7. Assignment problem and sequencing.

Exercise 7.1

1. A job production unit has four jobs P, Q, R, S Which can be manufactured on each of the four machine I, II, III And IV. The processing cost of each job for each machine Is given in the following table:

Jobs	Processing cost (in ₹)			
	Machines			
	I	II	III	IV
P	31	25	33	29
Q	25	24	23	21
R	19	21	23	24
S	38	36	34	40

Find the optimal assignment to minimize the total Processing cost.

Solution:

Step1. Subtract the smallest elements in each row from every element of it. New assignment matrix is obtained as follows:

Jobs	Processing cost (in ₹)			
	Machines			
	I	II	III	IV
P	6	0	8	4
Q	4	3	2	0
R	0	2	4	5
S	4	2	0	6

Step 2. Subtract the smallest elements in each column from every elements of it. New assignment matrix is obtained as above, because each column in it contains one zero.

Step 3. Cover all zeroes by minimum number of horizontal and vertical lines.

Jobs	Processing cost (in ₹) Machines			
	I	II	III	IV
P	6	0	8	4
Q	4	3	2	0
R	0	2	4	5
S	4	2	0	6

As the minimum number of straight lines required to Cover all zeros in the assignment matrix equals the Number of rows / columns. Optimal solution has Reached.

Step 4. Examine the rows one by one starting with the First row with exactly one zero is found. Mark the zero By enclosing it in (◻) *(in this case we are using **0**)* indicating assignment of the job.

Cross all the zeros in the same columns.

This step is shown in the following table:

Jobs	Processing cost (in ₹) Machines			
	I	II	III	IV
P	6	**0**	8	4
Q	4	3	2	**0**
R	**0**	2	4	5
S	4	2	**0**	6

It is observed that all the zeros are assigned and each row And each column contains exactly one assignment. Hence the optical (minimum) assignment schedule is.

Jobs	Machines	Processing cost (in ₹)
P	II	25
Q	IV	21
R	I	19
S	III	34

Hence, total (minimum) processing cost =
$25 + 21 + 19 + 34 =$ ₹ 99.

2. Five wagons are available at stations 1, 2, 3, 4 and 5.
These are required at 5 stations I, II, III, IV and V. The mileage between various stations are given in the table below. How should the wagons be transported so as to minimize the mileage covered?

	I	II	III	IV	V
1	10	5	9	18	11
2	13	9	6	12	14
3	7*	2	4	4	5
4	18	9	12	17	15
5	11	6	14	19	10

Solution:
Step 1. Subtract the smallest elements in each row from
Every element of that row.

Wagons	Mileage of Stations				
	I	II	III	IV	V
1	5	0	4	13	6
2	7	3	0	6	8
3	5	0	5	2	3
4	9	0	3	8	6
5	5	0	8	13	4

Step 2. Subtract the smallest elements of each column
From every elements of that column

Wagons	Mileage of Stations				
	I	II	III	IV	V
1	0	0	4	11	3
2	2	3	0	4	5
3	0	0	2	0	0
4	4	0	3	6	3
5	0	0	8	11	1

The number of lines covering all zeros (4) is not equal To order of matrix (5). So solution has not reached.

Step 3. Therefore, subtract the smallest uncovered Elements (1) from all uncovered elements and add it to All elements which lie at the intersection of two lines. All other elements on the line remain unchanged.

Wagons	Mileage of Stations				
	I	II	III	IV	V
1	0	0	4	10	2
2	2	3	0	3	4
3	1	1	3	0	0
4	4	0	3	5	2
5	0	0	8	10	0

The number of lines covering all zeros is equal to order of matrix.

Step 4. Hence, optical solution has reached. Therefore, the optimal assignment is made as follows:

Wagons	Mileage of Stations				
	I	II	III	IV	V
1	**0**	0	4	10	2
2	2	3	**0**	3	4
3	1	1	3	**0**	0
4	4	**0**	3	5	2
5	0	0	8	10	**0**

The optimal assignment is shown as follows:

Wagon	Station	Miles
1	I	10
2	III	6
3	IV	4
4	II	9
5	V	10

The minimum mileage covered = 10+6+4+9+10=39 miles

3. Five different machines can do any of the five required jobs, with different profits resulting from
Each assignment as shown below:

Wagons	Mileage of Stations				
	I	II	III	IV	V
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

Find the optimal assignment schedule.

