

Chapter - 10
Algebraic Expressions

Exercise

In each of the questions 1 to 16, out of the four options, only one is correct. Write the correct answer.

1. An algebraic expression containing three terms is called a

- (a) monomial (b) binomial (c) trinomial (d) All of these**

Solution:

- (c) trinomial

An algebraic expression containing one term is called monomial, two terms is called binomial and three terms is called trinomial.

2. Number of terms in the expression $3x^2y - 2y^2z - z^2x + 5$ is

- (a) 2 (b) 3 (c) 4 (d) 5**

Solution:-

- (c) 4

In the given expression there are 4 terms.

3. The terms of expression $4x^2 - 3xy$ are:

- (a) $4x^2$ and $-3xy$**
(b) $4x^2$ and $3xy$
(c) $4x^2$ and $-xy$
(d) x^2 and xy

Solution:-

- (a) $4x^2$ and $-3xy$

Terms in the expression $4x^2 - 3xy$ are $4x^2$ and $-3xy$.

4. Factors of $-5x^2y^2z$ are

- (a) $-5 \times x \times y \times z$ (b) $-5 \times x^2 \times y \times z$ (c) $-5 \times x \times x \times y \times y \times z$**
(d) $-5 \times x \times y \times z^2$

Solution:-

(c) $-5 \times x \times x \times y \times y \times z$

$-5x^2y^2z$ can be written as $-5 \times x \times x \times y \times y \times z$

5. Coefficient of x in $-9xy^2z$ is

- (a) $9yz$ (b) $-9yz$ (c) $9y^2z$ (d) $-9y^2z$

Solution:-

(d) $-9y^2z$

Coefficient is the numerical factor in a term. Sometimes, any factor in a term is called the coefficient of the remaining part of the term.

6. Which of the following is a pair of like terms?

- (a) $-7xy^2z$, $-7x^2yz$
 (b) $-10xyz^2$, $3xyz^2$
 (c) $3xyz$, $3x^2y^2z^2$
 (d) $4xyz^2$, $4x^2yz$

Solution:-

(b) $-10xyz^2$, $3xyz^2$

Like terms have the same algebraic factors.

7. Identify the binomial out of the following:

- (a) $3xy^2 + 5y - x^2y$
 (b) $x^2y - 5y - x^2y$
 (c) $xy + yz + zx$
 (d) $3xy^2 + 5y - xy^2$

Solution:-

(d) $3xy^2 + 5y - xy^2$

Expression with two unlike terms is called a 'Binomial'.

So,

$$3xy^2 + 5y - xy^2 = (3xy^2 - xy^2) + 5y$$

$$= 2xy^2 + 5y$$

8. The sum of $x^4 - xy + 2y^2$ and $-x^4 + xy + 2y^2$ is

- (a) Monomial and polynomial in y
(c) Trinomial and polynomial

- (b) Binomial and Polynomial
(d) Monomial and polynomial in x

Solution:-

- (a) Monomial and polynomial in y

$$\begin{aligned}\text{Sum of two expressions} &= (x^4 - xy + 2y^2) + (-x^4 + xy + 2y^2) \\ &= x^4 - xy + 2y^2 - x^4 + xy + 2y^2 \\ &= (x^4 - x^4) + (-xy + xy) + (2y^2 + 2y^2) \\ &= 0 + 0 + 4y^2 \\ &= 4y^2\end{aligned}$$

9. The subtraction of 5 times of y from x is

- (a) $5x - y$ (b) $y - 5x$ (c) $x - 5y$ (d) $5y - x$

Solution:-

- (c) $x - 5y$

10. $-b - 0$ is equal to

- (a) $-1 \times b$ (b) $1 - b - 0$ (c) $0 - (-1) \times b$ (d) $-b - 0 - 1$

Solution:-

- (a) $-1 \times b$

As,
 $-b - 0 = -b$

11. The side length of the top of square table is x. The expression for perimeter is:

- (a) $4 + x$ (b) $2x$ (c) $4x$ (d) $8x$

Solution:-

- (c) $4x$

We have,
Perimeter of the square = $4 \times \text{side}$

Also, side length of the top of square table is x.
Therefore,
Perimeter = $4 \times x$
 $= 4x$

12. The number of scarfs of length half metre that can be made from y metres of cloth is:

- (a) $2y$ (b) $y/2$ (c) $y + 2$ (d) $y + (1/2)$

Solution:-

- (a) $2y$

We have,

Length of scarf is half meter = $\frac{1}{2}$ m

So, the number of scarfs can be made from y meters of cloth = $\frac{y}{\frac{1}{2}}$
 $= 2y$

13. $123x^2y - 138x^2y$ is a like term of :

- (a) $10xy$ (b) $-15xy$ (c) $-15xy^2$ (d) $10x^2y$

Solution:-

- (d) $10x^2y$

$$123x^2y - 138x^2y = (123 - 138) x^2y \\ = -15 x^2y$$

So,
 $-15x^2y$ is a like term of $10x^2y$ as both contain x^2y .

14. The value of $3x^2 - 5x + 3$, when $x = 1$ is

- (a) 1 (b) 0 (c) -1 (d) 11

Solution:-

- (a) 1

We have,

$$x = 1$$

So,

$$3x^2 - 5x + 3 = (3 \times (1)^2) - (5 \times 1) + 3 \\ = 3 - 5 + 3 \\ = 6 - 5$$

$$= 1$$

15. The expression for the number of diagonals that we can make from one vertex of a n sided polygon is:

- (a) $2n + 1$ (b) $n - 2$ (c) $5n + 2$ (d) $n - 3$

Solution:-

- (d) $n - 3$

There are n vertices, and from each vertex you can draw $n-3$ diagonals, so the total number of diagonals that can be drawn is $(n-3)$.

16. The length of a side of square is given as $2x + 3$. Which expression represents the perimeter of the square?

- (a) $2x + 16$ (b) $6x + 9$ (c) $8x + 3$ (d) $8x + 12$

Solution:-

- (d) $8x + 12$

We have,

Perimeter of the square = $4 \times \text{side}$

Side length of the top of square table is $2x + 3$.

So,

$$\begin{aligned} \text{Perimeter} &= 4 \times (2x + 3) \\ &= (4 \times 2x) + (4 \times 3) \\ &= 8x + 12 \end{aligned}$$

In questions 17 to 32, fill in the blanks to make the statements true.

17. Sum or difference of two like terms is _____.

Solution:-

Sum or difference of two like terms is a like term.

18. In the formula, area of circle = πr^2 , the numerical constant of the expression πr^2 is _____.

Solution:-

In the formula, area of circle = πr^2 , the numerical constant of the expression πr^2 is π .

19. $3a^2b$ and $-7ba^2$ are _____ terms.

Solution:-

$3a^2b$ and $-7ba^2$ are like terms.

The terms having the same algebraic factors are called like terms.

20. $-5a^2b$ and $-5b^2a$ are _____ terms.

Solution:-

$-5a^2b$ and $-5b^2a$ are unlike terms.

Unlike terms have different algebraic factors.

21. In the expression $2\pi r$, the algebraic variable is _____.

Solution:-

In the expression $2\pi r$, the algebraic variable is r .

22. Number of terms in a monomial is _____.

Solution:-

Number of terms in a monomial is 1.

Expression with one term is called a 'Monomial'.

23. Like terms in the expression $n(n + 1) + 6(n - 1)$ are _____ and _____.

Solution:-

Like terms in the expression $n(n + 1) + 6(n - 1)$ are n and $6n$.

Considering the expression,

$$\begin{aligned} n(n + 1) + 6(n - 1) &= n^2 + n + 6n - 6 \\ &= n^2 + 7n - 6 \end{aligned}$$

Therefore, like terms are n and $6n$.

24. The expression $13 + 90$ is a _____.

Solution:

As,

$$13 + 90 = 103$$

103 is a constant term.

25. The speed of car is 55 km/hrs. The distance covered in y hours is _____.

Solution:

Given, speed of car = 55 km/h.

Distance = Speed \times Time

$$\begin{aligned}\text{Distance covered in } y \text{ hours} &= 55 \times y \\ &= 55y \text{ km}\end{aligned}$$

26. $x + y + z$ is an expression which is neither monomial nor _____.

