CHAPTER 6

INDEX NUMBERS

Irving Fisher

Irving Fisher (1867–1947) was an American Statistician born in New York and his father was a teacher. As a child, he had remarkable mathematical ability and a flair for invention. In 1891, Fisher received the first Ph.D in economics from Yale University. Fisher had shown particular talent and inclination for mathematics, but he found that economics offered greater scope for his ambition and social concerns. He made important contributions to economics including index numbers. He edited the *Yale Review* from 1896 to 1910 and was active in many learned societies, institutes, and welfare organizations. He was a president of the American Economic Association. He died in New York City in 1947, at the age of 80.

LEARNING OBJECTIVES

The students will able to

- ❖ understand the concept and purpose of Index Numbers.
- ❖ calculate the indices to measure price and quantity changes over period of time.
- ❖ understand different tests an ideal Index Number satisfies.
- ❖ understand consumer price Index Numbers.
- ❖ understand the limitations of the construction of Index Numbers.

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Introduction

Index number is a technique of measuring changes in a variable or a group of variables with respect to time, location or other characteristics. It is one of the most widely used statistical methods. Index number is a specialized average designed to measure the change in a group of related variables over a period of time. For example, the price of cotton in 2010 is studied with reference to its price in 2000. It is used to feel the pulse of the economy and it reveals the inflationary or deflationary tendencies. In reality, it is viewed as barometers of economic activity because if one wants to have an idea as to what is happening in an economy, he should check the important indicators like the index number of agricultural production, index number of industrial production, and the index number business activity *etc*., There are several types of index numbers and the students will learn them in this chapter.

6.1 DEFINITION AND USES OF INDEX NUMBERS

6.1.1 Definition

An Index Number is defined as a relative measure to compare and describe the average change in price, quantity value of an item or a group of related items with respect to time, geographic location or other characteristics accordingly.

*In the words of **Maslow** "An index number is a numerical value characterizing the change in complex economic phenomenon over a period of time or space"*

***Spiegal** defines, "An index number is a statistical measure designed to show changes in a variable on a group of related variables with respect to time, geographical location or other characteristics".*

*According to **Croxton and Cowden** "Index numbers are devices for measuring differences in the magnitude of a group of related variables".*

***Bowley** describes "Index Numbers as a series which reflects in its trend and fluctuations the movements of some quantity".*

6.1.2 Uses

The various uses of index numbers are:

Economic Parameters

The Index Numbers are one of the most useful devices to know the pulse of the economy. It is used as an indicator of inflanationary or deflanationary tendencies.

Measures Trends

Index numbers are widely used for measuring relative changes over successive periods of time. This enable us to determine the general tendency. For example, changes in levels of prices, population, production etc. over a period of time are analysed.

Useful for comparsion

The index numbers are given in percentages. So it is useful for comparison and easy to understand the changes between two points of time.

Help in framing suitable policies

Index numbers are more useful to frame economic and business policies. For example, consumer price index numbers are useful in fixing dearness allowance to the employees.

Useful in deflating

Price index numbers are used for connecting the original data for changes in prices. The price index are used to determine the purchasing power of monetary unit.

Compares standard of living

Cost of living index of different periods and of different places will help us to compare the standard of living of the people. This enables the government to take suitable welfare measures.

Special type of average

All the basic ideas of averages are employed for the construction of index numbers. In averages, the data are homogeneous (in the same units) but in index number, we average the variables which have different units of measurements. Hence, it is a special type of average.

6.2 TYPES OF INDEX NUMBERS

(i) Price Index Numbers

Price index is a 'Special type' of average which studies net relative change in the prices of commodities, expressed in different units. Here comparison is made in respect of prices. Price index numbers are wholesale price index numbers and retail price index numbers.

(i) Quantity Index Numbers

This number measures changes in volume of goods produced, purchased or consumed. Here, the comparison is made in respect of quantity or volume. For example, the volume of agricultural goods produced, consumed, import, export etc.

(ii) Value Index

Value index numbers study the changes in the total value of a certain period with the total value of the base period. For example, the indices of stock-in-made, purchase, sales profit *etc.*, are analysed here.

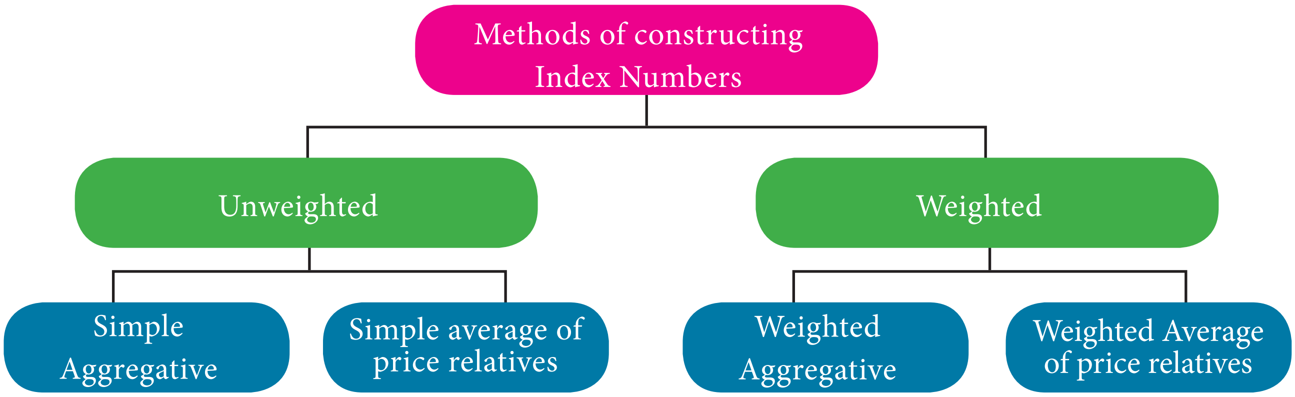
NOTE

The points and precautions that should be taken in the constructing index numbers are:

- determination of the purpose
- selection of the base period
- selection of commodities
- selection of price quotations
- selection of appropriate weight
- selection of an appropriate average
- selection of an appropriate formula

6.3 METHODS OF CONSTRUCTING INDEX NUMBERS

Different types of index number (price/quantity/value) can be classified as follows.



