# Lecture - 14

## Impedance Matching Technique :-

- Active (Bias)
  - CB Amp
  - CC Amp
- Passive
  - Series
    - $\lambda/4$ → Quaterwave transformer
  - Shunt → Stub matching

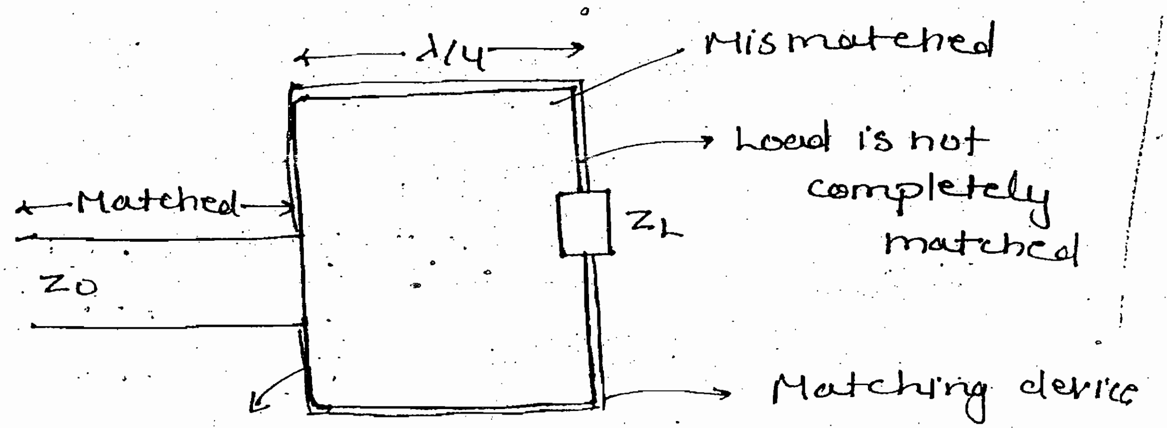
$$Z_0 = \frac{60 \ln(b/a)}{\sqrt{\epsilon_r}}$$
= few 10 Ω to 100 Ω

&

$Z_L$ = few Ω - kΩ

( $Z_L \neq Z_0$ Most often )

## Series ($\lambda/4$) Quaterwave transformer :-




The line is matched

The termination of $Z_0$ line $\doteq$ $Z_{in}$ of $\lambda/4$ matching device

$$= \frac{Z_0'^2}{Z_L} = Z_0$$

Design needs → $\boxed{Z_0' = \sqrt{Z_0 Z_L}}$

eg :- 50 Ω → line  100 Ω → load

$$\Gamma_{(load)} = \frac{100-50}{100+50} = \frac{1}{3}$$

Using a $\lambda/4$ transformer

$$Z_0' = \sqrt{50 \times 100} \simeq 70\ \Omega$$

$$\Gamma_{(at\ load)} = \frac{100-70}{100+70} < \frac{1}{3}.$$

Disadvantage:

i) The technique is suited for a narrow range of frequencies as the length of the transformer is frequency dependent.

ii) $Z_0' = \sqrt{Z_0 \cdot Z_L}$ → The technique is suited only for lossless line and resistive load.

## Shunt - Stub matching:-

A stub is S.C or O.C line of a pre-calculated length, placed at a precalculated position such that the line is matched from the stub to the source