Solution:

Since, $x + y + z$ has three terms, so it is trinomial.

Hence, $x + y + z$ is an expression which is neither monomial nor binomial.

27. If $(x^2y + y^2 + 3)$ is subtracted from $(3x^2y + 2y^2 + 5)$, then coefficient of y in the result is _____.

Solution:

$$\begin{aligned}(3x^2y + 2y^2 + 5) - (x^2y + y^2 + 3) &= 3x^2y + 2y^2 + 5 - x^2y - y^2 - 3 \\ &= 2x^2y + y^2 + 2\end{aligned}$$

Coefficient of $y = 2x^2$

28. $-a - b - c$ is same as $-a - (\quad)$.

Solution:

We have,

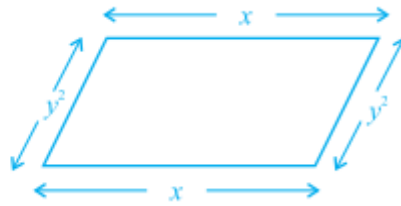
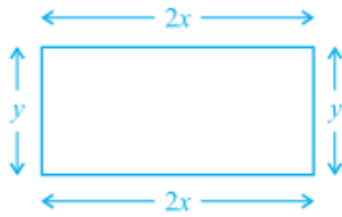
$$-a - b - c = -a - (b + c)$$

[taking common (-) minus sign]

So,

$$-a - b - c \text{ is same as } -a - (b + c).$$

29. The unlike terms in perimeters of following figures are _____ and _____.



Solution:

In Fig. (i),

$$\begin{aligned}\text{Perimeter} &= \text{Sum of all sides} \\ &= 2x + y + 2x + y \\ &= 4x + 2y\end{aligned}$$

In Fig. (ii),

$$\begin{aligned}\text{Perimeter} &= \text{Sum of all sides} \\ &= x + y^2 + x + y^2 \\ &= 2x + 2y^2\end{aligned}$$

Unlike terms in perimeters are $2y$ and $2y^2$.

30. On adding a monomial _____ to $-2x + 4y^2 + z$, the resulting expression becomes a binomial.

Solution:

We add $2x$, $-4y^2$ and $-z$ to the expression to make it binomial.

$$\begin{aligned}2x + (-2x + 4y^2 + z) &= 4y^2 + z \\ -4y^2 + (-2x + 4y^2 + z) &= -2x + z \\ -z + (-2x + 4y^2 + z) &= -2x + 4y^2\end{aligned}$$

So, on adding a monomial $2x$ or $-4y^2$ or $-z$ to $-2x + 4y^2 + z$, the resulting expression becomes a binomial.

31. $3x + 23x^2 + 6y^2 + 2x + y^2 + \dots = 5x + 7y^2$.

Solution:

Let,

$$\begin{aligned}(3x + 23x^2 + 6y^2 + 2x + y^2) + M &= 5x + 7y^2 \\ M &= (5x + 7y^2) - (3x + 23x^2 + 6y^2 + 2x + y^2) \\ M &= 5x + 7y^2 - 3x - 23x^2 - 6y^2 - 2x - y^2 \\ M &= 5x - 3x - 2x + 7y^2 - 6y^2 - y^2 - 23x^2 \\ M &= 0 + 0 - 23x^2 \\ &= -23x^2\end{aligned}$$

32. If Rohit has $5xy$ toffees and Shantanu has $20yx$ toffees, then Shantanu has _____ more toffees.

Solution:

Rohit has toffees = $5xy$

Shantanu has toffees = $20yx$

Difference = $20xy - 5xy$
 $= 15xy$

So,

Shantanu had $15xy$ more toffees.

In questions 33 to 52, state whether the statements given are True or False.

33. $1 + \left(\frac{x}{2}\right) + x^3$ is a polynomial.

Solution:

True

Expression with one or more than one term is called a polynomial.

34. $(3a - b + 3) - (a + b)$ is a binomial.

Solution:

False

35. A trinomial can be a polynomial.

Solution:

True

Trinomial is a polynomial, because it has three terms.

36. A polynomial with more than two terms is a trinomial.

Solution:

False

A polynomial with more than two terms can be trinomial or more. While a trinomial have exact three terms.

37. Sum of x and y is $x + y$.

Solution:

True

Sum of x and y is $x + y$.

38. Sum of 2 and p is $2p$.

Solution:

False

Sum of 2 and p is $2 + p$.

39. A binomial has more than two terms.

Solution:

False

Binomial has exactly two unlike terms.

40. A trinomial has exactly three terms.

Solution:

True

A trinomial has exactly three unlike terms.

41. In like terms, variables and their powers are the same.

Solution:

True

In like terms, algebraic factors are same.

42. The expression $x + y + 5x$ is a trinomial.

Solution:

False

As,

$$x + y + 5x = 6x + y$$

It is a binomial.

43. $4p$ is the numerical coefficient of q^2 in $-4pq^2$

Solution:

False

Numerical coefficient of q^2 in $-4pq^2 = -4$.

44. 5a and 5b are unlike terms.

Solution:

True

Because both the terms have different algebraic factors.

45. Sum of $x^2 + x$ and $y + y^2$ is $2x^2 + 2y^2$

Solution:

False

46. Subtracting a term from a given expression is the same as adding its additive inverse to the given expression.

Solution:

True

Because additive inverse is the negation of a number or expression.

47. The total number of planets of Sun can be denoted by the variable n.

Solution:

False

As, Sun has infinite planets around it.

48. In like terms, the numerical coefficients should also be the same.

Solution:

False

Example:

$-3x^2y$ and $4x^2y$ are like terms as they have same algebraic factor x^2y but have different numerical coefficients.

49. If we add a monomial and binomial, then answer can never be a monomial.

Solution:

False

50. If we subtract a monomial from a binomial, then answer is atleast a binomial.

Solution:

False

If we subtract a monomial from a binomial, then answer is atleast a monomial.

51. When we subtract a monomial from a trinomial, then answer can be a polynomial.

Solution:

True

When we subtract a monomial from a trinomial, then answer can be binomial or polynomial.

52. When we add a monomial and a trinomial, then answer can be a monomial.

Solution:

False

When we add a monomial and a trinomial, then it can be binomial or trinomial.

53. Write the following statements in the form of algebraic expressions and write whether it is monomial, binomial or trinomial.

(a) x is multiplied by itself and then added to the product of x and y .

(b) Three times of p and two times of q are multiplied and then subtracted from r .

(c) Product of p , twice of q and thrice of r .

(d) Sum of the products of a and b , b and c and c and a .

(e) Perimeter of an equilateral triangle of side x .

(f) Perimeter of a rectangle with length p and breadth q .

(g) Area of a triangle with base m and height n .

(h) Area of a square with side x .

(i) Cube of s subtracted from cube of t .

(j) Quotient of x and 15 multiplied by x .

(k) The sum of square of x and cube of z .

(I) Two times q subtracted from cube of q.

Solution:

(a) x is multiplied by itself and then added to the product of x and y.

From the question it is given that,
x is multiplied by itself = $x \times x$
 $= x^2$

Product of x and y = $x \times y$
 $= xy$

As per the condition in the question = $x^2 + xy$

Therefore, the obtained expression is binomial.

(b) Three times of p and two times of q are multiplied and then subtracted from r.

From the question it is given that,

Three times of p = $3p$
Two times of q = $2q$

Three times of p and two times of q are multiplied = $3p \times 2q$
 $= 6pq$

As per the condition in the question = $r - 6pq$

Therefore, the obtained expression is binomial.

(c) Product of p, twice of q and thrice of r.

As per the condition given in the question,
 $p \times 2q \times 3r = 6pqr$

Therefore, the obtained expression is monomial.

(d) Sum of the products of a and b, b and c and c and a.

Products of a and b, b and c and c and a = $(a \times b)$ and $(b \times c)$ and $(c \times a)$
Now,

Sum products of a and b, b and c and c and a = $ab + bc + ca$

Therefore, the obtained expression is trinomial.

(e) Perimeter of an equilateral triangle of side x.

$$\begin{aligned}\text{We know that, perimeter of triangle} &= \text{sum of all sides} \\ &= x + x + x \\ &= 3x\end{aligned}$$

Therefore, the obtained expression is monomial.

