Chapter 11. **Linear Programming**

6 Marks Questions

1. A housewife wishes to mix together two kinds of food, X and Y in such a way that the mixture contains atleast 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of 1 kg of food is given below :

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

1 kg of food X costs ₹ 6 and 1 kg of food Y costs ₹ 10. Formulate the above problem as a linear programming problem and find the least cost of the mixture which will produce the diet graphically. What value will you like to attach with this problem? **Value Based Question; Delhi 2014C**

Let the quantity of food X be x kg and the quantity of food Y be y kg.

∴ The objective function is to minimise,

$$Z = 6x + 10y.$$

Subject to the constraints are

$$x + 2y \geq 10 \quad \text{...(i)}$$

$$2x + 2y \geq 12 \quad \text{...(ii)}$$

$$3x + y \geq 8 \quad \text{...(iii)}$$

$$x, y \geq 0 \quad \text{...(iv)}$$

On considering the constraints as equations, we get

$$x + 2y = 10 \quad \text{...(v)}$$

$$x + y = 6 \quad \text{...(vi)}$$

$$3x + y = 8 \quad \text{...(vii)}$$

and $$x = 0, y = 0 \quad \text{...(viii)}$$

Firstly, draw the graph of line $x + 2y = 10$ **(1)**

x	0	10
y	5	0

(1)

On putting (0, 0) in the inequality $x + 2y \geq 10$, we get

$0 + 2(0) \geq 10 \Rightarrow 0 \geq 10$ [which is false]

So, the half plane is away from the origin.

Secondly, draw the graph of line $2x + 2y = 12$

x	0	6
y	6	0

On putting (0,0) in the inequality
$2x + 2y \geq 12$, we get
$2(0) + 2(0) \geq 12 \Rightarrow 0 \geq 12$ [which is false]

So, the half plane is away from the origin.

Thirdly, draw the graph of line $3x + y = 8$

x	0	8/3
y	8	0

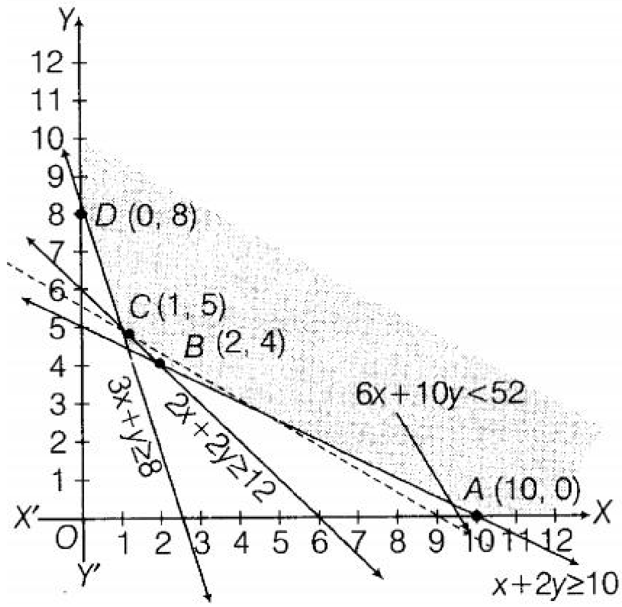
(1)

On putting (0, 0) in the inequality $3x + y \geq 8$, we get
$3(0) + 0 \geq 8 \Rightarrow 0 \geq 8$ [which is false]

So, the half plane is away from the origin.

The points of intersection of lines (v), (vi) and (vii) are $B(2, 4)$ and $C(1, 5)$

The graphical representation of these lines is given below :



(1)

The shaded region in the graph represents the feasible region which is unbounded and its extreme points are

$A(10, 0)$, $B(2, 4)$, $C(1, 5)$ and $D(0, 8)$

Now, the values of Z at extreme points are

Corner Points	Value of $Z = 6x + 10y$
$A(10, 0)$	$Z = 6(10) + 10(0) = 60$
$B(2, 4)$	$Z = 6(2) + 10(4) = 52$ (minimum)
$C(1, 5)$	$Z = 6(1) + 10(5) = 56$
$D(0, 8)$	$Z = 6(0) + 10(8) = 80$

(1)

As the feasible region is unbounded, therefore 52 may or may not be the minimum value of Z. For this, we draw a dotted graph of the inequality $6x + 10y < 52$ or $3x + 5y < 26$ and check, whether the resulting half plane has point in common with the feasible region or not. It can be seen that the feasible region has no common point with $3x + 5y < 26$.

Therefore, the minimum value of Z is 52 at $B(2, 4)$

Hence, the mixture should contain 2 kg of food X and 4 kg of food Y. The minimum cost of the mixture is ₹ 52.

This problum attaches the value of taking a healthy and well-balanced diet which has all the important vitamins in the right proportion.

2. If a young man rides his motor-cycle at 25 km per hour, he had to spend of 2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per hour, the petrol cost increases to ₹ 5 per km and rate of pollution also increases. He has ₹ 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as an L.PP Solve it graphically to find the distance to be covered with different speeds. What value is indicated in this question?

Value Baced Question; Delhi 2014C, 2013C

Let the young man covers x km at the speed of 25 km/h and y km at the speed of 40 km/h.

The total distance travelled is $x + y$, which we have to maximise under the certain constraints.

Here, objective function is $\max (Z) = x + y$

Cost constraints

Given, the cost of 1 km at the speed of 25 km/h = ₹2

∴ The cost of x km at the speed of 25 km/h

$$= 2x$$

Also, the cost of 1 km at the speed of 40 km/h = ₹5

∴ The cost of y km at the speed of 40 km/h = $5y$

So, the total cost of travel $(x + y)$ km = $2x + 5y$

Given, the driver has ₹100 to spend.

Hence, $2x + 5y \leq 100$ **(1)**

Time constraints

Total available time = 1 h

Time to travel a distance of 25 km = 1 h

∴ Time to travel a distance of x km = $\frac{x}{25}$ h

Also, time to travel a distance of 40 km = 1h

Time to travel a distance of y km $= \frac{y}{40}$ h

$\therefore$ The inequation represents time constraint is

$$\frac{x}{25} + \frac{y}{40} \leq 1$$

$\Rightarrow$ $$8x + 5y \leq 200 \qquad \textbf{(1)}$$

Thus, the LPP formed here objective function

$$\max(Z) = x + y$$

Subject to the constraints

$$2x + 5y \leq 100$$

$$8x + 5y \leq 200$$

and $$x, y \geq 0$$

Consider the inequalities as equations, we get

$$2x + 5y = 100 \qquad \text{...(i)}$$

$$8x + 5y = 200 \qquad \text{...(ii)}$$

and $$x, y = 0 \qquad \text{...(iii)}$$

Firstly, draw the graph of the line $2x + 5y = 100$.

x	0	50
y	20	0

Eq. (i) passes through the points (0, 20) and (50, 0).

On putting (0,0) in the inequality $2x + 5y \leq 100$, we get

$$2(0) + 5(0) \leq 100$$

$\Rightarrow$ $$0 \leq 100 \qquad \text{(true)}$$

So, the half plane is towards the origin.

Secondly, draw the graph of the line $8x + 5y = 200$.

x	0	25
y	40	0

Eq.(ii) passes through the points (0, 40) and (25, 0)

On putting (0,0) in the inequality

$8x + 5y \leq 200$, we get

$$8(0) + 5(0) \leq 200$$

$\Rightarrow \quad 0 \leq 200 \quad$ (true)

So, the half plane is towards the origin. **(1)**

Since, $x, y \geq 0$, so the feasible region lies in the first quadrant.

For determining the intersection point, on subtract Eq. (i) from Eq. (ii), we get

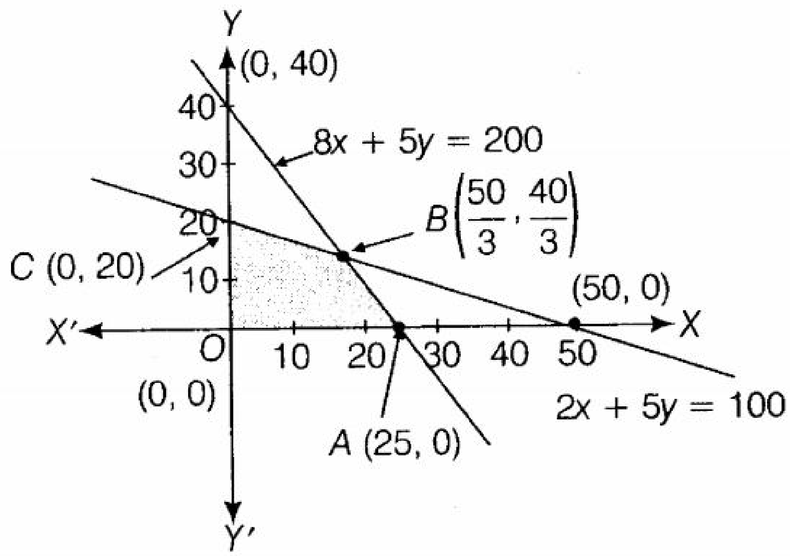
$$\begin{array}{r} 2x + 5y = 100 \\ \underset{-}{8x} \underset{-}{+} 5y = \underset{-}{200} \\ \hline x = \frac{50}{3} \end{array}$$

On putting $x = \frac{50}{3}$ in Eq. (i), we get

$$2 \times \left(\frac{50}{3}\right) + 5y = 100 \quad \Rightarrow \quad y = \frac{40}{3}$$

So, the point of intersection of Eqs. (i) and (ii) is $B\left(\frac{50}{3}, \frac{40}{3}\right)$.

Now, we draw all the lines on a graph paper and we get the feasible region *OABCO* which is bounded.



(1)

The corner points of the feasible region *OABC* are *O* (0,0), *A*(25,0), $B\left(\frac{50}{3}, \frac{40}{3}\right)$ and *C* (0,20).

Corner points	Value of $Z = x + y$
$O(0, 0)$	$Z = 0 + 0 = 0$
$A(25, 0)$	$Z = 25 + 0 = 25$
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	$Z = \frac{50}{3} + \frac{40}{3} = 30$ (maximum)
$C(0, 20)$	$Z = 0 + 20 = 20$

From the table, the maximum value of z is 30 at point $B\left(\frac{50}{3}, \frac{40}{3}\right)$. (1)

Hence, young man cover $\frac{50}{3}$ km at the speed of 25 km and $\frac{40}{3}$ km at the speed of 210km/h.

Also, he can travel maximum 130 km in 1 h.

Here, we find that on increasing the speed of motor-cycle, air pollution and expenditure on petrol are also increases. **(1)**

NOTE While plotting the graph, please be careful about the inequalities in which direction we have to plot

3. One kind of cake requires 200 g of flour and 25 g of fat, another kind of cake requires 100g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it as an LPP and solve it graphically. **All India 2014C, 2011C**

We can write the given data in tabular form as follows:

Cake	Flour	Fat
I kind	200 g	25 g
II kind	100 g	50 g
Total amount	5 kg or 5000g	1 kg or 1000g

(1)

Suppose the number of cakes of I kind be x and II kind be y.

Then, required LPP is max $(Z) = x + y$

Subject to the constraints

$$200x + 100y \le 5000 \Rightarrow 2x + y \le 50 \quad \text{...(i)}$$

[dividing both sides by 100]

$$25x + 50y \le 1000 \Rightarrow x + 2y \le 40 \quad \text{...(ii)}$$

[dividing both sides by 25]

and $x \ge 0, y \ge 0$ **(1)**

Let us consider the inequalities as equations, we get

$$2x + y = 50 \quad \text{...(iii)}$$

and $x + 2y = 40$ ...(iv)

Table for line $2x + y = 50$ is

x	25	0
y	0	50

So, it passes through the points (25, 0) and (0, 50).

Put (0,0) in $2x + y \le 50$, we get $0 \le 50$ (true)

So, the half plane is towards the origin.

Table for line $x + 2y = 40$ is

x	40	0
y	0	20

So, it passes through the points (40, 0) and (0, 20). **(1)**

Put (0,0) in $x + 2y \le 40$, we get

$0 \le 40$ (true)

So, the half plane is towards the origin.

Now, we solve Eqs.(iii) and (iv) to find the point of intersection.

On multiplying Eq.(iv) by 2 and then subtracting Eq. (iv) from Eq. (iii), we get

$$\begin{array}{r} 2x + y = 50 \\ 2x + 4y = 80 \\ - \quad - \quad\quad - \\ \hline -3y = -30 \end{array}$$

$\Rightarrow \qquad y = 10$

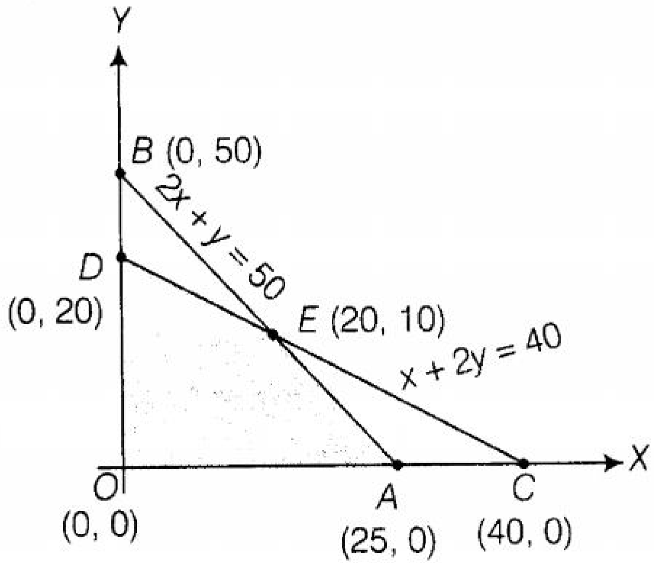
On putting $y = 10$ in Eq.(iv), we get

$$x + 20 = 40 \quad \Rightarrow \quad x = 20$$

So, the intersection point of lines is (20, 10).