### Design of a stub:-

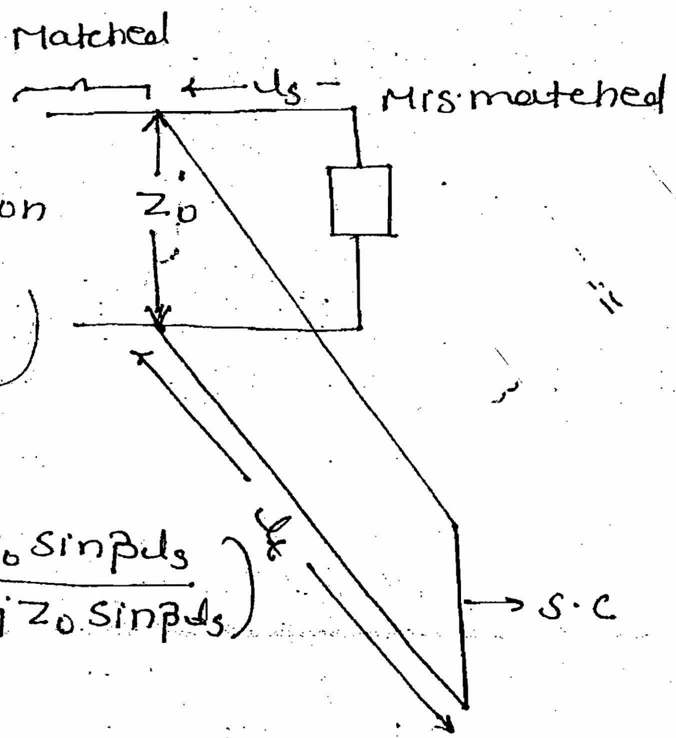
**Step-(1):-**

Identify a length $l_s$ on the line from the load where the impedance has a real part $Z_0$

$$Z(l_s) = Z_0 \pm jX$$

$l_s$ = stub position

$$l_s = \frac{\lambda}{2\pi}\tan^{-1}\left(\sqrt{\frac{Z_L}{Z_0}}\right)$$




$$Z(l_s) = Z_0\left(\frac{Z_L\cos\beta l_s + jZ_0\sin\beta l_s}{Z_0\cos\beta l_s + jZ_0\sin\beta l_s}\right)$$

$$Z(l_s) = \text{Real}\left[\frac{Z_L\cos\beta l_s + jZ_0\sin\beta l_s}{Z_0\cos\beta l_s + jZ_0\sin\beta l_s}\right] = 1$$

Step-(II):-

At this identified position place an equal and opposite reactance in shunt such that cancel to the existing reactance i.e.

$$Z(l_s) = Z_0 \pm jX \; (\mp jX)$$

$$Y(l_s) = Y_0 \pm jB \; (\mp jB)$$

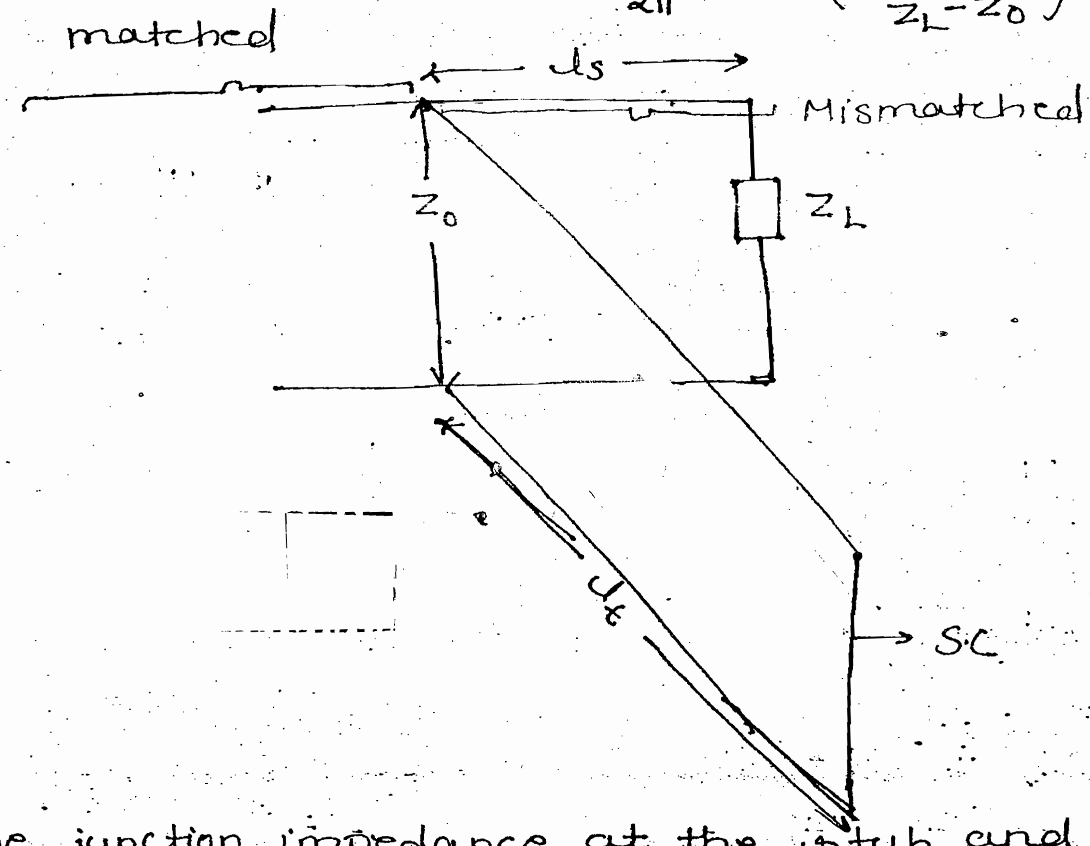
Stub

Step-(III):-

This reactance is realised from a SC or O.C stub of a finite length $l_t$ such that

