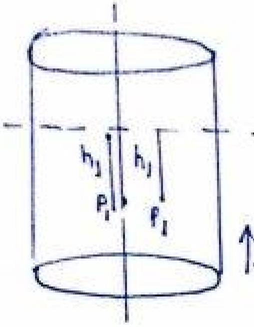
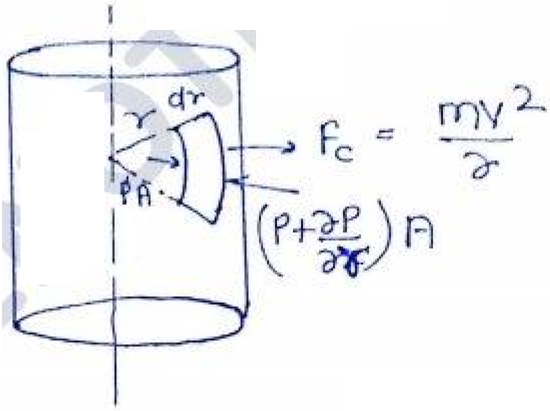
Vortex Motion (Dynamics)

Flow around circular path is known as vortex motion.



$P = f(z)$

$$\frac{\partial P}{\partial z} = -\omega \quad \text{—(i)}$$

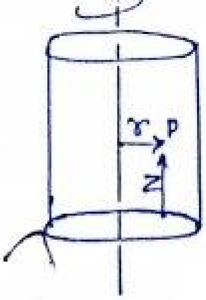


Equilibrium

$$\left(P+\frac{\partial P}{\partial r}dr\right)A = PA + f_c$$

$$\frac{\partial P}{\partial r}\,dr\cdot A = \frac{\rho(A\,dr)v^2}{r}$$

$$\frac{\partial P}{\partial r} = \frac{\rho v^2}{r} \quad \text{—(ii)}$$



Vortex

$P = f(r, z)$

$$dP = \frac{\partial P}{\partial r}dr + \frac{\partial P}{\partial z}dz$$

$$\boxed{dP = \frac{\rho v^2}{r}dr - \omega dz}$$ General Vortex Motion

$$dp = \frac{\rho V^2}{r} \cdot dr - w\, dz$$

<u>Free Vortex</u>

$$V_\theta = \frac{C}{r}$$

$$V_1 = \frac{C}{r_1}, \quad V_2 = \frac{C}{r_2}$$

$$\int_1^2 dp = \rho \int_1^2 \frac{c^2}{r^3}\, dr - w \int_1^2 dz$$

$$\boxed{\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2}$$

Bernoullie's eqn
(irrotational flow)

Forced Vortex Motion

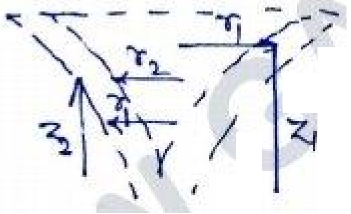
$$V = r\omega$$

$$V_1 = r_1\omega, \quad V_2 = r_2\omega$$

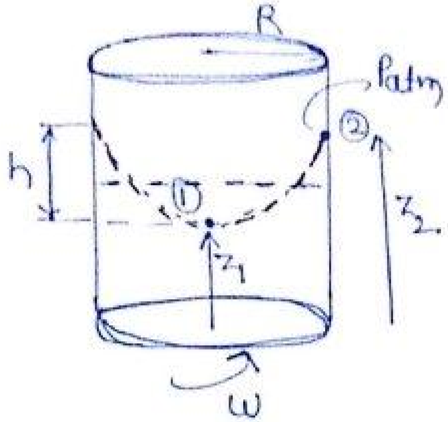
$$\int_1^2 dp = \rho\omega^2 \int_1^2 r\, dr - \rho g \int_1^2 dz$$

$$\boxed{\frac{P_1}{\rho g} - \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} - \frac{V_2^2}{2g} + z_2}$$

forced Vortex
motion eqn



Que Forced Vortex constant angular Velocity (Partially filled)



$$\frac{P_1}{\rho g} - \frac{r_1^2\omega^2}{2g} + z_1 = \frac{P_2}{\rho g} - \frac{r_2^2\omega^2}{2g} + z_2$$

