WB JEE

Engineering Entrance Exam

Solved Paper 2018

Physics

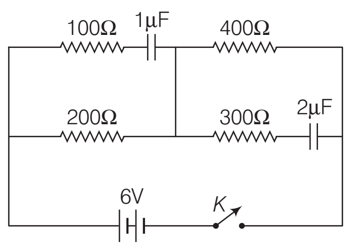
Category-I (Q. Nos. 1 to 30)

Only one answer is correct. Correct answer will fetch full marks 1. *Incorrect answer or any combination of more than one answer will fetch* $-\frac{1}{4}$ *marks. No answer will fetch* 0 *marks.*

1. Four resistors, $100\,\Omega$, $200\,\Omega$, $300\,\Omega$ and $400\,\Omega$ are connected to form four sides of a square. The resistors can be connected in any order. What is the maximum possible equivalent resistance across the diagonal of the square?

(a) 210Ω (b) 240Ω
(c) 300Ω (d) 250Ω

2. What will be current through the $200\,\Omega$ resistor in the given circuit, a long time after the switch K is made on?

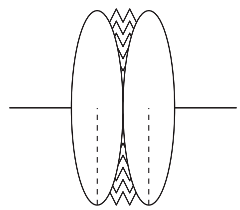


(a) Zero (b) 100 mA
(c) 10 mA (d) 1 mA

3. A point source is placed at coordinates (0, 1) in xy-plane. A ray of light from the source is reflected on a plane mirror placed along the X-axis and perpendicular to the xy-plane. The reflected ray passes through the point (3, 3). What is the path length of the ray from (0, 1) to (3, 3)?

(a) 5 (b) $\sqrt{13}$
(c) $2\sqrt{3}$ (d) $1+2\sqrt{3}$

4. Two identical equiconvex lenses, each of focal length f are placed side by side in contact with each other with a layer of water in between them as shown in the figure. If refractive index of the material of the lenses is greater than that of water, how the combined focal length F is related to f?



(a) $F > f$ (b) $\frac{f}{2} < F < f$
(c) $F < \frac{f}{2}$ (d) $F = f$

5. There is a small air bubble at the centre of a solid glass sphere of radius r and refractive index μ. What will be the apparent distance of the bubble from the centre of the sphere, when viewed from outside?

(a) r (b) $\frac{r}{\mu}$
(c) $r\left(1-\frac{1}{\mu}\right)$ (d) Zero

6. If Young's double slit experiment is done with white light, which of the following statements will be true?

(a) All the bright fringes will be coloured.
(b) All the bright fringes will be white.
(c) The central fringe will be white.
(d) No stable interference pattern will be visible.

7. How the linear velocity v of an electron in the Bohr orbit is related to its quantum number n?

(a) $v \propto \frac{1}{n}$ (b) $v \propto \frac{1}{n^2}$
(c) $v \propto \frac{1}{\sqrt{n}}$ (d) $v \propto n$

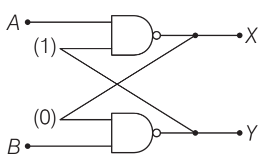
8. If the half-life of a radioactive nucleus is 3 days, nearly what fraction of the initial number of nuclei will decay on the third day? (Given, $\sqrt[3]{0.25} \approx 0.63$)

(a) 0.63 (b) 0.5 (c) 0.37 (d) 0.13

9. An electron accelerated through a potential of 10000 V from rest has a de-Broglie wave length λ. What should be the accelerating potential, so that the wavelength is doubled?

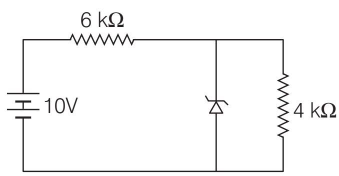
(a) 20000 V (b) 40000 V (c) 5000 V (d) 2500 V

10. In the circuit shown, inputs A and B are in states 1 and 0 respectively. What is the only possible stable state of the outputs X and Y?



(a) $X = 1, Y = 1$ (b) $X = 1, Y = 0$
(c) $X = 0, Y = 1$ (d) $X = 0, Y = 0$

11. What will be the current flowing through the $6\,k\Omega$ resistor in the circuit shown, where the breakdown voltage of the Zener is 6 V?



(a) $\frac{2}{3}$ mA (b) 1 mA (c) 10 mA (d) $\frac{3}{2}$ mA

12. In case of a simple harmonic motion, if the velocity is plotted along the X-axis and the displacement (from the equilibrium position) is plotted along the Y-axis, the resultant curve happens to be an ellipse with the ratio:

$$\frac{\text{major axis (along } X)}{\text{minor axis (along } Y)} = 20\pi$$

What is the frequency of the simple harmonic motion?

(a) 100 Hz (b) 20 Hz (c) 10 Hz (d) $\frac{1}{10}$ Hz

13. A block of mass m_2 is placed on a horizontal table and another block of mass m_1 is placed on top of it. An increasing horizontal force $F = \alpha t$ is exerted on the upper block but the lower block never moves as a result. If the coefficient of friction between the blocks is μ_1 and that between the lower block and the table is μ_2, then what is the maximum possible value of μ_1/μ_2?

(a) $\frac{m_2}{m_1}$ (b) $1 + \frac{m_2}{m_1}$ (c) $\frac{m_1}{m_2}$ (d) $1 + \frac{m_1}{m_2}$

14. In a triangle ABC, the sides AB and AC are represented by the vectors $3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, respectively. Calculate the angle $\angle ABC$.

(a) $\cos^{-1}\sqrt{\frac{5}{11}}$ (b) $\cos^{-1}\sqrt{\frac{6}{11}}$
(c) $\left(90° - \cos^{-1}\sqrt{\frac{5}{11}}\right)$ (d) $\left(180° - \cos^{-1}\sqrt{\frac{5}{11}}\right)$

15. The velocity (v) of a particle (under a force F) depends on its distance (x) from the origin (with $x > 0$) $v \propto \frac{1}{\sqrt{x}}$. Find how the magnitude of the force (F) on the particle depends on x?

(a) $F \propto \frac{1}{x^{3/2}}$ (b) $F \propto \frac{1}{x}$ (c) $F \propto \frac{1}{x^2}$ (d) $F \propto x$

16. The ratio of accelerations due to gravity $g_1 : g_2$ on the surfaces of two planets is 5 : 2 and the ratio of their respective average densities $\rho_1 : \rho_2$ is 2 : 1. What is the ratio of respective escape velocities $v_1 : v_2$ from the surface of the planets?

(a) 5 : 2 (b) $\sqrt{5} : \sqrt{2}$ (c) $5 : 2\sqrt{2}$ (d) 25 : 4

17. A spherical liquid drop is placed on a horizontal plane. A small disturbance causes the volume of the drop to oscillate. The time period of oscillation (T) of the liquid drop depends on radius (r) of the drop, density (ρ) and surface tension (S) of the liquid. Which among the following will be a possible expression for T (where, k is a dimensionless constant)?

(a) $k\sqrt{\frac{\rho r}{S}}$ (b) $k\sqrt{\frac{\rho^2 r}{S}}$ (c) $k\sqrt{\frac{\rho r^3}{S}}$ (d) $k\sqrt{\frac{\rho r^3}{S^2}}$

18. The stress along the length of a rod (with rectangular cross-section) is 1% of the Young's modulus of its material. What is the approximate percentage of change of its volume? (Poisson's ratio of the material of the rod is 0.3.)

(a) 3% (b) 1% (c) 0.7% (d) 0.4%

19. What will be the approximate terminal velocity of a rain drop of diameter 1.8×10^{-3} m, when density of rain water $\approx 10^3$ kgm^{-3} and the coefficient of viscosity of air $\approx 1.8 \times 10^{-5}$ N-sm^{-2}? (Neglect buoyancy of air)

(a) 49 ms^{-1} (b) 98 ms^{-1} (c) 392 ms^{-1} (d) 980 ms^{-1}

20. The water equivalent of a calorimeter is 10 g and it contains 50 g of water at 15°C. Some amount of ice, initially at -10°C is dropped in it and half of the ice melts till equilibrium is reached. What was the initial amount of ice that was dropped (when specific heat of ice $= 0.5$ cal gm^{-1} °C^{-1}, specific heat of water $= 1.0$ cal gm^{-1}°C^{-1} and latent heat of melting of ice $= 80$ cal gm^{-1})?

(a) 10 g (b) 18 g (c) 20 g (d) 30 g

21. One mole of a monoatomic ideal gas undergoes a quasistatic process, which is depicted by a straight line joining points (V_0, T_0) and $(2V_0, 3T_0)$ in a V-T diagram. What is the value of the heat capacity of the gas at the point (V_0, T_0)?

(a) R (b) $\frac{3}{2}R$

(c) $2R$ (d) 0

22. For an ideal gas with initial pressure and volume p_i and V_i respectively, a reversible isothermal expansion happens, when its volume becomes V_0. Then, it is compressed to its original volume V_i by a reversible adiabatic process. If the final pressure is p_f, then which of the following statement(s) is/are true?

(a) $p_f = p_i$ (b) $p_f > p_i$

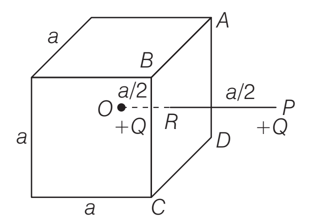
(c) $p_f < p_i$ (d) $\frac{p_f}{V_0} = \frac{p_i}{V_i}$

23. A point charge $-q$ is carried from a point A to another point B on the axis of a charged ring of radius r carrying a charge $+q$. If the point A is at a distance $\frac{4}{3}r$ from the centre of the ring and the point B is $\frac{3}{4}r$ from the centre but on the opposite side, what is the net work that need to be done for this?

(a) $-\frac{7}{5}\cdot\frac{q^2}{4\pi\varepsilon_0 r}$ (b) $-\frac{1}{5}\cdot\frac{q^2}{4\pi\varepsilon_0 r}$

(c) $\frac{7}{5}\cdot\frac{q^2}{4\pi\varepsilon_0 r}$ (d) $\frac{1}{5}\cdot\frac{q^2}{4\pi\varepsilon_0 r}$

24. Consider a region in free space bounded by the surfaces of an imaginary cube having sides of length a as shown in the figure. A charge $+Q$ is placed at the centre O of the cube. P is such a point outside the cube that the line OP perpendicularly intersects the surface $ABCD$ at R and also $OR = RP = a/2$. A charge $+Q$ is placed at point P also. What is the total electric flux through the five faces of the cube other than $ABCD$?



(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{5Q}{6\varepsilon_0}$

(c) $\frac{10Q}{6\varepsilon_0}$ (d) zero

25. Four equal charges of value $+Q$ are placed at any four vertices of a regular hexagon of side 'a'. By suitably choosing the vertices, what can be the maximum possible magnitude of electric field at the centre of the hexagon?

(a) $\frac{Q}{4\pi\varepsilon_0 a^2}$ (b) $\frac{\sqrt{2}\,Q}{4\pi\varepsilon_0 a^2}$

(c) $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 a^2}$ (d) $\frac{2Q}{4\pi\varepsilon_0 a^2}$

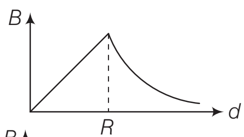
26. A proton of mass m moving with a speed v ($<< c$, velocity of light in vacuum) completes a circular orbit in time T in a uniform magnetic field. If the speed of the proton is increased to $\sqrt{2}\,v$, what will be time needed to complete the circular orbit?

(a) $\sqrt{2}\,T$ (b) T

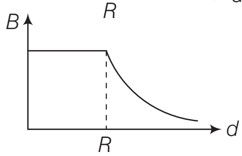
(c) $\frac{T}{\sqrt{2}}$ (d) $\frac{T}{2}$

27. A uniform current is flowing along the length of an infinite, straight, thin, hollow cylinder of radius R. The magnetic field B produced at a perpendicular distance d from the axis of the cylinder is plotted in a graph. Which of the following figures looks like the plot?

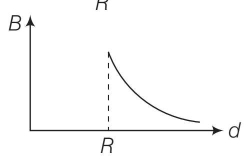
(a)



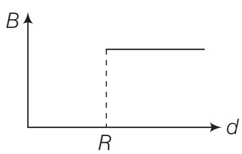
(b)



(c)



(d)



28. A circular loop of radius r of conducting wire connected with a voltage source of zero internal resistance produces a magnetic field B at its centre. If instead, a circular loop of radius $2r$, made of same material, having the same cross-section is connected to the same voltage source, what will be the magnetic field at its centre?

(a) $\frac{B}{2}$ (b) $\frac{B}{4}$ (c) $2B$ (d) B

29. An alternating current is flowing through a series L-C-R circuit. It is found that the current reaches a value of 1 mA at both 200 Hz and 800 Hz frequency. What is the resonance frequency of the circuit?

(a) 600 Hz (b) 300 Hz

(c) 500 Hz (d) 400 Hz

30. An electric bulb, a capacitor, a battery and a switch are all in series in a circuit. How does the intensity of light vary when the switch is turn on?

(a) Continues to increase gradually

(b) Gradually increases for sometime and then becomes steady

(c) Sharply rises initially and then gradually decreases

(d) Gradually increases for sometime and then gradually decreases

Category-II (Q. Nos. 31 to 35)

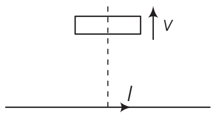
Only one answer is correct. Correct answer will fetch full marks 2. Incorrect answer or any combination of more than one answer will fetch $-\frac{1}{2}$ *marks. No answer will fetch 0 marks.*

31. A light charged particle is revolving in a circle of radius r in electrostatic attraction of a static heavy particle with opposite charge. How does the magnetic field B at the centre of the circle due to the moving charge depend on r?

(a) $B \propto \frac{1}{r}$ (b) $B \propto \frac{1}{r^2}$

(c) $B \propto \frac{1}{r^{3/2}}$ (d) $B \propto \frac{1}{r^{5/2}}$

32. As shown in the figure, a rectangular loop of a conducting wire is moving away with a constant velocity v in a perpendicular direction from a very long straight conductor carrying a steady current I. When the breadth of the rectangular loop is very small compared to its distance from the straight conductor, how does the emf. E induced in the loop vary with time t?



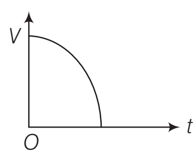
(a) $E \propto \frac{1}{t^2}$ (b) $E \propto \frac{1}{t}$ (c) $E \propto -\ln(t)$ (d) $E \propto \frac{1}{t^3}$

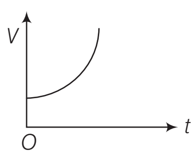
33. A solid spherical ball and a hollow spherical ball of two different materials of densities ρ_1 and ρ_2 respectively have same outer radii and same mass. What will be the ratio, the moment of inertia (about an axis passing through the centre) of the hollow sphere to that of the solid sphere?

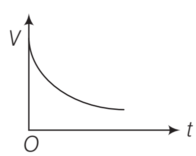
(a) $\frac{\rho_2}{\rho_1}\left(1-\frac{\rho_2}{\rho_1}\right)^{\frac{5}{3}}$ (b) $\frac{\rho_2}{\rho_1}\left[1-\left(1-\frac{\rho_2}{\rho_1}\right)^{\frac{5}{3}}\right]$

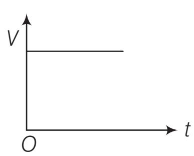
(c) $\frac{\rho_2}{\rho_1}\left(1-\frac{\rho_1}{\rho_2}\right)^{\frac{5}{3}}$ (d) $\frac{\rho_2}{\rho_1}\left[1-\left(1-\frac{\rho_1}{\rho_2}\right)^{\frac{5}{3}}\right]$

34. The insulated plates of a charged parallel plate capacitor (with small separation between the plates) are approaching each other due to electrostatic attraction. Assuming no other force to be operative and no radiation taking place, which of the following graphs approximately shows the variation with time (t) of the potential difference (V) between the plates?

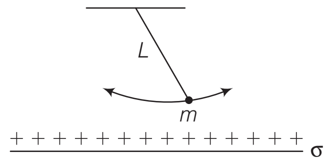
(a)


(b)


(c)


(d)


35. The bob of a pendulum of mass m, suspended by an inextensible string of length L as shown in the figure carries a small charge q. An infinite horizontal plane conductor with uniform surface charge density σ is placed below it. What will be the time period of the pendulum for small amplitude oscillations?



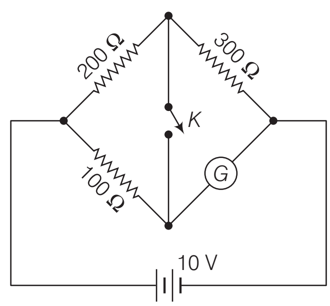
(a) $2\pi\sqrt{\frac{L}{\left(g-\frac{mq}{\varepsilon_0\sigma}\right)}}$ (b) $\sqrt{\frac{L}{\left(g-\frac{mq\sigma}{\varepsilon_0}\right)}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{L}{\left(g-\frac{q\sigma}{\varepsilon_0 m}\right)}}$ (d) $2\pi\sqrt{\frac{L}{\left(g-\frac{q\sigma}{\varepsilon_0 m}\right)}}$

Category-III (Q. Nos. 36 to 40)

One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also, no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers.

36. A non-zero current passes through the galvanometer G shown in the circuit when the key K is closed and its value does not change when the key is opened. Then, which of the following statement(s) is/are true?



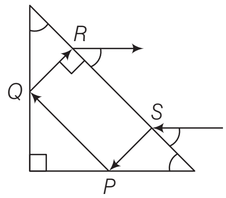
(a) The galvanometer resistance is infinite.

(b) The current through the galvanometer is 40 mA.

(c) After the key is closed, the current through the 200 Ω resistor is same as the current through the 300 Ω resistor.

(d) The galvanometer resistance is 150 Ω.

37. A ray of light is incident on a right angled isosceles prism parallel to its base as shown in the figure. Refractive index of the material of the prism is $\sqrt{2}$. Then, which of the following statement(s) is/are true?



(a) The reflection at P is total internal.
(b) The reflection at Q is total internal.
(c) The ray emerging at R is parallel to the ray incident at S.
(d) Total deviation of the ray is 150°.

38. The intensity of a sound appears to an observer to be periodic. Which of the following can be the cause of it?

(a) The intensity of the source is periodic
(b) The source is moving towards the observer
(c) The observer is moving away from the source
(d) The source is producing a sound composed of two nearby frequencies

39. Which of the following statements(s) is/are true?

"Internal energy of an ideal gas ...."

(a) decreases in an isothermal process.
(b) remains constant in an isothermal process.
(c) increases in an isobaric process.
(d) decreases in an isobaric expansion.

40. Two positive charges Q and $4Q$ are placed at points A and B respectively, where B is at a distance d units to the right of A. The total electric potential due to these charges is minimum at P on the line through A and B. What is (are) the distance (s) of P from A?

(a) $\frac{d}{3}$ units to the right of A
(b) $\frac{d}{3}$ units to the left of A
(c) $\frac{d}{5}$ units to the right of A
(d) d units to the left of A

Chemistry

Category-I (Q. Nos. 41 to 70)

Only one answer is correct. Correct will fetch full marks 1. Incorrect answer or any combination of more than one answer will detch – 1/4 marks. No answer will fetch 0 marks.

41. Cl_2O_7 is the anhydride of

(a) $HOCl$
(b) $HClO_2$
(c) $HClO_3$
(d) $HClO_4$

42. The main reason that $SiCl_4$ is easily hydrolysed as compared to CCl_4 is that

(a) Si—Cl bond is weaker than C—Cl bond
(b) $SiCl_4$ can form hydrogen bonds
(c) $SiCl_4$ is covalent
(d) Si can extend its coordination number beyond four

43. Silver chloride dissolves in excess of ammonium hydroxide solution. The cation present in the resulting solution is

(a) $[Ag(NH_3)_6]^+$
(b) $[Ag(NH_3)_4]^+$
(c) Ag^+
(d) $[Ag(NH_3)_2]^+$

44. The ease of hydrolysis in the compounds CH_3COCl (I), $CH_3—CO—O—COCH_3$ (II), $CH_3COOC_2H_5$ (III) and CH_3CONH_2 (IV) is of the order

(a) I > II > III > IV
(b) IV > III > II > I
(c) I > II > IV > III
(d) II > I > IV > III

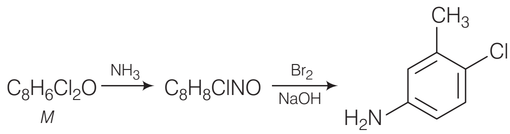
45. $CH_3—C \equiv C\,MgBr$ can be prepared by the reaction of

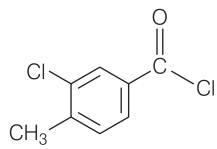
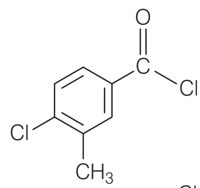
(a) $CH_3—C \equiv C—Br$ with $MgBr_2$
(b) $CH_3—C \equiv CH$ with $MgBr_2$
(c) $CH_3—C \equiv CH$ with KBr and Mg metal
(d) $CH_3—C \equiv CH$ with CH_3MgBr

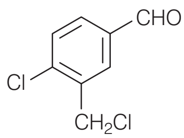
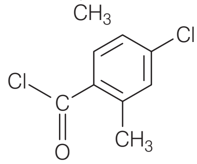
46. The number of alkene (s) which can produce 2-butanol by the successive treatment of (i) B_2H_6 in tetrahydrofuran solvent and (ii) alkaline H_2O_2 solution is

(a) 1
(b) 2
(c) 3
(d) 4

47. Identify '*M*' in the following sequence of reactions



(a)  (b) 

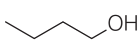
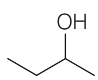
(c)  (d) 

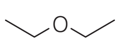
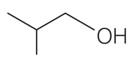
48. Methoxybenzene on treatment with HI produces

(a) iodobenzene and methanol
(b) phenol and methyl iodide
(c) iodobenzene and methyl iodide
(d) phenol and methanol

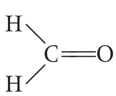
49. $\underset{N}{C_4H_{10}O} \xrightarrow[H_2SO_4]{K_2Cr_2O_7} C_4H_8O \xrightarrow[\text{Warm}]{I_2/NaOH} CHI_3$

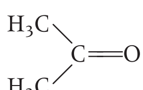
Here, *N* is

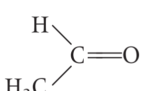
(a)  (b) 

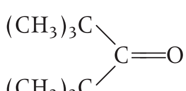
(c)  (d) 

50. The correct order of reactivity for the addition reaction of the following carbonyl compounds with ethylmagnesium iodide is


(I)


(II)


(III)


(IV)

(a) I > III > II > IV
(b) IV > III > II > I
(c) I > II > IV > III
(d) III > II > I > IV

51. If aniline is treated with conc. H_2SO_4 and heated at 200°C, the product is

(a) anilinium sulphate
(b) benzenesulphonic acid
(c) *m*-aminobenzenesulphonic acid
(d) sulphanilic acid