$$Z_{SC} = jZ_0 \tan\beta l_t \simeq -\text{Imag}[Z(l_s)]$$

$$l_t = \text{stub length} = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{\sqrt{Z_L Z_0}}{Z_L - Z_0}\right)$$




The junction impedance at the stub and the line purely real as $Z_0$ and hence matched from the stub to the source

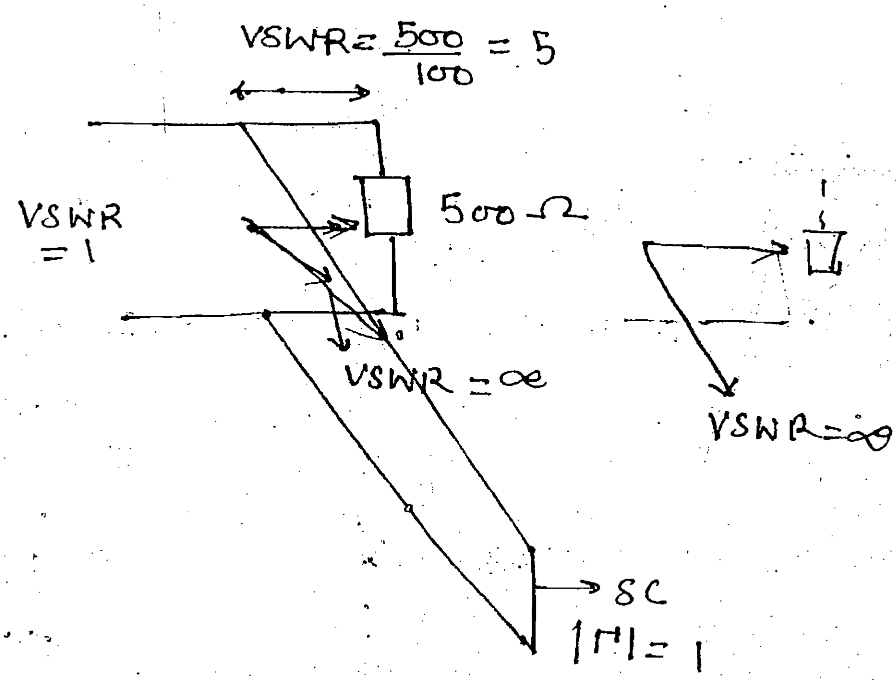
As the frequency on the line changes the stub length and stub position both have to adjusted which is mechanically inconvenient however in SC stub it is easy to using movable shorts

NOTE:-

For wide range of frequencies we use double stub matching where $l_{s_1}$ and $l_{s_2}$ are two stub position fixed and $l_{t_1}$ and $l_{t_2}$ are varied.

WORKBOOK:-

0.




1. $VSWR = 3$

$$|\Gamma|_V = \frac{3-1}{3+1} = \frac{1}{2}$$

$$\Gamma_P = -\frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{4} = 25\%$$

32. $$Z_0' = \sqrt{225 \times 250} = 240\,\Omega$$

33. $$Z_L = 75 - j40$$
$$Z_0 = 75$$

Note:-

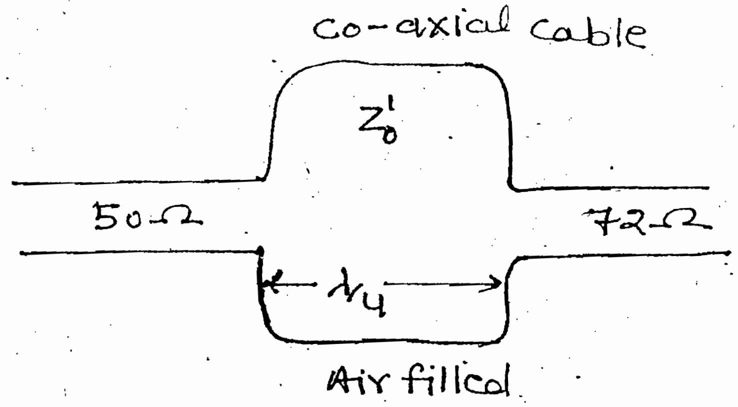
Stub can never be placed at node

4. OC → Stubs → More difficult to realize
SC → Stubs

Ans-(a)

5. → D

36.