6.3.1 Unweighted Index Numbers

An unweighted price Index Number measures the percentage change in price of a single item or a group of items between two periods of time. In unweighted index numbers, all the values taken for study are of equal importance. There are two methods in this category.

(i) Simple aggregative method:

Under this method the prices of different items of current year are added and the total is divided by the sum of prices of the base year items and multiplied by 100.

$$P_{01} = \frac{\sum p_1}{\sum p_0} \times 100$$

p_1 = Current year prices for various commodities
p_0 = Base year prices for various commodities
P_{01} = Price Index number

NOTE

The base period is the period against which comparison is made. Generally a year is taken as base period. The base period should be free from economic and natural disturbances.

Limitations of the simple aggregative method

(i) Relative importance of the commodities is not taken into account.

(ii) Highly priced items influence the index number

Example 6.1

Construct the Price Index Number for the year 1997, from the following information taking 1996 as base year.

Commodities	Price in 1996 (₹)	Price in 1997 (₹)
Rice	130	115
Wheat	80	65
Sugar	75	70
Ragi	95	90
Oil	105	105
Dal	35	20

Solution:

Construction of Price Index:

Commodities	Price in 1996 (₹) (p_0)	Price in 1997 (₹) (p_1)
Rice	130	115
Wheat	80	65
Sugar	75	70
Ragi	95	90
Oil	105	105
Dal	35	20
	$\sum p_0 = 520$	$\sum p_1 = 465$

$$P_{01} = \frac{\sum p_1}{\sum p_0} \times 100$$

$$= \frac{465}{520} \times 100 = 89.42$$

Price Index in 1997, when compared to 1996 has fallen by 10.58%

Example 6.2

Calculate Price Index Number for 2016 from the following data by simple aggregate method, taking 2016 as base year.

Commodities	Price per kg	
	2015	2016
Apple	100	140
Orange	30	40
Pomegranate	120	130
Guava	40	50

Solution:

Commodities	2015 (p_0)	2016 (p_1)
Apple	100	140
Orange	30	40
Pomegranate	120	130
Guava	40	50
Total	290	360

Price index: $P_{01} = \frac{\Sigma\, p_1}{\Sigma\, p_0} \times 100$

$$= \frac{360}{290} \times 100$$

$$= \frac{3600}{29}$$

$$P_{01} = 124.13\%$$

Price index for the year 2016 when compared to 2015 has been increased by 24.13%.

2. Simple average of price relative method

Under this method, first of all price relatives are obtained for the various items and then average of these relatives is obtained by using either arithmetic mean or geometric mean. Price relative is the price of the current year expressed as the percentage of the price of the base year. The formula for computing Index Number under this method on using Arithmetic mean and Geometric mean are given below.

If N is the member of items, p_1 is the price of the commodity with current year and p_0 is the price of the commodity in the base year then, the average Price Index Number is

(i) $$P_{01} = \frac{\sum \frac{p_1}{p_0} \times 100}{N}$$ (using Arithmetic mean)

(ii) $$P_{01} = \text{antilog} \frac{\sum \log\left(\frac{p_1}{p_0} \times 100\right)}{N}$$ (using Geometric mean)

Advantages of Average Price Index

1. It is not influenced by the extreme prices of items as equal importance is given to all items.
2. Price relatives are pure numbers; therefore the value of the average price relative index is not affected by the units of measurement of commodities included in the calculation of index numbers.

Limitations

1. Equal weights are assigned to every commodity included in the index. Each price relatives is given equal importance, but in actual practice, it is not true.
2. Arithmetic mean is very often used to calculate the average price relatives, but it has a few disadvantages. The use of geometric mean is difficult to calculate.

Example 6.3

Compute price index number by simple average of price relatives method using arithmetic mean and geometric mean.

Item	Price in 2001 (₹)	Price in 2002 (₹)
A	6	10
B	2	2
C	4	6
D	10	12
E	8	12

Solution:

Calculation of price index number by simple average of price relatives:

Item	Price in 2001 (₹) p_0	Price in 2002 (₹) p_1	$p = \frac{p_1}{p_0} \times 100$	$\log p$
A	6	10	166.7	2.2219
B	2	2	100.0	2.0000
C	4	6	150.0	2.1761
D	10	12	120.0	2.0792
E	8	12	150.0	2.1761
			$\sum p = 686.7$	$\sum \log p = 10.6533$

(i) Price relative index number based on arithmetic mean:

$$P_{01} = \frac{\sum \frac{p_1}{p_0} \times 100}{N} = \frac{\sum p}{N} = \frac{686.7}{5} = 137.34$$

(ii) Price relative index number based on geometric mean:

$$P_{01} = \text{antilog}\left(\frac{\sum \log p}{N}\right) = \text{antilog}\left(\frac{10.6533}{5}\right)$$

$$= \text{antilog}\,(2.13066)$$

$$= 135.1$$

Hence, the price index number based on arithmetic mean and geometric mean for the year 2002 are 137.34 and 135.1 respectively.

