EXERCISE 9.3 (NCERT)

QNo1: Find 20^{th} and n^{th} term of G.P $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}$ .....

Sol: Given G.P is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}$ ......

$\therefore \quad a = \frac{5}{2} \quad , \quad r = \frac{\frac{5}{4}}{\frac{5}{2}} = \frac{5}{4} \times \frac{2}{5} = \frac{1}{2}$

Now $T_n = ar^{n-1}$

$\Rightarrow \quad T_{20} = \frac{5}{2}\left(\frac{1}{2}\right)^{20-1} = \frac{5}{2} \times \frac{1}{2^{19}} = \frac{5}{2^{20}}$

and $T_n = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{5}{2^n}$.

QNo.2 Find 12^{th} term of G.P whose 8^{th} term is 192 and Common ratio is 2.

Sol: Here $a = ?$ and $T_8 = 192$, $r = 2$

Now $T_8 = 192$

$\Rightarrow \quad ar^7 = 192$

$\Rightarrow \quad a \times (2)^7 = 192 \quad \Rightarrow a = \frac{192}{2^7}$

Now $T_{12} = ar^{11} = \frac{192}{2^7} \times (2)^{11} = 192 \times 2^4 = 3072$.

QNo3: The 5^{th}, 8^{th} and 11^{th} terms of G.P are p, q and s, respectively. show that $q^2 = ps$.

Sol. Let a be the first term and r be the common ratio

Now $T_5 = ar^4 = p$

$T_8 = ar^7 = q$

$T_{11} = ar^{10} = s$.

Now $\therefore q^2 = (ar^7)^2 = a^2r^{14}$ ...(1)

and $ps = (ar^4)(ar^{10}) = a^2r^{14} = q^2$ [from (1)]

$\Rightarrow \quad q^2 = ps$.

Q.No 4 : The fourth term of a G.P is square of its second term and the first term is -3. Determine its 7^{th} term.

Sol : Here $a = -3$.

Also $T_4 = (T_2)^2$

$\Rightarrow a r^3 = (ar)^2$ where 'r' is common ratio.

$\Rightarrow -3r^3 = (-3r)^2$

$\Rightarrow -3r^3 = 9r^2 \Rightarrow r = -3$.

$\therefore T_7 = ar^6 = (-3)(-3)^6 = (-3)^7 = -2187$.

Q.No 5. Which term of following sequences :

(a) $2, 2\sqrt{2}, 4$ ...... is 128 (b) $\sqrt{3}, 3, 3\sqrt{3}$ ..... is 729

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ .... is $\frac{1}{19683}$?

Sol. (a) Given G.P is $2, 2\sqrt{2}, 4$ .....

$\therefore a = 2$, $r = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Let $T_n = 128$

$\Rightarrow ar^{n-1} = 128$

$\Rightarrow 2(\sqrt{2})^{n-1} = 128 \Rightarrow \left[(2)^{1/2}\right]^{n-1} = 64$

$\Rightarrow 2^{\frac{n-1}{2}} = 2^6 \quad [\because 2^6 = 64]$

$\Rightarrow \frac{n-1}{2} = 6 \Rightarrow n-1 = 12 \Rightarrow n = 13$

$\therefore$ 128 is 13^{th} term of given G.P.

(b) Given G.P is $\sqrt{3}, 3, 3\sqrt{3}$, ...

$\therefore a = \sqrt{3}$, $r = \frac{3}{\sqrt{3}} = \frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}} = \sqrt{3}$

Let $T_n = 729$

$\Rightarrow ar^{n-1} = 729$

$\Rightarrow \sqrt{3}(\sqrt{3})^{n-1} = 729 \Rightarrow (\sqrt{3})^n = 729$

$\Rightarrow (3^{1/2})^n = 3^6 \Rightarrow (3)^{n/2} = 3^6$

$\Rightarrow \frac{n}{2} = 6 \Rightarrow n = 12$

$\therefore$ 729 is the 12^{th} term of given G.P.

