Measurement of Mass

Mass

Mass is the basic property of all bodies. Mass of a body is defined as the quantity of matter possessed by it. It is represented by the symbol m.
The S.I. unit of mass is kilogram (kg). The C.G.S. unit of mass, generally used in laboratories, is gram (g).
1 gram (g) = 10^{-3} kilogram (kg) or 1 kg = 1000 g.

Kilogram

It is the mass of a platinum-iridium alloy cylinder (90% platinum, 10% iridium), having diameter equal to height, stored in a special vault in the International Bureau of Weights and | Measures at Severes, near Paris in France.

Weight

Weight of a body is defined as the force with which the body is attracted by the earth towards its centre. It is represented by the symbol W.
A body of mass m has weight W and is given by the relation, W = mg, where g is acceleration due to gravity. The S.I. unit of weight is newton. Weight is a vector quantity. Its direction is towards the centre of earth and along its radius.

Difference between mass and weight

Besides the difference in their definitions, the two quantities have following two differences :
1. Mass m is constant at all places. Weight W = m'g varies from place to place because g varies on earth surface.
2. Mass m is never zero. Weight W = mg becomes zero at the centre of the earth, g = 0, free fall state and in a satellite where a – g Apparent weight R = m(g – a)
=> R = m(g -g) => R = 0. It is called weightlessness.

Internal and gravitational masses of a body

Inertial Mass

Definition. The mass which offers inertia, is called inertial mass. It is represented by the symbol m_1.
Discussion. From the study of Newton's first law of motion, we know that inertia is a natural property of all the bodies. Because of inertia, effort (force) is needed to move a body from rest or to bring a body to rest from motion. Bodies with more mass need bigger efforts for bringing similar changes in their states of rest or of motion. It means that the mass of a body is a measure of quantity of inertia in it. Such a mass which gives measure of inertia, is the inertial mass. For this reason, this mass is also defined as the 'inertial-coefficient' of the body,
Measurement. From Newton's second law of motion, F = $m_1\alpha$, i.e., m_1 – F/a. This relation can be used for measuring the inertial mass of a body by applying a known force on the body and measuring the acceleration produced in it. But this direct

measurement is difficult because it needs friction and other forces to be zero for motion and accurate measurement of acceleration.

Gravitational Mass

(i) Definition. The mass which offers gravitational attraction, is called gravitational mass.It is represented by the symbol m_G.

(ii) Discussion. Due to gravity (a special case of gravitation) earth attracts every body towards its centre. This attraction makes the body fall towards the centre of the earth. Hence (force of gravity is more on bodies with more mass. It means that the mass of a body is a measure of force of gravity (weight) acting on the body. Such a mass which gives measure of weight, is the gravitational mass. For this reason, this mass (gravitational mass) is also called weight of the body (and the process of its determination is called 'weighing' not 'massing).

(iii) Measurement. From the weight-mass relation, $W = m_G g$, i.e., $m_G = W/g$. This relation can be used for measuring the gravitational mass of a body by measuring its weight at a place where value of acceleration due to gravity is known.

Relation between internal mass and gravitational mass of a body

Let a simple pendulum of length l, have a bob having inertial mass m1 and gravitational mass mG.

Let the bob be displaced by an angle 0 and distance x, from vertical position of rest

(Fig.3.01).

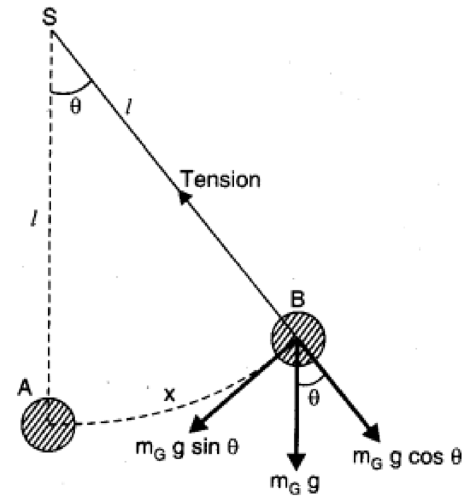

Fig. 3.01. Relation between inertial and gravitational mass.