(1/2)

Graph of above LPP is follows: **(1)**



From the graph, *OADE* is the feasible region.

The corner points of feasible region are $O(0, 0)$, $A(25, 0)$, $D(0, 20)$ and $E(20, 10)$, respectively.

Now, we evaluate Z at the corner points.

Corner points	Value of $Z = x + y$
$O(0, 0)$	$Z = 0 + 0 = 0$
$A(25, 0)$	$Z = 25 + 0 = 25$
$D(0, 20)$	$Z = 0 + 20 = 20$
$E(20, 10)$	$Z = 20 + 10 = 30$ (maximum)

(1)

From the table maximum number of cakes

$= 30$

Hence, 20 cakes of Ist kind and 10 cakes of IInd kind be prepared. **(1/2)**

4. A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹ 5760 to invest and has space for atmost 20 items for storage. An electronic sewing machine cost his ₹ 360 and a manually operated sewing machine ₹ 240. He can sell an electronic sewing machine at a profit of ₹ 22 and a manually operated sewing machine at a profit of ₹ 18. Assuming that he can sell all the items that he can buy, how should he invest his money in oder to maximise his profit? Make it as an LPP and solve it graphically. **Delhi 2014, 2009C; All India 2009**

Let the dealer purchased x electronic sewing machines and y manually operated sewing machines.

Now, we can construct the following table

Type of sewing machine	Number	Investment (in ₹)	Profit (in ₹)
Electronic	x	$360x$	$22x$
Manually	y	$240y$	$18y$
Total	$x+y$	$360x+240y$	$22x+18y$
Availability	20	5760	

(1)

Then, given LPP is

maximise, $Z = 22x + 18y$ ...(i)

Subject to constraints

$$x + y \leq 20 \quad \text{...(ii)}$$

$$360x + 240y \leq 5760$$

or $$3x + 2y \leq 48 \quad \text{...(iii)}$$

$$x \geq 0, y \geq 0 \quad \text{...(iv) (1)}$$

Firstly, draw the graph of the line,

$$x + y = 20 \quad \text{...(v)}$$

x	0	20

y	20	0

On putting (0, 0) in the inequality $x + y \leq 20$, we get

$0 + 0 \leq 20 \quad \Rightarrow \quad 0 \leq 20$, which is true.

So, the half plane is towards the origin.

Secondly, draw the graph of the line,

$$3x + 2y = 48 \qquad \text{...(vi)}$$

x	0	16
y	24	0

(1)

On putting (0, 0) in the inequality $3x + 2y \leq 48$, we get $0 \leq 48$, which is true.

So, the half plane is towards the origin.

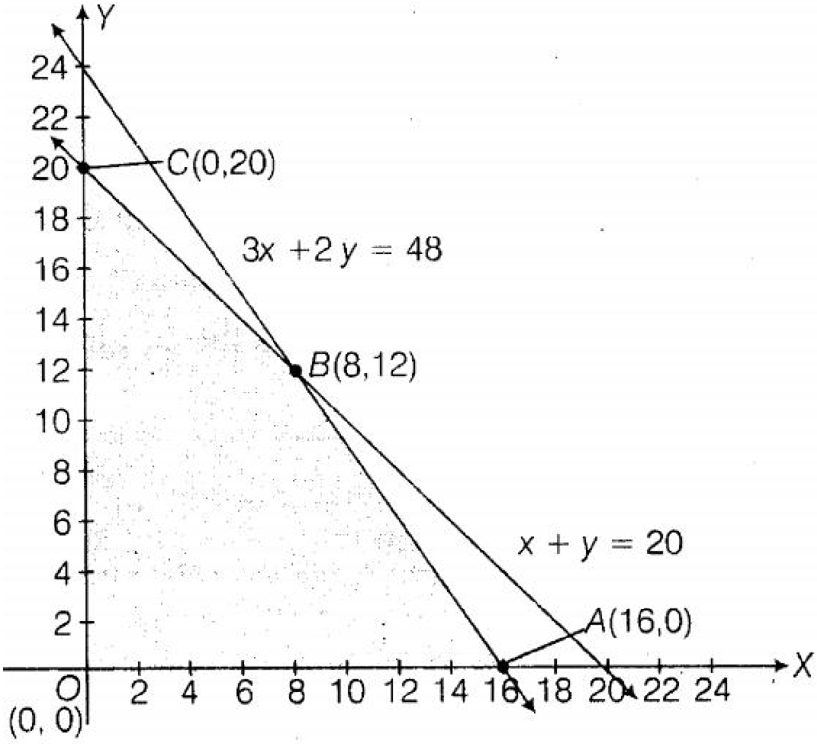
On solving Eqs. (v) and (vi), we get

$$x = 8 \text{ and } y = 12$$

So, the point of intersection of the lines (v) and (vi) is $B(8, 12)$.

Since, $x \geq 0$ and $y \geq 0$, so the feasible region lies in the first quadrant.

Graphical representation of the Eqs. is given below :



From the graph, feasible region is $OABCO$.

The corner points of the feasible region are $O(0, 0)$, $A(16, 0)$, $B(8, 12)$ and $C(0, 20)$.

The value of Z at these points are as follows

Corner Points	$Z = 22x + 18y$
$O(0, 0)$	$Z = 22(0) + 18(0) = 0$
$A(16, 0)$	$Z = 22 \times 16 + 0 = 352$
$B(8, 12)$	$Z = 22 \times 8 + 18 \times 12 = 392$ (maximum)
$C(0, 20)$	$Z = 22 \times 0 + 18 \times 20 = 360$

(1)

From the table, maximum value of $Z =$ ₹ 392 at point $B(8, 12)$.

Hence, dealer should purchased 8 electronic and 12 manually operated sewing machines to get maximum profit. **(1)**

5. A manufacturing company makes two types of teaching aids *A* and *B* of Mathematics for class XII. Each type of *A* requires 9 labour hours of fabricating and 1 labour hour for finishing. Each type of *B* requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30, respectively. The company makes a profit of ₹ 80 on each piece of type *A* and ₹ 120 on each piece of type *B*. How many pieces of type *A* and type *B* should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week?

All India 2014

Let the number of pieces of two types of teaching aids *A* and *B* be *x* and *y*, respectively.

We can write the given data in tabular form as follows:

Item	Number of pieces	Time on fabricating (in hr)	Time on finishing (in hr)	Profit (in ₹)
A	x	$9x$	x	$80x$
B	y	$12y$	$3y$	$120y$
Total	$x+y$	$9x+12y$	$x+3y$	$80x+120y$
Availability		180	30	

The profit on type *A* is ₹ 80 and type *B* is ₹ 120.

Thus, the required LPP is

$$\text{Maximise, } Z = 80x + 120y \quad \text{...(i)}$$

Subject to constraints

$$9x + 12y \leq 180 \quad \text{...(ii)}$$

$$x + 3y \leq 30 \quad \text{...(iii)}$$

$$x \geq 0, y \geq 0 \quad \text{...(iv)}$$

Let us consider the inequalities as equations, we get

$$9x + 12y = 180$$

$$x + 3y = 30$$

Now, table for line

$$9x + 12y = 180$$

or $$3x + 4y = 60 \quad \text{...(v)}$$

x	0	20
y	15	0

(1)

On putting (0, 0) in the inequality $9x + 12y \leq 180$, we get

$$9(0) + 12(0) \leq 180$$

$\Rightarrow$ $0 \leq 180$ [which is true]

So, the half plane is towards the origin.

Table for line $x + 3y = 30$ ...(vi)

x	0	30
y	10	0

(1)

On putting (0, 0) in the inequality $x + 3y \leq 30$, we get $0 + 3(0) \leq 30$

$\Rightarrow$ $0 \leq 30$ [which is true]

So, the half plane is towards the origin.

Also, $x \geq 0$ and $y \geq 0$

Thus, the feasible region lies in the first quadrant.

On multiplying Eq. (vi) by 3 and then subtracting from Eq. (v), we get

$$3x + 4y = 60$$

$$3x + 9y = 90$$

$$- \quad - \quad -$$

$$-5y = -30$$

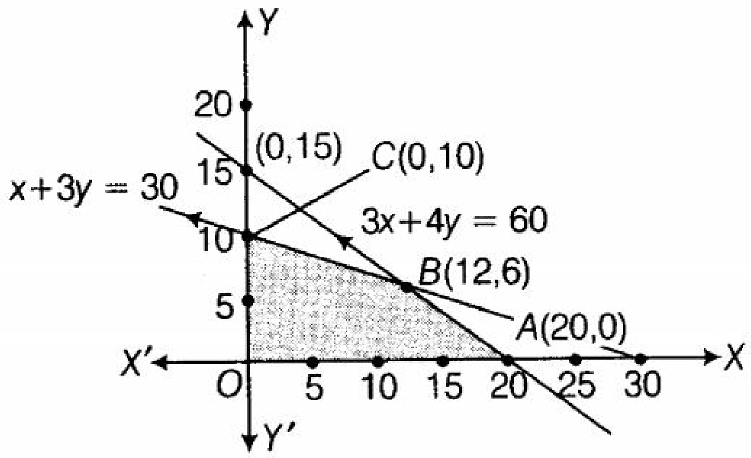
$\Rightarrow$ $y = 6$ **(1)**

From Eq. (vi), we get

$$x = 30 - 18 = 12$$

So, the point of intersection is (12, 6).

The graphical representation of the lines is given below :



From graph, feasible region is $OABC$, whose corner points are $O(0, 0)$, $A(20, 0)$, $B(12, 6)$ and $C(0, 10)$.

Corner points	**Value of $Z = 80x + 120y$**
$O(0, 0)$	$Z = 80(0) + 120(0) = 0$
$A(20, 0)$	$Z = 80(20) + 120(0) = 1600$
$B(12, 6)$	$Z = 80(12) + 120(6) = 1680$ (maximum)
$C(0, 10)$	$Z = 80(0) + 120(10) = 1200$

From the table, the maximum value of Z is ₹ 1680.

Also, 12 pieces of type A and 6 pieces of type B should be manufactured per week to get a maximum profit.

Hence, manufacturer earn maximum profit ₹ 1680 per week.

6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 h on the grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 h on the grinding/cutting machine and 2 h on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 h and the grinding/cutting machine for at most 12 h. The profit from the sale of a lamp is ₹ 25 and that from a shade is ₹ 15. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit? Formulate an LPP and solve it graphically.

Foreign 2014

Let the cottage industry manufacture x pedestal lamps and y wooden shades.

Therefore, $x \geq 0$ and $y \geq 0$.

The given information can be written in table form as given below :

	Pedestal lamps (x)	Wooden shades (y)	Time available
Grinding/ Cutting	2	1	≤ 12
Sprays	3	2	≤ 20
Profit	25	15	

Then, the required LPP is

$$\text{Maximise, } Z = 25x + 15y$$

Subject to constraints

$$2x + y \leq 12$$

$$3x + 2y \leq 20$$

$x \geq 0, y \geq 0$ [non-negative constraints]

Let us consider the inequalities as equations, we get

$$2x + y = 12 \quad \text{...(i)}$$

$$3x + 2y = 20 \quad \text{...(ii)}$$

Table for $2x + y = 12$

Table for 2x + y = 12

x	0	6
y	12	0

On putting (0,0) in inequality $2x + y \le 12$.
we get $2(0) + 0 \le 12 \Rightarrow 0 < 12$ (which is true)
So, the half plane is towards the origin.
Table for $3x + 2y = 20$

x	0	6.6
y	10	0

(1)

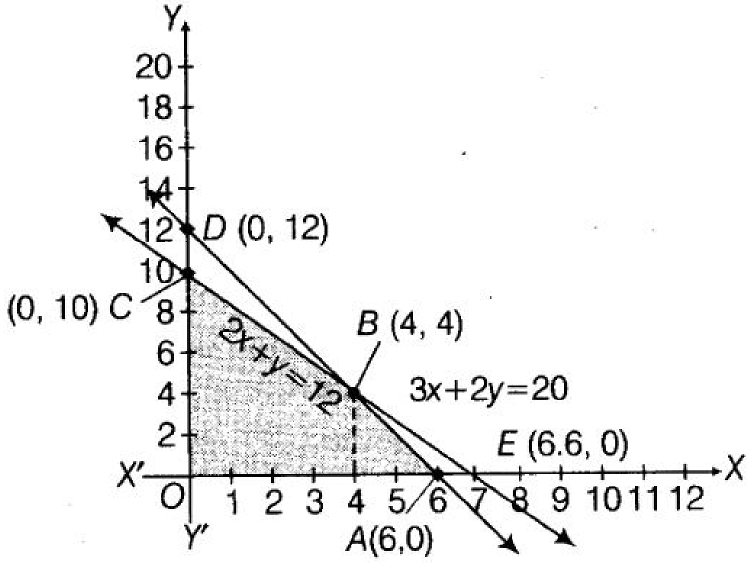
On putting (0,0) in inequality $3x + 2y \le 20$,
we get

$$3(0) + 2(0) \le 20 \quad \text{(which is true)}$$

So, the half plane is towards the origin.
Also, $x \ge 0,\ y \ge 0$
Thus, the feasible region lies in first quadrant.
On solving Eqs. (i) and (iii), we get

$$x = 4 \quad \text{and} \quad y = 4$$

So, the intersection point is (4,4). **(1)**
The graphical representation of the lines is given below:



From graph, the feasible region is $OABC$, whose corner points are $O(0,0)$ A (6,0) B (4,4) and C(0,10).

