Code No. 041

Roll No. ☐☐☐☐☐☐☐

- Please check that this question paper contains **4** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **33** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII
Sample Paper (Solved)

Time allowed: 3 hours *Maximum Marks: 80*

General Instructions:

PART A

Section I

All questions are compulsory. In case of internal choices attempt anyone.

1. Find the number of all onto functions from the set {1, 2, 3, …, 10} to itself.

Or

Let **R** be the relation in the set Z of integers given by R = {(a, b) : 2 divides $a - b$}. Show that the relation R transitive? Write the equivalence class [0].

2. A = {1, 2}. How many one-one functions from A to A possible? Also write them.

3. Write total number of functions from set A to set B, where set A = {1, 2, 3, 4}, set B = {a, b, c}.

Or

If X and Y are two sets having 2 and 3 elements respectively then find the number of functions from X to Y.

4. If for any 2 × 2 square matrix A, A(adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$.

5. Given A = $\begin{pmatrix} 4 & 2 & 5 \\ 2 & 0 & 3 \\ -1 & 1 & 0 \end{pmatrix}$, write the value of det. ($2AA^{-1}$).

Or

If $\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2\begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix}$ = I, where I is a 2 × 2 Unit matrix, find ($x - y$).

6. If 3A – B = $\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A.

7. Evaluate: $\int_{-2}^{2} (x^3 + 1)$.

Or

Find $\int x\, e^{(1+x^2)} dx$.

8. Write the angle between the vectors $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.

9. Find the general solution of the differential equation $\frac{dy}{dx} + 2y = e^{3x}$.

Or

Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$.

10. If $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$, then the values of p and q are?

11. The magnitude of projection of $(2\hat{i} - \hat{j} + \hat{k})$ on $(\hat{i} - 2\hat{j} + 2\hat{k})$ is ....

12. Vector of magnitude 5 units and in the direction opposite to $(2\hat{i} + 3\hat{j} - 6\hat{k})$ is ....

13. Find the direction cosines of the vector joining the points A(1, 2, –3) and B(–1, –2, 1) directed from B to A.

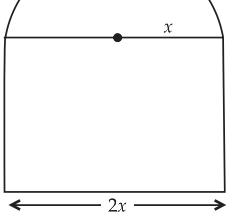
14. Find the vector equation for the line which passes through the point (1, 2, 3) and is parallel to the line $\frac{x-1}{-2} = \frac{1-y}{3} = \frac{3-z}{-4}$.

15. If A and B are two events such that P(A) = 0.2, P(B) = 0.4 and P(A ∪ B) = 0.5, then what is the value of P(A | B)?

16. An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. What is the probability that they are of the different colours?

Section II

Both the case-study based questions are compulsory. Attempt any 4 sub parts from each question (17 – 21) and (22 – 26). Each question carries 1 mark.



17. Case Study—An architect designs a House in which a window for a study room in designed in the form of a rectangle above which there is a semi-circle, so that maximum sunlight can enter into the room. The perimeter of the window is P and the length and breadth of the Rectangular portion of window is given by $2x$ and y respectively.

Based on the above information answer the following questions:

(*i*) What should be the relation between the variables.

(*a*) $x + \pi y = \text{P}$ (*b*) $2y + x\,(4 + \pi)$ (*c*) $\text{P} = 2(2y + x)$ (*d*) $\text{P} = 2(y + 2x)$

(*ii*) The area of the rectangular region of window expressed as a function of *x* is

(*a*) $x\,(\text{P} - x\,(4 + \pi))$ (*b*) $x\,(\text{P} + (4 - \pi)\,x)$ (*c*) $\text{P} + x\pi + 2x$ (*d*) $\text{P} - x\,(4 + \pi)$

(*iii*) The maximum value of Area of Rectangular region.

(*a*) $\frac{\text{P}^2}{4\,(16 + \pi)}$ (*b*) $\frac{\text{P}}{(4 + \pi^2)}$ (*c*) $\frac{\text{P}^2}{4\,(4 + \pi)}$ (*d*) $\frac{\text{P}}{2\,(2 + \pi)}$

(*iv*) The owner of the house wants to maximize the area of the whole window including the semi-circle head. For this to happen the value of *x* should be

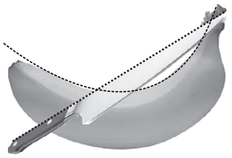
(*a*) $\frac{2\,(4 + \pi)}{(\text{P} + \pi)}$ (*b*) $\frac{-2\,(\text{P} + \pi)}{4 + \pi}$ (*c*) 0 (*d*) $\frac{(\text{P} + \pi)}{2\,(4 + \pi)}$

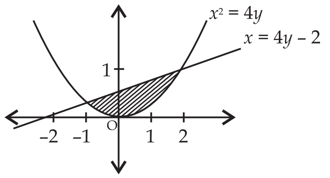
(*v*) Maximum area of entire window is:

(*a*) $\text{P}\left(\frac{\text{P} + \pi}{4 + \pi}\right)$ (*b*) $2x + \frac{\text{P} + \pi}{4 + \pi}$

(*c*) $\frac{4 + \pi}{6 + \pi} + x\pi$ (*d*) None of the above

18. **Case Study—*This image shows a banana with is overheaded by a knife.***





Answer the following questions:

(*i*) Present the equation of the line in term of *x*.

(*a*) $y = 4(x - 2)$ (*b*) $2y = 4(x + 2)$ (*c*) $y = \frac{x-2}{4}$ (*d*) $y = \frac{x+2}{4}$

(*ii*) Find the intersection points of line and parabola.

(*a*) $(-1, 1)\ (2, 1)$ (*b*) $\left(1, \frac{-1}{4}\right)(2, -1)$ (*c*) $\left(-1, \frac{1}{4}\right)(2, 1)$ (*d*) $(1, 1)\ (2, 2)$

(*iii*) Find the area of the line.

(*a*) $\frac{13}{2}$ sq. unit (*b*) $\frac{15}{8}$ sq. unit (*c*) $\frac{4}{3}$ sq. unit (*d*) 2 sq. unit

(*iv*) Find the Area of parabola.

(*a*) $\frac{1}{2}$ sq. unit (*b*) $\frac{2}{3}$ sq. unit (*c*) $\frac{3}{2}$ sq. unit (*d*) $\frac{3}{4}$ sq. unit

(*v*) Find the shaded region of the figure.

(*a*) $\frac{9}{8}$ sq. unit (*b*) $\frac{9}{4}$ sq. unit (*c*) $\frac{8}{9}$ sq. unit (*d*) $\frac{4}{3}$ sq. unit

PART B

Section III

19. Express $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$; where $-\frac{\pi}{4} < x < \frac{\pi}{4}$, in the simplest form.

20. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹ 15,000 per month, find their monthly incomes using matrix method.

Or, Show that all the diagonal elements of a skew symmetric matrix are zero.

21. If $y = \log(1 + 2t^2 + t^4)$, $x = \tan^{-1} t$, find $\frac{d^2y}{dx^2}$.

22. Separate the interval $\left[0, \frac{\pi}{2}\right]$ into subintervals in which the function $f(x) = \sin^4 x + \cos^4 x$ is strictly increasing or strictly decreasing.

23. Evaluate : $\int \frac{x+2}{\sqrt{x^2 + 5x + 6}}\, dx$ ***Or,*** Evaluate : $\int_1^4 \{|x-1| + |x-2| + |x-4|\}\, dx.$

24. Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above x-axis and between $x = -6$ to $x = 0$.

25. Solve the differential equation: $x\frac{dy}{dx} + y - x + xy \cot x = 0, x \neq 0$

26. For three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ if $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \times \vec{c} = \vec{b}$, then prove that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, $|\vec{b}| = |\vec{c}|$ and $|\vec{a}| = 1$.

27. Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the z-axis.

28. Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X.

Or, There are three coins, one is a two headed coin (having head on both the faces), another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows head. What is probability that it was the two headed coin?

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Consider $f : \mathbf{R} - \mathbf{R} - \left\{-\frac{4}{3}\right\} \to \text{R} - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective.

30. If $x = a(\cos 2\theta + 2\theta \sin 2\theta)$ and $y = a(\sin 2\theta - 2\theta \cos 2\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$.

31. Find 'a' and 'b', if the function given by $f(x) = \begin{cases} ax^2 + b, \text{ if } x < 1 \\ 2x + 1, \text{ if } x \geq 1 \end{cases}$ is differentiable at $x = 1$.

Or, Determine the values of 'a' and 'b' such that the following function is continuous at $x = 0$:

$$f(x) = \begin{cases} \dfrac{x + \sin x}{\sin(a+1)x}, & \text{if } -\pi < x < 0 \\ \quad 2 \quad , & \text{if } x = 0 \\ 2\dfrac{e^{\sin bx} - 1}{bx}, & \text{if } x > 0 \end{cases}$$

32. Find the equation of the normal to the curve $2y = x^2$, which passes through the point (2, 1).

33. Evaluate : $\int_{-1}^{1} |x \cos \pi x| \, dx.$

34. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 1, x + y \geq 1, x \geq 0, y \geq 0\}$.

Or, Using integration find the area of the following region : $\left\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\right\}$

35. Solve the following differential equation : $xy \log\left(\frac{y}{x}\right)dx + \left(y^2 - x^2 \log\left(\frac{y}{x}\right)\right)dy = 0$

Section V

All questions are compulsory. In case of internal choices attempt anyone.