(f) Perimeter of a rectangle with length p and breadth q.

$$\begin{aligned}\text{We know that, perimeter of rectangle} &= 2 (\text{length} + \text{breadth}) \\ &= 2 (p + q) \\ &= 2p + 2q\end{aligned}$$

Therefore, the obtained expression is binomial.

(g) Area of a triangle with base m and height n.

$$\begin{aligned}\text{We know that,} \\ \text{Area of triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times m \times n \\ &= \frac{1}{2} mn\end{aligned}$$

Therefore, the obtained expression is monomial.

(h) Area of a square with side x.

$$\begin{aligned}\text{We know that, area of square} &= \text{side} \times \text{side} \\ &= x \times x \\ &= x^2\end{aligned}$$

Therefore, the obtained expression is monomial.

(i) Cube of s subtracted from cube of t.

$$\begin{aligned}\text{As per the condition given in the question,} \\ t^3 - s^3\end{aligned}$$

Therefore, the obtained expression is binomial.

(j) Quotient of x and 15 multiplied by x.

Quotient of x and 15 = $x \div 15$

As per the condition given the question,

$$\begin{aligned}\text{Quotient of x and 15 multiplied by x} &= (x \div 15)x \\ &= \frac{x^2}{15}\end{aligned}$$

Therefore, the obtained expression is monomial.

(k) The sum of square of x and cube of z.

As per the condition given the question = $x^2 + z^3$

Therefore, the obtained expression is binomial.

(l) Two times q subtracted from cube of q.

As per the condition given the question = $q^3 - 2q$

Therefore, the obtained expression is binomial.

54. Write the coefficient of x^2 in the following:

(i) $x^2 - x + 4$

(ii) $x^3 - 2x^2 + 3x + 1$

(iii) $1 + 2x + 3x^2 + 4x^3$

(iv) $y + y^2x + y^3x^2 + y^4x^3$

Solution:

(i) $x^2 - x + 4$

The coefficient of x^2 in the given expression is 1.

Coefficient is the numerical factor in a term. Sometimes, any factor in a term is called the coefficient of the remaining part of the term.

(ii) $x^3 - 2x^2 + 3x + 1$

The coefficient of x^2 in the given expression is -2.

Coefficient is the numerical factor in a term. Sometimes, any factor in a term is called the coefficient of the remaining part of the term.

(iii) $1 + 2x + 3x^2 + 4x^3$

The coefficient of x^2 in the given expression is 3.

Coefficient is the numerical factor in a term. Sometimes, any factor in a term is called the coefficient of the remaining part of the term.

$$(iv) y + y^2x + y^3x^2 + y^4x^3$$

The coefficient of x^2 in the given expression is y^3 .

Coefficient is the numerical factor in a term. Sometimes, any factor in a term is called the coefficient of the remaining part of the term.

55. Find the numerical coefficient of each of the terms:

$$(i) x^3y^2z, xy^2z^3, -3xy^2z^3, 5x^3y^2z, -7x^2y^2z^2$$

$$(ii) 10xyz, -7xy^2z, -9xyz, 2xy^2z, 2x^2y^2z$$

Solution:

$$(i) x^3y^2z, xy^2z^3, -3xy^2z^3, 5x^3y^2z, -7x^2y^2z^2$$

Numerical coefficient of,

$$x^3y^2z = 1$$

$$xy^2z^3 = 1$$

$$-3xy^2z^3 = -3$$

$$5x^3y^2z = 5$$

$$-7x^2y^2z^2 = -7$$

$$(ii) 10xyz, -7xy^2z, -9xyz, 2xy^2z, 2x^2y^2z$$

Numerical coefficient of,

$$10xyz = 10$$

$$-7xy^2z = -7$$

$$-9xyz = -9$$

$$2xy^2z = 2$$

$$2x^2y^2z = 2$$

56. Simplify the following by combining the like terms and then write whether the expression is a monomial, a binomial or a trinomial.

$$(a) 3x^2yz^2 - 3xy^2z + x^2yz^2 + 7xy^2z$$

$$(b) x^4 + 3x^3y + 3x^2y^2 - 3x^3y - 3xy^3 + y^4 - 3x^2y^2$$

$$(c) p^3q^2r + pq^2r^3 + 3p^2qr^2 - 9p^2qr^2$$

$$(d) 2a + 2b + 2c - 2a - 2b - 2c - 2b + 2c + 2a$$

$$(e) 50x^3 - 21x + 107 + 41x^3 - x + 1 - 93 + 71x - 31x^3$$

Solution:

$$(a) 3x^2yz^2 - 3xy^2z + x^2yz^2 + 7xy^2z = 4x^2yz^2 + 4xy^2z \\ = 4(4x^2yz^2 + 4xy^2z)$$

It is binomial.

$$(b) x^4 + 3x^3y + 3x^2y^2 - 3x^3y - 3xy^3 + y^4 - 3x^2y^2 = x^4 - 3xy^3 + y^4$$

It is trinomial.

$$(c) p^3q^2r + pq^2r^3 + 3p^2qr^2 - 9p^2qr^2 = p^3q^2r + pq^2r^3 - 6p^2qr^2$$

It is trinomial.

$$(d) 2a + 2b + 2c - 2a - 2b - 2c - 2b + 2c + 2a = -2b + 2c + 2a$$

It is trinomial.

$$(e) 50x^3 - 21x + 107 + 41x^3 - x + 1 - 93 + 71x - 31x^3 = 60x^3 + 49x + 15$$

It is trinomial.

57. Add the following expressions:

$$(a) p^2 - 7pq - q^2 \text{ and } -3p^2 - 2pq + 7q^2$$

$$(b) x^3 - x^2y - xy^2 - y^3 \text{ and } x^3 - 2x^2y + 3xy^2 + 4y$$

$$(c) ab + bc + ca \text{ and } -bc - ca - ab$$

$$(d) p^2 - q + r, q^2 - r + p \text{ and } r^2 - p + q$$

$$(e) x^3y^2 + x^2y^3 + 3y^4 \text{ and } x^4 + 3x^2y^3 + 4y^4$$

$$(f) p^2qr + pq^2r + pqr^2 \text{ and } -3pq^2r - 2pqr^2$$

$$(g) uv - vw, vw - wu \text{ and } wu - uv$$

$$(h) a^2 + 3ab - bc, b^2 + 3bc - ca \text{ and } c^2 + 3ca - ab$$

$$(i) \frac{5}{8}p^4 + 2p^2 + \frac{5}{8}, \frac{1}{8} - 17p + \frac{9}{8}p^2, p^5 - p^3 + 7$$

$$(j) t - t^2 - t^3 - 14, 15t^3 + 13 + 9t - 8t^2; 12t^2 - 19 - 24t \text{ and } 4t - 9t^2 + 19t^3$$

Solution:

$$(a) p^2 - 7pq - q^2 \text{ and } -3p^2 - 2pq + 7q^2$$

$$p^2 - 7pq - q^2 + (-3p^2 - 2pq + 7q^2) = -2p^2 - 9pq + 6q^2$$

$$(b) x^3 - x^2y - xy^2 - y^3 \text{ and } x^3 - 2x^2y + 3xy^2 + 4y$$

$$x^3 - x^2y - xy^2 - y^3 + x^3 - 2x^2y + 3xy^2 + 4y = 2x^3 - 3x^2y + 2xy^2 + 4y - y^3$$

$$(c) ab + bc + ca \text{ and } -bc - ca - ab$$

$$ab + bc + ca + (-bc - ca - ab) = 0$$

$$(d) p^2 - q + r, q^2 - r + p \text{ and } r^2 - p + q$$

$$p^2 - q + r + q^2 - r + p + r^2 - p + q = p^2 + q^2 + r^2$$

$$(e) x^3y^2 + x^2y^3 + 3y^4 \text{ and } x^4 + 3x^2y^3 + 4y^4$$

$$x^3y^2 + x^2y^3 + 3y^4 + x^4 + 3x^2y^3 + 4y^4 = x^3y^2 + 4x^2y^3 + 7y^4 + x^4$$

