Quadratic Equations

INTRODUCTION

An equation of degree two is called a *quadratic equation*. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, c are real numbers, $a \neq 0$ and x is a real variable. Some examples of quadratic equations are $x^2 + 4x + 3 = 0$, $3x^2 - 4x + 5 = 0$ and $3x^2 + 2x - 3 = 0$.

Roots of a Quadratic Equation

A *root* of the equation $f(x) = 0$ is that value of x which makes $f(x) = 0$. In other words, $x = a$ is said to be a root of $f(x) = 0$, where $f(a)$ is the value of the polynomial $f(x)$ at $x = a$ and is obtained by replacing x by a in $f(x)$.

For example, -1 is a root of the quadratic equation $x^2 + 6x + 5 = 0$ because $(-1)^2 + 6(-1) + 5 = 0$.

Solution of a Quadratic Equation

If there is a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, the roots of this equation are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

Illustration 1 Solve the following quadratic equations

(i) $6x^2 + x - 2 = 0$

(ii) $2x^2 + x - 1 = 0$

Solution: (i) Using formula:

$$\text{The roots are } x = \frac{-1 \pm \sqrt{(1)^2 - 4(6)(-2)}}{2 \times 6}$$

$$= \frac{-1 \pm \sqrt{49}}{12} = \frac{6}{12}, \frac{-8}{12}$$

i.e., $\frac{1}{2}, \frac{-2}{3}$

Using factorization:

$$6x^2 + x - 2 = 0 \Leftrightarrow 6x^2 + 4x - 3x - 2 = 0$$

$$\Leftrightarrow 2x(3x+2) - 1(3x+2) = 0$$

$$\Leftrightarrow (2x+1)(3x+2) = 0$$

$$\Leftrightarrow x = \frac{1}{2} \text{ or, } x = -\frac{2}{3}$$

(ii) Using formula:

$$\text{The roots are } x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-1)}}{2 \times 2}$$

$$= \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4}$$

$$= \frac{2}{4}, \frac{-4}{4} \text{ i.e., } \frac{1}{2}, -1$$

Using factorization:

$$2x^2 + x - 1 = 0 \Leftrightarrow 2x^2 + 2x - x - 1 = 0$$

$$\Leftrightarrow 2x(x+1) - 1(x+1) = 0$$

$$\Leftrightarrow (2x-1)(x-1) = 0$$

$$\Leftrightarrow x = \frac{1}{2} \text{ or, } x = -1$$

Nature of Roots

A quadratic equation has exactly two roots may be real or imaginary or coincident.

If $ax^2 + bx + c$, $a \neq 0$, then $D = b^2 - 4ac$ is called *discriminant*.

1. If $D > 0$, then there are two distinct and real roots given by

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}, \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

2. If $D = 0$, then there is a repeated real root given by

$$\alpha = -\frac{b}{2a} \text{ i.e., roots are real and equal.}$$

3. If $D < 0$, then there are no real roots.

Note:

The roots are rational if $D > 0$ and D is a perfect square whereas the roots are irrational if $D > 0$ but D is not a perfect square.

Illustration 2 Find the nature of the roots of the equations

(i) $2x^2 + x - 1 = 0$

(ii) $x^2 + x + 1 = 0$

(iii) $x^2 + 5x + 5 = 0$

(iv) $\frac{4}{3}x^2 - 2x + \frac{3}{4} = 0$

Solution: (i) $D = (1)^2 - 4 \times 2 \times (-1) = 9 > 0$

Also, D is a perfect square.

So, the roots are real, distinct and rational.

(ii) $D = (1)^2 - 4 \times 1 \times 1 = -3 < 0$

So, the roots are imaginary.

(iii) $D = (5)^2 - 4 \times 1 \times 5 = 5 > 0$

Also, D is not a perfect square.

So, the roots are real, distinct and irrational.

(iv) $D = (-2)^2 - 4 \times \frac{4}{3} \times \frac{3}{4} = 0$

So, the roots are real and equal.

Illustration 3 For what value of k will the quadratic equation $kx^2 - 2\sqrt{5}\,x + 4 = 0$ have real and equal roots

Solution: $D = (-2\sqrt{5}\,)^2 - 4 \times k \times 4 = 20 - 16k$

The given equation will have real and equal roots if $D = 0$

i.e., $20 - 16k = 0$ or $k = \frac{20}{16} = \frac{5}{4}$

Note:

1. If $p + \sqrt{q}$ is a root of a quadratic equation, then its other root is $p - \sqrt{q}$.

Illustration 4 If $2 + \sqrt{3}$ is one root of a quadratic equation, find the other root

Solution: The other root is $2 - \sqrt{3}$

2. $ax^2 + bx + c$ can be expressed as a product of two linear factors only when $D \geq 0$

Illustration 5 For what value of k, the quadratic polynomial $kx^2 + 4x + 1$ can be factorized into two real linear factors

Solution: $D = (4)^2 - 4 \times k \times 1 = 16 - 4k$.

The given quadratic polynomial can be factorized into real linear factors if $D \geq 0$

i.e., $16 - 4k \geq 0$ or, $-4k \geq -6$ or $k \leq 4$

Relation Between Roots and Coefficients

Let α, β be the roots of the equation,

$$ax^2 + bx + c = 0$$

Then, sum of the roots

$$= \alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and product of the roots

$$= \alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Illustration 6 Find the sum and the product of the roots of the quadratic equation $2x^2 + 5\sqrt{3}\,x + 6 = 0$

Solution: Here, $a = 2$, $b = 5\sqrt{3}$, $c = 6$

$\therefore$ Sum of the roots $= -\frac{b}{a} = -\frac{5\sqrt{3}}{2}$

Product of the roots $= \frac{c}{a} = \frac{6}{2} = 3$

Formation of a Quadratic Equation with Given Roots

If α, β are the roots of a quadratic equation the equation can he written as

$$x^2 - (\alpha + \beta)\,x + \alpha\beta = 0$$

i.e., $x^2 -$ (sum of roots)x + product of roots = 0

Illustration 7 Find the quadratic equation whose roots are 5 and –6

Solution: Sum of roots $= 5 + (-6) = -1$,

Product of roots $= 5 \times (-6) = -30$

$\therefore$ The required quadratic equation is

$x^2 - (-1)x + (-30) = 0$ i.e., $x^2 + x - 30 = 0$

Practice Exercises

Difficulty Level-1
(Based on Memory)

1. If α and β are the roots of the quardratic equation $ax^2 + bx + c = 0$, then the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is:

(*a*) $\frac{2bc - a^3}{b^2c}$ (*b*) $\frac{3abc - b^3}{a^2c}$

(*c*) $\frac{3abc - b^2}{a^3c}$ (*d*) $\frac{ab - b^2c}{2b^2c}$

[Based on MAT, 2003]

2. If a, b are the two roots of a quadratic equation such that $a + b = 24$ and $a - b = 8$, then the quadratic equation having a and b as its roots is,

(*a*) $x^2 + 2x + 8 = 0$ (*b*) $x^2 - 4x + 8 = 0$

(*c*) $x^2 - 24x + 128 = 0$ (*d*) $2x^2 + 8x + 9 = 0$

[Based on MAT, 2003]

3. One-fourth of a herd of cows is in the forest. Twice the square root of the heard has gone to mountains and on the remaining 15 are on the banks of a river. The total number of cows is:

(*a*) 6 (*b*) 100

(*c*) 63 (*d*) 36

[Based on MAT, 2003]

4. If the roots of the equation

$$(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$$

for $a \neq 0$ are real and equal, then the value of $a^3 + b^3 + c^3$ is:

(*a*) abc (*b*) $3abc$

(*c*) zero (*d*) None of these

[Based on MAT, 2003]

5. If $2x^2 - 7xy + 3y^2 = 0$, then the value of $x:y$ is:

(*a*) 3:2 (*b*) 2:3

(*c*) 3:1 and 1:2 (*d*) 5:6

[Based on MAT, 2003]

6. If α and β are the roots of the equation $x^2 + 2x - 1 = 0$ and γ and δ are the roots of the equation $x^2 + 3x - 4 = 0$, then find the value of $(\alpha + \gamma)(\beta + \delta)(\alpha + \delta)(\beta + \gamma)$.

(*a*) –46 (*b*) –24

(*c*) 0 (*d*) –64

7. Find the quadratic equation whose roots are α and β, given that $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$ are the roots of the equation $14x^2 - 45x + 81 = 0$.

(*a*) $2x^2 - 8x + 7 = 0$ (*b*) $2x^2 - 5x + 7 = 0$

(*c*) $2x^2 - 8x + 5 = 0$ (*d*) $2x^2 - 5x + 5 = 0$

8. If α, β are the roots of $X^2 - 8X + P = 0$ and $\alpha^2 + \beta^2 = 40$, then the value of P is:

(*a*) 8 (*b*) 10

(*c*) 12 (*d*) 14

9. A man is $18X$ years old and his son is $2X^2$ years old. When he was $3X^2$ years old, his son was $X + 4$ years old. How old is he now?

(*a*) 68 years (*b*) 70 years

(*c*) 72 years (*d*) 74 years

10. In a family, eleven times the number of children is greater than twice the square of the number of children by 12. How many children are there?

(*a*) 3 (*b*) 4

(*c*) 2 (*d*) 5

11. For what values of k, the equation

$$x^2 + 2(k - 4)x + 2k = 0 \text{ has equal roots?}$$

(*a*) 8, 2 (*b*) 6, 4

(*c*) 12, 2 (*d*) 10, 4

[Based on IIFT, 2003]

12. The number of quadratic equations which are unchanged by squaring their roots is:

(*a*) 2 (*b*) 4

(*c*) 5 (*d*) 6

[Based on FMS (Delhi), 2002]

13. If α and β are the two roots of the equation $2x^2 - 7x - 3 = 0$, then find the value of $(\alpha + 2)(\beta + 2)$.

(*a*) 9 (*b*) –9.5

(*c*) 9.5 (*d*) 6

[Based on SCMHRD, 2002]

14. Given that α, γ are the roots of the equation $Ax^2 - 4x + 1 = 0$ and β, δ are the roots of the equation $Bx^2 - 6x + 1 = 0$, then the values of A and B, respectively, such that α, β, γ and δ are in H.P.