52. Which of the following electronic configuration is not possible?

(a) $n = 3, l = 0, m = 0$
(b) $n = 3, l = 1, m = -1$
(c) $n = 2, l = 0, m = -1$
(d) $n = 2, l = 1, m = 0$

53. The number of unpaired electrons in Ni (atomic number = 28) are

(a) 0 (b) 2 (c) 4 (d) 8

54. Which of the following has the strongest H-bond?

(a) O—H ... S (b) S—H ... O
(c) F—H ... F (d) F—H ... O

55. The half-life of C^{14} is 5760 years. For a 200 mg sample of C^{14}, the time taken to change to 25 mg is

(a) 11520 years (b) 23040 years
(c) 5760 years (d) 17280 years

56. Ferric ion forms a prussian blue precipitate due to the formation of

(a) $K_4[Fe(CN)_6]$ (b) $K_3[Fe(CN)_6]$
(c) $Fe(CNS)_3$ (d) $Fe_4[Fe(CN)_6]_3$

57. The nucleus $^{64}_{29}Cu$ accepts an orbital electron to yield,

(a) $^{65}_{28}Ni$ (b) $^{64}_{30}Zn$ (c) $^{64}_{28}Ni$ (d) $^{65}_{30}Zn$

58. How many moles of electrons will weigh one kilogram?

(a) 6.023×10^{23} (b) $\frac{1}{9.108} \times 10^{31}$
(c) $\frac{6.023}{9.108} \times 10^{54}$ (d) $\frac{1}{9.108 \times 6.023} \times 10^{8}$

59. Equal weights of ethane and hydrogen are mixed in an empty container at 25°C. The fraction of total pressure exerted by hydrogen is

(a) 1 : 2 (b) 1 : 1
(c) 1 : 16 (d) 15 : 16

60. The heat of neutralisation of a strong base and a strong acid is 13.7 kcal. The heat released when 0.6 mole HCl solution is added to 0.25 mole of NaOH is

(a) 3.425 kcal (b) 8.22 kcal
(c) 11.645 kcal (d) 13.7 kcal

61. A compound formed by elements X and Y crystallises in the cubic structure, where X atoms are at the corners of a cube and Y atoms are at the centre of the body. The formula of the compounds is

(a) XY (b) XY_2
(c) X_2Y_3 (d) XY_3

62. What amount of electricity can deposit 1 mole of Al metal at cathode when passed through molten $AlCl_3$?

(a) 0.3 F (b) 1 F
(c) 3 F (d) 1/3 F

63. Given the standard half-cell potentials ($E°$) of the following as

$$Zn \longrightarrow Zn^{2+} + 2e^-; \quad E° = +0.76 \text{ V}$$
$$Fe \longrightarrow Fe^{2+} + 2e^-; \quad E° = 0.41 \text{ V}$$

Then the standard e.m.f. of the cell with the reaction $Fe^{2+} + Zn \longrightarrow Zn^{2+} + Fe$ is

(a) – 0.35 V (b) + 0.35 V
(c) + 1.17 V (d) – 1.17 V

64. The following equilibrium constants are given

$$N_2 + 3H_2 \rightleftharpoons 2NH_3; K_1$$
$$N_2 + O_2 \rightleftharpoons 2NO; K_2$$
$$H_2 + \frac{1}{2}O_2 \rightleftharpoons H_2O; K_3$$

The equilibrium constant for the oxidation of 2 mole of NH_3 to give NO is

(a) $K_1 \cdot \frac{K_2}{K_3}$ (b) $K_2 \cdot \frac{K_3^3}{K_1}$
(c) $K_2 \cdot \frac{K_3^2}{K_1}$ (d) $K_2^2 \frac{K_3}{K_1}$

65. Which one of the following is a condensation polymer?

(a) PVC (b) Teflon
(c) Dacron (d) Polystyrene

66. Which of the following is present in maximum amount in 'acid rain'?

(a) HNO_3 (b) H_2SO_4
(c) HCl (d) H_2CO_3

67. Which of the set of oxides are arranged in the proper order of basic, amphoteric, acidic?

(a) SO_2, P_2O_5, CO (b) BaO, Al_2O_3, SO_2
(c) CaO, SiO_2, Al_2O_3 (d) CO_2, Al_2O_3, CO

68. Out of the following outer electronic configurations of atoms, the highest oxidation state is achieved by which one?

(a) $(n-1)d^8ns^2$ (b) $(n-1)d^5ns^2$
(c) $(n-1)d^3ns^2$ (d) $(n-1)d^5ns^1$

69. At room temperature, the reaction between water and fluorine produces

(a) HF and H_2O_2 (b) HF, O_2 and F_2O_2
(c) F^-, O_2 and H^+ (d) HOF and HF

70. Which of the following is least thermally stable?

(a) $MgCO_3$ (b) $CaCO_3$
(c) $SrCO_3$ (d) $BeCO_3$

Category-II (Q. Nos. 71 to 75)

Only one answer is correct. Correct answer will fetch full marks 2. *Incorrect answer or any combination of more than one answer will fetch –1/2 marks. No answer will fetch 0 marks.*

71. $[P] \xrightarrow{Br_2} C_2H_4Br_2 \xrightarrow[NH_3]{NaNH_2} [Q]$

$$[Q] \xrightarrow[Hg^{2+}, \Delta]{20\% H_2SO_4} [R] \xrightarrow{Zn\text{-}Hg/HCl} [S]$$

The species P, Q, R and S respectively are

(a) ethene, ethyne, ethanal, ethane
(b) ethane, ethyne, ethanal, ethene
(c) ethene, ethyne, ethanal, ethanol
(d) ethyne, ethane, ethene, ethanal

72. The number of possible organobromine compounds which can be obtained in the allylic bromination of 1-butene with N-bromosuccinimide is

(a) 1 (b) 2
(c) 3 (d) 4

73. A metal M (specific heat 0.16) forms a metal chloride with 65% chlorine present in it. The formula of the metal chloride will be

(a) MCl (b) MCl_2
(c) MCl_3 (d) MCl_4

74. During a reversible adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio $\frac{C_p}{C_V}$ for the gas is

(a) $\frac{3}{2}$ (b) $\frac{7}{2}$
(c) $\frac{5}{3}$ (d) $\frac{9}{7}$

75. $[X] + \text{dil. } H_2SO_4 \longrightarrow [Y]$: Colourless, suffocating gas

$[Y] + K_2Cr_2O_7 + H_2SO_4 \longrightarrow$ Green colouration of solution

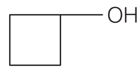
Then, $[X]$ and $[Y]$ are

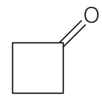
(a) SO_3^{2-}, SO_2
(b) Cl^-, HCl
(c) S^{2-}, H_2S
(d) CO_3^{2-}, CO_2

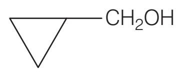
Category-III (Q. Nos. 76 to 80)

One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers.

76. The possible product(s) to be obtained from the reaction of cyclobutyl amine with HNO_2 is/are

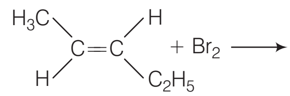
(a) 

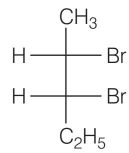
(b) 

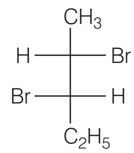
(c) 

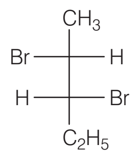
(d) $H_2C{=}CH_2$

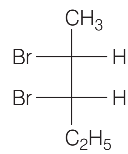
77. The major products obtained in the following reaction is/are



(a) 

(b) 

(c) 

(d) 

78. Which statements are correct for the peroxide ion?

(a) It has five completely filled anti-bonding molecular orbitals
(b) It is diamagnetic
(c) It has bond order one
(d) It is isoelectronic with neon

79. Among the following, the extensive variables are

(a) H (Enthalpy) (b) p (Pressure)
(c) E (Internal energy) (d) V (Volume)

80. White phosphorous P_4 has the following characteristics

(a) 6P—P single bonds
(b) 4P—P single bonds
(c) 4 lone pair of electrons
(d) P—P—P angle of 60°

Mathematics

Category-I (Q. Nos. 1 to 50)

Only one answer is correct. Correct answer will fetch full marks 1. Incorrect answer or any combination of more than one answer will fetch-1/4 marks. No answer will fetch 0 marks.

1. The approximate value of $\sin 31°$ is

(a) > 0.5 (b) > 0.6
(c) < 0.5 (d) < 0.4

2. Let $f_1(x) = e^x$, $f_2(x) = e^{f_1(x)}$, ..., $f_{n+1}(x) = e^{f_n(x)}$ for all $n \geq 1$. Then for any fixed n, $\frac{d}{dx} f_n(x)$ is

(a) $f_n(x)$
(b) $f_n(x) f_{n-1}(x)$
(c) $f_n(x) f_{n-1}(x) \ldots f_1(x)$
(d) $f_n(x) \ldots f_1(x) e^x$

3. The domain of definition of $f(x) = \sqrt{\dfrac{1-|x|}{2-|x|}}$ is

(a) $(-\infty, -1) \cup (2, \infty)$
(b) $[-1, 1] \cup (2, \infty) \cup (-\infty, -2)$
(c) $(-\infty, 1) \cup (2, \infty)$
(d) $[-1, 1] \cup (2, \infty)$

Here $(a, b) \equiv \{x : a < x < b\}$ and $[a, b] \equiv \{x : a \leq x \leq b\}$

4. Let $f : [a, b] \to R$ be differentiable on $[a, b]$ and $k \in R$. Let $f(a) = 0 = f(b)$. Also let $J(x) = f'(x) + kf(x)$. Then

(a) $J(x) > 0$ for all $x \in [a, b]$
(b) $J(x) < 0$ for all $x \in [a, b]$
(c) $J(x) = 0$ has at least one root in (a, b)
(d) $J(x) = 0$ through (a, b)

5. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$. Then $\lim\limits_{h \to 0} \dfrac{f(1-h) - f(1)}{h^3 + 3h}$

(a) does not exist (b) is $\frac{50}{3}$
(c) is $\frac{53}{3}$ (d) is $\frac{22}{3}$

6. Let $f : [a, b] \to R$ be such that f is differentiable in (a, b), f is continuous at $x = a$ and $x = b$ and moreover $f(a) = 0 = f(b)$. Then

(a) there exists at least one point c in (a, b) such that $f'(c) = f(c)$
(b) $f'(x) = f(x)$ does not hold at any point in (a, b)
(c) at every point of (a, b), $f'(x) > f(x)$
(d) at every point of (a, b), $f'(x) < f(x)$

7. Let $f : R \to R$ be a twice continuously differentiable function such that $f(0) = f(1) = f'(0) = 0$. Then

(a) $f''(0) = 0$
(b) $f''(c) = 0$ for some $c \in R$
(c) if $c \neq 0$, then $f''(c) \neq 0$
(d) $f'(x) > 0$ for all $x \neq 0$

8. If $\int e^{\sin x} \cdot \left[\dfrac{x\cos^3 x - \sin x}{\cos^2 x}\right] dx = e^{\sin x} f(x) + c$, where c is constant of integration, then $f(x)$ is equal to

(a) $\sec x - x$ (b) $x - \sec x$
(c) $\tan x - x$ (d) $x - \tan x$

9. If $\int f(x) \sin x \cos x \, dx = \dfrac{1}{2(b^2 - a^2)} \log f(x) + c$, where c is the constant of integration, then $f(x)$ is equal to

(a) $\dfrac{2}{(b^2 - a^2)\sin 2x}$ (b) $\dfrac{2}{ab \sin 2x}$
(c) $\dfrac{2}{(b^2 - a^2)\cos 2x}$ (d) $\dfrac{2}{ab\cos 2x}$

10. If $M = \displaystyle\int_0^{\pi/2} \frac{\cos x}{x+2} dx$, $N = \displaystyle\int_0^{\pi/4} \frac{\sin x \cos x}{(x+1)^2} dx$, then the value of $M - N$ is

(a) π (b) $\frac{\pi}{4}$ (c) $\frac{2}{\pi - 4}$ (d) $\frac{2}{\pi + 4}$

11. The value of the integral $I = \displaystyle\int_{1/2014}^{2014} \frac{\tan^{-1} x}{x} dx$ is

(a) $\frac{\pi}{4} \log 2014$ (b) $\frac{\pi}{2} \log 2014$
(c) $\pi \log 2014$ (d) $\frac{1}{2} \log 2014$

12. Let $I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$. Then

(a) $\frac{1}{2} \le I \le 1$ (b) $4 \le I \le 2\sqrt{30}$
(c) $\frac{\sqrt{3}}{8} \le I \le \frac{\sqrt{2}}{6}$ (d) $1 \le I \le \frac{2\sqrt{3}}{\sqrt{2}}$

13. The value of
$$I = \int_{\pi/2}^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx, \text{ is}$$

(a) 1 (b) π (c) e (d) $\frac{\pi}{2}$

14. The value of
$$\lim_{n\to\infty} \frac{1}{n}\left\{\sec^2 \frac{\pi}{4n} + \sec^2 \frac{2\pi}{4n} + \ldots + \sec^2 \frac{n\pi}{4n}\right\} \text{ is}$$

(a) $\log_e 2$ (b) $\frac{\pi}{2}$ (c) $\frac{4}{\pi}$ (d) e

15. The differential equation representing the family of curves $y^2 = 2d(x + \sqrt{d})$, where d is a parameter, is of

(a) order 2 (b) degree 2
(c) degree 3 (d) degree 4

16. Let $y(x)$ be a solution of
$(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ and $y(0) = -1$. Then $y(1)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{6}$ (d) -1

17. The law of motion of a body moving along a straight line is $x = \frac{1}{2}$ vt. x being its distance from a fixed point on the line at time t and v is its velocity there. Then

(a) acceleration f varies directly with x
(b) acceleration f varies inversely with x
(c) acceleration f is constant
(d) acceleration f varies directly with t

18. Number of common tangents of $y = x^2$ and $y = -x^2 + 4x - 4$ is

(a) 1 (b) 2
(c) 3 (d) 4

19. Given that n numbers of arithmetic means are inserted between two sets of numbers $a, 2b$ and $2a, b$ where $a, b \in R$. Suppose further that the mth means between these sets of numbers are same, then the ratio $a : b$ equals

(a) $n - m + 1 : m$ (b) $n - m + 1 : n$
(c) $n : n - m + 1$ (d) $m : n - m + 1$

20. If $x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$, then the value of x is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) 1 (d) 2

21. If $Z_r = \sin \frac{2\pi r}{11} - i\cos \frac{2\pi r}{11}$, then $\sum_{r=0}^{10} Z_r$ is equal to

(a) -1 (b) 0 (c) i (d) $-i$

22. If z_1 and z_2 be two non-zero complex numbers such that $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$, then the origin and the points represented by z_1 and z_2

(a) lie on a straight line
(b) form a right angled triangle
(c) form an equilateral triangle
(d) form an isosceles triangle

23. If $b_1 b_2 = 2(c_1 + c_2)$ and b_1, b_2, c_1, c_2 are all real numbers, then at least one of the equations $x^2 + b_1 x + c_1 = 0$ and $x^2 + b_2 x + c_2 = 0$ has

(a) real roots
(b) purely imaginary roots
(c) roots of the form $a + ib$ ($a, b \in R$, $ab \neq 0$)
(d) rational roots

24. The number of selection of n objects from $2n$ objects of which n are identical and the rest are different, is

(a) 2^n (b) 2^{n-1}
(c) $2^n - 1$ (d) $2^{n-1} + 1$

25. If $(2 \le r \le n)$, then ${}^nC_r + 2 \cdot {}^nC_{r+1} + {}^nC_{r+2}$ is equal to

(a) $2 \cdot {}^nC_{r+2}$ (b) ${}^{n+1}C_{r+1}$
(c) ${}^{n+2}C_{r+2}$ (d) ${}^{n+1}C_r$

26. The number $(101)^{100} - 1$ is divisible by

(a) 10^4 (b) 10^6
(c) 10^8 (d) 10^{12}

27. If n is even positive integer, then the condition that the greatest term in the expansion of $(1+x)^n$ may also have the greatest coefficient, is

(a) $\frac{n}{n+2} < x < \frac{n+2}{n}$ (b) $\frac{n}{n+1} < x < \frac{n+1}{n}$

(c) $\frac{n+1}{n+2} < x < \frac{n+2}{n+1}$ (d) $\frac{n+2}{n+3} < x < \frac{n+3}{n+2}$

28. If $\begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix} = A$, Then $\begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix}$ is

(a) A^2 (b) $A^2 - A + I_3$

(c) $A^2 - 3A + I_3$ (d) $3A^2 + 5A - 4I_3$

(I_3 denotes the det of the identity matrix of order 3)

29. If $a_r = (\cos 2r\pi + i\sin 2r\pi)^{1/9}$, then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to

(a) 1 (b) –1

(c) 0 (d) 2

30. If $S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$, then the value of $\sum_{r=1}^{n} S_r$ is independent of

(a) only x (b) only y

(c) only n (d) x, y, z and n

31. If the following three linear equations have a non-trivial solution, then

$$x + 4ay + az = 0$$
$$x + 3by + bz = 0$$
$$x + 2cy + cz = 0$$

(a) a, b, c are in AP (b) a, b, c are in GP

(c) a, b, c are in HP (d) $a + b + c = 0$

32. On R, a relation ρ is defined by $x\rho y$ if and only if $x - y$ is zero or irrational. Then,

(a) ρ is equivalence relation

(b) ρ is reflexive but neither symmetric nor transitive

(c) ρ is reflexive and symmetric but not transitive

(d) ρ is symmetric and transitive but not reflexive

33. On the set R of real numbers, the relation ρ is defined by $x\rho y$, $(x, y) \in R$.

(a) If $|x - y| < 2$, then ρ is reflexive but neither symmetric nor transitive.

(b) If $x - y < 2$, then ρ is reflexive and symmetric but not transitive.

(c) If $|x| \geq y$, then ρ is reflexive and transitive but not symmetric.

(d) If $x > |y|$, then ρ is transitive but neither reflexive nor symmetric.

34. If $f : R \to R$ be defined by $f(x) = e^x$ and $g : R \to R$ be defined by $g(x) = x^2$. The mapping $gof : R \to R$ be defined by $(gof)(x) = g[f(x)] \ \forall \ x \in R$. Then,

(a) gof is bijective but f is not injective

(b) gof is injective and g is injective

(c) gof is injective but g is not bijective

(d) gof is surjective and g is surjective

35. In order to get a head at least once with probability ≥ 0.9, the minimum number of times a unbiased coin needs to be tossed is

(a) 3 (b) 4 (c) 5 (d) 6

36. A student appears for tests I, II and III. The student is successful if he passes in tests I, II or I, III. The probabilities of the student passing in tests I, II and III are respectively p, q and 1/2. If the probability of the student to be successful is 1/2. Then

(a) $p(1+q) = 1$ (b) $q(1+p) = 1$

(c) $pq = 1$ (d) $\frac{1}{p} + \frac{1}{q} = 1$

37. If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$, then general value of θ is

(a) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$ (b) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$

(c) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$ (d) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$

(n is an integer)

38. If $0 \leq A \leq \frac{\pi}{4}$, then

$$\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$$

is equal to

(a) $\frac{\pi}{4}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

39. Without changing the direction of the axes, the origin is transferred to the point $(2, 3)$. Then the equation $x^2 + y^2 - 4x - 6y + 9 = 0$ changes to

(a) $x^2 + y^2 + 4 = 0$
(b) $x^2 + y^2 = 4$
(c) $x^2 + y^2 - 8x - 12y + 48 = 0$
(d) $x^2 + y^2 = 9$

40. The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α. The equation of the locus of the point P is

(a) $x^2 + y^2 + 4x + 6y + 9 = 0$
(b) $x^2 + y^2 - 4x + 6y + 9 = 0$
(c) $x^2 + y^2 - 4x - 6y + 9 = 0$
(d) $x^2 + y^2 + 4x - 6y + 9 = 0$

41. The point Q is the image of the point $P(1, 5)$ about the line $y = x$ and R is the image of the point Q about the line $y = -x$. The circumcentre of the ΔPQR is

(a) (5, 1) (b) (–5, 1)
(c) (1, – 5) (d) (0, 0)

42. The angular points of a triangle are $A(-1, -7)$, $B(5, 1)$ and $C(1, 4)$. The equation of the bisector of the angle $\angle ABC$ is

(a) $x = 7y + 2$ (b) $7y = x + 2$
(c) $y = 7x + 2$ (d) $7x = y + 2$

43. If one of the diameter of the circle, given by the equation $x^2 + y^2 + 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is $(2, -3)$, the radius of S is

(a) $\sqrt{41}$ unit (b) $3\sqrt{5}$ unit
(c) $5\sqrt{2}$ unit (d) $2\sqrt{5}$ unit

44. A chord AB is drawn from the point $A(0, 3)$ on the circle $x^2 + 4x + (y-3)^2 = 0$, and is extended to M such that $AM = 2AB$. The locus of M is

(a) $x^2 + y^2 - 8x - 6y + 9 = 0$
(b) $x^2 + y^2 + 8x + 6y + 9 = 0$
(c) $x^2 + y^2 + 8x - 6y + 9 = 0$
(d) $x^2 + y^2 - 8x + 6y + 9 = 0$

45. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 9y^2 = 9$, then the ratio $a^2 : b^2$ equals

(a) 8 : 1 (b) 1 : 8
(c) 9 : 1 (d) 1 : 9

46. Let A, B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as diameter, the slope of the line AB is

(a) $-\frac{1}{r}$ (b) $\frac{1}{r}$ (c) $\frac{2}{r}$ (d) $-\frac{2}{r}$

47. Let $P(at^2, 2at)$, Q, $R(ar^2, 2ar)$ be three points on a parabola $y^2 = 4ax$. If PQ is the focal chord and PK, QR are parallel where the co-ordinates of K is $(2a, 0)$, then the value of r is

(a) $\frac{t}{1-t^2}$ (b) $\frac{1-t^2}{t}$ (c) $\frac{t^2+1}{t}$ (d) $\frac{t^2-1}{t}$

48. Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line through P parallel to the Y-axis meets the circle $x^2 + y^2 = 9$ at Q, where P, Q are on the same side of the X-axis. If R is a point on PQ such that $\frac{PR}{RQ} = \frac{1}{2}$, then the locus of R is

(a) $\frac{x^2}{9} + \frac{9y^2}{49} = 1$ (b) $\frac{x^2}{49} + \frac{y^2}{9} = 1$
(c) $\frac{x^2}{9} + \frac{y^2}{49} = 1$ (d) $\frac{9x^2}{49} + \frac{y^2}{9} = 1$

49. A point P lies on a line through $Q(1, -2, 3)$ and is parallel to the line $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$. If P lies on the plane $2x + 3y - 4z + 22 = 0$, then segment PQ equals

