Plane Geometry

Section 1 Lines and Angles

Line A geometrical straight line is a set of points that extends endlessly in both the directions.

Axiom-1 A line contains infinitely many points.

Axiom-2 Through a given point, infinitely many lines pass.

Axiom-3 Given two distinct points A and B, there is one and only one line that contains both the points.

Parallel Lines If two lines have no point in common, they are said to be *parallel lines*

Intersecting Lines If two lines have a point in common, they are said to be *intersecting lines*. Two lines can intersect at the most at one point.

Line Segment and Ray A part (or portion) of a line with two end points is called a *line segment* and a part of a line with one end point is called a *ray*. A line segment $\overline{AB}$ and its length is denoted as AB. Ray AB (i.e., A towards B) is denoted as $\overrightarrow{AB}$ and ray BA (i.e., B towards A) is denoted as $\overrightarrow{BA}$.

Collinear Points Three or more than three points are said to be *collinear* if there is a line which contains them all.

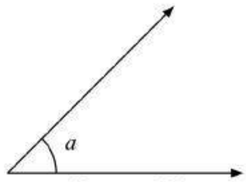
Concurrent Lines Three or more than three lines are said to be *concurrent* if there is a point which lies on all of them.

Angle An *angle* is a figure formed by two rays with a common initial point. The two rays forming an angle are called *arms* of the angle and the common initial point is called *vertex* of the angle.

Types of Angles

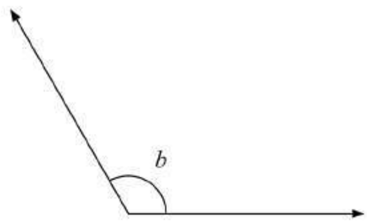
An angle is said to be:

(i) *Acute*, if $a < 90°$.

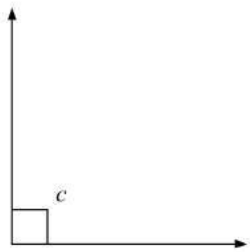


$0° < a < 90°$

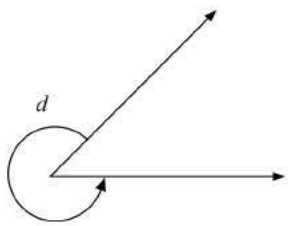
(ii) *Obtuse*, if $90° < b < 180°$.



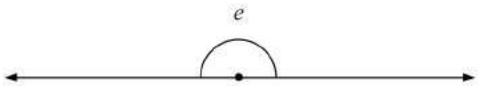
(iii) *Right angle*, if $c = 90°$



(iv) *Reflex angle*, if $180° < d < 360°$



(v) *Straight angle*, if $e = 180°$



(vi) *Complete angle*: An angle whose measure is 360°, is called a *complete angle*.

Complementary Angles Two angles, the sum of whose measures is 90°, are called *complementary angles,* e.g. 50° and 40° is a pair of complementary angles.

Supplementary Angles Two angles, the sum of whose measures is 180°, are called *supplementary angles*, e.g. 72° and 108° is a pair of supplementary angles.

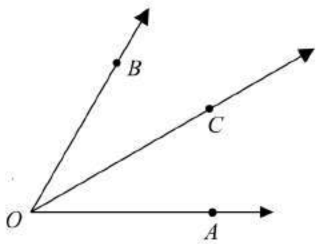
Adjacent Angles Two angles are called *adjacent angles* if

(i) they have the same vertex.

(ii) they have a common arm.

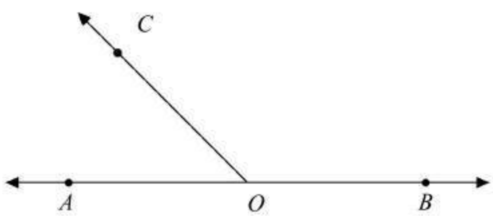
(iii) uncommon arms are on either side of the common arm.

E.g. $\angle AOC$ and $\angle BOC$ are adjacent angles.



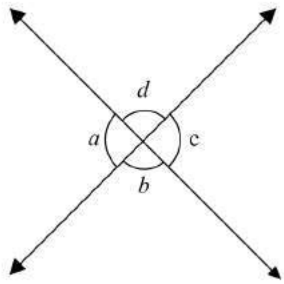
Linear Pair: Two adjacent angles are said to form a linear pair of angles if their non-common arms are two opposite rays.

E.g. $\angle AOC$ and $\angle BOC$ form a linear pair.



Linear Pair Axiom: If a ray stands on a line, then the sum of the two adjacent angles so formed is 180°. Conversely, if the sum of two adjacent angles is 180°; then the non-common arms of the angles are two opposite rays.

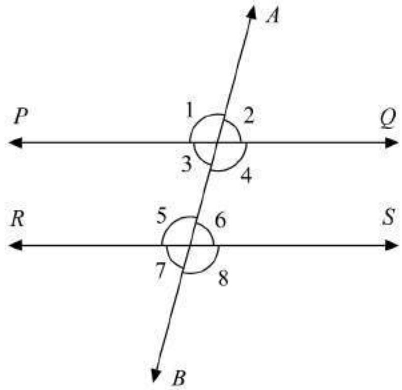
Vertically Opposite Angles: When two lines intersect, four angles are formed. The angles opposite to each other are called *vertically opposite angles*.



a and c are vertically opposite angles, $\angle a = \angle c$.

b and d are vertically opposite angles, $\angle b = \angle d$.

Angles made by a transversal* with two parallel lines Suppose $PQ \parallel RS$ and a transversal AB cuts them, then



(*a*) Pair of corresponding angles are
(1 and $\angle 5$), ($\angle 2$ and $\angle 6$),
($\angle 4$ and $\angle 8$) and ($\angle 3$ and $\angle 7$)

(*b*) Pair of alternate angles are
($\angle 3$ and $\angle 6$) and ($\angle 4$ and $\angle 5$)

(*c*) Pair of interior angles (consecutive interior angles or cointerior angles) on the same side of the transversal are
($\angle 3$ and $\angle 5$) and ($\angle 4$ and $\angle 6$)

KEY RESULTS TO REMEMBER

If two parallel lines are intersected by a transversal, then

(i) each pair of corresponding angles are equal.

(ii) each pair of alternate angles are equal.

(iii) interior angles on the same side of the transversal are supplementary.

*A line which intersects two or more lines at distinct points is called a transversal of the given lines.

SECTION 2 TRIANGLES

Triangle A plane figure bounded by three lines in a plane is called a *triangle*.

Types of Triangles (On the basis of sides)

Scalene triangle A triangle two of whose sides are equal is called a *scalene triangle*.

Isosceles triangle A triangle two of whose sides are equal in length is called an *isosceles triangle*.

Equilateral triangle A triangle all of whose sides are equal is called an *equilateral triangle*.

Types of Triangles (On the basis of angles)

Acute triangle A triangle, each of whose angle is acute, is called an *acute triangle* or *acute-angled triangle*.

Right triangle A triangle with one right angle is called a *right triangle* or a *right-angled triangle*.

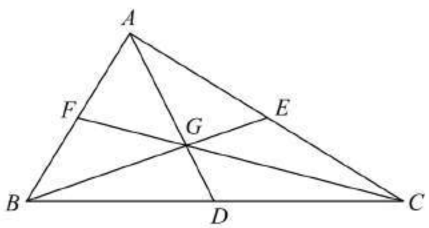
Obtuse triangle A triangle with one angle an obtuse angle, is known as *obtuse triangle* or *obtuse-angled triangle*.

Some Important Terms Related to a Triangle

1. **Median** The median of a triangle corresponding to any side is the line segment joining the midpoint of that side with the opposite vertex.

   In the figure given below, AD, BE and CF are the medians.

   The medians of a triangle are concurrent i.e., they intersect each other at the same point.

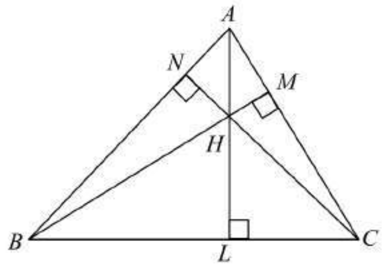


2. **Centroid** The point of intersection of all the three medians of a triangle is called its *centroid*.

   In the above figure G is the centroid of ΔABC.

   Note: The centroid divides a median in the ratio 2:1.

3. **Altitudes** The *altitude* of a triangle corresponding to any side is the length of perpendicular drawn from the opposite vertex to that side.



In the figure given above, AL, BM and CN are the altitudes.

Note: The altitudes of a triangle are concurrent.

4. **Orthocentre** The point of intersection of all the three altitudes of a triangle is called its *orthocentre*.

   In the figure given above H is the orthocentre of ΔABC.

   Note: The orthocentre of a right-angled lies at the vertex containing the right angle.

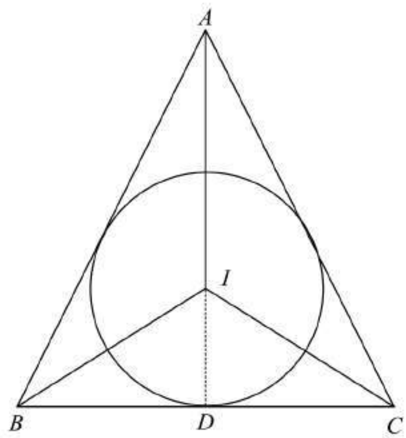
5. **Incentre of a triangle** The point of intersection of the internal bisectors of the angles of a triangle is called its *incentre*.

   In the figure given below, the internal bisectors of the angles of ΔABC intersect at I.

   $\therefore$ I is the Incentre of ΔABC.

   Let, $ID \perp BC$

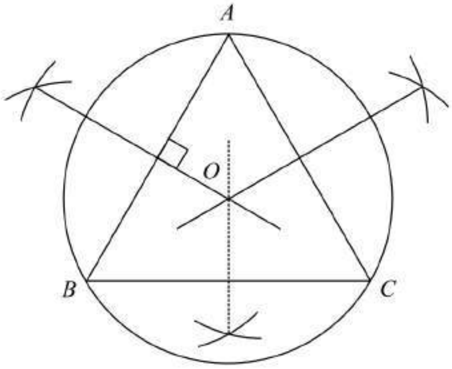
   Then, a circle with centre I and radius ID is called the *incircle* of ΔABC.



Note: The incentre of a triangle is equidistant from its sides.

6. **Circumcentre of a triangle** The point of intersection of the perpendicular bisectors of the sides of a triangle is called its *circumcentre*.

In the figure given below, the right bisectors of the sides of ΔABC intersect at O.



$\therefore$ O is the *circumcentre* of ΔABC with O as centre and radius equal to $OA = OB = OC$. We draw a circle passing through the vertices of the given Δ.

This circle is called the *circumcircle* of ΔABC.

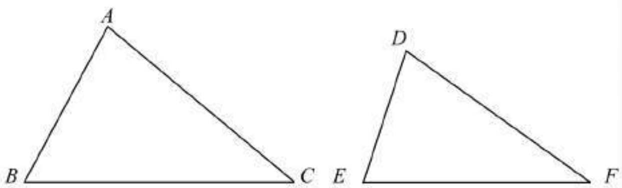
Note: The circumcentre of a triangle is *equidistant* from its *vertices*.

CONGRUENT TRIANGLES

Two triangles are *congruent* if and only if one of them can be superposed on the other, so as to cover it exactly.

Thus, congruent triangles are exactly identical

For example, If $\Delta ABC \cong \Delta DEF$ then we have



$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F;$

and $AB = DE, BC = EF$ and, $AC = DF$.

Similar Triangles

Congruent figures Two geometric figures having the same shape and size are known as *congruent figures*.

Similar figures Two figures (plane or solid) are said to be *similar* if they have the same shape irrespective of their sizes.

Note: Two similar figures may not be congruent as their size may be different.

For examples,

1. Any two lines segments are similar.
2. Any two equilateral triangles are similar.
3. Any two squares are similar.
4. Any two circles are similar.
5. Any two rectangles are similar.

Similar triangles Two triangles are similar if

(*a*) their corresponding angles are equal.

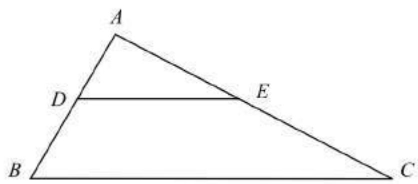
(*b*) their corresponding sides are proportional.

KEY RESULTS TO REMEMBER

1. The sum of all the angles round a point is equal to 360°.
2. Two lines parallel to the same line are parallel to each other.
3. The sum of three angles of a triangle is 180°.
4. If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles. (Exterior Angle Theorem)
5. If two sides of a triangle are unequal, the longer side has greater angle opposite to it.
6. In a triangle, the greater angle has the longer side opposite to it.
7. The sum of any two sides of a triangle is greater than the third side.
8. If a, b, c denote the sides of a triangle then
   (i) If $c^2 < a^2 + b^2$, triangle is acute angled.
   (ii) If $c^2 = a^2 + b^2$, triangle is right angled.
   (iii) If $c^2 > a^2 + b^2$, triangle is obtuse angled.
9. Two triangles are congruent if:
   (i) Any two sides and the included angle of one triangle are equal to any two sides and the included angle of the other triangle.
   (SAS congruence theorem)
   (ii) Two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.
   (ASA congruence theorem)
   (iii) The three sides of one triangle are equal to the corresponding three sides of the other triangle.
   (SSS congruence theorem)

> *Note:* Two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and the corresponding side of the other triangle.
>
> (RHS Congruence theorem)
>
> **10.** The line segments joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.
>
> **11. Basic Proportionality Theorem** If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

In the figure given below, In a ΔABC



If $DE \parallel BC$

Then, $\frac{AD}{DB} = \frac{AE}{EC}$

Illustration 1 In the figure given above, D and E are the points on the AB and AC respectively such that $DE \parallel BC$. If $AD = 8$ cm, $AB = 12$ cm and $AE = 12$ cm. Find CE

Solution: In ΔABC, $DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC} \text{ (Basic Proportionality Theorem)}$$

$$\Rightarrow \frac{8}{12-8} = \frac{12}{EC}$$

$$\Rightarrow \frac{8}{4} = \frac{12}{EC}$$

or $EC = 6$ cm

> **12.** If a line divides any two sides of a triangle in the same ratio, the line is parallel to the third side.

Explanation In the above figure (given in point 11). In ΔABC

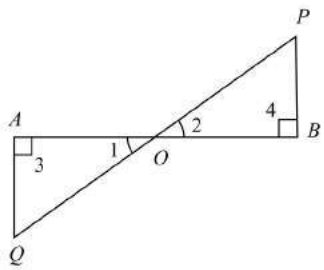
if $\frac{AD}{DB} = \frac{AE}{EC}$, then $DE \parallel BC$

Similarity Theorems

> **13. AAA Similarity** If in two triangles, corresponding angles are equal, then the triangles are similar.
>
> **Corollary** (AA-similarity): If two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar.

Illustration 2 In the figure given below, QA and PB are perpendiculars to AB. If $AO = 15$ cm, $BO = 9$ cm, $PB = 12$ cm, find AQ.

Solution:



In Δs AOQ and BOP

$\angle 1 = \angle 2$ [vertically opposite angles]

$\angle 3 = \angle 4$ [each 90°]

$\therefore \Delta AOQ \sim \Delta BOP$ [AA Similarity Criterion]

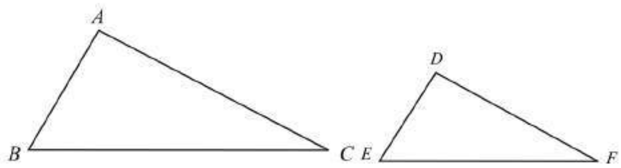
$$\therefore \frac{AO}{BO} = \frac{AQ}{BP} \text{ (corresponding sides of } \sim\Delta\text{s)}$$

or $\frac{15}{9} = \frac{AQ}{12}$

or $\frac{5}{1} = \frac{AQ}{4} \Rightarrow AQ = 20$ cm

> **14. SSS-Similarity** If the corresponding sides of two triangles are proportional then they are similar.

Explanation In Δs ABC and DEF,

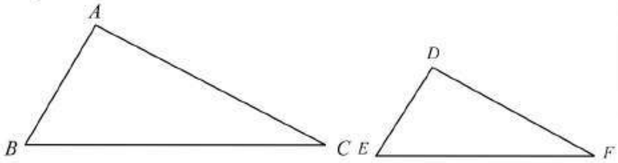


if $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Then, $\Delta ABC \sim \Delta DEF$ [SSS Similarity]

15. SAS-Similarity If in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.

Explanation In Δs ABC and DEF,



if $\angle A = \angle D$ and $\dfrac{AB}{DE} = \dfrac{AC}{DF}$

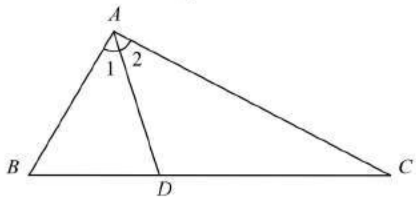
or $\angle B = \angle E$ and $\dfrac{AB}{DE} = \dfrac{BC}{EF}$

or $\angle C = \angle F$ and $\dfrac{AC}{DF} = \dfrac{BC}{EF}$,

then, $\Delta ABC \sim \Delta DEF$ [SAS-Similarity]

16. Internal Bisector Property The internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle.

Explanation In ΔABC, if $\angle 1 = \angle 2$

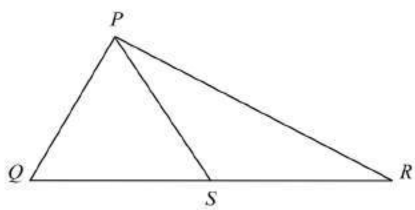


Then, $\dfrac{AB}{AC} = \dfrac{BD}{CD}$

17. If a line segment drawn from the vertex of an angle of a triangle to its opposite side divides it in the ratio of the sides containing the angle, then the line segment bisects the angle.

Illustration 3 In ΔPQR, $PQ = 6$ cm, $PR = 8$ cm, $QS = 1.5$ cm, $RS = 2$ cm

Solution:



$\therefore \dfrac{PQ}{PR} = \dfrac{6}{8} = \dfrac{3}{4}$ and $\dfrac{QS}{RS} = \dfrac{1.5}{2} = \dfrac{3}{4}$

Thus, $\dfrac{PQ}{PR} = \dfrac{QS}{RS}$

$\therefore$ PS is the bisector of $\angle P$

18. Pythagoras Theorem In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Explanation In a right ΔABC, right angled at B

$$AC^2 = AB^2 + BC^2$$

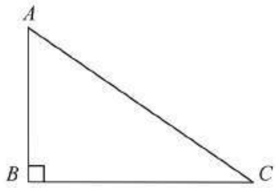
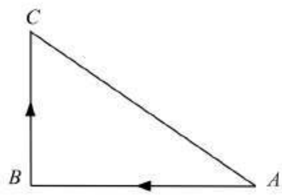


Illustration 4 A man goes 15 m west and then 8 m due north. How far is he from the starting point.

Solution: Let the initial position of the man be A.



Let, $AB = 15$ m and $BC = 8$ m

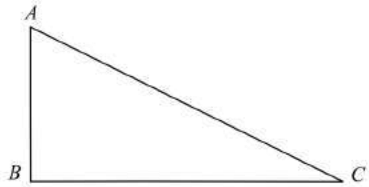
$$\begin{aligned} \therefore \quad AC^2 &= AB^2 + BC^2 \text{ (Pythagoras Theorem)} \\ &= (15)^2 + (8)^2 \\ &= 225 + 64 \\ &= 289 \\ AC &= \sqrt{289} \\ &= 17 \text{ m} \end{aligned}$$

Hence, the man is 17 m away from the starting point.

19. Converse of Pythagoras Theorem. In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

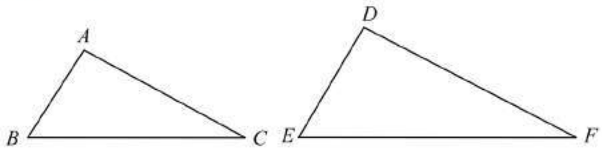
Explanation In a ΔABC if $AB^2 + BC^2 = AC^2$

Then, $\angle ABC = 90°$



> **20. Area Theorem** The ratio of the areas of two similar Δs is equal to the ratio of the squares of any two corresponding sides

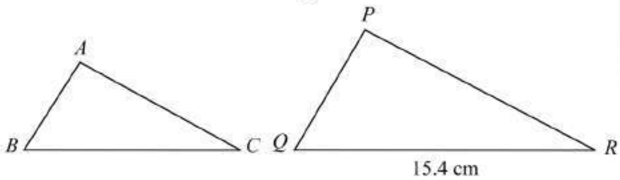
Explanation If $\Delta ABC \sim \Delta DEF$,



then, $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AB^2}{DE^2} = \dfrac{AC^2}{DF^2} = \dfrac{BC^2}{EF^2}$

Illustration 5 The areas of two similar Δs ABC and PQR are 64 cm^2 and 121 cm^2, respectively. If $QR = 15.4$ cm, find BC.

Solution: Since $\Delta ABC \sim \Delta PQR$



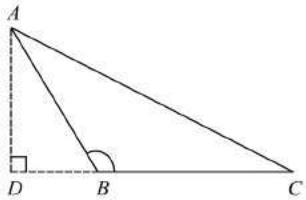
$\therefore \dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{BC^2}{QR^2}$ (Area Theorem)

i.e., $\dfrac{64}{121} = \dfrac{BC^2}{(15.4)^2} \Rightarrow \dfrac{8}{11} = \dfrac{BC}{15.4}$

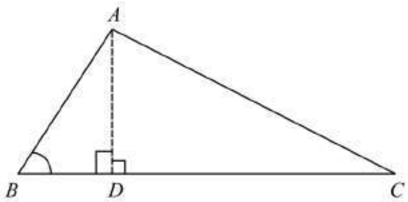
$\therefore BC = 11.2$ cm

> **21.** The ratio of the areas of two similar triangles is equal to the
> (i) ratio of the squares of the corresponding medians
> (ii) ratio of the squares of the corresponding altitudes
> (iii) ratio of the squares of the corresponding angle bisector segments
>
> **22.** If two similar triangles have equal areas, then the Δs are congruent.

> **23.** In two similar triangles, the ratio of two corresponding sides is same as the ratio of their perimeters.
>
> **24. Obtuse Angle Property** In a ΔABC, if $\angle B$ is obtuse then, $AC^2 = AB^2 + BC^2 + 2\, BC \times BD$ where $AD \perp BC$

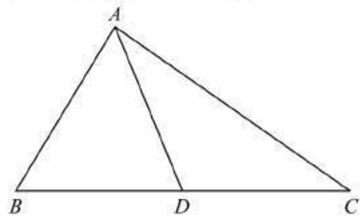


> **25. Acute Angle Property** In a ΔABC, if $\angle C$ is acute, then, $AB^2 = AC^2 + BC^2 - 2BC \times CD$ where $AD \perp BC$



> **26. Apollonius Theorem** The sum of the squares on any two sides of a triangle is equal to the sum of twice the square of the median, which bisects the third side and half the square of the third side.

Explanation In the given ΔABC,



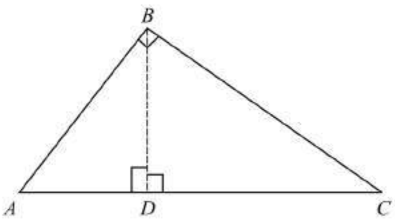
$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$$

or $$AB^2 + AC^2 = 2[AD^2 + BD^2]$$

> **27.** If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other. Also the square of the perpendicular is equal to the product of the lengths of the two parts of the hypotenuse.