36. If $\text{A} = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations:

$3x + 3y + 2z = 1; \quad x + 2y = 4; \quad 2x - 3y - z = 5$

Or, Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

37. Find the distance of point $-2\hat{i} + 3\hat{j} - 4\hat{k}$ from the line $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 3\hat{j} - 9\hat{k})$ measured parallel to the plane: $x - y + 2z - 3 = 0$.

Or, Find the coordinates of the point where the line through the points (3, –4, –5) and (2, –3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, –3) and (0, 4, 3).

38. Solve the following graphically and also find the maximum profit.

Maximum Profit, $\text{Z} = 24x + 18y$

Subject to the constraints: $2x + 3y \leq 10$; $3x + 2y \leq 10$; $x \geq 0, y \geq 0$.

Or, **(*a*) The corner points of the feasible region determined by the following system of linear inequalities:** $2x + y \leq 10$, $x + 3y \leq 15$, $x \geq 0$, $y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $\text{Z} = px + qy$, where $p, q > 0$. Find the condition of p and q so that the maximum of Z occurs at both (3, 4) and (0, 5).

(*b*) Solve the following graphically and also find the maximum profit.

Minimize and Maximize, $\text{Z} = 5x + 10y$

Subject to the constraints: $1x + 2y \leq 120$; $1x + 1y \geq 60$; $x - 2y \geq 8$; $x \geq 0, y \geq 0$.

Answer Sheet

SAMPLE PAPER

Code No. 041

Roll No.

MATHEMATICS

1. Total number of all onto functions from the set $\{1, 2, 3, \ldots, 10\}$ to itself is **10!**

Or

Let 2 divides $(a - b)$, and 2 divides $(b - c)$; where $a, b, c \in Z$

Let $a - b = 2p$ ...(*i*)

Let $b - c = 2q$ ...(*ii*)

Now, $a - c = a - b + b - c$

$= (a - b) + (b - c) = 2p + 2q$ **...[From (*i*) & (*ii*)**

$= 2(p + q)$

$\therefore$ 2 divides $a - c$, **Yes, relation R is transitive.**

Equivalence class $[0] = \{\mathbf{0, \pm 2, \pm 4, \pm 6, \ldots}\}$

2. Let $P = n(A) = 2$

$\therefore$ Number of one-one functions from A to A = P! = 2! = 2

They are $f_1 = \{\mathbf{(1, 1), (2, 2)}\}$ and $f_2 = \{\mathbf{(1, 2), (2, 1)}\}$

3. Let $p = n(A) = 4$ and $q = n(B) = 3$

Number of functions from A to B $= q^p = 3^4 = \mathbf{81}$

Or

Let $p = n(X) = 2$ and $q = n(Y) = 3$

Number of functions from X to Y $= q^p = 3^2 = \mathbf{9}$

4. $$A(adj\ A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ $\mathbf{|A| = 8}$ **... [$\because$ A (*adj* A) = | A | I**

5. As we know, $AA^{-1} = I$

$\therefore$ $2AA^{-1} = 2I$

$|2AA^{-1}| = |2I| = 8|I| = 8(1) = \mathbf{8}$ **[$\because$ |AI| = A^3 |I| and |I| = 1]**

Or

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2\begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + \begin{bmatrix} 2x & 0 \\ 2 & -4 \end{bmatrix} = I \qquad \Rightarrow \begin{bmatrix} 1 + 2x & 0 \\ y + 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$1 + 2x = 1,$ | $y + 2 = 0$

$\Rightarrow \quad 2x = 0$ | $\Rightarrow \; y = -2$

$\Rightarrow \quad x = 0$

$\therefore \quad \boldsymbol{x - y} = 0 - (-2) = \mathbf{2}$

6. **Given.** $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

$$\therefore \quad 3A - \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 3A = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow \quad A = \frac{1}{3}\begin{bmatrix} 9 & 3 \\ 3 & 6 \end{bmatrix} \qquad \therefore \; \mathbf{A} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

7. $$\int_{-2}^{2} (x^3 + 1)dx = \left[\frac{x^4}{4} + x\right]_{-2}^{2} = \left[\frac{2^4}{4} + 2\right] - \left[\frac{(-2)^4}{4} - 2\right]$$

$$= \left(\frac{16}{4} + 2\right) - \left(\frac{16}{4} - 2\right) = (6) - (2) = \mathbf{4}$$

Or

$$\int x\, e^{(1+x^2)} dx = \frac{1}{2}\int e^p dp \quad \Rightarrow \quad \frac{1}{2} e^p + c \quad \Rightarrow \quad \mathbf{\frac{1}{2} e^{(1+x^2)} + c}$$ **...[Let $p = 1 + x^2$, $dp = 2x\ dx$, $dp/2 = x\ dx$**

8. $\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})$

$\therefore$ The angle between the vector $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$ **= π.**

9. Given differential equation is $\frac{dy}{dx} + 2y = e^{3x}$

Here 'P' = 2, $Q = e^{3x}$

I.F $= e^{\int P\, dx} = e^{\int 2\, dx} = e^{2x}$

Hence, the solution is $y(\text{I.F}) = \int Q.(\text{I.F})\, dx$

$$y(e^{2x}) = \int e^{3x} . e^{2x} dx$$ [$\because a^m . a^n = a^{m+n}$

$$y(e^{2x}) = \int e^{5x}\, dx$$

$$y(e^{2x}) = \frac{e^{5x}}{5} + c$$

$$y = \frac{e^{5x}}{5e^{2x}} + \frac{c}{e^{2x}}$$

Therefore, $y = \frac{e^{3x}}{5} + ce^{-2x}$ is the general solution.

Or

9. $$\frac{dy}{dx} = x^3 e^{-2y}$$

$$\frac{dy}{e^{-2y}} = x^3\, dx$$

$$\int e^{2y} dy = \int x^3 dx$$ **...[Integrating both sides**

$$\therefore \quad \mathbf{\frac{e^{2y}}{2} = \frac{x^4}{4} + C}$$

10. **Given.** $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+p\hat{j}+q\hat{k}) = \vec{0}$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 6 & 27\\ 1 & p & q\end{vmatrix} = \vec{0}$$

Expanding along R_1, we have

$\Rightarrow \quad \hat{i}\,(6q-27p)-\hat{j}\,(2q-27)+\hat{k}\,(2p-6) = 0\hat{i}+0\hat{j}+0\hat{k}$

$\Rightarrow \quad -(2q-27) = 0$

$\Rightarrow \quad 2q = 27 \quad \therefore\ \boldsymbol{q = \frac{27}{2}}$

$2p-6 = 0$

$\Rightarrow \ 2p = 6 \qquad \therefore\ \boldsymbol{p} = \frac{6}{2} = \mathbf{3}$

11. Let $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}-2\hat{j}+2\hat{k}$

$\therefore$ **The magnitude of projection of $\vec{a}$ on $\vec{b}$** $= \dfrac{\vec{a}\,.\,\vec{b}}{|\vec{b}|} = \dfrac{(2\hat{i}-\hat{j}+\hat{k}).(\hat{i}-2\hat{j}+2\hat{k})}{\sqrt{(1)^2+(-2)^2+(2)^2}}$

$= \dfrac{2(1)-1(-2)+1(2)}{\sqrt{1+4+4}} = \dfrac{2+2+2}{\sqrt{9}} = \dfrac{6}{3} =$ **2 units**

12. Let $\vec{a} = 2\hat{i}+3\hat{j}-6\hat{k}$

Then, $-\vec{a} = -(2\hat{i}+3\hat{j}-6\hat{k})$

$\therefore$ **Required vector** $= \dfrac{5(-\vec{a})}{|\vec{a}|} = \dfrac{5(-2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{(2)^2+(3)^2+(-6)^2}} = \dfrac{5(-2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{49}} = \boldsymbol{\frac{5}{7}(-2\hat{i}-3\hat{j}+6\hat{k})}$

13. **Given.** A(1, 2, –3) and B(–1, –2, 1)
Direction ratios of BA:
$= 1-(-1), \quad 2-(-2), \quad -3-1$
$2, \quad 4, \quad -4$
or $\quad 1, \quad 2, \quad -2$
Here $a = 1, b = 2, c = -2$

Now, $\sqrt{a^2+b^2+c^2} = \sqrt{(1)^2+(2)^2+(-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

Direction cosines of BA are,

$l = \dfrac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \dfrac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \dfrac{c}{\sqrt{a^2+b^2+c^2}}$

$\boldsymbol{l = \frac{1}{3}}, \quad \boldsymbol{m = \frac{2}{3}}, \quad \boldsymbol{n = \frac{-2}{3}}$

14. Given line is $\dfrac{x-1}{-2} = \dfrac{y-1}{-3} = \dfrac{z-3}{4}$

Direction ratios of the given line are –2, –3 and 4
Given point is (1, 2, 3)

$\therefore \quad \vec{a} = 2\hat{i}+2\hat{j}+3\hat{k},\ \vec{b} = -2\hat{i}-3\hat{j}+4\hat{k}$

Vector equation of a line is, $\vec{r} = \vec{a}+\lambda\,\vec{b}$.