Example 6.4

Construct simple average price relative index number using arithmetic mean for the year 2012 for the following data showing the profit from various categories sold out in departmental stores.

Profit (per week)	2010	2012
Groceries	150600	170800
Cosmetics	70000	82000
Stationery items	12000	10800
Utensils	20000	18600

Solution: Index number uning Arithmertic Mean of price relatives

	Profit in 2010 (p_0)	Profit in 2012 (p_1)	p_1/p_0 x 100
Groceries	150600	170800	$\frac{170800}{150600} \times 100 = 11341$
Cosmetics	70000	82000	$\frac{82000}{70000} \times 100 = 117.14$
Stationery items	12000	10800	$\frac{10800}{12000} \times 100 = 90.00$
Utensils	20000	18600	$\frac{18600}{20000} \times 100 = 93.00$
		Total	413.55

$$\text{Simple average price relatives using A.M} = P_{01} = \frac{\sum \frac{p_1}{p_0} \times 100}{N}$$

$$= \frac{413.55}{4}$$

$$= 103.3875$$

$$P_{01} = 103.39$$

The average price relative index number using arithmetic mean for the year 2012 is 103.39

Example 6.5

Construct simple average price relative index number using geometric mean for the year 2015 for the data showing the expenditure in education of the children taking different courses.

Expenditure per year	2014	2015
B.Sc	24000	26000
B.Com	20000	22000
B.E	108000	12000
M.B.B.S	150000	168000

Solution:

Expenditure	Year 2014 (p_0)	Year 2015 (p_1)	$P = (p_1/p_0) \times 100$	log P
B.Sc	24000	26000	$\frac{26000}{24000} \times 100 = 108.33$	2.0346
B.Com	20000	22000	$\frac{22000}{20000} \times 100 = 110.00$	2.0414
B.E	108000	12000	$\frac{120000}{108000} \times 100 = 111.11$	2.0457
M.B.B.S	150000	168000	$\frac{168000}{150000} \times 100 = 112.00$	2.0492
				$\sum \log P = 8.1709$

$$P_{01} = \text{antilog}\left(\frac{\sum \log P}{N}\right)$$

$$= \text{antilog}\left(\frac{8.1709}{4}\right)$$

$$= \text{antilog}\ (2.04275)$$

$$= \text{antilog}\ (2.0428)$$

$$= 110.4$$

The average price relative index number using geometric mean for the year 2015 is 110.4

6.4 WEIGHTED INDEX NUMBERS

In computing weighted Index Numbers, the weights are assigned to the items to bring out their economic importance. Generally quanties consumed or value are used as weights.

Weighted index numbers are also of two types

(i) Weighted aggregative

(ii) Weighted average of price relatives

6.4.1 Weighted aggregate Index Numbers

In this method price of each commodity is weighted by the quantity sale either in the base year or in the current year. There are various methods of assigning weights and thus there are many methods of constructing index numbers. Some of the important formulae used under this methods are

a) Laspeyre's Index ($P_{01}{}^{L}$)

b) Paasche's Index ($P_{01}{}^{P}$)

c) Dorbish and Bowley's Index ($P_{01}{}^{DB}$)

d) Fisher's Ideal Index ($P_{01}{}^{F}$)

e) Marshall-Edgeworth Index ($P_{01}{}^{Em}$)

f) Kelly's Index ($P_{01}{}^{K}$)

a. Laspeyre's method

The base period quantities are taken as weights. The Index is

$$P_{01}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

b. Paasche's method

The current year quantities are taken as a weight. In this method, we use continuously revised weights and thus this method is not frequently used when the number of commodities is large. The Index is

$$P_{01}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

c. Dorbish and Bowley's method

In order in take into account the impact of both the base and current year, we make use of simple arithmetic mean of Laspeyre's and Paasche's formula

The Index is

$$P_{01}^{DB} = \frac{P_{01}^{L} + P_{01}^{P}}{2}$$

$$= \frac{\frac{\sum p_1q_0}{\sum p_0q_0} + \frac{\sum p_1q_1}{\sum p_0q_1}}{2} \times 100$$

d. Fisher's Ideal Index

It is the geometric mean of Laspeyre's Index and Paasche's Index, given by:

$$P_{01}^{F} = \sqrt{P_{01}^{L} \times P_{01}^{P}}$$

$$= \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100$$

e. Marshall-Edgeworth method

In this method also both the current year as well as base year prices and quantities are considered.

The Index is

$$P_{01}^{ME} = \frac{\sum p_1(q_0+q_1)}{\sum p_0(q_0+q_1)} \times 100$$

$$= \frac{\sum p_1q_0 + \sum p_1q_1}{\sum p_0q_0 + \sum p_0q_1} \times 100$$

f. Kelly's method

The Kelly's Index is

$$P_{01}^{K} = \frac{\sum p_1q}{\sum p_0q} \times 100, \qquad q = \frac{q_0+q_1}{2}$$

where q refers to quantity of some period, not necessarily of the mean of the base year and current year. It is also possible to use average quantity of two or more years as weights. This method is known as fixed weight aggregative index.