(c) Given G.P is $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ ....

Here $a = 1/3$, $r = \frac{1/9}{1/3} = \frac{1}{9}\times\frac{3}{1} = \frac{1}{3}$.

Let $T_n = \frac{1}{19683}$

$\Rightarrow \quad ar^{n-1} = \frac{1}{19683}$

$\Rightarrow \quad \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$

$\Rightarrow \quad \left(\frac{1}{3}\right)^{n-1} = \frac{1}{6561} \Rightarrow \left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^{8}$

$\Rightarrow \quad n-1 = 8 \Rightarrow n = 9$

$\therefore \quad \frac{1}{19683}$ is 9th term of given GP.

Q No 6: For what values of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P.?

Sol Since $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P.

$\therefore \quad \frac{x}{-\frac{2}{7}} = \frac{-\frac{7}{2}}{x}$

$\Rightarrow \quad x^2 = \left(-\frac{7}{2}\right)\left(-\frac{2}{7}\right)$

$\Rightarrow \quad x^2 = 1 \Rightarrow x = \pm\sqrt{1} = \pm 1$

Q No 7. Find the sum of G.P to indicated terms in Exercise 7 to 10

0.15, 0.015, 0.0015, ... 20 terms.

Sol. Given G.P. is $0.15, 0.015, 0.0015, \ldots\ldots$

or $\frac{15}{100}, \frac{15}{1000}, \frac{15}{10000} \ldots\ldots$

Here $a = \frac{15}{100}, \quad r = \frac{\frac{15}{1000}}{\frac{15}{100}} = \frac{15}{1000} \times \frac{100}{15} = \frac{1}{10} < 1$

$\therefore \quad S_{20} = \frac{\frac{15}{100}\left[1 - \left(\frac{1}{10}\right)^{20}\right]}{1 - \frac{1}{10}} \qquad \left[\because S_n = \frac{a(1-r^n)}{1-r}\right]$

$= \frac{15}{100} \times \frac{10}{9}\left[1 - \left(\frac{1}{10}\right)^{20}\right] = \frac{1}{6}\left[1 - (0.1)^{20}\right]$

Q No 8: $\sqrt{7}, \sqrt{21}, 3\sqrt{7}$ .... to n terms.

Sol. Given GP is $\sqrt{7}, \sqrt{21}, 3\sqrt{7} \ldots\ldots$

or $\sqrt{7}, \sqrt{3}\sqrt{7}, 3\sqrt{7} \ldots\ldots$

Here $a = \sqrt{7}, \quad r = \frac{\sqrt{3}\sqrt{7}}{\sqrt{7}} = \sqrt{3} > 1$

$\therefore \quad S_n = \frac{a[r^n - 1]}{r-1} = \frac{\sqrt{7}\left[(\sqrt{3})^n - 1\right]}{\sqrt{3} - 1}$

$$S_n = \frac{\sqrt{7}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}\left[3^{n/2}-1\right] = \frac{\sqrt{7}(\sqrt{3}+1)}{3-1}\left(3^{n/2}-1\right)$$

$$= \frac{\sqrt{7}}{2}(\sqrt{3}+1)(3^{n/2}-1)$$

Q No 09: $1, -a, a^2, -a^3 \ldots$ n terms (If $a \neq -1$)

Sol Let A be the first term and R be the common ratio.