(*a*) –5, 9 (*b*) 3/2, 5

(*c*) 3, 8 (*d*) None of these

15. A class decided to have a party for their class at a total cost of ₹720. Four students decided to stay out of the party. To meet the expenses the remaining students have to increase their share by ₹9. What is the original cost per student?

(*a*) ₹18 (*b*) ₹24

(*c*) ₹36 (*d*) ₹20

[Based on MAT (May), 2010]

16. Students of a class are made to stand in rows. If 4 students are extra in each row, there would be 2 rows less. If 4 students are less in each row, there would be 4 more rows. The number of students in the class is:

(*a*) 90 (*b*) 94

(*c*) 92 (*d*) 96

[Based on MAT (Feb), 2006]

17. The solutions of the $\sqrt{25-x^2} = x-1$ equation are:

(*a*) $x = 3$ and $x = 4$ (*b*) $x = 5$ and $x = 1$

(*c*) $x = -3$ and $x = 4$ (*d*) $x = 4$ and $x = -3$

[Based on MAT, 1999]

18. The value of x satisfying the equation

$$\sqrt{2x+3} + \sqrt{2x-1} = 2 \text{ is:}$$

(*a*) 3 (*b*) 2

(*c*) 1 (*d*) $\frac{1}{2}$

[Based on MAT, 1999]

19. One-fourth of a herd of cows is in the forest. Twice the square root of the heard has gone to mountains and on the remaining 15 are on the banks of a river. The total number of cows is:

(*a*) 6 (*b*) 100

(*c*) 63 (*d*) 36

20. A positive number when decreased by 4, is equal to 21 times the reciprocal of the number. The number is:

(*a*) 3 (*b*) 5

(*c*) 7 (*d*) 9

[Based on MAT, 2000]

21. For which value of k does the following pair of equations yield a unique solution for x such that the solution is positive?

$$x^2 - y^2 = 0$$
$$(x-k)^2 + y^2 = 1$$

(*a*) 2 (*b*) 0

(*c*) $\sqrt{2}$ (*d*) $-\sqrt{2}$

22. The sum of all the roots of $4x^3 - 8x^2 - 63x - 9 = 0$ is:

(*a*) 8 (*b*) 2

(*c*) –8 (*d*) –2

[Based on FMS, 2011]

23. The solution of $\sqrt{5x-1} + \sqrt{x-1} = 2$ is:

(*a*) $x = 1$ (*b*) $x = 2$

(*c*) $x = \frac{2}{3}$ (*d*) $x = 2, x = 1$

[Based on FMS, 2011]

24. Let $y = \cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\ldots}}}}$. What is the value of y?

(a) $\frac{\sqrt{13}+3}{2}$ (b) $\frac{\sqrt{13}-3}{2}$

(*c*) $\frac{\sqrt{15}+3}{2}$ (*d*) $\frac{\sqrt{15}-3}{2}$

25. For what values of k, the equation $x^2 + 2(k-4)x + 2k = 0$ has equal roots?

(*a*) 8, 2 (*b*) 6, 4

(*c*) 12, 2 (*d*) 10, 4

26. If the sum of the roots of the quadratic equation $px^2 + qx + r = 0$ is equal to the sum of the square of their reciprocals, mark all the correct statements.

(*a*) r/p, p/q and q/r are in AP

(*b*) p/r, q/p and r/q are in GP

(*c*) p/r, q/p and r/q are in HP

(*d*) p/r, q/p and r/q are in AP

[Based on ITFT, 2006]

27. If one root of the equation $ax^2 + bx + c = 0$ is double of the other, then $2b^2$ is equal to:

(*a*) $9\,ca$ (*b*) $c\sqrt{2}\,a$

(*c*) $2\sqrt{3}\,ac$ (*d*) None of these

[Based on ITFT, 2008]

28. One of the roots of the equation $x^2 - x + 3m = 0$ is double of one of the roots of the equation $x^2 - x + m = 0$. If $m \neq 0$, then find its value.

(*a*) 1 (*b*) –1

(*c*) 2 (*d*) –2

29. If 4 is a solution of the equation $x^2 + 3x + k = 10$, where k is a constant, what is the other solution?

(*a*) –18 (*b*) –7

(*c*) –28 (*d*) None of these

[Based on ATMA, 2006]

30. The coefficient of x in the equation $x^2 + px + p = 0$ was wrongly written as 17 in place of 13 and the roots thus found were – 2 and – 15. The roots of the correct equation would be:

(*a*) –4, –9 (*b*) –3, –10

(*c*) –3, –9 (*d*) –4, –10

[Based on ATMA, 2006]

31. If the roots, x_1 and x_2, of the quadratic equation $x^2 - 2x + c = 0$ also satisfy the equation $7x_2 - 4x_1 = 47$, then which of the following is true?

(*a*) $c = -15$ (*b*) $x_1 = 5, x_2 = 3$

(*c*) $x_1 = 4.5, x_2 = -2.5$ (*d*) None of these

32. If $-2\sqrt{3}$ is a root of the equation $x^2 + px - 6 = 0$ and the equation $x^2 + px + q$ has equal roots, then the value of q is:

(*a*) $\frac{3}{4}$ (*b*) $-\sqrt{3}$

(*c*) $\frac{4}{3}$ (*d*) $\sqrt{3}$

[Based on ATMA, 2008]

33. At what value of x it will give the minimum value of $x^2 - 4x + 7$?

(*a*) $x \geq 3$ (*b*) $x = 2$

(*c*) $x < 2$ (*d*) $x = 0$

[Based on ATMA, 2008]

34. Which of the following equations has a root in common with $x^2 - 6x + 5 = 0$?

(*a*) $x^2 + 1 = 0$ (*b*) $x^2 - 10x - 5 = 0$

(*c*) $x^2 - 2x - 3 = 0$ (*d*) $2x^2 - 2 = 0$

[Based on ATMA, 2008]

35. If $(x - 3)(2x + 1) = 0$, then possible value of $2x + 1$ are:

(*a*) 0 only (*b*) 0 and 3

(*c*) $-\frac{1}{2}$ and 3 (*d*) 0 and 7

36. If the roots of the quadratic equation $y^2 + My + N$ are N and M, then find the possible number of pairs of (M, N).

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

[Based on CAT, 2009]

37. A manager is not used to work in the decimal system. She says that there are 100 employees in the office of which 2 are men and 32 are women. Which number system does the manager use?

(*a*) 4 (*b*) 6

(c) 8 (*d*) 16

[Based on CAT, 2009]

38. The equation $2x^2 + 2(p+1)x + p = 0$, where p is real, always has roots that are:

(*a*) Equal

(*b*) Equal in magnitude but opposite in sign

(*c*) Irrational

(*d*) Real

[Based on CAT, 2010]

39. If the roots of the equation $(a^2 + b^2)x^2 + 2(b^2 + c^2)x + (b^2 + c^2) = 0$ are real, which of the following must hold true?

(*a*) $c^2 \geq a^2$ (*b*) $c^4 \geq a^2(b^2 + c^2)$

(*c*) $b^2 \geq a^2$ (*d*) $a^4 \leq b^2(a^2 + c^2)$

[Based on CAT, 2012]

40. If p is a prime number and m is a positive integer, how many solutions exist for the equation $p^6 - p = (m^2 + m + 6)(p - 1)$?

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) Infinite

[Based on CAT, 2012]

41. Three consecutive positive integers are raised to the first, second and third powers, respectively and then added. The sum so obtained is a perfect square whose square root equals the total of the three original integers. Which of the following best describes the minimum, say m, of these three integers?

(*a*) $1 \leq m \leq 3$ (*b*) $4 \leq m \leq 6$

(*c*) $7 \leq m \leq 9$ (*d*) $10 \leq m \leq 12$

(*e*) $13 \leq m \leq 15$

[Based on, 2008]

42. Two teams participating in a competition had to take a test in a given time. Team B chose the easier test with 300 questions, and teams A difficult test with 10% less questions. Team A completed the test 3 hours before schedule while team B completed it 6 hours before schedule. If team B answered 7 questions more than team A per hour, how many questions did team A answer per hour?

[Based on MAT, 2013]

(*a*) 15 (*b*) 18

(*c*) 21 (*d*) 24

43. A polynomial $ax^3 + bx^2 + cx + d$ intersects x-axis at 1 and −1 and y-axis at 2. The value of b is:

(*a*) –2 (*b*) 0

(*c*) 1 (*d*) 2

(*e*) Cannot be determined

[Based on XAT, 2014]

Difficulty Level-2
(Based on Memory)

1. Let p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$. What is the minimum possible value of $p^2 + q^2$?

(*a*) 0 (*b*) 3

(*c*) 4 (*d*) 5

[Based on CAT, 2003]

2. If both a and b belong to the set {1, 2, 3, 4} then the number of equations of the form $ax^2 + bx + 1 = 0$ having real roots is:

(*a*) 10 (*b*) 7

(*c*) 6 (*d*) 12

[Based on CAT, 2004]

3. The angry Arjun carried some arrows to fight Bheeshma Pitamaha. Hiding behind Shikhandi with half the arrows, he perished the arrows thrown by Pitamaha on him and with six other arrows he killed the chariot driver (sarathi) of Pitamaha. Then with one arrow each, he knocked down the chariot, the flag on the chariot and the bow of Pitamaha and finally with one more than four times the square root of the arrows, he laid down the Pitamaha unconscious on the arrow bed. Assuming he used all the arrows with him, the total number of arrows with Arjun was:

(*a*) 4 (*b*) 49
(*c*) 100 (*d*) 144

4. If $f(x, y, z)$ = sum of z terms of the sequence $x, x + y, x + 2y$ ..., what is the value of z if $f(1, 1, z) = 21$?

(*a*) 3 (*b*) 4
(*c*) 6 (*d*) 7

5. If l, m, n are real and $l = m$, then the roots of the equations $(l - m)x^2 - 5(l + m)x - 2(l - m) = 0$ are:

(*a*) Real and equal (*b*) Complex
(*c*) Real and unequal (*d*) None of these

6. Which of the following expressions cannot be equal to zero, when $X^2 - 2X = 3$?

(*a*) $X^2 - 7X + 6$ (*b*) $X^2 - 9$
(*c*) $X^2 - 4X + 3$ (*d*) $X^2 - 6X + 9$

[Based on SCMHRD Ent. Exam., 2003]

Directions (Q. 7 to 12): *In each of these questions, two equations I and II are given. You have to solve both the equations and give answer (a), if $p < q$ (b), if $p \le q$; (c), if $p = q$, (d), if $p \ge q$ and (e), if $p > q$.*

7. I. $p^2 - 18p + 77 = 0$
II. $3q^2 - 25q + 28 = 0$ **[Based on IRMA, 2002]**

8. I. $6q^2 + q - 1 - 0$
II. $6p^2 - 7p + 2 = 0$ **[Based on IRMA, 2002]**

9. I. $7p^2 + 6p - 1 = 0$
II. $32q^2 - 20q + 3 = 0$ **[Based on IRMA, 2002]**

10. I. $4p^2 = 9$
II. $2q^2 - 9q + 10 = 0$ **[Based on IRMA, 2002]**

11. I. $2p^2 - 12p + 16 = 0$
II. $q2 - 9q + 20 = 0$ **[Based on IRMA, 2002]**

12. If is a root of the equation $x^2 + px + q = 0$, where p and q are real, then (p, q) is:

(*a*) (2, 3) (*b*) (–2, 3)
(*c*) (4, 7) (*d*) (–4, 7)

[Based on FMS (Delhi), 2002]

13. A quadratic function $f(x)$ attains a maximum of 3 at $x = 1$. The value of the function at $x = 0$ is 1. What is the value of $f(x)$ at $x = 10$?

(*a*) –110 (*b*) –180
(*c*) –105 (*d*) –159

[Based on CAT, 2007]

14. If a is an integer, then for how many integer values of n can the quadratic equation $x^2 - (2a + 3)x + 4^n = 0$ have real and equal roots for x?

(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

15. If p, r are positive and are in AP, the roots of quadratic equation $px^2 + qx + r = 0$ are real for:

(*a*) $\left|\frac{r}{p} - 7\right| \ge 4\sqrt{3}$ (*b*) $\left|\frac{p}{r} - 7\right| \ge 4\sqrt{3}$
(*c*) all p and r (*d*) no p and r

16. The sum of all the real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$ is:

(*a*) 2 (*b*) 3
(*c*) 4 (*d*) None of these

17. Let a, b, c be real, if $ax^2 + bx + c = 0$ has two real roots α, β, where $\alpha < -1$ and $\beta > 1$, then the value of $1 + \frac{c}{a} + \left|\frac{b}{a}\right|$ is:

(*a*) less than zero (*b*) greater than zero
(*c*) equal to zero (*d*) equal to $b^2 - 4ac$

18. The values of a for which the quadratic equations $(1 - 2a)x^2 - 6ax - 1 = 0$ and $ax^2 - x + 1 = 0$ have at least one root in common are:

(*a*) $\frac{1}{2}, \frac{2}{9}$ (*b*) $0, \frac{1}{2}$
(*c*) $\frac{2}{9}$ (*d*) $0, \frac{1}{2}, \frac{2}{9}$

19. If the roots of $ax^2 + bx + c = 0$, $a > 0$, be each greater than unity, then:

(*a*) $a + b + c = 0$ (*b*) $a + b + c > 0$
(*c*) $a + b + c < 0$ (*d*) None of these

20. If n is such that $36 \le n \le 72$, then

$$x = \frac{n^2 + 2\sqrt{n}\,(n + 4) + 16}{n + 4\sqrt{n} + 4} \text{ satisfies}$$

(*a*) $20 < x < 54$ (*b*) $23 < x < 58$
(*c*) $25 < x < 64$ (*d*) $28 < x < 60$

21. A telecom service provider engages male and female operators for answering 1000 calls per day. A male operator can handle 40 calls per day whereas a female operator can handle 50 calls per day. The male and the female operators get a fixed wage of ₹250 and ₹300 per day respectively. In addition, a male operator gets ₹15 per call he answers and a female operator gets ₹10 per call she answers. To minimize the total cost, how many male operators should the service provider employ assuming he has to employ more than 7 of the 12 female operators available for the job?

(*a*) 15 (*b*) 14
(*c*) 12 (*d*) 10

22. The inequality of $p^2 + 5 < 5p + 14$ can be satisfied, if:

(*a*) $p \le 6, p = -1$ (*b*) $p = 6, p = -2$

(*c*) $p \le 6, p \le 1$ (*d*) $p \ge 6, p = 1$

[Based on SNAP, 2008]

23. Each boy contributed rupees equal to the number of girls and each girl contributed rupees equal to the number of boys in a class of 60 students. If the total contribution thus collected is ₹1600, how many boys are there in the class?

(*a*) 30 (*b*) 25

(*c*) 50 (*d*) Data inadequate

[Based on FMS (MS), 2006]

24. If the ratio between the roots of the equation $lx^2 + mx + n = 0$ is $p : q$, then the value of $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}}$ is:

(*a*) 4 (*b*) 3

(*c*) 0 (*d*) –1

[Based on FMS, 2005]

25. Vaidya and Vandana solved a quadratic equation. In solving it, Vaidya made a mistake in the constant term and obtained the roots as 6 and 2, while Vandana made a mistake in the coefficient of x only and obtained the roots as –7 and –1. The correct roots of the equation are:

(*a*) 6, 1 (*b*) 7, 2

(*c*) 6, 2 (*d*) 7, 1

[Based on FMS, 2006]

26. If x is a number satisfying the equation $\sqrt[3]{x+9} - \sqrt[3]{x-9} = 3$, then x^2 is between:

(*a*) 55 and 65 (*b*) 65 and 75

(*c*) 75 and 85 (*d*) 85 and 95

[Based on FMS, 2010]

27. The number of real values of x satisfying the equation $2^{2x^2 - 7x + 5} = 1$

(*a*) 1 (*b*) 2

(*c*) 4 (*d*) More than 4

[Based on FMS, 2010]

28. Which of the following sets of x-values satisfy the inequality $2x^2 + x < 6$

(*a*) $-2 < x < \frac{3}{2}$ (*b*) $x > \frac{3}{2}$ or $x < -2$

(*c*) $x < \frac{3}{2}$ (*d*) $\frac{3}{2} < x < 2$

[Based on FMS, 2010]

29. For what value (*s*) of k does the pair of equations $y = x^2$ and $y = 3x + k$ have two identical solutions:

(*a*) $-\frac{4}{9}$ (*b*) $\frac{9}{4}$

(*c*) $-\frac{9}{4}$ (*d*) $\frac{9}{4}$ or $-\frac{9}{4}$

[Based on FMS, 2010]

30. If $\frac{x^2 - bx}{ax - c} = \frac{m-1}{m+1}$ has roots which are numerically equal but of opposite signs, the value of m must be:

(*a*) $\frac{a-b}{a+b}$ (*b*) $\frac{a+b}{a-b}$

(*c*) c (*d*) $1/c$

[Based on FMS, 2011]

31. In solving a problem that reduces to a quadratic equation one student makes a mistake only in the constant term of the equation and obtains 8 and 2 for the roots. Another student makes a mistake only in the coefficient of the first degree term and finds – 9 and – 1 for the roots. The correct equation is:

(*a*) $x^2 - 10x + 9 = 0$ (*b*) $x^2 + 10x + 9 = 0$

(*c*) $x^2 - 10x + 16 = 0$ (*d*) $x^2 - 8x - 9 = 0$

[Based on FMS, 2011]

32. The values of y which will satisfy the equations,

$$2x^2 + 6x + 5y + 1 = 0$$
$$2x + y + 3 = 0$$

may be found by solving:

(*a*) $x^2 + 14y - 7 = 0$ (*b*) $y^2 + 8y + 1 = 0$

(*c*) $y^2 + 10y - 7 = 0$ (*d*) $y^2 + y - 12 = 0$

[Based on FMS, 2011]

33. If the roots of the equation $\frac{x+a}{x+a+c} + \frac{x+b}{x+b+c} = 1$ are equal in magnitude but opposite in sign, then:

(*a*) $c \ge a$ (*b*) $a \ge c$

(*c*) $a + b = 0$ (*d*) $a = b$

34. If one root is the square of the other root in the equation $x^2 + px + q = 0$, mark the correct relationship in the following options.

(*a*) $p^3 - q(3p + i) + q^2 = 0$

(*b*) $p^3 - q(3p - 1) + q^2 = 0$

(*c*) $p^3 + q(3p - 1) + q^2 = 0$

(*d*) $p^3 - q(3p - 1) - q^2 = 0$

[Based on ITFT, 2006]

35. Find the root of the quadratic equation $bx^2 - 2ax + a = 0$

(a) $\frac{\sqrt{b}}{\sqrt{b} \pm \sqrt{a-b}}$ (b) $\frac{\sqrt{a}}{\sqrt{b} \pm \sqrt{a-b}}$

(*c*) $\frac{\sqrt{a}}{\sqrt{a} \pm \sqrt{a-b}}$ (*d*) $\frac{\sqrt{a}}{\sqrt{a} \pm \sqrt{a+b}}$

[Based on IIFT, 2010]

36. If the common factor of $px^2 + qx + r$ and $qx^2 + px + r$ is $(x + 2)$, then:

(*a*) $p = q$ or $p + q + r = 0$ (*b*) $p = r$ or $p + q + r = 0$

(*c*) $q = r$ or $p + q + r = 0$ (*d*) $p = q = r$

[Based on XAT, 2006]

37. For which value of non-negative '*a*' will the system $x^2 - y^2 = 0$, $(x - a)^2 + y^2 = 1$ have exactly three real solutions?