$\therefore \quad \boldsymbol{\vec{r} = (\hat{i}+2\hat{j}+3\hat{k})+\lambda(-2\hat{i}-3\hat{j}+4\hat{k})}$

15. As we know, $P(A\cap B) = P(A)+P(B)-P(A\cup B) = 0.2+0.4-0.5 = 0.1$

$\therefore \quad \mathbf{P(A\,|\,B)} = \dfrac{P(A\cap B)}{P(B)} = \dfrac{0.1}{0.4} = \dfrac{1}{4} = \mathbf{0.25}$

16.

Red	Black
2	4

$\therefore$ **P(different colours)** $= \dfrac{{}^2C_1 + {}^4C_1}{{}^6C_2} = \dfrac{\frac{2}{1} \times \frac{4}{1}}{\frac{6 \times 5}{2 \times 1}} = \dfrac{2}{1} \times \dfrac{4}{1} \times \dfrac{2 \times 1}{6 \times 5} = \mathbf{\dfrac{8}{15}}$

17. (*i*) (*b*); We have, length, $l = y$; Breadth, $b = 2x$; radius of the semi-circle, $r = x$.

Given. Perimeter of full window $= 2(y + 2x) + \pi x$

$P = 2y + 4x + \pi x$

$P = \mathbf{2y + x(4 + \pi)}$

(*ii*) (*a*); We have, $P = 2y + x(4 + \pi)$ …[From point (*i*)

$2y = P - x(4 + \pi)$

$y = \dfrac{P - x(4 + \pi)}{2}$ …(M)

Now, Area of the Rectangular region, $A = l \times b = 2x.y$

$A = 2x\left(\dfrac{P - x(4 + \pi)}{2}\right)$ …[From (M)

$A = x\,\mathbf{(P - x(4 + \pi))}$

(*iii*) (*c*); We have, $A = x\,[P - x(4 + \pi))$ …[From point (*ii*)

Differentiating the above w.r.t. x, we have

$\dfrac{dA}{dx} = [P - x(4 + \pi)] + x[-(4 + \pi)]$

$= P - x(4 + \pi) - x(4 + \pi) = P - 2x(4 + \pi)$ …(N)

When $\dfrac{dA}{dx} = 0, 0 = P - 2x(4 + \pi)$ $\Rightarrow x = \dfrac{P}{2(4 + \pi)}$

Again differentiating, $\dfrac{d^2A}{dx^2} = -2(4 + \pi) < 0$ (–ve)(maximum)

Hence A is maximum at $x = \dfrac{P}{2(4 + \pi)}$.

Therefore, maximum value of Area, $A = \dfrac{P}{2(4 + \pi)}\left[P - \dfrac{P}{2(4 + \pi)}(4 + \pi)\right]$ …[From point (*ii*)

$= \dfrac{P}{2(4 + \pi)}\left[\dfrac{P}{2}\right] = \mathbf{\dfrac{P^2}{4(4 + \pi)}}$

(*iv*) (*d*); Area of the whole window, F = Area of Rectangle + Area of semi-circle

Let $F = A + \pi x$ …[From point (*ii*)

Differentiating the above w.r.t. x, we get

$\dfrac{dF}{dx} = \dfrac{dA}{dx} + \pi = P - 2x(4 + \pi) + \pi$ …[From (N)

When $\dfrac{dF}{dx} = 0, P - 2x(4 + \pi) + \pi = 0$

$\Rightarrow 2x = \dfrac{(P + \pi)}{(4 + \pi)}$ $\therefore$ $\mathbf{x = \dfrac{1}{2}\left(\dfrac{P + \pi}{4 + \pi}\right)}$

(*v*) (*d*); Maximum Area of the window, $F = x\,[P - x(4 + \pi)] + \pi x$ …[From point (*ii*)

$= \dfrac{1}{2}\left[\dfrac{(P + \pi)}{(4 + \pi)}\right]\left[P - \dfrac{1}{2}\dfrac{(P + \pi)(4 + \pi)}{(4 + \pi)}\right] + \dfrac{\pi}{2}\left[\dfrac{(P + \pi)}{(4 + \pi)}\right]$ …[From point (*iv*)

$= \dfrac{1}{2}\dfrac{(P + \pi)}{(4 + \pi)}\left[P - \dfrac{1}{2}(P + \pi)\right] + \dfrac{\pi}{2}\left[\dfrac{(P + \pi)}{(4 + \pi)}\right]$

$$= \frac{1}{2}\frac{(P+\pi)}{(4+\pi)}\left[\frac{2P-P-\pi)}{2}\right]+\frac{\pi}{2}\left[\frac{(P+\pi)}{(4+\pi)}\right]$$

$$= \frac{1}{2}\frac{(P+\pi)}{(4+\pi)}\cdot\frac{(P-\pi)}{2}+\frac{\pi}{2}\left[\frac{(P+\pi)}{4+\pi}\right]$$

$$= \frac{(P+\pi)}{(4+\pi)}\left[\frac{P-\pi}{4}+\frac{\pi}{2}\right]=\frac{(P+\pi)}{(4+\pi)}\frac{(P+\pi)}{4} \quad = \boldsymbol{\frac{(P+\pi)^2}{4(4+\pi)}}$$

18. **(*i*)** **(*d*)**; We have, $x = 4y - 2$

$\Rightarrow \; 4y = x + 2 \qquad \therefore \; \boldsymbol{y = \frac{x+2}{4}}$

(*ii*) **(*c*)**; **For point of intersection,**

$x = 4y - 2$ …(*i*)

$y = \frac{x^2}{4}$ …(*ii*)

$x = 4\left(\frac{x^2}{4}\right) - 2$ …[From (*ii*)

$\Rightarrow \; x = x^2 - 2$

$\Rightarrow \; x^2 - x - 2 = 0$

$\Rightarrow \; x^2 - (2x - 1x) - 2 = 0$

$\Rightarrow \; x^2 - 2x + 1x - 2 = 0$

$\Rightarrow \; x(x-2) + 1(x-2) = 0$

$\Rightarrow \; (x-2)(x+1) = 0$

$\Rightarrow \; x - 2 = 0 \quad \forall \quad x + 1 = 0$

$\Rightarrow \; x = 2 \quad \forall \quad x = -1$

From (*ii*), when $x = 2$, then $y = 1$

when $x = -1$, then $y = \frac{1}{4}$

(2, 1) and $\boldsymbol{\left(-1, \frac{1}{4}\right)}$

(*iii*) **(*b*)**; **Area of line** $= \int_{-1}^{2}\left(\frac{x+2}{4}\right)dx = \frac{1}{4}\left[\frac{x^2}{2}+2x\right]_{-1}^{2}$

$$= \frac{1}{4}\left[\frac{4}{2}+4-\frac{1}{2}+2\right] = \frac{1}{4}\left[8-\frac{1}{2}\right] = \frac{1}{4}\times\frac{15}{2} = \boldsymbol{\frac{15}{8}} \textbf{ sq. unit}$$

(*iv*) **(*d*)**; **Area of parabola** $= \int_{-1}^{2}\frac{x^2}{4}dx = \left[\frac{x^3}{12}\right]_{-1}^{2} = \frac{8}{12}-\frac{(-1)}{12}$

$$= \frac{8}{12}+\frac{1}{12} = \frac{9}{12} = \boldsymbol{\frac{3}{4}} \textbf{ sq. units}$$

(*v*) **(*a*)**; **Area of Shaded Region** = Area of line – Area of parabola

$$= \frac{15}{8}-\frac{3}{4} = \frac{15-6}{8} = \boldsymbol{\frac{9}{8}} \textbf{ sq. unit}$$ …[From point (*iii*) & (*iv*)

19. $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$

$= \sin^{-1}\left(\sin x.\frac{1}{\sqrt{2}} + \cos x.\frac{1}{\sqrt{2}}\right)$

$= \sin^{-1}\left(\sin x.\cos\frac{\pi}{4} + \cos x.\sin\frac{\pi}{4}\right)$

$= \sin^{-1}\left[\sin\left(x+\frac{\pi}{4}\right)\right] = \boldsymbol{x + \frac{\pi}{4}}$

…[**if** $\boldsymbol{\frac{-\pi}{4} < x < \frac{\pi}{4}}$

$\boldsymbol{-\frac{\pi}{4}+\frac{\pi}{4} < x + \frac{\pi}{4} < \frac{\pi}{4}+\frac{\pi}{4}}$

$\boldsymbol{0 < x + \frac{\pi}{4} < \frac{\pi}{2}}$

i.e. **Principal value**

20. The monthly incomes of Aryan and Babban are ₹ $3x$ and ₹ $4x$ respectively, and the monthly expenditures of Aryan and Babban are ₹ $5y$ and ₹ $7y$ respectively.