$$(f) p^2qr + pq^2r + pqr^2 \text{ and } -3pq^2r - 2pqr^2$$

$$p^2qr + pq^2r + pqr^2 + (-3pq^2r - 2pqr^2) = p^2qr - 2pq^2r - pqr^2$$

$$(g) uv - vw, vw - wu \text{ and } wu - uv$$

$$uv - vw + vw - wu + wu - uv = 0$$

$$(h) a^2 + 3ab - bc, b^2 + 3bc - ca \text{ and } c^2 + 3ca - ab$$

$$a^2 + 3ab - bc + b^2 + 3bc - ca + c^2 + 3ca - ab = a^2 + 2ab + 2bc + b^2 + 2ca + c^2$$

$$(i) \frac{5}{8}p^4 + 2p^2 + \frac{5}{8}, \frac{1}{8} - 17p + \frac{9}{8}p^2, p^5 - p^3 + 7$$

$$\frac{5}{8}p^4 + 2p^2 + \frac{5}{8} + \frac{1}{8} - 17p + \frac{9}{8}p^2 + p^5 - p^3 + 7 = \frac{5}{8}p^4 - 17p + \left(\frac{9}{8} + 2\right)p^2 + p^5 - p^3 + \left(7 + \frac{5}{8} + \frac{1}{8}\right)$$

$$= \frac{5}{8}p^4 - 17p + \left(\frac{25}{8}\right)p^2 + p^5 - p^3 + \left(\frac{31}{4}\right)$$

$$(j) t - t^2 - t^3 - 14, 15t^3 + 13 + 9t - 8t^2; 12t^2 - 19 - 24t \text{ and } 4t - 9t^2 + 19t^3$$

$$= t - t^2 - t^3 - 14 + 15t^3 + 13 + 9t - 8t^2 + 12t^2 - 19 - 24t + 4t - 9t^2 + 19t^3$$

$$= -10t - 6t^2 + 33t^3 - 20$$

58. Subtract:

(a) $-7p^2qr$ from $-3p^2qr$.

(b) $-a^2 - ab$ from $b^2 + ab$.

(c) $-4x^2y - y^3$ from $x^3 + 3xy^2 - x^2y$.

(d) $x^4 + 3x^3y^3 + 5y^4$ from $2x^4 - x^3y^3 + 7y^4$.

(e) $ab - bc - ca$ from $-ab + bc + ca$.

(f) $-2a^2 - 2b^2$ from $-a^2 - b^2 + 2ab$.

(g) $x^3y^3 + 3x^2y^2 - 7xy^3$ from $x^4 + y^4 + 3x^2y^2 - xy^3$.

(h) $2(ab + bc + ca)$ from $-ab - bc - ca$.

(i) $4.5x^5 - 3.4x^2 + 5.7$ from $5x^4 - 3.2x^2 - 7.3x$.

(j) $11 - 15y^2$ from $y^3 - 15y^2 - y - 11$.

Solution:

(a) $-7p^2qr$ from $-3p^2qr$.

$$\begin{aligned} -3p^2qr - (-7p^2qr) &= -3p^2qr + 7p^2qr \\ &= 4p^2qr \end{aligned}$$

(b) $-a^2 - ab$ from $b^2 + ab$.

$$\begin{aligned} b^2 + ab - (-a^2 - ab) &= b^2 + ab + a^2 + ab \\ &= b^2 + 2ab + a^2 \end{aligned}$$

(c) $-4x^2y - y^3$ from $x^3 + 3xy^2 - x^2y$.

$$x^3 + 3xy^2 - x^2y + 4x^2y + y^3 = x^3 + 3xy^2 + 3x^2y + y^3$$

(d) $x^4 + 3x^3y^3 + 5y^4$ from $2x^4 - x^3y^3 + 7y^4$.

$$2x^4 - x^3y^3 + 7y^4 - x^4 - 3x^3y^3 - 5y^4 = x^4 - 4x^3y^3 + 2y^4$$

(e) $ab - bc - ca$ from $-ab + bc + ca$.

$$-ab + bc + ca - ab + bc + ca = -2ab + 2bc + 2ca$$

(f) $-2a^2 - 2b^2$ from $-a^2 - b^2 + 2ab$.

$$-a^2 - b^2 + 2ab + 2a^2 + 2b^2 = a^2 + b^2 + 2ab$$

(g) $x^3y^3 + 3x^2y^2 - 7xy^3$ from $x^4 + y^4 + 3x^2y^2 - xy^3$.

$$x^4 + y^4 + 3x^2y^2 - xy^3 - x^3y^3 - 3x^2y^2 + 7xy^3 = x^4 + y^4 + 6xy^3 - x^3y^3$$

(h) $2(ab + bc + ca)$ from $-ab - bc - ca$.

$$\begin{aligned} -ab - bc - ca - 2(ab + bc + ca) &= -ab - bc - ca - 2ab - 2bc - 2ca \\ &= -3ab - 3bc - 3ca \end{aligned}$$

(i) $4.5x^5 - 3.4x^2 + 5.7$ from $5x^4 - 3.2x^2 - 7.3x$.

$$5x^4 - 3.2x^2 - 7.3x - 4.5x^5 + 3.4x^2 - 5.7 = 5x^4 - 4.5x^5 + 0.2x^2 - 7.3x - 5.7$$

(j) $11 - 15y^2$ from $y^3 - 15y^2 - y - 11$

$$y^3 - 15y^2 - y - 11 - 11 + 15y^2 = y^3 - y - 22$$

59.

(a) What should be added to $x^3 + 3x^2y + 3xy^2 + y^3$ to get $x^3 + y^3$?

(b) What should be added to $3pq + 5p^2q^2 + p^3$ to get $p^3 + 2p^2q^2 + 4pq$?

Solution:

(a)

According to question,

$$x^3 + y^3 - (x^3 + 3x^2y + 3xy^2 + y^3)$$

$$x^3 + y^3 - x^3 - 3x^2y - 3xy^2 - y^3$$

$$-3(x^2y + xy^2)$$

So, we have to add, $-3(x^2y + xy^2)$

(b)

According to question,

$$p^3 + 2p^2q^2 + 4pq - (3pq + 5p^2q^2 + p^3)$$

$$p^3 + 2p^2q^2 + 4pq - 3pq - 5p^2q^2 - p^3$$

$$-3p^2q^2 + pq$$

So, we have to add, $-3p^2q^2 + pq$

60.

(a) What should be subtracted from $2x^3 - 3x^2y + 2xyz + 3y^3$ to get $x^3 - 2x^2y + 3xy^2 + 4y^3$?

(b) What should be subtracted from $-7mn + 2m^2 + 3n^2$ to get $m^2 + 2mn + n^2$?

Solution:

(a)

According to question,

$$2x^3 - 3x^2y + 2xy^2 + 3y^3 - (x^3 - 2x^2y + 3xy^2 + 4y^3)$$

$$2x^3 - 3x^2y + 2xy^2 + 3y^3 - x^3 + 2x^2y - 3xy^2 - 4y^3$$

$$x^3 - x^2y - y^3 - xy^2$$

So we have to subtract, $x^3 - x^2y - y^3 - xy^2$

(b)

According to question,

$$-7mn + 2m^2 + 3n^2 - (m^2 + 2mn + n^2)$$

$$-7mn + 2m^2 + 3n^2 - m^2 - 2mn - n^2$$

$$-9mn + m^2 + 2n^2$$

So, we have to subtract, $-9mn + m^2 + 2n^2$

61. How much is $21a^3 - 17a^2$ less than $89a^3 - 64a^2 + 6a + 16$?

Solution:

Required expression is

$$\begin{aligned} 89a^3 - 64a^2 + 6a + 16 - (21a^3 - 17a^2) &= 89a^3 - 64a^2 + 6a + 16 - 21a^3 + 17a^2 \\ &= 89a^3 - 21a^3 - 64a^2 + 17a^2 + 6a + 16 \\ &= 68a^3 - 47a^2 + 6a + 16 \end{aligned}$$

62. How much is $y^4 - 12y^2 + y + 14$ greater than $17y^3 + 34y^2 - 51y + 68$?

Solution:

Required expression is

$$\begin{aligned} y^4 - 12y^2 + y + 14 - (17y^3 + 34y^2 - 51y + 68) &= y^4 - 12y^2 + y + 14 - 17y^3 - 34y^2 + 51y - 68 \\ &= y^4 - 17y^3 - 46y^2 + 52y - 54 \\ &= y^4 - 17y^3 - 46y^2 + 52y - 54 \end{aligned}$$

63. How much does $93p^2 - 55p + 4$ exceed $13p^3 - 5p^2 + 17p - 90$?

Solution:

Required expression is

$$\begin{aligned}93p^2 - 55p + 4 - (13p^3 - 5p^2 + 17p - 90) &= 93p^2 - 55p + 4 - 13p^3 + 5p^2 - 17p + 90 \\&= 93p^2 + 5p^2 - 55p - 17p + 4 + 90 - 13p^3 \\&= 98p^2 - 72p + 94 - 13p^3 \\&= -13p^3 + 98p^2 - 72p + 94\end{aligned}$$

So,

$93p^2 - 55p + 4$ is $-13p^3 + 9p^2 - 72p + 94$ exceed from $13p^3 - 5p^2 + 17p - 90$.