Example 6.6

Construct weighted aggregate index numbers of price from the following data by applying

1. Laspeyre's method
2. Paasche's method
3. Dorbish and Bowley's method
4. Fisher's ideal method
5. Marshall-Edgeworth method

Commodity	2016		2017	
	Price	Quantity	Price	Quantity
A	2	8	4	6
B	5	10	6	5
C	4	14	5	10
D	2	19	2	13

Solution:

Calculation of various indices

Commodity	2016		2017		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	Price p_0	Quantity q_0	Price p_1	Quantity q_1				
A	2	8	4	6	32	16	24	12
B	5	10	6	5	60	50	30	25
C	4	14	5	10	70	56	50	40
D	2	19	2	13	38	38	26	26
					$\sum p_1q_0 = 200$	$\sum p_0q_0 = 160$	$\sum p_1q_1 = 130$	$\sum p_0q_1 = 103$

(1) Laspeyre's Index:

$$P_{01}^{L} = \frac{\sum p_1q_0}{\sum p_0q_0} \times 100$$

$$= \frac{200}{160} \times 100 = 125$$

(2) Paasche's Index

$$P_{01}^{P} = \frac{\sum p_1q_1}{\sum p_0q_1} \times 100$$

$$= \frac{130}{103} \times 100 = 126.21$$

(3) Dorbish and Bowley's Index

$$P_{01}^{DB} = \frac{P_{01}^{L} + P_{01}^{P}}{2} = \frac{125 + 126.21}{2}$$

$$= 125.6$$

(4) Fisher's Ideal Index

$$P_{01}^{F} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$$

$$= \sqrt{\frac{200}{160} \times \frac{130}{103}} \times 100$$

$$= \sqrt{1.578} \times 100 = 1.2561 \times 100$$

= 125.61

(5) Marshall-Edgeworth method

$$P_{01}^{ME} = \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100$$

$$= \frac{200+130}{160+103} \times 100 = \frac{330}{263} \times 100$$

= 125.48

Example 6.7

Calculate the price indices from the following data by applying (1) Laspeyre's method (2) Paasche's method and (3) Fisher ideal number by taking 2010 as the base year.

Commodity	2010		2011	
	Prices	Quantities	Prices	Quantities
A	20	10	25	13
B	50	8	60	7
C	35	7	40	6
D	25	5	35	4

Solution: Calculations

p_0	q_0	p_1	q_1	$p_0 q_0$	$p_0 q_1$	$p_1 q_0$	$p_1 q_1$
20	10	25	13	200	260	250	325
50	8	60	7	400	350	480	420
35	7	40	6	245	210	280	240
25	5	35	4	125	100	175	140
				970	920	1185	1125

(1) Laspeyre's Index

$$P_{01}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

$$= \frac{1185}{970} \times 100$$

$$= 122.16$$

(2)) Paasche's Index

$$P_{01}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

$$= \frac{11254}{920} \times 100$$

$$= 122.28$$

(3) Fisher's Ideal Index

$$P_{01}^{F} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$$

$$= \sqrt{\frac{1185}{970} \times \frac{1125}{920}} \times 100$$

$$= \sqrt{1.2216 \times 1.2228} \times 100$$

$$= \sqrt{1.49377} \times 100$$

$$= 1.2222 \times 100$$

$$= 122.22$$

Example 6.8

Calculate the Dorbish and Bowley's price index number for the following data taking 2014 as base year.

Items	2014		2015	
	Prices (per kg)	Quantities (purchased)	Prices (per kg)	Quantities (purchased)
Oil	80	3	100	4
Pulses	35	2	45	3
Sugar	25	2	30	3
Rice	50	30	54	35
Cereals	35	2	40	3

Solution: Price Index by Dorbish and Bowley's Method

p_0	q_0	p_1	q_1	$p_0 q_0$	$p_0 q_1$	$p_1 q_0$	$p_1 q_1$
80	3	100	4	240	320	300	400
35	2	45	3	70	105	90	135
25	2	30	3	50	75	60	90
50	30	54	35	1500	1750	1620	1890
35	2	40	3	70	105	80	120
				1930	2355	2150	2635

$$P_{01}^{DB} = \frac{\dfrac{\sum p_1 q_0}{\sum p_0 q_0} + \dfrac{\sum p_1 q_1}{\sum p_0 q_1}}{2} \times 100$$

$$= \frac{1}{2}\left[\frac{2150}{1930} + \frac{2635}{2355}\right] \times 100$$

$$= \frac{1}{2}[1.1139 + 1.1188] \times 100$$

$$= \frac{1}{2}[2.2327] \times 100$$

= 1.1164 x 100 = 111.64

Example 6.9

Compute Marshall – Edgeworth price index number for the following data by taking 2016 as base year.

Items sold out in a men's wear	2016		2017	
	Prices	Quantity	Prices	Quantity
Shirts	700	150	900	175
Pants	1000	100	1200	150
Sandals	500	70	600	100
Shoes'	1500	50	1800	60
Belts	400	100	600	150
Watches	1200	300	1500	250

Solution: Price Index by Marshall-Edgeworth Method

p_0	q_0	p_1	q_1	$p_0 q_0$	$p_0 q_1$	$p_1 q_0$	$p_1 q_1$
700	150	900	175	105000	122500	135000	157500
1000	100	1200	150	100000	150000	120000	180000
500	70	600	100	35000	50000	42000	60000
1500	50	1800	60	75000	90000	90000	108000
400	100	600	150	40000	60000	60000	90000
1200	300	1500	250	360000	300000	450000	375000
				715000	772500	897000	970500

Marshall – Edgeworth Index:

$$P_{01}^{ME} = \frac{\sum p_1q_0 + \sum p_1q_1}{\sum p_0q_0 + \sum p_0q_1} \times 100$$

$$= \frac{897000 + 970500}{715000 + 772500} \times 100$$

$$= \frac{1867500}{1487500} \times 100$$

$$= 125.55$$

Example 6.10

Calculate a suitable price index form the following data.

Commodity	Quantity	Price	
		2007	2010
X	25	3	4
Y	12	5	7
Z	10	6	5

Solution:

In this problem, the quantities for both current year and base year are same. Hence, we can conlude Kelly's Index price number.