Solution:

Step 1. Since, it is a maximization problem, subtract Each of the elements in the given matrix from the largest Elements, which is 62 here. The assignment matrix is obtained
As follows:

Jobs	Processing cost (in ₹)				
	Machines				
	A	B	C	D	E
1	32	25	22	34	22
2	22	38	35	41	26
3	22	30	29	32	27
4	37	24	22	26	26
5	33	0	21	28	23

Step 2. Subtract the smallest elements in each row from
The every elements of that row. We get

Jobs	Processing cost (in ₹)				
	Machines				
	A	**B**	**C**	**D**	**E**
1	10	3	0	12	0
2	0	16	13	19	4
3	0	8	7	10	5
4	15	2	0	4	4
5	33	0	21	28	23

Step 3. Subtract the smallest elements in each column From the every elements of that column. We get

Jobs	Processing cost (in ₹)				
	Machines				
	A	B	C	D	E
1	10	3	0	8	0
2	0	16	13	15	4
3	0	8	7	6	5
4	15	2	0	0	4
5	33	0	21	24	23

Step 4. Cover zeros elements with minimum number of Straight lines. We get

Jobs	Processing cost (in ₹)				
	Machines				
	A	**B**	**C**	**D**	**E**
1	10	3	0	8	0
2	0	16	13	15	4
3	0	8	7	6	5
4	15	2	0	0	4
5	33	0	21	24	23

Since, number of straight lines covering all zeros is not
Equal to number of rows/ columns optimum solution
Has not reached.

Step 5. Select the smallest elements among the uncovered Elements, which is 4 here. Subtract it from each elements Of the uncovered elements and add it to the elements At the intersection of two lines, we get

Jobs	Processing cost (in ₹) Machines				
	A	B	C	D	E
1	14	3	0	8	0
2	0	12	9	11	0
3	0	4	3	2	1
4	19	2	0	0	4
5	37	0	21	24	23

Optimum assignment can be made as follows:

Jobs	Processing cost (in ₹) Machines				
	A	B	C	D	E
1	14	3	**0**	8	~~0~~
2	~~0~~	12	9	11	**0**
3	**0**	4	3	2	1
4	19	2	~~0~~	**0**	4
5	37	**0**	21	24	23

Optimum solution is shown as follows:

Jobs	Machines	Profit (in ₹)
1	C	40
2	E	36
3	A	40
4	D	36
5	B	62

Hence, total (maximum) profit
= 40 + 36+ 40+ 36+ 62
= ₹ 214.

4. Four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant Place A, B, C, D and E available. Because of limited Space, machine M_2 cannot be placed at C and M_3 Cannot be placed at A. the cost matrix is given below.

Machine	Places				
	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	-	5	4
M_3	-	6	9	6	2
M_4	9	3	7	2	3

Find the optimal assignment schedule.

Solution:

As the number of machines is less than the number of Places, the problem is unbalanced. It is balanced by Introducing a dummy machine M_5 with zero cost. As M_2 cannot be placed at C and M_3 cannot be placed At A, a very high cost say ∞ is assigned to the corresponding Elements.

Machine	Places				
	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	∞	5	4
M_3	∞	6	9	6	2
M_4	9	3	7	2	3
M_5	0	0	0	0	0

Step 1. Minimum elements of each row is subtracted From every elements of that row.

Machine	Places				
	A	B	C	D	E
M_1	0	2	6	1	2
M_2	3	0	∞	1	0
M_3	∞	4	7	4	0
M_4	7	1	5	0	1
M_5	0	0	0	0	0

Step 2. Minimum elements of each row is subtracted from Every elements of that row.

Machine	Places				
	A	**B**	**C**	**D**	**E**
M_1	0	2	6	1	2
M_2	3	0	∞	1	0
M_3	∞	4	7	4	0
M_4	7	1	5	0	1
M_5	0	0	0	0	0

Step 3. Since, the number of lines covering zeros is 5 And is equal to order of matrix 5, the optical solution Has reached. Optimal assignment can be made as follows.