Explanation In the figure given below,

ABC is a right triangle, right angled at B and $BD \perp AC$, then



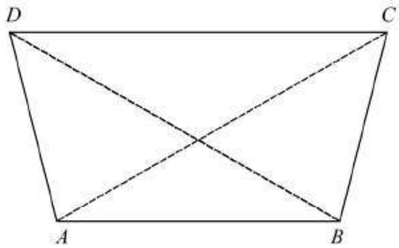
(i) $\Delta ADB \sim \Delta ABC$ (AA Similarity)

(ii) $\Delta BDC \sim \Delta ABC$ (AA Similarity)

(iii) $\Delta ADB \sim \Delta BDC$ also $BD^2 = AD \times CD$

SECTION 3 QUADRILATERALS AND PARALLELOGRAMS

Quadrilateral A plane figure bounded by four line segments AB, BC, CD and DA is called a *quadrilateral*, written as quad. $ABCD$ or $\angle ABCD$.



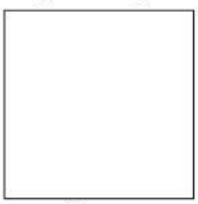
Various types of Quadrilaterals

(i) Parallelogram A quadrilateral in which opposite sides are parallel is called *parallelogram*, written as $\|_{gm}$.

(ii) Rectangle A parallelogram each of whose angles is 90° is called a *rectangle*, written as rect. $ABCD$.

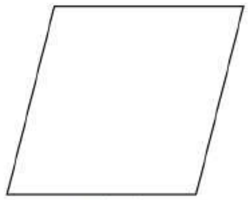
Rectangle

(iii) Square A rectangle having all sides equal is called a *square*.



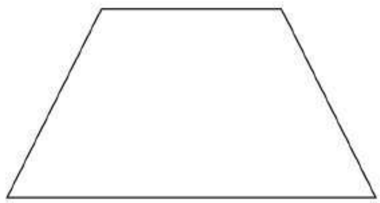
Square

(iv) Rhombus A quadrilateral having all sides equal is called a *rhombus*



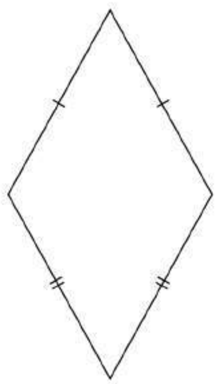
Rhombus

(v) Trapezium A quadrilateral in which two opposite sides are parallel and two opposite sides are non-parallel is called a *trapezium*.



Trapezium

(vi) Kite A quadrilateral in which pairs of adjacent sides are equal is known as *kite*.



Key Results to Remember

1. The sum of all the four angles of a quadrilateral is 360°.
2. In a parallelogram
   (i) opposite sides are equal.
   (ii) opposite angles are equal.
   (iii) each diagonal bisects the parallelogram.
   (iv) the diagonal bisect each other.
3. A quadrilateral is a $\|_{gm}$
   (i) if both pairs of opposite sides are equal.
   or (ii) if both pairs of opposite angles are equal.
   or (iii) if the diagonals bisect each other.
   or (iv) if a pair of opposite sides are equal and parallel.
4. The diagonals of a rectangle are equal.
5. If the diagonals of a $\|_{gm}$ are equal, it is a rectangle.
6. Diagonals of a rhombus are perpendicular to each other.
7. Diagonals of a square are equal and perpendicular to each other.
8. The figure formed by joining the mid-points of the pairs of consecutive sides of a quadrilateral is a $\|_{gm}$.
9. The quadrilateral formed by joining the mid-points of the consecutive sides of a rectangle is a rhombus.
10. The quadrilateral formed by joining the mid-points of the consecutive sides of a rhombus is a rectangle.
11. If the diagonals of a quadrilateral are perpendicular to each other, then the quadrilateral formed by joining the mid-points of its sides, is a rectangle.
12. The quadrilateral formed by joining the mid-points of the sides of a square, is also a square.

Section 4 Polygons

Polygon A closed plane figure bounded by line segments is called a *polygon.*

The line segments are called its *sides* and the points of intersection of consecutive sides are called its *vertices*. An angle formed by two consecutive sides of a polygon is called an *interior angle* or simply an *angle* of the polygon.

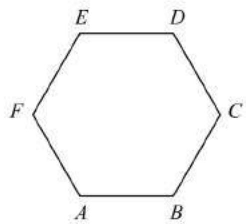
No. of sides	*Name*
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	
10	Decagon

A polygon is named according to the number of sides, it has.

In general, a polygon of n sides is called *n-gon.* Thus, a polygon having 18 sides is called 18-gon.

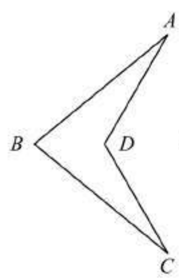
Diagonal of a Polygon Line segment joining any two non-consecutive vertices of a polygon is called its *diagonal*.

Convex Polygon If all the (interior) angles of a polygon are less than 180°, it is called a *convex polygon*. In the figure given below, *ABCDEF* is a convex polygon. In fact, it is a convex hexagon.



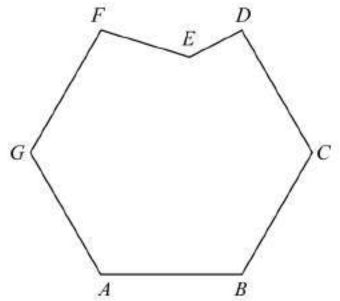
(In other words, a polygon is a convex polygon if the line segment joining any two points inside it lies completely inside the polygon).

Concave Polygon If one or more of the (interior) angles of a polygon is greater than 180° i.e., reflex, it is called

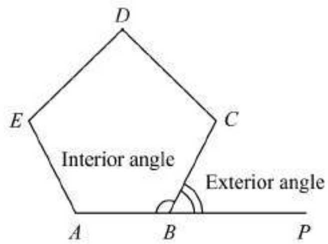


⇐ Concave polygon

concave (or re-entrant) *polygon* In the figure given below, *ABCDEFG* is a concave polygon. In fact, it is a concave heptagon.



Exterior Angle of Convex Polygon If we produce a side of polygon, the angle it makes with the next side is called an *exterior angle*. In the diagram given below, *ABCDE* is a pentagon. Its side *AB* has been produced to *P*, then $\angle CBP$ is an exterior angle.



Note: Corresponding to each interior angle, there is an exterior angle. Also, as an exterior angle and its adjacent interior angle make a straight line, we have **an exterior angle + adjacent interior angle = 180°**

Regular Polygon A polygon is called regular polygon if all of its sides have equal length and all its angles have equal size.

Thus, in a regular polygon

(i) all sides are equal in length.

(ii) all interior angles are equal in size.

(iii) all exterior angles are equal size.

Note: All regular polygons are convex.

KEY RESULTS TO REMEMBER

1. (*a*) If there is a polygon of *n* sides ($n \geq 3$), we can cut it into $(n-2)$ triangles with a common vertex and so *the sum of the interior angles of a polygon of n sides would be*

$$(n-2) \times 180° = (n-2) \times 2 \text{ right angles} = (2n-4) \text{ right angles}$$

(*b*) If there is a regular polygon of n sides ($n \geq 3$), then its each interior angle is equal to

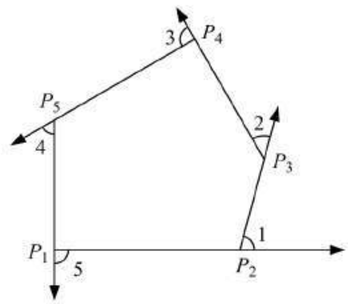
$$\left(\frac{2n-4}{n} \times 90\right)$$

(*c*) Each exterior angle of a regular polygon of *n* sides is equal to

$$= \left(\frac{360}{n}\right)^{\circ}$$

2. The sum of all the exterior angles formed by producing the sides of a convex polygon in the same order is equal to four right angles.

Explanation If in a convex polygon $P_1P_2P_3P_4P_5$, all the sides are produced in order, forming exterior angles $\angle 1, \angle 2, \angle 3, \angle 4$ and $\angle 5$, then $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 =$ 4 right angles.



3. If each exterior angle of a regular polygon is $x°$, then the number of sides in the polygon $= \frac{360°}{x}$.

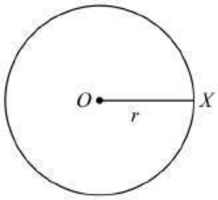
Note: Greater the number of sides in a regular polygon, greater is the value of its each interior angle and smaller is the value of each exterior angle.

4. If a polygon has *n* sides, then the number of diagonals of the polygon

$$= \frac{n(n-1)}{2} - n.$$

Section 5 Circles and Tangents

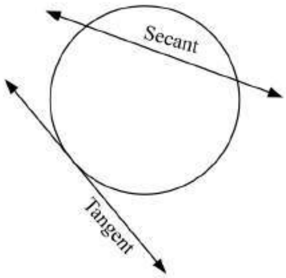
Circle A circle is a set of all those points in a plane, each one of which is at given constant distance from a given fixed point in the plane.



The fixed point is called the *centre* and the given constant distance is called the *radius* of the circle.

A circle with centre O and radius r is usually denoted by $C(O, r)$.

Tangent A line meeting a circle in only one point is called a *tangent* to the circle. The point at which the tangent line meets the circle is called the *point of contact*.

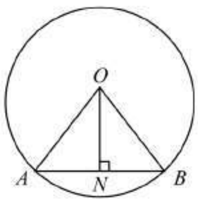


Secant A line which intersects a circle in two distinct points is called a *secant line*.

Key Results to Remember

1. The perpendicular from the centre of a circle to a chord bisects the chord.

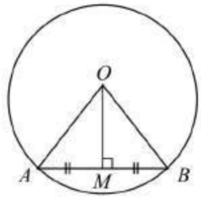
Explanation If $ON \perp AB$, then $AN = NB$.



Note: The converse of above theorem is true and can be stated as point 2.

2. The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.

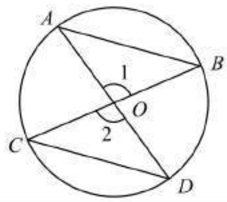
Explanation If $AM = MB$, then $OM \perp AB$.



Cor. The perpendicular bisectors of two chords of a circle intersect at its centre.

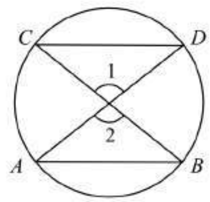
3. Equal chords of a circle subtend equal angles at the centre.

Explanation If $AB = CD$, then $\angle 1 = \angle 2$



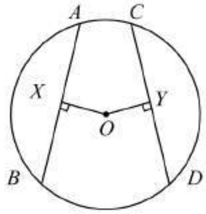
4. (Converse of above theorem) If the angles subtended by two chords at the centre of a circle are equal then the chords are equal.

Explanation If $\angle 1 = \angle 2$, then $AB = CD$



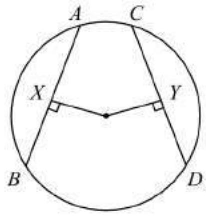
5. Equal chords of a circle are equidistant from the centre.

Explanation If the chords AB and CD of a circle are equal and if $OX \perp AB$ and $OY \perp CD$ then $OX = OY$.



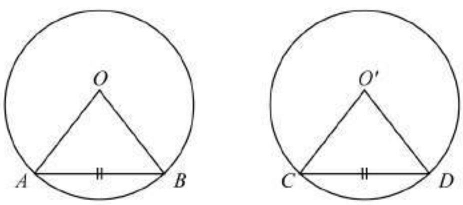
> **6.** (Converse above theorem) Chords equidistant from the centre of the circle are equal.

Explanation If $OX \perp AB$ $OY \perp CD$ and $OX = OY$, then chords $AB = CD$



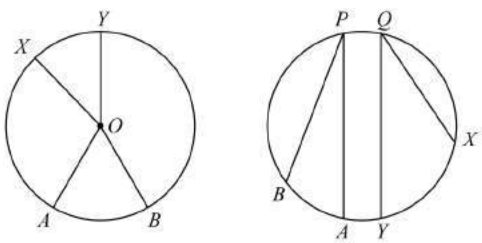
> **7.** In equal circles (or in the same circle), equal chords cut of equal arcs.

Explanation If the chords $AB = CD$, then arc AB = are CD.



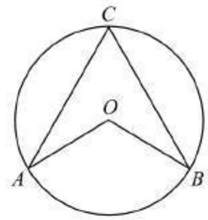
> **8.** In equal circles (or in the same circle) if two arcs subtend equal angles at the centre (or at the circum-ference), the arcs are equal.

Explanation If $\angle BOA = \angle XOY$, then arc AB = arc XY or if $\angle BPA = \angle XQY$, then arc AB = arc XY.



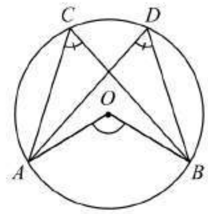
> **9.** The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.
>
> (The theorem is popularly known as Degree Measure Theorem).

Explanation A circle, centre O, with $\angle AOB$ at the centre, $\angle ACB$ at the circumference, standing on the same arc AB, then $\angle AOB = 2\angle ACB$



> **10.** Angles in the same segment of a circle are equal.

Explanation A circle, centre O, $\angle ACB$ and $\angle ADB$ are angles at the circumference, standing on the same arc, then

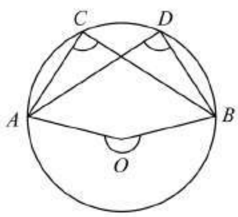


$\angle ACB = \angle ADB$

(angles in same arc)

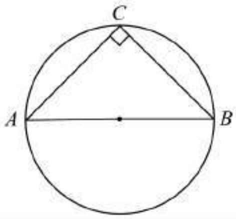
or

(angles in same segment)



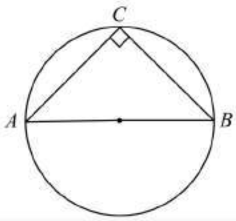
> **11.** The angle in a semicircle is a right angle.

Explanation In the figure given below $\angle ACB = 90°$

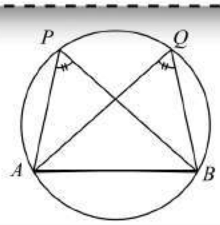


> **12.** (Converse of above theorem) The circle drawn with hypotenuse of a right triangle as diameter passes through its opposite vertex.

Explanation The circle drawn with the hypotenuse *AB* of a right triangle *ACB* as diameter passes through its opposite vertex *C*.



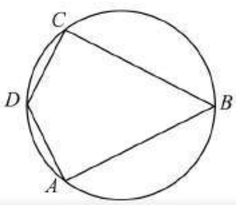
> **13.** If $\angle APB = \angle AQB$, and if *P*, *Q* are on the same side of *AB*, then *A*, *B*, *Q*, *P* are concyclic i.e., lie on the same circle.



> **14.** The sum of the either pair of the opposite angles of a cyclic quadrilateral is 180°.

Explanation If *ABCD* is a cyclic quadrilateral,

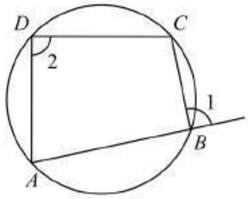
then $\angle A + \angle C = \angle B + \angle D = 180°$



> **15.** (Converse of above theorem) If the two angles of a pair of opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.

> **16.** If a side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

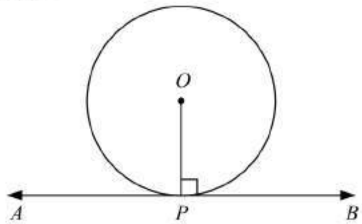
Explanation If the side *AB* of a cyclic quadrilateral *ABCD* is produced then $\angle 1 = \angle 2$.



THEOREMS ON TANGENTS

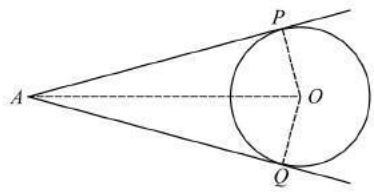
> **17.** A tangent at any point of a circle is perpendicular to the radius through the point of contact.

Explanation If *AB* is a tangent at a point *P* to a circle *C* (*O*, *r*) then $PO \perp AB$

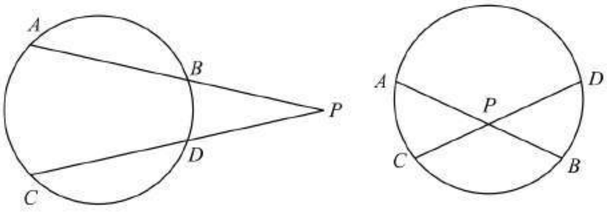


> **18.** (Converse of above theorem) A line drawn through the end of a radius and perpendicular to it, is a tangent to the circle.
>
> **19.** The lengths of two tangents drawn from an external point to a circle are equal.

Explanation If two tangents *AP* and *AQ* are drawn from a point A to a circle *C*(*O*, *r*), then $AP = AQ$

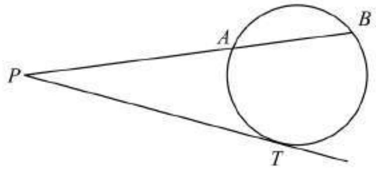


> **20.** If two chords *AB* and *CD* intersect internally (ii) or externally (i) at a point *P* then
>
> $$PA \times PB = PC \times PD$$

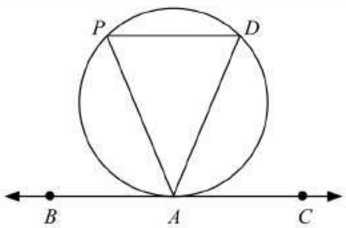


(i) (ii)

> **21.** If PAB is a secant to a circle intersecting the circle at A and B is a tangent segment then $PA \times PB = PT^2$ (refer the figure below).
>
> (popularly known as Tangent-Secant theorem)



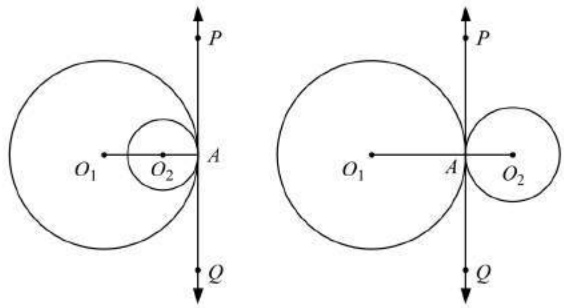
> **22.** Alternate Segment Theorem:
>
> In the figure below, if BAC is the tangent at A to a circle and if AD is any chord, then
>
> $\angle DAC = \angle APD$ and
>
> $\angle PAB = \angle PDA$
>
> (Angles in alternate segment)



Note: The converse of the above theorem is true.

> **23.** If two circles touch each other internally or externally, the point of contact lies on the line joining their centres.

Explanation If two circles with centre O_1 and O_2 which touch each other internally (i) or externally (ii), at a point A then the point A lies on the line $O_1 O_2$, i.e., three points A, O_1 and O_2 are collinear.

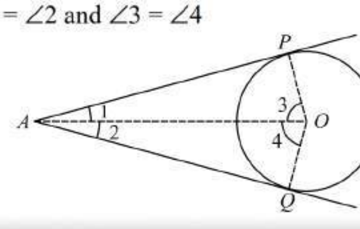


Some Useful Results

> 1. There is one and only one circle passing through three non-collinear points.
> 2. Two circles are congruent if and only if they have equal radii.
> 3. Of any two chords of a circle, the one which is greater is nearer to the centre.
> 4. Of any two chords of a circle, the one which is nearer to the centre is greater.
> 5. If two circles intersect in two points, then the line through the centres is the perpendicular bisector of the common chord.
> 6. Angle in a major segment of a circle is acute and angle in a minor segment is obtuse.
> 7. If two tangents are drawn to a circle from an external point then
>   (i) they subtend equal angles at the centre.
>   (ii) they are equally inclined to the segment, joining the centre to that point.

Explanation In a circle $C(O,r)$, A is a point outside it and AP and AQ are the tangents drawn to the circle

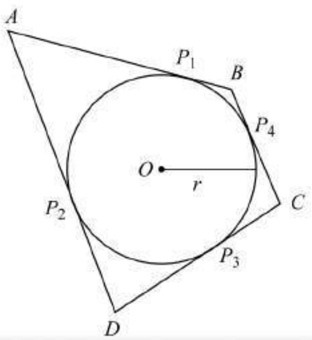
Then, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$



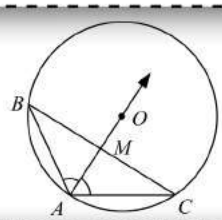
> 8. If a circle touches all the four sides of a quadrilateral then the sum of opposite pair of sides are equal.

Explanation If $ABCD$ is a circumscribed quadrilateral.

Then, $AB + CD = AD + BC$

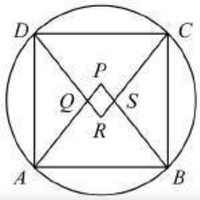


> **9.** If two chords AB and AC of a circle are equal, then the bisector of $\angle BAC$ passes through the centre O of the circle.



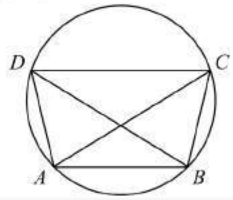
> **10.** The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.

Explanation If $ABCD$ is a cyclic quadrilateral in which AP, BP, CR and DR are the bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$, respectively, then quadrilateral $PQRS$ is also cyclic.



> **11.** A cyclic trapezium is isoceles and its diagonals are equal.

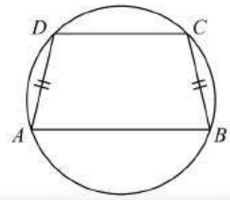
Explanation If ABC cyclic trapezium such that $AB \parallel DC$, then $AD = BC$ and $AC = BD$



> **12.** If two opposite sides of a cyclic quadrilateral are equal, then the other two sides are parallel.

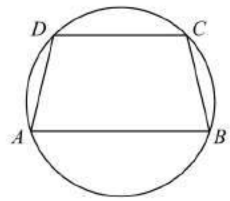
Explanation A cyclic quadrilateral $ABCD$ in which $AD = BC$

Then, $AB \parallel CD$



> **13.** An isoceles trapezium is always cyclic.

Explanation A trapezium $ABCD$ in which $AB \parallel CD$ and $AD = BC$



Then, $ABCD$ is a cyclic trapezium.

> **14.** Any four vertices of a regular pentagon are *concyclic* (lie on the same circle).

Practice Exercises

Difficulty Level-1
(Based on Memory)

1. The two sides of a right triangle containing the right angle measure 3 cm and 4 cm. The radius of the incircle of the triangle is:

(*a*) 3.5 cm (*b*) 1.75 cm
(*c*) 1 cm (*d*) 0.875 cm

[Based on MAT, 2003]

2. In a trapezium, one diagonal divides the other in the ratio 1:4. If the smaller of the two parallel sides is of length 3 cm, then the length of the other parallel side is:

(*a*) 9 cm (*b*) 12 cm
(*c*) 15 cm (*d*) None of these

3. The ratio of the sum of the squares of the sides of a triangle and that of the sum of the squares of its median is:

(*a*) 1:2 (*b*) 4:3

(*c*) 3:4 (*d*) 2:3

4. In a triangle ABC, the lengths of the sides AB, AC and BC are 3, 5 and 6 cm, respectively. If a point D on BC is drawn such that the line AD bisects the angle A internally, then what is the length of BD?