Corner points	Value of $Z = 25x + 15y$
$O(0, 0)$	$Z = 25(0) + 15(0) = 0$
$A(6, 0)$	$Z = 25(6) + 15(0) = 150$

B(4, 4)	$Z = 25(4) + 15(4) = 160$ (maximum)
C(0, 10)	$Z = 25(0) + 15(10) = 150$

From table, maximum value of Z is 160 at $B(4, 4)$. Hence, the manufacturer should produce 4 pedestal lamps and 4 wooden shades daily to maximise his profit. **(1)**

7. A cooperative society of farmers has 50 hec of land to grow two crops *A* and *B*. The profits from crops *A* and *B* per hectare are estimated as ₹ 10500 and ₹ 9000, respectively. To control weeds, a liquid herbicide has to be used for crops *A* and *B* at the rate of 20 L per hec and 10 L per hec, respectively. Further, not more than 800 L of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land.
Keeping in mind that the protection of fish and other wildlife is more important than earning profit. How much land should be allocated to each crop so as to maximise the total profit? Formulate the above as an LPP and solve it graphically. Do you agree with the message that the protection of wildlife is atmost necessary to preserve the balance in environment?.**Value Based Question; Delhi 2013**

Let x hec for crop *A* and y hec for crop *B* be allocated.

Then given data can be written in tabular form as given below:

Item	*A*	*B*
50 hec	x	y
Profit	10500	9000
800 L atmost	20 L/hec	10 L/hec

(1)

According to the question,
we get the following LPP:

maximize, $Z = 10500x + 9000y$

Subject to constraints

$$x + y \leq 50$$

$$20x + 10y \leq 800 \quad \text{...(ii)}$$

or $$2x + y \leq 80$$

and $$x, y \geq 0 \quad \textbf{(1/2)}$$

On considering the inequalities as equations, we get

$$x + y = 50 \quad \text{...(iii)}$$

$$20x + 10y = 800 \text{ or } 2x + y = 80 \quad \text{.....(iv)}$$

Table for line $x + y = 50$

x	0	50
y	50	0

On putting (0,0) in the inequality $x + y \leq 50$, we get

$$0 + 0 \leq 50$$

$\Rightarrow$ $$0 \leq 50 \quad \text{(true)}$$

So, the half plane is towards the origin.

Table for line $2x + y = 80$

x	0	40
y	80	0

On putting (0,0) in the inequality $2x + y \leq 80$, we get

$$2(0) + 1(0) \leq 80$$

$\Rightarrow$ $$0 \leq 80 \quad \text{(true)}$$

So, the half plane is towards the origin.

For determining the intersection point, subtracting Eq. (iii) from Eq. (iv), we get

$$\begin{array}{r} x + y = 50 \\ 2x + y = 80 \\ - \quad - \quad - \\ \hline x = 30 \end{array}$$

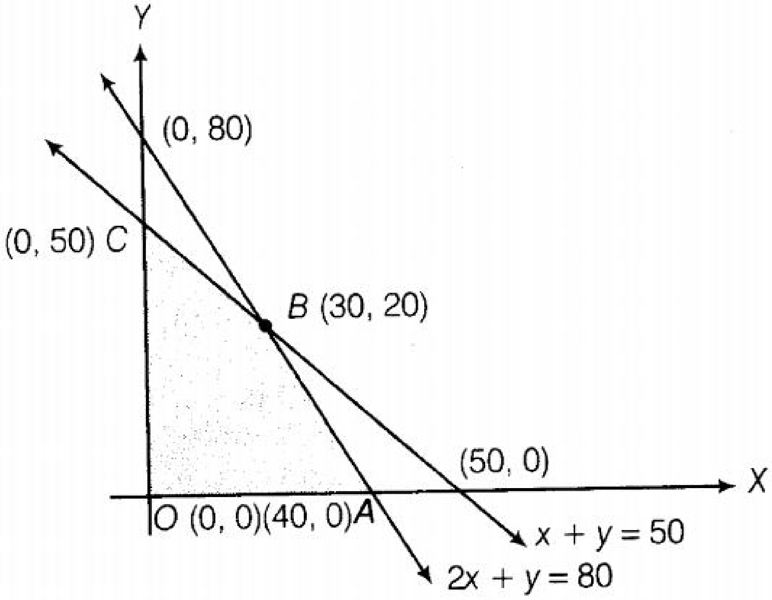
On putting $x = 30$ in Eq. (iii), we get

$$30 + y = 50$$

$\Rightarrow \qquad y = 20$

So, the intersection point is $B(30,20)$. **(1/2)**

Now, on plotting the points on graph paper, we get following graph:



(1)

From the graph, we observe that $OABC$ is the region which is bounded and extreme points are $O(0,0)$, $A(40, 0)$, $B(30,20)$ and $C(0, 50)$. **(1)**

Corner points	Value of $Z = 10500x + 9000y$
$O(0,0)$	$Z=0$
$A(40,0)$	$Z=420000$
$B(30,20)$	$Z=315000+180000$ $=495000$ (maximum)
$C(0,50)$	$Z=450000$

(1)

From table, maximum value of Z is 495000. Hence, for maximise the profit, the land allocated 30 hec for crop A and 20 hec for crop B.

Yes, I agree with the message that the protection of wildlife is atmost necessary to preserve the balance in environment. **(1)**

8. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods *A* and *B*. To produce one unit of *A*, 2 workers and 3 units of capital are required, while 3 workers and 1 unit of capital is required to produce one unit of *B*. If *A* and *B* are priced at ₹ 100 and ₹ 120 per unit respectively, then how should he use his resources to maximise the total revenue? Formulate the above as an LPP and solve it graphically.
Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate?.

Value Based Question; All India 2013

Let the manufacturer produces x units of goods A and y units of goods B.

Now, given data can be written in table form as given below:

	A	*B*	Required capacity
Workers	2	3	30
Capital	3	1	17
Price	100	120	

(1)

∴ The required LPP is

$$\max (Z) = 100x + 120y$$

Subject to constraints are

$$2x + 3y \leq 30 \quad \ldots(i)$$

$$3x + y \leq 17 \quad \ldots(ii)$$

and $x, y \geq 0$ **(1/2)**

Now, for solving the above LPP by graphical method, firstly we assume all the inequalities as equations. Then, we get

$$2x+3y=30 \quad \text{...(iii)}$$

and $$3x+y=17 \quad \text{...(iv)}$$

Table for line $2x + 3y = 30$ is

x	0	15
y	10	0

$\therefore$ Eq. (iii) passes through the points (0,10) and (15, 0)

On putting (0,0) in $2x + 3y \leq 30$, we get

$$0 \leq 30 \quad \text{(true)}$$

So, the half plane is towards the origin.

Table for line $3x + y = 17$ is

x	0	17/3
y	17	0

(1/2)

$\therefore$ Eq. (iv) passes through the points (0,17) and (17/3, 0).

On putting (0,0) in $3x + y \leq 17$, we get

$$0 \leq 17 \quad \text{(true)}$$

So, the half plane is towards the origin.

For determining the intersection point, multiplying Eq. (iv) by 3 and then subtracting from Eq. (iii), we get

$$2x+3y=30$$
$$\underset{-}{9x}+\underset{-}{3y}=\underset{-}{51}$$
$$-7x=-21$$

$\Rightarrow$ $$x=3$$

Then, from Eq. (iv), we get

$$y=17-3(3)$$
$$=17-9$$

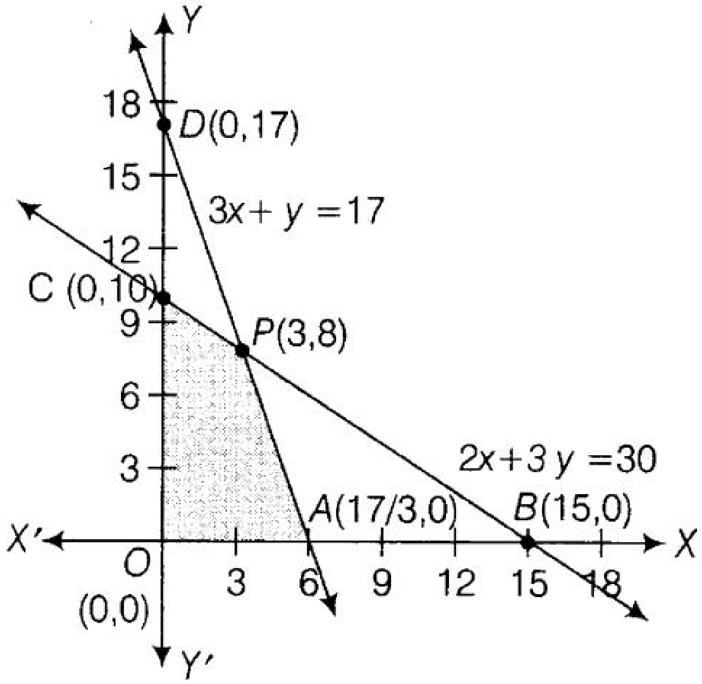
$\Rightarrow$ $$y=8$$

So, intersection point of both lines is P (3, 8).

(1)

Now, plotting these points on graph paper, we get the following graph:

get the following graph:



(1)

Here, the feasible region is *OAPCO*, whose extreme points are *O*, *A*, *P* and *C*.

Corner points	Value of $Z = 100x + 120y$
$O(0,0)$	$Z = 100 \times 0 + 120 \times 0 = 0 + 0 = 0$
$A\left(\frac{17}{3}, 0\right)$	$Z = 100 \times \frac{17}{3} + 120 \times 0$ $= \frac{1700}{3} = 566.66$
$C(0, 10)$	$Z = 100 \times 0 + 120 \times 10 = 1200$
$P(3, 8)$	$Z = 100 \times 3 + 120 \times 8$ $= 300 + 960 = 1260$ (maximum)

From table, maximum value of Z is 1260 at $P(3,8)$. **(1)**

Hence, a manufacturer will produce 3 units of goods *A* and 8 units of goods *B* to maximise the total revenue.

Yes, I agree with this view of the manufacturer because in our Indian constitution, according to right of equality clause, men and women to right of equality clause, men and women workers are equally efficient and so should be paid at the same rate. **(1)**

9. A manufacturer produces nuts and bolts. It takes 1 h of work on machine *A* and 3 h on machine *B* to produce package of nuts. It takes 3 h on machine *A* and 1 h on machine *B* to produce a package of bolts. He earns a profit of ₹ 17.50 per package on nuts and ₹ 7 per package on bolts. How many packages of each should be produced each day so as to maximise his profits, if he operates his machines for atmost 12 h a day? Formulate above as a Linear Programming Problem (LPP) and solve it graphically. **Delhi 2012, 2009C**

Let the manufacturer produces x nuts and y bolts. Then, given data can be written in tabular form as follows:

Item	Time on machine *A*	Time on machine *B*	Profit (in ₹)
Nuts (x)	1 h	3 h	17.50
Bolts (y)	3 h	1 h	7.00
	≤12 h	≤12 h	

(1)

∴ The required LPP is

Maximise profit, $Z = 17.50x + 7.00y$

Subject to constraints

$$x + 3y \le 12, \quad 3x + y \le 12$$

and $x, y \ge 0$ **(1)**

Let us consider the inequalities as equations, we get $x + 3y = 12$ ...(i)

and $3x + y = 12$ ...(ii)

Table for line $x + 3y = 12$ is

x	0	12
y	4	0

∴ Eq. (i) passes through the points (0, 4) and (12, 0).

On putting (0,0) in $x + 3y \leq 12$, we get

$$0 \leq 12 \quad \text{(true)}$$

So, the half plane is towards the origin.

Table for line $3x + y = 12$ is

x	0	4
y	12	0

∴ Eq. (ii) passes through the points (0, 12) and (4, 0).

On putting (0,0) in $3x + y \leq 12$, we get

$$0 \leq 12 \quad \text{(true)}$$

So, the half plane is towards the origin.