Machine	Places				
	A	B	C	D	E
M_1	**0**	2	6	1	2
M_2	3	**0**	∞	1	0
M_3	∞	4	7	4	**0**
M_4	7	1	5	**0**	1
M_5	0	0	**0**	0	0

The following optimal solution is obtained.

Machine	Places	Cost (in ₹)
M_1	A	4

M_2	B	4
M_3	C	2
M_4	D	2
M_5	E	0

Total cost = ₹ 12.

5. A company has a team of four salesman and there Are four district where the company wants to start? Its business. After taking into account the capabilities of salesman and the nature of district, the company Estimates that the profit per day in rupees for each Salesman in each district is as below.

Salesman	Districts			
	1	2	3	4
A	16	10	12	11
B	12	13	15	15
C	15	14	11	14
D	13	15	14	15

Find the assignment of salesman to various districts Which will yield maximum profit.

Solution:
Since, it is a maximization problem, subtract each of the elements In the matrix in the from the largest elements of the Matrix which is 16 here.

Salesman	Districts			
	1	2	3	4
A	0	6	4	5
B	4	3	1	1
C	1	1	5	2
D	3	2	2	1

Step 1. Subtract the minimum (Smallest) elements of each
Row from the elements of that row.

Salesman	Districts			
	1	2	3	4
A	0	6	4	5
B	3	2	0	0
C	0	0	4	1
D	2	1	1	0

Step 2. Subtract the smallest elements of each column
From the elements of that column.

Salesman	Districts			
	1	2	3	4
A	0	6	4	5
B	3	2	0	0
C	0	0	4	1
D	2	1	1	0

Step 3. Since, the number of lines covering zeros is 4 equal to the order of matrix

4. The optical solution has reached.)

Salesman	Districts			
	1	2	3	4
A	**0**	6	4	5
B	3	2	**0**	~~0~~
C	~~0~~	**0**	4	1
D	2	1	1	**0**

The following optimal solution is obtained.

Salesmen	Districts	Profit (in ₹)
A	1	16
B	3	15
C	2	15
D	4	15

Total profit = ₹ 61.

6. in the modification of a planet layout of a factory four New machine M_1, M_2, M_3, and M_4 are to be installed In a machine shop. There are five vacant places A, B, C, D And E available. Because of limited space machine M_2 cannot Be placed at c and machine M_3 cannot be placed at a. the cost Of locating a machine at a place (in hundred rupees) is a follows.

Machines	Locations				
	A	B	C	D	E
M_1	9	11	15	10	11
M_2	12	9	-	10	9
M_3	-	11	14	11	7
M_4	14	8	12	7	8

Find the optical assignment schedule.

Solution: As a number of machines is less than the number of Vacant places, the problem is unbalanced. It is balanced by Introduction of dummy machine M_5 with zero cost. Also machine M_2 cannot be placed at c and machine M_3 cannot be placed at A, a very high cost say ∞ is Assigned to the corresponding elements. We get

Machines	Locations				
	A	B	C	D	E
M_1	9	11	15	10	11
M_2	12	9	∞	10	9
M_3	∞	11	14	11	7
M_4	14	8	12	7	8
M_5	0	0	0	0	0

Subtract the smallest elements of each row from every Elements in that row. We get

Machines	Locations				
	A	B	C	D	E
M_1	0	2	6	1	2
M_2	3	0	∞	1	0
M_3	∞	4	7	4	0
M_4	7	1	5	0	1
M_5	0	0	0	0	0

Since, the smallest element in each column is zero, The resultant matrix is as given in the above table.
Since, number of straight lines covering all zeros is Equal to number of rows/ columns, the optimal solution has reached. The optimal assignment can be made as follows:

Machines	Locations				
	A	B	C	D	E
M_1	**0**	2	6	1	2
M_2	3	**0**	∞	1	0
M_3	∞	4	7	4	**0**
M_4	7	1	5	**0**	1
M_5	0	0	**0**	0	0

The following is the optimum solution obtained:

Machines	Locations	Cost (₹)
M_1	A	9
M_2	B	9
M_3	E	7
M_4	D	7

Total cost = ₹32

Exercise 7.2

1. A machine operator has to perform two operations, turning and threading on 6 different jobs. The time required to perform these operations (in minutes) for each job is known. Determine the order in which the jobs should be processed in order to minimize the total time required

To complete all the jobs. Also find the total processing time and idle times for turning and threading operations.