(*a*) 2 cm (*b*) 2.25 cm

(*c*) 2.5 cm (*d*) 3 cm

[Based on MAT, 2003]

5. In a triangle ABC, $\angle A = x^\circ$, $\angle B = y^\circ$ and $\angle C = (y + 20)^\circ$. If $4x - y = 10$, then the triangle is:

(*a*) Right angled (*b*) Obtuse angled

(*c*) Equilateral (*d*) None of these

[Based on MAT, 2003]

6. If one of the diagonals of a rhombus is equal to its side, then the diagonals of the rhombus are in the ratio:

(*a*) $\sqrt{3}:1$ (*b*) $\sqrt{2}:1$

(*c*) 3: 1 (*d*) 2:1

[Based on MAT, 2003]

7. If P and Q are the mid points of the sides CA and GB respectively of a triangle ABC, right angled at C. Then the value of $4\,(AQ^2 + BP^2)$ is equal to:

(*a*) $4\,BC^2$ (*b*) $5\,AB^2$

(*c*) $2\,AC^2$ (*d*) $2\,BC^2$

[Based on MAT, 2003]

8. In a quadrilateral $ABCD$, $\angle B = 90^\circ$ and $AD^2 = AB^2 + BC^2 + CD^2$, then $\angle ACD$ is equal to:

(*a*) 90° (*b*) 60°

(*c*) 30° (*d*) None of these

[Based on MAT, 2003]

9. $ABCD$ is a square, F is mid point of AB and E is a point on BC such that BE is one-third of BC. If area of $\Delta FBE = 108$ m^2, then the length of AC is:

(*a*) 63 m (*b*) $36\sqrt{2}$ m

(*c*) $63\sqrt{2}$ m (*d*) $72\sqrt{2}$ m

[Based on MAT, 2003]

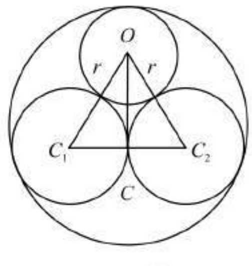
10. Two circles with radii 'a' and 'b' respectively touch each other externally. Let 'c' be the radius of a circle that touches these two circles as well as a common tagent to the two circles. Then:

(*a*) $\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$ (*b*) $\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}} = \frac{2}{\sqrt{c}}$

(*c*) $\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$ (*d*) None of these

[Based on MAT, 2002]

11. Two circles of unit radius touch each other and each of them touches internally a circle of radius two as shown in the following figure. The radius of the circle which touches all the three circles:

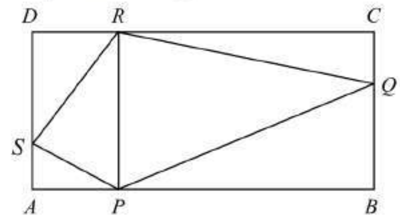


(*a*) 5 (*b*) $\frac{3}{2}$

(*c*) $\frac{2}{3}$ (*d*) None of these

[Based on MAT, 2002]

12. $ABCD$ is a parallelogram P, Q, R and S are points on sides AB, BC, CD and DA, respectively, such that $AP = DR$. If the area of the parallelogram $ABCD$ is 16 cm^2, then the area of the quadrilateral $PQRS$ is:



(*a*) 6 cm^2 (*b*) 6.4 cm^2

(*c*) 4 cm^2 (*d*) 8 cm^2

[Based on MAT, 2002]

13. Let ABC be an acute-angled triangle and CD be the altitude through C. If $AB = 8$ and $CD = 6$, then the distance between the mid-points of AD and BC is:

(*a*) 36 (*b*) 25

(*c*) 27 (*d*) 5

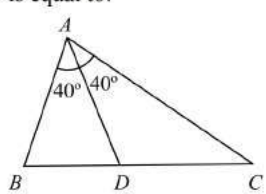
[Based on MAT, 2002]

14. The perimeters of two similar triangles ABC and PQR are 36 cm and 24 cm respectively. If $PQ = 10$ cm, the length of AB is:

(*a*) 16 cm (*b*) 12 cm

(*c*) 14 cm (*d*) 15 cm

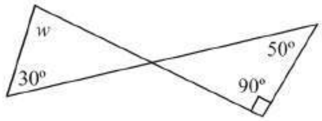
15. In the following figure, if $BC = 8$ cm, $AB = 6$ cm, $AC = 9$ cm, then DC is equal to:



(*a*) 7 cm (*b*) 4.8 cm
(*c*) 7.2 cm (*d*) 4.5 cm

[Based on MAT, 2001]

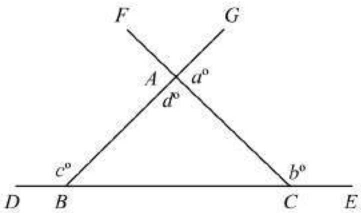
16. In the figure given below, what is the value of *w*?



(*a*) 100 (*b*) 110
(*c*) 120 (*d*) 130

Note: The diagram is not drawn to scale.

17. It is given that $d^o = 70^o$, $b^o = 120^o$. Then:



(*a*) $c^o = 130^o$ (*b*) $a^o = 110^o$
(*c*) Both (*a*) and (*b*) are correct
(*d*) Both (*a*) and (*b*) are wrong

18. The sum of the interior angles of a polygon is 1620°. The number of sides of the polygon are:

(*a*) 9 (*b*) 11
(*c*) 15 (*d*) 12

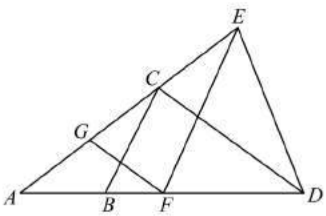
[Based on MAT, 2001]

19. How many sides a regular polygon has with its interior angle eight times its exterior angle?

(*a*) 16 (*b*) 24
(*c*) 18 (*d*) 20

[Based on MAT, 2001]

20. In the figure below, $AB = BC = CD = DE = EF = FG = GA$. Then $\angle DAE$ is approximately:



(*a*) 15° (*b*) 20°
(*c*) 30° (*d*) 25°

21. Find the distance of a perpendicular from the centre of a circle to the chord if the diameter of the circle is 30 cm and its chord is 24 cm.

(*a*) 6 cm (*b*) 7 cm
(*c*) 9 cm (*d*) 10 cm

[Based on I.P. Univ., 2002]

22. In a cyclic quadrilateral *ABCD*, $\angle A$ is double its opposite angle and the difference between the other two angles is one-third of $\angle A$. The minimum difference between any two angles of this quadrilateral is:

(*a*) 30° (*b*) 10°
(*c*) 20° (*d*) 40°

[Based on MAT (Sept), 2010]

23. Rama owns a piece of land in the shape of a right triangle. Its hypotenuse is 3 m more than twice the shortest side. If the perimeter of the piece of land is six times the shortest side, find the dimensions of the piece of land.

(*a*) 6, 15, 12 (*b*) 5, 12, 13
(*c*) 4, 9, 11 (*d*) None of these

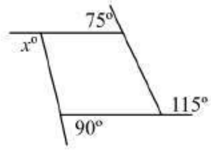
[Based on MAT (May), 2010]

24. An angle is equal to one-third of its supplement. Find its measure.

(*a*) 60° (*b*) 80°
(*c*) 90° (*d*) 45°

[Based on MAT (Sept), 2007]

25. The sides of a quadrilateral are extended to make the angles as shown below:



What is the value of *x*?

(*a*) 100 (*b*) 90
(*c*) 80 (*d*) 75

[Based on MAT, 1997]

26. The altitude of an equilateral triangle of side *a* is:

(*a*) $\frac{\sqrt{}}{}$ (*b*) $\frac{2a}{\sqrt{3}}$

(*c*) $\frac{a}{\sqrt{3}}$ (*d*) $a\sqrt{3}$

[Based on MAT, 1998]

27. Two isosceles triangles have equal vertical angles and their areas are in the ratio of 9:16. The ratio of their corresponding heights is:

(*a*) 1:2 (*b*) 2:3
(*c*) 3:4 (*d*) 4:5

[Based on MAT, 1998]

28. A circle of 30 cm diameter has a 24 cm chord. The distance of the perpendicular from the centre to the chord is:

(*a*) 9 cm (*b*) 5 cm

(*c*) 7 cm (*d*) 10 cm

[Based on MAT, 1998]

29. If the radius of a circle is tripled, then the ratio of the new circumference to the new diameter will be:

(*a*) $\frac{\pi}{2}$ (*b*) 9π

(*c*) 3π (*d*) π

[Based on MAT, 1998]

30. Two non-intersecting circles, one lying inside another, are of diameters *a* and *b*. The minimum distance between their circumferences is *c*. The distance between their centres is:

(*a*) $a - b - c$ (*b*) $a + b - c$

(*c*) $\frac{1}{2}(a - b - c)$ (*d*) $\frac{1}{2}(a - b) - c$

[Based on MAT, 1998]

31. A ladder reaches a window which is 12 m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 9 m high. Find the width of the street if the length of the ladder is 15 m.

(*a*) 21 m (*b*) 12 m

(*c*) 9 m (*d*) None of these

[Based on MAT, 1999]

32. A postman walks towards North a distance of 120 m to deliver a letter. He then goes towards East for a distance of 50 m for delivering another letter. The shortest distance between the two places is:

(*a*) 70 m (*b*) 120 m

(*c*) 130 m (*d*) 170 m

[Based on MAT, 1999]

33. In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. The distance between the chords is 23 cm. If length of one chord is 16 cm, then the length of the other is:

(*a*) 15 cm (*b*) 23 cm

(*c*) 30 cm (*d*) 34 cm

[Based on MAT, 1999]

34. The sides of a triangle measure 4 cm, 3.4 cm and 2.2 cm. Three circles are drawn with centres at *A*, *B* and *C* in such a way that each circle touches the other two. Then the diameters of these circles would measure (in cm):

(*a*) 1.11, 1.7, 5.0 (*b*) 1.6, 2.8, 5.2

(*c*) 1.5, 2.9, 5.2 (*d*) 1.6, 3.0, 5.0

[Based on MAT, 1999]

35. In a triangle *ABC*, $\angle A = 90°$ and *D* is mid-point of *AC*. The value of $BC^2 - BD^2$ is equal to:

(*a*) AD^2 (*b*) $2\ AD^2$

(*c*) $3\ AD^2$ (*d*) $4\ AD^2$

[Based on MAT, 2000]

36. *ABC* is a triangle with $\angle BAC = 60°$. A point *P* lies on one-third of the way from *B* to *C*, and *AP* bisects $\angle BAC$. $\angle APC =$:

(*a*) 30° (*b*) 45°

(*c*) 60° (*d*) 90°

37. A wire is in the form of a circle of radius 35 cm. If it is bent into the shape of a rhombus, what is the side of the rhombus?

(*a*) 32 cm (*b*) 70 cm

(*c*) 55 cm (*d*) 17 cm

[Based on MAT, 2000]

38. *ABC* is a triangle with $\angle CAB = 15°$ and $\angle ABC = 30°$. If *M* is the midpoint of *AB*, then $\angle ACM =$:

(*a*) 15° (*b*) 30°

(*c*) 45° (*d*) 60°

39. A triangle with sides 13 cm, 14 cm and 15 cm is inscribed in a circle. The radius of the circle inscribed in the triangle is:

(*a*) 2 cm (*b*) 3 cm

(*c*) 4 cm (*d*) 5 cm

[Based on FMS, 2006]

40. Two tangents are drawn to a circle from an exterior point *A*; they touch the circle at points *B* and *C*, respectively. A third tangent intersects segment *AB* in *P* and *AC* in *R*, and touches the circle at *Q*. If $AB = 20$, then the perimeter of triangle *APR* is:

(*a*) 42

(*b*) 40.5

(*c*) 40

(*d*) not determined by the given information

[Based on FMS, 2010]

41. Points *P* and *Q* are both in the line segment *AB* and on the same side of its midpoint. *P* divides *AB* in the ratio 2:3, and *Q* divides *AB* in the ratio 3:4. If $PQ = 2$, then the length of *AB* is:

(*a*) 70 (*b*) 75

(*c*) 80 (*d*) 85

[Based on FMS, 2010]

42. Triangle *ABD* is right angled at *B*. On *AD* there is a point *C* for which $AC = CD$ and $AB = BC$. The magnitude of angle, *DAB*, in degrees, is:

(a) $67\frac{1}{2}$ (b) 60

(c) 45 (d) 30

[Based on FMS, 2010]

43. A circular table is pushed to the corner of a room touching two perpendicular walls. If a point on the edge of the table facing the corner is 8 and 9 cm from the two walls, then the radius of the table (in cm) is:

(a) 29

(b) 17

(c) 5

(d) undeterminable from above

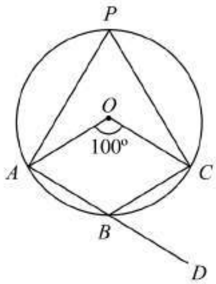
[Based on JMET, 2006]

44. AB and CD are two parallel chords of a circle such that $AB = 10$ cm and $CD = 24$ cm. If the chords are on opposite sides of the centre and the distance between them is 17 cm, what is the radius of the circle?

(a) 14 cm (b) 10 cm

(c) 13 cm (d) 15 cm

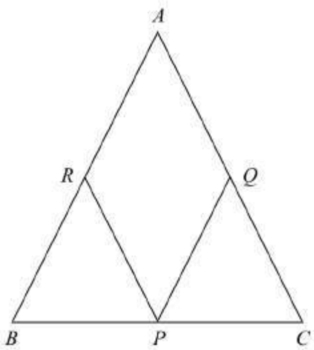
45. In the given figure, O is the centre of the circle. Find $\angle CBD$.



(a) 140° (b) 50°

(c) 40° (d) 130°

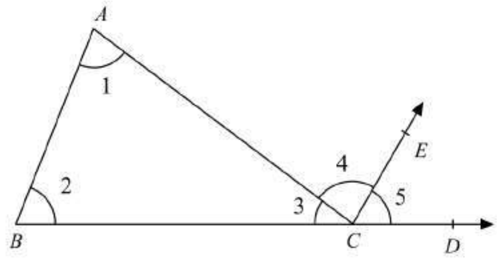
46. In a ΔABC, P, Q and R are the mid-points of sides BC, CA and AB, respectively. If $AC = 21$ cm, $BC = 29$ cm and $AB = 30$ cm. The perimeter of the quadrilateral $ARPQ$ is:



(a) 91 cm (b) 60 cm

(c) 51 cm (d) 70 cm

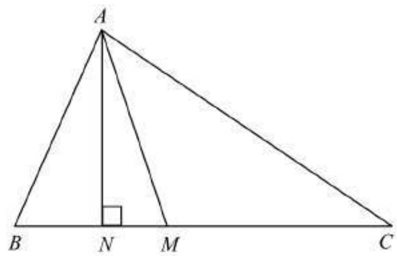
47. In the given figure, side BC of ΔABC is produced to form ray BD and $CE \parallel BA$. Then $\angle ACD$ is equal to:



(a) $\angle A - \angle B$ (b) $(\angle A + \angle B)$

(c) $\angle A + \angle B$ (d) $(\angle A - \angle B)$

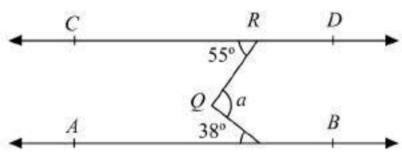
48. In the given figure, in a ΔABC, $\angle B = \angle C$. If AM is the bisector of $\angle BAC$ and $AN \perp BC$, then $\angle MAN$ is equal to:



(a) $\frac{1}{2}(\angle B + \angle C)$ (b) $\frac{1}{2}(\angle C - \angle B)$

(c) $\angle B + \angle C$ (d) $\frac{1}{2}(\angle B - \angle C)$

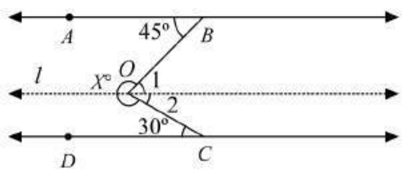
49. In figure, $AB \parallel CD$, $\angle a$ is equal to:



(a) 93° (b) 103°

(c) 83° (d) 97°

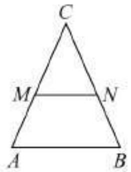
50. In the given figure, $AB \parallel CD$. Then X is equal to:



(a) 290° (b) 300°

(c) 280° (d) 285°

51. In the triangle *ABC*, *MN* is parallel to *AB*. Area of trapezium *ABNM* is twice the area of triangle *CMN*. What is ratio of *CM*:*AM*?

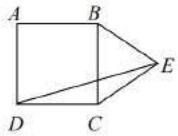


(a) $\frac{1}{\sqrt{3}+1}$ (b) $\frac{\sqrt{3}-1}{2}$

(c) $\frac{\sqrt{3}+1}{2}$ (d) None of these

[Based on SNAP, 2013]

52. If *ABCD* is a square and *BCF* is an equilateral triangle, what is the measure of the angle *DEC*?



(a) 15° (b) 30°

(c) 20° (d) 45°

[Based on SNAP, 2013]

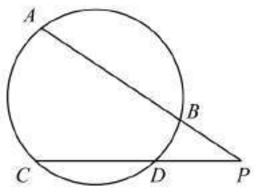
Difficulty Level-2 (Based on Memory)

1. In ΔABC, l $(AB) = (AB) = c$, $l(BC) = a$, $l(AC) = b$. If this triangle is inscribed in a circle, then find the ratio of arc (AB):arc(BC):arc (AC), if $a:b:c = 1:1:\sqrt{3}$.

(*a*) 1:4:1 (*b*) $\sqrt{3}$:1:1

(*c*) 1:$\sqrt{3}$:1 (*d*) 4:1:1

2. If in the following figure, *PA* = 8 cm, *PD* = 4 cm, *CD* = 3 cm, then *AB* is:



(*a*) 3.0 cm (*b*) 3.5 cm

(*c*) 4.0 cm (*d*) 4.5 cm

3. With the vertices of a ΔABC as centers, three circles are described, each touching the other two externally. If the sides of the triangle are 4, 6 and 8 cm, respectively, then the sum of the radii of the three circles equals:

(*a*) 10 (*b*) 14

(*c*) 12 (*d*) 9

4. The intercepts made by three parallel lines on a transverse line (l_1) are in the ratio 1:1. A second transverse line (l_2) making an angle of 30° with (l_1) is drawn. The corresponding intercepts on (l_2) are in the ratio:

(*a*) 1:1 (*b*) 2:1

(*c*) 1:2 (*d*) 1:3

5. The degree measure of each of the three angles of a triangle is an integer. Which of the following could not be the ratio of their measures?

(*a*) 2:3:4 (*b*) 3:4:5

(*c*) 5:6:7 (*d*) 6:7:8

6. Three lines are drawn in a plane. Which of the following could not be the total number of points of intersection?

(*a*) 0

(*b*) 1

(*c*) 2

(*d*) All of the above could be the total number of points of intersection

Directions (Q. 7 to 10): *Use the following information:*

ABC forms an equilateral triangle in which *B* is 2 Km from *A*. A person starts walking from *B* in a direction parallel to *AC* and stops when he reaches a point *D* directly east of *C*. He, then, reverses direction and walks till he reaches a point *E* directly south of *C*.

7. Then *D* is:

(*a*) 3 Km east and 1 Km north of *A*

(*b*) 3 Km east and $\sqrt{3}$ Km north of *A*

(*c*) $\sqrt{3}$ Km east and 1 Km south of *A*

(*d*) $\sqrt{3}$ Km west and 3 Km north of *A*

8. The total distance walked by the person is:

(*a*) 3 Km (*b*) 4 Km

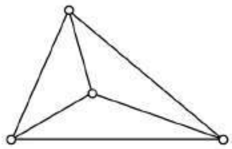
(*c*) $2\sqrt{3}$ Km (*d*) 6 Km

9. Consider the five points comprising the vertices of a square and the intersection point of its diagonals. How many triangles can be formed using these points?

(*a*) 4 (*b*) 6

(*c*) 8 (*d*) 10

10. Four cities are connected by a road network as shown in the figure. In how many ways can you start from any city and come back to it without travelling on the same road more than once?



(*a*) 8 (*b*) 12

(*c*) 16 (*d*) 20

Directions (Q. 11 and 12): *Answer the questions based on the following information.*

Rectangle *PRSU* is divided into two smaller rectangles *PQTU*, and *QRST* by the line *TQ*. *PQ* = 10 cm, *QR* = 5 cm and *RS* = 10 cm. Points *A*, *B*, *F* are within rectangle *PQTU*, and points *C*, *D*, *E* are within the rectangle *QRST*. The closest pair of points among the pairs (*A*, *C*), (*A*, *D*),(*A*, *E*), (*F*, *C*), (*F*, *D*), (*F*, *E*), (*B*, *C*), (*B*, *D*), (*B*, *E*) are $10\sqrt{3}$ cm apart.

11. Which of the following statements is necessarily true?

(*a*) The closest pair of points among the six given points cannot be (*F*, *C*).

(*b*) Distance between *A* and *B* is greater than that between *F* and *C*.

(*c*) The closest pair of points among the six given points is (*C*, *D*), (*D*, *E*) or (*C*, *E*).

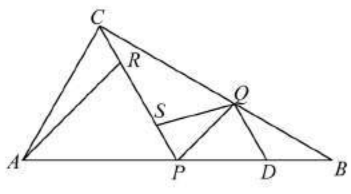
(*d*) None of the above.

12. $AB > AF > BF$; $CD > DE > CE$; and $BF = 6\sqrt{5}$. Which is the closest pair of points among all the six given points?

(*a*) *B*, *F* (*b*) *C*, *D*

(*c*) *A*, *B* (*d*) None of these

13. In the figure (not drawn to scale) given below, *P* is a point on *AB* such that *AP*:*PB* = 4:3. *PQ* is parallel to *AC* and *QD* is parallel to *CP*. In ΔARC, $\angle ARC = 90°$, and in ΔPQS, $\angle PSQ = 90°$. The length of *QS* is 6 cm. What is the ratio *AP*:*PD*?



(*a*) 10:3 (*b*) 2:1

(*c*) 7:3 (*d*) 8:3

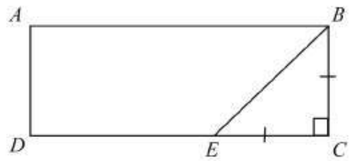
14. The radius of the circumcircle of an equilateral triangle of side 12 cm is:

(*a*) $(4/3)\sqrt{3}$ (*b*) $\sqrt{2}$

(*c*) $4\sqrt{3}$ (*d*) 4

[Based on IIFT, 2003]

15. In the diagram below, *ABCD* is a rectangle. The area of isosceles right triangle *BCE* is 14, and *DE* = 3*EC*. What is the area of *ABCD*?

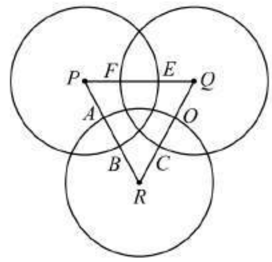


(*a*) 112 (*b*) 56

(*c*) 84 (*d*) $3\sqrt{28}$

[Based on SCMHRD Ent. Exam., 2003]

16. Below shown are three circles, each of radius 20 and centres at *P*, *Q* and *R*; further *AB* = 5, *CD* = 10 and *EF* = 12. What is the perimeter of the triangle *PQR*?



(*a*) 120 (*b*) 66

(*c*) 93 (*d*) 87

17. There is a circle of radius 1 cm. Each member of sequence of regular polygons S_1 (*n*), *n* = 4, 5 , 6, ..., where *n* is the number of sides of the polygon, is circumscribing the circle; and each member of the sequence of regular polygons S_2 (*n*), *n* = 4, 5, 6,..., where *n* is the number of sides of the polygon, is inscribed in the circle. Let L_1 (*n*) and L_2 (*n*) denote the perimeters of the corresponding polygons of $S_1(n)$ and $S_2(n)$, then $\frac{\{L_1(13) + 2\pi\}}{L_2(17)}$ is:

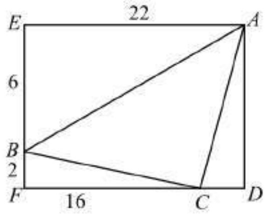
(*a*) greater than π/4 and less than 1

(*b*) greater than 1 and less than 2

(*c*) greater than 2

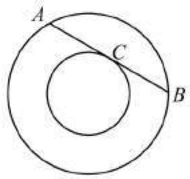
(*d*) less than π/4

18. In the given figure, *EADF* is a rectangle and *ABC* is a triangle whose vertices lie on the sides of *EADF* *AE* = 22, *BE* = 6, *CF* = 16 and *BF* = 2. Find the length of the line joining the mid-points of the sides *AB* and *BC*.