(a) $\sqrt{42}$ units (b) $\sqrt{32}$ units
(c) 4 units (d) 5 units

50. The foot of the perpendicular drawn from the point $(1, 8, 4)$ on the line joining the point $(0, -11, 4)$ and $(2, -3, 1)$ is

(a) (4, 5, 2) (b) (–4, 5, 2)
(c) (4, –5, 2) (d) (4, 5, –2)

Category-II (Q. No. 51 to 65)

Carry 2 marks each if only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

51. A ladder 20 ft long leans against a vertical wall. The top end slides downwards at the rate of 2 ft per second. The rate at which the lower end moves on a horizontal floor when it is 12 ft from the wall is

(a) $\frac{-8}{3}$ (b) $\frac{6}{5}$

(c) $\frac{3}{2}$ (d) $\frac{17}{4}$

52. For $0 \le p \le 1$ and for any positive a, b; let $I(p) = (a+b)^p$, $J(p) = a^P + b^P$, then

(a) $I(p) > J(p)$

(b) $I(p) \le J(p)$

(c) $I(p) < J(p)$ in $\left[0, \frac{p}{2}\right]$ and $I(p) > J(p)$ in $\left[\frac{p}{2}, \infty\right)$

(d) $I(p) < J(p)$ in $\left[\frac{p}{2}, \infty\right)$ and $J(p) < I(p)$ in $\left[0, \frac{p}{2}\right]$

53. Let $\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}$, $\vec{\beta} = \hat{i} - \hat{j} - \hat{k}$ and $\vec{\gamma} = -\hat{i} + \hat{j} - \hat{k}$ be three vectors. A vector $\vec{\delta}$, in the plane of $\vec{\alpha}$ and $\vec{\beta}$, whose projection on $\vec{\gamma}$ is $\frac{1}{\sqrt{3}}$, is given by

(a) $-\hat{i} - 3\hat{j} - 3\hat{k}$ (b) $\hat{i} - 3\hat{j} - 3\hat{k}$

(c) $-\hat{i} + 3\hat{j} + 3\hat{k}$ (d) $\hat{i} + 3\hat{j} - 3\hat{k}$

54. Let $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ be the three unit vectors such that $\vec{\alpha} \cdot \vec{\beta} = \vec{\alpha} \cdot \vec{\gamma} = 0$ and the angle between $\vec{\beta}$ and $\vec{\gamma}$ is 30°. Then $\vec{\alpha}$ is

(a) $2(\vec{\beta} \times \vec{\gamma})$ (b) $-2(\vec{\beta} \times \vec{\gamma})$

(c) $\pm 2(\vec{\beta} \times \vec{\gamma})$ (d) $(\vec{\beta} \times \vec{\gamma})$

55. Let z_1 and z_2 be complex numbers such that $z_1 \ne z_2$ and $|z_1| = |z_2|$. If $\text{Re}(z_1) > 0$ and $\text{Im}(z_2) < 0$, then $\frac{z_1 + z_2}{z_1 - z_2}$ is

(a) one (b) real and positive

(c) real and negative (d) purely imaginary

56. From a collection of 20 consecutive natural numbers, four are selected such that they are not consecutive. The number of such selections is

(a) 284×17 (b) 285×17

(c) 284×16 (d) 285×16

57. The least positive integer n such that $\begin{pmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}^n$ is an identity matrix of order 2 is

(a) 4 (b) 8

(c) 12 (d) 16

58. Let ρ be a relation defined on N, the set of natural numbers, as $\rho = \{(x, y) \in N \times N : 2x + y = 41\}$. Then

(a) ρ is an equivalence relation

(b) ρ is only reflexive relation

(c) ρ is only symmetric relation

(d) ρ is not transitive

59. If the polynomial

$$f(x) = \begin{vmatrix} (1+x)^a & (2+x)^b & 1 \\ 1 & (1+x)^a & (2+x)^b \\ (2+x)^b & 1 & (1+x)^a \end{vmatrix},$$

then the constant term of $f(x)$ is

(a) $2 - 3\cdot 2^b + 2^{3b}$ (b) $2 + 3\cdot 2^b + 2^{3b}$

(c) $2 + 3\cdot 2^b - 2^{3b}$ (d) $2 - 3\cdot 2^b - 2^{3b}$

[a and b are positive integers]

60. A line cuts the X-axis at $A(5, 0)$ and the Y-axis at $B(0, -3)$. A variable line PQ is drawn perpendicular to AB cutting the X-axis at P and the Y-axis at Q. If AQ and BP meet at R, then the locus of R is

(a) $x^2 + y^2 - 5x + 3y = 0$ (b) $x^2 + y^2 + 5x + 3y = 0$

(c) $x^2 + y^2 + 5x - 3y = 0$ (d) $x^2 + y^2 - 5x - 3y = 0$

61. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Let $B(1, 7)$ and $D(4, -2)$ be two points on the circle such that tangents at B and D meet at C. The area of the quadrilateral $ABCD$ is

(a) 150 sq units (b) 50 sq units

(c) 75 sq units (d) 70 sq units

62. Let $f(x)=\begin{cases}-2\sin x, & \text{if } x\le -\frac{\pi}{2}\\ A\sin x+B, & \text{if } -\frac{\pi}{2}<x<\frac{\pi}{2}\\ \cos x, & \text{if } x\ge \frac{\pi}{2}\end{cases}$. Then,

(a) f is discontinuous for all A and B
(b) f is continuous for all $A=-1$ and $B=1$
(c) f is continuous for all $A=1$ and $B=-1$
(d) f is continuous for all real values of A, B

63. The normal to the curve $y=x^2-x+1$, drawn at the points with the abscissa $x_1=0$, $x_2=-1$ and $x_3=5/2$

(a) are parallel to each other
(b) are pairwise perpendicular
(c) are concurrent
(d) are not concurrent

64. The equation $x\log x=3-x$

(a) has no root in (1, 3)
(b) has exactly one root in (1, 3)
(c) $x\log x-(3-x)>0$ in [1, 3]
(d) $x\log x-(3-x)<0$ in [1, 3]

65. Consider the parabola $y^2=4x$. Let P and Q be points on the parabola where $P(4,-4)$ and $Q(9,6)$. Let R be a point on the arc of the parabola between P and Q. Then, the area of ΔPQR is largest when

(a) $\angle PQR=90°$ (b) $R(4, 4)$
(c) $R\left(\frac{1}{4}, 1\right)$ (d) $R\left(1, \frac{1}{4}\right)$

Category-III (Q. Nos. 66 to 75)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked, ÷ actual number of correct answer. If any wrong option is marked or if, any combination including a wrong option is marked, the answer will considered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. Let $I=\int_0^1 \frac{x^3\cos 3x}{2+x^2}dx$. Then

(a) $-\frac{1}{2}<I<\frac{1}{2}$ (b) $-\frac{1}{3}<I<\frac{1}{3}$
(c) $-1<I<1$ (d) $-\frac{3}{2}<I<\frac{3}{2}$

67. A particle is in motion along a curve $12y=x^3$. The rate of change of its ordinate exceeds that of abscissa in

(a) $-2<x<2$ (b) $x=\pm 2$
(c) $x<-2$ (d) $x>2$

68. The area of the region lying above X-axis, and included between the circle $x^2+y^2=2ax$ and the parabola $y^2=ax, a>0$ is

(a) $8\pi a^2$ (b) $a^2\left(\frac{\pi}{4}-\frac{2}{3}\right)$
(c) $\frac{16\pi a^2}{9}$ (d) $\pi\left(\frac{27}{8}+3a^2\right)$

69. If the equation $x^2-cx+d=0$ has roots equal to the fourth powers of the roots of $x^2+ax+b=0$, where $a^2>4b$, then the roots of $x^2-4bx+2b^2-c=0$ will be

(a) both real
(b) both negative
(c) both positive
(d) one positive and one negative

70. On the occasion of Dipawali festival each student of a class sends greeting cards to others. If there are 20 students in the class, the number of cards sends by students is

(a) ${}^{20}C_2$ (b) ${}^{20}P_2$ (c) $2\times {}^{20}C_2$ (d) $2\times {}^{20}P_2$

71. In a third order matrix A, a_{ij} denotes the element in the ith row and jth column.

If $a_{ij}=0$ for $i=j$
$=1$ for $i>j$
$=-1$ for $i<j$

Then the matrix is

(a) skew symmetric (b) symmetric
(c) not invertible (d) non-singular

72. The area of the triangle formed by the intersection of a line parallel to X-axis and passing through $P(h, k)$, with the lines $y=x$ and $x+y=2$ is h^2. The locus of the point P is

(a) $x=y-1$ (b) $x=-(y-1)$
(c) $x=1+y$ (d) $x=-(1+y)$

73. A hyperbola, having the transverse axis of length $2\sin\theta$ is confocal with the ellipse $3x^2+4y^2=12$. Its equation is

(a) $x^2\sin^2\theta-y^2\cos^2\theta=1$
(b) $x^2\operatorname{cosec}^2\theta-y^2\sec^2\theta=1$
(c) $(x^2+y^2)\sin^2\theta=1+y^2$
(d) $x^2\operatorname{cosec}^2\theta=x^2+y^2+\sin^2\theta$

74. Let $f(x) = \cos\left(\frac{\pi}{x}\right)$, $x \neq 0$, then assuming k as an integer,

(a) $f(x)$ increases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

(b) $f(x)$ decreases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

(c) $f(x)$ decreases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

(d) $f(x)$ increases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

75. Consider the function $y = \log_a(x + \sqrt{x^2+1})$, $a > 0$, $a \neq 1$. The inverse of the function

(a) does not exist

(b) is $x = \log_{1/a}(y + \sqrt{y^2+1})$

(c) is $x = \sinh(y \log a)$

(d) is $x = \cosh\left(-y \log \frac{1}{a}\right)$

Answers

Physics

1. (d)	2. (c)	3. (a)	4. (b)	5. (d)	6. (c)	7. (a)	8. (d)	9. (d)	10. (c)
11. (a)	12. (c)	13. (b)	14. (a)	15. (c)	16. (c)	17. (c)	18. (d)	19. (b)	20. (c)
21. (c)	22. (b)	23. (b)	24. (a)	25. (c)	26. (b)	27. (c)	28. (b)	29. (d)	30. (c)
31. (d)	32. (a)	33. (d)	34. (a)	35. (d)	36. (b,c,d)	37. (a,c)	38. (a,d)	39. (b)	40. (a)

Chemistry

41. (d)	42. (d)	43. (d)	44. (a)	45. (d)	46. (b)	47. (b)	48. (b)	49. (b)	50. (a)
51. (d)	52. (c)	53. (b)	54. (c)	55. (d)	56. (d)	57. (c)	58. (d)	59. (d)	60. (a)
61. (a)	62. (c)	63. (b)	64. (b)	65. (c)	66. (b)	67. (b)	68. (b)	69. (c)	70. (d)
71. (a)	72. (a)	73. (b)	74. (a)	75. (a)	76. (a,c)	77. (a, d)	78. (b,c)	79. (a,c,d)	80. (a,c,d)

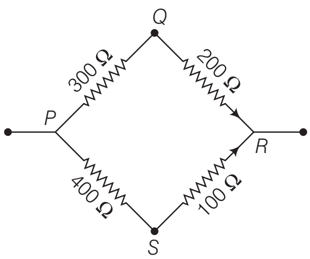
Mathematics

1. (a)	2. (c)	3. (b)	4. (c)	5. (c)	6. (a)	7. (b)	8. (b)	9. (c)	10. (d)
11. (b)	12. (c)	13. (b)	14. (c)	15. (c)	16. (c)	17. (c)	18. (b)	19. (d)	20. (c)
21. (b)	22. (c)	23. (a)	24. (a)	25. (c)	26. (a)	27. (a)	28. (a)	29. (c)	30. (d)
31. (c)	32. (c)	33. (d)	34. (c)	35. (b)	36. (a)	37. (a)	38. (c)	39. (b)	40. (d)
41. (d)	42. (b)	43. (a)	44. (c)	45. (a)	46. (c,d)	47. (d)	48. (a)	49. (a)	50. (d)
51. (a)	52. (b)	53. (c)	54. (c)	55. (d)	56. (a)	57. (b)	58. (d)	59. (a)	60. (a)
61. (c)	62. (b)	63. (c)	64. (b)	65. (c)	66. (b)	67. (c,d)	68. (b)	69. (a,d)	70. (b,c)
71. (a,c)	72. (a,b)	73. (b)	74. (a,c)	75. (c)					

Answer *with* Explanations

Physics

1. *(d)* For maximum equivalent resistance across the diagonal of the square, the given resistors connected as



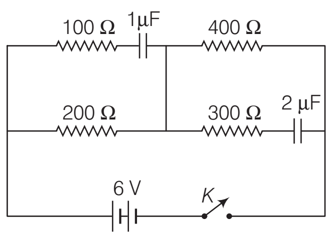
Resistance of PQR arm, $R_1 = 300 + 200 = 500\,\Omega$
Resistance of PSR arm, $R_2 = 400 + 100\,\Omega = 500\,\Omega$
The equivalent resistance between P and R.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

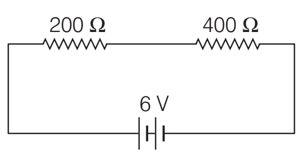
$$= \frac{1}{500} + \frac{1}{500} = \frac{1+1}{500}$$

$$\therefore \quad R_{eq} = \frac{500}{2} = 250\,\Omega$$

2. *(c)*



In steady state, arm having capacitors does not flow current. So, we can neglect them.
∴The given circuit reduces to



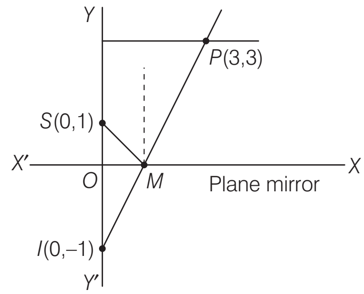
$$\therefore \quad R_{net} = 200 + 400 = 600\,\Omega$$

∴Current in circuit,

$$I = \frac{V}{R} = \frac{6}{600} = 0.01 \text{ A}$$

$$= 10\text{mA}$$

3. *(a)* Ray diagram for the question,



I is image of source S by the plane mirror placed perpendicularly along X-axis.

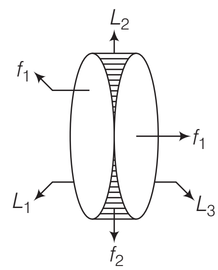
$$\because \quad SM = IM$$

$$\therefore PM + MS = PM + MI = PI$$

$$\therefore \quad PI = \sqrt{(3-0)^2 + (3+1)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

4. *(b)* The given combination of lenses



Here, f_1 = focal length of equiconvex lenses of glass.
f_2 = focal length of lens formed by water (concave).
The focal length of the combination.

$$\therefore \quad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{f_1} - \frac{1}{f_2} + \frac{1}{f_1} = \frac{2}{f_1} - \frac{1}{f_2}$$

$$\frac{1}{F} = \frac{2f_2 - f_1}{f_1 f_2} \Rightarrow F = \frac{f_1 f_2}{2f_2 - f_1} \qquad [\because f_3 = f_1]$$

$$F = \frac{f_1}{2 - \frac{f_1}{f_2}} \qquad \ldots\text{(i)}$$

Here, $\frac{1}{f_1} = (\mu_g - 1)\frac{2}{R}$, for L_1 and L_3

and $\frac{1}{f_2} = (\mu_w - 1)\left(-\frac{2}{R}\right)$, for L_2

Given, $\mu_w < \mu_g$

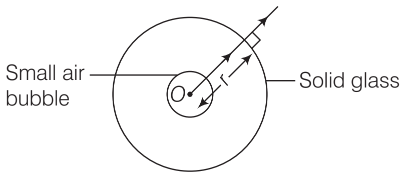
Thus, $\frac{f_1}{f_2} < 1$

So, $$F > \frac{f_1}{2} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{f_1}{2} < F < f_1 \quad \text{or} \quad \frac{f}{2} < F < f \qquad (\because f_1 = f)$$

5. *(d)*



As the object is at centre of the sphere.

$\therefore$ All rays will fall normally on surface, hence they do not deflect. Thus, a virtual image is formed at the centre O.

$\therefore$ Apparent depth = Real depth

6. *(c)* Young's double slit experiment with white light which consist wavelength from range 4000 Å to 7000 Å.

$\because$ At the centre of the screen, the path difference is zero for all wavelengths. The bright fringes of these wavelengths overlap at the centre. Thus, the white fringe at the centre is formed.

7. *(a)* $\because$ Linear velocity of an electron in a orbit of H like atom,

$$v = 2.18 \times 10^6 \times \frac{Z}{n} \text{ m/s} = \frac{c_0}{137} \cdot \frac{Z}{n}$$

where, Z = number of protons in nucleus,

n = principal quantum number

and c_0 = speed of light in free space,

Thus, we have

$$v \propto \frac{1}{n}$$

8. *(d)* Given, $t_{1/2} = 3$ days

Number of active nuclei remaining after time t,

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

After $t = 2$ days,

$$N_1 = N_0 \left(\frac{1}{2}\right)^{2/3} = \frac{N_0}{2^{2/3}} = \frac{N_0}{4^{1/3}} = 0.63 N_0$$

$\therefore$ $N_1 = 0.63 N_0$

After $t = 3$ days,

$$N_2 = \frac{N_0}{2} = 0.5 N_0$$

Number of nuclei that will decay on the 3rd day,

$N_3 = N_2 - N_1 = 0.63 N_0 - 0.5 N_0 = 0.13 N_0$

In the term, fraction is 0.13.

9. *(d)* $\because$ Kinetic energy of a electron due to accelerated by a potential V, KE = eV

$$\frac{1}{2} m_e v^2 = eV$$

$$\Rightarrow \quad \frac{1 \times p^2}{2m_e} = eV \qquad [\because p = mv]$$

$$\therefore \quad p = \sqrt{2eVm_e}$$

$\because$ de-Broglie wavelength of a particle having momentum p,

$$\lambda = \frac{h}{p}$$

According to question,

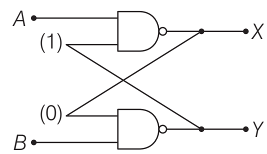
$$\frac{\lambda_1}{\lambda_2} = \frac{p_2}{p_1} = \frac{\sqrt{2\,eV_2 m_e}}{\sqrt{2\,eV_1 m_e}} = \sqrt{\frac{V_2}{V_1}}$$

$$\because \quad \lambda_2 = 2\lambda_1 \qquad \text{(given)}$$

$$\Rightarrow \quad \frac{\lambda_1}{2\lambda_1} = \sqrt{\frac{V_2}{10000}}$$

$$\therefore \quad \text{Potential } V_2 = \frac{10^4}{4} = 2500\,\text{V}$$

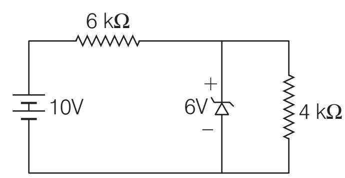
10. *(c)*



$$\therefore \quad X = \overline{AY} = \overline{Y} = 0$$

$$Y = \overline{BX} = 1$$

11. *(a)*



In the given circuit, the zener diode is used as a voltage regulating device. The voltage across 6 kΩ resistance is $(10 - 6)$ V = 4 V

Current through 6 kΩ resistor,

$$I = \frac{4\text{ V}}{6\text{k}\,\Omega} = \frac{4}{6 \times 10^3} = \frac{2}{3} \times 10^{-3} = \frac{2}{3}\text{mA}$$

12. *(c)* In representation of simple harmonic motion in ellipse as velocity along X-axis and displacement along Y-axis.

$$\because \quad \frac{\text{Major axis (along } X\text{ - axis)}}{\text{Minor axis (along } X\text{ - axis)}} = 20\pi$$

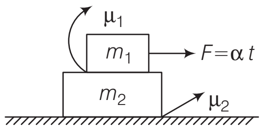
$\Rightarrow \qquad \frac{\omega A}{A} = 20\pi \Rightarrow \omega = 20\pi$

$\Rightarrow \qquad 2\pi n = 20\pi$

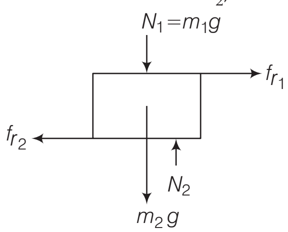
The frequency of the simple harmonic motion

$\therefore \qquad n = 10\,\text{Hz}$

13. *(b)* According to the question,



FBD of lower block of mass m_2,



$\therefore \qquad N_2 = m_2 g + N_1 = (m_1 + m_2)\, g$

$\because$ Lower block never moves.