Now, on multiplying Eq. (i) by 3 and then subtracting Eq. (ii) from Eq. (i), we get

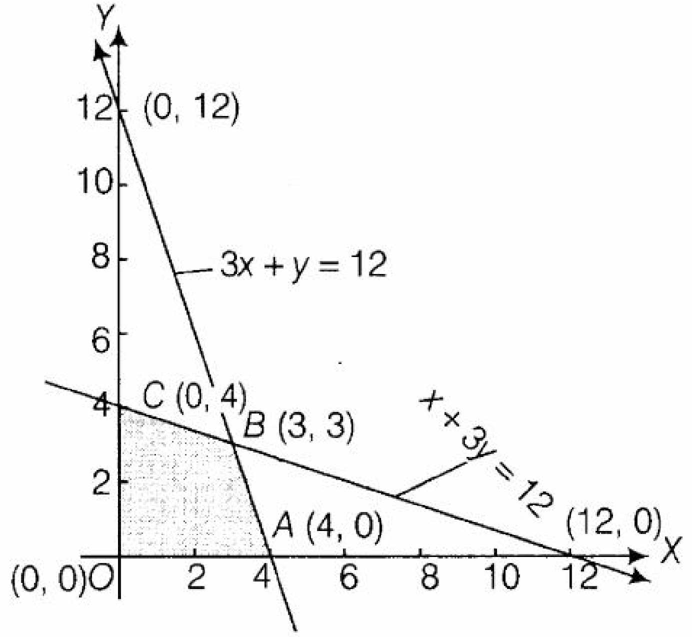
$$\begin{array}{r} 3x + 9y = 36 \\ \underline{\underset{-}{}3x \underset{-}{+} \ y = \underset{-}{}12} \\ 8y = 24 \end{array} \Rightarrow \quad y = 3$$

On putting $y = 3$ in Eq. (i), we get

$$x + 3(3) = 12 \quad \Rightarrow \quad x = 12 - 9 = 3$$

So, the point of intersection is (3, 3). **(1)**

Now, the graph of the system of inequalities is given as follows:



(1)

From the graph, we see that *OABC* is the feasible region.

The corner points of the feasible region are

$O(0, 0), C(0, 4), B(3, 3)$ and $A(4, 0)$ **(1)**

Now, evaluate profit Z at corner points.

Corner points	Value of $Z = 17.50\,x + 7.00\,y$
$O(0,0)$	$Z=17.50\,(0) + 7.00\,(0) = 0 + 0 = 0$
$A(4,0)$	$Z=17.50\,(4) + 7.00\,(0) = 70.00 + 0 = 70.00$
$B(3, 3)$	$Z=17.50\,(3) + 7.00\,(3) = 52.50 + 21.00$ $= 73.50$ (maximum)
$C(0, 4)$	$Z=17.50\,(0) + 7.00\,(4) = 0 + 28.00 = 28.00$

Hence, the profit is maximum, i.e. ₹ 73.50, when he produces 3 nuts and 3 bolts each day.

10. A dietician wishes to mix two types of foods in such a way that the vitamin contents of mixture contains atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C, while food II contains 1 unit per kg of vitamin A and 2 units per kg of vitamin C. It costs ₹ 5 per kg to purchase food I and ₹ 7 per kg to purchase food II. Find the minimum cost of such a mixture. Formulate above as an LPP and solve it graphically.

All India 2012

. The given data can be put in the tabular form as follows:

Food	Vitamin A	Vitamin C	Cost/Unit
I	2	1	₹ 5
II	1	2	₹ 7
Minimum requirement	atleast 8	atleast 10	

(1)

Suppose the diet contains x units of food I and y units of food II.

Then, the required LPP is min $(Z) = 5x + 7y$

Subject to constraints

$2x + y \geq 8,\ x + 2y \geq 10$ and $x \geq 0, y \geq 0$ **(1/2)**

On considering the inequalities as equations, we get

$$2x + y = 8 \quad \text{...(i)}$$

and

$$x + 2y = 10 \quad \text{...(ii)}$$

Table for line $2x + y = 8$ is

x	0	4
y	8	0

$\therefore$ The line $2x + y = 8$ passes through the points (0, 8) and (4, 0).

On putting (0,0) in $2x + y \geq 8$, we get

$$0 \geq 8 \quad \text{(false)}$$

So, the half plane is away from the origin.

Table for line $x + 2y = 10$ is

x	10	0
y	0	5

$\therefore$ The line $x + 2y = 10$ passes through the points (10, 0) and (0, 5).

On putting (0,0) in $x + 2y \geq 10$, we get

$$0 \geq 10 \qquad \text{(false)}$$

So, the half plane is away from the origin. **(1)**

On multiplying Eq. (i) by 2 and subtracting Eq. (ii) from Eq. (i), we get

$$4x + 2y = 16$$
$$\underline{\underset{-}{}x \underset{-}{+} 2y \underset{-}{=} 10}$$
$$3x = 6$$

$\Rightarrow$ $$x = 2$$

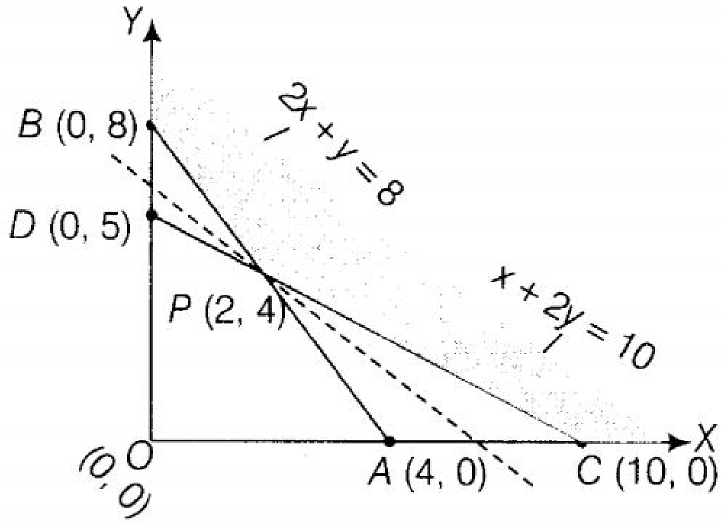
On putting $x = 2$ in Eq. (i), we get

$$2(2) + y = 8$$

$\Rightarrow$ $$y = 8 - 4 = 4$$

So, these lines intersect at P (2, 4). **(1½)**

Now, the graph of above LPP is as follows:



From the graph, the feasible region is BPC which is unbounded. **(1)**

Now, the table with corner point and value of Z is as follows:

Z is as follows.

Corner points	Value of $Z = 5x + 7y$
$C(10, 0)$	$Z = 5(10) + 7(0) = 50$
$P(2, 4)$	$Z = 5(2) + 7(4) = 10 + 28 = 38$ (minimum)
$B(0, 8)$	$Z = 5(0) + 7(8) = 0 + 56 = 56$

From table, the minimum value of Z is 38.

As the feasible region is unbounded, therefore 38 may or may not be the minimum value of Z. For this, we draw a dotted graph of the inequality $5x + 7y < 38$ and check whether the resulting half plane has point in common with the feasible region or not.

It can be seen that the feasible region has no common point with $5x + 7y < 38$.

Hence, the minimum cost is ₹ 38, when $x = 2$ and $y = 4$. **(1)**

11. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 h of machine time and 3 h of craftman's time in its making, while a cricket bat takes 3 h of machine time and 1 h of craftman's time. In a day, the factory has the availability of not more than 42 h of machine time and 24 h of craftman's time. If the profits on a racket and a bat are ₹ 20 and ₹ 10 respectively, then find the number of tennis rackets and cricket bats that the factory must manufacture to earn the maximum profit. Make it as an LPP and solve it graphically. **Delhi 2011**

- Let x be the number of tennis rackets and y be the cricket bats produced in one day in the factory.

  Given data can be written in tabular form as follows:

Item	**Number**	**Machine hours**	**Craftman's hours**	**Profit**
Tennis rackets	x	1.5	3	₹ 20
Cricket bats	y	3	1	₹ 10
Total		atmost 42	atmost 24	

(1)

According to the above table, the required LPP is

$$\max (Z) = 20x + 10y$$

Subject to constraints

$$1.5x + 3y \leq 42$$

$$3x + y \leq 24$$

and $\quad x \geq 0, y \geq 0$ **(1)**

Let us consider the inequalities as equations, we get

$$1.5x + 3y = 42 \quad \text{...(i)}$$

and $\quad 3x + y = 24 \quad$ ...(ii)

Table for line $1.5x + 3y = 42$ is

x	0	28
y	14	0

So, this line passes through the points (0, 14) and (28, 0).

On putting (0,0) in $1.5x + 3y \leq 42$, we get

$$0 \leq 42 \quad \text{(true)}$$

So, the half plane is towards the origin.

Table for line $3x + y = 24$ is

x	0	8
y	24	0

So, this line passes through the points (0, 24) and (8, 0). **(1)**

On putting (0,0) in $3x + y \leq 24$, we get

$$0 \leq 24 \quad \text{(true)}$$

So, the half plane is towards the origin.

On multiplying Eq. (i) by 2 and then subtracting Eq. (ii) from Eq (i), we get

$$\begin{array}{r} 3x + 6y = 84 \\ \underline{-3x \pm \ y = -24} \\ 5y = 60 \Rightarrow y = \frac{60}{5} = 12 \end{array}$$

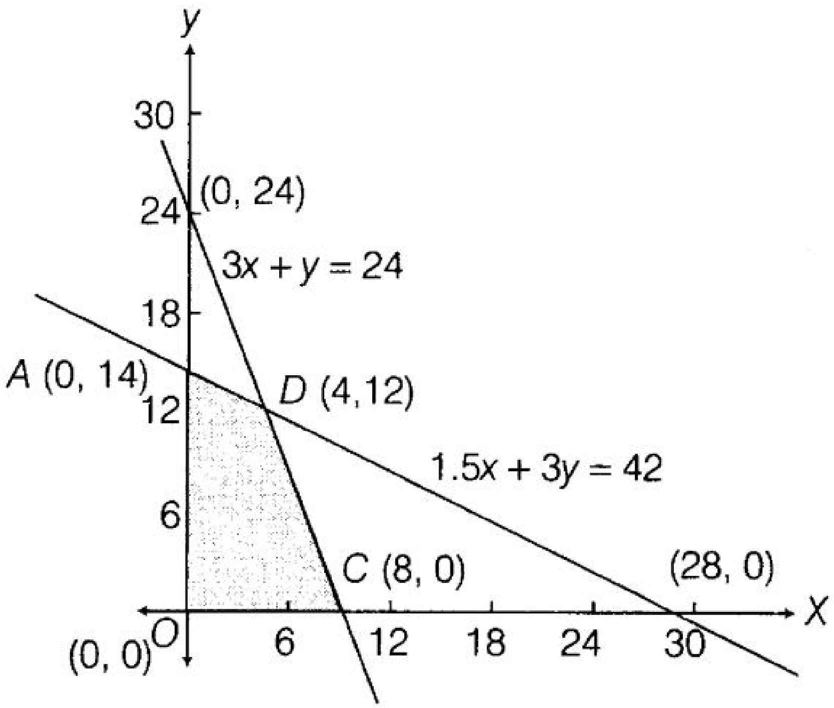
On putting the value of y in Eq. (ii), we get

$$3x + 12 = 24$$

$$\Rightarrow \quad 3x = 24 - 12 = 12 \quad \Rightarrow \quad x = 4$$

So, the intersection point is (4, 12). **(1)**

Now, plotting these points on graph paper, we get the following graph.



(1)

From the graph, the region $OADC$ is the feasible region, whose corner points are O (0, 0), A (0, 14), C (8, 0) and D (4, 12).

Now, evaluate function Z at corner points.

From table, maximum value is 200 at $D(4,12)$. Hence, for maximum profit of ₹ 200, 4 tennis rackets and 12 cricket bats must be produced. **(1)**

12. A merchant plans to sell two types of personal computers, a desktop model and a portable model that will cost ₹ 25000 and ₹ 40000, respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit, if he does not want to invest more than ₹ 70 lakh and his profit on the desktop model is ₹ 4500 and on the portable model is ₹ 5000. Make an LPP and solve it graphically. **All India 2011**

Let the merchant stocks x desktop computers and y portable computers. We construct the following table according to given data.

Type	Number	Cost per computer	Profit
Desktop	x	₹ 25000	₹ 4500 x
Portable	y	₹ 40000	₹ 5000 y
Total	at most 250	atmost 7000000	

(1)

Then, the required LPP is Maximise,

$$Z = 4500x + 5000y \quad \text{...(i)}$$

Subject to constraints

$$x + y \leq 250 \quad \text{...(ii)}$$

$$25000x + 40000y \leq 7000000$$

$$\Rightarrow \quad 5x + 8y \leq 1400 \quad \text{...(iii)}$$

[dividing both sides by 5000]

and $\quad x \geq 0, y \geq 0 \quad$ ...(iv) **(1)**

On considering the inequalities as equations, we get

$$x + y = 250 \quad \text{...(v)}$$

and $\quad 5x + 8y = 1400 \quad$ ...(vi)

Table for line $x + y = 250$ is

Table for line $x + y = 250$ is

x	0	250
y	250	0

So, this line passes through the points (0, 250) and (250, 0).

On putting (0,0) in $x + y \leq 250$, we get

$$0 \leq 250 \qquad \text{(true)}$$

So, the half plane is towards the origin.