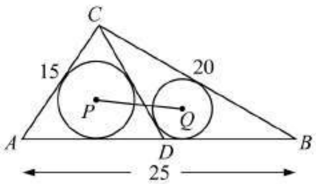
(*a*) $4\sqrt{2}$ (*b*) 5
(*c*) 3.5 (*d*) None of these

19. The line *AB* is 6 m in length and is tangent to the inner one of the two concentric circle at point *C*. It is known that the radii of the two circles are integers. The radius of the outer circle is:



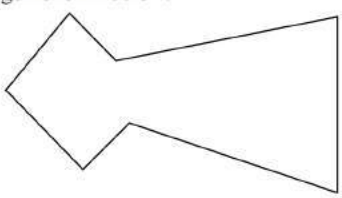
(*a*) 5 m (*b*) 4 m
(*c*) 6 m (*d*) 3 m

20. In the given figure, *ACB* is a right-angled triangle. *CD* is the altitude. Circles are inscribed within the triangles *ACD* and *BCD*, *P* and *Q* are the centres of the circles. The distance *PQ* is:



(*a*) 5 (*b*) $\sqrt{50}$
(*c*) 7 (*d*) 8

21. Find the sum of the degree measures of the internal angles in the polygon shown below.



(*a*) 600° (*b*) 720°
(*c*) 900° (*d*) 1080°

[Based on SCMHRD, 2002]

22. A semi-circle is drawn with *AB* as its diameter. From *C*, a point on *AB*, a line perpendicular to *AB* is drawn meeting the circumference of the semi-circle at *D*. Given that *AC* = 2 cm and *CD* = 6 cm, the area of the semi-circle (in cm^2) will be:

(*a*) $32\pi\ cm^2$ (*b*) $50\pi\ cm^2$
(*c*) $40.5\pi\ cm^2$ (*d*) $81\pi\ cm^2$

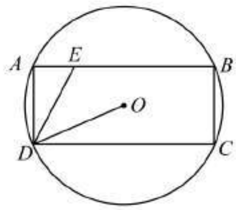
[Based on CAT, 2006]

23. An equilateral triangle *BPC* is drawn inside a square *ABCD*. What is the value of the angle *APD* in degrees?

(*a*) 75 (*b*) 90
(*c*) 12 (*d*) None of these

[Based on CAT, 2006]

24. In the figure below (not drawn to scale), rectangle *ABCD* is inscribed in the circle with centre at *O*. The length of side *AB* is greater than that of side *BC*. The ratio of the area of the circle to the area of the rectangle *ABCD* is $\pi:\sqrt{3}$. The line segment *DE* intersects *AB* at *E* such that $\angle ODC = \angle ADE$. What is the ratio *AE*:*AD*?



(*a*) $1:\sqrt{3}$ (*b*) $1:\sqrt{2}$
(*c*) $1:2\sqrt{3}$ (*d*) 1:2

25. Let the radius of each circular park be *r*, and the distance to be traversed by the sprinters *A*, *B* and *C* be *a*, *b* and *c*, respectively. Which of the following is true?

(*a*) $b - a = c - b = 3\sqrt{3}\,r$

(*b*) $b - a = c - b = \sqrt{3}\,r$

(*c*) $b = \dfrac{a+c}{2} = 2(1+\sqrt{3})r$

(*d*) $c = 2b - a = (2+\sqrt{3})r$

26. Sprinter *A* traverses distances A_1A_2, and A_3A_1 at average speeds of 20, 30 and 15, respectively. *B* traverses her entire path at a uniform speed of $(10\sqrt{3}+20)$. *C* traverses distances C_1C_2, C_2C_3 and C_3C_1 at average speeds of $\frac{40}{3}\left[\sqrt{3}+1\right]$, $\frac{40}{3}\left[\sqrt{3}+1\right]$ and 120, respectively. All speeds are in the same unit. Where would *B* and *C* be respectively, when *A* finishes her sprint?

(*a*) B_1, C_1
(*b*) B_3, C_3
(*c*) B_1, C_3
(*d*) B_1, somewhere between C_3 and C_1

27. Sprinters A, B and C traverse their respective paths at uniform speeds u, v and w respectively. It is known that u^2, v^2, w^2 is equal to Area A:Area B:Area C, where Area A, Area B and Area C are the areas of triangles $A_1A_2A_3$, $B_1B_2B_3$ and $C_1C_2C_3$, respectively. Where would A and C be when B reaches point B_3?

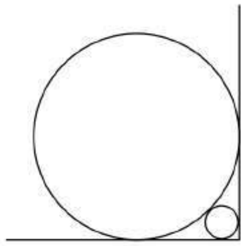
(*a*) A_2, C_3

(*b*) A_3, C_3

(*c*) A_3, C_2

(*d*) Somewhere between C_3 and C_1

28. A circle with radius 2 is placed against a right angle. Another smaller circle is also placed as shown in the given figure. What is the radius of the smaller circle?



(*a*) $3-2\sqrt{2}$ (*b*) $4-2\sqrt{2}$

(*c*) $7-4\sqrt{2}$ (*d*) $6-4\sqrt{2}$

29. If a parallelogram with area P, a rectangle with area R and a triangle with area T are all constructed on the same base and all have the same altitude, then a false statement is:

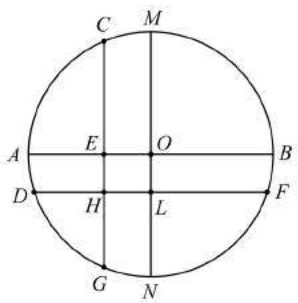
(*a*) $P = 2T$ (*b*) $T = \frac{1}{2}R$

(*c*) $P = R$ (*d*) None of these

30. In a circle of radius 10 cm, a chord is drawn 6 cm from the centre. If a chord half the length of the original chord was drawn, its distance in centimetres from the centre would be:

(*a*) $\sqrt{84}$ (*b*) 9 (*c*) 8 (*d*) 3π

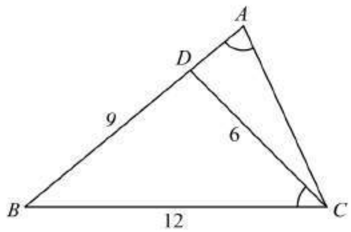
31. In the following figure, the diameter of circle is 3 cm. AB and MN are two diameters such that MN is perpendicular to AB. In addition, CG is perpendicular to AB such that AE:EB = 1:2 and DF is perpendicular to MN such that NL:LM = 1:2. The length of DH in cm is:



(*a*) $2\sqrt{2}-1$ (*b*) $\frac{(2\sqrt{2}-1)}{2}$

(*c*) $\frac{(3\sqrt{2}-1)}{2}$ (*d*) $\frac{(2\sqrt{2}-1)}{3}$

32. Consider the triangle ABC shown in the following figure where BC = 12 cm, DB = 9 cm, CD = 6 cm and $\angle BCD = \angle BAC$.



What is the ratio of 'the perimeter of the triangle ADC to that of the triangle BDC?

(*a*) $\frac{7}{9}$ (*b*) $\frac{8}{9}$

(*c*) $\frac{6}{9}$ (*d*) $\frac{5}{9}$

33. Three circles A, B and C have a common centre O. A is the inner circle, B middle circle and C is outer circle. The radius of the outer circle. OP cuts the inner circle at X and middle circle at Y such that $OX = XY = YP$. The ratio of the area of the region between the inner and middle circles to the area of the region between and outer circle is:

(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{5}$

(*c*) $\frac{3}{5}$ (*d*) $\frac{1}{5}$

34. Any five points are taken inside or on a square of side 1. Let a be the smallest possible number with the property that it is always possible to select one pair of points from these five such that the distance between them is equal to or less than a. Then a is:

(*a*) $\frac{\sqrt{3}}{3}$ (*b*) $\frac{\sqrt{2}}{2}$

(*c*) $\frac{2\sqrt{2}}{3}$ (*d*) 1

[Based on FMS, 2010]

35. The point A, B and C are on a circle O. The tangent line at A and the secant BC intersect at P, B lying between C and P. If $\overline{BC} = 20$ and $\overline{PA} = 10\sqrt{3}$, then $\overline{PB}$ equals:

(*a*) 5 (*b*) 10

(*c*) $10\sqrt{3}$ (*d*) 20

[Based on FMS, 2011]

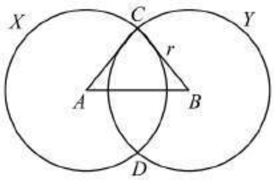
36. A rectangle inscribed in a triangle has its base coinciding with the base b of the triangle. If the altitude of the triangle is h, and the altitude x of the rectangle is half the base of the rectangle, then:

(*a*) $x = \frac{1}{2}h$ (*b*) $x = \frac{bh}{h+b}$

(*c*) $x = \frac{bh}{2h+b}$ (*d*) $x = \sqrt{\frac{hb}{2}}$

[Based on FMS, 2011]

37. Two circles X and Y with centres A and B intersect at C and D.

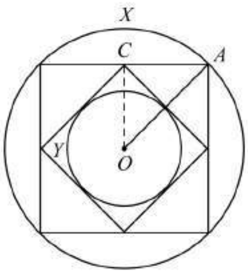


Area of circle X is 4 times area of circle Y. Then AB = ?

(*a*) $5r$ (*b*) $\sqrt{5}r$

(*c*) $3r$ (*d*) $\frac{\sqrt{5}}{2}r$

38. In the given figure, $OA = R$.



What is the ratio of areas of circle X and Y?

(*a*) 2:1 (*b*) 4:1

(*c*) 3:1 (*d*) 8:1

39. A pole has to be erected on the boundary of a circular park of diameter 13 m in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 m. The distance of the pole from one of the gate is

(*a*) 8 m (*b*) 8.25 m

(*c*) 5 m (*d*) None of the above

[Based on ITFT, 2008]

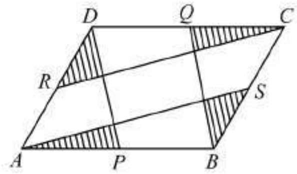
40. If D is the mid point of side BC of a ΔABC and AD is the perpendicular to AC, then

(*a*) $3AC^2 = BC^2 - AB^2$ (*b*) $3BC^2 = AC^2 - 3AB^2$

(*c*) $BC^2 + AC^2 = 5AB^2$ (*d*) None of these

[Based on ITFT, 2008]

41. In the parallelogram $ABCD$, P, Q, R and S are mid-points of the sides AB, CD, DA and BC respectively. AS, BQ, CR and DP are joined. Find the ratio of the area of the shaded region to the area of the parallelogram $ABCD$.



(*a*) $\frac{1}{5}$ (*b*) $\frac{1}{4}$

(*c*) $\frac{4}{15}$ (*d*) $\frac{1}{6}$

42. ABC is a triangle with $\angle CAB = 15°$ and $\angle ABC = 30°$. If M is the mid point of AB, then $\angle ACM =$

(*a*) 15° (*b*) 30°

(*c*) 45° (*d*) 60°

[Based on XAT, 2007]

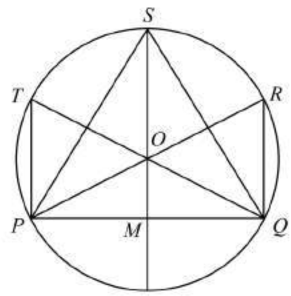
43. ABC is a triangle with $\angle BAC = 60°$. A point P lies on one-third of the way from B to C, and AP bisects $\angle BAC$. $\angle APC =$

(*a*) 30° (*b*) 45°

(*c*) 60° (*d*) 90°

[Based on XAT, 2007]

44. In the adjoining figure, the measure of $\angle POQ$ is equal to 128° and on extension SO is perpendicular to chord PQ of the circle with centre O. The measure of $\angle TPS$ is:



(*a*) 36° (*b*) 26°

(*c*) 34° (*d*) 32°

45. $ABCD$ is a rectangle with $AD = 10$. P is a point on BC such that $\angle APD = 90°$. If $DP = 8$, then the length of BP is:

(*a*) 3.6 (*b*) 6.4

(*c*) 5.2 (*d*) 4.8

[Based on XAT, 2008]

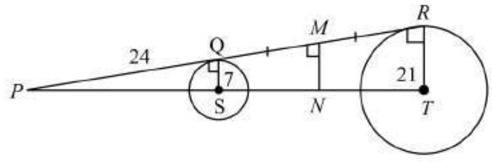
46. In a triangle ABC, $AB = 3$, $BC = 4$ and $CA = 5$. Point D is the midpoint of AB, point E is on segment AC and point F is on segment BC. If $AE = 1.5$ and $BF = 0.5$, then $\angle DEF =$

(*a*) 30° (*b*) 45°

(*c*) 60° (*d*) 75°

[Based on XAT, 2008]

47. In the given figure, $PQ = 24$. M is the mid-point of QR.

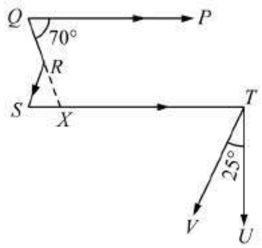


Also, $MN \perp PR$, $QS = 7$ and $TR = 21$, then $SN = ?$

(*a*) $\frac{25}{2}$ (*b*) 25

(*c*) 50 (*d*) Cannot be determined

48. In the given figure, $PQ \parallel ST$, $TV \parallel RS$ and $TU \perp ST$. Find $\angle QRS$.



(*a*) 120° (*b*) 125°

(*c*) 135° (*d*) None of these

49. In a quadrilateral $ABCD$, $BC = 10$, $CD = 14$, $AD = 12$ and $\angle CBA = \angle BAD = 60°$. If $AB = a + \sqrt{b}$, where a and b are positive integers, then $a + b$ is equal to:

(*a*) 193 (*b*) 201

(*c*) 204 (*d*) 207

[Based on XAT, 2009]

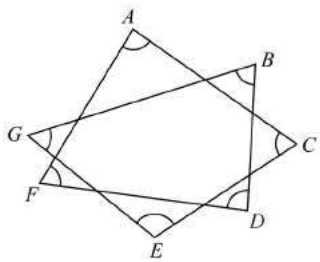
50. Two poles, of height 2 m and 3 m, are 5 m apart. The height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is

(*a*) 1.2 m (*b*) 1.0 m

(*c*) 5.0 m (*d*) 3.0 m

[Based on XAT, 2010]

51. All line segments are straight. Find the sum of the angles at the corners marked in the diagram.



(*a*) 360° (*b*) 450°

(*c*) 540° (*d*) 630°

52. A 25 ft long ladder is placed against the wall with its base 7 ft from the wall. The base of the ladder is drawn out so that the top comes down by half the distance that the base is drawn out. This distance is in the range:

(*a*) (2, 7) (*b*) (5, 8)

(*c*) (9, 10) (*d*) (3, 7)

[Based on XAT, 2011]

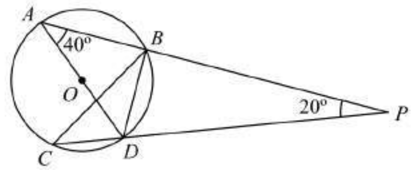
53. Consider a square $ABCD$ of side 60 cm. It contains arcs BD and AC drawn with centres at A and D, respectively. A circle is drawn such that it is tangent to side AB and the arcs BD and AC. What is the radius of the circle?

(*a*) 9 cm (*b*) 10 cm

(*c*) 12 cm (*d*) 15 cm

[Based on XAT, 2011]

54. PBA and PDC are two secants. AD is the diameter of the circle with centre at O. $\angle A = 40°$, $\angle P = 20°$. Find the measure of $\angle DBC$.



(*a*) 30° (*b*) 45°

(*c*) 50° (*d*) 40°

55. A straight line through point P of a triangle PQR intersects the side QR at the point S and the circum circle of the triangle PQR at the point T. If S is not the centre of the circum circle, then which of the following is true?

(*a*) $(1/PS) + (1/ST) < 2/\sqrt{(QS)(QR)}$

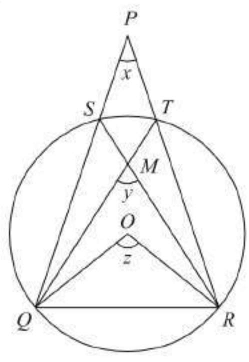
(*b*) $(1/PS) + (1/ST) < 4/QR$

(*c*) $(1/PS) + (1/ST) > 1/\sqrt{(QS)(QR)}$

(*d*) $(1/PS) + (1/ST) > 4/QR$

[Based on XAT, 2011]

56. In the given figure, O is the centre of the circle. Then $\angle x + \angle y$ is equal to:



(a) $2\angle z$ (b) $\frac{\angle z}{2}$

(c) $\angle z$ (d) None of these

57. In a right-angled triangle ΔPQR with $\overline{PQ} \neq \overline{QR}$, M is point on its hypotenuse PR, L and N are feet of the perpendiculars from M on PQ and QR, respectively. $\overline{LN}$ will be minimized when:

(a) ΔPQM and ΔPQR are similar

(b) M is the mid point of PR

(c) $m\angle PQM = m\angle MQR = 45°$

(d) $\overline{PM}:\overline{MR} = \overline{PQ}:\overline{QR}$

[Based on JMET, 2006]

58. In ΔQR, $\overline{PQ} = \overline{PR}$ and $m\angle QPR = 20°$, S is a point on PR such that $m\angle SQR = 60°$ and T is a point on PQ such that $m\angle TRQ = 50°$. Then $m\angle STR$ equals:

(a) 60° (b) 70°

(c) 80° (d) 90°

[Based on JMET, 2006]

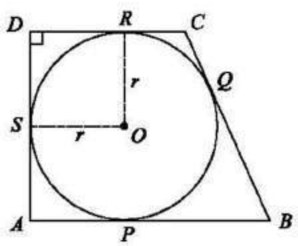
59. A city has a park shaped as a right-angled triangle. The length of the longest side of this park is 80 m. The Mayor of the city wants to construct three paths from the corner point opposite to the longest side such that these three paths divide the longest side into four equal segments. Determine the sum of the squares of the lengths of the three paths.

(a) 4000 m (b) 4800 m

(c) 5600 m (d) 6400 m

[Based on XAT, 2012]

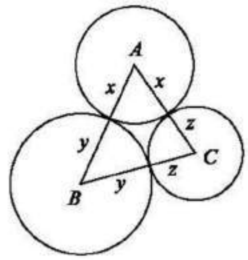
60. In the given figure, $ABCD$ is a quadrilateral in which $\angle O = 90°$. A circle $C(0, r)$ touches the sides AB, BC, CD and DA at P, Q, R, S respectively. If $BC = 38$ cm, $CD = 25$ cm and $BP = 27$ cm, find the value of r.



(a) 14 cm (b) 15 cm

(c) 10 cm (d) 16 cm

61. With the vertices of a ΔABC as centres, three circles are described, each touching the other two externally. If the sides of the Δ are 9 cm, 7 cm and 6 cm, find the radii of the circles.



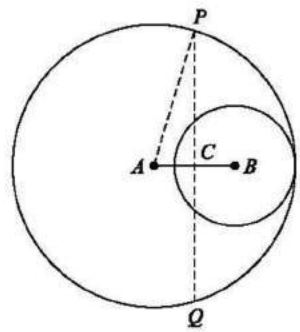
(a) 4 cm, 7 cm and 3 cm

(b) 7 cm, 5 cm and 2 cm

(c) 5 cm, 4 cm and 3 cm

(d) 4 cm, 5 cm and 2 cm

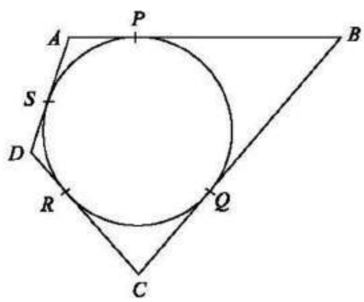
62. In the given figure, two circle with centres A and B of radii 5 cm and 3 cm touch each other internally. If the perpendicular bisector of segment AB meets the bigger circle in P and Q, find the length of PQ.



(a) $4\sqrt{6}$ cm (b) $\sqrt{24}$

(c) $8\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

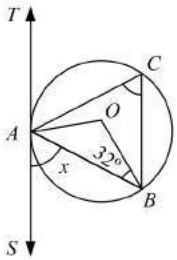
63. In the given figure, a circle touches all the four sides of quadrilateral $ABCD$ whose sides $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm. Find AD.



(a) 5 cm (b) 4 cm

(c) 3 cm (d) 2 cm

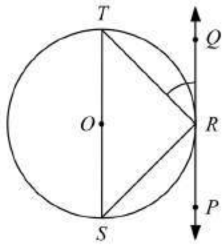
64. In the given figure, TAS is a tangent to the circle at the point A. If $\angle OBA = 32°$, what is the value of x?



(*a*) 64° (*b*) 40°

(*c*) 58° (*d*) 50°

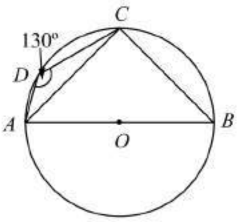
65. In the given figure, *ST* is a diameter of the circle with centre *O* and *PQ* is the tangent at a point *R*. If $\angle TRQ = 40°$, what is $\angle RTS$.



(*a*) 40° (*b*) 50°

(*c*) 60° (*d*) 30°

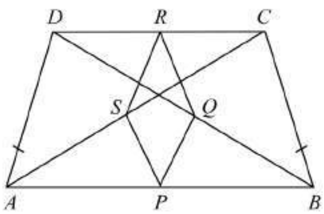
66. In the given figure, *ABCD* is a cyclic quadrilateral whose side *AB* is a diameter of the circle through *A*, *B* and *C*. If $\angle ADC = 130°$, find $\angle CAB$.