64. To what expression must $99x^3 - 33x^2 - 13x - 41$ be added to make the sum zero?

Solution:

Subtracting $99x^3 - 33x^2 - 13x - 41$ from 0, we get,

Required expression is

$$\begin{aligned}0 - (99x^3 - 33x^2 - 13x - 41) &= 0 - 99x^3 + 33x^2 + 13x + 41 \\&= -99x^3 + 33x^2 + 13x + 41\end{aligned}$$

So,

If we add $-99x^3 + 33x^2 + 13x + 41$ to $99x^3 - 33x^2 - 13x - 41$, the sum is zero.

65. Subtract $9a^2 - 15a + 3$ from unity.

Solution:

In order to find solution, we will subtract $9a^2 - 15a + 3$ from unity,

So,

Required expression:

$$\begin{aligned}1 - (9a^2 - 15a + 3) &= 1 - 9a^2 + 15a - 3 \\&= -9a^2 + 15a - 2\end{aligned}$$

66. Find the values of the following polynomials at $a = -2$ and $b = 3$:

(a) $a^2 + 2ab + b^2$

(b) $a^2 - 2ab + b^2$

(c) $a^3 + 3a^2b + 3ab^2 + b^3$

(d) $a^3 - 3a^2b + 3ab^2 - b^3$

(e) $(a^2 + b^2) / 3$

(f) $(a^2 - b^2) / 3$

(g) $(a/b) + (b/a)$

(h) $a^2 + b^2 - ab - b^2 - a^2$

Solution:

Putting,

$$a = -2$$

$$b = 3$$

We get,

$$(a) \ a^2 + 2ab + b^2 = (-2)^2 + 2(-2)(3) + 3^2 \\ = 1$$

$$(b) \ a^2 - 2ab + b^2 = 25$$

$$(c) \ a^3 + 3a^2b + 3ab^2 + b^3 = 1$$

$$(d) \ a^3 - 3a^2b + 3ab^2 - b^3 = -125$$

$$(e) \ (a^2 + b^2)/3 = \frac{13}{3}$$

$$(f) \ (a^2 - b^2)/3 = \frac{-5}{3}$$

$$(g) \ a/b + b/a = \frac{-13}{6}$$

$$(h) \ a^2 + b^2 - ab - b^2 - a^2 = 6$$

67. Find the values of following polynomials at $m = 1$, $n = -1$ and $p = 2$:

$$(a) \ m + n + p$$

$$(b) \ m^2 + n^2 + p^2$$

$$(c) \ m^3 + n^3 + p^3$$

$$(d) \ mn + np + pm$$

$$(e) \ m^3 + n^3 + p^3 - 3mnp$$

$$(f) \ m^2n^2 + n^2p^2 + p^2m^2$$

Solution:

Given,

$$m = 1,$$

$$n = -1 \text{ and}$$

$$p = 2$$

Putting $m = 1$, $n = -1$ and $p = 2$ in the given expressions, we get

$$(a) \ m + n + p = 1 - 1 + 2 \\ = 2$$

$$(b) \ m^2 + n^2 + p^2 = (1)^2 + (-1)^2 + (2)^2 \\ = 1 + 1 + 4 \\ = 6$$

$$\begin{aligned} \text{(c) } m^3 + n^3 + p^3 &= (1)^3 + (-1)^3 + (2)^3 \\ &= 1 - 1 + 8 \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{(d) } mn + np + pm &= (1)(-1) + (-1)(2) + (2)(1) \\ &= -1 - 2 + 2 \\ &= -1 \end{aligned}$$

$$\begin{aligned} \text{(e) } m^3 + n^3 + p^3 - 3mnp &= (1)^3 + (-1)^3 + (2)^3 - 3(1)(-1)(2) \\ &= 1 - 1 + 8 + 6 \\ &= 14 \end{aligned}$$

$$\begin{aligned} \text{(f) } m^2n^2 + n^2p^2 + p^2m^2 &= (1)^2(-1)^2 + (-1)^2(2)^2 + (2)^2(1)^2 \\ &= 1 + 4 + 4 \\ &= 9 \end{aligned}$$

68. If $A = 3x^2 - 4x + 1$, $B = 5x^2 + 3x - 8$ and $C = 4x^2 - 7x + 3$, then find:

(i) $(A + B) - C$ (ii) $B + C - A$ (iii) $A + B + C$

Solution:

Given,

$$A = 3x^2 - 4x + 1,$$

$$B = 5x^2 + 3x - 8$$

$$C = 4x^2 - 7x + 3$$

$$\begin{aligned} (A + B) - C &= (3x^2 - 4x + 1 + 5x^2 + 3x - 8) - (4x^2 - 7x + 3) \\ &= (3x^2 + 5x^2 - 4x + 3x + 1 - 8) - (4x^2 - 7x + 3) \\ &= (8x^2 - x - 7) - (4x^2 - 7x + 3) \\ &= 8x^2 - x - 7 - 4x^2 + 7x - 3 \\ &= 8x^2 - 4x^2 - x + 7x - 7 - 3 \\ &= 4x^2 + 6x - 10 \end{aligned}$$

$$\begin{aligned} B + C - A &= 5x^2 + 3x - 8 + 4x^2 - 7x + 3 - (3x^2 - 4x + 1) \\ &= (5x^2 + 4x^2 + 3x - 7x - 8 + 3) - (3x^2 - 4x + 1) \\ &= (9x^2 - 4x - 5) - (3x^2 - 4x + 1) \\ &= 9x^2 - 4x - 5 - 3x^2 + 4x - 1 \\ &= 9x^2 - 3x^2 - 4x + 4x - 5 - 1 \\ &= 6x^2 - 6 \end{aligned}$$

$$\begin{aligned} A + B + C &= 3x^2 - 4x + 1 + 5x^2 + 3x - 8 + 4x^2 - 7x + 3 \\ &= 3x^2 + 5x^2 + 4x^2 - 4x + 3x - 7x + 1 - 8 + 3 \\ &= 12x^2 - 8x - 4 \end{aligned}$$

69. If $P = -(x - 2)$, $Q = -2(y + 1)$ and $R = -x + 2y$, find a, when $P + Q + R = ax$.

Solution:

Given,

$$P = -(x-2),$$

$$Q = -2(y+1)$$

$$R = -x + 2y$$

Also,

$$P+Q + R = ax$$

On putting the values of P,Q and R on LHS, we get

$$-(x-2)+[-2(y+1)]+(-x+2y) = ax$$

$$-x+2 + (-2y-2)-x + 2y = ax$$

$$-x + 2 - 2y - 2 - x + 2y = ax$$

$$-x - x - 2y + 2y + 2 - 2 = ax$$

$$-2x = ax$$

By comparing LHS and RHS, we get,

$$a = -2$$

70. From the sum of $x^2 - y^2 - 1$, $y^2 - x^2 - 1$ and $1 - x^2 - y^2$, subtract $-(1 + y^2)$.

Solution:

Sum of $x^2 - y^2 - 1$, $y^2 - x^2 - 1$ and $1 - x^2 - y^2$

$$\begin{aligned} x^2 - y^2 - 1 + y^2 - x^2 - 1 + 1 - x^2 - y^2 &= x^2 - x^2 - x^2 - y^2 + y^2 - y^2 - 1 - 1 + 1 \\ &= -x^2 - y^2 - 1 \end{aligned}$$