$\therefore \qquad f_{r_2} \ge f_{r_1}$

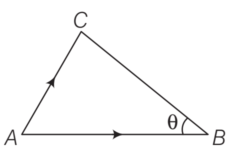
$\mu_2 N_2 \ge \mu_1 N_1 \Rightarrow \mu_2 (m_1 + m_2)\, g \ge \mu_1 m_1 g$

$\Rightarrow \qquad \frac{m_1 + m_2}{m_1} \ge \frac{\mu_1}{\mu_2}$

$\therefore \qquad \frac{\mu_1}{\mu_2} \le 1 + \frac{m_2}{m_1}$

$\therefore \qquad \left.\frac{\mu_1}{\mu_2}\right|_{\max} = 1 + \frac{m_2}{m_1}$

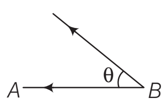
14. *(a)*



$\mathbf{AB} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathrm{AC} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\therefore \qquad \mathbf{BA} = -(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$\angle ABC$ is angle between BA and BC



$\mathbf{BC} = \mathbf{AC} + \mathbf{BA}$

$= \mathbf{AC} - \mathbf{AB} \qquad (\because \mathbf{BA} = -\mathbf{AB})$

$= \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} - (3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = -2\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$\therefore \qquad \mathbf{BA} \cdot \mathbf{BC} = |\mathbf{BA}||\mathbf{BC}|\cos\theta$

$-(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (-2\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$= (\sqrt{3^2 + 1^2 + 1^2}) \cdot (\sqrt{2^2 + 1^2}) \cos\theta$

$\Rightarrow \qquad 6 - 1 = \sqrt{11 \times 5} \cdot \cos\theta$

$\therefore \qquad \cos\theta = \frac{5}{\sqrt{55}} = \sqrt{\frac{5}{11}} = \theta = \cos^{-1} = \sqrt{\frac{5}{11}}$

15. *(c)* According to the question,

$$v \propto \frac{1}{\sqrt{x}}$$

or $\qquad v = \frac{k}{\sqrt{x}} \qquad \ldots(\text{i})$

$$\frac{dv}{dt} = \frac{d}{dt} \cdot \frac{K}{\sqrt{x}} = k \cdot \frac{-1}{2} \cdot x^{-\frac{1}{2} - 1} \cdot \frac{dx}{dt}$$

$$a = -\frac{k}{2} \cdot x^{-\frac{3}{2}} \cdot \frac{k}{\sqrt{x}} \left[\because \frac{dx}{dt} = v \text{ and } v = \frac{k}{\sqrt{x}} \text{ from Eq.(i)}\right]$$

$$= \frac{-k^2}{2} \cdot \frac{1}{x^2}$$

$\therefore$ Force, $F = \text{Mass} \times |\text{Acceleration}|$

$= m\frac{k^2}{2} \cdot \frac{1}{x^2} \qquad$ (magnitude)

$F = \frac{k^2}{2} \cdot \frac{m}{x^2}$

$\therefore \qquad F \propto \frac{1}{x^2}$

16. *(c)* $\because$ Escape velocity from a planet,

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R^2}}\, R$$

$= \sqrt{2gR} \qquad \ldots(\text{i})$

According due to gravity

$$g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3 \cdot \rho}{R^2} = \frac{4}{3} G\pi R\rho$$

$\therefore \qquad$ Radius $R = \frac{3g}{4\pi G\rho} \qquad \ldots(\text{ii})$

From Eqs. (i) and (ii), we get

$$v_e = \sqrt{2g \cdot \frac{3g}{4\pi G\rho}} = \sqrt{\frac{3}{2} \frac{g^2}{\pi G\rho}}$$

Thus, $\qquad v_e \propto \frac{g}{\sqrt{\rho}}$

$\therefore \qquad \frac{v_{e_1}}{v_{e_2}} = \frac{g_1}{\sqrt{\rho_1}} \times \frac{\sqrt{\rho_2}}{g_2} = \frac{5}{2} \times \frac{1}{\sqrt{2}}$

(given, $\frac{g_1}{g_2} = \frac{5}{2}$ and $\frac{\rho_1}{\rho_2} = 2 : 1$)

$= \frac{5}{2\sqrt{2}}$

17. *(c)* According to the question,

time period, $T \propto r^a \rho^b S^c$

$$T = kr^a \rho^b S^c \quad \text{...(i)}$$

Thus, putting dimension, we get

$$[T] = [L]^a \, [ML^{-3}]^b \, [MT^{-2}]^c$$

$$[T] = [M]^{b+c} \cdot [L]^{a-3b} \cdot [T]^{-2c}$$

Equating the dimensions of both sides, we get

$$b + c = 0, \; a - 3b = 0$$

and $-2c = 1$

$\therefore \qquad c = -\frac{1}{2}$

$\therefore \qquad b = \frac{1}{2}$

and $\qquad a = 3b = \frac{3}{2}$

Putting these value of a, b and c into Eq. (i), we get

$$T = k \cdot r^{\frac{3}{2}} \rho^{\frac{1}{2}} \cdot S^{-\frac{1}{2}}$$

$$= k\sqrt{\frac{\rho}{S} r^3}$$

18. *(d)* $\because$ Stress, $\frac{F}{\Delta A} = 1\%$ of $Y = \frac{Y}{100}$

But Young's modulus, $Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{F}{\Delta A}}{\frac{\Delta l}{l}}$

$\therefore \qquad Y = \frac{\frac{Y}{100}}{\frac{\Delta l}{l}} \qquad \left(\text{putting } \frac{F}{\Delta A} = \frac{Y}{100}\right)$

$\therefore \qquad \frac{\Delta l}{l} = \frac{1}{100}$

Poisson's ratio, $\sigma = \frac{-\frac{\Delta r}{r}}{\frac{\Delta l}{l}}$

$\therefore \qquad \frac{\Delta r}{r} = -\sigma \cdot \frac{\Delta l}{l} = \frac{-0.3}{100}$

$$\frac{\Delta r}{r} = \frac{-0.3}{100}$$

$\therefore$ Change in volume, $\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta l}{l}$

$$= \frac{2 \times (-0.3)}{100} + \frac{1}{100}$$

$$= \frac{1 - 0.6}{100} = \frac{0.4}{100}$$

$\therefore \qquad \Delta V\% = 0.4\%$

19. *(b)* Terminal velocity, $v = \frac{2}{9} r^2 \frac{(\rho - \sigma)}{\eta} g$

Neglecting buoyancy effect of the fluid,

$$v = \frac{2}{9} \cdot \frac{\rho}{\eta} r^2 g$$

Putting the given values, we get

$$v = \frac{2}{9} \times \frac{10^3 \times (0.9 \times 10^{-3})^2}{1.8 \times 10^{-5}} \times 9.8 = 98 \, \text{ms}^{-1}$$

20. *(c)* Let the mass of ice $= m$

Applying calorimetry principle,

heat given = heat taken

$$(m_1 + m_2) \, s_1 (t_1 - t) = \frac{mL}{2} + ms_2(t - t_2)$$

Putting values, we get

$$(10 + 50) \times 1 \times (15 - 0) = \frac{m}{2} \times 80 + m \times 0.5[0 - (-10)]$$

$$\Rightarrow 60 \times 15 = 40m + \frac{m}{2} \times 10 = 45m$$

$\therefore \qquad m = \frac{60 \times 15}{45} = 20 \text{ g}$

21. *(c)* Heat capacity of an ideal gas in a thermodynamic process,

$\because$ Ideal gas is monoatomic,

$$C_V = \frac{fR}{2} = \frac{3R}{2}$$

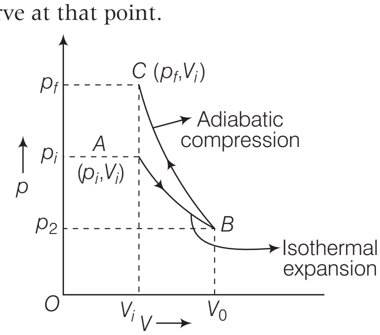
$$C_{\text{process}} = C_V + \frac{p}{n} \cdot \frac{dV}{dT}$$

$$= \frac{3}{2}R + \frac{p}{n} \cdot \frac{V_0}{2T}$$

$\therefore \quad C_{\text{process}} = \frac{3}{2}R + \frac{nRT}{n2T} \qquad [\because pV = nRT]$

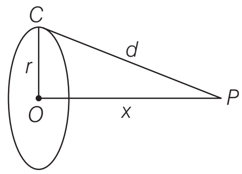
$$= \frac{3}{2}R + \frac{R}{2} = 2R$$

22. *(b)* In p-V diagram, the slope of an adiabatic curve at any point in steeper than that of isothermal curve at that point.



Thus, $p_f > p_i$

23. *(b)* A charged circular ring of radius R is shown in figure.



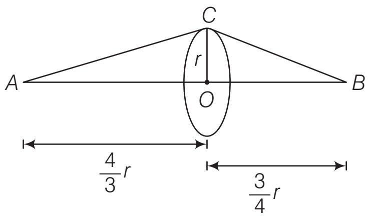
From the figure, $d = \sqrt{x^2 + r^2}$ ...(i)

$\therefore$ Potential due to a uniform ring of positive charge q at point P,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{d}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{x^2 + r^2}} \quad \text{[from Eq. (i)]}$$

Now,



$$\therefore \quad CB = \sqrt{OC^2 + OB^2} = \sqrt{r^2 + \left(\frac{3}{4}\right)^2 r^2} = \frac{5}{4}r$$

Similarly, $AC = \frac{5}{3}r$

$\therefore$ Work done in bringing a point charge $-q$ from A to B,

$$W = -q(V_B - V_A)$$

$$= -q\left[\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{CB} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{CA}\right]$$

$$= \frac{-1}{4\pi\varepsilon_0} \cdot q^2 \left[\frac{1}{\frac{5}{4}r} - \frac{1}{\frac{5}{3}r}\right]$$

$$= \frac{-1}{4\pi\varepsilon_0} \cdot \frac{q^2}{5r}[4 - 3] = -\frac{1}{5} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r}$$

24. *(a)* For surface $ABCD$ electric flux is zero. Because at surface $ABCD$ net electric field is zero.

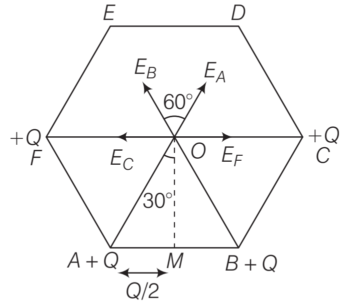
Using Gauss's law,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{\text{in}}}{\varepsilon_0}$$

Electric flux through the five faces of the cube,

$$\phi_E = \frac{Q}{\varepsilon_0}$$

25. *(c)*



(Regular hexagon)

In ΔAOM, $\sin 30° = \frac{AM}{AO}$

$$\therefore \quad AO = \frac{\frac{a}{2}}{1/2} = a$$

For maximum electric field at centre O charges should be placed at F, A, B and C.

$\because$ Electric field due to charges at F and C is equal and opposite at O.

$\therefore$ Net electric field at centre O due to charges at A and B.

Angle between E_A and E_B is 60°.

$\therefore E_{\text{net}}$ at O,

$$E_{\text{net}} = \sqrt{E_A^2 + E_B^2 + 2E_A E_B \cos 60°} \quad (\because E_A = E_B = E)$$

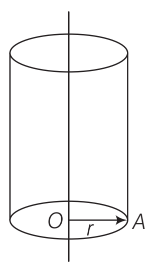
$$= \sqrt{E^2 + E^2 + 2E^2 \cdot \frac{1}{2}} \quad \left(\because \cos 60° = \frac{1}{2}\right)$$

$$= E\sqrt{3} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{a^2}\sqrt{3}$$

26. *(b)* $\because$ When a charged particle moves at a circular path in uniform magnetic field, it time period is independent from its speed.

$\therefore$ Time period of proton will not change with change in its speed.

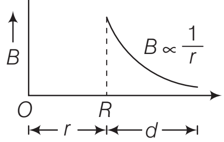
27. *(c)* $\because$ Magnetic field due to a hollow cylinder of radius r.



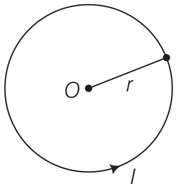
$$B_{\text{inside}} = 0 \text{ (for } d < r)$$

$$B_{\text{outside}} = \frac{\mu_0}{2\pi} \cdot \frac{i}{d} \text{ (for } d \geq r)$$

So, graph between B and d.



28. *(b)*



$$B = \frac{\mu_0}{2} \cdot \frac{I}{r}$$

$\because \qquad I_1 = \frac{V}{R_1}$

Case (I) $l_1 = 2\pi r$ (length of the loop),

$$R_1 = \rho \frac{l_1}{A} \text{ and } B_1 = \frac{\mu_0}{2} \cdot \frac{I_1}{r} \qquad \text{...(i)}$$

Case (II) $l_2 = 2\pi \cdot 2r = 2l_1$

$$R_2 = \rho \cdot \frac{l_2}{A} = \rho \cdot \frac{2l_1}{A} = 2R_1$$

$$I_2 = \frac{V}{R_2} = \frac{V}{2R_1} = \frac{I_1}{2}$$

$$B_2 = \frac{\mu_0}{2} \cdot \frac{I_2}{2r} = \frac{\mu_0}{2} \cdot \frac{I_1}{2 \cdot 2r} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{B_2}{B_1} = \frac{\frac{\mu_0}{2} \cdot \frac{I_1}{2 \cdot 2r}}{\frac{\mu_0}{2} \cdot \frac{I_1}{r}} = \frac{1}{4}$$

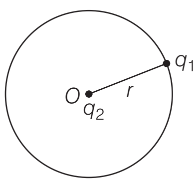
Magnetic field $B_2 = \frac{B_1}{4}$

29. *(d)* $\because$ Resonance frequency, $f_0 = \sqrt{f_1 f_2}$

$$= \sqrt{200 \times 800} = 400\,\text{Hz}$$

30. *(c)* Initially, there will be no voltage drop across capacitor, so intensity of bulb will rise sharply and gradually voltage drop across capacitor will increase as a result voltage drop across bulb decreases, so intensity of bulb will decreases.

31. *(d)*



$$F_{\text{centripetal}} = mr\omega^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$\therefore \qquad \omega^2 = \frac{1}{4\pi\varepsilon_0 m} \cdot \frac{q_1 q_2}{r^3} \Rightarrow \omega = \sqrt{\frac{q_1 q_2}{4\pi\varepsilon_0 m r^3}}$$

$$\omega \propto \frac{1}{r^{3/2}} \qquad \text{...(i)}$$

Current produced due to moving q_1,

$$i = \frac{\omega}{2\pi} q_1 \qquad \text{...(ii)}$$

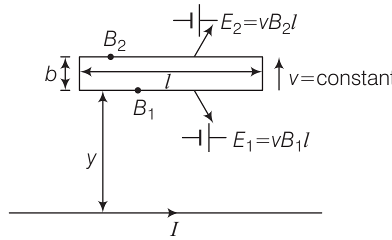
Thus, magnetic field due to motion of q_1 in circular path at point O

$$B = \frac{\mu_0}{2} \cdot \frac{i}{r} = \frac{\mu_0}{2} \cdot \frac{\omega q_1}{2\pi r} \qquad \text{[using Eq. (ii)]}$$

Hence, $B \propto \frac{\omega}{r}$

Magnetic field $B \propto \frac{1}{r^{5/2}}$ [using Eq. (i)]

32. *(a)*



$\therefore$ Motional emf in the loop,

$$E = E_1 - E_2 = vl(B_1 - B_2)$$

$$= vl\left[\frac{\mu_0}{2\pi} \cdot \frac{I}{y} - \frac{\mu_0 I}{2\pi(y+b)}\right]$$

$$= \frac{\mu_0}{2\pi} \cdot I \cdot vl \left[\frac{y+b-y}{y(y+b)}\right] = \frac{\mu_0}{2\pi} \cdot \frac{vlI}{y(y+b)}\, b$$

$\because \qquad b << y$

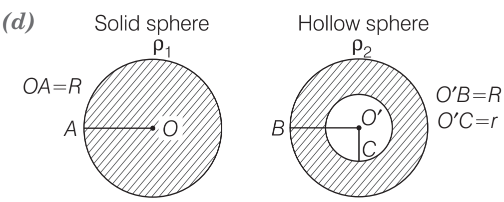
$\therefore \qquad E = \frac{\mu_0}{2\pi} \cdot \frac{vlI}{y^2} b$

$\because v = \text{constant} \qquad (\therefore y = vt)$

Thus, $\quad E = \frac{\mu_0}{2\pi} \cdot \frac{vlI}{v^2 t^2}\, b$

Hence, $\quad E \propto \frac{1}{t^2}$

33. *(d)*



$\therefore \quad M_1 = \rho_1 \frac{4}{3}\pi R^3 \text{ and } M_2 = \rho_2 \frac{4}{3}\pi(R^3 - r^3)$

$\therefore \quad \rho_1 \frac{4}{3}\pi R^3 = \rho_2 \frac{4}{3}\pi(R^3 - r^3)$

$\Rightarrow \quad \rho_1 R^3 = \rho_2(R^3 - r^3)$

$\Rightarrow \quad \frac{\rho_1}{\rho_2} = 1 - \frac{r^3}{R^3}$

$\therefore \quad \frac{r}{R} = \left(1 - \frac{\rho_1}{\rho_2}\right)^{\frac{1}{3}} \quad \ldots(\text{i})$

Moment of inertia,

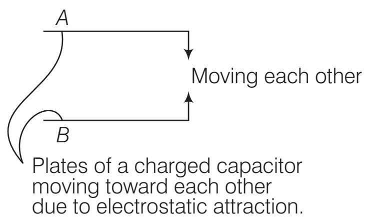
$$\frac{I_H}{I_S} = \frac{\frac{2}{5} M_2 \left[\frac{R^5 - r^5}{R^3 - r^3}\right]}{\frac{2}{5} M_1 R^2}$$

$$= \frac{\rho_2 \frac{4}{3}\pi (R^3 - r^3)\left[\frac{R^5 - r^5}{(R^3 - r^3)}\right]}{\rho_1 \frac{4}{3}\pi R^3 R^2}$$

$$= \frac{\rho_2}{\rho_1}\left[\frac{R^5 - r^5}{R^5}\right] = \frac{\rho_2}{\rho_1}\left[1 - \left(\frac{r}{R}\right)^5\right]$$

$$\therefore \quad \frac{I_H}{I_S} = \frac{\rho_2}{\rho_1}\left[1 - \left(1 - \frac{\rho_1}{\rho_2}\right)^{\frac{5}{3}}\right]$$

34. *(a)*



$$\text{Force} = \frac{1}{2} \cdot \frac{\sigma}{\varepsilon_0} q$$

$\therefore$ Relative acceleration of plates,

$$a_{\text{rel}} = \frac{2F}{M}$$

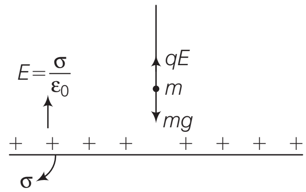
Potential difference, $V = Ed$ ...(i)

where, $d = d_0 - \frac{1}{2} a_{\text{rel}} t^2$ ($\because$ plates are moving)

$\therefore \quad V = E\left[d_0 - \frac{1}{2} a_{\text{rel}} t^2\right]$

This is a equation of a parabola with downward concavity.

35. *(d)*



$\therefore$ Apparent weight of the bob, $w' = mg - qE$

$$mg' = mg - qE$$

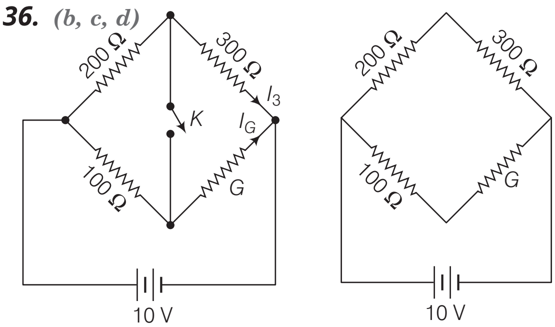
$$\therefore \quad g' = g - \frac{qE}{m}$$

$\because$ Time period of a pendulum,

$$T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$$

$$= 2\pi\sqrt{\frac{L}{g - \frac{qE}{m}}} = 2\pi\sqrt{\frac{L}{\left(g - \frac{q\sigma}{\varepsilon_0 m}\right)}}$$

36. *(b, c, d)*



$$\therefore \quad I_G = \frac{10}{100 + G}$$

$$300\, I_3 = \frac{10}{100 + G} G$$

$$I_3 = \frac{G}{30}\left[\frac{1}{100 + G}\right]$$

$$\therefore \quad I_3 + I_G = \frac{1}{(100 + G)}\left[10 + \frac{G}{30}\right] = \frac{(300 + G)}{(100 + G) \times 30}$$

According to the problem,

$$10 = \left(\frac{200}{3} + \frac{300G}{300 + G}\right)\left[\frac{(300 + G)}{(100 + G)\, 30}\right]$$

$[\because \; V = RI]$

$$10 = \frac{60000 + 1100G}{3} \times \frac{1}{100 + G} \times \frac{1}{30}$$

$\Rightarrow 900(100 + G) = 60000 + 1100\,G$

$\Rightarrow \quad 30000 = 200\,G$

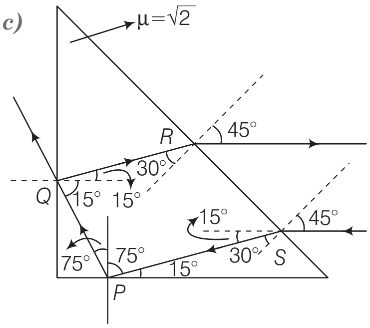
$\therefore \quad G = 150\,\Omega$

$$I_G = \frac{10}{100+150} = \frac{10}{250} = 40\,\text{mA}$$

As $\quad \dfrac{200}{100} = \dfrac{300}{150}$

So, $\quad I_{200} = I_{300}$

37. *(a, c)*



Critical angle $\theta_C = \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = 45°$

Snell's law at points, $(\because n_1 \sin\theta_1 = n_2 \sin\theta_2)$

$1 \sin 45° = \sqrt{2}\sin\theta$

$\therefore \quad \theta = 30°$

At P $\because$ incidence angle at $P = 75° > \theta_C$.

$\therefore$ TIR at point P.

At Q $\because$ incidence angle at $Q = 15° < \theta_C$.

$\therefore$ Partial reflection and refraction at Q.

As $SP \parallel QR$, emergent ray at R is parallel to incident ray at S.

$\therefore$ Net deviation $= 180°$.

38. *(a, d)* The intensity of a sound source appears to be periodic due to

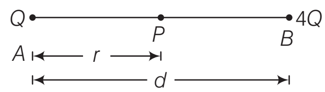
(i) source intensity is periodic.

(ii) source is producing a sound composed of two nearby frequencies.

39. *(b)* $\because$ Internal energy of an ideal gas depends only on the temperature of gas.

$$E_{\text{in}} \propto T$$

40. *(a)*



$\because V_P$ is minimum.

$\therefore \quad E_P = 0 \quad \Rightarrow E_A = E_B$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4Q}{(d-r)^2}$$

$$\Rightarrow \quad \frac{1}{r} = \frac{2}{d-r} \quad \Rightarrow \quad d - r = 2r$$

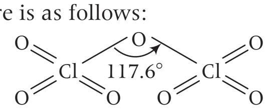
$\therefore$ Distance of P from the A $r = \dfrac{d}{3}$ units to the right of A.

Chemistry

41. *(d)* Cl_2O_7 is the anhydride of $HClO_4$. It reacts with water slowly to give perchloric acid.

$$Cl_2O_7 + H_2O \longrightarrow \underset{\text{Perchloric acid}}{2HClO_4}$$

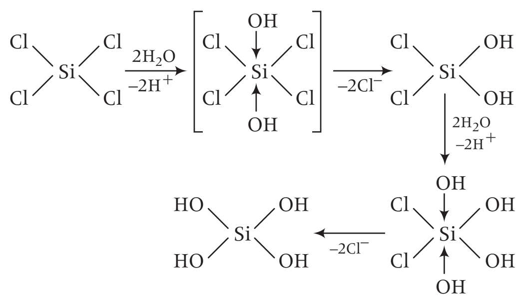
Its structure is as follows:



It is less reactive than other oxides of chlorine and does not react with P, S, coal or paper at room temperature.

42. *(d)* The main reason that $SiCl_4$ is easily hydrolysed as compared to CCl_4 is that Si can extend its coordination number beyond four because it has vacant d-orbital. On the other hand, carbon has no d-orbital thus, it cannot extend its coordination number beyond four, so its halides are not attacked (hydrolysed) by water. The vacant d-orbitals of Si can coordinate with water molecules and hence their halides are hydrolysed by water.

The hydrolysis of $SiCl_4$ occurs due to coordination of —OH with empty 3d-orbitals of Si-atom of $SiCl_4$ molecule.



