure does not change with trials, then the probability of getting exactly r success in n trials of an experiment is ${}^nC_r\, p^r q^{n-r}$, where 'p' is the probability of a success and q is the probability of a failure. Note that p + q = 1.

5. Expectation :

If a value M_i is associated with a probability of p_i , then the expectation is given by $\sum p_i M_i$.

6. Total Probability Theorem : $P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)$

7. Bayes' Theorem :

If an event A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots, B_n$ and the probabilities $P(A/B_1), P(A/B_2) \ldots P(A/B_n)$ are known, then $P(B_i / A) = \frac{P(B_i) \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)}$

$B_1, B_2, B_3, \ldots, B_n$

$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \ldots \cup (A \cap B_n)$

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \ldots + P(A \cap B_n) = \sum_{i=1}^{n} P(A \cap B_i)$$

8. Binomial Probability Distribution :

(i) Mean of any probability distribution of a random variable is given by : $\mu = \frac{\Sigma p_i x_i}{\Sigma p_i} = \Sigma p_i x_i$

(ii) Variance of a random variable is given by, $\sigma^2 = \Sigma (x_i - \mu)^2 \cdot p_i = \Sigma p_i x_i^2 - \mu^2$