(*a*) $-\sqrt{2}$ (*b*) 1

(*c*) $\sqrt{2}$ (*d*) 2

[Based on XAT, 2007]

38. If $0 < p < 1$, then roots of the equation $(1 - p)x^2 + 4x + p = 0$ are ...?

(*a*) both 0 (*b*) imaginary

(*c*) real and both positive (*d*) real and both negative

[Based on XAT, 2008]

39. The number of possible real solutions of y in equation $y^2 - 2y \cos x + 1 = 0$ is ...?

(*a*) 0 (*b*) 2

(*c*) 1 (*d*) 3

[Based on XAT, 2008]

40. Let a and b be the roots of the quadratic equation $x^2 + 3x - 1 = 0$. If $P_n = a^n + b^n$ for $n \geq 0$, then, for $n \geq 2$, P_n is equal to:

(*a*) $-3P_{n-1} + P_{n-2}$ (*b*) $3P_{n-1} + P_{n-2}$

(*c*) $-P_{n-1} + 3P_{n-2}$ (*d*) $P_{n-1} + 3P_{n+1}$

[Based on XAT, 2009]

41. If one of the roots of a quadratic equation is $\frac{115}{11+\sqrt{6}}$, then the quadratic equation must be:

(I) $x^2 + 22x + 115 = 0$

(II) $2x^2 + 44x + 115 = 0$

(III) $x^2 - 22x - 115 = 0$

(IV) $x^2 - 22x + 115 = 0$

(*a*) I only (*b*) II only

(*c*) III only (*d*) IV only

[Based on ATMA, 2008]

42. If a, b and c are roots of $x^3 - 6x^2 + 11x - 6 = 0$ and the roots of the equation $x^3 - px^2 + qx - r = 0$ are $a + b$, $b + c$ and $c + a$, then r equals:

(*a*) 40 (*b*) 50

(*c*) 60 (*d*) 70

[Based on JMET, 2009]

43. A man covers a certain distance on a toy train. If the train moved 4 Km/h faster, it would take 30 minutes less. If it moved 2 Km/h slower, it would have taken 20 minutes more. What is the distance covered?

(*a*) 65 Km (*b*) 60 Km

(*c*) 70 Km (*d*) 75 Km

[Based on MAT, 2013]

Answer Keys

Difficulty Level-1

1. (*b*)	**2.** (*c*)	**3.** (*d*)	**4.** (*b*)	**5.** (*c*)	**6.** (*a*)	**7.** (*b*)	**8.** (*c*)	**9.** (*c*)	**10.** (*b*)	**11.** (*a*)	**12.** (*a*)	**13.** (*c*)
14. (*c*)	**15.** (*c*)	**16.** (*d*)	**17.** (*d*)	**18.** (*d*)	**19.** (*d*)	**20.** (*c*)	**21.** (*c*)	**22.** (*b*)	**23.** (*a*)	**24.** (*d*)	**25.** (*a*)	**26.** (*a, c*)
27. (*a*)	**28.** (*d*)	**29.** (*b*)	**30.** (*b*)	**31.** (*a*)	**32.** (*a*)	**33.** (*b*)	**34.** (*d*)	**35.** (*d*)	**36.** (*c*)	**37.** (*b*)	**38.** (*d*)	**39.** (*a*)
40. (*b*)	**41.** (*a*)	**42.** (*b*)	**43.** (*a*)									

Difficulty Level-2

1. (*d*)	**2.** (*b*)	**3.** (*c*)	**4.** (*c*)	**5.** (*c*)	**6.** (*a*)	**7.** (*d*)	**8.** (*c*)	**9.** (*a*)	**10.** (*a*)	**11.** (*b*)	**12.** (*d*)	**13.** (*d*)
14. (*b*)	**15.** (*b*)	**16.** (*c*)	**17.** (*c*)	**18.** (*c*)	**19.** (*b*)	**20.** (*c*)	**21.** (*d*)	**22.** (*c*)	**23.** (*d*)	**24.** (*a*)	**25.** (*d*)	**26.** (*c*)
27. (*b*)	**28.** (*a*)	**29.** (*c*)	**30.** (*a*)	**31.** (*a*)	**32.** (*c*)	**33.** (*a*)	**34.** (*b*)	**35.** (*c*)	**36.** (*a*)	**37.** (*b*)	**38.** (*d*)	**39.** (*b*)
40. (*a*)	**41.** (*c*)	**42.** (*c*)	**43.** (*b*)									

Explanatory Answers

Difficulty Level-1

1. (*b*)

$$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$$

$$\therefore \quad \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)}{\alpha\beta}$$

$$= \frac{\left(-\frac{b}{a}\right)\left(\frac{b^2 - 2ac}{a^2} - \frac{c}{a}\right)}{\frac{c}{a}}$$

$$= \frac{-b(b^2 - 3ac)}{a^3} \times \frac{a}{c}$$

$$= \frac{3abc - b^3}{a^2c}.$$

2. (*c*) $a + b = 24,\ a - b = 8 \Rightarrow a = 16,\ b = 8$

$\therefore$ $ab = 128$

$\therefore$ Required equation is the one whose sum of the roots is 24 and product of the roots is 128

i.e., $x^2 - 24x + 128 = 0.$

3. (*d*) Suppose total number of cows = x

$\frac{1}{4}$ x of the cows are in forest,

$2\sqrt{x}$ have gone to mountains and 15 are on the banks of a river.

$\therefore$ $2\sqrt{x} + \frac{1}{4}x + 15 = x$

$\Rightarrow$ $2\sqrt{x} - \frac{3}{4}x + 15 = 0$

$\Rightarrow$ $8\sqrt{x} - 3x + 60 = 0$

$\Rightarrow$ $3(\sqrt{x})^2 - 8\sqrt{x} - 60 = 0$

$\Rightarrow$ $3(\sqrt{x})^2 - 18\sqrt{x} + 10\sqrt{x} - 60 = 0$

$\Rightarrow$ $3\sqrt{x}(\sqrt{x} - 6) + 10(\sqrt{x} - 6) = 0$

$\Rightarrow$ $\sqrt{x} = 6,\ \sqrt{x} = -\frac{10}{3}$

$\Rightarrow$ $x = 36.$

4. (*b*) Discriminant

$= [2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$

for the roots to be equal

$\therefore$ $a^4 + b^2c^2 - 2a^2bc - c^2b^2 + ac^3 + ab^3 - a^2bc = 0$

$\Rightarrow a^3 - 2abc + c^3 + b^3 - abc = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc.$

5. (*c*) $2\frac{x^2}{y^2} - 7\frac{x}{y} + 3 = 0$

$\Rightarrow$ $\frac{x}{y} = \frac{7 \pm \sqrt{49 - 24}}{2 \times 2}$

$= \frac{7 \pm 5}{4} = 3,\ \frac{1}{2}.$

6. (*a*) $(\alpha + \gamma)(\beta + \delta)(\alpha + \delta)(\beta + \gamma)$

$= [(\alpha + \gamma)(\beta + \gamma)][(\beta + \delta)(\alpha + \delta)]$

$= [\alpha\beta + \gamma(\alpha + \beta) + \gamma^2][\alpha\beta + \delta(\alpha + \beta) + \delta^2]$

$= (-1 - 2\gamma + \gamma^2)(-1 - 2\gamma + \delta^2)$

[Since γ and δ are the roots of the equation $x^2 + 3x - 4 = 0$]

$= (\gamma^2 + 3\gamma - 4 - 5\gamma + 3)(\delta^2 + 3\delta - 4 - 5\delta + 3)$

$= (-5\gamma + 3)(-5\delta + 3)$

$= (25\gamma\delta + 9 - 15(\delta + \gamma))$

$= (-100 + 9 + 45) = -46.$

7. (*b*) Given that $\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = \frac{45}{14}$

and, $\left(\alpha + \frac{1}{\beta}\right) \times \left(\beta + \frac{1}{\alpha}\right) = \frac{81}{14}$

$\Rightarrow$ $(\alpha + \beta) + \frac{[(\alpha + \beta)^2 - 2\alpha\beta]}{\alpha\beta} = \frac{45}{14}$

$\Rightarrow$ $\alpha\beta + \frac{1}{\alpha\beta} + 2 = \frac{81}{14}$

Solve for $\alpha\beta$ and $(\alpha + \beta)$, we get

$\alpha + \beta = \frac{5}{2}$ and $\alpha\beta = \frac{7}{2}$

Therefore, $2x^2 - 5x + 7 = 0.$

8. (*c*) We have, $\alpha + \beta = 8,\ \alpha\beta = P$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 64 - 2P = 40$ (given)

$\Rightarrow$ $P = 12.$

9. (*c*) By the condition given in question

$3X^2 - 18X = X + 4 - 2X^2$

$\Rightarrow$ $5X^2 - 19X - 4 = 0$

$\Rightarrow$ $(5X + 1)(X - 4) = 0$

$\Rightarrow$ $X = 4$ or $X = -\frac{1}{5}$

So, the present age of man = $18 \times 4 = 72$ years

Note: Can be directly obtained from options.

Man's present age is given as $18X$ years.

Therefore the answer should be divisible by 18 and from options only 72 is divisible by 18.

10. (*b*) Let the number of children in the family be X.

Given: $11X - 12 = 2X^2 \Rightarrow 2X^2 - 11X + 12 = 0$

Solving the quadratic equation, we get

$X = \frac{3}{2}$ or $X = 4$

Number of children can not be $\frac{3}{2}$. Hence the number of children is 4.

11. (*a*) For any quadratic equation, $ax^2 + bx + c = 0$, to have equal roots,

$b^2 - 4ac = 0$

$$\Rightarrow \quad [2(k-4)]^2 - 4 \times 2k = 0$$
$$\Rightarrow \quad (k-4)^2 - 2k = 0$$
$$\Rightarrow \quad k + 16 - 8k - 2k = 0$$
$$\Rightarrow \quad k^2 - 10k + 16 = 0$$
$$\Rightarrow \quad k = 8, 2.$$

12. (*a*) Let the equation be $ax^2 + bx + c = 0$

Suppose its roots are α and β.