Then component $m_G g \cos\theta$ of its weight balances tension in string and component $m_G g \sin\theta$ acts on bob as restoring force and produces acceleration in its inertial mass.

Restoring inward force on bob, $F = m_G g \sin\theta$

$$\text{Inward acceleration} = \frac{\text{Force}}{\text{Inertial mass}}$$

$$a = \frac{m_G g \sin\theta}{m_I}$$

$$a = \frac{m_G}{m_I} g \frac{x}{l} \qquad \left(\text{For small angle, } \sin\theta = \theta = \frac{x}{l}\right)$$

Since $\frac{m_G}{m_I}\frac{g}{l} = \text{constant}$

$$a \propto x$$

i.e., inward acceleration ∝ displacement. It means that the motion of the bob, when released, will be a simple harmonic motion (S.H.M.).

Time period T is given by

$$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

or

$$T = 2\pi\sqrt{\frac{x}{a}}$$

or $$T = 2\pi \sqrt{\frac{x}{\frac{m_G}{m_I}\frac{g}{l}x}}$$

$$= 2\pi \sqrt{\frac{m_I l}{m_G g}}$$

or $$T^2 = \frac{4\pi^2 m_I l}{m_G g}$$

or $$\frac{m_I}{m_G} = \frac{gT^2}{4\pi^2 l}.$$

The ratio can be found by using very accurate types of pendulums.
Observations carried over a period of about two decades have shown that the ratio is one with variation of only 1 part in 5 x 10^9. Hence for all purposes, the two kinds of masses are taken to be equal and same.

Determination of internal mass and gravitational mass

Determination of inertial mass. The inertial mass of a body can be determined experimentally by comparing the mass of the body with the mass of a set of standard bodies of known masses by observing the response of the masses to an external applied force.
The device used for this purpose is called inertial balance.
Determination of gravitational mass. The gravitational mass of a body can be determined experimentally by comparing the mass of the body with the mass of a set of standard bodies of known masses by observing the response of the masses to same force of gravity.
The devices used for this purpose are called : (i) Spring balance, (ii) Physical (beam) balance.

Internal balance

Introduction. It is a device used for determination of the inertial mass of a body. It works on principle of simple harmonic motion of the system.
Diagram.