Now,

Subtract $-(1 + y^2)$ from $-x^2 - y^2 - 1$

$$\begin{aligned} -x^2 - y^2 - 1 - [-(1 + y^2)] &= -x^2 - y^2 - 1 + 1 + y^2 \\ &= -x^2 - y^2 + y^2 - 1 + 1 \\ &= -x^2 \end{aligned}$$

71. Subtract the sum of $12ab - 10b^2 - 18a^2$ and $9ab + 12b^2 + 14a^2$ from the sum of $ab + 2b^2$ and $3b^2 - a^2$.

Solution:

According to question,

$$\begin{aligned} 12ab - 10b^2 - 18a^2 \text{ and } 9ab + 12b^2 + 14a^2 &= 12ab - 10b^2 - 18a^2 + 9ab + 12b^2 + 14a^2 \\ &= 12ab + 9ab - 10b^2 + 12b^2 - 18a^2 + 14a^2 \\ &= 21ab + 2b^2 - 4a^2 \end{aligned}$$

Also,

$$\begin{aligned} ab + 2b^2 \text{ and } 3b^2 - a^2 &= ab + 2b^2 + 3b^2 - a^2 \\ &= ab + 5b^2 - a^2 \end{aligned}$$

Now,

Subtracting $21ab + 2b^2 - 4a^2$ from $ab + 5b^2 - a^2$, we get

$$\begin{aligned}
 (ab + 5b^2 - a^2) - (21ab + 2b^2 - 4a^2) &= ab + 5b^2 - a^2 - 21ab - 2b^2 + 4a^2 \\
 &= ab - 21ab + 5b^2 - 2b^2 - a^2 + 4a^2 \\
 &= -20ab + 3b^2 + 3a^2 \\
 &= 3a^2 + 3b^2 - 20ab
 \end{aligned}$$

72. Each symbol given below represents an algebraic expression:

$$\triangle = 2x^2 + 3y, \quad \bigcirc = 5x^2 + 3x, \quad \square = 8y^2 - 3x^2 + 2x + 3y$$

The symbols are then represented in the expression:

$$\triangle + \bigcirc - \square$$

Find the expression which is represented by the above symbols.

Solution:

$$\triangle + \bigcirc - \square$$

Putting the values of above figures, we get,

$$\begin{aligned}
 \triangle + \bigcirc - \square &= 2x^2 + 3y + 5x^2 + 3x - (8y^2 - 3x^2 + 2x + 3y) \\
 &= 2x^2 + 3y + 5x^2 + 3x - 8y^2 + 3x^2 - 2x - 3y \\
 &= 10x^2 + x - 8y^2
 \end{aligned}$$

73. Observe the following nutritional chart carefully:

Food Item (Per Unit = 100g)	Carbohydrates
Rajma	60g
Cabbage	5g
Potato	22g
Carrot	11g
Tomato	4g
Apples	14g

Write an algebraic expression for the amount of carbohydrates in ‘g’ for

(a) y units of potatoes and 2 units of rajma

(b) 2x units tomatoes and y units apples.

Solution:

(a) By unitary method,

So,

1 unit of potatoes contain carbohydrates = 22 g

y units of potatoes contain carbohydrates = $22 \times y$
= $22y$ g

Similarly,

1 unit of rajma contain carbohydrates = 60 g

2 units of rajma contain carbohydrates = (60×2)
= 120 g

Hence, the required expression is $22y + 120$.

(b) By unitary method,

1 unit of tomatoes contain carbohydrates = 4 g

So,

2x units of tomatoes contain carbohydrates = $2x \times 4$
= $8x$ g

Similarly,

1 unit apples contain carbohydrates = 14 g

y units apples contain carbohydrates = $14 \times y$
= $14y$ g

Hence, the required expression is $8x + 14y$.

74. Arjun bought a rectangular plot with length x and breadth y and then sold a triangular part of it whose base is y and height is z. Find the area of the remaining part of the plot.

Solution:

Length of rectangular plot = x

Breadth of rectangular plot = y

Now,

Area of rectangular plot = xy

And,

Triangular plot with base y and height z is sold,

Area of triangular plot = $\frac{1}{2} \times y \times z$
= $\frac{1}{2} yz$

So,

Area of remaining part of the plot = Area of rectangular plot - Area of triangular plot

$$= xy - \frac{1}{2} yz$$

$$= y \left(x - \frac{1}{2} z \right)$$

75. Amisha has a square plot of side m and another triangular plot with base and height each equal to m. What is the total area of both plots?

Solution:

Side of square plot = m

Height and base of triangular plot = m

Area of square plot = m^2

and,

$$\begin{aligned}\text{Area of triangular plot} &= \frac{1}{2} \times m \times m \\ &= \frac{1}{2} m^2\end{aligned}$$

Total area of both plots = Area of square plot + Area of triangular plot

$$\begin{aligned}&= m^2 + \frac{m^2}{2} \\ &= \frac{3m^2}{2}\end{aligned}$$

76. A taxi service charges rupees 8 per km and levies a fixed charge of rupees 50. Write an algebraic expression for the above situation, if the taxi is hired for x km.

Solution:

According to question,

Taxi service charges Rs. 8 per km and fixed charge of Rs. 50.

If taxi is hired for x km.

Then,

$$\begin{aligned}\text{Algebraic expression for the situation} &= 8 \times x + 50 \\ &= 8x + 50\end{aligned}$$

Hence, the required expression is $8x + 50$.

77. Shiv works in a mall and gets paid rupees 50 per hour. Last week he worked for 7 hours and this week he will work for x hours. Write an algebraic expression for the money paid to him for both the weeks.

Solution:

Given,

Money paid to shiv = Rs. 50 per hr.

So,

$$\begin{aligned}\text{Money paid last week} &= \text{Rs. } 50 \times 7 \\ &= \text{Rs. } 350\end{aligned}$$

So,

$$\begin{aligned}\text{Money paid this week} &= \text{Rs. } 50 \times x \\ &= \text{Rs. } 50x\end{aligned}$$

$$\begin{aligned}\text{Total money paid to shiv} &= \text{Rs. } (350 + 50x) \\ &= \text{Rs. } 50(x + 7)\end{aligned}$$

78. Sonu and Raj have to collect different kinds of leaves for science project. They go to a park where Sonu collects 12 leaves and Raj collects x leaves. After some time Sonu loses 3 leaves and Raj collects $2x$ leaves. Write an algebraic expression to find the total number of leaves collected by both of them.

Solution:

We have,

$$\begin{aligned}\text{Sonu collected leaves} &= 12 - 3 \\ &= 9\end{aligned}$$

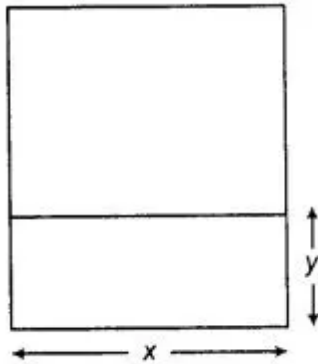
$$\begin{aligned}\text{Raj collected leaves} &= x + 2x \\ &= 3x\end{aligned}$$

So,

$$\text{Total leaves collected} = 9 + 3x$$

Hence, the required expression is $9 + 3x$.

79. A school has a rectangular play ground with length x and breadth y and a square lawn with side x as shown in the figure given below. What is the total perimeter of both of them combined together?



Solution:

Length of rectangular playground (AB) = x

Breadth of rectangular playground (BC) = y

Here,

FCDE is a square

So,

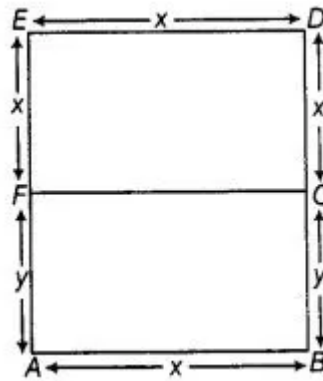
$$FC = CD = EF = DE = x$$

And,

ABCF is a rectangle,

$$AB = FC = x$$

$$BC = AF = y$$



Now,

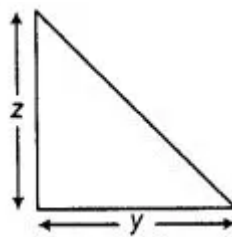
$$\begin{aligned} \text{Combined perimeter (Playground + lawn)} &= \text{Sum of all sides} \\ &= AB + BC + CD + DE + EF + FA \\ &= x + y + x + x + x + y \\ &= 4x + 2y \end{aligned}$$

80. The rate of planting the grass is rupees x per square metre. Find the cost of planting the grass on a triangular lawn whose base is y metres and height is z metres.