Commodity	q	p_0	p_1	p_0q	p_1q
X	25	3	4	75	100
Y	12	5	7	60	84
Z	10	6	5	60	50
				195	234

Kelly's price Index number:

$$P_{01}^{K} = \frac{\sum p_1q}{\sum p_0q} \times 100$$

$$= \frac{234}{195} \times 100$$

$$= 120$$

6.4.2 Weighted average of price relatives

The weighted average of price relatives can be computed by introducing weights into the unweighted price relatives. Here also, we may use either arithmetic mean or the geometric mean for the purpose of averaging weighted price relatives.

The weighted average price relatives using arithmetic mean:

If $p = \frac{p_1}{p_0} \times 100$ is the price relative index and $w = p_0q_0$ is attached to the commodity, then the weighed price relative index is

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$$P_{01} = \frac{\sum\left[\frac{p_1}{p_0}\times 100\right]\times p_0q_0}{\sum p_0q_0} \quad = \quad P_{01} = \frac{\sum wp}{\sum w}$$

The weighted average price relatives using geometric mean:

$$P_{01} = \text{antilog}\left(\frac{\sum w \log p}{\sum w}\right)$$

Example 6.11

Compute price index for the following data by applying weighted average of price relatives method using (i) Arithmetic mean and (ii) Geometric mean.

Item	p_0	q_0	p_1
Wheat	3.0	20 kg	4.0
Flour	1.5	40 kg	1.6
Milk	1.0	10 kg	1.5

Solution:

(i) Computation for the weighted average of price relatives using arithmatic mean.

Item	p_0	q_0	p_1	w	p	$\log p$	wp	$w \log p$
Wheat	3.0	20	4.0	60	133.3	2.1249	7998	127.494
Flour	1.5	40	1.6	60	106.7	2.0282	6402	121.692
Milk	1.0	10	1.5	10	150.0	2.1761	1500	21.761
				$\sum w = 130$			$\sum wp = 15900$	$\sum w \log p = 270.947$

$$P_{01} = \frac{\sum wp}{\sum w} = \frac{15,900}{130} = 122.31$$

This means that there has been a 22.31 % increase in prices over the base year.

(ii) Index number using geometric mean of price relatives is:

$$P_{01} = \text{Antilog}\ \frac{\sum w \log p}{\sum w} = \text{Antilog}\ \frac{270.947}{130}$$

$$= \text{Antilog } (2.084) = 121.3$$

This means that there has been a 21.3 % increase in prices over the base year.

6.4.3 Quantity Index Number

The quantity index number measures the changes in the level of quantities of items consumed, or produced, or distributed during a year under study with reference to another year known as the base year.

Laspeyre's quantity index:

$$Q_{01}^{L} = \frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100$$

Paasche's quantity index

$$Q_{01}^{P} = \frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100$$

Fisher's quantity index

$$Q_{01}^{F} = \sqrt{Q_{01}^{L} \times Q_{01}^{P}}$$

$$= \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}} \times 100$$

These formulae represent the quantity index in which quantities of the different commodities are weighted by their prices.

Example 6.12

Compute the following quantity indices from the data given below:

(i) Laspeyre's quantity index (ii) Paasche's quantity index and (iii) Fisher's quantity index

Commodity	1970		1980	
	Price	Total value	Price	Total value
A	10	80	11	110
B	15	90	9	108
C	8	96	17	340

Solution:

Since we are given the value and the prices, the quantity figures can be obtained by dividing the value by the price for each of the commodities.

Commodity	p_0	q_0	p_1	q_1	p_0q_0	p_1q_0	p_0q_1	p_1q_1
A	10	8	11	10	80	88	100	110
B	15	6	9	12	90	54	180	108
C	8	12	17	20	96	204	160	340
	Total				266	342	440	558

(i) Laspeyre's quantity index

$$Q_{01}^{L} = \frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100$$

$$= \frac{440}{266} \times 100$$

$$= 165.4$$

(ii) Paasche's quantity index

$$Q_{01}^{P} = \frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100$$

$$= \frac{558}{342} \times 100$$

$$= 163.15$$

(iii) Fisher's quantity index

$$Q_{01}^{F} = \sqrt{Q_{01}^{L} \times Q_{01}^{P}}$$

$$= \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}} \times 100$$

$$= \sqrt{\frac{440}{266} \times \frac{558}{342}} \times 100$$

$$= 1.6428 \times 100$$

$$= 164.28$$

6.4.4 Tests for Index numbers

Fisher has given some criteria that a good index number has to satisfy. They are called (i) Time reversal test (ii) Factor reversal test (iii) Circular test. Fisher has constructed in such a way that this index number satisfies all these tests and hence it is called Fisher's Ideal Index number.

Time reversal test

Fisher has pointed out that a formula for an index number should maintain time consistency by working both forward and backward with respect to time. This is called time reversal test. Fisher describes this test as follows.

"The test is that the formula for calculating an index number should be such that it gives the same ratio between one point of comparison and the other, no matter which of the two is taken as base or putting in another way the index number reckoned forward should be the reciprocal of that reckoned back ward". A good index number shoud satisfy the time reversal test.

This statement is expressed in the form of equation as $P_{01} \times P_{10} = 1$.

where

$$P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}}$$

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$$P_{10} = \sqrt{\frac{\sum p_0 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum p_0 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0}}$$

Hence, $P_{01} \times P_{10} = \sqrt{1} = 1$

Factor reversal test

This test is also suggested by Fisher According to the factor reversal test, the product of price index and quantity index should be equal to the corresponding value index.