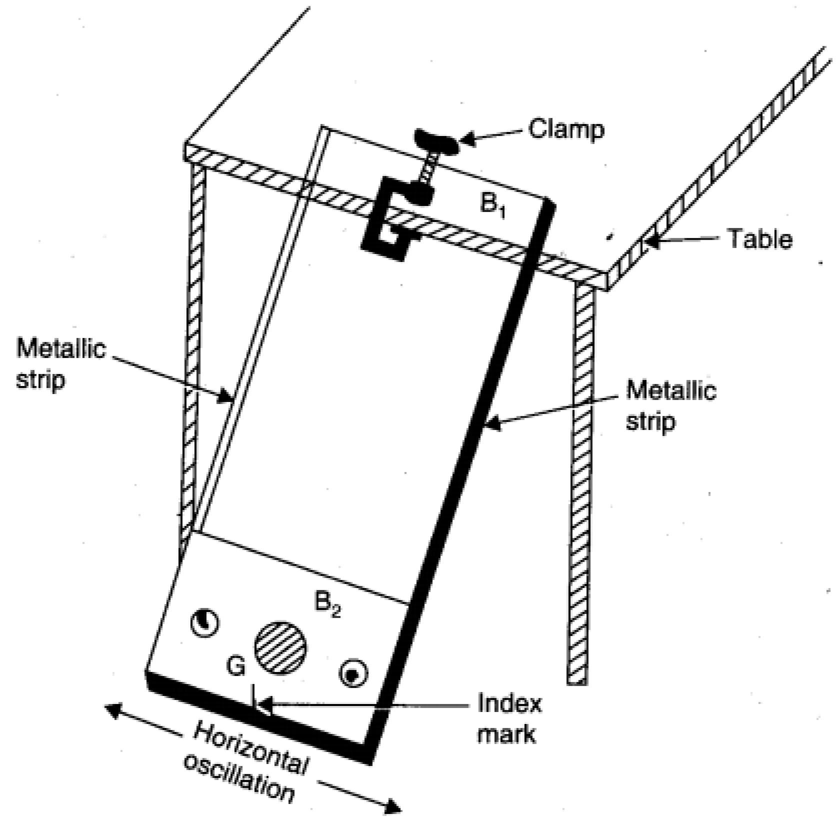

Fig. 3.02. Inertial balance.

Construction. It consists of two wooden boards B_1 and B_2, joined by two metallic strips with their edges vertical. One board B_1 is clamped with a table with the strips and board B_2 projecting out horizontally. The board B_2 has grooves of different sizes for placing bodies in them to avoid their slipping during motion. This board acts as pan of the balance.

Working. A body is put in the groove and board B_2 is pulled horizontally a little to one side and left. The board moves simple harmonically. The time period of the system depends upon the mass of the board $B_2$and the body or bodies in it. The time period can be measured.

Theory. If m be the total mass of the board B_2 and body or bodies in it and K be the spring constant of the two metallic strips, then time period T is given by the relation,

$$T = 2\pi\sqrt{\frac{m}{K}} \qquad \text{or} \qquad T^2 = 4\pi^2\frac{m}{K}$$

or

$$m = \frac{K}{4\pi^2}T^2 \qquad i.e., \quad m \propto T^2.$$

Knowing K and observing T, m can be calculated.

Calibration of balance. Since the formula includes unknown quantities like K and mass of the board B_2, the balance is calibrated to eliminate them.

Time period of system is found with empty board B2 and then after putting bodies of known masses (100 g, 200 g,…, 500 g). A graph is plotted taking mass of body kept along X-axis and (time period)2 along Y-axis. The graph comes to be a straight line (Fig. 3.03).

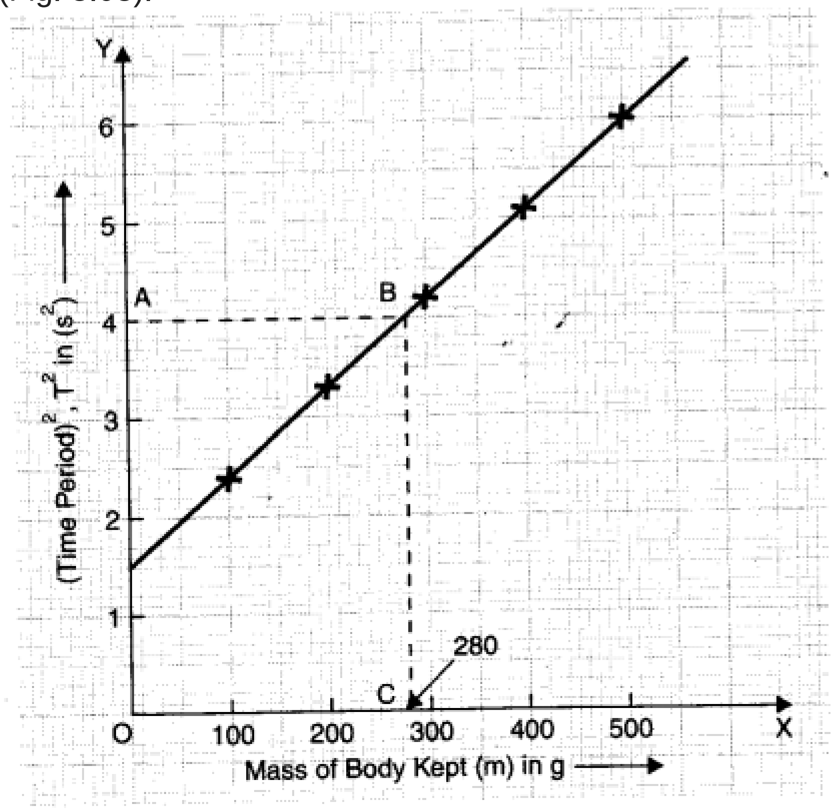

Fig. 3.03. Calibration of inertial balance.