$\therefore \quad 3x - 5y = 15000$

$4x - 7y = 15000$

Writing in matrix form

$$\begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

$$\text{A} \quad \text{X} = \text{B}$$

$$\text{A}^{-1}(\text{AX}) = \text{A}^{-1}\text{B} \qquad \text{...[Pre-multiplying by A}^{-1}$$

$$\text{IX} = \text{A}^{-1}\text{B}$$

$$\text{X} = \text{A}^{-1}\text{B} \qquad \ldots(i)$$

$|\text{A}| = -21 + 20 = -1 \neq 0$

$\therefore \quad \text{A}^{-1}$ does exist.

$$adj\ \text{A} = \begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix}$$

$$\text{A}^{-1} = \frac{1}{|\text{A}|}.adj\ \text{A} = \frac{1}{-1}\begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$$

From (*i*), $\quad \text{X} = \text{A}^{-1}\text{B}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 15000 \\ 15000 \end{bmatrix} = \begin{bmatrix} 105000 - 75000 \\ 60000 - 45000 \end{bmatrix} = \begin{bmatrix} 30000 \\ 15000 \end{bmatrix}$$

$\therefore \quad x =$ ₹30,000 and $y =$ ₹15000

The monthly income of Aryan = ₹ $3x$ = **₹90,000**

The monthly income of Babban = ₹ $4x$ = **₹1,20,000**

Or

Let $\text{A} = [a_{ij}]$ be a skew-symmetric matrix.

$a_{ij} = -a_{ji}$ for all i, j

$a_{ii} = -a_{ii}$ ...[for all values of *i.e.*, $i = j$

$a_{ii} + a_{ii} = 0$

$2a_{ii} = 0$

$a_{ii} = 0$ for all values of i.

$\Rightarrow \quad \boldsymbol{a_{11} = a_{22} = a_{33} = \ldots = a_{nn} = 0}$

21. **Given:** $y = \log(1 + 2t^2 + t^4)$

$y = \log(1 + t^2)^2$

$y = 2 \log (1 + t^2)$ **...[$\because \log x^n = n \log x$**

Differentiating both sides w.r.t. t, we get $\dfrac{dy}{dt} = \dfrac{2(2t)}{1+t^2} = \dfrac{4t}{1+t^2}$ ...(*i*)

Also we have, $x = \tan^{-1}t$

Differentiating both sides w.r.t t, $\dfrac{dx}{dt} = \dfrac{1}{1+t^2}$ ...(*ii*)

Now, From (*i*) & (*ii*), $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\frac{4t}{1+t^2}}{\frac{1}{1+t^2}} = 4t$ ...(*iii*)

$\therefore \quad \dfrac{\boldsymbol{d^2y}}{\boldsymbol{dx^2}} = \dfrac{d}{dt}(4t) \times \dfrac{dt}{dx} = 4 \times (1 + t^2) = \mathbf{4(1 + t^2)}$ ...[From (*ii*)

22. $f(x) = \sin^4 x + \cos^4 x, \qquad \left(0, \frac{\pi}{2}\right)$

Differentiating both sides w.r.t. x, we have

$$\begin{aligned} f'(x) &= 4\sin^3 x \,.\, \cos x + 4\cos^3 x \,.\, (-\sin x) \\ &= 4 \sin x \cos x\, (\sin^2 x - \cos^2 x) \\ &= -2 \,.\, 2\sin x \cos x\, (\cos^2 x - \sin^2 x) \\ &= -2 \,.\, \sin 2x \,.\, \cos 2x = -\sin 4x \end{aligned}$$

...[$\because$ 2 sin θ cos θ = sin 2θ

When $f'(x) = 0$

$-\sin 4x = 0 \qquad \Rightarrow \sin 4x = 0$

$\Rightarrow \quad 4x = 0, \pi, 2\pi, \ldots \quad \Rightarrow x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \ldots$

Intervals	*Checking point*	*Sign of –sin 4x*	*Sign of f′(x)*	*Nature of f(x)*
$\left(0, \frac{\pi}{4}\right)$	$x = \frac{\pi}{6}$	–ve as $0 < 4x < \pi$	≤ 0	decreasing
$\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$	$x = \frac{\pi}{2}$	+ve as $\pi < 4x < 2\pi$	≥ 0	increasing

So, $f(x)$ is strictly decreasing on $\left[0, \frac{\pi}{4}\right]$ and strictly increasing on $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

23.

$$\int \frac{x+2}{\sqrt{x^2+5x+6}}\,dx = \frac{1}{2}\int \frac{2(x+2)}{\sqrt{x^2+5x+6}}\,dx = \frac{1}{2}\int \frac{(2x+4+5-5)}{\sqrt{x^2+5x+6}}\,dx$$

$$= \frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+6}}\,dx - \frac{1}{2}\int \frac{dx}{\sqrt{x^2+5x+6}}$$

...[Let $p = x^2 + 5x + 6$, $dp = (2x + 5)\, dx$

$$= \frac{1}{2}\int \frac{dp}{\sqrt{p}} - \frac{1}{2}\int \frac{dx}{\sqrt{x^2+5x+\left(\frac{5}{2}\right)^2+6-\left(\frac{5}{2}\right)^2}}$$

$$= \frac{1}{2}\int p^{-1/2}dp - \frac{1}{2}\int \frac{dx}{\sqrt{\left(x+\frac{5}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$

$$= \frac{1}{2}\cdot\frac{2}{1}p^{1/2} - \frac{1}{2}\log\left| x+\frac{5}{2}+\sqrt{\left(x+\frac{5}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right| + C$$

$$\left[\because \int \frac{dx}{\sqrt{x^2-a^2}} = \log\left| x+\sqrt{x^2-a^2}\right| + C\right.$$

$$= \boldsymbol{\sqrt{x^2+5x+6} - \frac{1}{2}\log\left| x+\frac{5}{2}+\sqrt{x^2+5x+6}\right| + C}$$

Or

$$|x-1| = \begin{cases} -(x-1), & x < 1 \\ (x-1), & x \ge 1 \end{cases}$$

$$|x-2| = \begin{cases} -(x-2), & x < 2 \\ (x-2), & x \ge 2 \end{cases}$$

$$|x-4| = \begin{cases} -(x-4), & x < 4 \\ (x-4), & x \ge 4 \end{cases}$$

$$\text{Let } I = \int_1^4 |x-1| + |x-2| + |x-4| \; dx$$

$$= \int_1^4 |x-1|\,dx + \int_1^4 |x-2|\,dx + \int_1^4 |x-4|\,dx$$

$$= \int_1^4 (x-1)\,dx + \int_1^2 -(x-2)\,dx + \int_2^4 (x-2)\,dx + \int_1^4 -(x-4)\,dx$$

$$= \left(\frac{x^2}{2} - x\right)_1^4 - \left(\frac{x^2}{2} - 2x\right)_1^2 + \left(\frac{x^2}{2} - 2x\right)_2^4 - \left(\frac{x^2}{2} - 4x\right)_1^4$$

$$= \left[\left(\frac{16}{2} - 4\right) - \left(\frac{1}{2} - 1\right)\right] - \left[\left(\frac{4}{2} - 4\right) - \left(\frac{1}{2} - 2\right)\right] + \left[\left(\frac{16}{2} - 8\right) - \left(\frac{4}{2} - 4\right)\right] - \left[\left(\frac{16}{2} - 16\right) - \left(\frac{1}{2} - 4\right)\right]$$

$$= \left(4 + \frac{1}{2}\right) - \left(-2 + \frac{3}{2}\right) + (0 + 2) - \left(-8 + \frac{7}{2}\right)$$

$$= \frac{9}{2} + \frac{1}{2} + 2 + \frac{9}{2} = \mathbf{\frac{23}{2}}$$

24. Given curve is $y = |x + 3|$

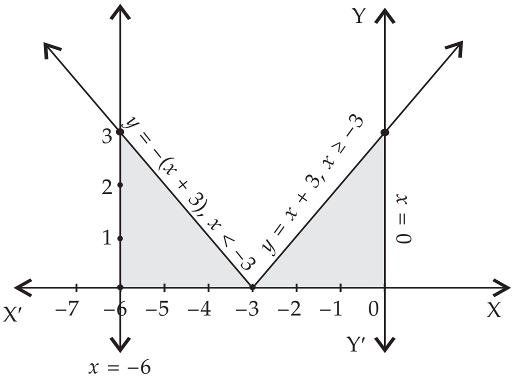
$$|x + 3| = \begin{cases} -(x + 3), & x < -3 \\ (x + 3), & x \geq -3 \end{cases}$$

$y = -(x + 3), x < -3;$

x	-4	-5	-6
y	1	2	3

$y = (x + 3), x \geq -3$

x	0	-1	-2	-3
y	3	2	1	0



$$\textbf{Area of the shaded region} = 2\int_{-3}^{0} (x + 3)\; dx = 2\left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

$$= 2\left[\left(\frac{0}{2} + 3(0)\right) - \left(\frac{9}{2} - 9\right)\right] = 2\left[-\left(\frac{9 - 18}{2}\right)\right] = \textbf{9 sq. units}$$

25. $x\frac{dy}{dx} + y - x + xy \cot x = 0, x \neq 0$

Dividing both sides by x, we get

$$\frac{dy}{dx} + \frac{y}{x} - 1 + y \cot x = 0 \qquad \Rightarrow \quad \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$$

Comparing with $\frac{dy}{dx}$ + Py = Q, we have 'P' = $\frac{1}{x}$ + cot x, Q = 1

$$\text{I.F.} = e^{\int \mathrm{P}\,dx} = e^{\int\left(\frac{1}{x}+\cot x\right)dx} = e^{\log x+\log|\sin x|}$$

$$= e^{\log|x\sin x|} = x\sin x \qquad [\because e^{\log|x|} = x]$$

Hence the solution is y(I.F.) = $\int$ Q.(I.F.) dx

$$y(x\sin x) = \int 1.(\underset{\text{I}}{x}\ \underset{\text{II}}{\sin} x)\ dx$$

$$= x(-\cos x) - \int 1(-\cos x)\ dx$$

$$y(x\sin x) = -x\cos x + \sin x + c$$

$$y = \frac{-x\cos x}{x\sin x} + \frac{\sin x}{x\sin x} + \frac{c}{x\sin x} \qquad \therefore \quad \boldsymbol{y = \frac{1}{x} - \cot x + \frac{c}{x\sin x}}$$

26. $\vec{a} \times \vec{b} = \vec{c}$ **(given)**

$\therefore \quad \vec{c} \perp \vec{a}$ and $\vec{c} \perp \vec{b}$

$\vec{a} \times \vec{c} = \vec{b}$ **(given)**

$\therefore \quad \vec{b} \perp \vec{a}$ and $\vec{b} \perp \vec{c}$

$\therefore \quad \vec{a} \perp \vec{b} \perp \vec{c}$

$\therefore \quad \vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors.