In Fisher's words "Just as each formula should permit the interchange of two times without giving inconsistent results so it ought to permit interchanging the prices and quantities without giving inconsistent results. i.e, the two results multiplied together should give the true ratio".

This statement is expressed as follows:

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

$$\text{Now, } P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}}$$

$$Q_{01} = \sqrt{\frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_1 q_0}}$$

$$\text{Hence, } P_{01} \times Q_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_1 q_0}}$$

$$= \sqrt{\frac{\sum p_1 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_0}}$$

$$= \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

Circular Test

It is an extension of time reversal test. The time reversal test takes into account only two years. The current and base years. The circular test would require this property to holdgood for any two years. An index number is said to satisfy the circular test when there are three indices, P_{01}, P_{12} and P_{20}, such that $P_{01} \times P_{12} \times P_{20} = 1$.

Laspeyres, Paasche's and Fisher's ideal index numbers do not satisfy this test.

Example 6.13

The table below gives the prices of base year and current year of 5 commodities with their quantities. Use it to verify whether Fisher's ideal index satisfies time reversal test.

Commodity	Base year		Current year	
	Unit price (₹)	Quantity	Unit price (₹)	Quantity
A	4	40	5	60
B	5	50	10	70
C	8	65	12	80
D	6	20	6	90
E	7	30	10	75

Solution:

Index number by Fisher's ideal index method

Commodity	p_0	q_0	p_1	q_1	p_0q_0	p_0q_1	p_1q_0	p_1q_1
A	4	40	5	60	160	240	200	300
B	5	50	10	70	250	350	500	700
C	8	65	12	80	520	640	780	960
D	6	20	6	90	120	540	120	540
E	7	30	10	75	210	525	300	750
					1260	2295	1900	3250

$$P_{01} = \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}}$$

$$= \sqrt{\frac{1900}{1260} \times \frac{3250}{2295}}$$

$$P_{10} = \sqrt{\frac{\sum q_0 p_1}{\sum q_1 p_1} \times \frac{\sum q_0 p_0}{\sum q_1 p_0}}$$

$$= \sqrt{\frac{2295}{3250} \times \frac{1260}{1900}}$$

Hence, $P_{01} \times P_{10} = \sqrt{\frac{1900}{1260} \times \frac{3250}{2295} \times \frac{2295}{3250} \times \frac{1260}{1900}}$

$$= \sqrt{1} = 1$$

Fisher's Index number satisties time reveral test.

Example 6.14

Calculate the price index and quantity index for the following data by Fisher's ideal formula and verify that it statisfies the factor reversal test.

Commodity	Base year		Current year	
	Price (₹)	Quantity (`000 tonnes)	Price (₹)	Quantity (`000 tonnes)
A	40	70	40	32
B	50	84	30	80
C	60	58	25	50

Solution

Commodity	p_0	q_0	p_1	q_1	p_1q_1	p_1q_0	p_0q_0	$p_0\,q_1$
A	40	70	40	32	1280	2800	2800	1280
B	50	84	30	80	2400	2520	4200	4000
C	60	58	25	50	1250	1450	3480	3000
					5930	6770	10480	8280

Factor Reversal test: $P_{01} \times Q_{01} = \dfrac{\sum p_1q_1}{\sum p_0q_0}$

$$P_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}}$$

$$Q_{01} = \sqrt{\frac{\sum q_1p_0}{\sum q_0p_0} \times \frac{\sum q_1p_1}{\sum q_0p_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1} \times \frac{\sum q_1p_0}{\sum q_0p_0} \times \frac{\sum q_1p_1}{\sum q_0p_1}}$$

$$= \sqrt{\frac{6770}{10480} \times \frac{5930}{8280} \times \frac{8280}{10480} \times \frac{5930}{6770}}$$

$$= \left(\sqrt{\frac{5930}{10480}}\right)^2$$

$$= \frac{5930}{10480}$$

$$= \frac{\sum p_1q_1}{\sum p_0q_0}$$

Hence, Fisher ideal index number satisfies the factor reversal test

6.5 CONSUMER PRICE INDEX NUMBERS

Consumer Price Index Numbers are computed with a view of study the effect of changes in prices on the people as consumers. These indices give the average increase in the expenses if it is designed to maintain the standard of living of base year. General index numbers fail to give an indea about the effect of the change in the general price level on the cost of living of different classes of people since a given change in the price level affects different classes of people differently.

The consumer price indices are of great significance and is given below

1. This is very useful in wage negotiations, wage contracts and dearness allowance adjustments in many countries.
2. At Government level the index numbers are used for wage policy, price policy, rent control, taxation and general economic policies.
3. Change in the purchasing power of money and real income can be measured.
4. Index numbers are also used for analyzing market price for particular kind of goods and services.

Note: *Consumer price index numbers are also called as cost of living index numbers.*

Methods of constructing consumer price Index

There are two methods of constructing consumer price index. They are:

1. Aggregate Expenditure method (or) Aggregate method
2. Family Budget method or method of weighted relative method.

1. Aggregate Expenditure method

This method is based upon the Laspeyre's method. It is widely used. The quantities of commodities consumed by a particular group in the base year are the weight.

Thus, consumer price index number $= \dfrac{\sum p_1q_0}{\sum p_0q_0} \times 100$

2. Family budget method (or) Method of weight relatives method

This method estimates an aggregate expenditure of an average family on various items and it is weighted. It is given by

consumer price index $= \dfrac{\sum wp}{\sum w}$

where

$$p = \frac{p_1}{p_0} \times 100 \text{ for each item and } w = p_0q_0$$

The family budget method is the same as "weighted average price relative method" which we have studied earlier.