Calculation. For finding unknown mass of a body, the body is kept on board B2 and time period of system is found. Square of this time period is represented by point A (Fig. 3.03). Aline AB is drawn parallel to X-axis cutting the straight line calibration curve at point B. From B, BC is drawn parallel to Y-axis, cutting X-axis at point C. The point C represents the unknown mass of the body, which comes to be 280 g.

Spring balance

Introduction. It is a device used for the determination of gravitational mass of a body, t It works on the principle of Hooke's law of elasticity which states that when a body is suspended from a vertical spring, the body produces extension in the length of the spring proportional is its gravitational mass.

W=kl

Diagram.

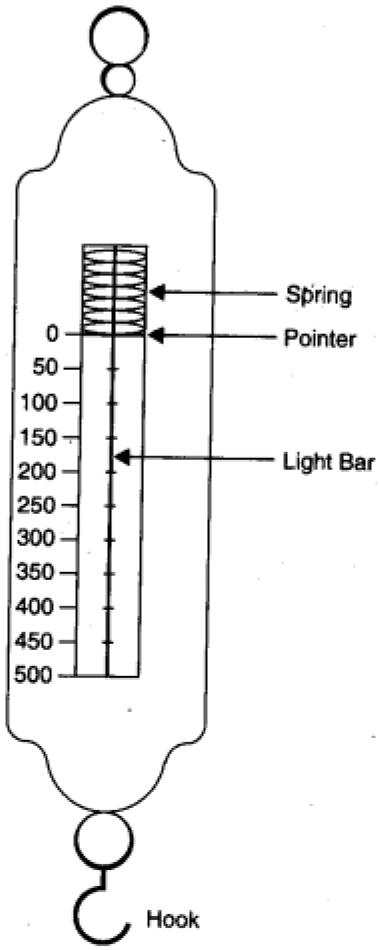

Fig. 3.04. Spring balance.

Construction. It consists of a vertical steel spring. Its upper end is fixed and the lower end carries a pointer. A light bar is attached to the lower end of the spring which carries a hook at its lower end. The body whose gravitational mass is to be determined is suspended from this hook. The spring is enclosed in a metallic case on which a graduated brass plate is fixed. The graduations are either in gram or in kilogram. The pointer projects out of a long narrow rectangular slit in the front plate. It moves down along the graduated scale when the spring is stretched by suspending a body from the lower hook.

Zero error and zero correction. The value of one smallest division on scale is found to know the least count of the balance. The hook is pulled down a little and then left. If the pointer returns back to zero of scale, there is no zero error. If the pointer stays below zero, the zero error is positive. It is equal to the reading of the position of the pointer. If the pointer goes above zero mark, zero error is negative. To know this negative zero error, a body of known mass is suspended from the hook. The pointer moves down to indicate its mass which is less than the actual mass. The difference in mass gives value of the negative zero error.

Sign of zero error is changed to convert it into zero correction. The zero correction is then algebraically added to observed values of masses to know correct masses.

Working. The body whose gravitational mass is to be determined, is suspended from the hook of the balance. The body stretches the spring due to its weight and the pointer moves down. The weight is proportional to the gravitational mass (weight $W = m_G g$). The position of the pointer gives value of the gravitational mass of the body.

Changes in calibration. Since $W – m_G g$, for same gravitational mass m_G, weight W will change from place to place due to changes in value of g. It means that scale calibrated at one place will not be accurate at other places.

Hence we say that spring balance measures weight of a body which has different values at different places.

Comparison of gravitational masses. Since $W_1 = m_{G1} g$, and $W_2 = m_{G2} g$, we have

$$\frac{m_{G1}}{m_{G2}} = \frac{W_1}{W_2}$$

At all places, the ratio of gravitational masses of two bodies will be same as ratio of their weights. Hence by comparing an unknown gravitational mass with a known gravitational mass by a spring balance, the unknown gravitational mass can be determined.