(*a*) 40° (*b*) 50°

(*c*) 30° (*d*) 130°

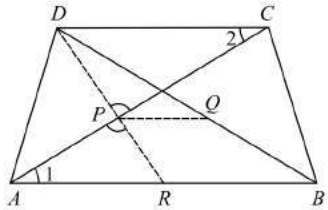
67. *ABCD* is trapezium in which $AB \parallel DC$ and $AD = BC$. If *P*, *Q*, *R*, *S* be respectively the mid-point of *BA*, *BD* and *CD*, *CA*, then *PQRS* is a:



(*a*) Rhombus (*b*) Rectangle

(*c*) Parallelogram (*d*) Square

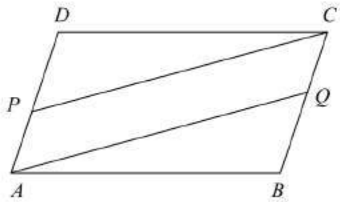
68. *ABCD* is a trapezium and *P*, *Q* are the mid-points of the diagonals *AC* and *BD*. Then *PQ* is equal to:



(*a*) $\frac{1}{2}$ (*AB*) (*b*) $\frac{1}{2}$ (*CD*)

(*c*) $\frac{1}{2}$ (*AB* – *CD*) (*d*) $\frac{1}{2}$ (*AB* + *CD*)

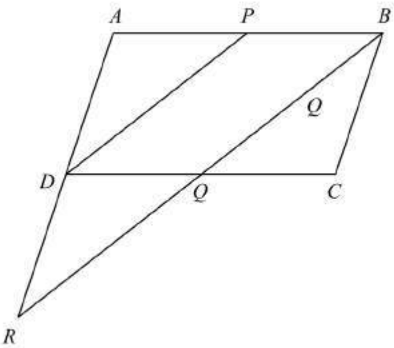
69. *ABCD* is a parallelogram. *P* is a point on *AD* such that $AP = \frac{1}{3}AD$ and *Q* is a point on *BC* such that $CQ = \frac{1}{3}BC$. Then *AQCP* is a:



(*a*) Parallelogram (*b*) Rhombus

(*c*) Rectangle (*d*) Square

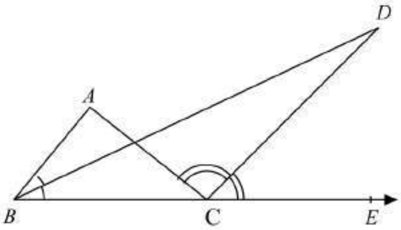
70. *P* is the mid-point of side *AB* to a parallelogram *ABCD*. A line through *B* parallel to *PD* meets *DC* at *Q* and *AD* produced at *R*. Then *BR* is equal to:



(*a*) *BQ* (*b*) $\frac{1}{2}$

(*c*) 2*BQ* (*d*) None of these

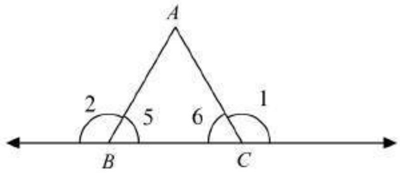
71. In the figure, *BD* and *CD* are angle bisectors of $\angle ABC$ and $\angle ACE$, respectively, then $\angle BDC$ is equal to:



(*a*) ∠*BAC* (*b*) 2 ∠*BAC*

(*c*) $\frac{1}{2}$ ∠*BAC* (*d*) $\frac{1}{3}$ ∠*BAC*

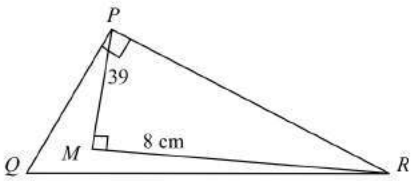
72. In the given figure, the side *BC* of a Δ*ABC* is produced on both sides, then ∠1 + ∠2 is equal to:



(*a*) ∠*A* + 180° (*b*) 180° – ∠*A*

(*c*) $\frac{1}{2}$(∠*A* + 180°) (*d*) ∠*A* + 90°

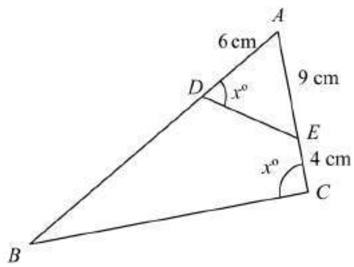
73. In the given figure ∠*QPR* = 90°, *QR* = 26 cm, *PM* = 6 cm, *MR* = 8 cm and ∠*PMR* = 90°, find the area of Δ*PQR*.



(*a*) 180 cm^2 (*b*) 240 cm^2

(*c*) 120 cm^2 (*d*) 150 cm^2

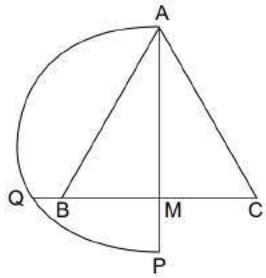
74. In the given figure, find the length of *BD*.



(*a*) 13.5 cm (*b*) 12 cm

(*c*) 14.5 cm (*d*) 15 cm

75. In the figure alongside, Δ*ABC* is an equilateral triangle with area *S*. *M* is the mid-point of *BC* and *P* is a point on *AM* extended such that *MP* = *BM*. If the semi-circle on *AP* intersects *CB* extended at *Q* and the area of a square with *MQ* as a side is *T*, which of the following is true?

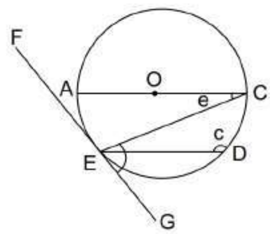


(*a*) $T = \sqrt{2}S$ (*b*) $T = S$

(*c*) $T = \sqrt{3}S$ (*d*) $T = 2S$

[Based on CAT, 2011]

76. In the figure alongside, *O* is the centre of the circle and *AC* is the diameter. The line *FEG* is tangent to the circle at E. If ∠*GEC* = 52°, find the value of ∠*e* + ∠*c*.



(*a*) 154° (*b*) 156°

(*c*) 166° (d) 180°

[Based on CAT, 2011]

77. If α and β are the roots of the quadratic equation $x^2 - 10x + 15 = 0$, then find the quadratic equation whose roots are $\left(\alpha + \frac{\alpha}{\beta}\right)$ and $\left(\beta + \frac{\beta}{\alpha}\right)$.

(*a*) $15x^2 + 71x + 210 = 0$

(*b*) $5x^2 - 22x + 56 = 0$

(*c*) $3x^2 - 44x + 78 = 0$

(*d*) Cannot be determined

[Based on CAT, 2012]

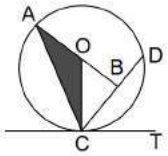
78. In the given diagram *CT* is tangent at *C*, making an angle of 45° with *CD*. *O* is the centre of circle, *CD* = 10 cm. What is the perimeter of the shaded region (Δ*AOC*) approximately?

(a) 25 cm (b) 26 cm

(c) 27 cm (d) 28 cm

[Based on CAT, 2013]

79. In the adjoining figure, the diameter of the larger circle is 20 cm and the smaller circle touches internally the larger circle at *P* and passes through *O*, the centre of the larger circle. Chord *SP* cuts the smaller circle at *R* and *OR* is equal to 8 cm. What is the length of chord *SP*?



(*a*) 9 cm (*b*) 6 cm

(*c*) 12 cm (*d*) 14 cm

[Based on CAT, 2013]

80. Triangle *ABC* is a right angled triangle. *D* and *E* are mid points of *AB* and *BC* respectively.

I. $AE = 19$

II. $CD = 22$

III. Angle *B* is a right angle.

[Based on XAT, 2014]

Which of the following statements would be sufficient to determine the length of AC?

(*a*) Statement I and statement II

(*b*) Statement I and statement III

(*c*) Statement II and III

(*d*) Statement III alone.

(*e*) All three statements.

81. There are two circles C_1 and C_2 of radii 3 and 8 units respectively. The common internal tangent, *T*, touches the circle at points P_1 and P_2 respectively. The line joining the centers of the circles intersects *T* at *X*. The distance of *X* from the center of the smaller circle is 5 units. What is the length of the line segment $P_1 P_2$?

(*a*) ≤ 13 (*b*) >13 and ≤ 14

(*c*) >14 and ≤ 15 (*d*) >15 and ≤ 16

(*e*) >16

[Based on XAT, 2014]

82. In quadrilateral *PQRS*, $PQ = 5$ units, $QR = 17$ units, $RS = 5$ units, and $PS = 9$ units. The length of the diagonal *QS* can be:

(*a*) >10 and <12 (*b*) >12 and <14

(*c*) >14 and <16 (*d*) >16 and <18

(*e*) Cannot be determined

[Based on XAT, 2014]

83. Circle C_1 has a radius of 3 units. The line segment *PQ* is the only diameter of the circle which is parallel to the *X* axis. *P* and *Q* are points on curves given by the equation $y = a^x$ and $y = 2a^x$ respectively, where $a < 1$. The value of a is:

(*a*) $\frac{1}{\sqrt[6]{2}}$ (*b*) $\frac{1}{\sqrt[6]{3}}$

(*c*) $\frac{1}{\sqrt[3]{6}}$ (*d*) $\frac{1}{\sqrt{6}}$

(*e*) None of the above

[Based on XAT, 2014]

Answer Keys

Difficulty Level-1

1. (*c*)	**2.** (*b*)	**3.** (*b*)	**4.** (*b*)	**5.** (*a*)	**6.** (*a*)	**7.** (*b*)	**8.** (*a*)	**9.** (*b*)	**10.** (*c*)	**11.** (*c*)	**12.** (*d*)	**13.** (*d*)
14. (*d*)	**15.** (*b*)	**16.** (*b*)	**17.** (*c*)	**18.** (*b*)	**19.** (*c*)	**20.** (*d*)	**21.** (*c*)	**22.** (*b*)	**23.** (*b*)	**24.** (*d*)	**25.** (*c*)	**26.** (*a*)
27. (*c*)	**28.** (*a*)	**29.** (*d*)	**30.** (*d*)	**31.** (*a*)	**32.** (*c*)	**33.** (*c*)	**34.** (*b*)	**35.** (*c*)	**36.** (*c*)	**37.** (*c*)	**38.** (*c*)	**39.** (*c*)
40. (*c*)	**41.** (*a*)	**42.** (*b*)	**43.** (*d*)	**44.** (*c*)	**45.** (*d*)	**46.** (*c*)	**47.** (*c*)	**48.** (*d*)	**49.** (*a*)	**50.** (*d*)	**51.** (*c*)	**52.** (*a*)

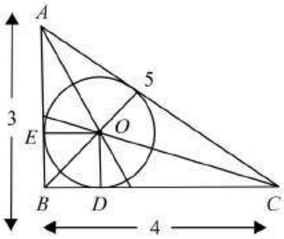
Difficulty Level-2

1. (*d*)	**2.** (*d*)	**3.** (*d*)	**4.** (*a*)	**5.** (*d*)	**6.** (*d*)	**7.** (*b*)	**8.** (*d*)	**9.** (*c*)	**10.** (*b*)	**11.** (*d*)	**12.** (*d*)	**13.** (*c*)
14. (*c*)	**15.** (*a*)	**16.** (*c*)	**17.** (*c*)	**18.** (*b*)	**19.** (*a*)	**20.** (*c*)	**21.** (*b*)	**22.** (*b*)	**23.** (*d*)	**24.** (*a*)	**25.** (*a*)	**26.** (*c*)
27. (*b*)	**28.** (*d*)	**29.** (*a, c*)	**30.** (*a*)	**31.** (*b*)	**32.** (*a*)	**33.** (*c*)	**34.** (*b*)	**35.** (*b*)	**36.** (*c*)	**37.** (*b*)	**38.** (*b*)	**39.** (*c*)
40. (*a*)	**41.** (*a*)	**42.** (*c*)	**43.** (*d*)	**44.** (*d*)	**45.** (*a*)	**46.** (*b*)	**47.** (*b*)	**48.** (*c*)	**49.** (*c*)	**50.** (*a*)	**51.** (*c*)	**52.** (*d*)
53. (*b*)	**54.** (*a*)	**55.** (*d*)	**56.** (*c*)	**57.** (*c*)	**58.** (*c*)	**59.** (*c*)	**60.** (*a*)	**61.** (*d*)	**62.** (*a*)	**63.** (*c*)	**64.** (*c*)	**65.** (*b*)
66. (*a*)	**67.** (*a*)	**68.** (*c*)	**69.** (*a*)	**70.** (*c*)	**71.** (*c*)	**72.** (*a*)	**73.** (*c*)	**74.** (*a*)	**75.** (*b*)	**76.** (*c*)	**77.** (*c*)	**78.** (*c*)
79. (*c*)	**80.** (*e*)	**81.** (*c*)	**82.** (*b*)	**83.** (*a*)								

Explanatory Answers

Difficulty Level-1

1. (*c*) If the in circle of a triangle ABC touches BC at D, then $|BD - CD| = |AB - AC|$



In our case, $AC = 5$, $AB = 3$

$\Rightarrow \quad AC - AB = 2$

$\therefore \quad CD - BD = 2$

In our case, $BC = 4$

$\Rightarrow \quad BD + DC = 4$ and $-BD + DC = 22$

$\Rightarrow \quad CD = 3$

$\Rightarrow \quad BD = 1$

$= OE =$ Radius of the incircle.

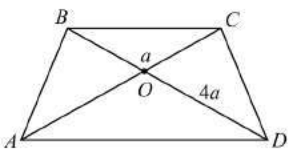
2. (*b*) Let, $BC \parallel AD$

ΔAOD is similar to ΔBOC

$$\Rightarrow \quad \frac{BC}{AD} = \frac{OB}{OD} = \frac{1}{4}$$

Also, $\quad BC = 3$ cm (given)

$\Rightarrow \quad AD = 4 \times 3 = 12$ cm



3. (*b*) Let the medians be m_1, m_2 and m_3 and the opposite sides be a_1, a_2 and a_3.

Then, we have

$$m_1^2 + 2\left(\frac{1}{2}a_1\right)^2 = a_2^2 + a_3^2;$$

$$m_2^2 + 2\left(\frac{1}{2}a_2\right)^2 = a_1^2 + a_2^2$$

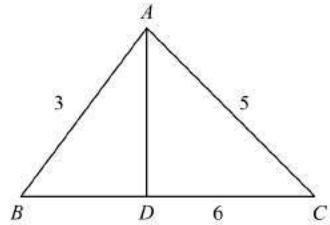
$$m_3^2 + 2\left(\frac{1}{2}a_3\right)^2 = a_1^2 + a_2^2$$

$$\therefore \quad 2m_1^2 + 2m_2^2 + 2m_3^2 + 2 \times \frac{1}{4}(a_1^2 + a_2^2 + a_3^2)$$

$$= 2(a_1^2 + a_2^2 + a_3^2)$$

$$\therefore \quad \frac{a_1^2 + a_2^2 + a_3^2}{m_1^2 + m_2^2 + m_3^2} = \frac{4}{3}.$$

4. (*b*)



$BD{:}DC = 3{:}5$

$\therefore$ Divided $BC = 6$ in the ratio 3:5

$\Rightarrow \quad BD = 2.25$, $CD = 3.75$.

5. (*a*) $\quad x + y + (y + 20) = 5 \Rightarrow x + 2y = 160$

$4x - y = 10 \Rightarrow y = 70, x = 20$

$\therefore$ The angles of the triangle are 20º, 70º, 90º.

6. (*a*)

7. (*b*) $\quad AQ^2 = AC^2 + QC^2$

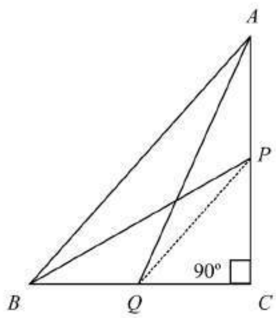
$BP^2 = BC^2 + CP^2$

$AQ^2 = BP^2 = (AC^2 + BC^2) + (QC^2 + CP^2)$

$= AB^2 + PQ^2$

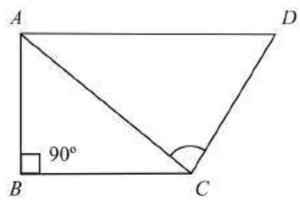
$$= AB^2 + \left(\frac{1}{2}AB\right)^2 \quad \left[\because PQ = \frac{1}{2}AB\right]$$

$$= \frac{5}{4}AB^2 = 4\,(AQ^2 + BP^2) = 5AB^2.$$

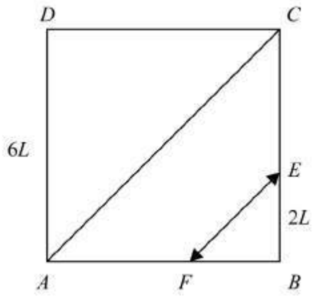


8. (*a*) $\quad AB^2 + BC^2 + CD^2 = AC^2 + CD^2 = AD^2$

$\Rightarrow \quad \angle ACD = 90º$.



9. (*b*) Let the side of the square be $6L$



Then, $\frac{1}{2} \times 3L \times 2L = 108 \Rightarrow L = 6$

$\therefore$ Side of the square = 60 m

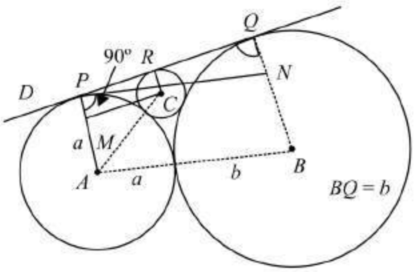
$\Rightarrow \quad AC^2 = AD^2 + DC^2$

$= (36)^2 + (36)^2 = 2 \times (36)^2$

$\Rightarrow \quad AC = 36\sqrt{2}$.

10. (*c*) $\quad PR = MC = \sqrt{AC^2 - AM^2}$

$\therefore \quad \sqrt{(a+c)^2 - (a-c)^2} = 2\sqrt{ac}$



Similarly, $QR = 2\sqrt{bc}$

Now, $\quad PQ = PR + RQ$

$= 2\sqrt{ac} + 2\sqrt{bc} \qquad (1)$

Draw PN Parallel to AB

$\therefore \quad PN = AB = a + b,$

$QN = BQ - BN = b - a$

$\therefore \quad PQ^2 = PN^2 - QN^2$

$= (a+b)^2 - (a-b)^2 = 4ab$

$\Rightarrow \quad PQ = 2\sqrt{ab} \qquad (2)$

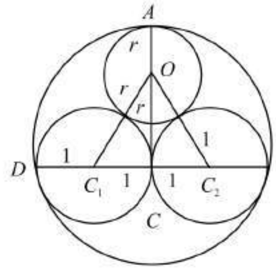
$\therefore$ From (1) and (2)

$\Rightarrow \quad \frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$.

11. (*c*) $\quad CC_1 = 1,\ OC_1 = 1 + r$

$OC = AC - AO = CD - AO = 2 - r$

[AC and CD are the radii of the bigger circle]



$\therefore \quad CO_1{}^2 = CC_1{}^2 + OC^2$

$\Rightarrow \quad (1 + r)^2 = 1^2 + (2 - r)^2$

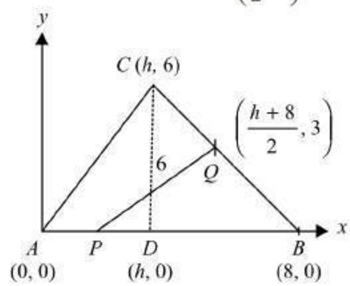
$\Rightarrow \quad r = \frac{2}{3}$.

12. (*d*) Area of $(\Delta PRS + \Delta PQR)$

$= \frac{1}{2} AD(AP + PB)$

$= \frac{1}{2} AD \times AB = 8 \text{ cm}^2$.

13. (*d*) Let $AD = h$ coordinates of P are $\left(\frac{h}{2}, 0\right)$.



$CD = 6.$

$PQ = \sqrt{\left(\frac{h}{2} - \left(\frac{h}{2}\right) + (0.3)\right)} = \sqrt{16 + 9} = 5.$

14. (*d*)

15. (*b*) Using the sine formula, we have

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\therefore \quad \frac{\sin 40^\circ}{BD} = \frac{\sin(40^\circ + C)}{6}$

$\Rightarrow \quad \frac{\sin 40^\circ}{B - DC} = \frac{\sin[180 - (40 + C)]}{6}$

$= \frac{\sin(140 - C)}{6} \qquad (1)$

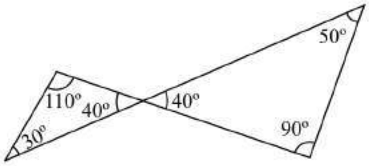
Also $\frac{\sin 40^\circ}{DC} = \frac{\sin(140^\circ - C)}{9} \qquad (2)$

$\therefore$ From (1) and (2) give

$\frac{DC}{8 - DC} = \frac{9}{6} = \frac{3}{2}$

$\Rightarrow \quad 5DC = 24 \Rightarrow DC = 4.8.$

16. (*b*)



17. (*c*)

18. (*b*) The sum of the interior angles of a polygon of *n* sides

$$= (2n-4)\times\frac{\pi}{2}$$

$$\therefore\ (2n+4)\times\frac{\pi}{2} = 1620 \Rightarrow n = 11.$$

19. (*c*) Let *n* be the number of sides of the polygon.

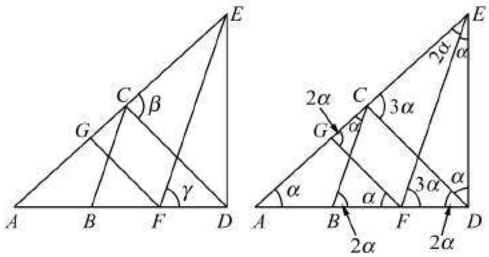
∴ Interior angle = 8 × Exterior angle.

$$\Rightarrow \frac{(2n-4)\times\pi/2}{n} = 8\times\frac{2\pi}{n}$$

$$\Rightarrow \qquad n = 18.$$

20. (*d*) Let $\angle EAD = \alpha$, $\angle AFG = \alpha$ and also $\angle ACB = \alpha$.

Hence, $\angle CBD = 2\alpha$ (exterior angle to ΔABC). Since $CB = CD$, hence $\angle CDB = 2\alpha$



$\angle FGC = 2\alpha$ (exterior angle to ΔAFG).

Since $GF = EF$, $\angle FEG = 2\alpha$

Now, $\angle DCE = \angle DEC = \beta$(say)

Then, $\angle DEF = \beta - 2\alpha$

Since, $\angle DCB = 180^\circ - (\alpha + \beta)$.

Therefore, in ΔDCB, $180^\circ - (\alpha+\beta) + 2\alpha + 2\alpha = 180^\circ$ or $\beta = 3\alpha$. Further,

$\angle EFD = \angle EDF = \gamma$ (say)

Then, $\angle EDC = \gamma - 2d$.

If CD and EF meet in P, then

$\angle FPD = 180^\circ - 5\alpha(\beta = 3\alpha)$

Now in ΔPED, $180^\circ - 5\alpha + \gamma + 2\alpha = 180^\circ$ or $\gamma = 3\alpha$

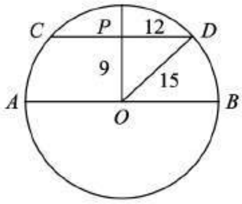
Therefore, in ΔEFD,

$\alpha + 2\gamma = 180^\circ$ or $\alpha + 6\alpha = 180^\circ$

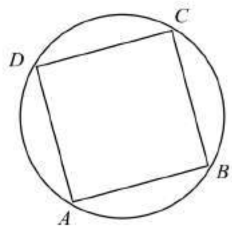
or $\alpha = 26^\circ$ or approximately 25°

21. (*c*) *OP* is perpendicular from the centre of the circle on the chord *CD*.

$$OP^2 = \sqrt{(15)^2 - (12)^2} = 9 \text{ cm.}$$



22. (*b*) Given $\angle A = 2\angle C$



Also, $\angle A + \angle C = 180^\circ$

and $\angle B + \angle D = 180^\circ$

$\therefore \qquad \angle C = 60^\circ$

and $\qquad \angle A = 120^\circ$

Now, $\angle B - \angle D = \frac{1}{3}\angle A = 40^\circ$

$\angle B = 110^\circ$

$\angle D = 70^\circ$

Minimum difference is between $\angle A$ and $\angle B$ or $\angle C$ and $\angle D$ which is 10°.

23. (*b*) Let the shortest side be *x* m.

The hypotenuse = $2x + 3$

Let third side = y

$\therefore \qquad x + y + 2x + 3 = 6x$

$\therefore \qquad y = 3x - 3$

Now, $(x)^2 + (3x-3)^2 = (2x+3)^2$

$\Rightarrow \quad x^2 + 9x^2 + 9 - 18x = 4x^2 + 9 + 12x$

$\Rightarrow \qquad 6x^2 - 30x = 0$

$\therefore \qquad x = 5$ m

as $\qquad x \neq 0$

∴ Three sides are 5, 12 and 13.

24. (*d*) Let the angle be *x*.