Given. $\vec{a} \times \vec{b} = \vec{c}$

$$|\vec{a}\times\vec{b}| = |\vec{c}|$$

$$|\vec{a}||\vec{b}|\sin\frac{\pi}{2} = |\vec{c}| \qquad \ldots[\because \boldsymbol{\vec{a}\perp\vec{b}}$$

$$|\vec{a}||\vec{b}|.1 = |\vec{c}| \qquad \ldots(i)$$

Given. $\vec{a} \times \vec{c} = \vec{b}$

$$|\vec{a}\times\vec{c}| = |\vec{b}|$$

$$|\vec{a}||\vec{c}|\sin\frac{\pi}{2} = |\vec{b}| \qquad \ldots[\because \boldsymbol{\vec{a}\perp\vec{b}}$$

$$|\vec{a}||\vec{c}|.1 = |\vec{b}| \qquad \ldots(ii)$$

$$|\vec{a}||\vec{a}||\vec{b}| = |\vec{b}| \qquad \ldots[\text{From } (i)$$

$$|\vec{a}|^2 = \frac{|\vec{b}|}{|\vec{b}|} = 1 \Rightarrow |\vec{a}|^2 = 1 \qquad \Rightarrow |\vec{a}| = 1$$

Putting the value of $|\vec{a}|$ in (i), we have

$$1|\vec{c}|.1 = |\vec{b}|$$

$$|\vec{c}| = |\vec{b}| \quad \text{or} \quad |\vec{b}| = |\vec{c}|$$ **(Hence proved)**

27. Vector equation of the line passing through (1, 2, 3) and (–3, 4, 3) is

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}) \qquad \ldots[\text{where } \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{b} = -3\hat{i} + 4\hat{j} + 3\hat{k}$$

$$\Rightarrow \quad \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda[(-3-1)\hat{i} + (4-2)\hat{j} + (3-3)\hat{k}]$$

$$\Rightarrow \quad \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-4\hat{i} + 2\hat{j}) \qquad \ldots(i)$$

Equation of z-axis is $\vec{r} = \mu\hat{k}$ …(ii)

Since $(-4\hat{i} + 2\hat{j}) \cdot \hat{k} = 0$

$\therefore$ **Line (i) is $\perp$ to z-axis.**

28. First 7 natural numbers are 1, 2, 3, 4, 5, 6, 7.

$$S = \begin{Bmatrix} (1,2) & (1,3) & (1,4) & (1,5) & (1,6) & (1,7) \\ (2,1) & (2,3) & (2,4) & (2,5) & (2,6) & (2,7) \\ (3,1) & (3,2) & (3,4) & (3,5) & (3,6) & (3,7) \\ (4,1) & (4,2) & (4,3) & (4,5) & (4,6) & (4,7) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,6) & (5,7) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,7) \\ (7,1) & (7,2) & (7,3) & (7,4) & (7,5) & (7,6) \end{Bmatrix} \text{ i.e. 42 ways}$$

Let X denotes the smaller of the two numbers obtained.
So X can take values 1, 2, 3, 4, 5, 6.

$P(X = 1) = \frac{12}{42} = \frac{6}{21}$ or $\frac{2}{7}$ …**[favourable cases (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (1, 7) (2, 1) (3, 1) (4, 1) (5, 1) (6, 1) (7, 1) *i.e.* 12 ways**

$P(X = 2) = \frac{10}{42} = \frac{5}{21}$ …**[favourable cases (2, 3), (2, 4), (2, 5), (2, 6), (2, 7) (3, 2), (4, 2), (5, 2), (6, 2), (7, 2) *i.e.* 10 ways**

$P(X = 3) = \frac{8}{42} = \frac{4}{21}$ …**[favourable cases (3, 4) (3, 5), (3, 6) (3, 7) (4, 3), (5, 3), (6, 3), (7, 3) *i.e.* 8 ways**

$P(X = 4) = \frac{6}{42} = \frac{3}{21}$ or $\frac{1}{7}$ …**[favourable cases (4, 5) (4, 6), (4, 7), (5, 4), (6, 4), (7, 4) *i.e.* 6 ways**

$P(X = 5) = \frac{4}{42} = \frac{2}{21}$ …**[favourable cases (5, 6) (5, 7) (6, 5), (7, 5) *i.e.* 4 ways**

$P(X = 6) = \frac{2}{42} = \frac{1}{21}$ …**[favourable cases (6, 7), (7, 6) *i.e.* 2 ways**

Probability distribution is

X	1	2	3	4	5	6
P(X)	$\frac{6}{21}$ or $\frac{2}{7}$	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{3}{21}$ or $\frac{1}{7}$	$\frac{2}{21}$	$\frac{1}{21}$

Or

Let E_1 : Two headed coin is chosen
E_2 : Coin chosen is biased
E_3 : Coin chosen is unbiased
A : Coin shows head

$$P(E_1) = \frac{1}{3},\ P(E_2) = \frac{1}{3},\ P(E_3) = \frac{1}{3}$$

$$P(A \mid E_1) = 1; \quad P(A \mid E_2) = 75\% \text{ or } \frac{75}{100} = \frac{3}{4}; \quad P(A \mid E_3) = \frac{1}{2}$$

Using Baye's theorem,

$$\therefore \quad \mathbf{P(E_1 \mid A)} = \frac{P(E_1) \times P(A \mid E_1)}{[P(E_1) \times P(A \mid E_1)] + [P(E_2) \times P(A \mid E_2)] + [P(E_3) \times P(A \mid E_3)]}$$

$$= \frac{\frac{1}{3} \times 1}{\left(\frac{1}{3} \times 1\right) + \left(\frac{1}{3} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{3}{12} + \frac{1}{6}} = \frac{\frac{1}{3}}{\frac{4+3+2}{12}} = \frac{1}{3} \times \frac{12}{9} = \mathbf{\frac{4}{9}}$$

29. $f(x) = \frac{4x+3}{3x+4}$

For *f* is one-one,

Let $x_1, x_2 \in R$

$$F(x_1) = F(x_2)$$

$$\frac{4x_1 + 3}{3x_1 + 4} = \frac{4x_2 + 3}{3x_2 + 4}$$

$\Rightarrow$ $(4x_1 + 3)(3x_2 + 4) = (4x_2 + 3)(3x_1 + 4)$

$\Rightarrow$ $12x_1x_2 + 16x_1 + 9x_2 + 12 = 12x_1x_2 + 16x_2 + 9x_1 + 12$

$\Rightarrow$ $16x_1 - 9x_1 = 16x_2 - 9x_2$

$\Rightarrow$ $7x_1 = 7x_2$

$\Rightarrow$ $x_1 = x_2$ $\therefore$ ***f* is one-one.**

For *f* is onto,

Let y be any element of R

$$y = f(x)$$

$$y = \frac{4x+3}{3x+4}$$

$$3xy + 4y = 4x + 3$$

$$3xy - 4x = 3 - 4y$$

$$x(3y - 4) = 3 - 4y$$

$$x = \frac{3-4y}{3y-4}$$

We have, $f(x) = \frac{4x+3}{3x+4}$

$$f\left(\frac{3-4y}{3y-4}\right) = \frac{4\left(\frac{3-4y}{3y-4}\right) + 3}{3\left(\frac{3-4y}{3y-4}\right) + 4} = \frac{\frac{12 - 16y + 9y - 12}{3y-4}}{\frac{9 - 12y + 12y - 16}{3y-4}} = \frac{-7y}{-7} = y$$

$f(x) = y$ $\therefore$ ***f* is onto.**

30. We are given, $x = a(\cos 2\theta + 2\theta \sin 2\theta)$

Differentiating both sides w.r.t. θ, we have

$$\frac{dx}{d\theta} = a[-2\sin 2\theta + 2(\sin 2\theta + 2\theta \cos 2\theta)]$$

$\Rightarrow$ $\frac{dx}{d\theta} = a[-2\sin 2\theta + 2\sin 2\theta + 4\theta \cos 2\theta)]$

$\Rightarrow$ $\frac{dx}{d\theta} = a[4\theta \cos 2\theta]$ …(*i*)