Physical balance or beam balance

Introduction. It is a device used for the determination of gravitational mass of a body. It works on the fact that at one place, bodies having equal weights have equal gravitational masses.

It also makes use of principle of moments. According to the principle, bodies of equal weights when suspended from a beam at equal distance, on the two sides of its fulcrum, keep the beam horizontal. It is essentially a lever of the first order having the power arm and weight arm equal.

Diagram.

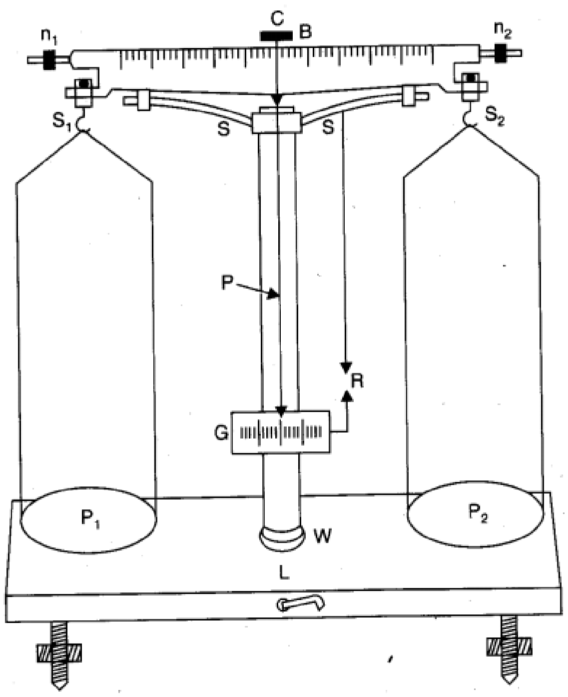

Fig. 3.05. Physical balance.

Construction. Its main parts are :

1. The central pillar. A metallic hollow pillar W is fixed upright (vertical) in the middle of a rectangular wooden base board. The board has two levelling screws at the front corner and small vertical pillar at the middle of the back side. There is a plumb line R, suspended along the sides of the pillar, which has its lower tip pointing to a metal cone. With its help, the vertical setting of the pillar is tested.
2. The beam. It is a rigid bar B, often of girder construction, made of nickel plated iron or brass or aluminium. It has a central knife edge or fulcrum of steel or agate (a kind of hard stone) turned downwards. This rests on a flat agate surface fixed to the top of a vertical support within the central pillar. At both the ends of the beam are sharp knife edges of agate or steel which are turned upwards. The ends are also provided with two small adjustable screws n_1 and n_2.
3. Scale pans. Two exactly similar pans P_1 and P_2 of equal weights and size are suspended by means of hooks S_1 and S_2 from agate planes supported on knife edges at the ends of the beam.
4. Arresting arrangement. When not in use, the beam of the balance is kept arrested by means of an arrangement known as arresting arrangement. In this arrangement, the beam rests on two pillars, S, S, projecting from and fixed to the top end of the pillar W. By means of a handle L, the vertical support of the beam may be raised.
5. The pointer and scale. A light pointer P is attached in the middle of the beam with pointed end vertical downwards. There is also small graduated scale G fixed at the foot of the pillar. When the beam is free, the pointed end moves to and fro over the scale. When the balance is adjusted, the pointer rests on the central mark of the scale generally marked ‘zero’. The pointer carries a small sliding nut (not shown in the figure) used to lower or raise the centre of gravity of the beam and the pointer.

The balance is enclosed in a glass case so that the air currents should not disturb the weighings. The front glass door can be raised or shut easily by a side attachment. (Before starting to use the balance, the students should carefully understand the function of each part of it).

Resting point of a balance

Definition. Resting point (R.P.) of a balance is defined as the point on the scale, at which the pointer will stay at rest when the beam is in raised position. In final adjustment the resting point should always be within one division on either side of the middle mark on the scale.