Jobs	1	2	3	4	5	6
Time for turning	3	12	5	2	9	11
Time for threading	8	10	9	6	3	1

Solution:

Jobs	Time required (in minutes)	
	For turning	For threading
1	3	8
2	12	10
3	5	9
4	2	6
5	9	3
6	11	1

Step 1. Here, min. $(M_{i1}, M_{i2}) = 1$, which corresponds to threading
Therefore, job 6 is operated at last.

					6

The problem now reduced to five jobs 1, 2, 3, 4, 5.
Here, min $(M_{i1}, M_{i2}) = 2$, which corresponds to turning
Therefore, job 4 is operated first of all for turning.

4					6

The problem now reduced to four jobs 1, 2, 3, 5.
Here, min $(M_{i1}, M_{i2}) = 3$, which corresponds to both Turning and threading.
Therefore, job 1 is operated first next to job 4 and job 5 is
Operated at last next to job 6.

4	1			5	6

The problem now reduced to two jobs 2 and 3 Here, min $(M_{i1}, M_{i2}) = 5$, which corresponding to turning
Therefore, job 3 is operated next to job 1.

4	1	3		5	6

Now, remaining job 2 is operated next to job 3. Thus, The optimal sequence of job is obtained as follows:

4	1	3	2	5	6

The minimum elapsed time can be computed as follow:

Jobs Sequences	For turning		For threading		Idle time for threading
	Time in	Time out	Time in	Time out	
4	0	2	2	8	2
1	2	5	8	16	0
3	5	10	16	25	0
2	10	22	25	35	0
5	22	31	35	38	4
6	31	42	42	43	0
Total idle time for threading					**06**

From the above table,
The minimum (optimum) total elapsed time
T = 43 minutes.
Idle time for turning
= T – Sum of the processing time of all six jobs on turning
= 43 – 42 = 1 minutes.
Idle time for threading = 6 months.

2. A company has three jobs in hand. Each of these must be processed through two departments, in the order AB where
Department A: Press shop and
Department B: Finishing
The table below give the number of days required
By each job in each department.

Jobs	I	II	III
Department A	8	6	5
Department B	8	3	4

Find the sequences in which the three jobs should be processed so as to take minimum time to finish
All the three jobs. Also find idle time for the both the departments.

Jobs	Department	
	A	B
I	8	8
II	6	3
III	5	4

Step 1. Min (M_{i1}, M_{i2}) = 3 which corresponds of Department B. Therefore job II is proceed in the last.

		II

The problem now reduced to two jobs me and III. Here, min. (M_{i1}, M_{i2}) = 4 which corresponds to the
Department B. Therefore, job III is processed in the last next to job III and then job I is processed.
Thus, the optimal sequences of jobs is obtained as follows.

I	III	II

Minimum elapsed time can be computed as follows.

Jobs Sequences	Department A		Department B		Idle time for B
	Time in	Time out	Time in	Time out	
I	0	8	8	16	8
III	8	13	16	20	0
II	13	19	20	23	0
Total idle time for threading B					8

From the above table, minimum total elapsed time to finish all three jobs
T= 23 days
Idle time for the department A
= T – {sum of the processing time to finish
All jobs in A}
= 23 – 19 = 4 Days
Idle time of the department B = 8 days

3. An insurance company receive three type of policy application bundles daily from its head office for data entry and tiling, the time (in minutes) required for each type for these two operations is
Given in the following table:

Policy	1	2	3
Data entry	90	120	180
Filing	140	110	100

Find the sequences that minimizes the total time required to complete the entire task. Also find the total elapsed time idle time for each operation:
Solution:

Policy	Time required in minutes	
	Data Entry	Filing
I	90	140
II	120	110
III	180	100

Min (M_{i1}, M_{i2}) = 90 which corresponds to job data entry
Therefore, policy 1 is processed first in sequences.