Now, $y = a(\sin 2\theta - 2\theta \cos 2\theta)$

Differentiating both sides w.r.t. θ, we have

$$\frac{dy}{d\theta} = a[2\cos 2\theta - 2(\cos 2\theta - 2\theta \sin 2\theta)]$$

$\Rightarrow \quad \dfrac{dy}{d\theta} = a(2\cos 2\theta - 2\cos 2\theta + 4\theta \sin 2\theta)$

$\Rightarrow \quad \dfrac{dy}{d\theta} = a(4\theta \sin 2\theta)$ …(*ii*)

Now, $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \times \dfrac{d\theta}{dx}$ **…[From (*i*) & (*ii*)**

$$\frac{dy}{dx} = \frac{a(4\theta \sin 2\theta)}{a(4\theta \cos 2\theta)} = \tan 2\theta$$

Differentiating both sides w.r.t. x, we have

$\dfrac{d^2y}{dx^2} = 2\sec^2 2\theta . \dfrac{d\theta}{dx} \Rightarrow \dfrac{d^2y}{dx^2} = 2\sec^2 2\theta \left(\dfrac{1}{a.4\theta \ \cos 2\theta} \right)$ **…[From (*i*)**

$$\therefore \quad \left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{8}} = 2\sec^2 2\left(\frac{\pi}{8}\right)\left(\frac{1}{a.4\left(\frac{\pi}{8}\right)\cos 2\left(\frac{\pi}{8}\right)}\right)$$

$$= 2\sec^2\left(\frac{\pi}{4}\right)\left(\frac{1}{a\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{4}\right)}\right) = 2\left(\sqrt{2}\right)^2\left(\frac{1}{\frac{\pi a}{2}\times\frac{1}{\sqrt{2}}}\right) = \frac{4\times 2\sqrt{2}}{\pi a} = \boldsymbol{\frac{8\sqrt{2}}{\pi a}}$$

31. As f is differentiable at $x = 1$.

$\therefore$ f is continuous at 1.

R.H.L. = $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} (2x+1) = 2(1) + 1 = 3$

L.H.L. = $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} (ax^2 + b) = a(1)^2 + b = a + b$

Since the function is continuous, L.H.L. = R.H.L.

$a + b = 3$ $[\because f(1) = 3$ …(*i*)

$\mathrm{L}f'(1) = \lim\limits_{h\to 0^+} \dfrac{f(1-h) - f(1)}{-h} \quad \Rightarrow \lim\limits_{h\to 0^+} \dfrac{a(1-h)^2 + b - 3}{-h}$

$\Rightarrow \lim\limits_{h\to 0^+} \dfrac{a + ah^2 - 2ah + b - 3}{-h} \quad \Rightarrow \lim\limits_{h\to 0^+} \dfrac{ah^2 - 2ah}{-h}$ **…[Using (*i*)**

$\Rightarrow \lim\limits_{h\to 0^+} (-ah + 2a) = \boldsymbol{2a}$

Also, $\mathrm{R}f'(1) = \lim\limits_{h\to 0^+} \dfrac{f(1+h) - f(1)}{h} = \lim\limits_{h\to 0^+} \dfrac{2(1+h) + 1 - 3}{h} = \mathbf{2}$

As f is differentiable at 1, we have $2a = 2 \Rightarrow$ ***a* = 1** and ***b* = 2.**

Or

As the function is continuous at $x = 0$, we have

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

L.H.L. $\lim\limits_{h\to 0^-} \dfrac{x + \sin x}{\sin(a+1)x}$

$$= \lim_{h\to 0^-} \frac{x\left(1 + \frac{\sin x}{x}\right)}{\sin(a+1)x}$$

$$= \lim_{x\to 0^-} \frac{1 + \frac{\sin x}{x}}{\frac{\sin(a+1)}{(a+1)}.(a+1)}$$

$$= \frac{2}{a+1}$$

R.H.L. $\lim\limits_{x\to 0^+} 2\dfrac{e^{\sin bx} - 1}{bx}$

$$= \lim_{x\to 0^+} 2\frac{e^{\sin bx} - 1}{\sin bx} \times \frac{\sin bx}{bx}$$

$$= 2$$

For the function to be continuous at 0, we must have L.H.L. = R.H.L.

$$\therefore \quad \frac{2}{a+1} = 2 \Rightarrow a + 1 = 1 \Rightarrow \boldsymbol{a = 0}$$

Therefore, *b* may be any real number other than 0.

32. **Find the equation of the normal to the curve $2y = x^2$, which passes through the point (2, 1).**

Let the normal be at (x_1, y_1) to the curve $2y = x^2$ …(*i*)

$$2y = x^2 \qquad \Rightarrow \qquad y = \frac{1}{2}x^2$$

Differentiating both sides w.r.t. x, we have

$$\frac{dy}{dx} = \frac{1}{2}.2x = x$$

Slope of normal at $(x_1, y_1) = \dfrac{-1}{\left(\dfrac{dy}{dx}\right)_{(x_1,\, y_1)}} = \dfrac{-1}{x_1}$

Equation of normal at (x_1, y_1) is $y - y_1$ = slope of normal $(x - x_1)$

$$y - y_1 = \frac{-1}{x_1}(x - x_1) \qquad \ldots(ii)$$

Point (2, 1) lies on (*i*), $1 - y_1 = \dfrac{-1}{x_1}(2 - x_1)$

$$\Rightarrow \quad x_1 - x_1y_1 = -2 + x_1 \Rightarrow x_1y_1 = 2 \qquad \ldots(iii)$$

Also, point (x_1, y_1) lies on the given curve (*i*)

$$2y_1 = {x_1}^2 \qquad \Rightarrow y_1 = \frac{1}{2}{x_1}^2$$

Putting the value of y_1 in (*iii*), we have

$$x_1\left(\frac{1}{2}{x_1}^2\right) = 2 \qquad \Rightarrow {x_1}^3 = 2^2$$

Taking cube root on both sides,

$$x_1 = 2^{2/3}$$

From (*iii*), $2^{2/3}.y_1 = 2 \quad \Rightarrow y_1 = \dfrac{2}{2^{2/3}} = 2^{1/3}$

Putting the value of x_1 and y_1 in (*i*), we have

$$y - 2^{1/3} = \frac{-1}{2^{2/3}}(x - 2^{2/3})$$

$$2^{2/3}.y - 2 = -x + 2^{2/3}$$

$\boldsymbol{x + 2^{2/3}.y = 2 + 2^{2/3}}$ **is the required equation of the normal.**

33. Let $f(x) = |x \cos \pi x|$

$f(-x) = |-x \cos(-\pi x)| = |-x \cos \pi\, x|$ $[\because \cos(-\theta) = \cos\theta]$

$= x \cos \pi x = f(x)$

Let $\mathrm{I} = \displaystyle\int_{-1}^{1} |x \cos \pi x|\, dx$

$$= 2\int_0^1 |x \cos \pi x| \qquad \left[\because \int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx \text{ if } f(-x) = f(x)\right.$$

$$\left[\because |x\cos\pi x| = \begin{cases} +x\cos\pi x, \text{ if } 0 \le x \le \dfrac{1}{2} \\ -x\cos\pi x, \text{ if } \dfrac{1}{2} \le x \le 1 \end{cases}\right.$$

$$= 2\int_0^{\frac{1}{2}} x\cos \pi x\, dx + \int_{\frac{1}{2}}^{1} - x\cos \pi x\, dx \qquad \left[\because \int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \text{ ...where } a < c < b\right.$$

$$= 2\int_0^{\frac{1}{2}} x\cos \pi x\, dx - 2\int_{\frac{1}{2}}^{1} x\cos \pi x\, dx$$

Integrating by parts taking x as first function, we have

$$= 2\left[x\frac{\sin \pi x}{\pi} - \int 1.\frac{\sin \pi x}{\pi}dx\right]_0^{\frac{1}{2}} - 2\left[x\frac{\sin \pi x}{\pi} - \int 1.\frac{\sin \pi x}{\pi}dx\right]_{\frac{1}{2}}^{1}$$

$$= 2\left[\frac{x}{\pi}\sin \pi x + \frac{1}{\pi^2}\cos \pi x\right]_0^{\frac{1}{2}} - 2\left[\frac{x}{\pi}\sin \pi x + \frac{1}{\pi^2}\cos \pi x\right]_{\frac{1}{2}}^{1}$$

$$= 2\left[\left(\frac{1}{2\pi}\sin\frac{\pi}{2} + \frac{1}{\pi^2}\cos\frac{\pi}{2}\right) - \left(0 + \frac{1}{\pi^2}\cos 0\right)\right] - 2\left[\left(\frac{1}{\pi}\sin \pi + \frac{1}{\pi^2}\cos \pi\right) - \left(\frac{1}{2\pi}\sin\frac{\pi}{2} + \frac{1}{\pi^2}\cos\frac{\pi}{2}\right)\right]$$

$$= 2\left[\frac{1}{2\pi}(1) + \frac{1}{\pi^2}(0) - \frac{1}{\pi^2}\right] - 2\left[\frac{1}{\pi^2}(-1) - \left(\frac{1}{2\pi}(1) + \frac{1}{\pi^2}(0)\right)\right] \qquad \left[\begin{array}{l}\because \sin \pi = 0,\ \cos \pi = -1, \\ \text{and } \sin\frac{\pi}{2} = 1,\ \cos\frac{\pi}{2} = 0\end{array}\right.$$

$$= 2\left[\frac{1}{2\pi} - \frac{1}{\pi^2}\right] - 2\left[\frac{-1}{\pi^2} - \frac{1}{2\pi}\right] = 2\left[\frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{1}{\pi^2} + \frac{1}{2\pi}\right] = 2\left(\frac{2}{2\pi}\right) = \boldsymbol{\frac{2}{\pi}}$$