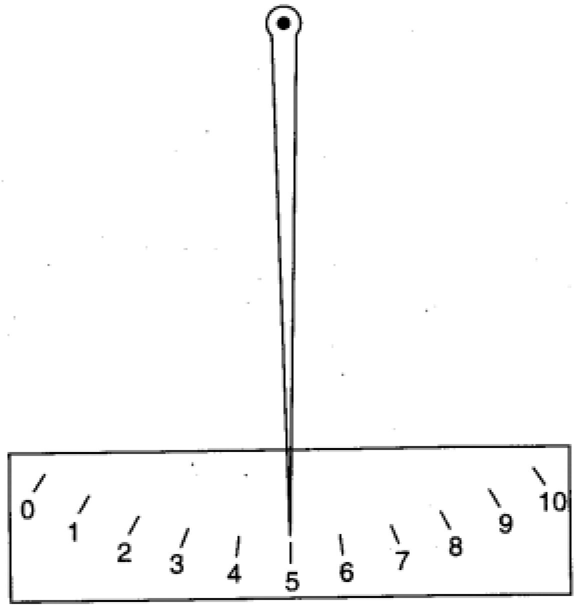

Fig. 3.06. Resting point of the balance.

Determination. In a sensitive balance, the pointer generally takes a long time to come to rest and hence the R.P. is determined by observations made during its motion (method of oscillations).

When beam is free, the pointer will swing across the scale, resting point will be that point about which the pointer swings equally on both sides. For greater accuracy, turning points of the pointer are noted on the scale. Three consecutive observations are taken, first on the left (L_f > then on the right (R) and again on the left L_2, taking zero on the extreme left.

Then,

$$\text{R.P.} = \frac{1}{2}\left(\frac{L_1 + L_2}{2} + R\right)$$

The observations can also be taken first on right (R_1), then on left (L) and again on the right (R_2).

Now

$$\text{R.P.} = \frac{1}{2}\left(\frac{R_1 + R_2}{2} + L\right)$$

Example. Let $L_1 = 2, R = 8,$ and $L_2 = 3$

Then

$$\text{R.P.} = \frac{1}{2}\left(\frac{2+3}{2} + 8\right)$$

$$= \frac{1}{2}(2.5 + 8) = \mathbf{5.25.}$$

Metre scale as a beam balance

Introduction. Like a physical balance, a metre scale can be used as a beam balance making use of the same principle of moments.

Besides it has adjustable power arm and weight arm whose length can be adjusted.
Diagram.

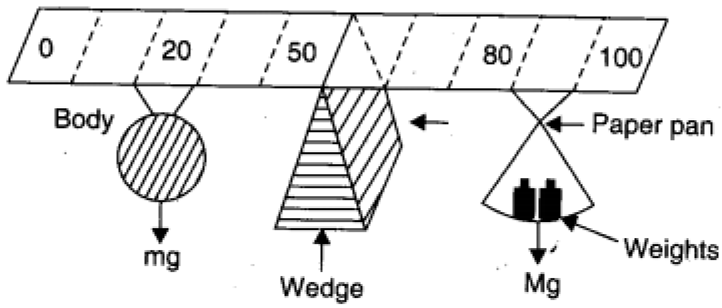

Fig. 3.07. Metre-scale balance. (Power and weight arms of equal length.)

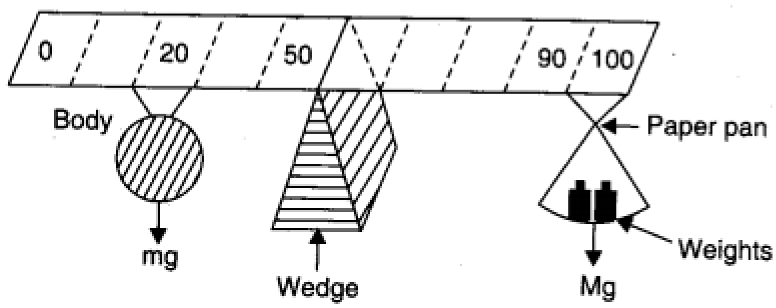

Fig. 3.08. Metre-scale balance. (Power and weight arms of unequal length.)