$\therefore$ $A = 1$ and $R = \frac{-a}{1} = -a$.

$$\therefore S_n = \frac{A[1-R^n]}{1-R} = \frac{1[1-(-a)^n]}{1-(-a)} = \frac{1-(-a)^n}{1+a}.$$

Q No 10 $x^3, x^5, x^7 \ldots$ n terms $(x \neq \pm 1)$

Sol. Here $a = x^3$, $r = \frac{x^5}{x^3} = x^2$

$$\therefore S_n = \frac{x^3[1-(x^2)^n]}{1-x^2} \quad \left[\because S_n = \frac{a[1-r^n]}{1-r}\right]$$

$$= \frac{x^3[1-x^{2n}]}{1-x^2}.$$

Q No 11. Evaluate $\sum_{k=1}^{11}(2+3^k)$

Sol.
$$\sum_{k=1}^{11}(2+3^k) = \sum_{k=1}^{11} 2 + \sum_{k=1}^{11} 3^k = 22 + (3^1+3^2+3^3+\ldots 3^{11})$$

$$= 22 + \frac{3(3^{11}-1)}{3-1} = 22 + \frac{3}{2}(3^{11}-1)$$

Q No 12: The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Sol.: Let the first 3 terms be $\frac{a}{r}, a, ar$.

$$\therefore \left(\frac{a}{r}\right)+(a)+(ar) = \frac{39}{10} \quad \text{and} \quad \left(\frac{a}{r}\right)(a)(ar) = 1$$

$$\Rightarrow \left(\frac{a}{r}\right)+(a)+(ar) = \frac{39}{10} \quad \text{and} \quad a^3 = 1 \quad \Rightarrow a = 1$$

$$\Rightarrow \frac{1}{r}+1+r = \frac{39}{10} \Rightarrow 10+10r+10r^2 = 39r$$

$$\therefore 10r^2 - 29r + 10 = 0$$

$$\Rightarrow r = \frac{29 \pm \sqrt{841-400}}{20} = \frac{29 \pm \sqrt{441}}{20} = \frac{29 \pm 21}{20} = \frac{50}{20}, \frac{8}{20} = \frac{5}{2}, \frac{2}{5}$$

When $r = \frac{5}{2}$, terms are $\frac{2}{5}, 1, \frac{5}{2}$

When $r = \frac{2}{5}$, terms are $\frac{5}{2}, 1, \frac{2}{5}$.

Q No 13 How many terms of GP $3, 3^2, 3^3 \dots$ are needed to given sum 120?

Sol: Let n be the no. of GP $3, 3^2, 3^3 \dots$ makes the sum 120

$\therefore \quad S_n = \frac{a(r^n-1)}{r-1}$ gives

$120 = \frac{3(3^n-1)}{3-1} \Rightarrow 120 \times 2 = 3(3^n-1)$

$\Rightarrow 240 = 3^{n+1} - 3$

$\Rightarrow 243 = 3^{n+1}$

$\Rightarrow 3^5 = 3^{n+1} \Rightarrow n+1 = 5 \Rightarrow n = 5-1 \Rightarrow n = 4.$

Q No 14: The Sum of first three terms of GP is 16 and sum of next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

Sol.: Let a be first term and r be the common ratio of G.P.

From given conditions

$a + ar + ar^2 = 16$ ie $a[1+r+r^2] = 16$ --- (1)

and $ar^3 + ar^4 + ar^5 = 128$ ie $ar^3[1+r+r^2] = 128$ --- (2)

Dividing (2) by (1)

$\Rightarrow r^3 = \frac{128}{16} \Rightarrow r^3 = 8 \Rightarrow r = 2$

$\therefore$ From (1) $a(1+2+2^2) = 16$

$\Rightarrow a(7) = 16 \Rightarrow a = \frac{16}{7}$

Now $S_n = \frac{a(r^n-1)}{r-1} = \frac{\frac{16}{7}(2^n-1)}{2-1} = \frac{16}{7}(2^n-1)$

Q No. 15. Given a GP with $a = 729$ and 7^{th} term 64, determine S_7

Sol. Here $a = 729$

Now $T_7 = 64 \Rightarrow ar^6 = 64 \Rightarrow 729 r^6 = 64$

$\Rightarrow r^6 = \frac{64}{729} \Rightarrow r^6 = \left(\frac{2}{3}\right)^6 \Rightarrow r = \frac{2}{3}$

Now $S_7 = \frac{a(1-r^7)}{1-r} = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-\frac{2}{3}} = 729 \times \frac{3}{1}\left(1 - \frac{128}{2187}\right)$

$= 729 \times 3 \times \left(\frac{2187-128}{2187}\right) = 2187 - 128 = 2059$

Q No 16: find a GP for which sum of first two terms is -4 and fifth term is 4 times the third term.