Example 6.15

Calculate the consumer price index number for 2015 on the basis of 2000 from the following data by using (i) the Aggregate expenditure method (ii) the family budget (or) weighted relatives method.

Commodity	Quantity	Price	
		2000	2015
Wheat	20	15	20
Rice	8	20	24
Ghee	2	160	200
Sugar	4	40	40

Solution

(i) Calculation of cost of living index number on the basis of Aggregate expenditure method.

Commodity	q_0	p_0	p_1	p_0q_0	p_1q_0
Wheat	20	15	20	300	400
Rice	8	20	24	160	192
Ghee	2	160	200	320	400
Sugar	4	40	40	160	160
Total				940	1152

Consumer price index number for 2015

$$P_{01} = \frac{\sum p_1q_0}{\sum p_0q_0} \times 100$$

$$= \frac{1152}{940} \times 100$$

$$\approx 112.6$$

(ii) Calculation of consumer price index number according to family budget method or weighted relative method

Commodity	q_0	p_0	p_1	$p = \frac{p_1}{p_0} \times 100$	$w = p_0q_0$	wp
Wheat	20	15	20	400/3	300	40000
Rice	8	20	24	120	160	19200
Sugar	2	160	200	125	320	40000
Ghee	4	40	40	100	160	16000
					940	115200

Consumer price index number for 2015

$$P_{01} = \frac{\sum wp}{\sum w} = \frac{115200}{940}$$

$$\approx 122.6$$

POINTS TO REMEMBER

- ❖ Index numbers are barometers of an Economy.
- ❖ It is a specilized average designed to measure the changes in a group of variables over time.
- ❖ The different types of Index Numbers are Price Index Number, Quantity Index Number and Value Index Number.
- ❖ Index numbers are classified as simple aggregative and weighted aggregative.
- ❖ The base period must be free from natural calamities.
- ❖ Laspeyeres, Paasches, Dorbish and Bowley, Fisher's ideal and Kelly's are weighed index numbers.
- ❖ Index numbers generally satisfied three tests – Time reversal, factor reversal and circular.
- ❖ Fisher's ideal index number satisfies both time and factor reversal tests.
- ❖ Many index numbers do not satisfy circular test.
- ❖ Cost of living index numbers is useful to the Government for policy making etc.

EXERCISE 6

I. Choose the best answer.

1) In simple aggregate method, the aggregate price of all items in the given year is expressed as percentage of the same in the
   (a) current year (b) base year
   (c) Quarterly (d) half yearly

2) If the index for 1990 to the base 1980 is 250, the index number for 1980 to the base 1990 is
   (a) 4 (b) 400 (c) 40 (d) 4000

3) If Laspeyre's price index is 324 and Paasche's price index is 144, then Fisher's ideal index is
   (a) 234 (b) 243 (c) 261 (d) 216

4) The index that satisfies factor reversal test is
   (a) Paasche's Index (b) Laspeyre's Index
   (c) Fisher's Ideal Index (d) Walsh price index

5) The Dorbish-Bowley's price index is the
   (a) geometric mean of Laspeyre's and Paasche's Price indices
   (b) arithmetic mean of Laspeyre's and Paasche's Price indices
   (c) weighted mean of Laspeyre's and Paasche's Price indices
   (d) weighted mean of Laspeyre's and Paasche's quantity indices

6) The condition for the time reversal test to hold good with usal notation is
   (a) $P_{01} \times P_{10} = 1$ (b) $P_{01} - P_{10} = 1$ (c) $P_{01} + P_{10} = 1$ (d) $P_{01}/P_{10} = 1$

7) The geometric mean of Laspeyre's and Paasche's price indices is also known as
   (a) Dorbish – Bowley's price index (b) Kelly's price index
   (c) Fisher's price index (d) Walsh price index

8) The index number for 1985 to the base 1980 is 125 and for 1980 to the base1985 is 80. The given indices satisfy
   (a) circular test (b) factor reversal test
   (c) time reversal test (d) Marshall-Edgeworth test

9) The consumer price index numbers for 1981 and 1982 to the base 1974 are 320 and 400 respectively. The consumer price index for 1981 to the base 1982 is
   (a)80 (b)128 (c)125 (d) 85

10) The consumer price index in 2000 increases by 80% as compared to the base 1990. A person I 1990 getting Rs. 60,000 per annum should now get:
   (a)Rs. 1,08,000 p.a. (b)Rs. 1,02,000 p.a.
   (c)Rs. 1,18,000 p.a. (d) Rs. 1,80,000 p.a.

II. Give very short answer to the following questions.

11. Define index number?
12. Write the uses of Index numbers.
13. Define base period.
14. State the types of Index numbers.
15. Point out the difference between weighted and unweighted index numbers.
16. Define weighted index number.
17. What is circular test?
18. State the methods of constructing consumer price index.

III. Give short answer to the following questions.

19. Give the diagrammatic representation of different types of index number.
20. Write the advantages of average price index.
21. State the methods of weighted aggregate index numbers.
22. What is the difference between the price index and quantity index numbers?
23. Write short notes on consumer price index.