Construction (Arrangement). The metre scale is balanced by putting its 50 cm mark over the sharp edge of a heavy broad wedge which works as fulcrum. In this position the weight of the metre scale and reaction of the wedge, balance each other.
Working. The body is tied to a strong and light thread loop and suspended on the left of the wedge on some fixed mark. (Say 20 cm in diagram)
A light paper pan is suspended by a strong and light thread on the right. Weights are put on the pan. The position of the loop of the pan and weight in it are so adjusted that the metre scale becomes horizontal again.
Position of thread of the loops and the amount of weights in the pan are noted.
Mass of the body is calculated using the following theory.
Theory. If m andM be the mass of the body and mass of the weight used and a1 anda2 be the distances of their loops from wedge. Then, power (mass) arm = a_1; weight arm = a_2

From principle of moments $mg\,a_1 = Mg\,a_2$

or $$m = \frac{M\,a_2}{a_1}$$, which can be calculated.

Two different methods
(i)Arm lengths fixed, and equal, weight adjustable.
The thread loops are suspended at position forming both arm of equal length. Weights in the paper pan are adjusted till the metre scale becomes horizontal. (Fig. 3.07).
In this case $a_1 = a_2 = a$

Hence, $mga_1 = Mga_2$
or m = M
A physical balance makes use of this method.
(ii) Masses and power arm fixed, weight arm adjustable.
.Mass is suspended at a fixed distance a1.
Length of power arm is adjusted by moving weight loop thread in and out till the metre scale becomes horizontal (Fig. 3.08). ,
In this case $a_1 = a, a_2 = A$
Hence mg a1 = Mg a_2, becomes mg a = Mg A
or m = M A/a

Weight box

It is a wooden box containing a set of weights along with a forceps used with a physical balance. They are all arranged in circular grooves made in order of increasing weights (Fig. 3.09).

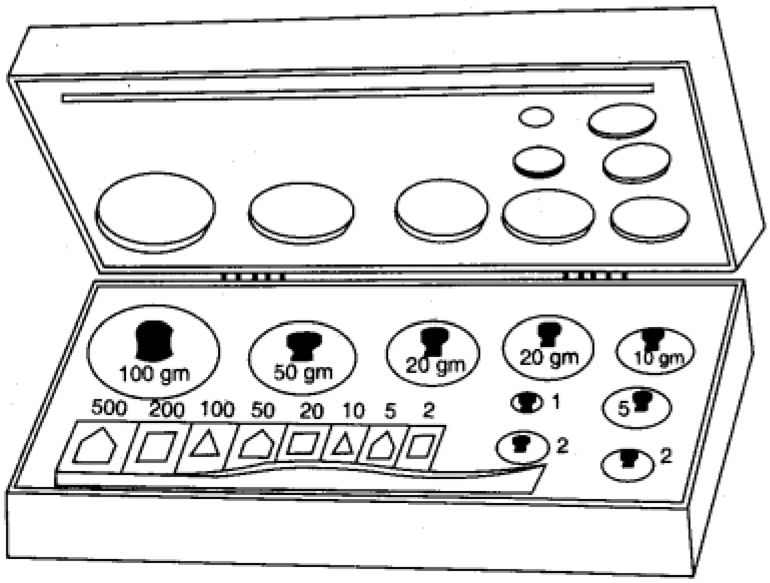

Fig. 3.09. Weight box.

The weights with 1 g to 100 g are cylindrical in shape and are provided with a small spherical knob at the top to help in lifting them with forceps. The number engraved on each weight gives value of its weight in grams. In addition to these weights, there are fractional weights of different shapes made of aluminium and marked in milligrams. Weight of 500,50 and 5 mg are pentagonal, 200, 20 and 2 mg are rectangular and 100, 10 and 1 mg are triangular in shape.