Table for line $5x + 8y = 1400$ is

x	280	0
y	0	175

So, this line passes through the points (280, 0) and (0, 175). **(1)**

On putting (0,0) in $5x + 8y \leq 1400$, we get

$$\Rightarrow \qquad 0 \leq 1400 \qquad \text{(true)}$$

So, the half plane is towards the origin.

Now, on multiplying Eq. (v) by 5 and then subtracting Eq. (vi) from Eq. (v), we get

$$\begin{aligned} 5x + 5y &= 1250 \\ 5x + 8y &= 1400 \\ - \quad - \quad &\quad - \\ \hline -3y &= -150 \end{aligned}$$

$$\Rightarrow \qquad y = 50$$

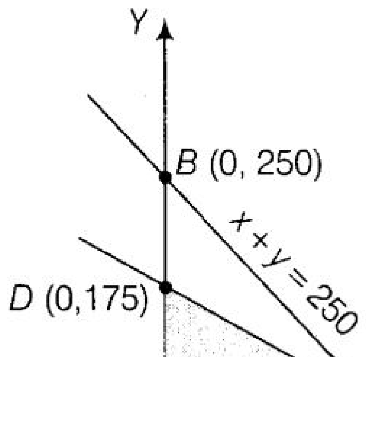
On putting $y = 50$ in Eq.(v), we get

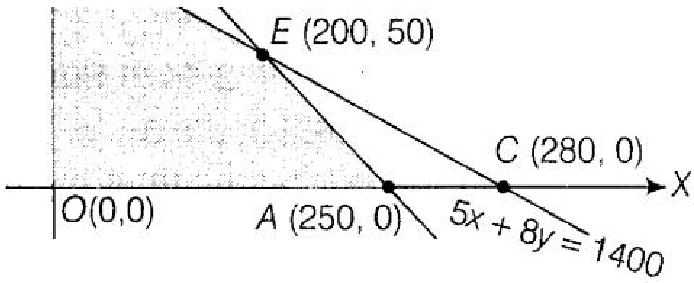
$$x + 50 = 250$$

$$\Rightarrow \qquad x = 200$$

So, the intersection point is (200, 50). **(1)**

Graph of above LPP is given below :





(1)

From the graph, *AODE* is the feasible region.
The corner points of the feasible region are $O(0, 0)$, $A(250, 0)$, $E(200, 50)$ and $D(0, 175)$.

Corner points	Value of $Z = 4500x + 5000y$
$O(0,0)$	$Z = 4500(0) + 5000(0) = 0$
$A(250,0)$	$Z = 4500(250) + 5000(0) = 1125000$
$E(200,50)$	$Z = 4500(200) + 5000(50) = 1150000$ (maximum)
$D(0,175)$	$Z = 4500(0) + 5000(175) = 875000$

From the table, maximum value of Z is 1150000 at $E(200,50)$
Hence, the profit is maximum, i.e. ₹ 1150000, when 200 desktop computers and 50 portable computers are stocked. **(1)**

13. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of grinding/cutting machine and a sprayer. It takes 2 h on the grinding/cutting machine and 3 h on the sprayer to manufacture a pedestal lamp. It takes 1 h on the grinding/cutting machine and 2 h on the sprayer to manufacture a shade. On any day, the sprayer is available for atmost 20 h and the grinding/cutting machine for atmost 12 h. The profit from the sale of a lamp is ₹ 5 and that from a shade is ₹ 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit? Make an LPP and solve it graphically. **Foreign 2011**

Let the number of pedestal lamps sold be x and the number of wooden shades sold be y.

The given data can be written into tabular form as follows:

	Pedestal lamps	Wooden shades	Total hours available
Grinding/ Cutting	2	1	atmost 12
Sprayer	3	2	atmost 20
Profit	₹ 5	₹ 3	

(1)

∴ The required LPP is

$$\max (Z) = 5x + 3y$$

Subject to constraints

$$2x + y \leq 12 \quad \text{...(i)}$$

$$3x + 2y \leq 20$$

and $$x, y \geq 0 \quad \text{...(ii)}$$

On considering the inequalities as equations, we get

$$2x + y = 12 \quad \text{...(iii)}$$

and $$3x + 2y = 20 \quad \text{...(iv)}$$

Now, table for line $2x + y = 12$ is **(1)**

x	6	0
y	0	12

So, it passes through the points (6, 0) and (0, 12).

On putting (0,0) in $2x + y \leq 12$, we get

$$0 \leq 12 \quad \text{(true)}$$

So, the half plane is towards the origin.

Table for line $3x + 2y = 20$ is

x	20/3	0
y	0	10

So, it passes through the points (— , 0) and

So, it passes through the points $\left(\frac{20}{3}, 0\right)$ and

(0, 10). **(1)**

On putting (0,0) in $3x + 2y \leq 20$, we get

$0 \leq 20$ (true)

So, the half plane is towards the origin.

Now, on multiplying Eq. (iii) by 2 and subtracting Eq. (iv) from it, we get

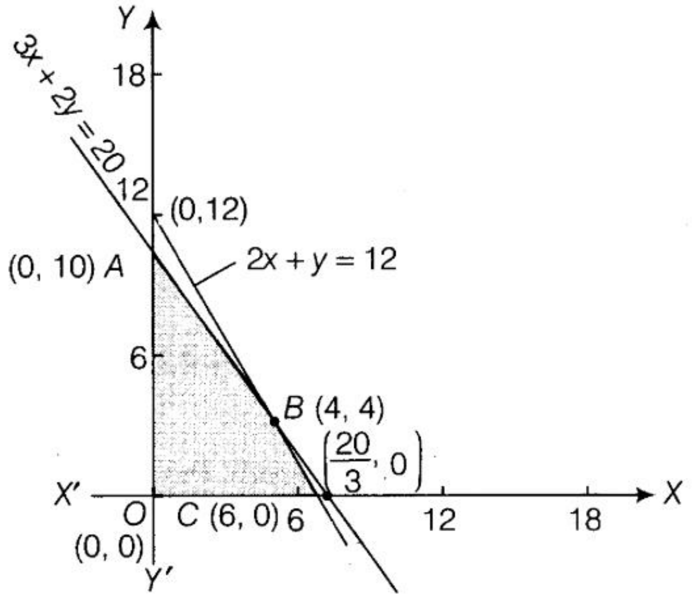
$$4x - 3x = 24 - 20 \quad \Rightarrow \quad x = 4$$

On putting $x = 4$ in Eq. (iii), we get

$$2(4) + y = 12 \Rightarrow y = 12 - 8 = 4 \quad \textbf{(1/2)}$$

So, lines intersect at point (4, 4).

Now, graph of above LPP is as follows:



(1)

From the graph, region *OABC* is the feasible region. The corner points of the feasible region are *O*(0, 0), *A* (0, 10), *B*(4, 4) and *C* (6, 0).

(1/2)

Corner points	**Value of $Z = 5x + 3y$**
$O(0,0)$	$Z = 5(0) + 3(0) = 0$
$A(0,10)$	$Z = 5(0) + 3(10) = 30$
$B(4,4)$	$Z = 5(4) + 3(4) = 20 + 12 = 32$ (maximum)
$C(6,0)$	$Z = 5(6) + 3(0) = 30$

From the table, maximum value is 30 at *B* (4,4).

Hence, the maximum profit is ₹ 32, when 4 lamps and 4 shades are sold. **(1)**

14. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food II contains 1 unit per kg of vitamin A and 2 units per kg of vitamin C. It costs ₹ 50 per kg to purchase food I and ₹ 70 per kg to purchase food II. Formulate the problem as a linear programming problem to minimise the cost of such mixture and find the minimise cost graphically. **Delhi 2011C**

Suppose the diet contains x units of food I and y units of food II.

The given data can be written in the tabular form as follows:

Food	Vitamin A	Vitamin C	Cost / Unit
I	2	1	₹ 50
II	1	2	₹ 70
Mixtrue requirements	atleast 8	atleast 10	

(1)

Then, the required LPP is

$$\min (Z) = 50x + 70y$$

Subject to constraints

$2x + y \geq 8,\ x + 2y \geq 10$ and $x \geq 0, y \geq 0$ **(1)**

On considering the inequalities as equations, we get

$$2x + y = 8 \quad \text{... (i)}$$

and $$x + 2y = 10 \quad \text{... (ii)}$$

Table for line $2x + y = 8$ is

x	0	4
y	8	0

$\therefore$ The line $2x + y = 8$ passes through the points (0, 8) and (4, 0).

On putting (0,0) in $2x + y \geq 8$, we get

$0 \geq 8$ (false)

So, the half plane is away from the origin.

Table for line $x + 2y = 10$ is

x	10	0
y	0	5

(1)

$\therefore$ The line $x + 2y = 10$ passes through the points (10, 0) and (0, 5).

On putting (0, 0) in $x + 2y \geq 10$, we get

$0 \geq 10$ (false)

So, the half plane is away from the origin.

On multiplying Eq. (i) by 2 and subtracting Eq. (ii) from Eq. (i), we get

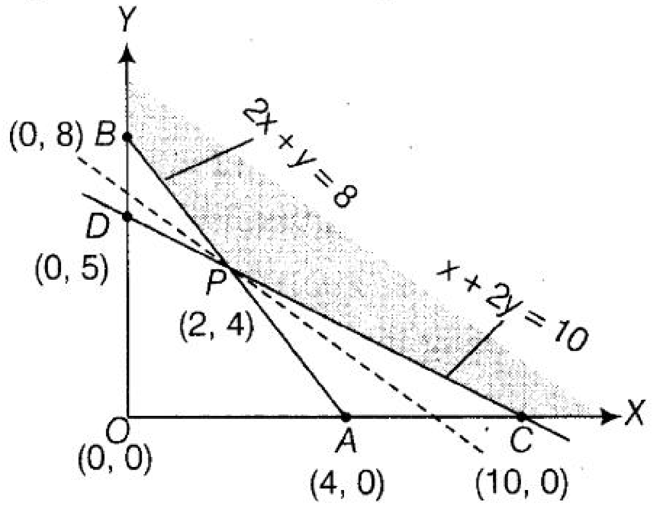
$$\begin{array}{r} 4x + 2y = 16 \\ \underline{\underset{-}{}x \underset{-}{+} 2y = \underset{-}{}10} \\ 3x = 6 \end{array} \Rightarrow x = 2$$

On putting $x = 2$ in Eq. (i), we get

$$2(2) + y = 8 \Rightarrow y = 8 - 4 = 4$$

So, these lines intersect at P (2, 4).

Graph of above LPP is given as follows:



(1)

From the graph, the feasible region is BPC which is unbounded.

The corner points of the feasible region are B (0, 8), P (2, 4) and C (10, 0)

Corner points	Value of $Z = 50x + 70y$
C (10, 0)	$Z = 50(10) + 70(0) = 500$
P (2, 4)	$Z = 50(2) + 70(4) = 380$ (minimum)

$B(0, 8)$	$Z = 50(0) + 70(8) = 560$

From the table, minimum value of Z is 380.**(1)**
As the feasible region is unbounded, therefore 380 may or may not be the minimum value of Z. For this, we draw a dotted line of the inequality $50x + 70y < 380$ and check whether the resulting half plane has point in common with the feasible region or not.
It can be seen that the feasible region has no common point with $50x + 70y < 380$.

Hence, the minimum cost of the mixture is ₹ 380, when food I contains 2 units and food II contains 4 units. **(1)**

15. A library has to accommodate two different types of books on a shelf. The books are 6 cm and 4 cm thick and weight 1 kg and $1\frac{1}{2}$ kg each, respectively. The shelf is 96 cm long and atmost can support a weight of 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books? Make it as an LPP and solve it graphically. **All India 2010C**

Let two types of books be x and y, respectively. The given data can be written in tabular form as follows:

Types of books	**Thickness** (cm)	**Weight** (kg)
x	6	1
y	4	$1\frac{1}{2}=\frac{3}{2}$
	atmost 96	atmost 21

(1)

The required LPP is Maximise $Z = x + y$

Subject to constraints

$$6x + 4y \le 96$$

$\Rightarrow$ $3x + 2y \le 48$ ...(i)

[dividing both sides by 2]

$$x + \frac{3}{2}y \le 21$$

$\Rightarrow$ $2x + 3y \le 42$ ...(ii)

[multiplying both sides by 2]

and $x, y \ge 0$ **(1)**

On considering the inequalities as equations, we get

$3x + 2y = 48$ ...(iii)

and $2x + 3y = 42$ ...(iv)

Table for line $3x + 2y = 48$ is

x	0	16
y	24	0

$\therefore$ It passes through the points (0, 24) and (16, 0).

On putting (0,0) in $3x + 2y \le 48$, we get

$0 \le 48$ (true)

So, the half plane is towards the origin.

Table for line $2x + 3y = 42$ is

x	0	21

y	14	0

$\therefore$ It passes through the points (0, 14) and (21, 0).

(1/2)

On putting (0,0) in $2x + 3y \leq 42$, we get

$$0 \leq 42 \qquad \text{(true)}$$

So, the half plane is towards the origin.