43. *(d)* Silver chloride dissolves in excess of ammonium hydroxide solution. The cation present in the resulting solution is $[Ag(NH_3)_2]^+$

In aqueous solution, silver chloride exist as Ag^+ and Cl^-. Ag^+ present in solution reacts with NH_3 to form $[Ag(NH_3)_2]^+$.

$$Ag^+Cl^- + 2NH_3 \longrightarrow [Ag(NH_3)_2]^+Cl^-$$

44. *(a)* The ease of hydrolysis of carbonyl compounds depend on the group attached to the carbonyl carbon. Among the given options, chlorine (—Cl) group is electron withdrawing group which makes the carbonyl group electrophilic. Hence, hydrolysis can occur most easily. The order for electron withdrawing tendency among given options is as follows:

$$—Cl > —OCOCH_3 > —OC_2H_5 > —NH_2$$

So, the correct order for the ease of hydrolysis in the given compounds is as follows:

$$\underset{I}{CH_3COCl} > \underset{II}{CH_3COOCOCH_3} > \underset{III}{CH_3COOC_2H_5} > \underset{IV}{CH_3CONH_2}$$

45. *(d)* $CH_3—C\equiv CMgBr$ can be prepared by the reaction of $CH_3—C\equiv CH$ with CH_3MgBr. This method is used in preparation of higher alkynes from lower alkynes. The chemical equation for the formation of $CH_3—C\equiv CMgBr$ is given below

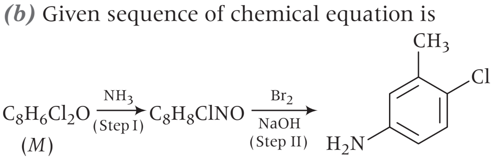
$$CH_3—C\equiv CH + \overset{-}{C}H_3\overset{2+}{Mg}\overset{-}{Br} \longrightarrow CH_3—C\equiv \overset{-}{C}\overset{2+}{Mg}X^-$$

46. *(b)* The number of alkene(s) that can produce 2-butanol by the successive treatment of B_2H_6 in tetrahydrofuran solvent and alkaline H_2O_2 solution is 2. The alkenes are *cis*-but -2- ene and *trans*-but-2-ene. The hydration product is obtained in accordance with opposite Markownikoff's rule.

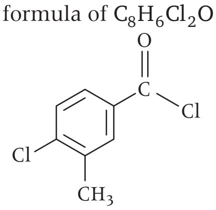
In this reaction, addition takes place through the initial formation of π-complex which changes into a cyclic four centre transition state with the addition of boron atom to the less hindered carbon atom.

$$\underset{\text{But-2-ene}}{H_3C—HC=CH—CH_3} \xrightarrow[\text{(ii) } H_2O_2/OH^-]{\text{(i) } B_2H_6} \underset{\text{Butan-2-ol}}{CH_3—\underset{\underset{OH}{|}}{CH}—CH_2CH_3}$$

47. *(b)* Given sequence of chemical equation is

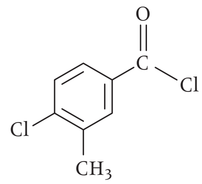


The structural formula of $C_8H_6Cl_2O(M)$ will be

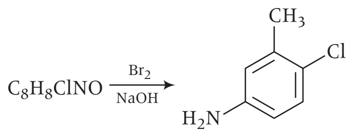


The various step involves in the above road map can be explain as follows

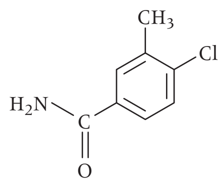
Step-I It involves the formation of an amide on reaction of ammonia and acetyl chloride so, *M* must be



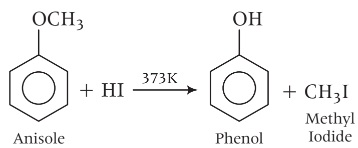
Step-II It involves the Hofmann-bromamide reaction.



In this reaction, migration of an alkyl or aryl group takes place from carbonyl carbon of the amide to the N-atom. So, C_8H_8ClNO must be an amide with structural formula.



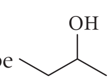
48. *(b)* Methoxybenzene on treatment with HI produces phenol and methyl iodide. This is due the more stable aryl oxygen bond. Here, methyl phenyl oxonium ion is formed by the protonation of ether. The bond between $O—CH_3$ is weaker than the bond between $O—C_6H_5$ because the carbon of phenyl group is sp^2-hybridised and there is a partial double bond character. Thus, the reaction yields phenol and alkyl halide.



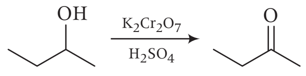
(Methoxy benzene)

49. *(b)* Given sequence of chemical equation is

$$\underset{(N)}{C_4H_{10}O} \xrightarrow[\text{(step I)}]{K_2Cr_2O_7 \atop H_2SO_4} C_4H_8O \xrightarrow[\text{(step II)}]{I_2/NaOH \atop \text{Warm}} CHI_3$$

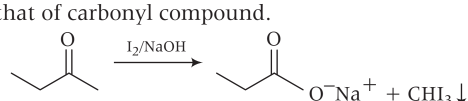
Here, *N* will be  (IUPAC name = Butan-2-ol)

Step I It involves the oxidation of secondary alcohol to corresponding alcohol in the presence of $K_2Cr_2O_7$ and H_2SO_4.



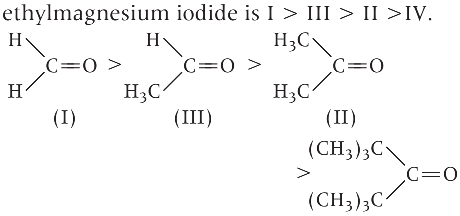
$[C_4H_{10}O]$ (N) $\quad$ $[C_4H_8O]$

Step II It involves the iodoform reaction. Ketone formed in step I have one methyl group linked to the carbonyl carbon atom are oxidised by sodium hypohalite to sodium salts of corresponding carboxylic acids having one carbon atom less than that of carbonyl compound.



$[C_4H_8O]$ $\quad$ (Yellow ppt.)

50. *(a)* The correct order of reactivity for the addition reaction of the given carbonyl compounds with ethylmagnesium iodide is I > III > II >IV.



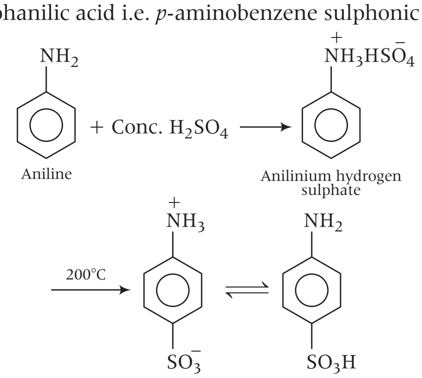
(I) > (III) > (II) > (IV)

This order can be explained on the basis of following two factors:

(i) **Inductive effect** Greater the number of alkyl (electron releasing) groups attached to carbonyl group, greater will be the electron density on carbonyl carbon. Thus, it lowers the attack of nucleophile and hence, reactivity decreases.

(ii) **Steric effect** As the number of alkyl group attached to carbonyl carbon increases, the attack of nucleophile on carbonyl group becomes more and more difficult due to steric hinderance.

51. *(d)* When aniline is treated with conc. H_2SO_4 followed by heating at 200° C, the product obtained is sulphanilic acid i.e. *p*-aminobenzene sulphonic acid



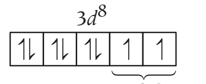
52. *(c)* $n = 2$, $l=0$, $m = -1$ is not a possible electronic configuration. With $n = 2$ and $l=0$, the orbital is $2s$ having only one m_l value i.e. 0. As the value of $m_l = -l$ to $+l$

Other options are correct.

	Orbital
$n = 3, l = 0, m = 0$	$3s$
$n = 3, l = 1, m = -1$	$3p$
$n = 2, l = 1, m = 0$	$2p$

53. *(b)* The number of unpaired electrons in Ni (atomic number=28) are 2. It can be easily concluded from the electronic configuration of Ni.

Electronic configuration of
$Ni = 1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^8$

Orbitals of Ni in $3d$ = 

Unpaired electrons

54. *(c)* The strongest H-bond is F—H------ F because in this case, a hydrogen is bonded to a most electronegative atom i.e. fluorine. In all other options, hydrogen is bonded to oxygen and sulphur that are less electronegative atom than fluorine.

The correct electronegativity order of elements are as follows: F > O > S

55. *(d)* Given,

Half-life of C^{14}, $t_{1/2} = 5760$ years

Initial concentration of sample of C^{14}, $N_0 = 200$ mg

Final concentration of sample of C^{14}, $N_t = 25$ mg

The given decay is radioactive and all radioactive decay follows first order kinetics.

$$t_{1/2} = \frac{0.693}{\lambda}$$

$$\therefore \quad \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{5760} \text{yr}^{-1}$$

We know that, for first order reaction

$$t = \frac{2.303}{\lambda} \log \frac{N_0}{N_t} = \frac{2303}{\frac{0.693}{5760}} \log \frac{[200]}{[25]}$$

$$= \frac{2303 \times 5760}{0.693} \log 8$$

$$= 17,286.78 \text{ yrs} \approx 17,280 \text{ yrs}$$

Therefore, the time taken to change 200 mg to 25 mg is 17, 286.78 yr, which is very close to option (*d*) i.e. 17280 yrs.

Alternative method

$$17280 \text{ years} \left\{ \begin{array}{l} 200 \text{ mg sample} \\ \big\downarrow t_{1/2} = 5760 \text{ years} \\ 100 \text{ mg} \\ \big\downarrow t_{1/2} = 5760 \text{ years} \\ 50 \text{ mg} \\ \big\downarrow t_{1/2} = 5760 \text{ years} \\ 25 \text{ mg} \end{array} \right.$$

56. *(d)* Ferric ion forms a prussian blue precipitate due to the formation of $Fe_4[Fe(CN)_6]_3$. This complex is formed during the determine action of presence of nitrogen in the given sample. In this method, to portion of sodium fusion extract, freshly prepared ferrous sulphate, $FeSO_4$ solution is added and warmed. Then about 2 to 3 drops of $FeCl_3$ solution are added and acidified with conc. HCl. The appearance of a prussian blue colour indicate the presence of nitrogen.

$$FeSO_4 + 2NaOH \longrightarrow Fe(OH)_2 + Na_2SO_4$$

$$6NaCN + Fe(OH)_2 \longrightarrow \underset{\text{Sodium ferrocyanide}}{Na_4[Fe(CN)_6]} + 2NaOH$$

$$3\,Na_4[Fe(CN)_6] + 4FeCl_3 \longrightarrow \underset{\substack{\text{Ferric ferrocyanide} \\ \text{(prussian blue)}}}{Fe_4[Fe(CN)_6]_3} + 12NaCl$$

57. *(c)* The nucleus ${}^{64}_{29}Cu$ accepts an orbital electron to yield ${}^{64}_{28}Ni$. The atomic number of Cu is 29, which is equal to the number of electrons and also equal to the number of protons.

When ${}^{64}_{29}Cu$ accepts an orbital electron then electrons subtract from the atomic number, i.e. $29 - 1 = 28$

$${}^{64}_{29}Cu + {}_{-1}e^0 \longrightarrow {}^{64}_{28}Ni$$

58. *(d)* Mass of an electron $= 9.108 \times 10^{-31}$ kg

Mass of one mole of electron

$$= (9.108 \times 10^{-31} \times 6.023 \times 10^{23}) \text{ kg}$$

Then, number of mole of electron in 1 kg

$$= \frac{1}{9.108 \times 6.023 \times 10^{-8}}$$

$$= \frac{1}{9.108 \times 6.023} \times 10^{8} \text{ mole of } e^-$$

59. *(d)* Given, equal weights of ethane and hydrogen are mixed in an empty container at 25°C.

$$\text{Initial gram weight} = \begin{array}{cc} C_2H_6 & H_2 \\ w\text{g} & w\text{g} \end{array}$$

$$\text{Number of moles} = \frac{w}{30} \qquad \frac{w}{2}$$

According to Henry's law,

$$\frac{p_{H_2}}{p_{\text{total}}} = \chi_{H_2} = \frac{n_{H_2}}{n_{H_2} + n_{C_2H_6}} = \frac{\frac{w}{2}}{\frac{w}{2} + \frac{w}{30}}$$

$$\frac{p_{H_2}}{p_{\text{total}}} = \frac{\frac{w}{2}}{\frac{15w + w}{30}} = \frac{w}{2} \times \frac{30}{16w} = \frac{15}{16}$$

So, the fraction of total pressure exerted by hydrogen is 15 : 16.

60. *(a)* Given, the heat of neutralisation of a strong base and a strong acid is 13.7 kcal.

The reaction of neutralisation is as follows

$$\underset{1 \text{ mol}}{HCl} + \underset{1 \text{ mol}}{NaOH} \longrightarrow NaCl + H_2O;\ \Delta H = -13.7 \text{ kcal}$$

According to question,

$$\underset{0.6 \text{ mol}}{HCl} + \underset{0.25 \text{ mol}}{NaOH} \longrightarrow NaCl + H_2O \qquad \ldots (i)$$

In equation (i), NaOH acts as a limiting reagent. For 1 mole of NaOH and 1 mole of HCl, heat of neutralisation = 13.7 kcal.

$\therefore$ For 0.25 mole of NaOH and 0.6 mole of HCl, heat of neutralisation $= 13.7 \times 0.25 \Rightarrow 3.425$ kcal

61. *(a)* Number of X atoms at the corners = 8

Number of X atoms per unit cell $= 8 \times \frac{1}{8} = 1$ atom

Number of Y atoms at the centre of the body = 1 atom

Hence, the formula of the compound is XY.

62. *(c)* The number of electrons involved in the reaction are three as shown below

$$Al^{3+} + 3e^- \longrightarrow Al$$

It means the conversion of every aluminium ion to aluminium atom requires three electrons.

Therefore, the amount of electricity required for one mole of Al^{3+} ions = 3F.

63. *(b)* Given,

$$Zn \longrightarrow Zn^{2+} + 2e^-;\ E^\circ = +0.76\text{ V} \qquad \ldots (i)$$

$$Fe \longrightarrow Fe^{2+} + 2e^-;\ E^\circ = +0.41\text{ V} \qquad \ldots (ii)$$

On reversing the above equation (i) and (ii), we get

$$Zn^{2+} + 2e^- \longrightarrow Zn;\ E^\circ = -0.76\text{ V}$$

$$Fe^{2+} + 2e^- \longrightarrow Fe;\ E^\circ = -0.41\text{ V}$$

[where, E° = standard reduction potential]

To find, the standard emf of the cell with the reaction.

$$\underbrace{Fe^{2+} + \overbrace{Zn \longrightarrow Zn^{2+}}^{\text{Oxidation (anode)}} + Fe}_{\text{Reduction (cathode)}}$$

So, $E^\circ_{cell} = E^\circ_{Fe^{2+}/Fe} - E^\circ_{Zn^{2+}/Zn}$

$= -0.41\text{ V} + 0.76\text{V} = +0.35\text{V}$

64. *(b)* Given,

$$N_2 + 3H_2 \rightleftharpoons 2NH_3;\ K_1 = \frac{[NH_3]^2}{[N_2][H_2]^3} \quad \dots\text{(i)}$$

$$N_2 + O_2 \rightleftharpoons 2NO;\ K_2 = \frac{[NO]^2}{[N_2][O_2]} \quad \dots\text{(ii)}$$

$$H_2 + \frac{1}{2}O_2 \rightleftharpoons H_2O;\ K_3 = \frac{[H_2O]}{[H_2][O_2]^{1/2}} \quad \dots\text{(iii)}$$

The chemical equation for the oxidation of 2 mol of NH_3 to give NO is

$$2NH_3 + \frac{5}{2}O_2 \rightleftharpoons 2NO + 3H_2O \quad \dots\text{(iv)}$$

To get the equation (iv) from equation (i), (ii) and (iii) following steps are followed:

Reversing equation (i), we get

$$2NH_3 \rightleftharpoons N_2 + 3H_2 \text{ so } K' = \frac{1}{K_1} \quad \dots\text{(v)}$$

Multiplying equation (iii) by 3, we get

$$3H_2 + \frac{3}{2}O_2 \rightleftharpoons 3H_2O \text{ so } K' = K_3^3 \quad \dots\text{(vi)}$$

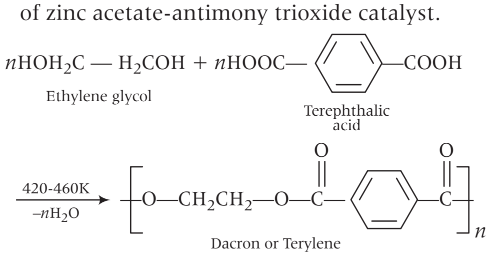
Adding equation (ii) and (vi)

$$N_2 + 3H_2 + \frac{5}{2}O_2 \rightleftharpoons 2NO + 3H_2O$$

so, $K' = K_2 \cdot K_3^3$ ... (vii)

On combining (v) and (vii), we get the required equation having equilibrium constant $K' = K_2 \cdot \frac{K_3^3}{K_1}$.

65. *(c)* Dacron is a condensation polymer. It is also known as terylene. It is a polymer obtained by condensation reaction between ethylene glycol and terephthalic acid at 420-460 K in the presence of zinc acetate-antimony trioxide catalyst.



66. *(b)* H_2SO_4 (sulphuric acid) is present in maximum amount in 'acid rain'. Oxides of nitrogen and sulphur, released into the atmosphere from thermal power plants, industries and automobiles are the main sources of acid rain. These oxides on oxidation followed by hydrolysis give sulphuric acid and nitric acid that alongwith HCl are responsible for the acidity of rain. The oxidation reaction is catalysed by particulate matter present in the polluted atmosphere.

$$2SO_2(g) + O_2(g) + 2H_2O(l) \longrightarrow 2H_2SO_4(aq)$$
$$4NO_2(g) + O_2(g) + 2H_2O(l) \longrightarrow 4HNO_3(aq)$$

67. *(b)* The correct set of oxides arranged in the proper order of basic, amphoteric, acidic are BaO, Al_2O_3, SO_2.

Oxides of non-metallic elements are **acidic** such as CO_2, NO_2, SO_2 etc. Oxides of less electropositive elements (such as BeO, Al_2O_3, Bi_2O_3, ZnO etc.) are **amphoteric** i.e. these behaves as acids toward strong bases and as bases towards strong acids. Oxides of electropositive elements (Na_2O, CaO, Tl_2O, BaO etc.) are **basic** and contain discrete O^{2-} ions.

68. *(b)* Out of the given outer electronic configuration of atoms, the highest oxidation state is achieved by $(n-1)d^5ns^2$ i.e. 7.

A large number of oxidation state is due to the fact the $(n-1)d$-electrons may get involved along with ns electrons in bonding as electrons in $(n-1)d$- orbitals are in an energy state comparable to ns -electrons.

Oxidation state of other options are as follows:

Electronic configuration	Oxidation state
$(n-1)d^8ns^2$	+2, +3, +4
$(n-1)d^3ns^2$	+2, +3, +4, +5
$(n-1)d^5ns^1$	+2, +3, +4, +5, +6

69. *(c)* At room temperature, the reaction between water and fluorine produces F^-, O_2 and H^+.

Fluorine, being non-polar molecule, readily dissolves with water and forms mixture of oxygen and ozone as shown below

$$2F_2 + 2H_2O \longrightarrow 4HF + O_2$$
$$3F_2 + 3H_2O \longrightarrow 6HF + O_3$$

HF exist as H^+ and F^- in a solution.

70. *(d)* $BeCO_3$ is least thermally stable. The thermal stability of carbonates increases down the group i.e from Be to Ba.

$$\underset{(523\text{ K})}{BeCO_3} < \underset{(813\text{ K})}{MgCO_3} < \underset{(1173\text{ K})}{CaCO_3} < \underset{(1562\text{ K})}{SrCO_3} < \underset{(1633\text{ K})}{BaCO_3}$$

$BeCO_3$ is unstable to the extent that it is stable only in atmosphere of CO_2. These carbonates show reversible decomposition in closed container.

$$MCO_3 \rightleftharpoons MO + CO_2$$

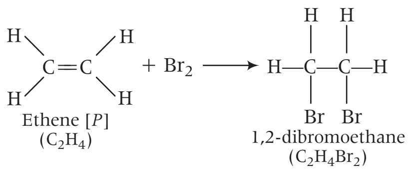
More stable the oxide is formed, lesser will be stability of carbonates.

71. *(a)* Given reactions are

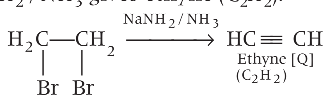
$$[P] \xrightarrow{Br_2} C_2H_4Br_2 \xrightarrow[NH_3]{NaNH_2} [Q]$$

$$[Q] \xrightarrow[Hg^{2+}, \Delta]{20\%/ H_2SO_4} [R] \xrightarrow{Zn\text{-}Hg/HCl} [S]$$

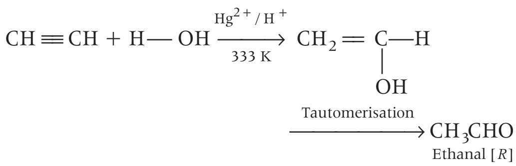
$[P]$ is an alkene with two carbon atoms i.e. ethene. When it reacts with Br_2, addition reaction occurs and $C_2H_4Br_2$ is formed.



$C_2H_4Br_2$ (1, 2-dibromoethane) on reaction with $NaNH_2 / NH_3$ gives ethyne (C_2H_2).



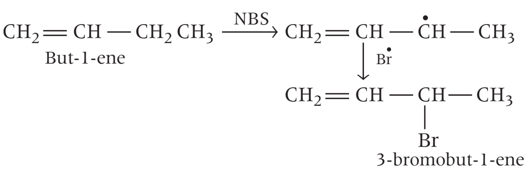
Ethyne $[Q]$ in presence of 20% H_2SO_4, Hg^{2+} at 333 K gives ethanal.



Ethanal $[R]$ undergoes reduction in presence of Zn-Hg / HCl to give ethane $[S]$

$$\underset{\substack{\text{Ethanal}\\[R]}}{CH_3CHO} \xrightarrow[\text{Clemmensen reduction}]{\text{Zn- Hg/ HCl}} \underset{\substack{\text{Ethane}\\[S]}}{CH_3—CH_3}$$

72. *(a)* The number of possible organobromine compounds which can be obtained in the allylic bromination of but-l-ene with N-bromosuccinimide is 1.



73. *(b)* Given, specific heat = 0.16

Let metal chloride be MCl_x then,

$$\frac{6.4}{\text{specific heat}} = \text{Atomic weight of metal}$$

$$\frac{6.4}{0.16} = \text{Atomic weight of metal}$$

Atomic weight = 40

40 is the atomic weight of calcium. According to question, metal chloride (MCl_x) have ≈ 65% chlorine present in it.

$$\frac{x \times \text{Atomic weight of chlorine}}{40 + x \times \text{Atomic weight of chlorine}} \times 100 = 65$$

$$\frac{x \times 35.5}{40 + x \times 35.5} \times 100 = 65$$

$$x = 2.09 \approx 2 \text{ (approx.)}$$

So, the formula of metal chloride will be MCl_2.