$Z_0' = \sqrt{50 \cdot 72}$

$= 60\Omega$

$= \dfrac{60 \ln(b/a)}{\sqrt{\epsilon_R}}$

$\ln(b/a) = 1 \Rightarrow \dfrac{b}{a} = e = 2.71 \Rightarrow b = 27\text{ mm}$

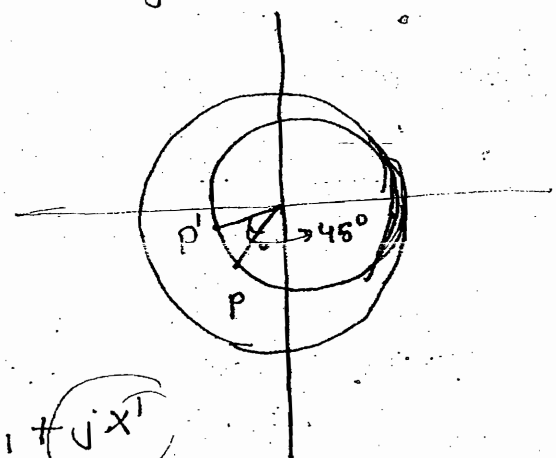
37. $0.5 - j0.3 = \dfrac{Z_L}{Z_0} =$ Normalized value

Actual value $= (0.5 - j0.3) \times 50$

$= 25 - j15$

38. Clocking on

R = constant




Z at P $= R + jX_1$

Z' at P' $= R + jX_2$

with $X_2 > X_1$

Z' at P' $= R + jX_1 + jX'$ → inductive

Ans – (a)

39. (I) $\dfrac{50 + j50}{50} = 1 + j1$ (1 → R, 1 → X)

(II) $\dfrac{10 + j10}{50} = 0.2 + j0.2$ (0.2 → R, 0.2 → X)

(III) $\dfrac{25 - j50}{50} = 0.5 - j1$ (0.5 → R, 1 → X)

40. (I) one revolution for 360° – length – $\lambda/2$

↓

All impedances repeat for $\lambda/2$

(II) Clockwise movement $X \uparrow$

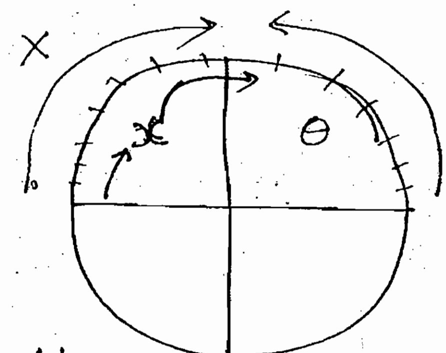
towards generator $x \uparrow$

$$Z(x) = Z_0 \left[ \frac{Z_L \cos\beta x + j Z_0 \sin\beta x}{Z_0 \cos\beta x + j Z_L \sin\beta x} \right]$$

$$= j Z_0 \tan\beta x$$

$x \uparrow \quad X \uparrow$

(III) $\Gamma(x) = \Gamma(0) e^{-j2\beta x}$




Three scales

$\rightarrow X$

$\rightarrow \theta$

$\rightarrow x$

Three scales on the Boundary

(I) $X$ – Reactance of the load

(II) $x$ – Length on the line

(III) $\Gamma$'s phase

All the three are related scales

$\rightarrow$ one is sufficient

(IV) Impedance (Wrong statement)

Conventional :—

$$V(x) = \frac{V_L(Z_L+Z_0)}{2Z_L} \left[ e^{\gamma x} + \frac{Z_L - Z_0}{Z_L + Z_0} e^{-\gamma x} \right]$$

$$= V_L \cosh \gamma x$$

If $\gamma = j\beta \quad V(x) = V_L \cos\beta x$ — (1)

OR

$$V(x) = V_L \cosh\gamma x + I_L Z_0 \sinh\gamma x$$

$$V(x) = V_L \cosh\gamma x$$

$$I(x) = I_L \cosh(\gamma x) + \frac{V_L}{Z_0} \sinh(\gamma x)$$

$$I(x) = \frac{V_L}{Z_0} \sinh(\gamma x)$$ [illegible]