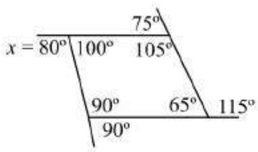
Supplement of this angle = $180 - x$

$$\therefore \qquad x = \frac{1}{3}(180 - x)$$

$$\Rightarrow \qquad 3x = 180 - x$$

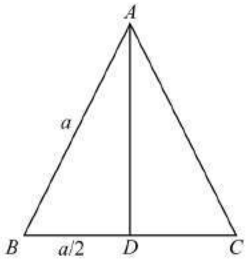
$$\Rightarrow \qquad x = 45^\circ.$$

25. (*c*)



26. (a) Let ABC be equilateral triangle of side A. From A draw AD altitude on BC from a right-angle triangle ABD,

$$AD^2 = AB^2 - BD^2$$

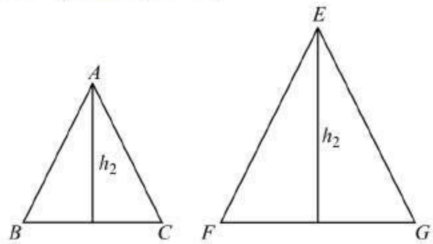


$$= a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$$

$\therefore$ Altitude, $AD = \frac{\sqrt{3}}{2}a.$

27. (c) Let h_1, h_2 be the heights of two isosceles triangles.

According to the question,



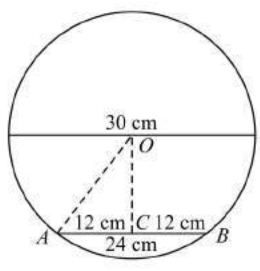
$$\frac{\frac{1}{2}\times BC\times h_1}{\frac{1}{2}\times FG\times h_2} = \frac{9}{16}$$

or $\quad \frac{BC}{FG}\times\frac{h_1}{h_2} = \frac{9}{16}$

or $\quad \left(\frac{h_1}{h_2}\right)^2 = \frac{}{16}$

$\therefore \quad \frac{h_1}{h_2} = \frac{3}{4}.$

28. (a)



Let OC be the perpendicular from the centre O on the chord AB.

From right-angled ΔOAC,

$$OA^2 = AC^2 + OC^2$$

or $\quad OC^2 = OA^2 - AC^2 = (15)^2 - (12)^2$

$$= 225 - 144 = 81$$

$\therefore \quad OC = \sqrt{81} = 9$ cm.

29. (d) Let radius of circle $= x$

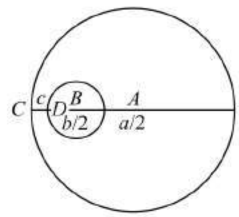
Increased radius $= 3x$

$\therefore$ New circumference $= 2\pi(3x) = 6\,\pi x$

Hence, ratio of new circumference to new diameter

$$= \frac{6\pi x}{2(3x)} = \frac{6\pi x}{6x} = \pi.$$

30. (d)



A and B are the centres of the respective circles.

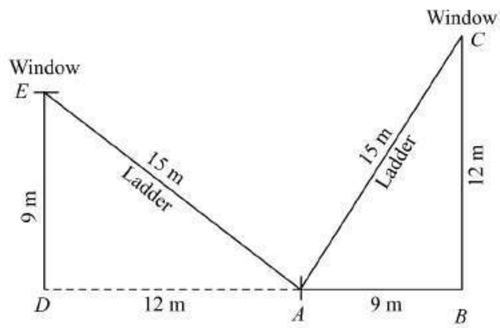
$$BA = AC - BC$$

$$= \frac{a}{2} - (CD + DB)$$

$$= \frac{a}{2} - \left(x + \frac{b}{2}\right)$$

$$= \frac{a}{2} - c - \frac{b}{2}.$$

31. (a)



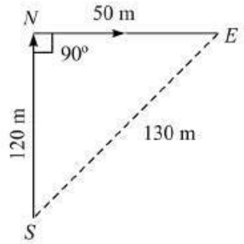
$$AB = \sqrt{(15)^2 - (12)^2} = 9$$

$$AD = \sqrt{(15)^2 - (9)^2} = 12$$

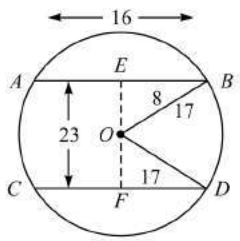
$\therefore$ Width of the street

$$= AB + DA = 21 \text{ m}$$

32. (*c*)



33. (*c*)



$AB = 16$ cm

$\therefore \quad EB = 8$ cm

Now Δ *OEB* is a right-angled triangle.

$\therefore \quad OE = \sqrt{17^2 - 8^2} = \sqrt{289 - 64}$

$= \sqrt{225} = 15$ cm

$\because \quad FE = 23$ cm

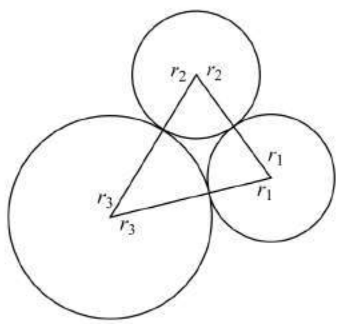
$\therefore \quad OF = 23 - 15 = 8$ cm

Now, Δ *OFD* is a right-angled triangle.

$\therefore \quad FD = \sqrt{17^2 - 8^2} = 15$ cm

$\therefore$ Length of the chord $CD = 2 \times 15 = 30$ cm

34. (*b*) Let r_1, r_2 and r_3 be the radius of three circles then



$r_1 + r_2 = 2.2 \quad (1)$

$r_2 + r_3 = 3.4 \quad (2)$

$r_1 + r_3 = 4.0 \quad (3)$

Adding (1), (2) and (3), we have

$2(r_1 + r_2 + r_3) = 9.6$

$\therefore r_1 + r_2 + r_3 = 4.8 \quad (4)$

$\therefore \quad r_3 = (4.8 - 2.2) = 2.6;$

$r_1 = (4.8 - 3.4) = 1.4;$

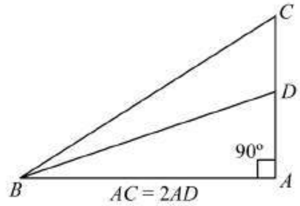
$r_2 = (4.8 - 4) = 0.8$

$\therefore$ the diameters are

$2 \times 1.4 = 2.8; 2 \times 0.8$

$= 1.6$ and $2 \times 2.6 = 5.2$

35. (*c*)



$BC^2 - BD^2 = (AB^2 + AC^2) - (AB^2 + AD^2) = AC^2 - AD^2$

$= (AC + AD)(AC - AD)$

$= (2AD + AD)(2AD - AD) = 3AD^2$

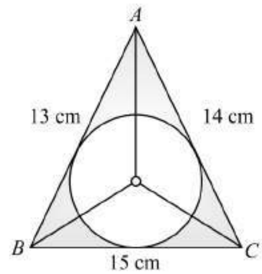
36. (*c*)

37. (*c*) Perimeter of the circle $= 2 \times \frac{22}{7} \times 35 = 220$ cm

$\therefore$ Side of the rhombus $= \frac{220}{4} = 55$ cm.

38. (*c*)

39. (*c*)



Here, $\quad s = \frac{13 + 14 + 15}{2} = \frac{42}{2} = 21$

$\therefore \quad A = \sqrt{s(s-a)(s-b)(s-c)}$

Putting the value, we get $A = 84$

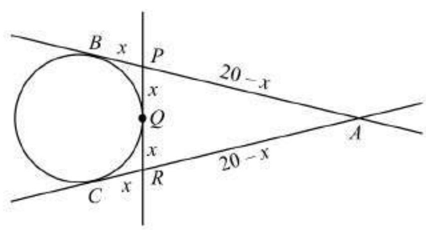
We know

Inscribed triangle area $= r\,s$

$\therefore \quad 84 = r \times 21$

$\Rightarrow \quad r = 4$ cm

40. (*c*)



Two tangents drawn from a point to a circle are congruent.

$\therefore \quad AB = AC, PB = PQ$ and $QR = RC$

Also, by symmetry of the figure,

$PQ = PR$

Let, $\quad PB = PQ = QR = CR = x$

As $\quad AB = 20, AP = AB - BP = 20 - x$

$\therefore \quad AR = 20 - x$

$\therefore$ Perimeter of

$\Delta APR = AP + PR + AR$

$= 20 - x + 2x + 20 - x = 40$ units

41. (*a*) P divides AB in the ratio of 2:3, $AP = 2x$ and $PB = 3x$

Q divides AB in the ratio 3:4

$\therefore\ AQ = 3y$ and $QR = 4y$

Now, $\quad AB = AP + PB + AQ + QB$

$\therefore \quad 2x + 3x = 3y + 4y$

$\therefore \quad 5x = 7y \qquad (1)$

It is given that $PQ = 2$

$PQ = AQ - AP$

$\therefore \quad 2 = 3y - 2x \qquad (2)$

From (1) and (2),

$\therefore\ 3y - 2\left(\frac{7}{5}y\right) = 2$

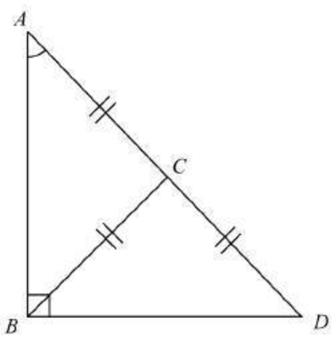
$\therefore\ 3y - \frac{14}{5}y = 2$

$\therefore\ 15y - 14y = 10$

$\therefore \quad y = 10$

So, length of $AB = 7y = 70$

42. (*b*)



As C is the mid point of the hypotenuse AD, it is the circumcentre.

$\therefore \quad AC = CD = BC$

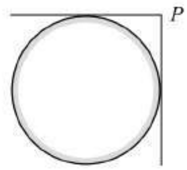
But $\quad AB = BC$

$\therefore$ In ΔABC, $AB = AC = BC$

$\therefore\ \Delta BAC$ is an equilateral triangle,

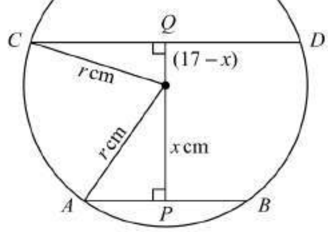
$\angle DAB = \angle BAC = 60°$

43. (*d*)



We know that tangents from a point are equidistant from the circle. Hence, they cannot have different values.

44. (*c*)



In right Δs OAP and OQC, we have

$OA^2 = OP^2 + AP^2$ and $OC^2 = OQ^2 + CQ^2$

$r^2 = x^2 + 5^2$ and $r^2 = (17 - x)^2 + 12^2$

$\Rightarrow \quad x^2 + 25 = 289 + x^2 - 34x + 144$

$\Rightarrow \quad 34x = 408 \Rightarrow x = 12$ cm

$\therefore \quad r^2 = 122 + 52 = 169$

$\therefore \quad r = 13$ cm

45. (*d*) $\quad \angle AOC = 2\angle APC$

$\therefore \quad \angle APC = 50°$

Also, $ABCP$ is a cyclic quadrilateral

$\therefore \quad \angle ABC = \angle APC$

$\therefore \quad \angle ABC = 50°$

$\therefore \quad \angle CBD = 180 - 50 = 130°$

46. (*c*) ABC is a Δ and P, Q, R are the mid-points of sides BC, CA and AB respectively

$\therefore \quad PQ \parallel AB$

and $\quad PQ = \frac{1}{2}AB = \frac{1}{2}(30)$

$= 15$ cm

Similarly, $RP \parallel AC$

and $RP = \frac{1}{2}AC = \frac{1}{2}(21) = 10.5$ cm

$\therefore$ Perimeter of $ARPQ$

$= (AR + RP + PQ + QA)$ cm

$= (15.0 + 10.5 + 15.0 + 10.5)$ cm

$= 51$ cm

47. (*c*) $CE \parallel BA$ and AC is the transversal

$\therefore \quad \angle 4 = \angle 1$ (alt. int. $\angle s$)

Again, $CE \parallel BA$ and BD is the transversal

$\therefore \quad \angle 5 = \angle 2$ (corr. $\angle s$)

$\therefore \quad \angle 4 + \angle 5 = \angle 1 + \angle 2$

$\therefore \quad \angle ACD = \angle A + \angle B$

48. (*d*) Since AM is the bisector of $\angle A$,

$\therefore \quad \angle MAB = \frac{1}{2}\angle A \quad (1)$

In rt-angled ΔANB, we have:

$\angle B + \angle NAB = 90°$

$\Rightarrow \quad \angle NAB = 90° - \angle B \quad (2)$

$\therefore \quad \angle MAN = \angle MAB - \angle NAB$

$= \frac{1}{2}\angle A - (90° - \angle B)$

$= \frac{1}{2}\angle A - 90° + \angle B$

$= \frac{1}{2}\angle A - \frac{1}{2}(\angle A + \angle B + \angle C) + \angle B$

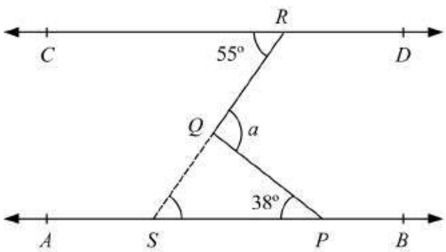
$\left(\because \frac{1}{2}(\angle A + \angle B + \angle C) = 90°\right)$

$= \frac{1}{2}(\angle B - \angle C)$

49. (*a*) $CD \parallel AB$ (Given)

Produce RQ to meet AB in S

$\angle CRS = \angle PSR$ (at. int. $\angle s$)



But $\angle CRS = 55°$

$\therefore \quad \angle PSR = 55°$

Now in QSP

$\angle QSP + \angle QPS + \angle PQS = 180°$

$55° + 38° + \angle SQP = 180°$

$\therefore \quad \angle SQP = 180° - 93°$

$= 87°$

But angle a and $\angle PQS$ are linear

$\therefore \quad \angle a = 180° - 87°$

$\angle a = 93°$

50. (*d*) Through O, draw a line l parallel to both AB and CD. Then

$\angle 1 = 45°$ (alt. $\angle S$)

and $\angle 2 = 30°$ (alt. $\angle S$)

$\therefore \quad \angle BOC = \angle 1 + \angle 2$

$= 45° + 30° = 75°$

So, $X = 360° - \angle BOC$

$= 360° - 75° = 285°$

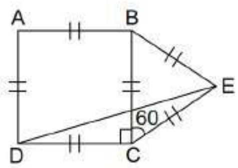
Hence, $X = 285°$.

51. (*c*) $\frac{ar(\Delta CMN)}{ar(\Delta ABNM)} = \frac{1}{2}$

$\therefore \frac{ar(\Delta CMN)}{ar(\Delta CAB)} = \frac{1}{3}$

$\Rightarrow \frac{MN}{AB} = \frac{CM}{CA} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{CM}{MA} = \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{2} \quad MA = (CA - CM)$



52. (*a*) In $\Delta DEC, \angle DCE = 90° + 60° = 150°$

$\angle CDE = \angle DEC = \frac{180 - 150}{2} = 15°$

Difficulty Level-2

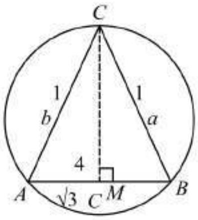
1. (*d*) $a:b:c = 1:1:\sqrt{3}$

ΔCAB is an isosceles triangle with $AC = CB$

Let M be the mid point of AB

$\therefore \quad CM \perp AB$ and $AM = MB = \dfrac{\sqrt{3}}{2}$

$\therefore \quad \dfrac{\text{AM}}{\text{AC}} = \dfrac{\sqrt{3}}{2} \Rightarrow \angle A = 30^\circ$



Similarly, $\quad \angle B = 30^\circ$ and $\angle C = 120^\circ$

$\therefore \quad \text{arc}(AB) = 2\angle C$, $\text{arc}(AC) = 2\angle B$,

$\text{arc}(BC) = 2\angle A$

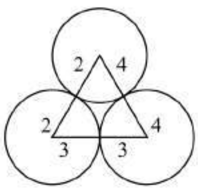
$\therefore \quad \text{arc}(AB):\text{arc}(BC):\text{arc}(AC)$

$= 2 \times 120 : 2 \times 30 : 2 \times 30$

$= 4:1:1$

2. (*d*)

3. (*d*)



Sum of radii = 2 + 3 + 4 = 9 cm.

4. (*a*)

5. (*d*)

6. (*d*)

7. (*b*) Since ABC is an equilateral triangle, then each side of the triangle would be 2 Km each. Required distance would be the altitude of the triangle

$\Rightarrow \quad \dfrac{\sqrt{3}}{4}(2)^2 = \dfrac{1}{2} \times 2 \times \text{Altitude}$

$\Rightarrow \quad \text{Altitude} = \sqrt{3}$ km

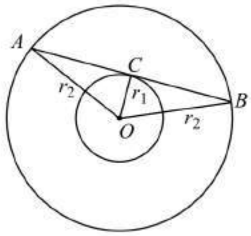
8. (*d*) Since each side of the triangle is 2 Km each, hence required distance is $BD + DB + BE = 6$ Km

9. (*c*) To form a triangle, 3 points out of 5 can be chosen in ${}^5C_3$ ways

$= \dfrac{5 \times 4 \times 3}{1 \times 2 \times 3} = 10$ ways

But of these, the 3 points using on the 2 diagonals will be collinear. So $(10 - 2) = 8$

10. (*b*) Starting from A, the possible roots are



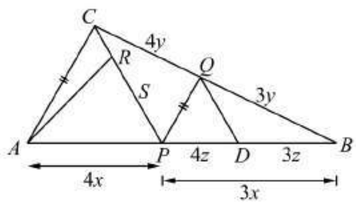
AOBA, ACOBA, ACBA, AOCBA,
AOCA, AOBCA, ABOA, ABOCA,
ABCA, ABCOA, ACOA, ACBOA.

11. (*d*) Cannot be determined

12. (*d*) Cannot be determined

13. (*c*) $\quad PD = \dfrac{4z}{7z} \times 3x$

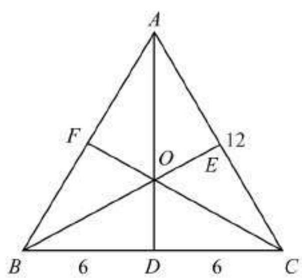
$= \dfrac{12x}{7}$



$AP = 4x$

$\therefore \quad AP:PD = 4x : \dfrac{12x}{7} = 7:3$

14. (*c*) Circumcircle of a triangle is the point of intersection of the perpendicular bisectors of the sides of the triangle.



O is the circumcentre of the ΔABC, whose sides

$$AB = BC = CA = 12 \text{ cm}$$

$\therefore$ From ΔADC

$$AC^2 = AD^2 + CD^2$$

$$\Rightarrow \quad AD = \sqrt{(12)^2 - 6^2} = 6\sqrt{3}$$

Since, triangle is equilateral, therefore circumcentre = centroid

$\therefore \quad AO:OD = 2:1$

i.e., $\quad AO = 4\sqrt{3}$,

$OD = 2\sqrt{3} \quad [\because AD = 6\sqrt{3}]$

$\therefore$ Radius of the circumcircle

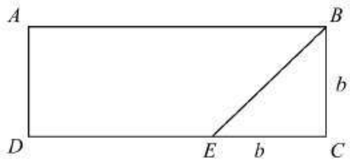
$$= 4\sqrt{3} = OA = OB = OC.$$

15. (*a*) Area of $\Delta BCE = \frac{1}{2} \times b \times b$

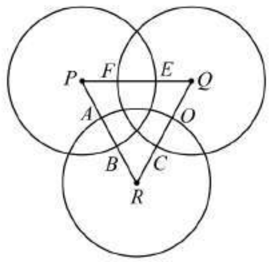
$\Rightarrow \quad b^2 = 28$

Area of rectangle

$$ABCD = (DE + EC) \times b = 4EC \times b$$
$$= 4b^2 = 112.$$



16. (*c*) $\quad AR = AB + BR = 20$



$\Rightarrow \quad 5 + BR = 20$ or $BR = 15$

Similarly, $PA = 15$

Also $PE = PF + FE = 20 \Rightarrow PF + 12 = 20$

$\Rightarrow \quad PF = 8 = EQ$

Similarly, we find QO and CR, they will come out to be 10.

$\therefore$ Perimeter of $\Delta PQR = 93$

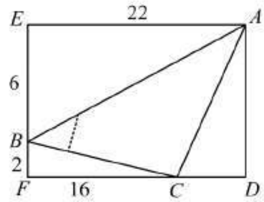
17. (*c*) The perimeter of any polygon circumscribing a circle is always greater than the circumference of the circle and the perimeter of any polygon inscribed in a circle is always less than the circumference of the circle.

Since, the circle is of radius 1, its circumference will be 2π.

Hence, $L_1(13) > 2\pi$ and $L_2(17) < \pi$

So, $\{L_1(13) + 2\pi\} > 4$ and hence $\frac{\{L_1(13) + 2\pi\}}{L_2(17)}$ will be greater than 2.

18. (*b*) $\quad EF = AD = 8 \quad (\therefore EADF$ is a rectangle)



$$CD = (22 - 16) = 6$$

So in the right-angled triangle ADC, $AD = 84$ and $CD = 6$.

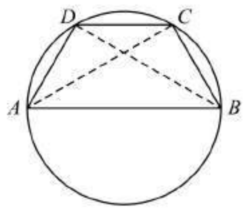
$\therefore \quad AC = 10$

$\therefore$ length of the line joining the mid-points of AB and

$$BC = \frac{1}{2}(10) = 5$$

($\because$ the length of the line joining the mid-point of two sides of triangle is half the 3rd)

19. (*a*) Perpendicular drawn from the centre bisects the chord, hence $AC = BC = 3$ m. Using options, we find that if the radius of outer circle is 5 m. only then the radius of inner circle is an integer.



$$r_1^2 = (5)^2 - (3)^2 = 16$$

$\Rightarrow \quad r_1 = 4$

Hence $\quad r_1 = 4m$ and $r_2 = 5m$

20. (*c*) In ΔCAD, $CD^2 = (15)^2 - (25 - x)^2$

$$= 225 - 625 - x^2 + 50x \quad (1)$$

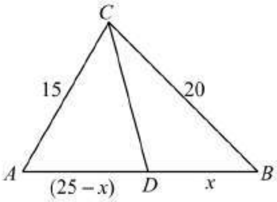
In ΔCDB, $CD^2 = 400 - x^2 \quad (2)$

From (1) and (2),

$$50x - x^2 - 400 = 400 - x^2$$

$\Rightarrow \quad x = 16$ cm.

Hence, $AD = 9$ cm, $BD = 16$ cm and $CD = 12$ cm



Now for ΔCAD,

$$\text{Area} = \frac{1}{2} \times 9 \times 12 = 54 \text{ cm}$$

and $$s = \frac{15+12+9}{2} = 18 \text{ cm}$$

$\therefore$ $$\text{radius } r_1 = \frac{\text{Area}}{s} = \frac{54}{18} = 3\text{cm}$$

For ΔCDB,

$$\text{Area} = \frac{1}{2} \times 16 \times 12 = 96 \text{ cm}$$

and $$s = \frac{16+12+20}{2} = 24 \text{ cm}$$

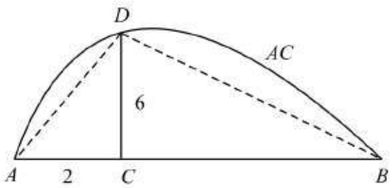
$\therefore$ $$\text{radius } r_2 = \frac{\text{Area}}{s} = \frac{96}{24} = 4$$

Hence, $r_1 + r_2 = 4 + 3 = 7$ cm.

21. (*b*) The sum of the internal angles of a polygon of n sides $= (n-2) \times 180°$.

If $n = 7$, then the sum of the interior angles of the given polygon $= (7-2) \times 180° = 900°$.