Then, $$\alpha + \beta = -\frac{b}{a} \quad (1)$$
$$\alpha\beta = \frac{c}{a} \quad (2)$$

Also, $$\alpha^2 + \beta^2 = -\frac{b}{a} \quad (3)$$
$$\alpha^2\beta^2 = \frac{c}{a} \quad (4)$$

From (2) and (4), we get

$$\Rightarrow \quad \alpha\beta = 1 \text{ or } c = a$$

From (1), we get

$$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{b^2}{a^2}$$
$$\Rightarrow \quad -\frac{b}{a} + 2 = \frac{b^2}{a^2}$$
$$\Rightarrow \quad b^2 + ab - 2a^2 = 0$$

or, $$b = \frac{-a \pm \sqrt{a^2 + 8a^2}}{2} = a, -2a.$$

13. (*c*) $$\alpha + \beta = \frac{7}{2}$$
$$\alpha\beta = \frac{-3}{2}$$
$$\therefore \quad (\alpha + 2)(\beta + 2) = \alpha\beta + 2\alpha + 2\beta + 4$$
$$= \alpha\beta + 2(\alpha + \beta) + 4$$
$$= \frac{3}{2} + 2 \times \frac{7}{2} + 4$$
$$= \frac{-3}{2} + 11 = \frac{19}{2} = 9.5.$$

14. (*c*) Let us consider choice (*a*). When we put the values of *A* and *B* respectively, we get the values of α, β, γ and δ as – 1, 1/3, 1/5, 1/3, which are not in H.P. So, this option is not correct.

Now for our convenience we consider choice (*c*), then by substituting the values of *A* and *B*, we get the value of α, β, γ and δ as 1, 1/2, 1/3 and 1/4 which are in H.P. Hence, this could be the correct choice.

15. (*c*) Let original cost per student be ₹x.

$$\therefore \quad \text{Total number of students} = \frac{720}{x}$$
$$\left(\frac{720}{x} - 4\right) \times (x + 9) = 720$$
$$\Rightarrow \quad 720 - 4x + \frac{6480}{x} - 36 = 720$$
$$\Rightarrow \quad x^2 + 9x - 1620 = 0$$
$$\Rightarrow \quad x^2 + 45x - 36x - 1620 = 0$$
$$\Rightarrow \quad (x + 45)(x - 36) = 0$$
$$\Rightarrow \quad x = ₹36 \qquad (\because x \neq 45)$$

16. (*d*) Let no. of rows be x and no. of students in each row be n.

Then, total no. of students = xn

Again, $(n + 4)(x - 2) = (n - 4)(x + 4) = xn$

$$\Rightarrow \quad n = 12 \text{ and } x = 8$$
$$\therefore \quad \text{No. of students} = 12 \times 8 = 96.$$

17. (*d*) $$\sqrt{25 - x^2} = x - 1$$
$$\Rightarrow \quad 25 - x^2 = (x - 1)^2$$
$$\Rightarrow \quad 25 - x^2 = x^2 + 1 - 2x$$
$$\Rightarrow \quad 2x^2 - 2x - 24 = 0$$
$$\Rightarrow \quad x^2 - x - 12 = 0$$
$$\Rightarrow \quad (x - 4)(x + 3) = 0$$
$$\Rightarrow \quad x = 4, x = -3.$$

18. (*d*) $x = \frac{1}{2}$ satisfies the given equation.

19. (*d*) Suppose total number of cows = x

$\frac{1}{4}x$ of the cows are in forest,

$2\sqrt{x}$ have gone to mountains and 15 are on the banks of a river.

$$\therefore \quad 2\sqrt{x} + \frac{1}{4}x + 15 = x$$
$$\Rightarrow \quad 2\sqrt{x} - \frac{3}{4}x + 15 = 0$$
$$\Rightarrow \quad 8\sqrt{x} - 3x + 60 = 0$$
$$\Rightarrow \quad 3(\sqrt{x})^2 - 8\sqrt{x} - 60 = 0$$
$$\Rightarrow \quad 3(\sqrt{x})^2 - 18\sqrt{x} + 10\sqrt{x} - 60 = 0$$
$$\Rightarrow \quad 3\sqrt{x}(\sqrt{x} - 6) + 10(\sqrt{x} - 6) = 0$$
$$\Rightarrow \quad \sqrt{x} = 6, \ \sqrt{x} = -\frac{10}{3}$$
$$\Rightarrow \quad x = 36.$$

20. (*c*) Suppose the positive number is x.

According to the question,

$$x - 4 = 21 - \frac{1}{x}$$

or, $x^2 - 4x = 21$

or, $x^2 - 4x - 21 = 0$

or, $x(x-7) + 3(x-7) = 0$

or, $(x-7)(x+3) = 0$

$\therefore$ $x = 7$ or $x = -3$

Since x is positive

$\therefore$ $x = 7$.

21. (*c*) $y^2 = x^2$

$2x^2 - 2kx + k^2 - 1 = 0$

$D = 0$

$\Rightarrow$ $4k^2 = 8k^2 - 8 \Rightarrow 4k^2 = 8$

$\Rightarrow$ $k = \sqrt{2}$.

22. (*b*) Sum of the roots $= -\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$$= \frac{(-8)}{4} = 2.$$

23. (*a*) $\sqrt{5x-1} + \sqrt{x-1} = 2$

$\sqrt{5x-1} = 2 - \sqrt{x-1}$

Squaring both sides,

$5x - 1 = 4 + x - 1 - 2\sqrt{x-1}$

$4x - 4 = -2\sqrt{x-1}$

$\Rightarrow$ $2x - 2 = -\sqrt{x-1}$

Squaring both sides,

$4x^2 + 4 - 8x = x - 1$

$4x^2 - 9x + 5 = 0 \Rightarrow (4x-5)(x-1) = 0$

$x = \dfrac{5}{4}$ or, $x = 1$

$x = \dfrac{5}{4}$ does not satisfy the original equation.

24. (*d*) Since $y = \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{3 + \dots}}}}$

$\Rightarrow$ $y = \cfrac{1}{2 + \cfrac{1}{3 + y}}$

$\Rightarrow$ $y = \dfrac{3 + y}{6 + 2y + 1}$

or, $2y^2 + 7y = 3 + y$ or, $2y^2 + 6y - 3 = 0$

$\therefore$ $y = \dfrac{-6 \pm \sqrt{36 + 24}}{4}$

$= \dfrac{-3 \pm \sqrt{15}}{2}$

As the contained fraction is positive, $y = \dfrac{\sqrt{15} - 3}{2}$.

25. (*a*) For any quadractic equation, $ax^2 + bx + c = 0$, to have equal roots,

$b^2 - 4ac = 0 \Rightarrow [2(k-4)]^2 - 4 \times 2k = 0$

$\Rightarrow$ $(k-4)^2 - 2k = 0$

$\Rightarrow$ $k + 16 - 8k - 2k = 0$

$\Rightarrow$ $k^2 - 10k + 16 = 0$

$\Rightarrow$ $k = 8, 2$.

26. (*a*, *c*) Option (*a*) is correct.

Option (*b*) is incorrect.

Option (*c*) as P, Q, R in AP their reciprocals are in HP.

Option (*d*) is wrong as these terms in HP, not in AP.

Hence, Options (*a*) and (*c*) are correct.

27. (*a*) Let the roots of the given equation are α and β.

Given, $\alpha = 2\beta$ (1)

Given equation be $x^2 + bx + c = 0$

Sum of the roots $(\alpha + \beta) = -\dfrac{b}{a}$

and product of roots $(\alpha \times \beta) = \dfrac{c}{a}$

From $\alpha + \beta = -\dfrac{b}{a}$

$2\beta + \beta = -\dfrac{b}{a}$ [by Eq. (1)]

$\Rightarrow$ $3\beta = -\dfrac{b}{a} \Rightarrow \beta = -\dfrac{b}{3a}$

$\beta^2 = \dfrac{b^2}{9a^2}$ (2)

Now, $\alpha \times \beta = \dfrac{c}{a}$

$2\beta \times \beta = \dfrac{c}{a} \Rightarrow 2\beta^2 = \dfrac{c}{a}$

$\Rightarrow$ $\beta^2 = \dfrac{c}{2a}$ (3)

From Eqs. (2) and (3) $\dfrac{b^2}{9a^2} = \dfrac{c}{2a}$

$$2b^2 = \frac{9a^2 + c}{a} = 9ca$$

$\therefore \quad 2b^2 = 9ca.$

28. (*d*) Let α, β be the roots of the equation

$$x^2 - x + m = 0$$

$\therefore \quad \alpha + \beta = 1, \alpha\beta = m \quad (1)$

Let 2α, γ be the roots of the equation

$$x^2 - x + 3m = 0$$

$\therefore \quad 2\alpha + \gamma = 1, 2\alpha\gamma = 3m \quad (2)$

$(1) \Rightarrow \alpha + \frac{m}{\alpha} = 1$

$(2) \Rightarrow \alpha^2 - \alpha + m = 0 \quad (3)$

$\Rightarrow \quad 2\alpha + \frac{3m}{2\alpha} = 1$

$\Rightarrow \quad 4\alpha^2 - 2\alpha + 3m = 0 \quad (4)$

Equations (3) and (4)

$\Rightarrow \quad m = 2\alpha$

$\therefore \quad 2\alpha\gamma = 3m \Rightarrow \gamma = 3$

$\therefore \quad 2\alpha + \gamma = 1 \Rightarrow \alpha = -1$

$\alpha + \beta = 1 \Rightarrow \beta = 2$

$\therefore \quad \alpha = -1, \beta = 2, \gamma = 3 \Rightarrow m = -2.$

29. (*b*) Equation $x^2 + 3x + k = 0$ puting $x = 4$

$$16 + 12 + k = 0$$

$$k = -28$$

By option method, put $x = -7$

$$x^2 + 3x + k = (-7)^2 + 3(-7) - 28$$
$$= 49 - 21 - 28$$
$$= 0 \text{ satisfy equation.}$$

$\Rightarrow \quad x = -7.$

30. (*b*) Equation $x^2 + px + q = 0$

If coefficient of x was wrong the product = 30

$\therefore$ Roots of correct equation are $-3, -10 \rightarrow 30$.