1		

The problem now reduced to two positions 2 and 3.
Here, min (M_{i1}, M_{i2}) = 100, which corresponds to job Filing. Therefore, policy 2 is placed last.

1		3

Now, min (M_{i1}, M_{i2}) = 110, which corresponds to job Filing. Therefore, policy 2 is placed next to policy 1,

1	2	3

Hence, the optimal sequences is 1 → 2 → 3
Total elapsed time to obtain as follows.

Policy Sequences	Data Entry		Filing		Idle time for filing
	Time in	Time out	Time in	Time out	
I	0	90	90	230	90
III	90	210	230	340	0
II	210	390	390	490	50
Total idle time for filing					**140**

Total elapsed time T = 490 minutes. Idle time for the data entry
= T – total time of data entry
= 490 – 390 = 100 minutes
Idle time for filling = 90 + 50
= 140 minutes.

4. There are five jobs, each of which must go through two machines in the order XY. Processing times (in hours) are given below. Determine the sequence for the jobs that will minimize the total elapsed time. Also find the Total elapsed time and idle time for each machine.

Jobs	A	B	C	D	E
Machine X	10	2	18	6	20
Machine Y	4	12	14	16	8

Solution:

Jobs	Processing time (in hours)	
	Machine X	Machine Y
A	10	4
B	2	12
C	18	14
D	6	16
E	20	8

Min (M_{i1}, M_{i2}) = 2, which corresponds to machine X.
Therefore, job B is processed first.

B				

The problem now reduced to jobs A, C, D, and E.
Here, min (M_{i1}, M_{i2}) = 4, which corresponds to machine Y.
Therefore jobs A is processed last.

B				

The problem now reduces to jobs C, D, and E.
Here, Min. (M_1, M_2) = 6, which corresponds to Machine X.
Therefore, job D is processed next to job B.

B				

The problem now reduces to jobs C and E.
Here, Min. (M_1, M_2) = 8, which corresponds to Machine Y.
Therefore, job E is processed last next to job A and job C is
Processed next to job D.

B	D	C	E	A

Total elapsed time is obtained as follows.

Job Sequences	Machine X		Machine Y		Idle time for Machine Y
	Time in	Time out	Time in	Time out	
B	0	2	2	14	2
D	2	8	14	30	0
C	8	26	30	44	0
E	26	46	46	54	2
A	46	56	56	60	2
Total idle time for Y					**6**

Total elapsed time T = 60 hours.
Idle time for machine Y = 6 hours.
Idle time for machine X
= T- total processing time of X.

= 60 – 56
= 4 HOURS.

5. Find the sequence that minimizes the total elapsed time to complete the following jobs in the order AB.
Find the total elapsed time and idle times for both the machines.

Jobs	I	II	III	IV	V	VI	VII
Machine A	7	16	19	10	14	15	5
Machine B	12	14	14	10	16	5	7

Solution:

Jobs	Time	
	Machine A	Machine B
I	7	12
II	16	14
III	19	14
IV	10	10
V	14	16
VI	15	5
VII	5	7

Here, Min. $(M_{i1}, M_{i2}) = 5$, which corresponds to both Machines A and B.
Therefore, job VII is processed first and job V1 is processed last.

VII						VI

The problem now reduces to jobs me, II, III, IV and V.
Here, Min. $(M_{i1}, M_{i2}) = 7$, which corresponds to machine A.
Therefore, job I is processed next to job VII.

VII	I					VI

The problem now reduces to jobs II, III, IV and V.
Here, Min. $(M_{i1}\ M_{i2}) = 10$, which corresponds to both
Machines A and 3. Therefore, job IV is processed next to job I.

VII	I	IV				VI

The problem now reduces to jobs II, III and V.
Here, Min. (M_{i1} M_{i2}) = 14, which corresponds to both
Machines A and B. Therefore, job V is processed next to job IV.

VII	I	IV	V			VI

The problem now reduces to jobs II and III.
Here, Min. (M_{i1} M_{i2}) = 14, which corresponds to Machines B.
Therefore, job II is processed in the last next to job VI
And job III is processed last next to job II.