Sol. Let a be the first term and r be the common ratio.

ATQ $a + ar = -4. \Rightarrow a(1+r) = -4.$ --- (1)

and. $T_5 = 4T_3 \Rightarrow ar^4 = 4ar^2 \Rightarrow r^2 = 4 \Rightarrow r = \pm 2$

When $r = 2$, from (i) $a(1+2) = -4 \Rightarrow a = -\frac{4}{3}$

When $r = -2$, from (i) $a(1-2) = -4 \Rightarrow a = 4$.

When $r = 2$, $a = -\frac{4}{3}$, then GP is $-\frac{4}{3}, -\frac{8}{3}, -\frac{16}{3}, \ldots$

When $r = -2$, $a = 4$, then GP is $4, -8, 16, -32, \ldots$

Q No 17 If the 4th, 10th and 16th terms of a GP are x, y and z respectively. Prove that x, y, z are in GP.

Sol. Let a be the first term and r be common ratio of GP.

From given conditions.

$$x = ar^3 \quad \text{---(1)}$$
$$y = ar^9 \quad \text{---(2)}$$
$$z = ar^{15} \quad \text{---(3)}$$

Now x, y, z will be in GP

if $y^2 = xz$

ie. if $(ar^9)^2 = (ar^3)(ar^{15})$

ie if $a^2r^{18} = a^2r^{18}$ which is true.

Hence the result.

Q No 18: Find the sum to 'n' terms of the sequences $8, 88, 888, \ldots$

Sol. Let S_n be the required sum.

$$S_n = 8 + 88 + 888 + \ldots \text{ to } n \text{ terms}.$$
$$= 8[1 + 11 + 111 + \ldots \text{ to } n \text{ terms}]$$
$$= \frac{8}{9}[9 + 99 + 999 + \ldots \text{ to } n \text{ terms}]$$
$$= \frac{8}{9}[(10-1) + (100-1) + (1000-1) + \ldots \text{ to } n \text{ terms}]$$
$$= \frac{8}{9}[(10 + 100 + 1000 + \ldots \text{ to } n \text{ term}) - n]$$
$$= \frac{8}{9}\left[\frac{10(10^n - 1)}{10-1} - n\right] = \frac{8}{9}\left[\frac{10^{n+1} - 10 - 9n}{9}\right]$$
$$= \frac{8}{81}\left[10^{n+1} - 10 - 9n\right]$$

Q No 19: Find the sum of the products of the corresponding terms of sequences $2, 4, 8, 16, 32$ and $128, 32, 8, 2, \frac{1}{2}$.

Sol: Given two sequences are

$2, 4, 8, 16, 32$ and $128, 32, 8, 2, \frac{1}{2}$.

Let $S = (2)(128) + (4)(32) + (8)(8) + (16)(2) + (32)(\frac{1}{2})$

$= 256 + 128 + 64 + 32 + 16$

$$= \frac{256\left(1-\left(\frac{1}{2}\right)^5\right)}{1-\frac{1}{2}} \qquad \left[\because S_n = \frac{a(1-r^n)}{1-r} \quad \text{Here } a = 256,\ r = \frac{1}{2},\ n = 5\right]$$

$$= \frac{256\left(1-\frac{1}{32}\right)}{\frac{1}{2}} = 256 \times 2 \times \frac{31}{32} = 16 \times 31 = 496$$

Q No 20: Show that the products of the corresponding terms of sequences $a, ar, ar^2 \ldots ar^{n-1}$ and $A, AR, AR^2 \ldots AR^{n-1}$ form a GP and find common ratio.