31. (*a*) We have, $7x_2 - 4x_1 = 47$

and, $x_1 + x_2 = 2$

On solving, $11x_1 = 55$ or $x_1 = 5$

$\therefore \quad x_2 = -3$

$\therefore \quad c = -15.$

32. (*a*) Given that $-2\sqrt{3}$ is a root of

$$x^2 + px - 6 = 0 \quad (1)$$

$\therefore$ $-2\sqrt{3}$ must satisfy Eq. (1)

Put $x = -2\sqrt{3}$ in Eq. (1), we have

$$(-2\sqrt{3})^2 + p(-2\sqrt{3}) - 6 = 0$$

$\Rightarrow \quad 12 - 2\sqrt{3}p - 6 = 0$

$\Rightarrow \quad 6 = 2\sqrt{3}p$

$\therefore \quad p = \frac{6}{2\sqrt{3}} = \sqrt{3}$

Another given equation is

$$x^2 + px + q = 0$$

$\Rightarrow \quad x^2 + \sqrt{3}x + q = 0 \quad (2)$

Also, given that Eq. (2) has equal roots let α be the root of Eq. (2)

Then, sum of roots = $\frac{-b}{a} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$

i.e., $(\alpha + \alpha) = -\sqrt{3}$

$\therefore \quad \alpha = \frac{-\sqrt{3}}{2} \quad (3)$

and product of roots = $\frac{c}{a}$

$\Rightarrow \quad \alpha \times \alpha = \frac{q}{1}$

$$\alpha^2 = q$$

$\therefore \quad q = \left(\frac{-\sqrt{3}}{2}\right)^2$ [by Eq. (3)]

$\therefore \quad q = \frac{3}{4}.$

33. (*b*) Suppose $y = x^2 - 4x + 7$

For minimum value of y, $\frac{dy}{dx}$ must be 0.

i.e., $\frac{d}{dx}(x^2) - 4\frac{d(x)}{dx} + \frac{d(7)}{dx} = 0$

$\Rightarrow \quad 2x - 4 + 0 = 0$

$\Rightarrow \quad 2x = 4 \Rightarrow x = 2.$

34. (*d*) $x^2 - 6x + 5 = 0$

$\Rightarrow \quad x^2 - 5x - x + 5 = 0$

$\Rightarrow \quad x(x - 5) - 1(x - 5) = 0$

$\Rightarrow \quad (x - 5)(x - 1) = 0$

$\Rightarrow \quad x = 5 \text{ or, } 1$

$\therefore$ Roots of given equation are 5 and 1.

Now, roots of equation (4)

i.e., $2x^2 - 2 = 0$

$\Rightarrow \quad 2x^2 = 2, x^2 = 1$

$\therefore \quad x = \pm 1$

Which is common with root of Eq. (1).

35. (*d*) Given, $(x - 3)(2x + 1) = 0$

Then, $(x - 3) = 0$

$\Rightarrow \quad x = 3$

and, $(2x + 1) = 0$

If $x = 3$, then $(2x + 1) = 7$

$\therefore$ Possible values of $(2x + 1)$ are 0 and 7.

36. (*c*) As M and N are the roots of $y^2 + My + N = 0$,

$M + N = -M$ and $MN = N$

$MN = N \Rightarrow N = 0$ or $M = 1$

If $N = 0$, then $M = -M \Rightarrow M = 0$

If $M = 1$, then $N = -2M \Rightarrow N = -2$

That is two (M, N) pairs $(0, 0)$ and $(1, -2)$ are possible.

37. (*b*) Let the required number system be N, then

$$(100)_N = (24)_N + (32)_N$$

$$N^2 = 2N + 4 + 3N + 2 = 5N + 6$$

$\Rightarrow N^2 - 5N - 6 = 0$

$\Rightarrow (N + 1)(N - 6) = 0$

So, $N = 6$ as -1 is not possible.

38. (*d*) We have,

$$a = 2, b = 2(p+1) \text{ and } c = p.$$

Therefore, the discriminant is

$$[2(p+1)]^2 - 4 \cdot 2 \cdot p$$
$$= 4(p+1)^2 - 8p$$
$$= 4[(p+1)^2 - 2p]$$
$$= 4[(p^2 + 2p + 1) - 2p]$$
$$= (4p^2 + 1)$$

For any real value of p, $4(p^2 + 1)$ will always be positive as p^2 cannot be negative for real p.

Hence, the roots of the quadratic equation are real.

39. (*a*) Since the roots of the given equation are real, therefore

$$(2(b^2 + c^2))^2 - 4(a^2 + b^2)(b^2 + c^2) \geq 0$$

$\Rightarrow (b^2 + c^2) - (a^2 + b^2) \geq 0$

$\Rightarrow c^2 \geq a^2$.

40. (*b*) We have,

$$\frac{p^6 - p}{p - 1} = m^2 + m + 6$$

$\Rightarrow p^5 + p^4 + p^3 + p^2 + p = m(m+1) + 6$

41. (*a*) Let the three consecutive positive integers be

$(n-1)$, n and $(n+1)$

Given, $n + 1 + n^2 + (n+1)^3 = (3n)^2$

$\Rightarrow n^3 + 4n^2 + 4n = 9n^2$

$\Rightarrow n^2 - 5n + 4 = 0$

$\Rightarrow n = 1$ or $n = 4$

Since, the three integers are positive, the value of 'n' cannot be equal to 1, therefore the value of $n = 4$ or $m = n - 1 = 3$

Hence, three consecutive integers are 3, 4 and 5

Hence, option (a) is the correct choice.

42. (*b*) Number of questions for team A = 300

Also, number of questions for team B 90% of 300

$$= \frac{90 \times 300}{100} = 270$$

Now, let the questions attempted per hour by A = x

Then, questions attempted per hour by B = $x + 7$

We are given,

$$\frac{270}{x} + 3 = \frac{300}{x+7} + 6$$

$$\frac{270}{x} = \frac{300}{x+7} + 3$$

$$\frac{270}{x} = \frac{300 + 3(x+7)}{x+7}$$

$$270(x+7) = (300 + 3x + 21)x$$

$$270x + 1890 = 321x + 3x^2$$

$$3x^2 + 51x - 1890 = 0$$

$$x^2 + 17x - 630 = 0$$

$$x^2 + 35 - 18x - 630 = 0$$

$$x(x+35) - 18(x+35) = 0$$

$$(x-18)(x+35) = 0$$

$\Rightarrow x = 18, -35$

So, team A attempted 18 questions per h.

($\therefore$ Negative value of x is not permitted)

43. (*a*) $ax^3 + bx^2 + cx + d$ intersect x-axis at 1 and -1

Hence when $x = 1$ or -1 then

$ax^3 + bx^2 + cx + d = 0$

$\therefore a + b + c + d = 0$ (1)

$-a + b - c + d = 0$ (2)

on adding (1) and (2), we get

$2(b + d) = 0$

$\Rightarrow b + d = 0$

Since $ax^3 + bx^2 + cx + d$ intersects y-axis at 2

Hence when $x = 0$, then $ax^3 + bx^2 + cx + d = 0$

$\therefore 0 + d = 2$

$\Rightarrow d = 2$

$\therefore b = -2$.

Difficulty Level-2

1. (*d*) Sum of the root $= p + q = \dfrac{-\{-(\alpha - 2)\}}{1}$

$= \alpha - 2$

Product of the roots

$$= pq = \frac{-(\alpha + 1)}{1} = -(\alpha + 1)$$

Now, $p^2 + q^2 = (p + q)^2 - 2pq$

$= (\alpha - 2)^2 - 2(-1)(\alpha + 1)$

$= \alpha^2 - 4\alpha + 4 + 2\alpha + 2$

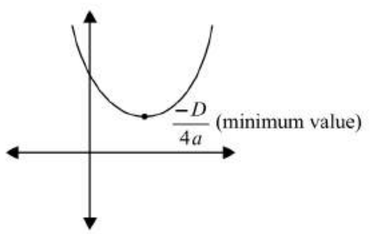
$= \alpha^2 - 2\alpha + 6$

We have to find the minimum possible value of $a^2 - 2a + 6$.

$$D = (-2)^2 - 4 \times 1 \times 6 = -20$$

and coefficient of α^2 is +ve

∴ Rough diagram of $\alpha^2 - 2\alpha + 6$ is



∴ minimum value $= \dfrac{-D}{4a} = \dfrac{-(-20)}{4/1} = \dfrac{20}{4} = 5.$

2. (*b*) $ax^2 + bx + 1 = 0$ has real roots if $b^2 - 4a \geq 0$

Value of a	Corresponding value of b for which $b^2 - 4a \geq 0$	No. of ways
1	2, 3, 4	3
2	3, 4	2
3	4	1
4	4	1

Total no. of ways = 7.

3. (*c*) Let Arjun has x arrows. According to the given condition $\frac{1}{2}x + 6 + 1 + 1 + 1 + 1 + 4\sqrt{x} = x \Rightarrow 8\sqrt{x} - 20$. Squaring both the sides, we get $64x = x^2 - 40x + 400 \Rightarrow x^2 - 104x + 400 = 0 \Rightarrow (x - 4)(x - 100) = 0 \Rightarrow x = 4$ or $x = 100$. x has to be greater than 4 because Arjun killed the chariot driver of Pitamaha with six arrows. Hence, $x = 100$.

4. (*c*) $f(1, 1, z) = 1 + 2 + 3 + ... + z$ terms

Given, $\dfrac{z(z+1)}{2} = 21$

$\Rightarrow \quad z^2 + z - 42 = 0$

or, $(z + 7)(z - 6) = 0$

$\therefore \quad z = 6.$

5. (*c*) Here, $B^2 - 4AC = 25(l + m)^2 + 8(l - m)^2 > 0$

⇒ Roots are real and unequal.