34. We have, $x^2 + y^2 \leq 1$ and $x + y \geq 1$

Let $x^2 + y^2 = 1$

$y^2 = 1 - x^2$

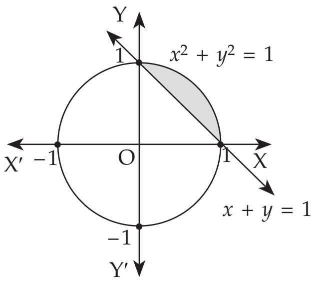
$y = \pm\sqrt{1 - x^2}$

x	0	±1
y	±1	0

Let $x + y = 1$

$y = 1 - x$

x	0	1
y	1	0



$\therefore$ **Area of shaded region** $= \int_0^1 \sqrt{1 - x^2}\, dx - \int_0^1 (1 - x)\, dx$

$$= \frac{1}{2}\left(x\sqrt{1 - x^2} + (1)^2\sin^{-1}\frac{x}{1}\right)_0^1 - \left[x - \frac{x^2}{2}\right]_0^1 \qquad \text{...}\left[\because \int\sqrt{a^2 - x^2}dx = \frac{1}{2}\left(x\sqrt{a^2 - x^2} + a^2\sin^{-1}\frac{x}{a}\right) + c\right.$$

$$= \frac{1}{2}\left[\left(1\sqrt{1 - 1} + \sin^{-1}(1)\right) - \left(0\sqrt{1 - 0} + \sin^{-1} 0\right)\right] - \left[\left(1 - \frac{1}{2}\right) - (0 - 0)\right]$$

$$= \frac{1}{2}\left(0 + \frac{\pi}{2} - 0 + 0\right) - \left(\frac{1}{2} - 0\right) = \left(\boldsymbol{\frac{\pi}{4} - \frac{1}{2}}\right)\textbf{sq. units}$$

Or

Given equations of curves are

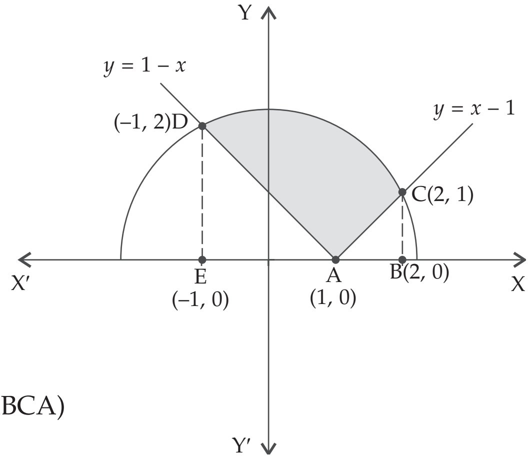
$$y = |x - 1| \text{ and } y = \sqrt{5 - x^2}$$

$$\Rightarrow \quad y = \begin{cases} x - 1 & \text{if } x \geq 1 \\ -(x - 1) & \text{if } x < 1 \end{cases}$$

$$\Rightarrow \quad y = \begin{cases} x - 1 & \text{if } x \geq 1 \\ 1 - x & \text{if } x < 1 \end{cases} \qquad \ldots(i)$$

and $y^2 = 5 - x^2$

$$\therefore \quad x^2 + y^2 = 5 \qquad \ldots(ii)$$



The curve (*i*) is a modulus function and curve (*ii*) is a circle with centre (0, 0) and radius $\sqrt{5}$.

Now plot the graphs of curves (*i*) and (*ii*).

The points of intersection of curves (*i*) and (*ii*) are C(2, 1) and D(–1, 2).

Required area of the shaded region is,

= Area of (EABCDE) – Area of (AEDA) – Area of (ABCA)

$$= \int_{-1}^{2} \sqrt{5 - x^2}\, dx - \int_{-1}^{1} (1 - x)\, dx - \int_{1}^{2} (x - 1)\, dx$$

$$= \left[\frac{x}{2}\sqrt{5 - x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x - \frac{x^2}{2}\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_{1}^{2} \qquad \ldots\left[\because \int (\boldsymbol{a^2 - x^2})\boldsymbol{dx} = \frac{\boldsymbol{x}}{\mathbf{2}}\sqrt{\boldsymbol{a^x - x^2}} + \frac{\boldsymbol{a^2}}{\mathbf{2}}\mathbf{sin^{-1}}\frac{\boldsymbol{x}}{\boldsymbol{a}}\right.$$

$$= \left\{\left[\frac{2}{2}\sqrt{5-4} + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right] - \left[-\frac{1}{2}\sqrt{5-1} + \frac{5}{2}\sin^{-1}\frac{-1}{\sqrt{5}}\right]\right\} - \left[\left(1 - \frac{1}{2}\right) - \left(-1 - \frac{1}{2}\right)\right] - \left[\left(\frac{4}{2} - 2\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= \left[1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + \frac{1}{2} \times 2 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right] - \left[\frac{1}{2} + \frac{3}{2}\right] - \left[0 + \frac{1}{2}\right]$$

$$= 1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}} - 2 - \frac{1}{2}$$

$$= -\frac{\mathbf{1}}{\mathbf{2}} + \frac{\mathbf{5}}{\mathbf{2}}\left(\mathbf{sin^{-1}}\frac{\mathbf{2}}{\sqrt{\mathbf{5}}} + \mathbf{sin^{-1}}\frac{\mathbf{1}}{\sqrt{\mathbf{5}}}\right) \textbf{ sq. units}$$

35.

$$xy \log\left(\frac{y}{x}\right)dx + \left(y^2 - x^2 \log\frac{y}{x}\right) dy = 0$$

$$\left(y^2 - x^2 \log\frac{y}{x}\right) dy = -xy \log\left(\frac{y}{x}\right) dx$$

$$\left(\frac{dy}{dx}\right) = \frac{-xy \log\left(\frac{y}{x}\right)}{y^2 - x^2 \log\left(\frac{y}{x}\right)}$$

$$\frac{dy}{dx} = \frac{-\frac{y}{x}\log\left(\frac{y}{x}\right)}{\left(\frac{y}{x}\right)^2 - \log\left(\frac{y}{x}\right)} \qquad \ldots\textbf{[Dividing numerator and denominator by } \boldsymbol{x^2}$$

$$v + x\frac{dy}{dx} = \frac{-v \log v}{v^2 - \log v} \qquad \ldots\left[\begin{array}{l} \textbf{Let } \boldsymbol{y = vx \Rightarrow \frac{y}{x} = v} \\ \boldsymbol{\because \ \frac{dy}{dx} = v.1 + x.\frac{dv}{dx}} \end{array}\right.$$

$$x\frac{dy}{dx} = \frac{-v \log v}{v^2 - \log v} - \frac{v}{1} = \frac{-v \log v - v^3 + v \log v}{v^2 - \log v} = \frac{-v^3}{-(\log v - v^2)}$$

$$\frac{\log v - v^2}{v^3}\, dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int\left(\frac{\log v}{v^3}-\frac{v^2}{v^3}\right)dv = \int\frac{dx}{x}$$

$$\Rightarrow \quad \int \underset{\text{I}}{\log v}\,.\,\underset{\text{II}}{v^{-3}}\,dv - \int\frac{1}{v}\,dv = \int\frac{dx}{x}$$

$$\Rightarrow \quad \log v.\frac{v^{-2}}{-2} - \int\frac{1}{v}\times\frac{v^{-2}}{-2}\,dv - \log\ |v| = \log|x| + c$$

$$\Rightarrow \quad \frac{-1}{2v^2}\log v + \frac{1}{2}\int v^{-3}\,dv - \log|v| = \log|x| + c$$

$$\Rightarrow \quad \frac{-\log v}{2v^2} + \frac{1}{2}.\frac{v^{-2}}{-2} - \log\ |v| = \log|x| + c$$

$$\Rightarrow \quad \frac{-\log v}{2v^2} - \frac{1}{4v^2} - \log\ |v| = \log|x| + c$$

$$\Rightarrow \quad \frac{-\log\left(\frac{y}{x}\right)}{2\,\frac{y^2}{x^2}} - \frac{1}{\frac{4y^2}{x^2}} - \log\left|\frac{y}{x}\right| - \log|x| = c$$

$$\Rightarrow \quad \frac{-x^2\log\left(\frac{y}{x}\right)}{2y^2} - \frac{x^2}{4y^2} - \log|y| + \log|x| - \log x = c$$

$$\Rightarrow \quad \frac{-x^2\log\left(\frac{y}{x}\right)}{2y^2} - \frac{x^2}{4y^2} - \log|y| - c = 0$$

$$\Rightarrow \quad \frac{x^2\log\frac{y}{x}}{2y^2} + \frac{x^2}{4y^2} + \log|y| + c = 0 \qquad \text{...[dividing both sides by –1}$$

$$\Rightarrow \quad \frac{2x^2\log\frac{y}{x} + x^2 + 4y^2\,(\log\ |\ y\ | + c)}{4y^2} = 0$$

$$\Rightarrow \quad \boldsymbol{x^2\left[1 + 2\log\left(\frac{y}{x}\right)\right] + 4y^2[\log|y| + c] = 0}$$

36. We have, $A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$

$|A| = 3(-2 - 0) - 1(-3 + 6) + 2(0 - 4) = -6 - 3 - 8 = -17 \neq 0$ $\therefore$ A^{-1} exists.