Now, on multiplying Eq. (iii) by 2 and Eq. (iv) by 3 and then subtracting Eq. (iv) from Eq. (iii), we get

$$\begin{array}{r} 6x + 4y = 96 \\ \underline{\underset{-}{6x} \;\underset{-}{+}9y = \underset{-}{126}} \\ -5y = -30 \end{array}$$

$$\Rightarrow \qquad y = 6$$

On putting $y = 6$ in Eq. (iii), we get

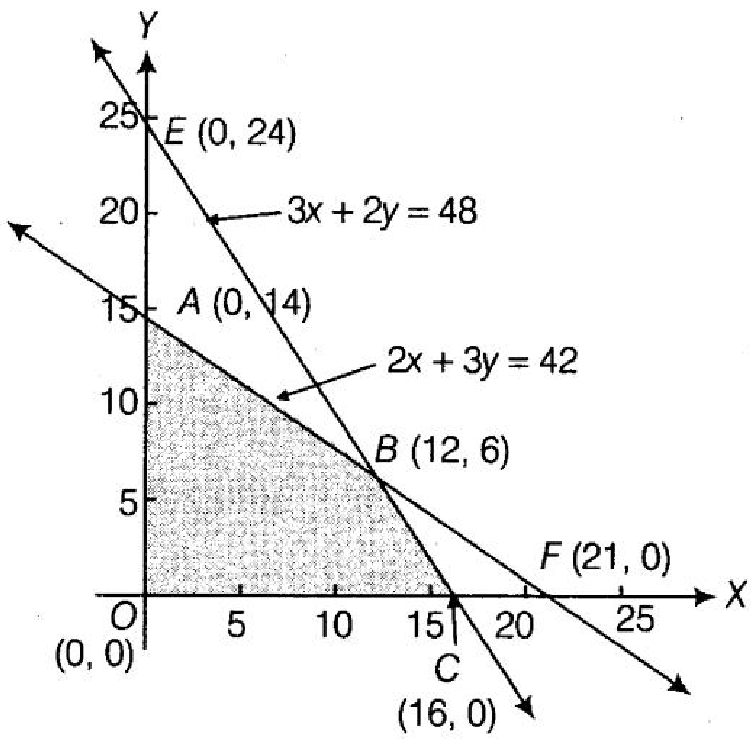
$$3x + 2(6) = 48$$

$$\Rightarrow \qquad 3x = 48 - 12$$

$$\Rightarrow \qquad x = 12$$

So, the point of intersection is $(12, 6)$. **(1/2)**

Now, the graph of above LPP is as follows:



(1)

From the graph, $OABC$ is the feasible region.

The corner points of feasible region are $O(0, 0)$, $A(0, 14)$, $B(12, 6)$ and $C(16, 0)$, respectively. **(1/2)**

Corner points	**Value of $Z = x + y$**
$O(0, 0)$	$Z = 0 + 0 = 0$
$A(0, 14)$	$Z = 0 + 14 = 14$
$B(12, 6)$	$Z = 12 + 6 = 18$ (maximum)
$C(16, 0)$	$Z = 16 + 0 = 16$

From table, maximum value of Z is 18. **(1)**

Hence, the maximum number of books is 18 and number of books of I type is 12 and books of II type is 6. **(1/2)**

16. A dealer deals in two items A and B. He has ₹ 15000 to invest and a space to store atmost 80 pieces. Item A costs him ₹ 300 and item B costs him ₹ 150. He can sell items A and B at profits of ₹ 40 and ₹ 25, respectively. Assuming that he can sell all that he buys, formulate the above as a linear programming problem for maximum profit and solve it graphically. **Delhi 2010C**

Let the number of items sold of A and B be x and y, respectively. The given data can be written in tabular form as follows:

Item	Cost (₹)	Profit (₹)
$A(x)$	300	40
$B(y)$	150	25
atmost 80	atmost 15000	

(1)

The required LPP is Maximise, $Z = 40x + 25y$

Subject to constraints

$$x + y \leq 80$$

$$300x + 150y \leq 15000$$

$\Rightarrow$ $$2x + y \leq 100$$

[dividing both sides by 150]

and $x, y \geq 0$ **(1)**

On considering the inequalities as equations, we get

$$x + y = 80 \quad \text{...(i)}$$

and $$2x + y = 100 \quad \text{...(ii)}$$

Table for line $x + y = 80$ is

x	0	80
y	80	0

$\therefore$ It passes through the points (0, 80) and (80, 0).

On putting (0,0) in $x + y \leq 80$, we get

$0 \leq 80$ (true)

So, the half plane is towards the origin.

Table for line $2x + y = 100$ is

x	0	50
y	100	0

(1)

$\therefore$ It passes through the points (0, 100) and (50, 0).

On putting (0,0) in $2x + y \leq 100$, we get

$0 \leq 100$ (true)

So, the half plane is towards the origin.

So, the half plane is towards the origin.

On multiplying Eq. (i) by 2 and subtracting Eq. (ii) from Eq. (i), we get

$$\begin{array}{r} 2x + 2y = 160 \\ 2x + y = 100 \\ - \quad - \quad - \\ \hline y = 60 \end{array}$$

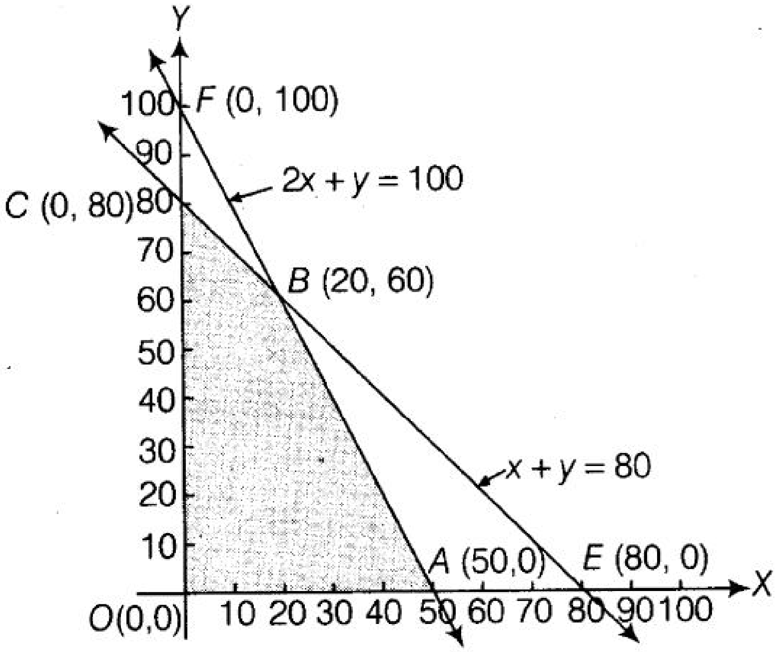
On putting $y = 60$ in Eq. (i), we get

$$x + 60 = 80$$

$\Rightarrow$ $$x = 20$$

So, the point of intersection is (20, 60). **(1/2)**

Now, the graph of the given system of inequalities is as follows:



(1)

From the graph, *OABC* is the feasible region.

The corner points of the feasible region are *O*(0, 0), *A* (50, 0), *B*(20, 60) and C(0,80), respectively. **(1/2)**

Corner points	Value of $Z = 40x + 25y$
O (0, 0)	$Z = 0 + 0 = 0$
A (50,0)	$Z = 2000 + 0 = 2000$
B (20,60)	$Z = 800 + 1500 = 2300$ (maximum)
C (0,80)	$Z = 0 + 2000 = 2000$

From table, maximum value of Z is 2300 at *B* (20,60). **(1)**

Hence, the dealer has maximum profit ₹ 2300, when he sell 20 items of A type and 60 items of B type.

17. One kind of cake requires 300 g of flour and 15 g of fat, another kind of cake requires 150 g of flour and 30 g of fat. Find the maximum number of cakes which can be made from 7.5 kg of flour and 600 g of fat, assuming that there is no shortage of other ingredients used in making the cakes. Make it as an LPP and solve it graphically.

HOTS ; All India 2010

Let the number of cakes of first kind be x and the number of cakes of second kind be y.

The given data can be written in tabular form as follows:

Type of cakes	**Flour** (in g)	**Fat** (in g)
First kind (x)	300	15
Second kind (y)	150	30
	atmost 7.5 kg or 7500	atmost 600

(1)

The required LPP is

$$\max (Z) = x + y$$

Subject to constraints

$$300x + 150y \leq 7500$$

$\Rightarrow$ $$2x + y \leq 50$$

[dividing both sides by 150]

$$15x + 30y \leq 600$$

$\Rightarrow$ $$x + 2y \leq 40$$

[dividing both sides by 15]

and $x \geq 0, y \geq 0$ **(1)**

On considering the inequalities as equations, we get

$$2x + y = 50 \quad \text{...(i)}$$

and $x + 2y = 40$ ...(ii)

Table for line $2x + y = 50$ is

x	0	25
y	50	0

$\therefore$ It passes through the points (0, 50) and (25, 0).

On putting (0,0) in $2x + y \le 50$, we get

$$0 \le 50 \quad \text{(true)}$$

So, the half plane is towards the origin.

Table for line $x + 2y = 40$ is

x	0	40
y	20	0

$\therefore$ It passes through the points (0, 20) and (40, 0). **(1)**

On putting (0,0) in $x + 2y \le 40$, we get

$$0 \le 40 \quad \text{(true)}$$

So, the half plane is towards the origin.

On multiplying Eq. (ii) by 2 and subtracting Eq. (ii) from Eq. (i), we get

$$y - 4y = 50 - 80$$

$$\Rightarrow \quad -3y = -30y$$

$$\Rightarrow \quad y = 10$$

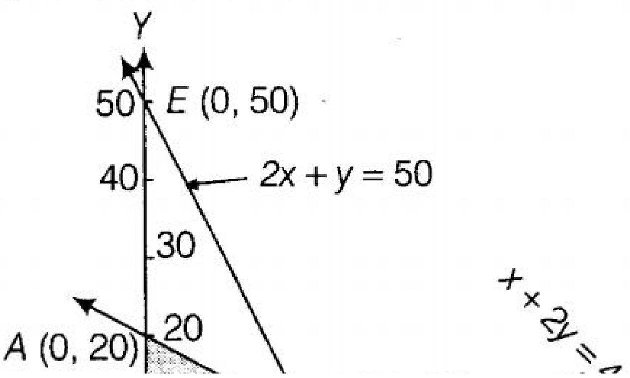
On putting $y = 10$ in Eq. (i), we get

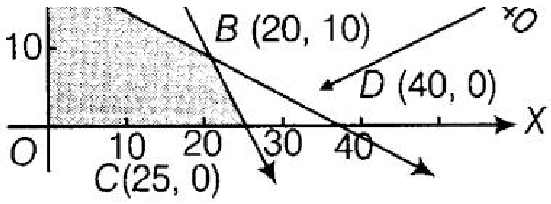
$$2x + 10 = 50 \quad \Rightarrow \quad 2x = 40$$

$$\Rightarrow \quad x = 20 \quad \textbf{(1/2)}$$

So, the point of intersection is (20, 10).

Now, the graph of the given system of inequalities is as follows:





(1)

From the graph, $OABC$ is the feasible region.

The corner points of the feasible region are $O(0, 0)$, $A(0, 20)$, $B(20, 10)$ and $C(25, 0)$, respectively. **(1/2)**

Corner points	Value of $Z = x + y$
$O(0, 0)$	$Z = 0 + 0 = 0$
$A(0, 20)$	$Z = 0 + 20 = 20$
$B(20, 10)$	$Z = 20 + 10 = 30$ (maximum)
$C(25, 0)$	$Z = 25 + 0 = 25$

From table, the maximum value of Z is 30 at B (20,10).

Hence, maximum number is 30, when number of cake of first kind of cakes is 20 and second kind is 10. **(1)**

18. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 h to make a ring and 30 min to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, then find the number of rings and chains that should be manufactured per day so as to earn the maximum profit. Make it as an LPP and solve it graphically. **HOTS; Delhi 2010**

Let the firm manufactures x gold rings and y chains per day. The given data can be written in tabular form as follows:

Items	Time taken	Profit (in ₹)
Gold rings (x)	1 h	300
Chains (y)	30 min = 1/2 h	190
atmost 24	atmost 16 h	

The required LPP is

Maximise, $Z = 300x + 190y$

Subject to constraints

$$x + y \leq 24$$

$$x + \frac{1}{2}y \leq 16$$

$\Rightarrow$ $$2x + y \leq 32$$

[multiplying both sides by2]

and $x \geq 0, y \geq 0$ **(1)**

On considering the inequations as equations, we get

$$x + y = 24 \quad \text{...(i)}$$

and $$2x + y = 32 \quad \text{...(ii)}$$

Table for line $x + y = 24$ is

x	0	24
y	24	0

$\therefore$ It passes through the points (0, 24) and (24, 0).

On putting (0,0) in $x + y \leq 24$, we get

$$0 \leq 24 \quad \text{(true)}$$

So, the half plane is towards the origin.

Table for line $2x + y = 32$ is

x	0	16
y	32	0

(1)

$\therefore$ It passes through the points (0, 32) and (16, 0).

On putting (0,0) in

$2x + y \leq 32$, we get

$$0 \leq 32 \quad \text{(true)}$$

So, the half plane is towards the origin.