74. *(a)* For a reversible adiabatic process,

$$pT^{\frac{\gamma}{1-\gamma}} = \text{constant} \qquad \ldots \text{(i)}$$

According to question, during a reversible adiabatic process the pressure of a gas is found to be proportional to the cube of its absolute temperature.

$$p \propto T^3$$

$$pT^{-3} = \text{constant} \qquad \ldots \text{(ii)}$$

Equating equation (i) and (ii), we get

$$\frac{\gamma}{1-\gamma} = -3$$

$$\gamma = -3(1-\gamma)$$

$$\gamma = -3 + 3\gamma$$

$$-2\gamma = -3$$

$$\gamma = \frac{3}{2}$$

As we know, the ratio of molar heat capacities at constant pressure and constant volume is represented by γ. So, the ratio of $\frac{C_p}{C_V}$ for the gas is $\frac{3}{2}$.

75. *(a)* According to question,

$[X]$ + Dil. $H_2SO_4 \longrightarrow [Y]$:
Colourless, suffocating gas

$[Y] + K_2Cr_2O_7 + H_2SO_4 \longrightarrow$
Green colouration of solution.

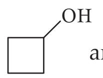
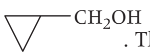
All sulphites when treated with dil. H_2SO_4 gives colourless and suffocating sulphur dioxide gas.

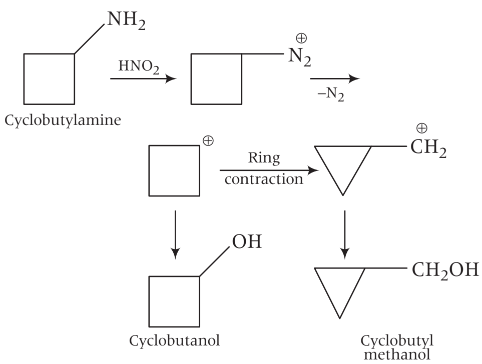
$$\underset{\substack{\text{Sulphite ion}\\[X]}}{SO_3^{2-}(s)} + \text{Dil.}H_2SO_4(aq) \longrightarrow SO_4^{2-}(aq) + \underset{\substack{\text{Sulphur}\\ \text{dioxide}[Y]}}{SO_2(g)} + H_2O(l)$$

SO_2 gas [Y] turns acidified potassium dichromate paper green due to reduction of Cr (VI) to Cr (III).

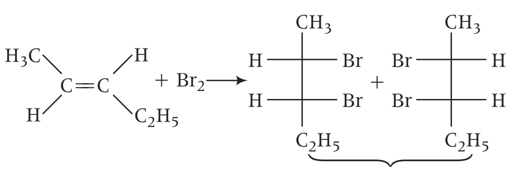
$$K_2Cr_2O_7(aq) + H_2SO_4(aq) + \underset{[Y]}{3SO_2(g)} \longrightarrow K_2SO_4(aq) + \underset{\text{Green}}{Cr_2(SO_4)_3(aq)} + H_2O(l)$$

76. ***(a, c)*** The possible product(s) to be obtained from the reaction of cyclobutyl amine with HNO_2 is/are

cyclobutanol =  and cyclopropylmethanol = . The chemical reactions involved are as follows:

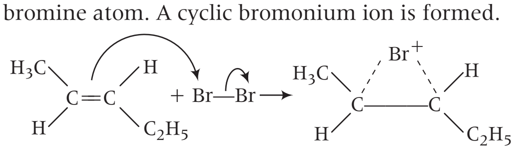


77. ***(a, d)*** The major product(s) obtained in the given reaction are



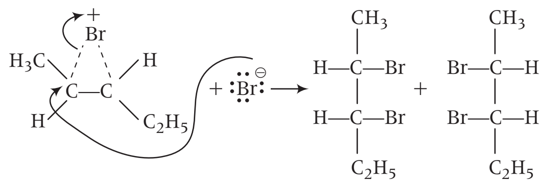
Enantiomers

The mechanism for the above given reaction is as follows. When the π-electrons of the alkene approach a molecule of Br_2, one of the bromine atom accepts them and releases the shared electrons to the other bromine atom. A cyclic bromonium ion is formed.



(Cyclic bromonium ion)

In next step, Br^- attacks a carbon atom of the bromonium ion. This release the strain in the three membered ring and form a *vicinal* dibromide.



78. ***(b, c)*** The correct statements for peroxide ion (O_2^{2-}) are that it is diamagnetic and it has bond order one.

Electronic configuration of O_2^{2-} (peroxide ion) is

$$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, [\pi 2p_x^2 = \pi 2p_y^2], [\pi^* 2p_x^2 = \pi^* 2p_y^2], \sigma^* 2p_z^0$$

$\because$ Bond order =

$$\frac{\text{No. of electrons present in bonding} - \text{No. of electrons present in non-bonding}}{2}$$

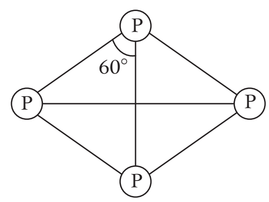
$$\therefore \quad BO = \frac{10-8}{2} = \frac{2}{2} = 1$$

Number of unpaired electrons = 0

So, O_2^{2-} is diamagnetic in nature.

79. ***(a, c, d)*** Among the given options, the extensive variables are H (enthalpy), E (internal energy), V (volume).These variables have values that depends upon the quantity or size of matter present in the system. Other examples are heat capacity, entropy, free energy, length and mass.

80. ***(a, c, d)*** White phosphorus (P_4) has 6P—P single bonds, 4 lone pairs of electrons and P—P—P angle of 60°. It is a translucent white waxy solid. It has large atomic size and less electronegativity. So, it forms single bond instead of $p\pi$ - $p\pi$ multiple bond. It consists of discrete tetrahedral P_4 molecule as shown in the figure.



Mathematics

1. *(a)* We know that, $\sin 30° = \frac{1}{2} = 0.5$

In 1st quadrant $\sin x$ is increasing function.

$\therefore \quad \sin 31° > \sin 30°$

$\Rightarrow \quad \sin 31° > 0.5$

2. *(c)* We have, $f_1(x) = e^x$

$$f_2(x) = e^{f_1(x)}$$

$$\cdots\cdots \quad \cdots\cdots \quad \cdots\cdots \quad \cdots\cdots$$
$$\cdots\cdots \quad \cdots\cdots \quad \cdots\cdots \quad \cdots\cdots$$

$$f_{n+1}(x) = e^{f_n(x)}$$

Now, $\quad f_n(x) = e^{f_{n-1}(x)}$

On taking log both sides, we get

$$\log\{f_n(x)\} = f_{n-1}(x) \log e$$

$$\Rightarrow \quad \frac{d}{dx}\log(f_n(x)) = \frac{d}{dx} f_{n-1}(x) \qquad (\because \log e = 1)$$

$$\Rightarrow \quad \frac{1}{f_n(x)} \frac{d}{dx} f_n(x) = f'_{n-1}(x)$$

$$\Rightarrow \quad \frac{d}{dx} f_n(x) = f_n(x) f'_{n-1}(x) \qquad \ldots \text{(i)}$$

Now, $\quad f_1'(x) = e^x$

and $\quad f_2(x) = e^{f_1(x)}$

$\Rightarrow \log f_2(x) = f_1(x) \log e = f_1(x)$

$$\Rightarrow \quad \frac{1}{f_2(x)} \cdot f_2'(x) = f_1'(x)$$

$$\Rightarrow \quad f_2'(x) = f_2(x) \cdot f_1'(x)$$

$$= f_2(x) \cdot e^x \qquad (\because f_1'(x) = e^x)$$

$$= f_2(x) \cdot f_1(x) \qquad [\because e^x = f_1(x)] \ldots \text{(ii)}$$

From Eq. (i),

$$\frac{d}{dx} f_n(x) = f_n(x) \cdot f_{n-1}(x) \; \ldots\ldots \; f_1(x) \text{ [using Eq. (ii)]}$$

3. *(b)* We have, $f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$

$f(x)$ is defined for all x satisfying

$$\frac{1-|x|}{2-|x|} \ge 0 \quad \Rightarrow \quad \frac{|x|-1}{|x|-2} \ge 0$$

$\Rightarrow \quad |x| \le 1 \text{ or } |x| > 2$

$\Rightarrow \quad x \in [-1, 1] \text{ or } x \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow \quad x \in (-\infty, -2) \cup [-1, 1] \cup (2, \infty)$

4. *(c)* We have, $f : [a, b] \longrightarrow R$ be differentiable on $[a, b]$ and $k \in R$, also $f(a) = 0 = f(b)$

and $\quad J(x) = f'(x) + kf(x)$

Let $g(x) = kxf(x)$ which is continuous in $[a, b]$ and differentiable in (a, b) such that

$$g(a) = 0 = g(b)$$

Then, for every $c \in (a, b)$, $g'(c) = 0$

(by Rolle's theorem)

Now, $\quad g'(x) = kf(x) + kxf'(x)$

$\Rightarrow \quad g'(c) = kf(c) + kcf'(c)$

$\Rightarrow \quad kf(c) + kcf'(c) = 0$

$\Rightarrow \quad f(x) = 0$, for every $x = c \in (a, b)$

$\therefore \quad J(x) = 0$ has atleast one root in (a, b).

5. *(c)* We have,

$$f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$$

$$\therefore \quad f(1-h) = 3(1-h)^{10} - 7(1-h)^8$$
$$+ 5(1-h)^6 - 21(1-h)^3 + 3(1-h)^2 - 7$$
$$= 3(1 - 10h + 45h^2 - 120h^3 + \ldots\ldots + h^{10})$$
$$- 7(1 - 8h + 28h^2 - 56h^3 + \ldots\ldots + h^8)$$
$$+ 5(1 - 6h + 15h^2 - 20h^3 + \ldots\ldots + h^6)$$
$$- 21(1 - 3h + 3h^2 - h^3)$$
$$+ 3(1 - 2h + h^2) - 7$$

$$\Rightarrow f(1-h) = -24 + 53h + h^2(-46) + h^3(-47) + \ldots\ldots$$

and $\quad f(1) = -24$

$$\therefore \quad \lim_{h \to 0} \frac{f(1-h) - f(1)}{h^3 + 3h}$$

$$= \lim_{h \to 0} \frac{-24 + 53h + h^2(-46) + h^3(-47) + \ldots - (-24)}{h(h^2 + 3)}$$

$$= \lim_{h \to 0} \frac{53h + h^2(-46) + h^3(-47) + \ldots}{h(h^2 + 3)}$$

$$= \lim_{h \to 0} \frac{53 + h(-46) + h^2(-47) + \ldots}{h^2 + 3} = \frac{53}{3}$$

6. *(a)* Let $g(x) = e^{-x} f(x)$

such that $\quad g(a) = 0, \; g(b) = 0$

and $g(x)$ is continuous and differentiable.

Then, for atleast one value of $c \in (a, b)$ such that $g'(c) = 0$

Now, $\quad g'(x) = e^{-x} f'(x) + (-e^{-x}) f(x)$

$\Rightarrow \quad g'(c) = e^{-c} f'(c) + (-e^{-c}) f(c) = 0$

$\Rightarrow \quad e^{-c} f'(c) = e^{-c} f(c) \quad \Rightarrow \quad f'(c) = f(c)$

7. *(b)* We have,

$f: R \to R$ be a twice continuously differentiable function such that $f(0) = f(1) = f'(0) = 0$

Now, for atleast one value of $c_1 \in (0, 1)$,

$$f'(c_1) = 0 \qquad \text{(by Rolle's theorem)}$$

Again, $f'(0) = 0 = f'(c_1)$

$\Rightarrow$ $f''(c) = 0$ for some $c \in (0, c_1)$

(by Rolle's theorem)

8. *(b)* We have,

$$\int e^{\sin x}\left(\frac{x\cos^3 x - \sin x}{\cos^2 x}\right) dx = e^{\sin x} f(x) + c$$

$$\Rightarrow \int e^{\sin x}(x\cos x - \sec x \tan x)\, dx = e^{\sin x} f(x) + c$$

$$\Rightarrow \int e^{\sin x}(x\cos x - 1 + 1 - \sec x \tan x)\, dx = e^{\sin x} f(x) + c$$

$$\Rightarrow \int [e^{\sin x}\cos x(x - \sec x) + e^{\sin x}(1 - \sec x \tan x)]\, dx = e^{\sin x} f(x) + c$$

$$\Rightarrow \int \frac{d}{dx}\{e^{\sin x}(x - \sec x)\} dx = e^{\sin x} f(x) + c$$

$$\Rightarrow e^{\sin x}(x - \sec x) = e^{\sin x} f(x) + c$$

$$\Rightarrow f(x) = x - \sec x$$

9. *(c)* We have,

$$\int f(x) \sin x \cos x\, dx = \frac{1}{2(b^2 - a^2)} \log(f(x)) + c$$

$$\Rightarrow f(x)\sin x \cos x = \frac{1}{2(b^2 - a^2)} \cdot \frac{1}{f(x)} \cdot f'(x)$$

$$\Rightarrow f(x) \sin 2x = \frac{1}{b^2 - a^2} \cdot \frac{f'(x)}{f(x)}$$

$$\Rightarrow \sin 2x = \frac{1}{b^2 - a^2} \frac{f'(x)}{(f(x))^2}$$

$$\Rightarrow \int \sin 2x\, dx = \frac{1}{b^2 - a^2} \int \frac{f'(x)}{(f(x))^2} dx$$

$$\Rightarrow \frac{-\cos 2x}{2} = \frac{1}{b^2 - a^2} \cdot \left(\frac{-1}{f(x)}\right)$$

$$\Rightarrow \frac{\cos 2x\,(b^2 - a^2)}{2} = \frac{1}{f(x)}$$

$$\Rightarrow f(x) = \frac{2}{(b^2 - a^2)\cos 2x}$$

10. *(d)* Given, $M = \int_0^{\pi/2} \frac{\cos x}{(x + 2)} dx$

and
$$N = \int_0^{\pi/4} \frac{\sin x \cos x}{(x + 1)^2} dx = \int_0^{\pi/4} \frac{1}{2} \cdot \frac{\sin 2x}{(x + 1)^2} dx$$

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

$$\therefore \quad N = \int_0^{\pi/2} \frac{\sin t}{4(t/2 + 1)^2} dt = \int_0^{\pi/2} \frac{\sin t}{(t + 2)^2} dt$$

$$\therefore \quad M - N = \int_0^{\pi/2} (\cos x) \cdot \frac{1}{(x + 2)} dx - \int_0^{\pi/2} \frac{\sin x}{(x + 2)^2} dx$$

$$= \left(\frac{\sin x}{x + 2}\right)_0^{\pi/2} - \int_0^{\pi/2} - \frac{\sin x}{(x + 2)^2} dx - \int_0^{\pi/2} \frac{\sin x}{(x + 2)^2} dx$$

$$= \frac{\sin \pi/2}{\pi/2 + 2} = \frac{1}{\frac{\pi + 4}{2}} = \frac{2}{\pi + 4}$$

11. *(b)* We have,

$$I = \int_{1/2014}^{2014} \frac{\tan^{-1} x}{x} dx \qquad \text{... (i)}$$

Let $x = \frac{1}{t}$

$$\Rightarrow \quad dx = \frac{-1}{t^2} dt$$

Now,
$$I = \int_{2014}^{1/2014} \frac{\tan^{-1}(1/t)}{1/t}\left(\frac{-1}{t^2} dt\right)$$

$$= \int_{1/2014}^{2014} \frac{\cot^{-1} t}{t} dt$$

$$= \int_{1/2014}^{2014} \frac{\cot^{-1} x}{x} dx \qquad \text{... (ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{1/2014}^{2014} \frac{\pi/2}{x} dx = \frac{\pi}{2}(\log x)_{1/2014}^{2014}$$

$$= \frac{\pi}{2}(\log 2014 - \log 1/2014)$$

$$\Rightarrow \quad I = \frac{\pi}{4}\left(\log 2014 - \log \frac{1}{2014}\right)$$

$$= \frac{\pi}{4}(\log 2014 + \log 2014)$$

$$= \frac{\pi}{4}(2 \log 2014) = \frac{\pi}{2} \log 2014$$

12. *(c)* We have,

$$I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$$

Since, $\frac{\sin x}{x}$ is a decreasing function.

$$\therefore \quad \frac{\pi}{12} \times \frac{\sin \pi/3}{\pi/3} \le I \le \frac{\pi}{12} \times \frac{\sin \pi/4}{\pi/4}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{8} \le I \le \frac{\sqrt{2}}{6}$$

13. *(b)* Let

$$I = \int_{\pi/2}^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$$

$$= \int_0^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx - \int_0^{\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx \quad \text{...(i)}$$

$$I = \int_0^{5\pi/2} \frac{e^{\tan^{-1}(\cos x)}}{e^{\tan^{-1}(\cos x)} + e^{\tan^{-1}(\sin x)}} dx - \int_0^{\pi/2} \frac{e^{\tan^{-1}(\cos x)}}{e^{\tan^{-1}(\cos x)} + e^{\tan^{-1}(\sin x)}} dx \quad \text{...(ii)}$$

$$\left[\text{using, } \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{5\pi/2} dx - \int_0^{\pi/2} dx$$

$$\Rightarrow \quad 2I = (x)_0^{5\pi/2} - (x)_0^{\pi/2}$$

$$\Rightarrow \quad 2I = \frac{5\pi}{2} - \frac{\pi}{2} = 2\pi \quad \Rightarrow \quad I = \pi$$

14. *(c)* We have,

$$\lim_{n \to \infty} \frac{1}{n}\left\{\sec^2 \frac{\pi}{4n} + \sec^2 \frac{2\pi}{4n} + \dots + \sec^2 \frac{n\pi}{4n}\right\}$$

$$= \lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{n} \sec^2\left(\frac{r\pi}{4n}\right) = \int_0^1 \sec^2\left(\frac{\pi x}{4}\right)$$

$$= \frac{4}{\pi}\left[\tan\left(\frac{\pi x}{4}\right)\right]_0^1$$

$$= \frac{4}{\pi} \times 1 = \frac{4}{\pi}$$

15. *(c)* Given, $\quad y^2 = 2d\,(x + \sqrt{d}) \quad$ ... (i)

$$\Rightarrow \quad 2y\,y_1 = 2d \quad \Rightarrow \quad d = y\,y_1$$

From Eq. (i),

$$y^2 = 2y\,y_1\,(x + \sqrt{y\,y_1})$$

$$\Rightarrow \quad y^2 - 2y\,y_1 x = \sqrt{y\,y_1} \cdot 2y\,y_1$$

$$\Rightarrow (y^2 - 2y\,y_1 x)^2 = 4(y\,y_1)^3$$

So, degree of above equation is 3.

16. *(c)* We have,

$$(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{2x}{1 + x^2}\right) y = \frac{4x^2}{1 + x^2}$$

Here, IF $= e^{\int \frac{2x}{1+x^2}} = e^{\log(1+x^2)} = 1 + x^2$

$$\therefore \quad y(1 + x^2) = \int (1 + x^2) \times \frac{4x^2}{(1 + x^2)} dx + C$$

$$\Rightarrow \quad y(1 + x^2) = \int 4x^2 dx + C$$

$$\Rightarrow \quad y(1 + x^2) = \frac{4x^3}{3} + C$$

$$\Rightarrow \quad y(1 + x^2) = \frac{4x^3}{3} - 1 \qquad [y(0) = -1]$$

$$\Rightarrow \quad y = \frac{4x^3}{3(1 + x^2)} - \frac{1}{1 + x^2}$$

$$\therefore \quad y(1) = \frac{4}{6} - \frac{1}{2} = \frac{1}{6}$$

17. *(c)* We have, $\quad x = \frac{1}{2} vt$

$$\Rightarrow \quad x = \frac{1}{2}\frac{dx}{dt} t \qquad \left[\because v = \frac{dx}{dt}\right]$$

$$\Rightarrow \quad \frac{2\,dt}{t} = \frac{dx}{x}$$

$$\Rightarrow \quad 2 \cdot \int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow \quad 2\log|t| + \log|c| = \log|x|$$

$$\Rightarrow \quad \log(t^2 \cdot c) = \log x$$

$$\Rightarrow \quad x = t^2 c$$

$$\Rightarrow \quad \frac{dx}{dt} = 2tc$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = 2c$$

$\Rightarrow$ acceleration f is constant.

18. *(b)* We have,

equation of parabola $y = x^2$

Let $P(\alpha, \alpha^2)$ is a point on the parabola,

$$\therefore \quad y - \alpha^2 = 2\alpha\,(x - \alpha)$$

$$\because \quad \left[\because \frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx_{(\alpha, \alpha^2)}} = 2\alpha\right]$$

$$\Rightarrow \quad y = 2\alpha x - \alpha^2$$

Also, given $y = -x^2 + 4x - 4$

$\therefore\quad -x^2 + 4x - 4 = 2\alpha x - \alpha^2$

$\Rightarrow\quad x^2 + 2x(\alpha - 2) + (4 - \alpha^2) = 0$

Discriminant = 0

$4(\alpha - 2)^2 - 4(4 - \alpha^2) = 0$

$\Rightarrow\quad (\alpha - 2)^2 - (4 - \alpha^2) = 0$

$\Rightarrow\quad \alpha^2 - 4\alpha + 4 - 4 + \alpha^2 = 0$

$\Rightarrow\quad \alpha^2 - 2\alpha = 0$

$\Rightarrow\quad \alpha = 0,\ \alpha = 2$

19. *(d)* $2b = (n+2)$th form

$= a + (n + 2 - 1)\, d$

$\Rightarrow\quad 2b = a + (n+1)\, d$

$\Rightarrow\quad d = \dfrac{2b - a}{n+1}$

$\therefore\quad m$ th mean $= a + m\left(\dfrac{2b-a}{n+1}\right)$

and also, $\quad b = (n+2)$th form

$= 2a + (n + 2 - 1)\, d$

$= 2a + (n+1)\, d$

$\Rightarrow\quad d = \dfrac{b - 2a}{n+1}$

$\therefore$ m th mean $= 2a + m\left(\dfrac{b-2a}{n+1}\right)$

According to the question,

$$a + m\left(\frac{2b-a}{n+1}\right) = 2a + m\left(\frac{b-2a}{n+1}\right)$$

$\Rightarrow\quad \dfrac{a}{b} = \dfrac{m}{n+1-m}$

20. *(c)* We have,

$x + \log_{10}(1 + 2^x) = x\log_{10} 5 + \log_{10} 6$

$\Rightarrow\quad \log_{10}(1 + 2^x) = x\log_{10} 5 + \log_{10} 6 - x$

$= \log_{10} 5^x + \log_{10} 6 - x\log_{10} 10$

$= \log_{10}(5^x \cdot 6) - \log_{10} 10^x$

$\Rightarrow\quad \log_{10}(1 + 2^x) = \log_{10}\left(\dfrac{5^x \cdot 6}{10^x}\right)$

$\Rightarrow\quad 1 + 2^x = \dfrac{5^x \cdot 6}{10^x} = \dfrac{6}{2^x}$

$\Rightarrow\quad 2^x(1 + 2^x) = 6$

$\Rightarrow\quad t(1 + t) = 6 \qquad (\text{let } 2^x = t)$

$\Rightarrow\quad t^2 + t - 6 = 0$

$\Rightarrow\quad (t+3)(t-2) = 0$

$\Rightarrow\quad t + 3 = 0$

or $\quad t - 2 = 0$

$\Rightarrow\quad t = 2 \qquad [\because \text{ neglect } t = -3]$

$\Rightarrow\quad 2^x = 2 \quad\Rightarrow\quad x = 1$

21. *(b)* We have, $Z_r = \sin\dfrac{2\pi r}{11} - i\cos\dfrac{2\pi r}{11}$

$= -i\left(\cos\dfrac{2\pi r}{11} + i\sin\dfrac{2\pi r}{11}\right)$

$= -ie^{\frac{i2\pi r}{11}}$

$\therefore\quad \displaystyle\sum_{r=0}^{10} Z_r = -i\sum_{r=0}^{10} e^{\frac{i2\pi r}{11}}$

$= -i \times 0 = 0$

22. *(c)* We know that, if z_1, z_2 and z_3 are the vertices of an equilateral triangle. Then,

$z_1^2 + z_2^2 + z_3^2 - z_1z_2 - z_2z_3 - z_3z_1 = 0 \qquad \ldots\text{(i)}$

Now, but we have

$\dfrac{z_1}{z_2} + \dfrac{z_2}{z_1} = 1$

$\Rightarrow\quad z_1^2 + z_2^2 = z_1z_2$

$\Rightarrow\quad z_1^2 + z_2^2 - z_1z_2 = 0$

Here, $\quad z_3 = 0$

Hence, given points form an equilateral triangle.