Principle of weighting

A physical balance determines gravitational mass of a body making use of principle of moments. It has both arms of beam of equal length and mass. The two pans also have equal weights. Hence the beam, when made free, remains horizontal with empty pans. A body having gravitational mass m_{G1} is placed in the left pan and a standard weight of gravitational mass m_{G2} is put in the right pan to keep the beam horizontal. Then,

Weight (m_{G1} g) in left pan = weight (m_{G2} g) in right pan
or $m_{G1} = m_{G2}$
i. e., gravitational mass of the body in left pan = gravitational mass of the standard weights in right pan.
Hence a physical balance determines gravitational mass of a body by equating it with gravitational mass of standard weight.

Precautions for weighting with a physical balance

1. Examine the balance and see that its every part is at the proper place.
2. Level the balance by means of levelling screws provided at the base so that tip of the plumb line is exactly above the needle point.
3. See that the pans are clean and dry.
4. Close the shutter and raise the beam by means of the handle. See that the pointer oscillates equally on both sides of the central mark. If not, proper adjustment be made by turning the screws n_1 and $n_2$at the ends of the beam. Thus, bring the Resting Point on the central mark.
5. Observe the swing of the pointer by fixing your eye perpendicular to the scale to avoid parallax.
6. If the beam does not swing when released, start it by giving a slight jerk to one of the pans.
7. See that weight box is complete. The body to be weighed must be placed in the left hand
   pan and the standard weights in the right hand pan.
8. Determine the resting point of unloaded balance {i.e., balance with empty pans).
9. Always handle the weights with the forceps. Do not touch them with fingers.
10. Always place the weights on the pans or remove them when the beam is in resting position.
11. To begin with, use a suitable heavier weight and then try the smaller weights in the descending order of magnitude.
    (The beam need not be raised to the full extent until the milligram weights are being used.)
12. Do not raise or lower the beam with jerk.
13. Close the case for the final adjustment of the weight or while showing to the examiner.
14. Large swings of the pans may be stopped by carefully touching the pans with the finger while the beam is arrested.
15. When the equilibrium is attained, take out weights one by one, count them and make a record of it in your note-book. Replace them in their proper places in the weight box. Do not take the pan out for this purpose, nor place the weight on the base of the case.
16. Do not weigh a body when it is hot or very cold to avoid convection currents.
17. Do not place the powder, liquid and chemically active substances directly in the pan.
18. Do not use a balance for weighing objects heavier than the heaviest it has been constructed to weigh which is nearly 250 g.

Density and specific gravity

Density of a substance is defined as its mass per unit volume. It is represented by the symbol ρ.

$$\text{Thus, density} = \frac{\text{Mass of the substance}}{\text{Volume of the substance}}$$

If M be the mass of a body and V be its volume, then its density ρ is given by

$$\rho = \frac{M}{V}$$

Density of a body is measured in gram per cubic centimetre (g-cm^{-3}) in C.G.S. system and kilogram per cubic metre (kg-m^{-3}) in S.I.

Density of a substance varies with temperature, hence the temperature at which density is measured, must be specified.

Density of common substances is given in Table 6 in Appendix.

Specific gravity (S.G.) *of a substance is defined as the ratio of the mass of given volume of the substance to the mass of same volume of water at 4°C.*

$$\text{S.G.} = \frac{\text{Mass of volume } V \text{ of substance}}{\text{Mass of volume } V \text{ of water at } 4°\text{C}} = \frac{\text{Density of the substance}}{\text{Density of water at } 4°\text{C}}$$

$$= \frac{\text{Density of the substance}}{\text{Density of water at room temp. } t°\text{C}} \times \frac{\text{Density of water at } t°\text{C}}{\text{Density of water at } 4°\text{C}}$$

i.e., S.G. = Observed S.G. × S.G. of water at t°C.

Since, the density of water at 4°C in C.G.S. system is 1 g-cm^{-3}, so the specific gravity of a substance is numerically equal to its density in C.G.S. system *e.g.*, Density of silver is 10.5 g-cm^{-3} so its S.G. is 10.5.

Specific gravity is also called **relative density** (R.D.).