24. Calculate index number from the following data by simple aggregate method taking prices of 2015 as base.

Commodity	Units	Price per kg	
		2015	2016
Wheat	Quintal	200	250
Rice	Quintal	300	400
Pulses	Quintal	400	500
Milk	Litre	2	3
Clothing	Meter	3	5

25. Compute (i) Laspeyre's (ii) Paasche's index numbers for 2010 from the following

Commodity	Price		Quantity	
	2002	2010	2002	2010
A	4	6	8	7
B	3	5	10	8
C	2	4	14	12
D	5	7	19	11

26. Calculate Fisher's ideal index method for the following data.

Commodity	2000		2001	
	Price	Quantity	Price	Quantity
A	2	7	3	5
B	5	11	6	10
C	3	14	5	11
D	4	16	4	18

IV. Give detail answer to the following questions.

27. Calculate the simple aggregate price index for the year 2013, and 2014 taking 2012 as the base year.

Categories of employees	Salary per month		
	2012	2013	2014
A	6000	6500	7200
B	12000	14000	16000
C	50000	64000	80000
D	70000	78000	84000

28 Construct the price indices from the following data by applying (1) Laspeyre's method (2) Paasche's method and (3) Fisher ideal number by taking 2010 as the base year.

Commodity	2010		2011	
	Price (₹)	Quantity	Price (₹)	Quantity
A	15	15	22	12
B	20	5	27	4
C	4	10	7	5

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29. Construct (1) Laspeyre's index, (2) Paasche's index, (3) Marshall-Edgeworth index, and (4) Fisher ideal index for the following data taking 2014 as base year

Items	year 2014		year 2015	
	Price	Quantity	Price	Quantity
A	6	50	10	56
B	2	100	2	120
C	4	60	6	60
D	10	30	12	24
E	8	40	12	36

30. Construct Marshall–Edgeworth price index number for the following data taking 2016 as base year

Commodity	year 2016		year 2017	
	Price	Quantity	Price	Quantity
A	4	58500	6	62000
B	3.5	15630	5.5	13050
C	3	26230	5	25000
D	2.5	11360	4	10000
E	2	30000	3	31500

31. A popular consumer co-operative store located in a labour colory reported the average monthly data on prices and quantities sold of a group of selected items of mass consumption as follows.

Items	Jan 2015		Jan 2018	
	Prices ₹ Per kg	Quantity sold kg	Prices Per kg	Quantity sold kg
Veg oil	26	40	31	45
Sugar	28	90	32	100
Rice	16	120	19	20
Wheat	15	110	18	130

Compute the following indices.

(a) Laspeyre's price index for 2018 using 2015 us base year.

(b) Paache's price index for 2018 using 2015 as base year.

32. From the data given, in problem. obtain the following

(a) Laspeyre's quantity index for 2018 using 2015 as the base year.

(b) Paasche's quantity index for 2018 using 2015 as the base

(c) Compute Index number using Fisher's formula and show it satisfies time reversal test and factor reversal test

Commodity	Base year		Current year	
	Price	Quantity	Price	Quantity
A	10	12	12	15
B	7	15	5	20
C	5	24	9	20
D	16	5	14	5

33. Construct the consumer price index number of 2015 on the from the following data using (i) the average expenditure method and (ii) the family budget method.

Commodity	Quantity consumed in 2014	Price in 2014	Price in 2015
A	6 Quintal	5	6
B	6 Quintal	6	7
C	1 Quintal	5	6
D	6 Quintal	6	7
E	4 kg	7	8
F	6 kg	8	9

34. An enquiry into the budgets of the middle class families in a city in India gave the following information.

Expenses on	Food	Rent	Clothing	Fuel	Mise
	35%	15%	20%	10%	20%
Price in 2014	450	90	225	75	120
Price in 2015	435	90	195	69	135

What change in the cost of living figures of 2015 has taken place as compared to 2014?

35. Construct the cost of living index of 2014 using family budget method.

Expenses	%	base year (2000)	year 2004
Food	40	150	174
Rent	15	50	60
Clothing	15	100	125
Fuel	10	20	25
Misc	20	60	90

36. Construct the index of 2014 from the following data for the year 2012 taking 2011 as base year as base using i) arithmetic mean and ii) geometric mean.

Item	Price (₹) in 2014	Price (₹) in 2015
A	6	10
B	2	2
C	4	6
D	10	12
E	8	12

37. Compute price index for the following data by applying weighted average of price relative method using i) arithmetic mean and ii) geometric mean.

Item	Price (₹) in 2006	Price (₹) in 2007	Quantity in 1996
A	2	2.5	40
B	3	3.25	20
C	1.5	1.75	10

ANSWERS

I. **1.** (b) **2.** (c) **3.** (d) **4.** (c) **5.** (b)

6. (a) **7.** (c) **8.** (c) **9.** (a) **10.** (a)

II. 24. 127.96%

25. $P_{01}^{L} = 155.14$, $P_{01}^{P} = 158.01$

26. $P_{01}^{F} = 124.33$

III. 27. for 2013 price index = 117.75 and for 2014 price index = 135.65

28. $P_{01}^{L} = 146.5$ $P_{01}^{P} = 145.35$, $P_{01}^{F} = 145.96$

29. $P_{01}^{L} = 139.7$, $P_{01}^{P} = 139.8$, $P_{01}^{ME} = 139.8$, $P_{01}^{F} = 139.8$

30. $P_{01}^{ME} = 154.18$

31. $P_{01}^{L} = 117.53$ $P_{01}^{P} = 117.22$

32. a) $Q_{01}^{L} = 110.58$ b) Q_{01}^{P} 104.95 c) $Q_{01}^{F} = 107.73$, satisfies time reversal test and factor reversal test.

33. (i)115.84 (ii) 115.84

34. Cost of living index = 96.43% , there is a decrease of 3.57% as compared to the prices in the year 2014.

35. 122.12

36. $P_{01} = 137.34$, $P_{01} = 134.99$

37. $P_{01} = 117.74$, $P_{01} = 117.4$