23. *(a)* We have equations

$x^2 + b_1 x + c_1 = 0$

$D_1 = b_1^2 - 4c_1$

and $\quad x^2 + b_2 x + c_2 = 0$

$D_2 = b_2^2 - 4c_2$

Now, $D_1 + D_2 = b_1^2 + b_2^2 - 4(c_1 + c_2)$

$= b_1^2 + b_2^2 - 2b_1b_2 \qquad [\because b_1b_2 = 2(c_1 + c_2)]$

$= (b_1 - b_2)^2 \geq 0$

$\Rightarrow$ At least one of D_1 and D_2 are non-negative real roots.

24. *(a)* Number of ways of selection of n objects from $2n$ objects, where as n objects are identical in out of $2n$ objects.

n identical and no different object = 1 ways

$= {}^nC_0$

$n-1$ identical and 1 different object $= 1 \times {}^nC_1$

$n-2$ identical and 2 different object $= 1 \times {}^nC_2$

....

0 identical and n different objects $= 1 \times {}^nC_n$

$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n$

25. *(c)* ${}^nC_r + 2\cdot {}^nC_{r+1} + {}^nC_{r+2}$

$$= {}^nC_r + {}^nC_{r+1} + {}^nC_{r+1} + {}^nC_{r+2}$$

$$= {}^{n+1}C_{r+1} + {}^{n+1}C_{r+2}$$

$$(\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$$

$$= {}^{n+2}C_{r+2}$$

26. *(a)* $(101)^{100} - 1 = (1+100)^{100} - 1$

$$= (1 + {}^{100}C_1 \cdot 100 + {}^{100}C_2 100^2 + \ldots\ldots) - 1$$

$$= {}^{100}C_1 100 + {}^{100}C_2 (100)^2 + {}^{100}C_3 (100)^3 + \ldots\ldots + {}^{100}C_{100}(100)^{100}$$

$$= 10^4 (1 + {}^{100}C_2 + {}^{100}C_3 10^2 + \ldots . + {}^{100}C_{100}(100)^{98}$$

$$= 10^4 (1 + \text{an integer multiple of } 10)$$

27. *(a)* For greatest term of $(1+x)^n$, we have

$$\frac{n}{2} < \frac{n+1}{1+x} < \frac{n}{2} + 1$$

$$\Rightarrow \quad \frac{n}{2} < \frac{n+1}{1+x} \text{ and } \frac{n+1}{1+x} < \frac{n}{2} + 1$$

$$\Rightarrow \quad 1 + x < \frac{n+1}{n/2} \text{ and } \frac{n+1}{\frac{n}{2}+1} < 1 + x$$

$$\Rightarrow \quad x < \frac{n+1-n/2}{n/2}$$

$$\text{and} \quad \frac{n+1-(n/2-1)}{\frac{n+2}{2}} < x$$

$$\Rightarrow \quad x < \frac{n+2}{n} \text{ and } \frac{n}{n+2} < x$$

$$\Rightarrow \quad \frac{n}{n+2} < x < \frac{n+2}{n}$$

28. *(a)* We have,

$$A = \begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix}$$

$$= -1\,(1+12) - 7\,(2+9)$$

$$= -13 - 77 = -90 \qquad \ldots \text{(i)}$$

And let
$$B = \begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix}$$

$$= 3 \times 5 \begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ -7 & -1 & -1 \end{vmatrix}$$

$$= 15 \begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ 0 & 0 & -6 \end{vmatrix} (R_3 \to R_3 - R_2)$$

$$= 15\,\{0 - 0 - 6\,(-13 - 77)\}$$

$$= 15\,\{(-6)\,(-90)\} = 90 \times 90 \qquad \ldots \text{(ii)}$$

From Eqs. (i) and (ii),

$$B = A^2$$

29. *(c)* We have,

$$a_r = (\cos 2r\pi + i \sin 2r\pi)^{1/9} = e^{\frac{2r\pi i}{9}}$$

$$\text{Now, } \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} & e^{\frac{6\pi i}{9}} \\ e^{\frac{8\pi i}{9}} & e^{\frac{10\pi i}{9}} & e^{\frac{12\pi i}{9}} \\ e^{\frac{14\pi i}{9}} & e^{\frac{16\pi i}{9}} & e^{\frac{18\pi i}{9}} \end{vmatrix}$$

$$= e^{\frac{2\pi i}{9}} \times e^{\frac{8\pi i}{9}} \begin{vmatrix} 1 & e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} \\ 1 & e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} \\ e^{\frac{14\pi i}{9}} & e^{\frac{16\pi i}{9}} & e^{\frac{18\pi i}{9}} \end{vmatrix}$$

$$= e^{\frac{2\pi i}{9}} \times e^{\frac{8\pi i}{9}} \times 0 \qquad (\because R_1 \text{ and } R_2 \text{ are identical})$$

$$= 0$$

30. *(d)* We have, $S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2 - 1 & y & n^2(2n+3) \\ 4r^3 - 2nr & z & n^3(n+1) \end{vmatrix}$

$$\Rightarrow \quad \sum_{r=1}^{n} S_r = \begin{vmatrix} 2\sum_{r=1}^{n} r & x & n(n+1) \\ \sum_{r=1}^{n} (6r^2 - 1) & y & n^2(2n+3) \\ \sum_{r=1}^{n} (4r^3 - 2nr) & z & n^3(n+1) \end{vmatrix}$$

$$= \begin{vmatrix} n(n+1) & x & n(n+1) \\ n^2(2n+3) & y & n^2(2n+3) \\ n^3(n+1) & z & n^3(n+1) \end{vmatrix}$$

$$= 0 \qquad (\because C_1 \text{ and } C_3 \text{ are identical})$$

Hence, $\sum_{r=1}^{n} S_r$ is independent of x, y, z and n.

31. *(c)* For non-trivial solution,

We have, $\begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0$

$$\Rightarrow \quad 1(3bc - 2bc) - (4ac - 2ac) + 1(4ab - 3ab) = 0$$

$\Rightarrow \quad bc - 2ac + ab = 0$

$\Rightarrow \quad bc + ab = 2ac$

$\Rightarrow \quad b(a + c) = 2ac$

$\Rightarrow \quad b = \dfrac{2ac}{a + c}$

$\Rightarrow a, b, c$ are in HP.

32. ***(c)*** On the set R,

$x\rho y \Rightarrow x - y$ is zero or irrational number.

Now, $\quad x\rho x$

$\Rightarrow \quad x - x = 0$

$\Rightarrow \rho$ is reflexive.

If $x\rho y \Rightarrow x - y$ is zero or irrational.

$= -(y - x)$ is zero or irrational.

$\Rightarrow y\rho x$ is zero or irrational.

$\Rightarrow \rho$ is symmetric.

And if

$x\rho y \Rightarrow x - y$ is 0 or irrational.

$y\rho z \Rightarrow y - z$ is 0 or irrational.

Then, $(x - y) + (y - z) = x - z$ may be 0 or rational.

$\Rightarrow \rho$ is not transitive.

33. ***(d)*** On the set R of real numbers

For reflexive,

$x\rho x \Rightarrow (x, x) \in R$

$\Rightarrow x > |x|$ which is not true.

$\Rightarrow \rho$ is not reflexive.

For symmetric,

$(x, y) \in R \Rightarrow x > |y|$

and $(y, x) \in R \Rightarrow y > |x|$

So, $x > |y| \neq y > |x|$

$\Rightarrow \rho$ is not symmetric.

For transitive,

$(x, y) \in R \Rightarrow x > |y|, (y, z) \in R \Rightarrow y > |z|$

$\Rightarrow \quad x > |z| \Rightarrow (x, z) \in R$

$\Rightarrow \rho$ is transitive.

34. ***(c)*** We have, $f : R \to R$, defined by $f(x) = e^x$

and $g : R \to R$ defined by $g(x) = x^2$

Now, We have

$$\begin{aligned}(gof)(x) &= g(f(x)) \\ &= g(e^x) \\ &= (e^x)^2 \\ &= e^{2x}, \ \forall x \in R\end{aligned}$$

$\Rightarrow gof$ is injective and g is neither injective nor surjective.

$\Rightarrow gof$ is injective but $g(x)$ is not bijective.

35. ***(b)*** We have, $P(H) = \dfrac{1}{2}, P(T) = \dfrac{1}{2}$

Now, $P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^nC_0 (p)^0 (q)^n = 1 - \left(\frac{1}{2}\right)^n$$

$\because \quad 1 - \dfrac{1}{2^n} \geq 0.9$

$\Rightarrow \quad 0.1 \geq \dfrac{1}{2n}$

$\Rightarrow \quad \dfrac{1}{10} \geq \dfrac{1}{2^n}$

$\Rightarrow \quad 2^n \geq 10$

$\Rightarrow \quad n \geq 4$

$\therefore$ Minimum number of tossed = 4

36. ***(a)*** Let X be the event that student will be successful.

X_1 be the event that student will be pass in test-I.

X_2 be the event that student will be pass in test-II.

X_3 be the event that student will be pass in test-III.

$$\therefore \quad P(X) = P(X_1 \cap X_2 \cap X'_3) + P(X_1 \cap X'_2 \cap X_3) + P(X_1 \cap X_2 \cap X_3)$$

$$\Rightarrow P(X) = P(X_1)\cdot P(X_2)\cdot P(X'_3) + P(X_1)\cdot P(X'_2)\cdot P(X_3) + P(X_1)\cdot P(X_2)\cdot P(X_3)$$

$\Rightarrow \quad \dfrac{1}{2} = p\cdot q\cdot\dfrac{1}{2} + p(1 - q)\dfrac{1}{2} + pq\dfrac{1}{2}$

$\Rightarrow \quad \dfrac{1}{2} = p\cdot q\cdot\dfrac{1}{2} + p\cdot\dfrac{1}{2} - p\cdot q\cdot\dfrac{1}{2} + p\cdot q\cdot\dfrac{1}{2}$

$\Rightarrow \quad \dfrac{1}{2} = p\cdot\dfrac{1}{2} + p\cdot q\cdot\dfrac{1}{2}$

$\Rightarrow \quad (p + pq) = 1$

$\Rightarrow \quad p(1 + q) = 1$

37. ***(a)*** We have, $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$\Rightarrow \quad \sin 6\theta + \sin 2\theta + \sin 4\theta = 0$

$\Rightarrow \quad 2\sin 4\theta\cdot\cos 2\theta + \sin 4\theta = 0$

$\Rightarrow \quad \sin 4\theta(2\cos 2\theta + 1) = 0$

$\Rightarrow \quad \sin 4\theta = 0$ or $2\cos 2\theta + 1 = 0$

$\Rightarrow \quad 4\theta = n\pi$ or $\cos 2\theta = \dfrac{-1}{2} = \cos\dfrac{2\pi}{3}$

$\Rightarrow \quad \theta = \dfrac{n\pi}{4}$ or $2\theta = 2n\pi \pm \dfrac{2\pi}{3}$

$\Rightarrow \quad \theta = \dfrac{n\pi}{4}$ or $\theta = n\pi \pm \dfrac{\pi}{3}$, where $n \in z$

38. ***(c)*** We have, $0 \leq A \leq \dfrac{\pi}{4}$

$$\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$$

$$= \tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}\left(\frac{\cot A + \cot^3 A}{1 - \cot^4 A}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\cdot\frac{2\tan A}{1-\tan^2 A}\right) + \tan^{-1}\left(\frac{\tan A}{\tan^2 A - 1}\right)$$

$$= \tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right) - \tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)$$

$$= 0$$

39. *(b)* Put $x = x' + 2$

and $y = y' + 3$

$\therefore \quad x^2 + y^2 - 4x - 6y + 9 = 0$

$\Rightarrow \quad (x'+2)^2 + (y'+3)^2 - 4(x'+2) - 6(y'+3) + 9 = 0$

$\Rightarrow \quad x'^2 + 4 + 4x' + y'^2 + 9 + 6y' - 4x' - 8 - 6y' - 18 + 9 = 0$

$\Rightarrow \quad x'^2 + y'^2 - 4 = 0$

$\Rightarrow \quad x^2 + y^2 = 4$

40. *(d)* We have equation of circle

$x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$

Here, $C \equiv (-2, 3)$

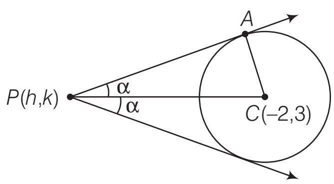
Radius $= \sqrt{(-2)^2 + (3)^2 - (9\sin^2\alpha + 13\cos^2\alpha)}$

$= \sqrt{4 + 9 - 9\sin^2\alpha - 13\cos^2\alpha}$

$= \sqrt{13 - 13(1-\sin^2\alpha) - 9\sin^2\alpha}$

$= \sqrt{13\sin^2\alpha - 9\sin^2\alpha}$

$= \sqrt{4\sin^2\alpha} = 2\sin\alpha$



Here, $\sin\alpha = \frac{AC}{PC}$

$\Rightarrow \quad PC\sin\alpha = AC$

$\Rightarrow \quad PC^2\sin^2\alpha = AC^2 = (2\sin\alpha)^2$

$\Rightarrow \quad [(h+2)^2 + (k-3)^2]\sin^2\alpha = 4\sin^2\alpha$

$\Rightarrow \quad (h+2)^2 + (k-3)^2 = 4$

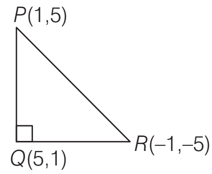
$\Rightarrow \quad h^2 + 4 + 4h + k^2 + 9 - 6k = 4$

$\Rightarrow \quad h^2 + k^2 + 4h - 6k + 9 = 0$

Hence, locus of a point is

$$x^2 + y^2 + 4x - 6y + 9 = 0$$

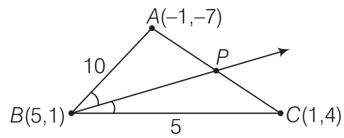
41. *(d)* Given, point $P(1, 5)$ image of the point $P(1, 5)$ about the line $y = x$ is $Q(5, 1)$ and image of the point Q on line $y = -x$ is $R(-1, -5)$



$\therefore$ Required circumcentre = Mid-point of P and R

$$= \left(\frac{1-1}{2}, \frac{5-5}{2}\right) = (0, 0)$$

42. *(b)* Here, $AB = \sqrt{(5+1)^2 + (1+7)^2} = \sqrt{36+64} = 10$



$BC = \sqrt{(1-5)^2 + (4-1)^2} = \sqrt{16+9} = 5$

By angle bisector theorem,

$$AP : CP = 10 : 5 = 2 : 1$$

$$\therefore \; P\left(\frac{2\times 1 + 1\times(-1)}{2+1}, \frac{2\times 4 + 1\times(-7)}{2+1}\right) = P\left(\frac{1}{3}, \frac{1}{3}\right)$$

Required equation of BP is

$$y - 1 = \frac{\frac{1}{3} - 1}{\frac{1}{3} - 5}(x - 5)$$

$\Rightarrow \quad y - 1 = \frac{-2}{-14}(x-5)$

$\Rightarrow \quad 7y - 7 = x - 5$

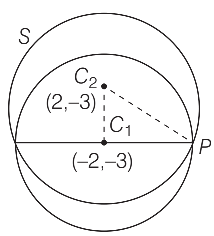
$\Rightarrow \quad 7y = x + 2$

43. *(a)* Given, equation of circle is

$$x^2 + y^2 + 4x + 6y - 12 = 0$$

Centre $C(-2, -3)$

and radius $= \sqrt{(-2)^2 + (-3)^2 + 12} = 5$



$$x^2 + y^2 + 4x + 6y - 12 = 0$$

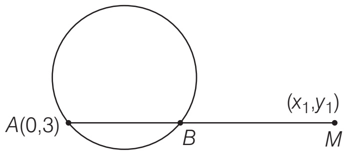
$\therefore \quad C_1C_2 = \sqrt{(2+2)^2 + (-3+3)^2}$

$$= \sqrt{(4)^2 + (0)^2} = 4$$

$\therefore$ Radius of circle, S

$$= \sqrt{(4)^2 + (5)^2} = \sqrt{16+25} = \sqrt{41} \text{ unit}$$

44. *(c)* Given, $AM = 2AB$



$\Rightarrow B$ is mid-point of AM.

$\therefore$ Coordinate of B is $\left(\dfrac{0+x_1}{2}, \dfrac{3+y_1}{2}\right)$

$$= \left(\frac{x_1}{2}, \frac{y_1+3}{2}\right)$$

Since, B lies on the circle $x^2 + 4x + (y-3)^2 = 0$

$$\therefore \left(\frac{x_1}{2}\right)^2 + 4\left(\frac{x_1}{2}\right) + \left(\frac{y_1+3}{2} - 3\right)^2 = 0$$

$$\Rightarrow \quad \frac{x_1^2}{4} + 2x_1 + \left(\frac{y_1-3}{2}\right)^2 = 0$$

$$\Rightarrow \quad \frac{x_1^2}{4} + 2x_1 + \frac{y_1^2 + 9 - 6y_1}{4} = 0$$

$$\Rightarrow \quad x_1^2 + y_1^2 + 8x_1 - 6y_1 + 9 = 0$$

Hence, locus of a point is

$$x^2 + y^2 + 8x - 6y + 9 = 0$$

45. *(a)* Given equation of ellipses is $x^2 + 9y^2 = 9$

$$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{1} = 1$$

Here, $\quad a = 3, b = 1$

$$c = \sqrt{(3)^2 - (1)^2} = \sqrt{8}$$

$\therefore$ Eccentricity of ellipse, $e = \dfrac{c}{a}$

$$\Rightarrow \quad e = \frac{\sqrt{8}}{3}$$

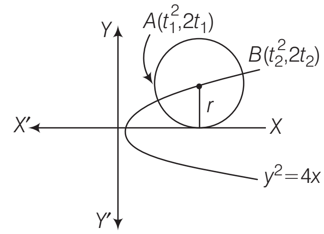
$\therefore$ Eccentricity of hyperbola $= \dfrac{3}{\sqrt{8}}$

$$\Rightarrow \quad 1 + \frac{b^2}{a^2} = \frac{9}{8}$$

$$\Rightarrow \quad \frac{b^2}{a^2} = \frac{1}{8}$$

$$\Rightarrow \quad a^2 : b^2 = 8 : 1$$

46. *(c, d)* Centre of circle $= \left(\dfrac{t_1^2 + t_2^2}{2}, (t_1 + t_2)\right)$



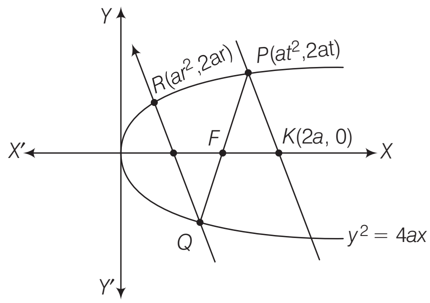
Since, circle touch the x-axis, so equation of tangent is $y = 0$

$\because$ Radius = Perpendicular distance from centre to the tangent

$\Rightarrow$ Radius $= |t_1 + t_2| = r$

Slope of $AB = \dfrac{2}{t_1 + t_2} = \dfrac{2}{\pm r}$

47. *(d)*



Here, coordinate of Q will be $\left(\dfrac{a}{t^2}, \dfrac{-2a}{t}\right)$.

Slope of $QR = \dfrac{2}{r - \dfrac{1}{t}}$

Slope of $PK = \dfrac{2at}{at^2 - 2a} = \dfrac{2t}{t^2 - 2}$

Since, Slope of QR = Slope of PK

$$\therefore \quad \frac{2}{r - \dfrac{1}{t}} = \frac{2t}{t^2 - 2}$$

$$\Rightarrow \quad r = \frac{t^2 - 1}{t}$$

48. *(a)* Since, point P on the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

$\therefore \quad P(3\cos\theta, 2\sin\theta)$

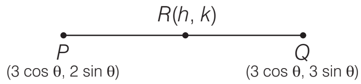
Now, equation of line parallel of Y-axis is

$$x = 3\cos\theta$$

and above line meets circle at Q

$\therefore \quad Q(3\cos\theta, 3\sin\theta)$

Given, $\frac{PR}{RQ} = \frac{1}{2}$



$\therefore\ h = \frac{3\cos\theta + 6\cos\theta}{3},\ \ k = \frac{3\sin\theta + 4\sin\theta}{3}$

$\Rightarrow \quad h = 3\cos\theta,\ \ k = \frac{7}{3}\sin\theta$

$\Rightarrow \quad \cos\theta = h/3,\ \ \sin\theta = \frac{3k}{7}$

Now, $\cos^2\theta + \sin^2\theta = h^2/9 + \frac{9k^2}{49} = 1$

Hence, locus of a point is $\frac{x^2}{9} + \frac{9y^2}{49} = 1$

49. *(a)* Equation of line through $Q\,(1, -2, 3)$ and parallel to the line $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$

is $\quad \frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = \lambda \quad$ (say)

Since, point P lies on above line.

$\therefore \quad P(\lambda + 1, 4\lambda - 2, 5\lambda + 3)$

Since, P lies on the given plane.