VII	I	IV	V	III	II	VI

Total elapsed time is obtained as follows.

Job Sequences	Machine A		Machine B		Idle time for Machine B
	Time in	Time out	Time in	Time out	
VII	0	5	5	12	5
I	5	12	12	24	0
IV	12	22	24	34	0
V	22	36	36	52	2
III	36	55	55	69	3
II	55	71	71	85	2
VI	71	86	86	91	1
Total idle time for B					**13**

Optimal sequences of jobs is
VII → I → IV → V → III → II → VI
Total elapsed time T = 91 units
Idle time for machine B = 13 units
Idle time for machine A
= T – processing time of A
= 91 - 86
= 5 units.

6. Find the Optimal sequence that minimizes total time Required to complete the following jobs in the order ABC. The processing times are given in hours.

I)

Jobs	I	II	III	IV	V	VI	VII

Machine A	6	7	5	11	6	7	12
Machine B	4	3	2	5	1	5	3
Machine C	3	8	7	4	9	8	7

Solution:

Here, Min. (A) =5, Min. (C) =3 and Max. (B)=5.
Since, Min. (A) 2 Max. (B) Is satisfied, the problem can
Be converted into 7 jobs and 2 machine problem.
NOW, the two fictitious machines are such that
G=A+B and H=B+C
Then the problem can be written as

Jobs	Machines	
	G=A+B	H=B+C
~~I~~	~~10~~	~~7~~
II	10	11
~~III~~	~~7~~	~~9~~
~~IV~~	~~16~~	~~9~~
~~V~~	~~7~~	~~10~~
VI	12	13
VII	15	10

Here, Min. (G_i, H_i) = 7, which corresponds to both
Machines G and H.
Therefore, job III is processed first and job V is process
Second and job I is processed at the last.

III	V					I

OR

V	III					I

The problem now reduces to jobs II, IV, VI and VII.
Here, Min. (G_i, H_i) = 9, which corresponds to machine H.
Therefore, job IV is processed in the last next to job I.

III	V				IV	I

OR

V	III				IV	I

The problem now reduces to jobs II, VI and VII.
Here, Min. (G_i, H_i) = 10, which corresponds to both
Machine G and H.
Therefore job II is processed next to job V and job VII
Is processed in the last next to job IV.

III	V	II		VII	IV	I

OR

V	III	II		VII	IV	I

The problem new reduced to only one job VI. It is
Processed next to job II.
∴ The following optimal sequences is obtained.

III	V	II	VI	VII	IV	I

OR

V	III	II	VI	VII	IV	I

Total elapsed time is obtained as follow.

Job Sequences	Machine A		Machine B		Machine C		Idle time for Machine C
	Time in	Time out	Time in	Time out	Time in	Time out	
III	0	5	5	7	7	14	7
V	5	11	11	12	14	23	0
II	11	18	18	21	23	31	0
VI	18	25	25	30	31	39	0
VII	25	37	37	40	40	47	1
IV	37	48	48	53	53	57	6
I	48	54	54	58	58	61	1
Total idle time for Machine C							**15**

Total elapsed time T = 61 hours
Idle time for Machine A

= T Total processing time of Machine A
= 61 54 = 7 hours Idle time for Machine B
= T Total processing time of Machine B
= 61 23 = 38 hours
Idle time for Machine C = 15 hours.

ii)

Jobs	1	2	3	4	5
Machine A	5	7	6	9	5
Machine B	2	1	4	5	3
Machine C	3	7	5	6	7

Solution:
Here, min (A) =5, min (C) = 3, max. (B) = 5.
Since, Min. (A) 2 Max. (B) Is satisfied, the problem Can Be converted into 5 Jobs and 2 machine problem.
Now, the two fictitious machines are such that
G=A+B and H=B+C
The problem can be written as the following 5 jobs and 2 machine problem.

Jobs	Machines	
	G=A+B	H=B+C
1	7	5
2	8	8
3	10	9
4	14	11
5	8	10

Min. (G_{i1} H_{i2}) = 5, which corresponds to H.
Therefore, job1 is processed last.