Sol. The given sequences are

$a, ar, ar^2 \ldots ar^{n-1}$ and $A, AR, AR^2 \ldots AR^{n-1}$

On multiplying the corresponding terms we get

$$aA,\ arAR,\ ar^2AR^2, \ldots ar^{n-1}AR^{n-1}$$

Now $\frac{arAR}{aA} = rR$ and $\frac{ar^2AR^2}{arAR} = rR \ldots$

Which is common ratio.

Clearly $aA, arAR, ar^2AR^2 \ldots ar^{n-1}AR^{n-1}$ is a GP with common Ratio rR.

Q No. 21 Find four numbers forming a geometric progression in which third term is greater than the first term by 9 and second term is greater than the 4th by 18.

Soln: Let four numbers in GP be a, ar, ar^2, ar^3

From the given condition

$$ar^2 - a = 9 \quad \text{--- (1)}$$

and $ar - ar^3 = 18$

or $ar^3 - ar = -18$ --- (2)

Multiplying (1) by 'r' we get

$$ar^3 - ar = 9r \quad \text{--- (3)}$$

Subtracting (3) from (2) we get

$$0 = 9r + 18 \Rightarrow 9r = -18 \Rightarrow r = -2$$

From (1) $4a - a = 9 \Rightarrow 3a = 9$ or $a = 3$

$\therefore$ The required nos. are $3, 3(-2), 3(-2)^2, 3(-2)^3$

ie $3, -6, 12, -24$.

Q No 22 If p^{th}, q^{th} and r^{th} terms of a GP are a, b, c respectively Prove that $a^{q-r}\, b^{r-p}\, c^{p-q} = 1$

Sol. Let A be the first term and R be the common ratio of GP

ATQ
$$a = AR^{p-1}$$
$$b = AR^{q-1}$$
$$c = AR^{r-1}$$

$$\text{LHS} = a^{q-r}\, b^{r-p}\, c^{p-q}$$
$$= (AR^{p-1})^{q-r}\,(AR^{q-1})^{r-p}\,(AR^{r-1})^{p-q}$$
$$= A^{q-r}\, R^{(p-1)(q-r)}\, A^{r-p}\, R^{(q-1)(r-p)}\, A^{p-q}\, R^{(r-1)(p-q)}$$
$$= A^{q-r+r-p+p-q}\, R^{(pq-pr-q+r)+(qr-qp-r+p)+(rp-rq-p+q)}$$
$$= A^0 R^0$$
$$= 1 \times 1 = 1 = \text{RHS}.$$

Q No. 23 If the first and n^{th} term of a GP are a and b respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Sol. Let r be the common ratio of GP
$$\therefore T_n = b = ar^{n-1}$$

Also
$$P = (a)(ar)(ar^2)\cdots(ar^{n-1})$$
$$= a^n\, r^{[1+2+3+\cdots(n-1)]}$$
$$= a^n\, r^{\frac{(n-1)(n-1+1)}{2}}$$
$$= a^n\, r^{\frac{(n-1)(n)}{2}}$$

$$\left[\because 1+2+3 \cdots n+1 = \frac{n-1}{2}\{2+(n-1-1)\times 1\} = \frac{(n-1)(n)}{2}\right.$$

$$\therefore P^2 = \left[a^n\, r^{\frac{n(n-1)}{2}}\right]^2 = a^{2n}\, r^{n(n-1)}$$
$$= a^n \cdot a^n \cdot r^{(n)(n-1)} = a^n (ar^{n-1})^n$$
$$= [a \cdot (ar^{n-1})]^n = (ab)^n.$$

$$\therefore P^2 = (ab)^n.$$

Q.No. 24 Show that ratio of the sum of first n terms of a GP to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