22. (*b*)



$CD^2 = AC \times CB$

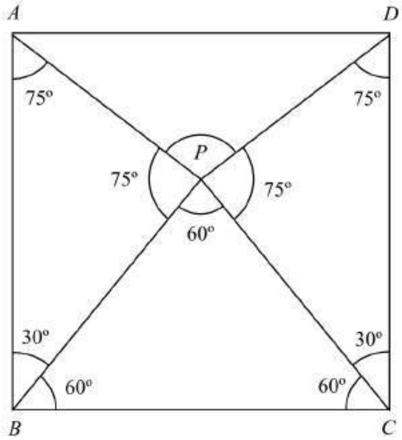
$(6)^2 = 2 \times CB$

$CB = 18$

$AB = AC + CB = 18 + 2 = 20$

$$\text{Area of semicircle} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \times (10)^2 = 50\,\pi \text{ cm}^2$$

23. (*d*) ΔBPC is an equilateral triangle



$\therefore$ $\angle CPD = \angle CDP = 75°$

Similarly, $\angle BAP = \angle BPA = 75°$

Hence, $\angle APD = 360° - (75° + 75° + 60°)$

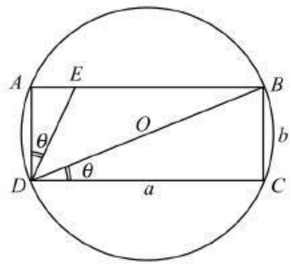
$= 360° - 210° = 150°$

24. (*a*) We have $\dfrac{\pi R^2}{ab} = \dfrac{\pi}{\sqrt{3}}$

$\Rightarrow$ $\sqrt{3}R^2 = ab$ (*a*)

From ΔDBC,

$$\tan\theta = \frac{BC}{DC} = \frac{b}{a} \quad (b)$$



From ΔDAE,

$$\tan\theta = \frac{AE}{AD} = \frac{AE}{b} \quad (c)$$

From (*b*) and (*c*), we get

$$\frac{AE}{AD} = \frac{b}{a}$$

From triangle DBC, $4R^2 = a^2 + b^2$

$\Rightarrow$ $4R^2 = a^2 + \dfrac{3R^4}{a^2}$

$\Rightarrow$ $a^4 - 4R^2a^2 + 3R^4 = 0$

$\Rightarrow$ $a^4 - 3R^2a^2 - R^2a^2 + 3R^4 = 0$

$\Rightarrow$ $a^2(a^2 - 3R^2) - R^2(a^2 - 3R^2) = 0$

$\Rightarrow$ $a^2 = R^2$ and $a^2 = 3R^2$

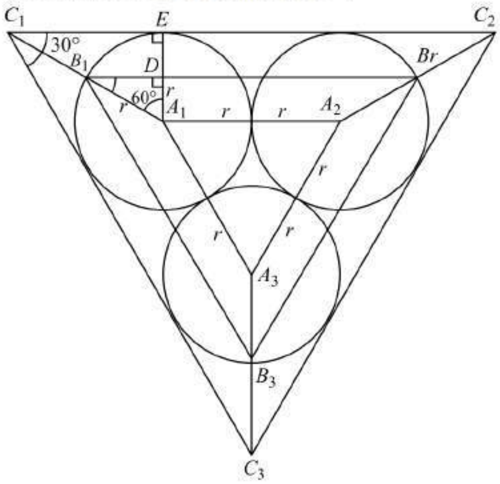
□ $\quad a = R$ and $a = \sqrt{3}R$

and $\quad b = \sqrt{3}R$, when $a = R$

$b = R$, when $a = \sqrt{3}R$

Hence, required ratio is $1 : \sqrt{3}$.

25. (*a*) Given radius of each circular park $= r$



$\Rightarrow$ Distance travelled by

$$A = a = 3 \times 2r = 6r$$

ΔA_1B_1D is a 30°, 60°, 90° triangle.

So, $\quad B_1D = \dfrac{\sqrt{3}r}{2}$

$$\Rightarrow \quad B_1B_2 = 2r + 2 \times \frac{\sqrt{3}}{2} r = r\left(2 + \sqrt{3}\right)$$

$\Rightarrow$ Distance travelled by B

$$= b = 3 \times r\left(2 + \sqrt{3}\right) = 3r\left(2 + \sqrt{3}\right)$$

ΔA_1C_1E is a 30°, 60°, 90° triangle.

So, $\quad C_1E = \sqrt{3}r$

$$\Rightarrow \quad C_1C_2 = 2r + 2\sqrt{3}r = 2r(1 + \sqrt{3})$$

$\Rightarrow$ Distance travelled by $C = c$

$$= 3 \times 2r\,(1 + \sqrt{3}) = 6r(1 + \sqrt{3})$$

Now, $\quad b - a = 3\sqrt{3}r$ and $c - b = 3\sqrt{3}r$.

26. (*c*) Time required by *A* to finish her sprint

$$= \frac{2r}{20} + \frac{2r}{30} + \frac{2r}{15} = \frac{3r}{10}$$

Now, distance travelled by *B* in this time

$$= \frac{3r}{10} \times (10\sqrt{3} + 20) = s\,3(r + 2\sqrt{3})$$

So, *B* will be at B_1

Now, distance travelled by *C* in this time

$$= \frac{40}{3}(1 + \sqrt{3}) \times \frac{3r}{10} = 4r\,(1 + \sqrt{3})$$

So, *C* will be on point C_3.

27. (*b*) $$u^2 = \frac{\sqrt{3}}{4} \times (2r)^2 \Rightarrow u = 3^{1/4}.r$$

$$v^2 = \frac{\sqrt{3}}{4} \times \{r(2 + \sqrt{3})\}^2$$

$$\Rightarrow \quad v = \frac{3^{1/4}\,(2 + \sqrt{3})r}{2}$$

$$w^2 = \frac{\sqrt{3}}{4} \times \{2r(1 + \sqrt{3})\}^2$$

$$\Rightarrow \quad w = 3^{1/4}.r(1 + \sqrt{3})$$

Time required by *B* to reach B_3

$$= \frac{2r\,(2 + \sqrt{3}) \times 2}{3^{1/4}\,r(2 + \sqrt{3})} = \frac{4}{3^{1/4}}$$

Distance covered by *A* in this time

$$= 3^{1/4}.r.\frac{4}{3^{1/4}} = 4r$$

So, A will be at A_3.

Distance covered by *C* in this time

$$= 3^{1/4}.r.(1 + \sqrt{3}) \times \frac{4}{3^{1/4}}$$

$$= 4r(1 + \sqrt{3})$$

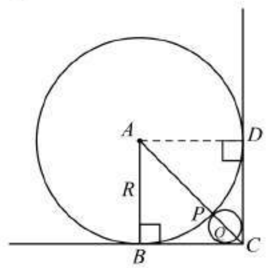
So, *C* will be at C_3.

28. (*d*) Let the radii of the bigger and smaller circles be *R* and *r*, respectively.

$\therefore$ In the figure, $AB = AD = R$

As $\quad \angle ADC = 90^\circ$; $\angle ABC = 90^\circ$ and $\angle DCB = 90^\circ$

$\therefore$ *ABCD* is square.



$\therefore \quad BC = R$ and $AC = \sqrt{2}R$ and

$$AC = AP + PQ + QC$$

$$= R + r + QC$$

($QC = \sqrt{2}$ can be proved in the same ways as we proved $AC = \sqrt{2}R$)

$\therefore \quad r = \dfrac{(\sqrt{2}-1)R}{\sqrt{2}+1}$

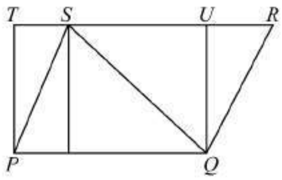
Rationalizing the denominator, we get

$$r = 2(3-2\sqrt{2})R$$

Given $R = 2$, we get

$$r = (3-2\sqrt{2})$$
$$= 6-4\sqrt{2}.$$

29. (*a*, *c*) Given



Area of $PQRS = P$, Area of $PQUT = R$ and Area of $PSQ = T$

Now, area of $\Delta = \dfrac{1}{2}$ Area of parallelogram $PQRS$

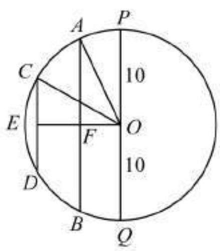
hence option (*a*) is true. Also, area of parallelogram = Area of rectangle (constructed on the same base)

Hence options (*a*) and (*c*) are correct.

30. (*a*) OE is the required distance

$$AF = \sqrt{(10)^2-(6)^2} = 8$$

$\therefore \quad CE = 4$

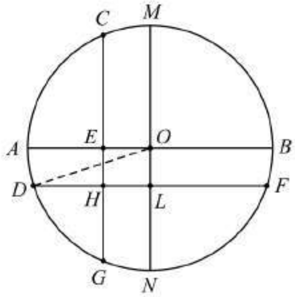


Now, in ΔOCE

$$OE^2 = OC^2 - CE^2$$
$$= 10^2 - 4^2 = 84$$

$\therefore \quad OE = \sqrt{84}$

31. (*b*) $\quad HL = OE = \dfrac{1}{2}$



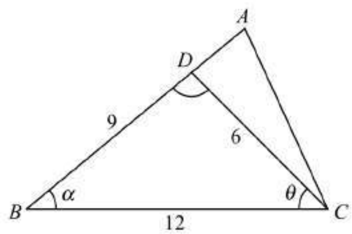
$$DL = DH + HL$$
$$DL = DH + \frac{1}{2}$$
$$OB = AO = \text{radius} = 1.5$$
$$DO^2 = OL^2 + DL^2$$
$$\left(\frac{3}{2}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(DH+\frac{1}{2}\right)^2$$

$\Rightarrow \quad \left(DH+\dfrac{1}{2}\right)^2 = 2$

$\Rightarrow \quad DH = \sqrt{2}-\dfrac{1}{2} = \dfrac{2\sqrt{2}-1}{2}$

32. (*a*) Here, $\angle ACB = \theta + 180 - (2\theta + \alpha) = 180 - (\theta + \alpha)$

So, here we can say that ΔBCD and ΔABC will be similar.



According to property of similarity

$$\frac{AB}{12} = \frac{12}{9}$$

Hence, $AB = 16$,

$$\frac{AC}{6} = \frac{12}{9}$$

$\Rightarrow \quad AC = 8$

Hence, $\quad AD = 7, AC = 8$

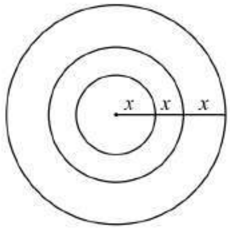
Now, $\quad \dfrac{\text{Perimeter of } \Delta ADC}{\text{Perimeter of } \Delta BDC} = \dfrac{6+7+8}{9+6+12}$

$$= \frac{21}{27} = \frac{7}{9}$$

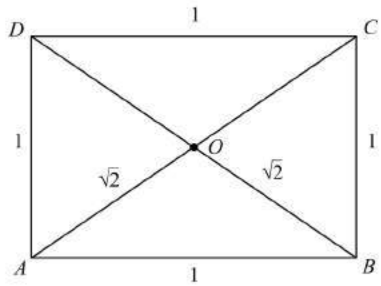
33. (*c*) Area of a circle = πr^2

$\therefore$ Required ratio = $\dfrac{\pi(2x^2)-\pi(x^2)}{\pi(3x)^2-\pi(2x)^2}$

$= \dfrac{\pi x^2(4-1)}{\pi x^2(9-4)} = \dfrac{3}{5}$



34. (*b*) Two points on or inside the square will be at the maximum distance when they are on two opposite vertices. Let us select 4 points on the vertices of the square. Then, the distance between any two of them is 1 or $\sqrt{2}$.



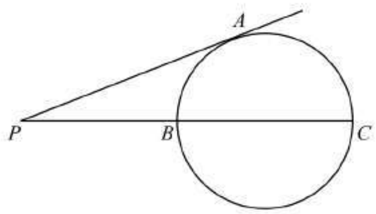
In the square $AB = BC = CD = DA = 1$

and $AC = BD = \sqrt{2}$

Now we select the fifth point such that it is at the maximum possible distance from each of the other four points. Such a point lies on the point of intersection of the diagonals and its distance from each of the other four points is $\sqrt{2}/2$.

Any other point on or inside the square will be at a distance less than $\sqrt{2}/2$ from at least one of the other four points.

35. (*b*) $(PB)(PC) = (PA)^2$



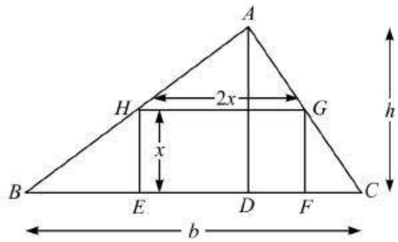
$x(x+20) = (10\sqrt{3})^2$

$x^2 + 20x - 300 = 0$

$(x+30)(x-10) = 0$

$x = 10$

36. (*c*) ΔBEH and ΔBDA are similar



$$\frac{BE}{BD} = \frac{HE}{AD}$$

$$\frac{BE}{BD} = \frac{x}{h} \qquad (1)$$

ΔCFG and ΔCDA are similar

$$\frac{CF}{CD} = \frac{FG}{DA}$$

$$\frac{CF}{CD} = \frac{x}{h} \qquad (2)$$

From Eqs. (1) and (2),

$$BE = \frac{x}{h}BD \text{ and } CF = \frac{x}{h}CD$$

$$BE + CF = b - 2x$$

$$b - 2x = \frac{x}{h}(BD + CD)$$

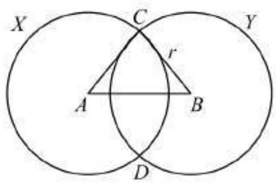
$$= \frac{x}{h}(b)$$

$$bh - 2hx = xb$$

$$x(b + 2h) = bh$$

$$x = \frac{bh}{b+2h}$$

37. (*b*) $\angle ACB = 90°$



[angle at the point of intersection to the centres of the circles.]

$$BC = r$$

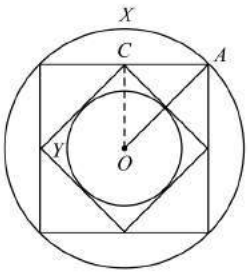
$$AC = 2r \quad \text{(as area of X = 4 area of Y)}$$

$$\therefore \quad AB = \sqrt{r^2 + 4r^2} = \sqrt{5}r$$

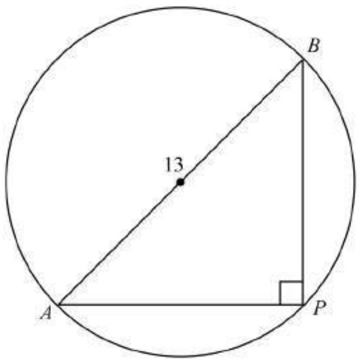
38. (*b*) $OA = R,$

$$\therefore \quad OC = \frac{R}{\sqrt{2}} \text{ and } \therefore OB = \frac{R}{2}$$

$$\therefore \quad \frac{\text{Area of circle } A}{\text{Area of circle } B} = \frac{\pi R^2}{\pi\left(\frac{R}{2}\right)^2} = \frac{4}{1}$$



39. (*c*) Let P be the required location of the pole. Let the distance of the pole from the gate B be x m i.e., $BP = x$ m.



Now the difference of the distance of the pole from the two gates

$$= AP - BP$$

$$= 7 \text{ m. (given)}$$

$$\therefore \quad AP = (x + 7) \text{ m}$$

Since $\angle P$ is an angle in semi-circle

$$\angle P = 90°$$

and $AB = 13$ m (diameter of circle)

$\therefore$ In right-angled $\Delta\, ABP$

$$AB^2 = AP^2 + BP^2$$

$$\Rightarrow \quad (13)^2 = (x + 7)^2 + x^2$$

$$\Rightarrow \quad 169 = x^2 + 14x + 49 + x^2$$

$$\Rightarrow \quad 2x^2 + 14x + 49 - 169 = 0$$

$$\Rightarrow \quad 2x^2 + 14x - 120 = 0$$

$$\Rightarrow \quad x^2 + 7x - 60 = 0$$

$$\Rightarrow \quad x^2 + 12x - 5x - 60 = 0$$

$$\Rightarrow \quad x(x + 12) - 5(x + 12) = 0$$

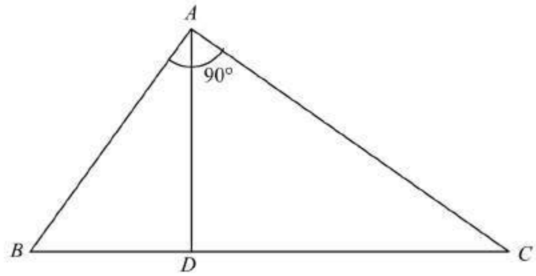
$$\Rightarrow \quad (x + 12)(x - 5) = 0$$

$$x = 5 \text{ or, } -12$$

(which is not possible)

$\therefore$ Required distance be 5 m

40. (*a*)



$\Delta\, ABC$ is a right-angled triangle with $\angle A = 90°$

$$\therefore \quad AD^2 = CD^2 = AC^2$$

In $\Delta\, ABC$, according to Apollonius theorem

$$= 2\, AD^2 + 2\, BD^2 = AB^2 + AC^2$$

$$= 2(CD^2 - AC^2) + 2BD^2 = AB^2 + AC^2$$

$$\therefore \quad BD = CD$$

$$\therefore \quad BD^2 = CD^2 = \frac{BC^2}{4}$$

$$\Rightarrow = \frac{BC^2}{2} - 2AC^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

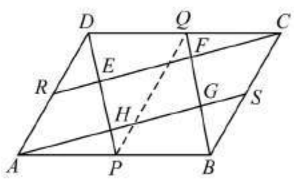
$$\Rightarrow = BC^2 - AB^2 = 3AC^2$$

41. (*a*) Q and P are mid points of DC and AB respectively

= area of ΔADP = aera of ΔDPQ = area of ΔPQB

= area of $\Delta QBC = \frac{1}{4}$

(area of parallelogram $ABCD$)



In ΔADH,

$$DR = RA,\ RE \parallel AH \Rightarrow DE = EH$$

In ΔABG, $AP = PB = \frac{1}{2}AB$, $PH \parallel BG$

$\Rightarrow$ $PH = \frac{1}{2}BG$

But $BG = DE$ (by symmetry)

$\Rightarrow$ $DE = EH = 2.HP$

$\Rightarrow$ $DE = \frac{2}{5}DP$

In ΔADP, $DR = \frac{1}{2}DA$, $DE = \frac{2}{5}DP$

$\Rightarrow$ area of $\Delta RDE = \frac{1}{2}\times\frac{2}{5}\times$ area of ΔADP

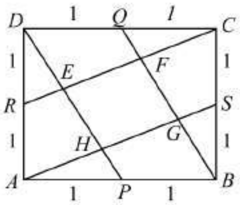
$= \frac{1}{2}\times\frac{2}{5}\times\frac{1}{4}\times$ area of parallelogram $ABCD$

$= \frac{1}{20}\times$ area of parallelogram $ABCD$

Hence, $\frac{\text{area of shaded region}}{\text{parallelogram } ABCD} = 4\times\frac{1}{20} = 1:5$

Short-cut Method

A square is a parallelogram.



Since all triangles are similar,

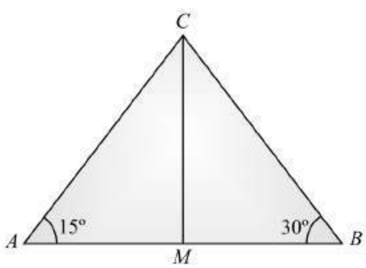
$$DE = \frac{2}{\sqrt{5}} \text{ and } ER = \frac{1}{\sqrt{5}}$$

$$\text{area of } \Delta DER = \frac{1}{2}\times\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{5}} = \frac{1}{5}$$

$\Rightarrow$ area of shaded region $= 4\times\frac{1}{5} = \frac{4}{5}$

$\therefore$ Ratio $= \frac{\frac{4}{5}}{2\times 2} = \frac{1}{5}$

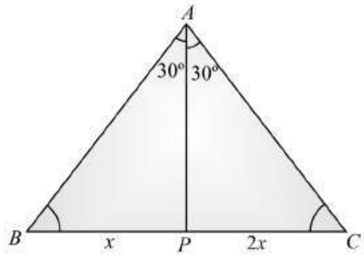
42. (*c*)



$$\angle C = 180° - (15 + 30°) = 135°$$

Since, M is the mid point, $\angle ACB$ is divided in the ratio of 30:15 or 2:1. Hence, $\angle ACM = 45°$.

43. (*d*) Using Angle bisector theorem



Using sine formula,

$$\frac{AC}{\sin B} = \frac{BA}{\sin C} \Rightarrow \frac{\sin C}{\sin B} = \frac{1}{2}$$

Since, ABC is a triangle.

$\therefore$ $\angle C = 30°$ and $\angle B = 90°$

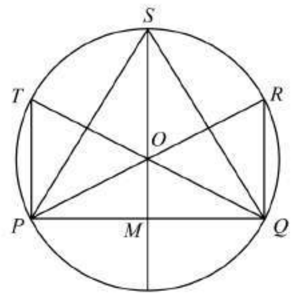
$\left[\text{as } \sin 30° = \frac{1}{2}, \sin 90° = 1\right]$

So, in ΔAPC,

$$\angle APC = 180° - (30° + 30°) = 120°$$

44. (*d*) Since $OP = OQ = r$

$\therefore$ $\angle OPQ = \angle OQP = \frac{180° - 128°}{2} = 26°$



Again $SO \perp PQ$, so $\angle OMQ = 90°$

$\Rightarrow$ $\angle MOQ = 180° - 90° - 26° = 64°$

$\Rightarrow$ $\angle QOS = 180° - 64° = 116°$

$\Rightarrow$ $\angle OQS = \angle OSQ$

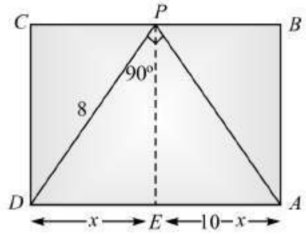
$$= \frac{180° - 116°}{2} = 32$$

[Since $OQ = OS = r$]

$\Rightarrow$ $\angle TPS = \angle OQS = 32°$

(Angles by same arc).

45. (*a*)



In ΔPDA, $AP^2 = 10^2 - 8^2 = 36$

$\Rightarrow \quad AP = 6$

In ΔDPE and ΔPEA,

$$(PE)^2 = 8^2 - x^2 \text{ and } (PE)^2$$

$$= 6^2 - (10 - x)^2$$

$\therefore \quad 64 - x^2 = 36 - (10 - x)^2$

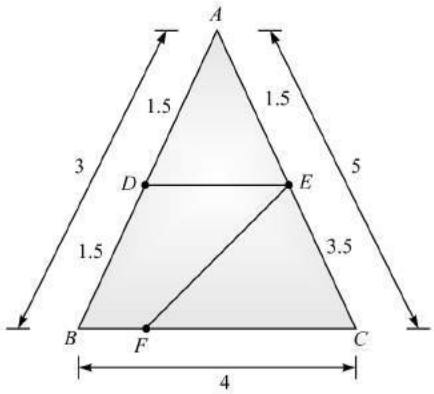
$\Rightarrow \quad x = 6.4 = PE$

$$PB = E - A = 10 - x$$

$$PB = 10 - 6.4 = 3.6$$

$\therefore \quad PB = 3.6$

46. (*b*)



In the given $\Delta\, ABC$

$$AB = 3,\ BC = 4$$

and $\quad CA = 5,\ AD = BD = 1.5$

$$AE = 1.5$$

$\therefore \quad CE = 5 - 1.5 = 3.5,\ BC = 4$

$\therefore \quad \angle DEF = (90° + 45°)\dfrac{1}{3} = 135 \times \dfrac{1}{3} = 45°$

47. (*b*) $\Delta PQS \sim \Delta PMN \sim \Delta PRT$

$\therefore$ N is the mid-point of ST.