$a_{11} = (-2 + 0) = -2$; $a_{21} = -(-1 - 0) = 1$; $a_{31} = (-3 - 4) = -7$

$a_{12} = -(-3 + 6) = -3$; $a_{22} = (-3 - 4) = -7$; $a_{32} = -(-9 - 6) = 15$

$a_{13} = (0 - 4) = -4$; $a_{23} = -(0 - 2) = 2$; $a_{33} = (6 - 3) = 3$

$$\text{Adj } A = \begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|} . \text{Adj } A = \frac{1}{-17}\begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix}$

Writing it in matrix form

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 2 & -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$$

$$\begin{aligned} (A^t) \quad X &= B \\ X &= (A^t)^{-1} . B \\ X &= (A^{-1})^t . B \end{aligned}$$

[$\because (A^t)^{-1} = (A^{-1})^t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-17}\begin{bmatrix} -2 & -3 & -4 \\ 1 & -7 & 2 \\ -7 & 15 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = \frac{1}{-17}\begin{bmatrix} -2-12-20 \\ 1-28+10 \\ -7+60+15 \end{bmatrix} = \frac{1}{-17}\begin{bmatrix} -34 \\ -17 \\ 68 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$$

$\therefore$ $\quad$ **$x = 2, y = 1, z = -4$**

Or

Let $\quad A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} AB &= 8I \\ AB . B^{-1} &= 8I . B^{-1} \\ AI &= 8B^{-1} \\ \frac{1}{8}A &= B^{-1} \end{aligned}$$

[Post multiplying by B^{-1}]

($\because BB^{-1} = I$)

…(*i*)

Writing the given equation in matrix form

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{aligned} BX &= C \\ X &= B^{-1}C \\ X &= \frac{1}{8}AC \end{aligned}$$

[From (*i*)]

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$\therefore$ $\quad$ **$x = 3, y = -2, z = -1$**

37. Let the line $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+1}{-9} = \lambda$

$x - 1 = \lambda$	$y - 2 = 3\lambda$	$z + 1 = -9\lambda$
$x = \lambda + 1$	$y = 3\lambda + 2$	$z = -9\lambda - 1$

The general point is $(\lambda + 1, 3\lambda + 2, -9\lambda - 1)$.

The Direction Ratios of the line parallel to the plane and passing through the point

$$= (\lambda + 1 + 2, 3\lambda + 2 - 3, -9\lambda - 1 + 4)$$
$$= (\lambda + 3, 3\lambda - 1, -9\lambda + 3)$$

and $(\lambda + 3)(1) + (3\lambda - 1)(-1) + (-9\lambda + 3)2 = 0$

$\Rightarrow \quad \lambda + 3 - 3\lambda + 1 - 18\lambda + 6 = 0$

$\Rightarrow \quad -20\lambda + 10 = 0$

$\Rightarrow \quad \lambda = \frac{1}{2}$

$\therefore$ The point of intersection is $\left(\frac{3}{2}, \frac{7}{2}, \frac{-11}{2}\right)$.

Required Distance = $\sqrt{\left(\frac{3}{2}+2\right)^2 + \left(\frac{7}{2}-3\right)^2 + \left(-\frac{11}{2}+4\right)^2} = \frac{\sqrt{59}}{2}$ **units**

Or

Let A(3, −4, −5), B (2, −3, 1),

Direction ratios of AB are $2 - 3, \ -3 + 4, \ 1 + 5$

$-1, \quad 1, \quad 6$

Equation of line AB is $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = p$ (Let)

General point of line AB is $Q(-p + 3, p - 4, 6p - 5)$

Equation of plane through points (1, 2, 3), (4, 2, − 3) and (0, 4, 3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ 4-1 & 2-2 & -3-3 \\ 0-1 & 4-2 & 3-3 \end{vmatrix} = 0 \qquad \Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0$

Expanding along R_1, we have $(x - 1)(12) - (y - 2)(-6) + (z - 3)(6) = 0$

Dividing both sides by 6, $2(x - 1) + (y - 2) + z - 3 = 0$

$\Rightarrow \quad 2x - 2 + y - 2 + z - 3 = 0 \qquad \Rightarrow \quad 2x + y + z = 7$ …(*i*)

Point Q lies in equation (*i*),

$\Rightarrow \quad 2(-p + 3) + p - 4 + 6p - 5 = 7 \qquad \Rightarrow \quad -2p + 6 + p - 4 + 6p - 5 = 7$

$\Rightarrow \quad 5p = 10 \qquad \Rightarrow \quad p = 2$

Hence, the required point Q is $(-2 + 3, 2 - 4, 12 - 5) =$ **(1, −2, 7).**

38. Maximise Profit, $Z = 24x + 18y$

Subject to the constraints,

$2x + 3y \le 10$

$3x + 2y \le 10, \ x \ge 0, y \ge 0$

Let $2x + 3y = 10$

x	0	2	5
y	$\frac{10}{3}$	2	0

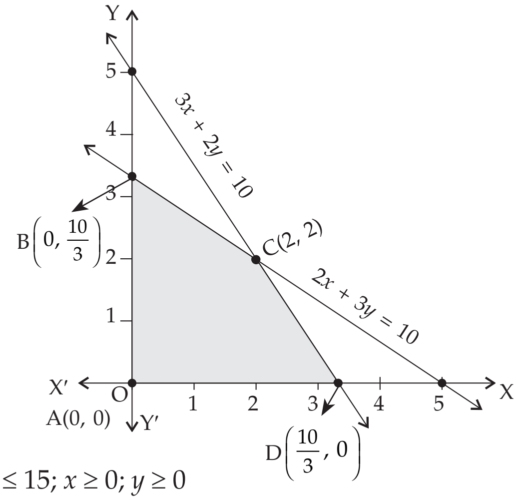
Let $3x + 2y = 10$

x	0	2	$\frac{10}{3}$
y	5	2	0

Corner Points	$Z = 24x + 18y$	
A(0, 0)	0	
$B\left(0, \frac{10}{3}\right)$	$0 + 18\left(\frac{10}{3}\right) = 60$	
C(2, 2)	$48 + 36 =$ **84**	←**Maximum**
$D\left(\frac{10}{3}, 0\right)$	$24\left(\frac{10}{3}\right) + 0 = 80$	

$\therefore \quad x = 2$ and $y = 2$

Maximum profit = ₹ 84



Or

(*a*) **Given.** Maximise $Z = px + qy$

Subject to the constraints: $2x + y \leq 10;\ x + 3y \leq 15;\ x \geq 0;\ y \geq 0$

Given that Z is maximum at (3, 4) and (0, 5)

At (3, 4), we have

Maximum value of $Z = 3p + 4q$ …(*i*)

At (0, 5), we have

Maximum value of $Z = 5q$ …(*ii*)

From (*i*) and (*ii*), we have $3p + 4q = 5q$

$\Rightarrow$ **$3p = q$ which is the required condition of p and q.**

(*b*) Minimise and maximise $Z = 5x + 10y$

Subject to constraints:

$x + 2y \leq 120, \quad x + y \geq 60, \quad x - 2y \geq 0, \quad x, y \geq 0$

Let $x + 2y = 120$

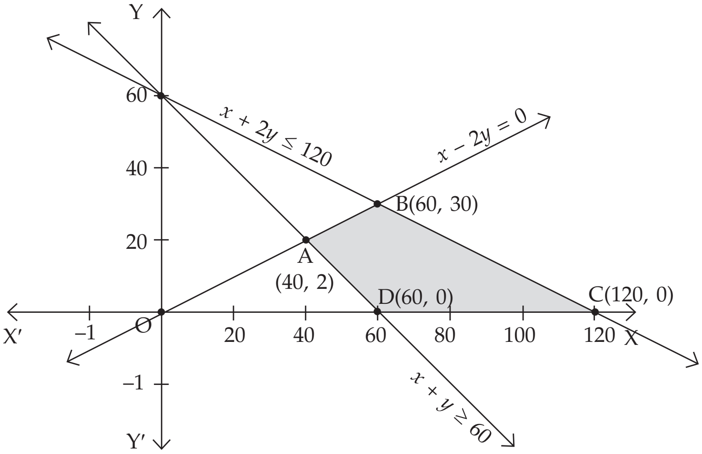
x	0	120
y	60	0

$x + y = 60$

x	0	60
y	60	0

$x - 2y = 0$

x	0	60
y	0	30



Corner Points	$Z = 5x + 10y$	
A(40, 20)	400	
B(60, 30)	600	Maximum (Multiple Optional Solutions)
C(120, 0)	600	
D(60, 0)	300	(Minimum)

$\therefore$ **Minimum value of Z** = 300 at (60, 0) at **$x = 60, y = 0$.**

❐❐❐❐