Sol. Let a be the first term and r be the common ratio

$$S_n = \frac{a(1-r^n)}{1-r}.$$

Let S be the sum of terms from $(n+1)^{th}$ term to $(2n)^{th}$ term

$$\therefore S = ar^n + ar^{n+1} + \cdots ar^{2n-1} = r^n\left[a + ar + ar^2 \cdots ar^{n-1}\right]$$

$$\therefore S = \frac{ar^n\left[1-r^n\right]}{1-r}$$

$\therefore$ Required Ratio

$$\frac{S_n}{S} = \frac{\frac{a(1-r^n)}{1-r}}{\frac{ar^n(1-r^n)}{1-r}} = \frac{a(1-r^n)}{(1-r)} \times \frac{(1-r)}{ar^n(1-r)}$$

$$= \frac{1}{r^n}.$$

Q.No. 25. If a, b, c, d are in GP, show that $(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$.

Sol: Let r be the common ratio.

Then $\frac{b}{a} = r \Rightarrow b = ar$

$\frac{c}{b} = r \Rightarrow c = br = (ar).r = ar^2$

$\frac{d}{c} = r \Rightarrow d = cr = (ar^2)r = ar^3$

$$\text{LHS} = (a^2+b^2+c^2)(b^2+c^2+d^2)$$
$$= (a^2 + a^2r^2 + a^2r^4)(a^2r^2 + a^2r^4 + a^2r^6)$$
$$= a^2\left[1+r^2+r^4\right] . a^2r^2\left[1+r^2+r^4\right] = a^4r^2\left[1+r^2+r^4\right]^2$$

$$\text{RHS} = (ab+bc+cd)^2 = \left[a.ar + ar.ar^2 + ar^2.ar^3\right]^2$$
$$= \left[a^2r + a^2r^3 + a^2r^5\right]^2 = (a^2r)^2\left[1+r^2+r^4\right]^2$$
$$= a^4r^2\left[1+r^2+r^4\right]^2 = \text{LHS}$$

Hence the result.

Q No 26 Insert two numbers between 3 and 81 so that resulting sequence is ~~an~~ a GP.

Sol : Let G_1, G_2 be the numbers so that

$3, G_1, G_2, 81$ are in GP. with common ratio r.

$\Rightarrow T_4 = 81$

$\Rightarrow 3r^3 = 81 \Rightarrow r^3 = 27$

$\Rightarrow r^3 = 3^3 \Rightarrow r = 3.$

$\therefore G_1 = 3r = 3 \times 3 = 9$

$G_2 = 3r^2 = 3 \times 3 \times 3 = 27.$

$\therefore$ Required Nos are 9, 27.

Q No 27. Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be Geometric Mean between a and b.

Sol. We know that Geometric Mean between a and $b = \sqrt{ab}$

$$\therefore \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab}$$

or. $$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{a^{1/2}\, b^{1/2}}{1}$$

$$\Rightarrow a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}} b^{\frac{1}{2}} + b^{n+\frac{1}{2}} a^{\frac{1}{2}} \quad [\text{cross-multiplying}]$$

$$\Rightarrow a^{n+1} - a^{\left(n+\frac{1}{2}\right)} b^{1/2} = b^{n+\frac{1}{2}} a^{\frac{1}{2}} - b^{n+1}$$

$$\Rightarrow a^{n+\frac{1}{2}}\left[a^{\frac{1}{2}} - b^{\frac{1}{2}}\right] = b^{n+\frac{1}{2}}\left[a^{\frac{1}{2}} - b^{\frac{1}{2}}\right]$$

Dividing both sides by $a^{\frac{1}{2}} - b^{\frac{1}{2}} \neq 0$ we get.

$$a^{n+\frac{1}{2}} = b^{n+\frac{1}{2}}$$

Or $$\frac{a^{n+\frac{1}{2}}}{b^{n+\frac{1}{2}}} = 1 \Rightarrow \left(\frac{a}{b}\right)^{n+\frac{1}{2}} = \left(\frac{a}{b}\right)^0 \quad \left[\because \left(\frac{a}{b}\right)^0 = 1\right]$$

$$\Rightarrow n + \frac{1}{2} = 0 \Rightarrow n = -\frac{1}{2}.$$

Q.No. 28 The sum of two numbers is 6 times the Geometric Mean. Show that the numbers are in Ratio $(3+2\sqrt{2}):(3-2\sqrt{2})$

Sol. Let a and b be two numbers.