				1

The problem now reduces to four jobs 2, 3, 4, 5. Here,
Min. (G_{i1}, H_{i2}) = 8, which corresponds to G and H both.
Therefore, job 2 is processed first of all and then job 5 is Processed.

2	5			1

Or 5, 2 ..., 1
The problem now reduces to two jobs 3 and 4. Here,
Min. (G_{i1}, H_{i2}) = 9, which corresponds to H. Therefore,
Job 3 is processed in the last next to job 1.

2	5		3	1

Or 5, 2 ...., 3, 1
Now, the remaining job 4 must be processed next to job 5.
Thus, the optimal sequence of jobs is obtained as
Follows:

2	5	4	3	1

Or 5, 2, 4, 3, 1
The minimum elapsed time can be computed as follows:

Job Sequences	Machine A		Machine B		Machine C		Idle time for Machine C
	Time in	Time out	Time in	Time out	Time in	Time out	
2	0	7	7	8	8	15	8
5	7	12	12	15	15	22	0
4	12	21	21	26	26	32	4
3	21	27	27	31	32	37	0
1	27	32	32	34	37	40	0
Total idle time for Machine C							**12**

From the above table:
The minimum (Optimum) total elapsed time
T = 40 hours.
Idle time for machine A
= T {Sum of processing time} of five jobs on A
= 40 32 = 8 hours Idle time for machine B
= T {Sum of processing time} of five jobs on B
=40-{2+1 +4+5+3}
= 40 -15 = 25 hours
Idle time for machine C = 12 hours.

7. A publisher produces 5 books on Mathematics. The books have to go through composing, printing

And binding done by 3 machines P, Q, R. The time schedule for the entire task in proper unit is as
Follows:

Books	A	B	C	D	E
Machine P	4	9	8	6	5
Machine Q	5	6	2	3	4
Machine R	8	10	6	7	11

Determine the optimum time required to finish the Entire task.

Solution:
Here, Min. (P) =4, Min. (R) =6 and Max. (Q)=6.
Since, Min. (R) 2 Max. (Q) Is satisfied, the problem can
Be converted into 5 jobs and 2 machine problem and two
Factious machines are G=P+Q and H=Q+R
The problem can be written as follows:

Jobs	Machines	
	G=P+Q	H=Q+R
A	9	13
B	15	16
C	10	8
D	9	10
E	9	15

Min (G_{i1}, H_{i2}) = 8, which corresponds to H.
Therefore, book C is processed at the last.

				C

The problem now reduced to four jobs A, B, D and E.
Here, min (G_{i1}, H_{i2}) = 9, which corresponds to G.
Therefore, either of the books A or D or E is processed
First of all and the remaining next to book A.

A	D	E		C

OR
A → E → D, D → A → E, E → A → D, D→ E → A, E → D → A.
Now the remaining book B is processed next to book E.
Thus, the optimal sequences of jobs is obtained as follows:

A	D	E	B	C

OR
A→E→D→B→C OR D→A→E→B→C OR
E→A→D→B→C OR D→E→A→B→S OR E→D→A→B→C.
Considering the first sequences of jobs, the minimum
Elapsed time can be computed as follows.

Job Sequences	Machine P		Machine Q		Machine R		Idle time for Machine R
	Time in	Time out	Time in	Time out	Time in	Time out	
A	0	4	4	9	9	17	9
D	4	10	10	13	17	24	0
E	10	15	15	19	24	35	4
B	15	24	24	30	35	45	0
C	24	32	32	34	45	51	0
Total idle time for Machine C							**9**

From the above table:
The minimum (optimum) total elapsed time
T= 51 hours.
Idle time for machine P
= T – sum of processing time of five jobs on P
= 51 – 32 = 19 hours.
Idle time for machine Q
= T – sum of processing time of five jobs on Q
= 51 – { 5 + 6 + 2 + 3 + 4 }
= 51 – 20 = 31 hours.
Idle time for machine R
= T – sum of processing time of five jobs on R
= 51 – {8 + 10 + 6 + 7 + 11} = 51 – 42
= 9 hours.