Solution:

Base = y metres

Height = z metres



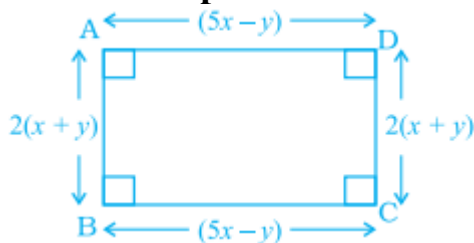
$$\begin{aligned} \text{Area of triangular lawn} &= \frac{1}{2} \times y \times z \\ &= \frac{1}{2} yz \text{ metres} \end{aligned}$$

Now,

$$\text{Cost of planting grass on lawn} = \frac{1}{2} yz \times x$$

$$= \text{Rs. } \frac{1}{2}xyz$$

81. Find the perimeter of the figure given below:



Solution:

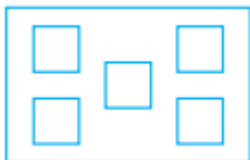
We have,

Perimeter is the sum of all sides.

Perimeter of the given figure = AB + BC + CD + DA

$$\begin{aligned} &= (5x - y) + 2(x + y) + (5x - y) + 2(x + y) \\ &= 5x - y + 2x + 2y + 5x - y + 2x + 2y \\ &= 5x + 2x + 5x + 2x - y + 2y - y + 2y \\ &= 14x + 2y \end{aligned}$$

82. In a rectangular plot, 5 square flower beds of side $(x + 2)$ metres each have been laid (see figure given below). Find the total cost of fencing the flower beds at the cost of rupees 50 per 100 metres :



Solution:

Side of one square flower bed = $x + 2$

Perimeter of one square flower bed = 4 x side
 $= 4(x + 2)$

Now,

Total perimeter of 5 such square flower beds = $5[4(x + 2)]$
 $= 20(x + 2)$

Now,

Cost of fencing of 100 m = Rs. 50

Cost of 1 m = $\frac{50}{100}$

Cost of $20(x + 2)$ = $\frac{50}{100}[20(x + 2)]$
 $= \text{Rs. } 10x + 20$

83. A wire is $(7x - 3)$ metres long. A length of $(3x - 4)$ metres is cut for use. Now, answer the following questions:

(a) How much wire is left?

(b) If this left out wire is used for making an equilateral triangle. What is the length of each side of the triangle so formed?

Solution:

Given,

Length of wire = $(7x - 3)$ m

Wire cut for use has length = $(3x - 4)$ m

$$\begin{aligned}\text{(a) Left wire} &= (7x - 3) - (3x - 4) \\ &= 7x - 3 - 3x + 4 \\ &= 7x - 3x - 3 + 4 \\ &= (4x + 1)\text{m}\end{aligned}$$

$$\text{(b) Left wire} = (4x + 1)\text{m}$$

Also,

Perimeter of equilateral triangle = Length of wire left

$$3 \times (\text{side}) = 4x + 1$$

$$\text{side} = \frac{4x + 1}{3}$$

$$\text{side} = \frac{1}{3} (4x + 1)\text{m}$$

84. Rohan's mother gave him rupees $3xy^2$ and his father gave him rupees $5(xy^2 + 2)$. Out of this total money he spent rupees $(10 - 3xy^2)$ on his birthday party. How much money is left with him?

Solution:

Given,

Amount given to Rohan by his mother = Rs. $3xy^2$ and

Amount given to Rohan by his father = Rs. $5(xy^2 + 2)$


So,

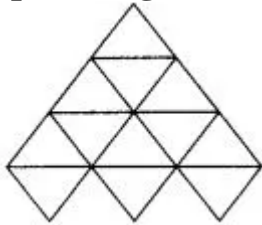
$$\begin{aligned}\text{Total amount Rohan have} &= [(3xy^2) + (5xy^2 + 10)] \\ &= \text{Rs. } [3xy^2 + 5xy^2 + 10] \\ &= \text{Rs. } (8xy^2 + 10)\end{aligned}$$

Total amount spent by Rohan = Rs. $(10 - 3xy^2)$

After spending,

$$\begin{aligned}\text{Rohan have left money} &= (8xy^2 + 10) - (10 - 3xy^2) \\ &= 11 xy^2\end{aligned}$$

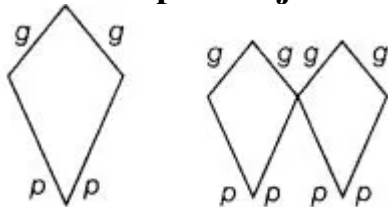
85. (i) A triangle is made up of 2 red sticks and 1 blue sticks . The length of a red stick is given by r and that of a blue stick is given by b . Using this information, write an expression for the total length of sticks in the pattern given below:



(ii) In the given figure, the length of a green side is given by g and that of the red side is given by p .



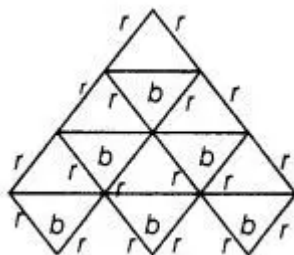
Write an expression for the following pattern. Also write an expression if 100 such shapes are joined together.



Solution:

- (i) Length of red stick = $2r$
Length of blue stick = b

Total number of red sticks = 18
Total number of blue sticks = 6



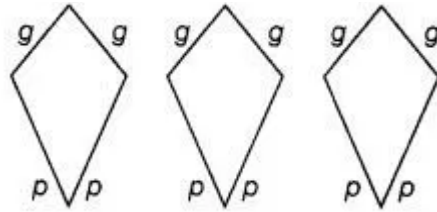
Total length of sticks = $18r + 6b$
= $6(3r+b)$

Required expression = $6(3r+b)$

(ii) Length of green side = g

Length of red side = p

When we take three figures,



Total length of 3 figures = $3(2g + 2p)$

$$= 6g + 6p$$

If 100 such shapes are joined together = $100(2g + 2p)$

$$= 200g + 200p$$

$$= 200(g+p)$$

Required expression is = $200(g+p)$

86. The sum of first n natural numbers is given by $(1/2)n^2 + (1/2)n$. Find

(i) The sum of first 5 natural numbers.

(ii) The sum of first 11 natural numbers.

(iii) The sum of natural numbers from 11 to 30.

Solution:

$$\text{Sum of first } n \text{ natural numbers} = \frac{1}{2}n^2 + \frac{1}{2}n$$

$$(i) \text{ Sum of first 5 natural numbers} = \frac{1}{2}n^2 + \frac{1}{2}n$$

$$= \frac{1}{2}(5^2) + \frac{1}{2}(5)$$

$$= \frac{25}{2} + \frac{5}{2}$$

$$= \frac{30}{2}$$

$$= 15$$

$$\begin{aligned}
 \text{(ii) Sum of first 11 natural numbers} &= \frac{1}{2}n^2 + \frac{1}{2}n \\
 &= \frac{1}{2}(11^2) + \frac{1}{2}(11) \\
 &= \frac{121}{2} + \frac{11}{2} \\
 &= \frac{132}{2} \\
 &= 66
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) Sum of natural numbers from 11 to 30} &= \text{Sum of first 30 natural numbers} - \text{Sum of first 10 natural numbers} \\
 &= \frac{1}{2}(30^2) + \frac{1}{2}(30) - \left[\frac{1}{2}(10^2) + \frac{1}{2}(10) \right] \\
 &= \frac{900}{2} + \frac{30}{2} - \frac{100}{2} - \frac{10}{2} \\
 &= 450 + 15 - 50 - 5 \\
 &= 410
 \end{aligned}$$

87. The sum of squares of first n natural numbers is given by

$\frac{1}{6}(n+1)(2n+1)$ or $\frac{1}{6}(2n^3 + 3n^2 + n)$. Find the sum of squares of the first 10 natural numbers.