$\therefore \quad 2(\lambda + 1) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$

$\Rightarrow \quad 2\lambda + 2 + 12\lambda - 6 - 20\lambda - 12 + 22 = 0$

$\Rightarrow \quad -6\lambda + 6 = 0$

$\Rightarrow \quad \lambda = 1$

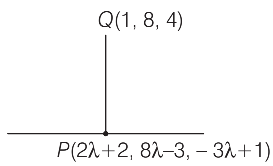
$\therefore \quad P(2, 2, 8)$

$\therefore \quad PQ = \sqrt{(2-1)^2 + (2+2)^2 + (3-8)^2}$

$\Rightarrow \quad PQ = \sqrt{1 + 16 + 25} = \sqrt{42}$

50. *(d)* Equation of line joining points $(0, -11, 4)$ and $(2, -3, 1)$ is

$$\frac{x-2}{2} = \frac{y+3}{8} = \frac{z-1}{-3} = \lambda \quad \text{(say)}$$



Let P is any point of the above line then coordinate of P is $(2\lambda + 2, 8\lambda - 3, -3\lambda + 1)$.

$\therefore$ DR's of PQ is $(2\lambda + 1, 8\lambda - 11, -3\lambda - 3)$

Now, $(2\lambda + 1)\,(2) + (8\lambda - 11)\,(8) + (-3\lambda - 3)\,(-3) = 0$

$\Rightarrow \quad 4\lambda + 2 + 64\lambda - 88 + 9\lambda + 9 = 0$

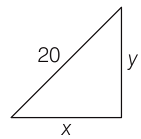
$\Rightarrow \quad 77\lambda - 77 = 0$

$\Rightarrow \quad \lambda = 1$

$\therefore$ Required foot of perpendicular,

$P(4, 5, -2)$

51. *(a)*



$$x^2 + y^2 = (20)^2 = 400$$

We have, $\quad \frac{dy}{dt} = 2\text{ ft /sec}$

When $\quad x = 12$

then $\quad (12)^2 + y^2 = 400$

$\Rightarrow \quad 144 + y^2 = 400$

$\Rightarrow \quad y^2 = 400 - 144 = 256$

$\Rightarrow \quad y = 16$

Now, $\quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\Rightarrow \quad x\frac{dx}{dt} = -y\frac{dy}{dt}$

$\Rightarrow \quad 12\left(\frac{dx}{dt}\right) = -16(2)$

$\Rightarrow \quad \frac{dx}{dt} = \frac{-8}{3}$

52. *(b)* Here, let $\quad p = \frac{1}{m}$

then $\quad (a^p + b^p)^{1/p} = (a^{1/m} + b^{1/m})^m$

$= a + b + k,\ k \ge 0$

$\therefore \quad a^p + b^p \ge (a + b)^p \ge (a + b)$

$\Rightarrow \quad J(p) \ge I\,(p)$

53. *(c)* We have,

$$\vec{\delta} = \vec{\alpha} + \lambda\vec{\beta}$$

$$= (\hat{i} + \hat{j} + \hat{k}) + \lambda\,(\hat{i} - \hat{j} - \hat{k})$$

$\Rightarrow \quad \vec{\delta} = (1 + \lambda)\hat{i} + (1 - \lambda)\hat{j} + (1 - \lambda)\hat{k}$

Given, $\quad \frac{\vec{\delta}.\vec{\gamma}}{|\vec{\gamma}|} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{(1 + \lambda)\ \ (-1) + (1 - \lambda)\,(1) + (1 - \lambda)\,(-1)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \lambda = -2$

$\therefore \quad \vec{\delta} = -\hat{i} + 3\hat{j} + 3\hat{k}$

54. *(c)* $\vec{\alpha} = \lambda\left(\vec{\beta} \times \vec{\gamma}\right) = \lambda(|\vec{\beta}|\ |\vec{\gamma}|\ |\sin 30°|$

$$\Rightarrow \quad |\vec{\alpha}| = |\lambda|\left(|\beta|\ |\vec{\gamma}| \cdot \frac{1}{2}\right)$$

$$\Rightarrow \quad 1 = |\lambda| \cdot 1 \cdot 1 \cdot \frac{1}{2}$$

$$\Rightarrow \quad |\lambda| = 2$$

$$\Rightarrow \quad \lambda = \pm 2$$

$$\therefore \quad \vec{\alpha} = \pm 2\left(\vec{\beta} \times \vec{\gamma}\right)$$

55. *(d)* Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\text{Re}(z_1) > 0 \Rightarrow x_1 > 0$$

and $\quad \text{Im}(z_2) < 0$

$\Rightarrow \quad y_2 < 0$

Given, $\quad |z_1| = |z_2|$

$\Rightarrow \quad |z_1|^2 = |z_2{}^2|$

$\Rightarrow \quad z_1\bar{z}_1 = z_2\bar{z}_2$

Now, $\left(\dfrac{z_1 + z_2}{z_1 - z_2}\right) + \overline{\left(\dfrac{z_1 + z_2}{z_1 - z_2}\right)}$

$$= \left(\frac{z_1 + z_2}{z_1 - z_2}\right) + \left(\frac{\bar{z}_1 + \bar{z}_2}{\bar{z}_1 - \bar{z}_2}\right)$$

$$= \frac{z_1\bar{z}_1 + z_2\bar{z}_1 - z_1\bar{z}_2 - z_2\bar{z}_2 + z_1\bar{z}_1 + z_1\bar{z}_2 - z_2\bar{z}_1 + z_2\bar{z}_2}{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)}$$

$$= \frac{2(|z_1|^2 - |z_2|^2)}{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)} = 0 \qquad (\because |z_1|^2 = |z_2|^2)$$

$= \dfrac{z_1 + z_2}{z_1 - z_2}$ is purely imaginary.

56. *(a)* Given, numbers are 1, 2, 3 20

Here, number of ways of selecting four consecutive numbers $= 17$

$\therefore$ Required number of selecting 4 non-consecutive numbers $= {}^{20}C_4 - 17$

$$= \frac{20 \times 19 \times 18 \times 17}{4 \times 3 \times 2 \times 1} - 17$$

$$= 285 \times 17 - 17$$

$$= 284 \times 17$$

57. *(b)* We have, $\begin{pmatrix} \cos \pi/4 & \sin \pi/4 \\ -\sin\dfrac{\pi}{4} & \cos\dfrac{\pi}{4} \end{pmatrix}^n$

Let $A = \begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\ -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{pmatrix}$

$$\Rightarrow \quad A = \begin{pmatrix} k & k \\ -k & k \end{pmatrix} \qquad \left(\text{where, } k = \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow A^2 = \begin{pmatrix} k & k \\ -k & k \end{pmatrix}\begin{pmatrix} k & k \\ -k & k \end{pmatrix} = \begin{pmatrix} 0 & 2k^2 \\ -2k^2 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 0 & 2k^2 \\ -2k^2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 2k^2 \\ -2k^2 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} -4k^4 & 0 \\ 0 & -4k^4 \end{pmatrix} = \begin{pmatrix} -4 \times \dfrac{1}{4} & 0 \\ 0 & -4 \times \dfrac{1}{4} \end{pmatrix}$$

$$\Rightarrow \quad A^4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow \quad A^8 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

58. *(d)* We have, $\rho = \{(x, y) \in N \times N : 2x + y = 41\}$

For reflexive,

$x\rho x \Rightarrow 2x + x = 41$

$\Rightarrow \quad 3x = 41$

$\Rightarrow \quad x = \dfrac{41}{3} \notin N$

So, ρ is not reflexive.

For symmetric,

$x\rho y \Rightarrow 2x + y = 41$

and $\quad y\rho x \Rightarrow 2y + x = 41$

$\Rightarrow \quad x\rho y \neq y\rho x$

So, ρ is not symmetric.

For transitive,

$x\rho y \Rightarrow 2x + y = 41$

and $\quad y\rho z \Rightarrow 2y + z = 41$

$\Rightarrow x \not\rho z$

$\Rightarrow \rho$ is not transitive.

59. *(a)* Given, $f(x) = \begin{vmatrix} (1+x)^a & (2+x)^b & 1 \\ 1 & (1+x)^a & (2+x)^b \\ (2+x)^b & 1 & (1+x)^a \end{vmatrix}$

For constant term put $x = 0$

$$f(0) = \begin{vmatrix} 1 & 2^b & 1 \\ 1 & 1 & 2^b \\ 2^b & 1 & 1 \end{vmatrix}$$

$$= 1(1 - 2^b) - 2^b(1 - 2^{2b}) + 1(1 - 2^b)$$

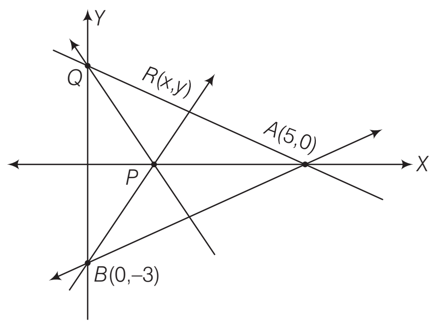
$$= 1 - 2^b - 2^b + 2^{3b} + 1 - 2^b$$

$$= 2 - 3 \cdot 2^b + 2^{3b}$$

60. *(a)* Equation of line AB is

$$\frac{x}{5}+\frac{y}{-3}=1$$

$$\Rightarrow \quad 3x-5y=15$$



Perpendicular line to AB is

$$5x+3y=\lambda$$

Coordinate of P is $\left(\frac{\lambda}{5}, 0\right)$

and coordinate of Q is $(0, \lambda/3)$

Now, equation of line AQ is

$$x/5+\frac{y}{\lambda/3}=1$$

$$\Rightarrow \quad \frac{x}{5}+\frac{3y}{\lambda}=1$$

$$\Rightarrow \quad \frac{3y}{\lambda}=1-\frac{x}{5}$$

$$\Rightarrow \quad \frac{1}{\lambda}=\frac{1}{3y}\left(1-\frac{x}{5}\right) \quad \text{... (i)}$$

and equation of line BP is

$$\frac{x}{\lambda/5}+\frac{y}{-3}=1$$

$$\Rightarrow \quad \frac{5x}{\lambda}-\frac{y}{3}=1$$

$$\Rightarrow \quad \frac{1}{\lambda}=\frac{1}{5x}\left(\frac{y}{3}+1\right) \quad \text{... (ii)}$$

From Eqs. (i) and (ii),

$$\frac{1}{3y}\left(1-\frac{x}{5}\right)=\frac{1}{5x}\left(\frac{y}{3}+1\right)$$

$$\Rightarrow \quad 5x\left(1-\frac{x}{5}\right)=3y\left(\frac{y}{3}+1\right)$$

$$\Rightarrow \quad 5x-x^2=y^2+3y$$

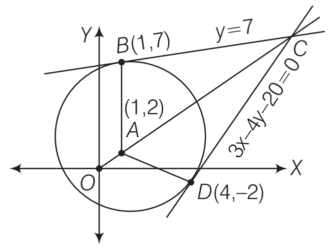
$\Rightarrow x^2+y^2-5x+3y=0$

which is a circle.

61. *(c)* Given, equation of circle is

$$x^2+y^2-2x-4y-20=0$$

Center $(1, 2)$ and radius $=\sqrt{(1)^2+(2)^2+20}=5$



Coordinate of intersecting point of tangents at B and D is $C(16, 7)$.

$\therefore$ Area of quadrilateral $ABCD$

$$=2\times \text{ar}(\Delta ABC)$$

$$=2\times\frac{1}{2}\times 15\times 5=75 \text{ sq units}$$

62. *(b)* At $\quad x=-\dfrac{\pi}{2}$

$$\text{LHL}=-2$$

$$\text{RHL}=-A+B$$

For continuity, LHL = RHL = $f(-\pi/2)$

$$\Rightarrow \quad -A+B=2 \quad \text{... (i)}$$

At $x=\pi/2$

$$\text{LHL}=A+B$$

$$\text{RHL}=0$$

For continuity, LHL = RHL = $f(\pi/2)$

$$\Rightarrow \quad A+B=0 \quad \text{... (ii)}$$

On solving Eqs. (i) and (ii), we get

$$A=-1 \text{ and } B=1$$

63. *(c)* Given equation of curve,

$$y=x^2-x+1$$

$$\Rightarrow \quad \frac{dy}{dx}=2x-1$$

Slope of normal $=\dfrac{1}{1-2x}$

Now, at $x_1=0, y_1=1$

$\therefore$ Slope of normal at $(0, 1)=1$

$\therefore$ Equation of normal, $y-1=1(x-0)$

$$\Rightarrow \quad x-y+1=0 \quad \text{.. (i)}$$

At $x_2=-1, y_2=3$

Slope of normal at $(-1, 3)=\dfrac{1}{3}$

Equation of normal, $y-3=\dfrac{1}{3}(x+1)$

$$\Rightarrow \quad 3y-9=x+1$$

$\Rightarrow \quad x - 3y + 10 = 0 \quad \text{... (ii)}$

At $\quad x_3 = \frac{5}{2}, y_3 = \frac{19}{4}$

Slope of normal at $\left(\frac{2}{5}, \frac{19}{4}\right) = -\frac{1}{4}$

Equation of normal, $y - \frac{19}{4} = -\frac{1}{4}\left(x - \frac{5}{2}\right)$

$\Rightarrow \quad x + 4y = \frac{43}{2}$

$\Rightarrow \quad 2x + 8y = 43 \quad \text{... (iii)}$

Here, intersecting point of Eqs. (i) and (ii) is $\left(\frac{7}{2}, \frac{9}{2}\right)$ and normal (iii) passes through it. Hence, normals are concurrent.

64. *(b)* Let $\quad f(x) = x \log x + x - 3$

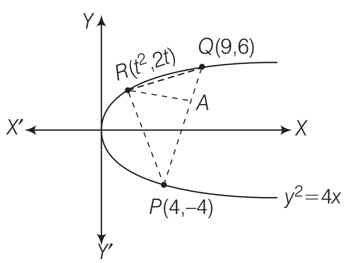
$\Rightarrow \quad f'(x) = x \cdot \frac{1}{x} + \log x + 1$

$\Rightarrow \quad f'(x) = \log x + 2 > 0$

$\Rightarrow \quad f(1) = -2$ and $f(3) = 3\log 3$, $f(1) \cdot f(3) < 0$

Hence, exactly one root in $x \in (1, 3)$ as $f(x) > 0$

65. *(c)* Equation of PQ is $2x - y = 12$



Perpendicular distance

$$AR = \left|\frac{2t^2 - 2t - 12}{\sqrt{5}}\right| = \frac{2}{\sqrt{5}}(t-3)(t+2)$$

AR is Maximum, at $t = \frac{1}{2}$

$\therefore R$ is $\left(\frac{1}{4}, 1\right)$

66. *(b)* $I = \int_0^1 \frac{x^3 \cos 3x}{2 + x^2} dx$

Here, $\quad -1 < \cos 3x < 1$

$\Rightarrow \quad -x^3 < x^3 \cos 3x < x^3$

$\Rightarrow \quad \frac{-x^3}{x^2} < \frac{-x^3}{x} < \frac{-x^3}{2+x^2} < \frac{x^3 \cos 3x}{2+x^2}$

$$< \frac{x^3}{2+x^2} < \frac{x^3}{x} < \frac{x^3}{x^2}$$

$\Rightarrow \quad \int_0^1 -x^2\, dx < I < \int_0^1 x^2 dx$

$\Rightarrow \quad \left(\frac{-x^3}{3}\right)_0^1 < I < \left(\frac{x^3}{3}\right)_0^1$

$\Rightarrow \quad \frac{-1}{3} < I < \frac{1}{3}$

67. *(c, d)* Given, curve, $12y = x^3$

and $\quad \frac{dy}{dt} > \frac{dx}{dt} \quad \text{... (i)}$

Now, $\quad 12\frac{dy}{dt} = 3x^2 \frac{dx}{dt} \quad \text{... (ii)}$

From Eqs. (i) and (ii), we get

$$3x^2 \frac{dx}{dt} > 12\frac{dx}{dt}$$

$\Rightarrow \quad 3x^2 > 12$

$\Rightarrow \quad x^2 - 4 > 0$

$\Rightarrow \quad (x-2)(x+2) > 0$

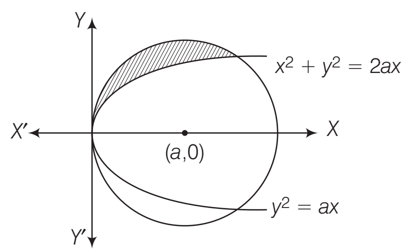
$\Rightarrow \quad x \in (-\infty, -2) \cup (2, \infty)$

68. *(b)* Given, equation of circle

$$x^2 + y^2 = 2ax$$

$\Rightarrow \quad (x-a)^2 + y^2 = a^2$

and equation of parabola is $y^2 = ax$, $a > 0$



Intersection points of circle and parabola

$\Rightarrow \quad x^2 + ax = 2ax$

$\Rightarrow \quad x^2 = ax$

$\Rightarrow \quad x^2 - ax = 0$

$\Rightarrow \quad x(x-a) = 0$

$\Rightarrow \quad x = 0, a$

Intersecting points are $(0, 0)$ and (a, a).

$\therefore$ Required area $= \frac{\pi a^2}{4} - \int_0^a \sqrt{ax}\, dx$

$$= \frac{\pi a^2}{4} - \sqrt{a}\left(\frac{x^{3/2}}{3/2}\right)_0^a$$

$$= \frac{\pi a^2}{4} - \frac{2a^2}{3} = a^2\left(\frac{\pi}{4} - \frac{2}{3}\right)$$

69. ***(a, d)*** Let α and β are the roots of $x^2 + ax + b = 0$ and the roots of $x^2 - cx + d = 0$ are α^4 and β^4.

Now, $\alpha + \beta = -a, \alpha\beta = b$ ... (i)

and $\alpha^4 + \beta^4 = c, \alpha^4\beta^4 = d$ ... (ii)

From Eqs. (i) and (ii),

$$b^4 = d \text{ and } \alpha^4 + \beta^4 = c$$

$$(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = c$$

$$\Rightarrow [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2 = c$$

$$\Rightarrow (a^2 - 2b)^2 - 2b^2 = c$$

$$\Rightarrow 2b^2 + c = (a^2 - 2b)^2$$

$$\Rightarrow 2b^2 + c = a^4 + 4b^2 - 4a^2b$$

$$\Rightarrow 2b^2 - c = 4a^2b - a^4$$

$$\Rightarrow 2b^2 - c = a^2(4b - a^2)$$

and for equation $x^2 - 4bx + 2b^2 - c = 0$

$$D = (4b)^2 - 4(1)(2b^2 - c)$$

$$= 16b^2 - 8b^2 + 4c = 8b^2 + 4c$$

$$= 4(2b^2 + c) = 4(a^2 - 2b)^2 > 0 = \text{real root}$$

Now, $f(0) = 2b^2 - c = a^2(4b - a^2) < 0 \quad (\because a^2 > 4b)$

= roots are opposite in sign

70. ***(b, c)*** Required number of ways $= {}^{20}C_2 \times 2!$

$$= {}^{20}P_2$$

71. ***(a, c)***

$$A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow A' = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} = -A$$

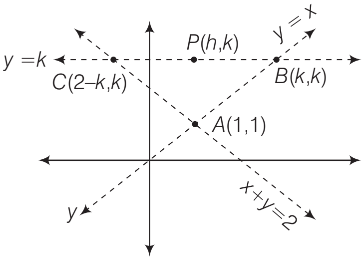
$\Rightarrow A$ is a skew-symmetric matrix.

$$|A| = 0 + 1(0 + 1) - 1(1 - 0)$$

$$= 0 + 1 - 1 = 0 = \text{Singular}$$

$\Rightarrow A$ is not invertible.

72. ***(a, b)*** Given, ar $(\Delta ABC) = h^2$



$$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ 2-k & k & 1 \end{vmatrix} = \pm h^2$$

$$\Rightarrow 1(k - k) - 1(k - 2 + k) + 1(k^2 - 2k + k^2) = \pm 2h^2$$

$$\Rightarrow -(2k - 2) + (2k^2 - 2k) = \pm 2h^2$$

$$\Rightarrow 2 - 2k + 2k^2 - 2k = \pm 2h^2$$

$$\Rightarrow 2k^2 - 4k + 2 = \pm 2h^2$$

$$\Rightarrow k^2 - 2k + 1 = \pm h^2$$

Hence, locus of a point is

$$\Rightarrow (k - 1)^2 = h^2$$

$$y - 1 = \pm x$$

$$\Rightarrow x = y - 1 \text{ or } x = -(y - 1)$$

73. ***(b)*** Given, $2a_1 = 2\sin\theta$

$$\Rightarrow a_1 = \sin\theta$$

and $3x^2 + 4y^2 = 12$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$$

Here, $a^2 = 4$ and $b^2 = 3$

$$\therefore b^2 = a^2(1 - e^2)$$

$$\Rightarrow 3 = 4(1 - e^2)$$

$$\Rightarrow e^2 = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow e = \frac{1}{2}$$

Focus, $F(ae, 0) = F\left(2 \times \frac{1}{2}, 0\right)$

$$= F(1, 0)$$

For hyperbola foci are same

$$a_1e_1 = ae = 1$$

$$\therefore (\sin\theta)e_1 = 1$$

$$\Rightarrow e_1 = \operatorname{cosec}\theta$$

and $b_1^2 = a_1^2(e_1^2 - 1) = a_1^2e_1^2 - a_1^2$

$$\Rightarrow b_1^2 = 1 - \sin^2\theta = \cos^2\theta$$

$$\frac{x^2}{a_1^2} - \frac{y^2}{b_1^2} = 1$$

$$\Rightarrow \frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$$

$$\Rightarrow x^2\operatorname{cosec}^2\theta - y^2\sec^2\theta = 1$$

74. *(a, c)* $f(x) = \cos\left(\frac{\pi}{x}\right)$

$$\Rightarrow \quad f'(x) = -\sin\left(\frac{\pi}{x}\right)\left(\frac{-\pi}{x^2}\right) = \frac{\pi}{x^2}\sin\frac{\pi}{x}$$

For increasing function, $f'(x) > 0$

$$\Rightarrow \sin\left(\frac{\pi}{x}\right) > 0 \Rightarrow 2k\pi < \frac{\pi}{x} < (2k+1)\pi$$

$$\Rightarrow \quad \frac{1}{2k} > x > \frac{1}{2k+1}$$

For decreasing function, $f'(x) < 0$

$$\Rightarrow \quad \sin\left(\frac{\pi}{x}\right) < 0$$

$$\Rightarrow \frac{\pi}{x} \in [(2k+1)\pi, (2k+2)\pi] \Rightarrow x \in \left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$$

75. *(c)* Given, $y = \log_a (x + \sqrt{x^2+1}), a > 0, a \neq 1$

$$\Rightarrow \quad a^y = (x + \sqrt{x^2+1})$$

$$\Rightarrow \quad a^{-y} = \frac{1}{x + \sqrt{x^2+1}}$$

$$= \sqrt{x^2+1} - x$$

$$\Rightarrow a^y - a^{-y} = 2x \Rightarrow x = \frac{a^y - a^{-y}}{2}$$

$$\Rightarrow \quad f^{-1}(y) = \frac{e^{y\log a} - e^{-y\log a}}{2}$$

$$\Rightarrow \quad f^{-1}(y) = \sinh(y\log a) \quad \left(\because \frac{e^x - e^{-x}}{2} = \sinh(x)\right)$$