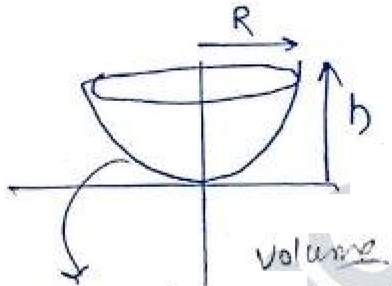
$$r_1 = 0 \;,\; r_2 = R$$

$$P_1 = P_2 = P_{atm}.$$

$$z_2 - z_1 = \frac{\omega^2 R^2}{2g}$$

$$\boxed{h = \frac{\omega^2 R^2}{2g}}$$

$h \propto r^2$



parabolide volume $V_p = \frac{\pi R^2 h}{2}$

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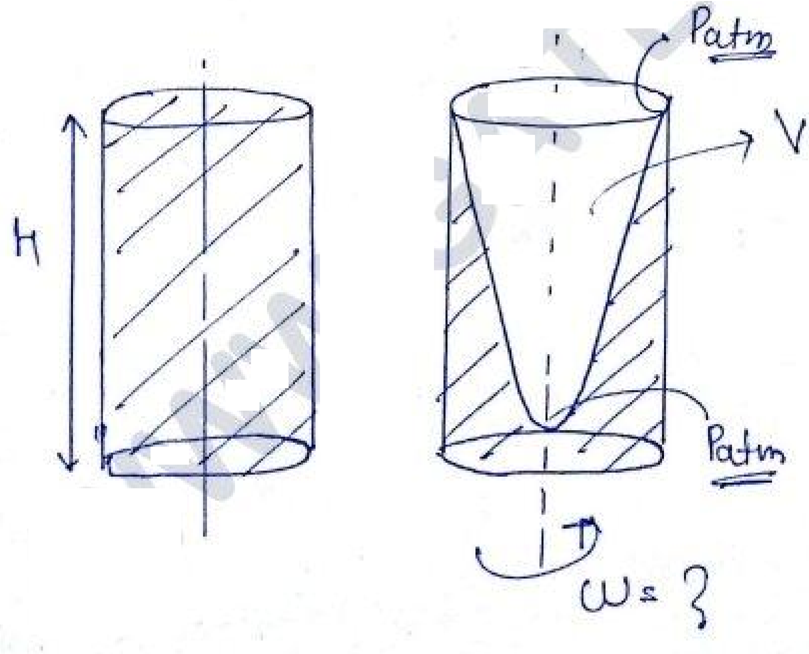
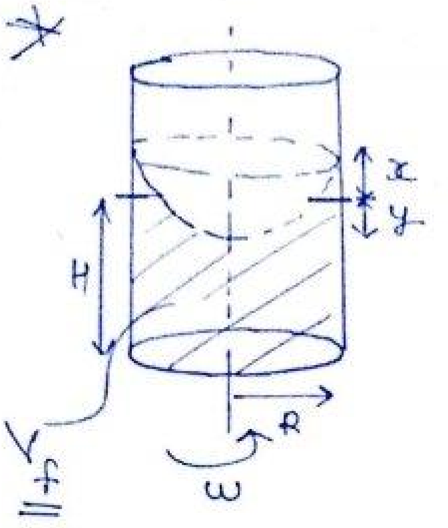


$$H = \frac{\omega^2 R^2}{2g}$$

$$\boxed{\omega = \sqrt{\frac{2gH}{R^2}}}$$

*



<u>No spillage</u>

$$V_i = \pi R^2 H$$

$$V_f = \pi R^2 (H+x) - \frac{1}{2}\pi R^2 (x+y)$$

$V_i = V_f$ No Spillage

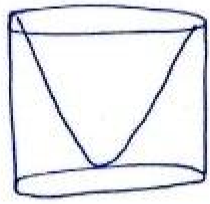
$$\pi R^2 H = \pi R^2 H + \pi R^2 x - \frac{1}{2}\pi R^2 x - \frac{\pi R^2}{2} y$$