6. (*a*)

$X^2 - 2X - 3 = (X - 3)(X + 1)$

(*a*) $\Rightarrow X^2 - 7X + 6 = (X - 6)(X - 1)$

(*b*) $\Rightarrow X^2 - 9 = (X + 3)(X - 3)$

(*c*) $\Rightarrow X^2 - 4X + 3 = (X - 3)(X - 1)$

(*d*) $\Rightarrow X^2 - 6X + 9 = (X - 3)^2$.

7. (*d*) $p^2 - 18p + 77 = 0$

$\Rightarrow \quad p = 11, p = 7$

$3q^2 - 25q + 28 = 0$

$\Rightarrow \quad 3q^2 - 21q - 4q + 28 = 0$

$\Rightarrow \quad 3q(q - 7) - 4(q - 7) = 0$

$\Rightarrow \quad (3q - 4)(q - 7) = 0$

$\Rightarrow \quad q = \dfrac{4}{3}, 7.$

$\therefore \quad p \geq q.$

8. (*c*) $6q^2 + q - 1 - 0$

$\Rightarrow \quad 6q^2 + 3q - 2q - 1 = 0$

$\Rightarrow \quad 3q(2q + 1) - (2q + 1) = 0$

$\Rightarrow \quad (3q - 1)(2q + 1) = 0$

$\Rightarrow \quad q = \dfrac{1}{3}, \dfrac{-1}{2}$

$6p^2 - 7p + 2 = 0$

$\Rightarrow \quad 6p^2 - 4p - 3p + 2 = 0$

$\Rightarrow \quad 2p(3p - 2) - (3p - 2) = 0$

$\Rightarrow \quad p = \dfrac{1}{2}, p = \dfrac{2}{3}$

$\therefore \quad p > q.$

9. (*a*) $$7p^2 + 6p - 1 = 0$$
$$\Rightarrow \quad 7p^2 + 7p - p - 1 = 0$$
$$\Rightarrow \quad 7p(p+1) - (p+1) = 0$$
$$\Rightarrow \quad p = \frac{1}{7}, p = -1$$
$$32q^2 - 20q + 3 = 0$$
$$\Rightarrow \quad 32q^2 - 12q - 8q + 3 = 0$$
$$\Rightarrow \quad 4q(8q-3) - 1(8q-3) = 0$$
$$\Rightarrow \quad q = \frac{3}{8}, \frac{1}{4}$$
$$\therefore \quad p < q.$$

10. (*a*) $$4p^2 = 9 \Rightarrow p = \pm\frac{3}{2}$$
$$2q^2 - 9q + 10 = 0$$
$$\Rightarrow \quad 2q^2 - 5q - 4q + 10 = 0$$
$$\Rightarrow \quad q(2q-5) - 2(2q-5) = 0$$
$$\Rightarrow \quad q = \frac{5}{2}, 2$$
$$\therefore \quad p < q.$$

11. (*b*) $$2p^2 - 12p + 16 = 0$$
$$\Rightarrow \quad p^2 - 6p + 8 = 0$$
$$\Rightarrow \quad (p-4)(p-2) = 0$$
$$\Rightarrow \quad p = 4, 2$$
$$q^2 - 9q + 20 = 0$$
$$\Rightarrow \quad q = 5, 4$$
$$\therefore \quad p \le q.$$

12. (*d*) If $2 + i\sqrt{3}$ is a root the equation
$$x^2 + px + q = 0,$$
then its other root will be $2 - i\sqrt{3}$
$$\therefore \quad x^2 + px + q = (x - 2 - i\sqrt{3})(x - 2 + i\sqrt{3})$$
$$= (x-2)^2 - (i\sqrt{3})^2$$
$$= x^2 + 4 - 4x + 3 = x^2 - 4x + 7$$
$$\therefore \quad p = -4, q = 7.$$

13. (*d*) $$f(x) = ax^2 + bx + c$$
f attains a maximum at $x = 1$
$$f(x) = 0$$
$$\Rightarrow \quad 2ax + b = 0 \Rightarrow x = \frac{-b}{2a} = 1 \Rightarrow b = -2a$$
$$\max f(x) = 3$$
$$a + b + c = 3$$
$$c - a = 3$$
$$f(0) = 1 \Rightarrow c = 1, a = -2$$
$$f(x) = -2x^2 + 4x + 1$$
$$f(10) = -2(10)^2 + 4(10) + 1$$
$$= -159.$$

14. (*b*) Let the equal roots be m and m
$$\Rightarrow \quad b^2 - 4ac = 0$$
$$\Rightarrow \quad (2a+3)^2 = 4^{n+1}$$
$$\Rightarrow \quad 2a + 3 = 2^{n+1} \text{ or, } -(2^{n+1})$$
Only possible solution for (a, n) are $(-1, -1)$ or $(-2, -1)$ So, only one possible value of n exists.

15. (*b*) $p, q\ r$ are in A.P.
$$q = \frac{p+r}{2} \qquad [p + r = 2p]$$
For the real roots $q^2 - 4pr \ge 0$
$$\Rightarrow \quad \left(\frac{p+r}{2}\right)^2 - 4pr$$
$$\ge \quad 0 \Rightarrow p^2 + r^2 - 14pr \ge 0$$
$$\Rightarrow \quad \left(\frac{p}{2}\right)^2 - 14\left(\frac{p}{r}\right) + 1 \ge 0$$
$$\Rightarrow \quad \left(\frac{p}{r} - 7\right)^2 \ge 48$$
$$\Rightarrow \quad \left|\frac{p}{r} - 7\right| \ge 4\sqrt{3}.$$

16. (*c*) The given equation is $|x-2|^2 + |x-2| = 0$
Let us assume $|x - 2| = m$
then, $m^2 + m - 2 = 0$
$$\Rightarrow \quad (m-1)(m+2) = 0$$
Only admissible value is
$$m = 1 \qquad [m \ne -2 \text{ as } m \ge 0]$$
$$\therefore \quad |x-2| = 1$$
$$\Rightarrow \quad x - 2 = 1 \Rightarrow x = 3$$
or, $-(x-2) = 1 \Rightarrow x = 1$
Hence, $x = 1, 3$
$\therefore$ Sum of the roots of equation $= 1 + 3 = 4$.

17. (*c*) Assume some values of α, β conforming the basic constraints of the problem.
e.g., $\alpha = -2$, $\beta = 8$, then the equation becomes
$x^2 - 6x - 16$
$$\Rightarrow \quad b = -5 \text{ and } c = -16$$
$$\therefore \quad 1 + \frac{c}{a} + \left|\frac{b}{a}\right| = 1 - 16 + 6 = -9$$
$\therefore$ The value of the expression is negative, hence choice (*a*) is correct.

Note:

Since $\alpha < 1$ and $\beta > 1$

$$\therefore \quad \alpha\beta < 1 \Rightarrow \frac{c}{a} < 1$$

Further the product of any two numbers (n_1, n_2 $\neq 0$) is less then the sum of the number if any one of them is negative.

So, $\alpha\beta < \alpha + \beta$ ($\because$ Here $\alpha\beta$ is negative)

$$\therefore \quad \frac{c}{a} < \left|\frac{b}{a}\right|$$

but $\frac{c}{a}$ is numerically greater than $\frac{b}{a}$

$$\Rightarrow \quad 1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0.$$

18. (*c*) For $a = 0$ or $a = \frac{1}{2}$ one of the quadratic equation becomes linear, So, $a \neq 0, a \neq \frac{1}{2}$.

Hence, the only answer is $a = \frac{2}{9}$.

19. (*b*) Let $f(x) = ax^2 + bx + c$. Since 1 lies outside the roots of $f(x) = 0$. So,

$$af(1) > 0 \Rightarrow f(1) > 0 \qquad (a > 0)$$

$$\Rightarrow \quad a + b + c > 0.$$

20. (*c*) $n^2 + 2\sqrt{n}(n+4) + 16$

$$= n^2 + 2n\sqrt{n} + 8\sqrt{n} + 16$$

$$= n\sqrt{n}(\sqrt{n}+2) + 8(\sqrt{n}+2)$$

$$= (\sqrt{n}+8)(\sqrt{n}+2)$$

$$= (n\sqrt{n}+2)[(\sqrt{n})^3 + (2)^3]$$

$$= (\sqrt{n}+2)(\sqrt{n}+2)(n - 2\sqrt{n} + 4)$$

$$\therefore \quad x = \frac{n^2 + 2\sqrt{n}(n+4) + 16}{n + 4\sqrt{n} + 4}$$

$$= \frac{(\sqrt{n}+2)^2(n - 2\sqrt{n} + 4)}{(\sqrt{n}+2)^2}$$

$$= n - 2\sqrt{n} + 4$$

If $n = 36$ then $x = 36 - 2\sqrt{36} + 4 = 28$

If $n = 72$ then $x = 72 - 2\sqrt{72} + 4 \approx 59$

$$\Rightarrow \quad 25 < x < 64.$$

21. (*d*) There are two equations to be formed

$$40\,m + 50\,f = 1000$$

$$250\,m + 300\,f + 40 \times 15\,m + 50 \times 10 \times f = A$$

$$850\,m + 8000\,f = A$$

m and f are the number of males and females A is amount paid by the employer.

Then, the possible value of $f = 8, 9, 10, 11, 12$

If, $f = 8$

$m = 15$

If $f = 9, 10, 11$ then m will not be an integer while $f = 12$, then m will be 10.

22. (*c*)

23. (*d*) Let number of boys $= x$

Girls $= 60 - x$

According to question,

$$2x(60 - x) = 1600$$

$$x^2 - 60x - 800 = 0$$

$\Rightarrow$ On solving, we get two values of x but we cannot determine which value is that of boys and which value is that of girls, hence data is inadequate.

24. (*a*) $lx^2 + mx + n = 0$

Let us consider $x^2 + 4x + 4 = 0$

Then, $\frac{p}{q} = 1$ and $\frac{n}{l} = 4$

$$\therefore \quad \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = \sqrt{1} + \sqrt{1} + \sqrt{4} = 4.$$

25. (*d*) $\alpha + \beta = 6 + 2 = 8$

$\alpha \times \beta = 7$

$\therefore \quad \alpha = 7$ and $\beta = 1$.