Now, on subtracting Eq. (i) from Eq. (ii), we get

$$2x - x = 32 - 24$$

$$\Rightarrow \qquad x = 8$$

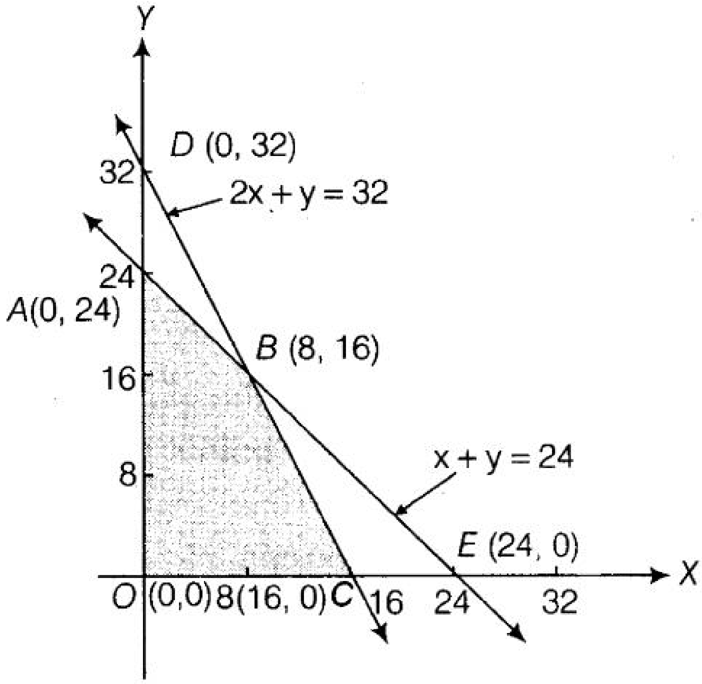
On putting $x = 8$ in Eq. (i), we get

$$8 + y = 24$$

$$\Rightarrow \qquad y = 16$$

So, the point of intersection is (8, 16). **(1/2)**

Now, the graph of the system of inequalities is given as follows:



(1)

From the graph, *OABC* is feasible region.

The corner points of feasible region are $O(0, 0)$, $A(0, 24)$, $B(8, 16)$ and $C(16, 0)$. **(1/2)**

Corner points	**Value of $Z = 300x + 190y$**
$O(0, 0)$	$Z = 300(0) + 190(0) = 0 + 0 = 0$
$A(0, 24)$	$Z = 300(0) + 190(24) = 0 + 4560 = 4560$
$B(8, 16)$	$Z = 300(8) + 190(16) = 2400 + 3040 = 5440$ (maximum)
$C(16, 0)$	$Z = 300(16) + 190(0) = 4800 + 0 = 4800$

From table the maximum value of Z is 5440 at B (8,16),

Hence, the manufacturers earns the maximum profit ₹ 5440, when he manufactures 8 gold rings and 16 chains per day. **(1)**

19. A man has ₹ 1500 for purchasing wheat and rice. A bag of rice and a bag of wheat cost ₹ 180 and ₹ 120, respectively. He has a storage capacity of only 10 bags. He earns a profit of ₹ 11 and ₹ 9 per bag of rice and wheat, respectively. Formulate the problem as an LPP to find the number of bags of each type he should buy for getting maximum profit and solve it graphically.

All India 2009C, 2008

Let x be the number of rice bags and y be the number of wheat bags purchase per day.

The given data can be written in the tabular form as follows:

Purchase	Bags	Cost (per bag)	Profit cost
Rice bag	1	₹ 180	₹ 11
Wheat bag	1	₹ 120	₹ 9
Requirement	10	₹ 1500	

(1)

The required LPP is Maximise, $Z = 11x + 9y$

Subject to constraints

$$180x + 120y \le 1500 \Rightarrow 3x + 2y \le 25$$

[dividing both sides by 60]

$x + y \le 10$ and $x, y \ge 0$ **(1)**

On considering the inequalities as equations, we get

$$3x + 2y = 25 \quad \text{...(i)}$$

and $$x + y = 10 \quad \text{...(ii)}$$

Table for line $3x + 2y = 25$ is

x	0	25/3
y	25/ 2 or 12.5	0

$\therefore$ It passes through the points (0, 12.5) and (25/3, 0).

On putting (0,0) in $3x + 2y \le 25$, we get $0 \le 25$ (true)

So, the half plane is towards the origin.

Table for line $x + y = 10$ is

x	0	10
y	10	0

$\therefore$ It passes through the points (0, 10) and (10, 0). **(1)**

On putting (0,0) in $x + y \leq 10$, we get

$$0 \leq 10 \quad \text{(true)}$$

So, the half plane is towards the origin.

On multiplying Eq. (ii) by 2 and subtracting Eq. (ii) from Eq. (i), we get

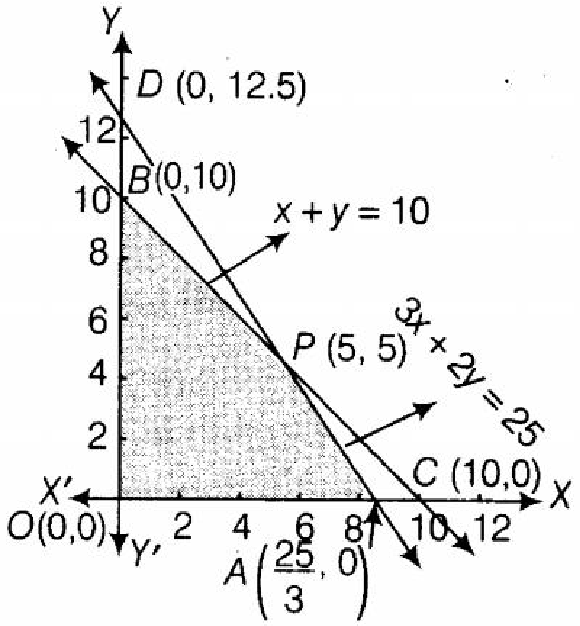
$$3x - 2x = 25 - 20 \Rightarrow x = 5$$

On putting $x = 5$ in Eq. (ii), we get

$$5 + y = 10 \Rightarrow y = 5$$

So, the intersection point is $P(5, 5)$. **(1/2)**

Now, the graph of the system of inequation is given as follows:



(1)

From the graph, $OAPB$ is the feasible region. The corner points of feasible region are $O(0, 0)$, $A\left(\frac{25}{3}, 0\right)$, $P(5, 5)$ and $B(0, 10)$, respectively.

Corner points	Value of $Z = 11x + 9y$
$O(0,0)$	$Z = 11(0) + 9(0) = 0$
$A\left(\frac{25}{3}, 0\right)$	$Z = 91.7 + 0 = 91.7$
$P(5, 5)$	$Z = 55 + 45 = 100$ (maximum)
$B(0,10)$	$Z = 0 + 90 = 90$

From table, maximum value of Z is 100 at $P(5,5)$. **(1/2)**

Hence, maximum profit is ₹ 100, when he purchase 5 rice bags and 5 wheat bags. **(1)**

20. A dealer wishes to purchase a number of fans and sewing machines. He has only ₹ 5760 to invest and has space for atmost 20 items. A fan costs ₹ 360 and a sewing machine costs ₹ 240. He can sell a fan at a profit of ₹ 22 and a sewing machine at a profit of ₹ 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximise his profit? Formulate the problem as an LPP and solve it graphically. **All India 2009; Delhi 2009C**

Suppose a dealer purchase x fans and y sewing machines.

The given data can be written in the tabular form as follows:

Purchase	Items	Costs (per item)	Profit cost
Fan (x)	1	₹ 360	₹ 22
Sewing machines (y)	1	₹ 240	₹ 18
	atmost 20	atmost ₹ 5760	

(1)

The required LPP is Maximum, $Z = 22x + 18y$

Subject to constraints are

$$x + y \leq 20$$

$$360x + 240y \leq 5760$$

$$\Rightarrow \quad 3x + 2y \leq 48$$

[dividing both sides by 120]

and $\quad x, y \geq 0$ **(1)**

On considering the inequalities as equations, we get

$$x + y = 20 \quad \text{...(i)}$$

and $\quad 3x + 2y = 48 \quad$ ...(ii)

Table for line $x + y = 20$ is

x	0	20
y	20	0

$\therefore$ It passes through the points (0, 20) and (20, 0).

On putting (0,0) in $x + y \leq 20$, we get

$$0 \leq 20 \quad \text{(true)}$$

So, the half plane is towards the origin.

Table for line $3x + 2y = 48$ is

x	0	16
y	24	0

$\therefore$ It passes through the points (0, 24) and (16, 0). **(1)**

On putting (0,0) in $3x + 2y \leq 48$, we get

$$0 \leq 48 \qquad \text{(true)}$$

So, the half plane is towards the origin.

On multiplying Eq. (i) by 3 and subtracting Eq. (ii) from Eq. (i), we get

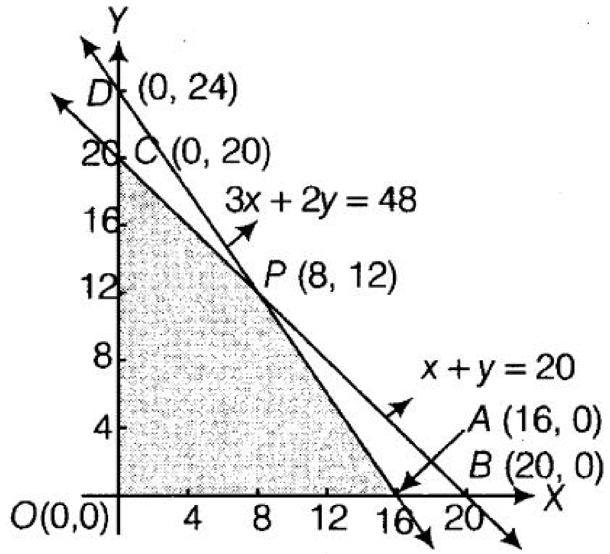
$3y - 2y = 60 - 48 \quad \Rightarrow \quad y = 12$

On putting $y = 12$ in Eq. (i), we get

$x + 12 = 20 \quad \Rightarrow \quad x = 8$

So, point of intersection is $P(8, 12)$. **(1/2)**

Now, the graph of the system of inequations is given as:



(1)

From graph, *OAPC* is the feasible region.

∴ The corner points of feasible region are $O(0, 0)$, $A(16, 0)$, $P(8, 12)$ and $C(0, 20)$. **(1/2)**

Corner points	Value of $Z = 22x + 18y$
$O(0,0)$	$Z = 22(0) + 18(0) = 0$
$A(16,0)$	$Z = 352 + 0 = 352$
$P(8,12)$	$Z = 176 + 216 = 392$ (maximum)
$C(0,20)$	$Z = 0 + 360 = 360$

From table, the maximum value of Z is 392 at $P(8,12)$.

Hence, a dealer purchase 8 fans and 12 sewing machines to get maximum profit of ₹ 392. **(1)**

21. Two tailors *A* and *B* earn ₹ 150 and ₹ 200 per day, respectively. *A* can stitch 6 shirts and 4 pants per day, while *B* can stitch 10 shirts and 4 pants per day. How many days shall each work, if it is desired to produce atleast 60 shirts and 32 pants at a minimum labour cost? Make it as an LPP and solve the problem graphically. **Delhi 2009C; 2008C**

Do same as Ques 14.

Required linear programming problem is

$$\min(Z) = 150x + 200y \quad \textbf{(1)}$$

Subject to constraints

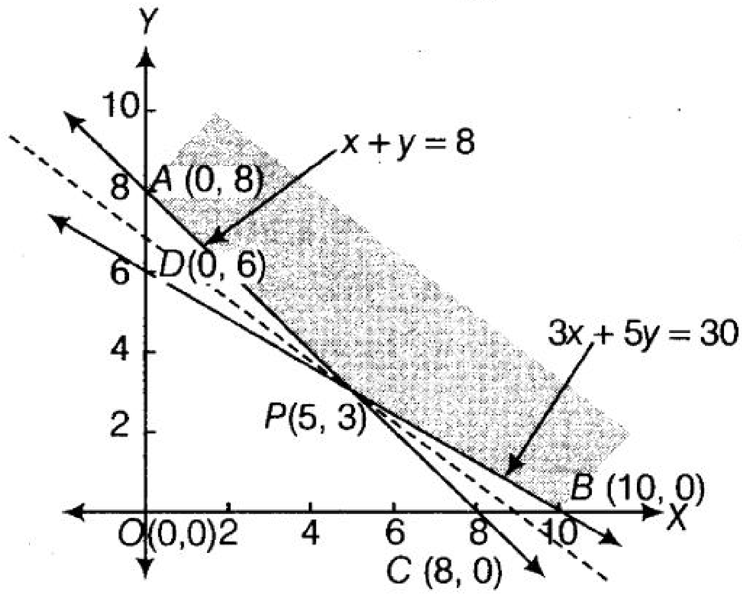
$$6x + 10y \geq 60 \quad \Rightarrow \quad 3x + 5y \geq 30$$

[dividing both sides by 2]

$$4x + 4y \geq 32 \quad \Rightarrow \quad x + y \geq 8$$

[dividing both sides by 4]

and $x \geq 0, y \geq 0$ **(1)**



(1)

(1/2)

Hence, minimum labour cost is ₹ 1350, when tailor *A* works for 5 days and tailor *B* works for 3 days. **(1)**

22. A diet is to contain atleast 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs ₹ 4 per unit and food F_2 costs ₹ 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of mixture of these two foods and also meets the minerals nutritional requirements.