ATQ $a+b = 6\sqrt{ab}$ $\Rightarrow$ $\frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$

Applying componendo and dividendo, we get

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{4}{2} = \frac{2}{1}$$

or $$\left(\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^2 = \left(\frac{\sqrt{2}}{1}\right)^2$$

or $$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$$

Again applying componendo and dividendo,

$$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

or $$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$ or $$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

Squaring both sides

$$\frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

ie $$\frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}.$$

$\therefore$ $a:b = 3+2\sqrt{2} : 3-2\sqrt{2}$.

Q.No. 29. If A and G be A.M and G.M. respectively between the two positive nos., prove that the numbers are

$$A \pm \sqrt{(A+G)(A-G)}$$

Sol. : Let a and be two positive numbers

AIQ

$A = \frac{a+b}{2} \Rightarrow a+b = 2A$

and $G = \sqrt{ab} \Rightarrow ab = G^2$

Now $A^2 - G^2 = \left(\frac{a+b}{2}\right)^2 - ab$

$= \frac{(a+b)^2 - 4ab}{4}$

$= \frac{a^2+b^2+2ab-4ab}{4} = \frac{a^2+b^2-2ab}{4}$

$= \frac{(a-b)^2}{4}.$

$\therefore \sqrt{A^2-G^2} = \frac{a-b}{2}$ ---(1)

Also $A = \frac{a+b}{2}.$ ---(2)

Adding (1) and (2)

$A + \sqrt{A^2-G^2} = \frac{a-b+a+b}{2} = \frac{2a}{2} = a.$

Subtracting (1) from (2)

$A - \sqrt{A^2-G^2} = \frac{a+b}{2} - \frac{a-b}{2} = \frac{a+b-a+b}{2} = \frac{2b}{2} = b$

$\therefore$ The required Nos are $A + \sqrt{(A+G)(A-G}$ and.

$A - \sqrt{(A+G)(A-G)}$ or $A \pm \sqrt{(A+G)(A-G)}$

Q No. 30: The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and nth hour?

Sol. Here $a = 30$ $r = 2$

Number of bacteria at the end of 2nd hr

$= ar^2 = 30 \times 2 \times 2 = 120$

Number of bacteria at the end of 4th hr

$= ar^4 = 30(2)^4 = 30 \times 16 = 480$

No. of bacteria at the end of nth hour

$= a r^n = 30(2)^n$.

QNo. 31 What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually.

Sol. Let a be the amount deposited = Rs 500

$r = 10\%$.

T_1 = Amount after one year $= 500 + 500 \times \frac{10}{100}$

$= 500\left(1 + \frac{10}{100}\right)$

T_2 = Amount after 2 years $= 500\left(1 + \frac{10}{100}\right)^2$.

$\therefore T_{10}$ = Amount after 10 years $= 500\left(1 + \frac{10}{100}\right)^{10}$

$= 500(1 + 0.1)^{10} = 500(1.1)^{10}$

QNo. 32: If AM and GM of roots of quadratic equation are 8 and 5 respectively then obtain the quadratic equation.

Sol. Let roots of quadratic equation be a and b.

ATQ. $AM = 8 \Rightarrow \frac{a+b}{2} = 8 \Rightarrow a+b = 16$

$GM = 5 \Rightarrow \sqrt{ab} = 5 \Rightarrow ab = 25$

The equation with roots a, b is

$x^2 - (a+b)x + ab = 0$ $\quad [x^2 - Sx + P = 0]$

ie $x^2 - 16x + 25 = 0$

#