$$\boxed{x = y}$$

<u>Ques 26</u>

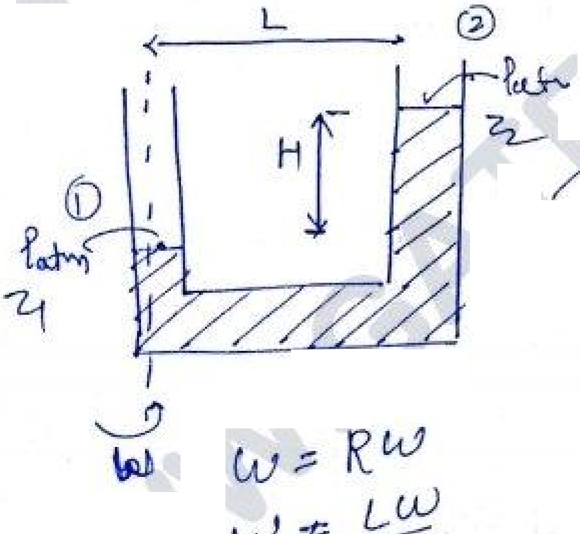
<u>Pg 31</u> <u>(c)</u>

Pg 31 <u>27(a)</u>



$V_p = \frac{1}{2} V_c$

<u>Q28</u>



$r_1 = 0$

$$\cancel{\frac{p_1}{w}} - \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{p_2}{w}} - \frac{V_2^2}{2g} + z_2$$

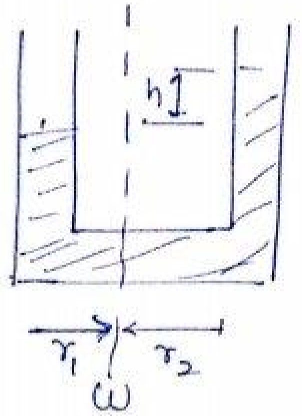
$$z_2 - z_1 = H = \frac{\omega^2 R^2}{2g}$$

$w = R\omega$

$w = \underline{L\omega}$

$$H = \frac{\omega^2 L^2}{2g}$$

<u>if</u> axis of rotation change



$v_1 = r_1 \omega$

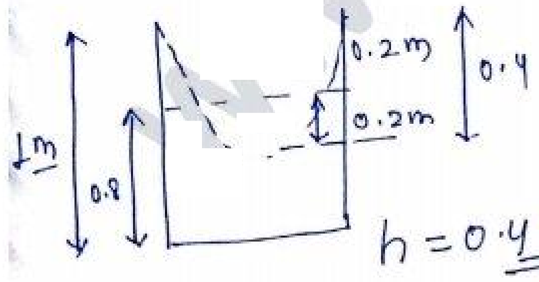
$v_2 = r_2 \omega$

$$\cancel{\frac{P_1}{\rho g}} - \frac{(r_1 \omega)^2}{2g} + z_1 = \cancel{\frac{P_2}{\rho g}} - \frac{(r_2 \omega)^2}{2g} + z_2$$

<u>Q.25</u>

$$H = \frac{\omega^2 R^2}{2g} \qquad \omega = \sqrt{\frac{2gH}{R^2}}$$

$$\omega = \sqrt{\frac{2 \times 9.81 \times 0.4}{(0.3/2)^2}}$$

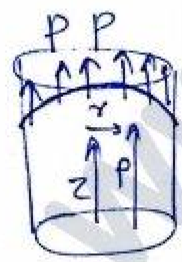


$$\omega = 18.65 \text{ rad/s}$$

$h = 0.4 \text{ m}$

*

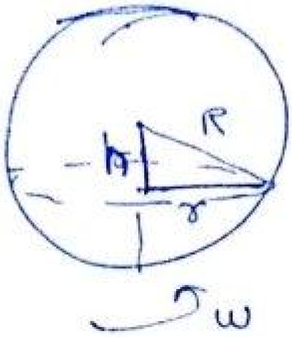
$$\cancel{\frac{P_1}{\rho g}} - \cancel{\frac{r_1^2 \omega^2}{2g}} + \cancel{z_1} = \frac{P}{\rho g} - \frac{r_1^2 \omega^2}{2g} + z_2$$



$$P/\rho g = \frac{r^2 \omega^2}{2g} \qquad \boxed{P = \frac{\rho r^2 \omega^2}{2}}$$

$P \propto r^2$

+



$$P = \rho g(h+R) + \frac{\rho r^2 \omega^2}{2}$$

$$P = \rho g(h+R) + \frac{\rho (R^2 - h^2) \omega^2}{2}$$

find h when P_{max}