26. (*c*) Let, $a = x + 9$ and $b = x - 9$

$\therefore$ The given equation is,

$$a^{\frac{1}{3}} - b^{\frac{1}{3}} = 27$$

Cubing both the sides we get,

$$a - b - 3a^{\frac{1}{3}} b^{\frac{1}{3}}\left(a^{\frac{1}{3}} - b^{\frac{1}{3}}\right) = 3$$

$$\Rightarrow \quad a - b - 9a^{\frac{1}{3}} b^{\frac{1}{3}} = 27$$

$$\Rightarrow \quad x + 9 - x + 9 - 9(x+9)^{\frac{1}{3}}(x-9)^{\frac{1}{3}} = 27$$

$$\Rightarrow \quad 18 - 9(x+9)^{\frac{1}{3}}(x-9)^{\frac{1}{3}} = 27$$

$$\Rightarrow -9(x+9)^{\frac{1}{3}}(x-9)^{\frac{1}{3}} = 9$$

$$\Rightarrow (x+9)^{\frac{1}{3}}(x-9)^{\frac{1}{3}} = -1$$

$$\Rightarrow (x+9)(x-9) = -1$$

$$\Rightarrow x^2 - 81 = -1$$

$$\Rightarrow x^2 = 80$$

$$\Rightarrow 75 < x^2 < 85.$$

27. (*b*) $2^{2x^2-7x+5} = 1 = 2^0$

$$\Rightarrow 2x^2 - 7x + 5 = 0$$

$$\Rightarrow 2x^2 - 5x - 2x + 5 = 0$$

$$\Rightarrow x(2x-5) - 1(2x-5) = 0$$

$$\Rightarrow (2x-5)(x-1) = 0$$

$$x = 1 \text{ or } 5/2$$

So, there are two real values of x which satisfy the equation.

28. (*a*) $2x^2 + x < 6$

$$\Rightarrow 2x^2 + x - 6 < 0$$

$$\Rightarrow 2x^2 + 4x - 3x - 6 < 0$$

$$\Rightarrow 2x(x+2) - 3(x+2) < 0$$

$$\Rightarrow (2x-3)(x+2) < 0$$

$$\therefore -2 < x < 3/2.$$

29. (*c*) $y = x^2$ and $y = 3x + k$

$$\therefore x^2 = 3x + k$$

$$\Rightarrow x^2 - 3x - k = 0$$

This equation has two identical solutions when the discriminant of the equation is 0.

$$\therefore (-3)^2 - 4(-k) = 0$$

$$\Rightarrow 9 + 4k = 0$$

$$\therefore k = -9/4.$$

30. (*a*)

$$\frac{x^2 - bx}{ax - c} = \frac{m-1}{m+1}$$

$$(x^2 - bx)(m+1) = (ax - c)(m-1)$$

$$x^2m + x^2 - bxm - bx = axm - ax - cm + c$$

$$x^2(m+1) + x(a - am - bm - b) + cm - c = 0$$

Since, the roots are numerically equal but opposite in sign, so the sum of the roots will be zero.

$$a - am - bm - b = 0$$

$$-m(a+b) = b - a$$

$$m = \frac{a-b}{a+b}.$$

31. (*a*) The student obtained the roots as 8 and 2, when he made a mistake only with the constant term, i.e., the coefficient of x that he obtained was correct.

Thus, the correct sum of the roots = 10

In the same way, the correct value of the constant term is the product of the roots $=(-9)(-1) = 9$

Thus, the quadratic equation $= x^2 - 10x + 9 = 0$.

32. (*c*) $2x^2 + 6x + 5y + 1 = 0$ (1)

$2x + y + 3 = 0$ (2)

In the options, all the equations involved have only y in them. So, we take x in terms of y from one equation and substitute it in the other. From Eq. (2),

$$x = -\left(\frac{y+3}{2}\right)$$

Substituting the value of x in Eq. (1),

$$\frac{2(y+3)^2}{4} - \frac{6(y+3)}{2} + 5y + 1 = 0$$

$$\frac{y^2 + 6y + 9}{2} - 3(y+3) + 5y + 1 = 0$$

$$y^2 + 6y + 9 + 4y - 16 = 0$$

$$\Rightarrow x^2 + 10y - 7 = 0.$$

33. (*a*) $a = -b$, or $a + b = 0$

Use discriminant, $D = b^2 - 4ac$.

34. (*b*) Since, one root is the square of the other root in equation

$$x^2 + px + q = 0$$

$$\therefore p^3 - q(3p-1) + q^2 = 0.$$

35. (*c*) Given, $bx^2 - 2ax + a = 0$

The roots are

$$x = \frac{2a \pm \sqrt{4a^2 - 4ab}}{2b}$$

$$= \frac{a + \sqrt{a^2 - ab}}{b} \text{ and } \frac{a - \sqrt{a^2 - ab}}{b}$$

Consider $= \dfrac{a + \sqrt{a^2 - ab}}{b}$, rationalise this

$$\frac{a + \sqrt{a^2 - ab}}{b} \times \frac{a - \sqrt{a^2 - ab}}{a - \sqrt{a^2 - ab}} = \frac{a^2 - (a^2 - ab)}{b(a - \sqrt{a^2 - ab})}$$

$$= \frac{a}{a - \sqrt{a^2 - ab}}$$

$$= \frac{\sqrt{a}}{\sqrt{a} - \sqrt{a-b}}$$

Hence, the roots are

$$\frac{\sqrt{a}}{\sqrt{a}-\sqrt{a-b}} \text{ and } \frac{\sqrt{a}}{\sqrt{a}+\sqrt{a-b}}$$

The most likely answer is option (*c*).

36. (*a*) Since $x + 2$ is a factor of both the polynomials.

Put, $x + 2 = 0 \Rightarrow x = -2$

Let, $f(x) = px^2 + qx + r$

and, $g(x) = qx^2 + px + r$

$\therefore$ $f(-2) = 0 \Rightarrow p(-2)^2 + q(-2) + r = 0$

$\Rightarrow$ $4p - 2q + r = 0$ (1)

and, $g(-2) = 0 \Rightarrow q(-2)^2 + p(-2) + r = 0$

$\Rightarrow$ $4q - 2p + r = 0$ (2)

Adding Eqs. (1) and (2), we get $p + q + r = 0$

Subtracting Eq. (2) from Eq. (1), we get $p = q$

37. (*b*) Given, $x^2 - y^2 = 0$

$\Rightarrow$ $x^2 = y^2$ (1)

and, $(x-a)^2 + y^2 = 1$

$\Rightarrow$ $(x-a)^2 + x^2 = 1$

$\Rightarrow$ $2x^2 - 2ax + a^2 - 1 = 0$

For x to be real, $D > 0$

$4a^2 - 8(a^2 - 1) > 0$

$\Rightarrow$ $4a^2 - 8a^2 + 8 > 0$

$\Rightarrow$ $4a^2 < 8$

$\Rightarrow$ $a^2 < 2$

As a is non-negative, $a = 1$.

38. (*d*) $(1-p)x^2 + 4x + p = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1-p)p}}{2(1-p)}$$

$\therefore$ Roots of the equation will be real and both negative.

39. (*b*) $y^2 - 2y\cos x + 1 = 0$

Now, $D = (-2\cos x)^2 - 4 = -4\sin^2 x < 0$

Hence, no real solution of y exists.

40. (*a*) Here, $a + b = -3$ and $a \times b = -1$

$\therefore$ $P_2 = a^2 + b^2 = (a+b)^2 - 2ab$

$= 9 - (-2) = 11$

Now go through options

For (*a*) $-3P_{n-1} + P_{n-2} = P_n$

$\Rightarrow$ $P_2 = -3(P_1) + P_0$

$= -3(a+b) + (a^0 + b^0)$

$= -3(-3) + 1 + 1 = 9 + 2 = 11.$

41. (*c*) Given one root is $\frac{115}{11+\sqrt{6}}$

$\Rightarrow \frac{115}{11+\sqrt{6}} \times \left(\frac{11-\sqrt{6}}{11-\sqrt{6}}\right)$

$\Rightarrow \frac{115(11-\sqrt{6})}{121-6} \Rightarrow 11 - \sqrt{6}$

$\therefore$ Other root is $= 11 + \sqrt{6}$

$\therefore$ Equation will be

$x^2 - (11 - \sqrt{6} + 11 + \sqrt{6})x$

$+ (11 - \sqrt{6})(11 + \sqrt{6}) = 0$

$\Rightarrow$ $x^2 - 22x + (121 - 6) = 0$

$\Rightarrow$ $x^2 - 22x + 115 = 0$

42. (*c*) Roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ are 1, 2 and 3

$\therefore$ Roots of the equation $x^3 - px^2 + qx - r$ are 3, 4 and 5

$\therefore$ $r = 60$ [$\because$ constant term be the product of roots].

43. (*b*) Let the distance covered be d km and speed of train be x Km/h

We are given,

$\frac{d}{x} - \frac{d}{x+4} = \frac{30}{60}$

$\Rightarrow \frac{d(x+4-x)}{x(x+4)} = \frac{1}{2}$

$\Rightarrow$ $8d = x^2 + 4x$ (1)

and $\frac{d}{x-2} - \frac{d}{x} = \frac{20}{60}$

$\Rightarrow d\left[\frac{x-x+2}{x(x-2)}\right] = \frac{1}{3}$

$\Rightarrow$ $6d = x^2 - 2x$ (2)

From Eqs. (1) and (2), we get

$\frac{x^2+4x}{8} = \frac{x^2-2x}{6}$

$\Rightarrow 6x^2 + 24x = 8x^2 - 16x$

$\Rightarrow 2x^2 - 40x = 0$

$\therefore 2x(x-20) = 0$

$\therefore$ $x = 20$ Km/h

Hence, distance covered $= \frac{(20)^2 - 2 \times 20}{6}$

$= 60$ Km.