Solution:

Given,

$$\text{Sum of squares of first n natural numbers} = \frac{1}{6}(n+1)(2n+1)$$

So,

The sum of squares of first 10 natural numbers [putting n=10]

$$\begin{aligned}
 \text{Sum} &= \frac{1}{6}(10)(10+1)(2 \times 10+1) \\
 &= \frac{1}{6} \times 10 \times 11 \times 21 \\
 &= 385
 \end{aligned}$$

88. The sum of the multiplication table of natural number 'n' is given by 55 × n. Find the sum of

(a) Table of 7 (b) Table of 10 (c) Table of 19

Solution:

Given,

$$\text{Sum of multiplication table of n natural numbers} = 55 \times n$$

- (a) Sum of table of 7 = 55×7
 $= 385$ [putting $n = 7$]
 (b) Sum of table of 10 = 55×10
 $= 550$ [putting $n = 10$]
 (c) Sum of table of 19 = 55×19
 $= 1045$ [putting $n = 19$]

89.

If $\triangle x = 2x + 3$, $\square x = \frac{3}{2}x + 7$ and $\bigcirc x = x - 3$,

then find the value of :

(i) $2 \triangle 6 + \square 3 - \bigcirc 1$

(ii) $\frac{1}{2} \square 2 + \bigcirc 8 - 3 \triangle 0$

Solution:

$$\begin{aligned} \text{(i)} \quad 2 \triangle 6 + \square 3 - \bigcirc 1 &= 2 \times (2 \times 6 + 3) + \left(\frac{3}{2} \times 3 + 7 \right) - (1 - 3) \\ &= 2 \times 15 + \frac{23}{2} + 2 \\ &= 32 + \frac{23}{2} \\ &= \frac{87}{2} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \frac{1}{2} \square 2 + \bigcirc 8 - 3 \triangle 0 &= \frac{1}{2} \left(\frac{3}{2} \times 2 + 7 \right) + (8 - 3) - 3(2 \times 0 + 3) \\ &= \frac{1}{2} (10) + 5 - 3(3) \\ &= 5 + 5 - 9 \\ &= 1 \end{aligned}$$

90.

If $\triangle x = \frac{3}{4}x - 2$ and $\diamond x = x + 6$, then find the value of:

(i) $\triangle 10 - \diamond 4$

(ii) $2 \diamond 12 - \frac{3}{2} \triangle 1$

Solution:

$$\begin{aligned} \text{(i)} \quad \triangle 10 - \diamond 4 &= \frac{3}{4} \times 10 - 2 - 4 - 6 \\ &= \frac{30}{4} - 12 \\ &= \frac{30 - 48}{4} \\ &= \frac{-18}{4} \\ &= \frac{-9}{2} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 2 \diamond 12 - \frac{3}{2} \triangle 1 &= 2 \times (12 + 6) - \frac{3}{2} \left(\frac{3}{4} \times 1 - 2 \right) \\ &= 36 - \frac{3}{2} \left(-\frac{5}{4} \right) \\ &= 36 + \frac{15}{8} \\ &= \frac{303}{8} \end{aligned}$$

Translate each of the following algebraic expressions Question 91 to 94 into words.

91. $4b - 3$

Solution:

Three subtracted from four times b.

92. $8(m + 5)$

Solution:

Eight times the sum of m and 5.

93. $7/(8 - x)$

Solution:

Quotient on dividing seven by the difference of eight and $x(x < 8)$.

94. $17(16/w)$

Solution:

Seventeen times quotient of sixteen divided by w.

95. (i) Critical Thinking Write two different algebraic expressions for the word phrase “ $(1/4)$ of the sum of x and 7.”

(ii) What’s the Error? A student wrote an algebraic expression for “5 less than a number n divided by 3” as $(n/3) - 5$. What error did the student make?

(iii) Write About it Shashi used addition to solve a word problem about the weekly cost of commuting by toll tax for rupees 15 each day. Ravi solved the same problem by multiplying. They both got the correct answer. How is this possible?

Solution:

First expression = $\frac{1}{4}(x+7)$

We know, the addition is commutative.

So, it can also be written as = $\frac{1}{4}(7 + x)$

Since, the expression of 5 less than a number n = $n-5$

So, 5 less than a number n divided by 3 will be written = $\frac{n-5}{3}$

And, student make an error of quotient.

By addition method,

Total weekly cost = $(15+15+15+15+15+15+15)$
= Rs. 105

By multiplication method,

$$\begin{aligned}
 \text{Total weekly cost} &= \text{Cost of one day} \times \text{Seven days} \\
 &= 15 \times 7 \\
 &= \text{Rs. } 105
 \end{aligned}$$

96. Challenge Write an expression for the sum of 1 and twice a number n . If you let n be any odd number, will the result always be an odd number?

Solution:

Let the number be n .

So, according to the statement, the expression can be written as $= 2n+1$.

Yes, the result is always an odd number, because when a number is multiplied by 2, it becomes even and addition of 1 in that even number makes it an odd number.

97. Critical Thinking Will the value of $11x$ for $x = -5$ be greater than 11 or less than 11? Explain.

Solution:

We have,

$$\begin{aligned}
 11x &= 11 \times (-5) \\
 &= -55
 \end{aligned}$$

$$[x = -5]$$

Clearly, $-55 < 11$.

So, the value is greater than 11.

98. Match Column I with Column II in the following:

Column I	Column II
1. The difference of 3 and a number squared	(a) $4 - 2x$
2. 5 less than twice a number squared	(b) $N^2 - 3$
3. Five minus twice the square of a number	(c) $2n^2 - 5$
4. Four minus a number multiplied by 2	(d) $5 - 2n^2$
5. Seven times the sum of a number and 1	(e) $3 - n^2$
	(f) $2(n + 6)$
	(g) $7(n + 1)$
	(h) $n^2 + 6$

6. A number squared plus 6 7. 2 times the sum of a number and 6 8. Three less than the square of a number	
--	--

Solution:

1 – (e) Let the number be n
So, the equation is : $3 - n^2$

2 – (c) Let the number be n
So, the equation is : $2n^2 - 5$

3 – (d) Let the number be n
So, the equation is : $5 - 2n^2$

4 – (a) Let the number be x
So, the equation is : $4 - 2x$

5 – (g) Let the number be n
So, the equation is : $7(n+1)$

6 – (h) Let the number be n
So, the equation is : (n^2+6)

7 – (f) Let the number be n
So, the equation is : $2(n+6)$

8 – (b) Let the number be n
So, the equation is : $(n^2 - 3)$

99. At age of 2 years, a cat or a dog is considered 24 “human” years old. Each year, after age 2 is equivalent to 4 “human” years. Fill in the

expression $[24 + \text{ } (a - 2)]$ so that it represents the age of a cat or dog in human years. Also, you need to determine for what ‘a’ stands for. Copy the chart and use your expression to complete it.

Age	$[24 + \text{ } (a - 2)]$	Age (Human Years)
2		
3		
4		
5		
6		

Solution:

The expression is $[24 + 4(a - 2)]$

Let,

'a' represents the present age of dog or cat.

Age	$[24 + 4(a - 2)]$	Age (human years)
2	$[24 + 4(2 - 2)]$	24
3	$[24 + 4(3 - 2)]$	28
4	$[24 + 4(4 - 2)]$	32
5	$[24 + 4(5 - 2)]$	36
6	$[24 + 4(6 - 2)]$	40

100. Express the following properties with variables x, y and z.

- (i) Commutative property of addition
- (ii) Commutative property of multiplication
- (iii) Associative property of addition
- (iv) Associative property of multiplication
- (v) Distributive property of multiplication over addition

Solution:

We have,

Commutative property of addition,

$$a + b = b + a$$

So,

Required expression is $x + y = y + x$

And,

Commutative property of multiplication,

$$a \times b = b \times a$$

So,

Required expression is $x \times y = y \times x$

Now,

Associative property of addition,

$$a + (b + c) = (a + b) + c$$

So,

Required expression is $x + (y + z) = (x + y) + z$

And,

Associative property of multiplication,

$$a \times (b \times c) = (a \times b) \times c$$

So,

Required expression is $x \times (y \times z) = (x \times y) \times z$

Now,

Distributive property of multiplication over addition,

$$a \times (b + c) = a \times b + a \times c$$

So,

Required expression is $x \times (y + z) = x \times y + x \times z$