Also, in ΔPQS,

$$(PS)^2 = (PQ)^2 + (QS)^2 =$$

$\therefore \quad PS^2 = 576 + 49 = 625$

$\therefore \quad PS = 25$

As $\Delta PQS \sim \Delta PRT$,

$\Rightarrow \quad \dfrac{QS}{RT} = \dfrac{PQ}{PR} = \dfrac{PS}{PT} = \dfrac{1}{3}$

$\therefore \quad PR = 3 \times PQ = 72$

and $\quad PT = 3 \times PS = 75$

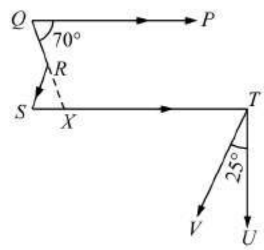
$\therefore \quad ST = PT - PS = 50$

$\therefore \quad SN = \dfrac{50}{2} = 25$

48. (*c*) Join RX

$\angle RXS = \angle PQR = 70°$ (Alternative angles)

Also, $\angle STV = \angle RSX$ (Alternative angles)



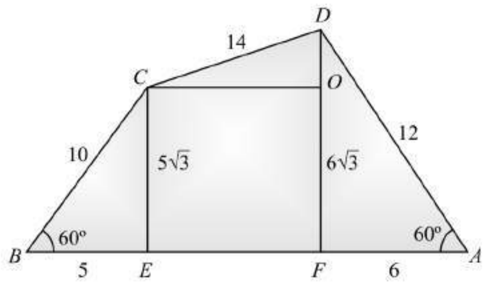
But $\quad \angle STV = 90° - 25° = 65°$

$\therefore \quad \angle RSX = 65°$

$$\angle QRS = \angle RSX + \angle RXS$$

$$= 70° + 65° = 135°$$

49. (*c*)



In $\Delta\, ADF$, $\sin 60° = \dfrac{DF}{AD}$

$\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{DF}{12}$

$\Rightarrow \quad DF = 6\sqrt{3}$

Similarly, in $\Delta\, BCE$

$$CE = 5\sqrt{3}$$

$\therefore$ In quadrilateral $COEF$

$\therefore \quad OF = 5\sqrt{3}$

$\therefore$ In $\Delta\, COD,\ OD = \sqrt{3}$

$\therefore \qquad OC = \sqrt{196-3}$

$= \sqrt{193}$

$\therefore \qquad EF = \sqrt{193}$

By using trigonometry we know $BE = 5$ and $AF = 6$

$\therefore \qquad AB = EF + AF + BE$

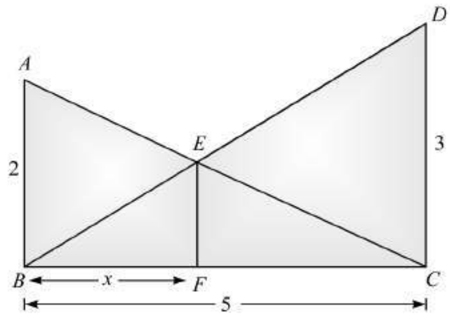
$= \sqrt{193} + 6 + 5 = 11 + \sqrt{193}$

$= a + \sqrt{b}$ (given)

$\therefore \qquad a = 11,\ \sqrt{b} = 193$

$\therefore \qquad a + b = 11 + 193 = 204$

50. (*a*)



$$\frac{AB}{EF} = \frac{BC}{CF} \Rightarrow \frac{2}{EF} = \frac{5}{5-x}$$

$\Rightarrow \qquad 5EF = 10 - 2x \qquad (1)$

$$\frac{CD}{EF} = \frac{BC}{BF} \Rightarrow \frac{3}{EF} = \frac{5}{x}$$

$\Rightarrow \qquad 5EF = 3x \qquad (2)$

On equating (1) and (2), we get $BF = 2$

Now, $\qquad \frac{AB}{EF} = \frac{5}{3} \Rightarrow EF$

$= 6/5 = 1.2$ metres

51. (*c*) Sum of the angles of the seven triangles

$= 180° \times 7$

$= 1260°$

The base angles of the triangles are the exterior angles of the seven-sided polygon.

Their sum $= 2 \times 360°$

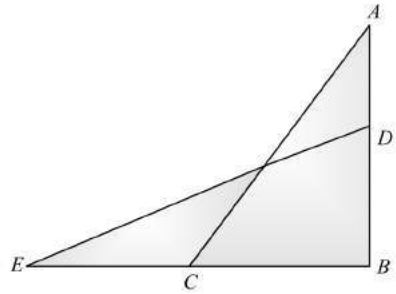
$= 720°$

$\therefore$ The sum of the angles at the vertices marked

$= 1260° - 720°$

$= 540°$

52. (*d*) Let the initial position of the ladder $= AC$.



The base C is drawn out, by $2x$ to E. As a result the top A comes down by x to D.

$BC = 7,\ AC = 25$, so $AB = 24$

In ΔDBE,

$DB^2 + BE^2 = DE^2$

$(24 - x)^2 + (7 + 2x)^2 = 25^2$

$24^2 - 48x + x^2 + 7^2 + 28x + 4x^2 = 25^2$

$5x^2 = 20x$

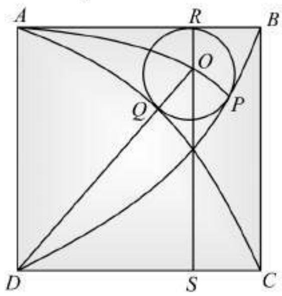
$\Rightarrow \qquad x = 4 \qquad (\text{as } x \neq 0))$

So, $\qquad EC = 8$

The interval (5, 8) does not include 8.

So, we have to select option (*d*).

53. (*b*) Let the side of the square $= 1$



Let the radius of the circle $= x$

$AO = AP - OP = 1 - x$

and $\qquad OR = x$

$DO = DQ + QO = 1 + x$

and $\qquad OS = 1 - x$

So, $(1 - x)^2 - x^2 = (1 + x)^2 - (1 - x)^2$

$$1 = 6x \Rightarrow x = \frac{1}{6}$$

As $\qquad AB = 60,\ OR = \frac{1}{6} \times 60 = 10$

54. (*a*) In ΔADP

Ext $\quad \angle ADC =$ Interior $(\angle A + \angle P)$

$= 40° + 20° = 60°$

$\therefore \qquad \angle ABC = \angle ADC = 60$

Since AD is the diameter

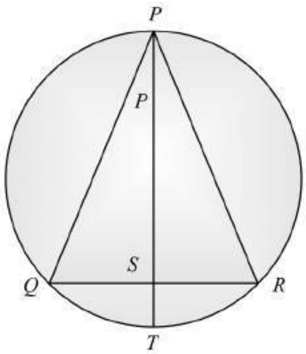
$\Rightarrow \quad \angle ABD = 90°$

$\therefore \quad \angle DBC = \angle ABD - \angle ABC$

$= 90° - 60° = 30°$

55. (*d*) $(PS)(ST) = (QS)(SR)$

$AM > GM$



$$\frac{\frac{1}{PS}+\frac{1}{ST}}{2} \geq \frac{1}{\sqrt{(PS)(ST)}}$$

$$\frac{1}{PS}+\frac{1}{ST} \geq \frac{2}{\sqrt{(PS)(ST)}}$$

$$\frac{1}{PS}+\frac{1}{ST} \geq \frac{2}{\sqrt{(QS)(SR)}}$$

Again $\frac{(QS)+(SR)}{2} \geq \sqrt{(QS)+(SR)}$

$$\frac{QR}{2} \geq \sqrt{(QS)+(SR)}$$

$$\frac{1}{PS}+\frac{1}{ST} > \frac{2}{\frac{QR}{2}} \quad (\because S \text{ is not the centre})$$

$$\frac{1}{PS}+\frac{1}{ST} \geq \frac{4}{QR}$$

56. (*c*) $\angle QSR = \angle QTR = \frac{z}{2}$

$\therefore \quad \angle PSM = \angle PTM = 180° - \frac{z}{2}$

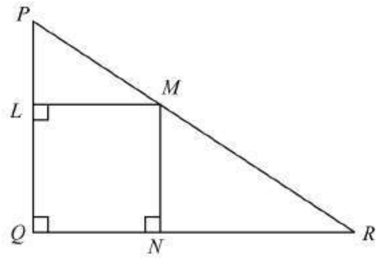
Also, $\angle SMR = y$

$\therefore$ In quadrilateral *PSMT*

$$180 - \frac{z}{2} + 180 - \frac{z}{2} + y + x = 360.$$

$\Rightarrow x + y = z$

57. (*c*)

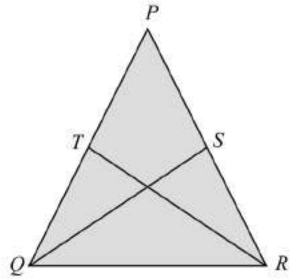


LN will be minimum only when $LM = MN$

$\Rightarrow$ *QLMN* is a square.

$\Rightarrow$ In ΔLMN, $m\angle PQM = m\angle MQR = 45°$

58. (*c*) Sine, $PQ = PR$



$\angle TQS = 80° - 60° = 20°$,

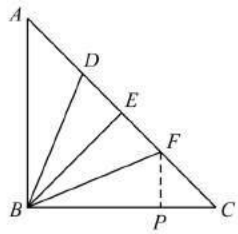
$\angle TRS = 80° - 50° = 30°$

$\angle QTR = 180° - 80° = 100°$,

$\angle RSQ = 180° - 80° - 60° = 40°$,

$\angle STR = 80°$

59. (*c*)



Let *ABC* be the park and *BD*, *BE* and *BF* be the paths such that

$$AD = DE = EF = FC = 20 \text{ m}$$

Let $AB = x$ and $BC = y$.

Let *FP* be perpendicular from *F* to *BC*.

Then, $FP = \frac{1}{4}x$ and

$$BP = \frac{3y}{4}$$

$$\Rightarrow \quad BF = \sqrt{\left(\frac{1}{4}x\right)^2 + \left(\frac{3}{4}y\right)}$$

or, $\quad BF^2 = \frac{1}{16}x^2 + \frac{9}{16}y^2$

Similarly,

$$BE^2 = \frac{1}{4}x^2 + \frac{1}{4}y^2$$

$$BD^2 = \frac{1}{16}x^2 + \frac{9}{16}y^2$$

$$\Rightarrow BF^2 = BE^2 + BD^2$$

$$= x^2\left(\frac{1}{16} + \frac{4}{16} + \frac{9}{16}\right) + y^2\left(\frac{9}{16} + \frac{4}{16} + \frac{1}{16}\right)$$

$$= \frac{14}{16}(x^2 + y^2)$$

$$= \frac{14}{16}(80 \times 80) \qquad [\because \quad (x^2 + y^2) = 80^2]$$

$$= 5600$$

60. (*a*) $\quad OR = OS, OR \perp DR$ and $OS \perp DS$

$\therefore$ $ORDS$ is a square

Also, $\quad BP = BQ, CQ = CR$ and $DR = DS$

$\therefore \quad BQ = BP = 27$ cm

$\Rightarrow \quad BC - CQ = 27$ cm

$\Rightarrow \quad 38 - CQ = 27$

$\Rightarrow \quad CQ = 11$ cm

$\Rightarrow \quad CR = 11$ cm

$\Rightarrow \quad CD - DR = 11$

$\Rightarrow \quad 25 - DR = 11$

$\Rightarrow \quad DR = 14$ cm

$\Rightarrow \quad r = 14$ cm

61. (*d*) Let $AB = 9$ cm, $BC = 7$ cm and $CA = 6$ cm

Then, $\quad x + y = 9$ cm

$y + z = 7$ cm

$z + x = 6$ cm

Adding, we get

$2(x + y + z) = 22$

$\Rightarrow \quad x + y + z = 11$

$\therefore \quad z = (11 - 9) = 2, x = (11 - 7) = 4$

and $\quad y = (11 - 6) = 5$

Hence, the radii of the given circles are 4 cm, 5 cm and 2 cm, respectively

62. (*a*) If two circles touch internally, then distance between their centres is equal to the difference of their radii.

$\therefore \quad AB = (5 - 3)$ cm $= 2$ cm

Also, the common chord PQ is the $\perp$ bisector of AB

$\therefore \quad AC = CB = 1$ cm

In rt. ΔACP, we have

$$AP^2 = AC^2 + CP^2$$

$\Rightarrow \quad 25 - 1 = CP^2$

$\therefore \quad CP = \sqrt{24}$ cm

Hence, $\quad PQ = 2CP = 2 = 4\sqrt{6}$ cm

63. (*c*) Since $ABCD$ is a *circumscribed* quadrilateral

$\therefore \quad AB + CD = BC + AD$

$\Rightarrow \quad 6 + 4 = 7 + AD$

$\therefore \quad AD = 10 - 7 = 3$cm

64. (*c*) $\quad OA = OB \Rightarrow \angle OAB = \angle OBA = 32°$

$\therefore \quad \angle OAB + \angle OBA = 32° + 32° = 64°$

$\therefore \quad \angle AOB = 180 - 64 = 116°$

$\Rightarrow \quad \angle ACB = \angle AOB = 58°$

(Degree Measure Theorem)

Also, $\quad \angle ACB = \angle BAS$

(angles in alternate segments)

$\therefore \quad \angle BAS = x = 58°$

65. (*b*) Since ST is a diameter

$\therefore \quad \angle TRS = 90°$

Also, $\quad \angle TRQ = \angle TSR$

(angles in alternate segments)

$\therefore \quad \angle TSR = 40°$

Hence, $\angle STR = 50°$

66. (*a*) Since $ABCD$ is a cyclic quadrilateral

$\therefore \quad \angle ADC + \angle ABC = 180°$

$\Rightarrow 130° + \angle ABC = 180°$

$\Rightarrow \quad \angle ABC = 50°$

Also, $\quad \angle ACB = 90°$

$\therefore$ In ΔABC,

$$\angle ACB + \angle ABC + \angle CAB = 180° \text{ (ASP)}$$

$\Rightarrow$ $90° + 50° + \angle CAB = 180°$

$\Rightarrow$ $\angle CAB = 40°$

67. (*a*) In ΔBDC, Q and R are the mid-points of BD and CD respectively.

$\therefore$ $QR \parallel BC$ and $QR = \frac{1}{2} BC$

Similarly, $PS \parallel BC$ and $PS = \frac{1}{2} BC$

$\therefore$ $PS \parallel QR$ and $PS = QR$

[each equal to 1/2 BC]

Similarly, $PQ \parallel SR$ and $PQ = SR$

[each equal to 1/2 AD]

$\therefore$ $PS = QR = SR = PQ$ [$\because$ $AD = BC$]

Hence, $PQRS$ is a rhombus.

68. (*c*) Since $AB \parallel DC$ and transversal AC cuts them at A and C cresp.

$\therefore$ $\angle 1 = \angle 2$ (1)

[$\because$ Alternate angles are equal.]

Now, in ΔAPR and ΔDPC, $\angle 1 = \angle 2$

$AP = CP$ [$\because$ P is the mid-point of AC]

and $\angle 3 = \angle 4$ [Vertically opposite angles].

So, $\Delta APR \cong \Delta DPC$ [ASA].

$\Rightarrow$ $AR = DC$ and $PR = DP$ (2)

Again, P and Q are the mid-points of sides DR and DB respectively in ΔDRB.

$\therefore$ $PQ = \frac{1}{2} RB = \frac{1}{2}(AB - AR)$.

[$\because$ $AR = DC$].

$\therefore$ $PQ = \frac{1}{2}(AB - DC)$

69. (*a*) $ABCD$ is a parallelogram.

$\Rightarrow$ $AD = BC$ and $AD \parallel BC$

$\Rightarrow$ $\frac{1}{3} AD = \frac{1}{3} BC$ and $AD \parallel BC$

$\Rightarrow$ $AP = CQ$ and $AP \parallel CQ$

Thus, $APCQ$ is a quad. Such that one pair of opposite side AP and CQ are parallel and equal.

Hence, $APCQ$ is a parallelogram.

70. (*c*) In ΔARB, P is the mid-point of AB and $PD \parallel BR$.

$\Rightarrow$ D is the mid-point of AR.

$\because$ $ABCD$ is a parallelogram

$\Rightarrow$ $DC \parallel AB \Rightarrow DQ \parallel AB$

Thus, in ΔRAB, D is the mid-point of AR and $DQ \parallel AB$.

$\therefore$ Q is the mid-point of

$RB \Rightarrow BR = 2BQ$.

71. (*c*) In ΔABC, $\angle ACE = \angle ABC + \angle BAC$

Similarly in ΔBCD,

$\angle BDC = \angle DCE - \angle DBC$

[Ext. angle prop. of a Δ]

But $\angle DCE = \frac{1}{2} \angle ACE$ and

$\frac{1}{2} \angle DBC = \frac{1}{2} \angle ABC$

Now, $\angle BDC = \angle DCE - \angle DBC$

$= \frac{1}{2} \angle ACE - \frac{1}{2} \angle ABC$

$= \frac{1}{2} (\angle ACE - \angle ABC)$

$= \frac{1}{2} (\angle ACE + \angle BAC - \angle ACE)$

$\therefore$ $\angle BDC = \frac{1}{2} \angle BAC$

72. (*a*) $\angle 1 = \angle A + \angle 5$ and

$\angle 2 = \angle A + \angle 6$

[Ext. angle prop. of a Δ]

$\angle 1 + \angle 2 = 2\angle A + \angle 5 + \angle 6$

$= 2\angle A + (180° - \angle A)$

$= \angle A + 180°$

The given question can be restated as the sum of two exterior angles exceeds $\angle A$ of the ΔABC by 2 right angles.

73. (*c*) $PR = \sqrt{PM^2 + MR^2}$

$= \sqrt{36+64} = 10$ cm

$PQ = \sqrt{QR^2 - PR^2}$

$= \sqrt{26^2 - 10^2} = 24$ cm

$\therefore$ ar(ΔPQR) = 1 × 10 × 12 = 120 cm^2

74. (*a*) In ΔADE and ΔABC

$\angle A = \angle A$ [common]

$\angle ADE = \angle ACB = x°$ (Given)

$\therefore$ $\Delta ADE \sim \Delta ACB$ (AA Similarly)

$\frac{AD}{AC} = \frac{AD}{AC}$

(corresponding sides of ~ Δs are proportional)

$\frac{6}{13} = \frac{9}{AB}$

$AB = \frac{39}{2} = 19.5$ cm

Hence $BD = AB - AD$

$= 19.5 - 6 = 13.5$ cm

75. (*b*) $\angle PQA$ is a right angle being an angle in a semi-circle.

$MP \cdot MA = MQ^2$

If $BM = 1$, $MP = 1$ and $AM = \sqrt{3}$,

$\because$ AM is the median and $MQ = 3^{\frac{1}{4}}$

So,

$T = MQ^2 = \sqrt{3}$ and $S = \frac{\sqrt{3}}{4}(4) = \sqrt{3}$.

$\therefore T = S$.

76. (*c*) We have $\angle GEC = 52°$

By alternate segment theorem,

$\angle OAE = \angle GEC = 52°$

$\therefore \angle OCE = 180° - 90° - 52° = 38°$

As $\angle AEC$ is an angle in a semicircle.

Now ACDE is a cyclic quadrilateral.

$\therefore \angle C = 180° - 52° = 128°$

$\therefore \angle C + \angle C = 38° + 128° = 166°$.

77. (*c*) The coefficient of x in the new equation is

$-\left[\left(\alpha + \frac{\alpha}{\beta}\right) + \left(\beta + \frac{\beta}{\alpha}\right)\right]$

$= -\left[\alpha + \beta + \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right]$

$= -\left[10 + \frac{100-30}{15}\right]$

$= -\frac{44}{3}$

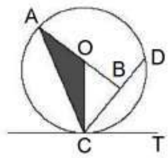
The constant term of the equation is,

$\left(\alpha + \frac{\alpha}{\beta}\right) \times \left(\beta + \frac{\beta}{\alpha}\right)$

$= \alpha\beta + \alpha + \beta + 1$

$= 15 + 10 + 1$

$= 26.$



78. (*c*) We have $\angle OCT = 90°$, $\angle DCT = 45°$ and $\angle OCB = 45°$

Since ΔBOC is a right-angled triangle

$\therefore \angle COB = 45°$

$\angle AOC = 180° - 45° = 135°$

As $CD = 10$

$BC = 5$ cm $= OB$

In ΔOBC,

$OC = 5\sqrt{2} = OA$

In ΔAOC,

$AC^2 = OA^2 + OC^2 - 2OA\ OC \cos 135°$

$= 2(OA)^2 - 2(OA)^2 \cos 135°$

$= 2(5\sqrt{2})^2 - 2(5\sqrt{2})^2 \times \frac{-1}{\sqrt{2}}$

$= 100 + \frac{100}{\sqrt{2}}$

$AC^2 = 170.70$

$AC = 13$ cm

$\therefore$ Perimeter of $\Delta AOC = AC + OC + AO$

$= 13 + 5\sqrt{2} + 5\sqrt{2}$

$= 13 + 10 \times 1.414$

$= 27$ cm(approx)

79. (*c*) In smaller circle, OP is the diameter of the circle.

So,

$\angle ORP = 90°$

$OP = 10$ cm (radius of bigger circle)

$OR = 8$ cm.

In ΔOPR

$OP^2 = OR^2 + RP^2$

$\Rightarrow \quad 10^2 = 8^2 + RP^2$

$\Rightarrow \quad RP^2 = 100 - 64$

$\Rightarrow \quad RP = \sqrt{36} = 6\text{cm}$

Also, $OR \perp SP$, so it passes through the centre.

$\therefore SP = 2\ RP = 2 \times 6 = 12$ cm.

80. (*e*) Using AE = 19, CD = 22, ∠B = 90°, we can find the length of AC.

$$AB^2 + \left(\frac{BC}{2}\right)^2 = (19)^2 \quad ...(i)$$

$$\left(\frac{AB}{2}\right)^2 + BC^2 = (22)^2 \quad ...(ii)$$

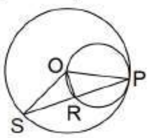
On adding (i) and (ii), we get

$$AB^2 + BC^2 = \frac{4}{5}(361 + 484)$$

$\Rightarrow \quad AC^2 = 676$

$\Rightarrow \quad AC = 26$

81. (*c*) Let A, B be the centres of the two circles with radius 3 cm, 8 cm respectively



$P_1X = \sqrt{25 - 9} = 4$

Now$\Delta AP_1X \sim BP_2X$

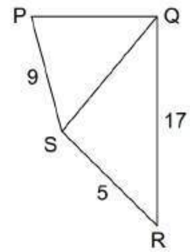
$\Rightarrow \quad AP_1 / BP_2 = P_1X / P_2X$

$\Rightarrow \quad 3/8 = 4/P_2X$

$P_2X = 10.66$

$P_1P_2 = P_1X + P_2X = 14.66$

82. (*b*) Let $QS = x$, we get the following figure



In any triangle, the sum of any two sides is always greater than the third side and the difference of any two sides is always smaller than the third side. Hence,

In QRS,

$x + 5 > 17$

$\Rightarrow \quad x > 12$ (1)

In PQRS

$x < 9 + 5$

$\Rightarrow \quad x > 14$ (2)

Combining (i) and (ii),we get

$12 < x < 14$

83. (*a*) Since PQ is the diameter and parallel to x-axis, there fore points P and Q have the same y-coordinate but their x – coordinates differ by 6 units. (Since the diameter = 6 units)

We have two possible cases:

Case 1:

$$y = a^x = 2a^{(x+6)}\delta \quad \Rightarrow \quad a = \frac{1}{6\sqrt{2}}$$

Case2 :

$$y = 2a^x = a^{(x+6)} \quad \Rightarrow \quad a = 2^{1/6}$$

But it is given that a < 1

$\therefore$ Case 2 can not be possible.