Delhi 2009

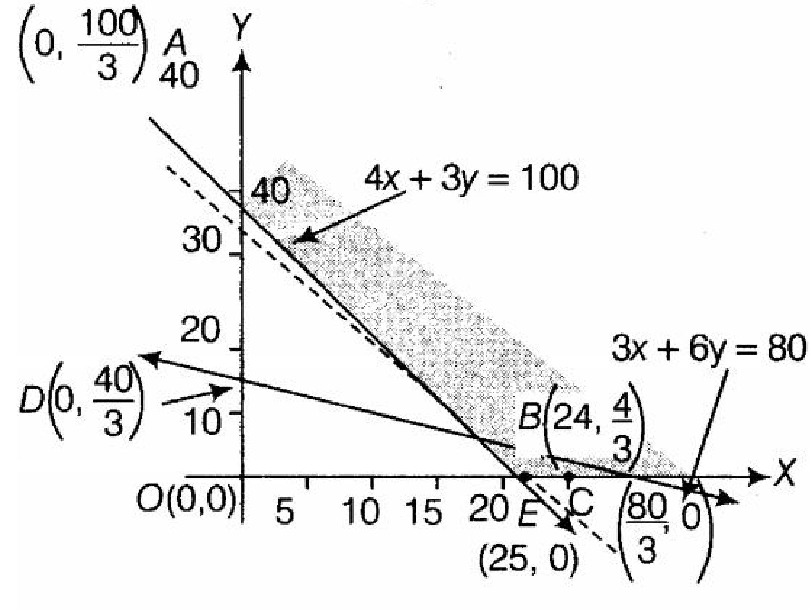
Do same as Que 14.

Required LPP is Minimise $z = 4x + 6y$

Subject to constraints

$$3x + 6y \geq 80,\ 4x + 3y \geq 100$$

and $$x, y \geq 0$$ **(1)**



Hence, the minimum cost is ₹ 104, when diet contains 24 units of food F_1 and $\frac{4}{3}$ units of food F_2. **(1)**

23. A diet for a sick person must contains atleast 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods *A* and *B* are available at a cost of ₹ 5 and ₹ 4 per unit, respectively. One unit of food *A* contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of food *B* contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods *A* and *B* should be used to have least cost, but it must satisfy the requirements of the sick person? Formulate the question as an LPP and solve it graphically. **All India 2008**

Let the sick person requires x units of food *A* and y units of food *B*.

The given data can be written into tabular form as follows :

Food	Vitamins	Minerals	Calories	Cost
$A(x)$	200	1	40	5
$B(y)$	100	2	40	4
	atleast 4000	atleast 50	atleast 1400	

$\therefore$ Required LPP is Minimise, $Z = 5x + 4y$

Subject to constraints

$$200x + 100y \geq 4000 \Rightarrow 2x + y \geq 40$$

[dividing both sides by 100]

$$x + 2y \geq 50$$

$$40x + 40y \geq 1400 \Rightarrow x + y \geq 35$$

[dividing both sides by 40]

and $x, y \geq 0$

On considering the inequalities as equations, we get

$$2x + y = 40 \quad \ldots\text{(i)}$$

$$x + 2y = 50 \quad \ldots\text{(ii)}$$

and $x + y = 35 \quad \ldots\text{(iii)}$

Table for line $2x + y = 40$ is

x	0	20

y	40	0

$\therefore$ It passes through the points (0, 40) and (20, 0).
On putting (0,0) in $2x + y \geq 40$ we get

$$0 \geq 40 \qquad \text{(false)}$$

So, the half plane is away from the origin.

Table for line $x + 2y = 50$ is

x	0	50
y	25	0

$\therefore$ It passes through the points (0, 25) and (50, 0).

On putting (0, 0) in $x + 2y \geq 50$, we get

$$0 \geq 50 \qquad \text{(false)}$$

So, the half plane is away from the origin.

Table for line $x + y = 35$ is

x	0	35
y	35	0

$\therefore$ It passes through the points (0,35) and (35, 0). **(1)**

On putting (0,0) in $x + y \geq 35$, we get

$$0 \geq 35 \qquad \text{(false)}$$

So, the half plane is away from the origin.

On multiplying Eq. (i) by 2 and subtracting Eq. (ii) from Eq. (i), we get

$$4x - x = 80 - 50$$

$$\Rightarrow \quad 3x = 30 \quad \Rightarrow \quad x = 10$$

On putting $x = 10$ in Eq. (ii), we get

$$10 + 2y = 50$$

$$\Rightarrow \quad 2y = 40 \quad \Rightarrow \quad y = 20$$

So, point of intersection of line (i) and (ii) is C (10, 20).

On multiplying Eq. (iii) by 2 and subtracting Eq. (iii) from Eq. (ii), we get

$$x - 2x = 50 - 70$$

$$\Rightarrow \quad -x = -20 \quad \Rightarrow \quad x = 20$$

On putting $x = 20$ in Eq. (ii), we get

$$20 + 2y = 50$$

$$\Rightarrow \quad 2y = 30 \quad \Rightarrow \quad y = 15$$

$\Rightarrow \quad 2y = 30 \quad \Rightarrow \quad y = 15$

So, point of intersection of lines (i) and (iii) is C (20,15).

On subtracting Eq. (iii) from Eq. (i), we get

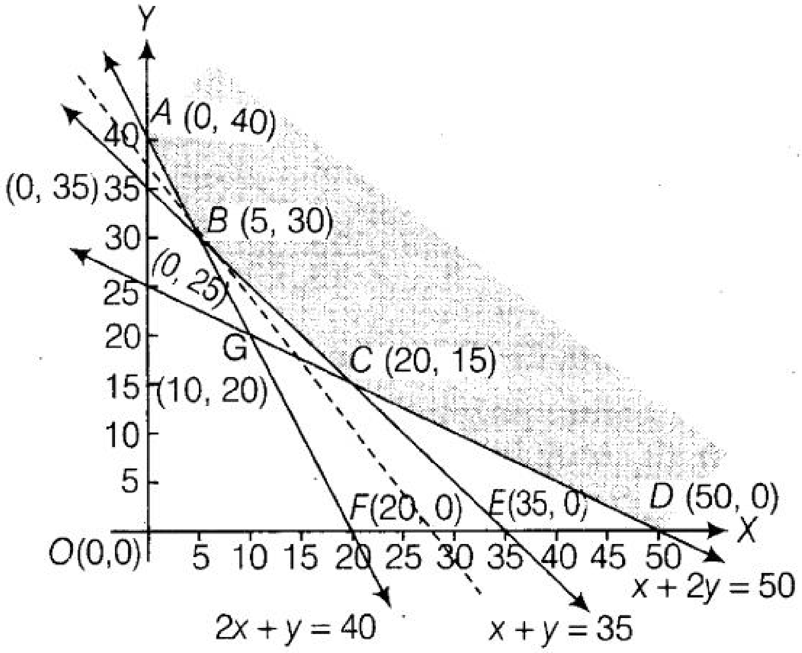
$$2x - x = 40 - 35 \quad \Rightarrow \quad x = 5$$

On putting $x = 5$ in Eq. (iii), we get

$$5 + y = 35 \quad \Rightarrow \quad y = 30$$

So, point of intersection of lines (i) and (iii) is $B(5, 30)$. **(1)**

Now, graph of above LPP is given below:



(1)

From the graph, *ABCD* is the feasible region which is bounded.

Corner points of feasible region are

A (0, 40), *B* (5, 30), *C* (20, 15) and *D* (50, 0).

Corner points	Value of $Z = 5x + 4y$
$A(0, 40)$	$Z = 0 + 160 = 160$
$B(5, 30)$	$Z = 25 + 120 = 145$ (least cost)
$C(20, 15)$	$Z = 100 + 60 = 160$
$D(50, 0)$	$Z = 250 + 0 = 250$

From the table, minimum value of Z is 145 at B(5,30). **(1)**

As the feasible region is unbounded, therefore 145 may or may not be the minimum value of Z. For this, we draw a dotted line of the inequality $5x + 4y < 145$ and check, whether the resulting half plane has point in common with the feasible region or not.

It can be seen that the feasible region has no common point with $5x + 4y < 145$.

Hence, least cost is ₹ 145, when sick person requires 5 units of food A and 30 units of food B. **(1)**

24. A factory owner purchases two types of machines A and B for his factory. The requirements and the limitations for the machines are as follows :

Machines	Area occupied	Labour force	Daily output (in units)
A	$1000\ m^2$	12 men	60
B	$1200\ m^2$	8 men	40

He has maximum area of $9000\ m^2$ available and 72 skilled labourers who can operate both the machines. How many machines of each type should be bought to maximise the daily output? **Delhi 2008**

Let the number of machines of type A be x and the machines of type B be y.

The given data can be written in the tabular form as follows:

Machines	Area occupied	Labour force	Daily output (in units)
$A(x)$	$1000\ m^2$	12 men	60
$B(y)$	$1200\ m^2$	8 men	40
Maximum	$9000\ m^2$	72 men	

Then given LPP is Maximise, $Z = 60x + 40y$ **(1)**
Subject to constraints

$1000x + 1200y \leq 9000 \Rightarrow 5x + 6y \leq 45$

[dividing both sides by 200]

$12x + 8y \leq 72 \Rightarrow 3x + 2y \leq 18$

[dividing both sides by 4]

and $x, y \geq 0$ **(1)**

On considering the inequalities as equations, we get

$$5x + 6y = 45 \quad \text{...(i)}$$

and

$$3x + 2y = 18 \quad \text{...(ii)}$$

Table for line $5x + 6y = 45$ is

x	0	9
y	7.5	0

$\therefore$ It passes through the points (0, 7.5) and (9, 0).

On putting (0,0) in $5x + 6y \leq 45$, we get

$$0 \leq 45 \quad \text{(true)}$$

So, the half plane is towards the origin.

Table for line $3x + 2y = 18$ is

x	0	6
y	9	0

$\therefore$ It passes through the points (0, 9) and (6, 0). **(1)**

On putting (0,0) in $3x + 2y \leq 18$, we get

$$0 \leq 18 \quad \text{(true)}$$

So, the half plane is towards the origin.

Now, on multiplying Eq. (ii) by 3 and then subtracting Eq. (i) from it, we get

$$9x - 5x = 54 - 45$$

$$\Rightarrow \quad 4x = 9 \quad \Rightarrow \quad x = \frac{9}{4}$$

On putting $x = \frac{9}{4}$ in Eq. (ii), we get

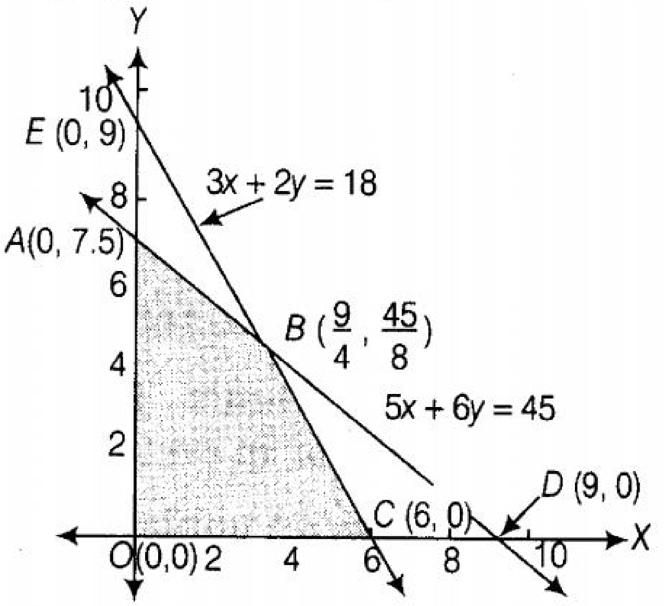
$$3\left(\frac{9}{4}\right) + 2y = 18$$

$$\Rightarrow \quad 2y = 18 - \frac{27}{4}$$

$$\Rightarrow \quad y = \frac{45}{8}$$

So, point of intersection is $B\left(\frac{9}{4}, \frac{45}{8}\right)$. **(1/2)**

The graph of above LPP is as follows:



(1)

From the graph, $OABC$ is the feasible region, whose corner points are

$O(0,0), A\ (0,\ 7.5), B\left(\frac{9}{4}, \frac{45}{8}\right)$ and $C\ (6, 0)$.

Corner points	Value of $Z = 60x + 40y$
$A\ (0, 7.5)$	$Z = 0 + 300 = 300$
$B\left(\frac{9}{4}, \frac{45}{8}\right)$	$Z = 135 + 225 = 360$ (maximum)
$C\ (6, 0)$	$Z = 360 + 0 = 360$
$O\ (0, 0)$	$Z = 0$

From the table, maximum value of Z, is 360 at $B\left(\frac{9}{4}, \frac{45}{8}\right)$ and C(6,0) **(1/2)**

Hence, the maximum output is at B and C. But the number of machines cannot be in fraction. Hence, number of machines of type $A = 6$ and number of machines of type $B = 0$. **(1)**