If $\gamma = j\beta$ $\quad I(x) = \frac{jV_L}{Z_0} \sin \beta x$

$V_L = 12V$

$Z_0 = 60\Omega$

$x = \lambda/4$

$l = \lambda/2$

$\rightarrow$ if $\gamma = j\beta$ $\quad V(x) = V_L \cos \beta x$

$$Z_L = 100 - \frac{j}{\omega C} = 100 - j180\Omega$$

$$= 100 - \frac{j}{2 \times \pi \times 10^8 \times \frac{1}{36\pi \times 10^9}}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \text{VSWR}$$

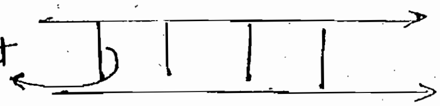
$$Z_{max} = Z_0 (SWR)$$

$$Z_{min} = Z_0 / SWR$$

# Wave Guides :-

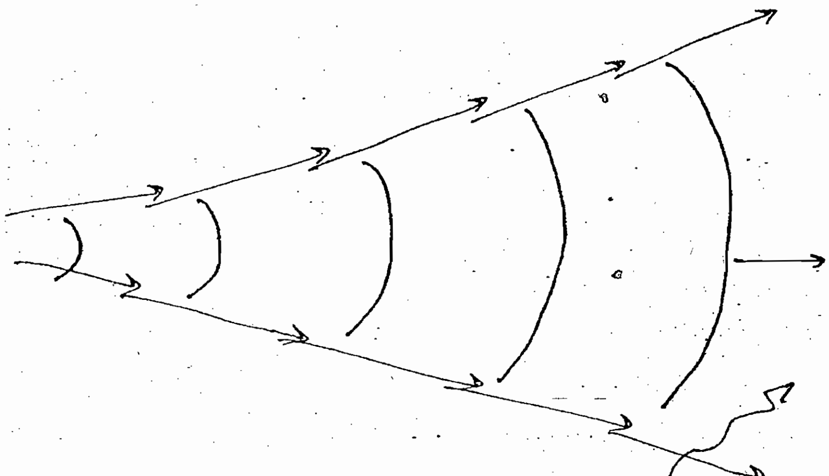
Uniform Plane Wave $\longrightarrow$ $E(z,t)_x$ , $H(z,t)_y$

Wavefront

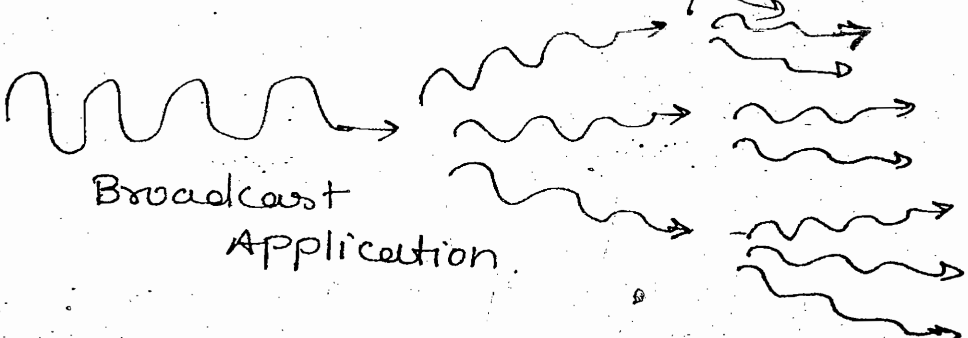


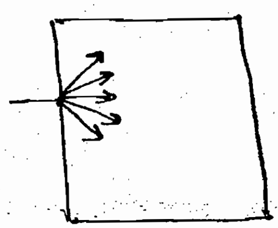
Dispersive Beams $\longrightarrow$ $E(x,y,z,t)_{(x,y,z)}$

$H(x,y,z,t)_{(x,y,z)}$



Spherical Wavefront



Scattering
Diffraction
Diffusion

→ All practical EM waves are dispersive in nature and hence they obey Huygen's wave principle that every ray is a source of secondary emission. This is the cause of diffusion, diffraction and scattering properties of EM Waves

→ This is an advantage in broadcast application but a serious limitation in point to point communication. Hence waveguide are used to restrict the wave with a specific bound

$E(x,z,t)$ }
$H(x,z,t)$ } one dimension restriction in $x$, one dimension propogation in $z$

↓

Parallel plane waveguide

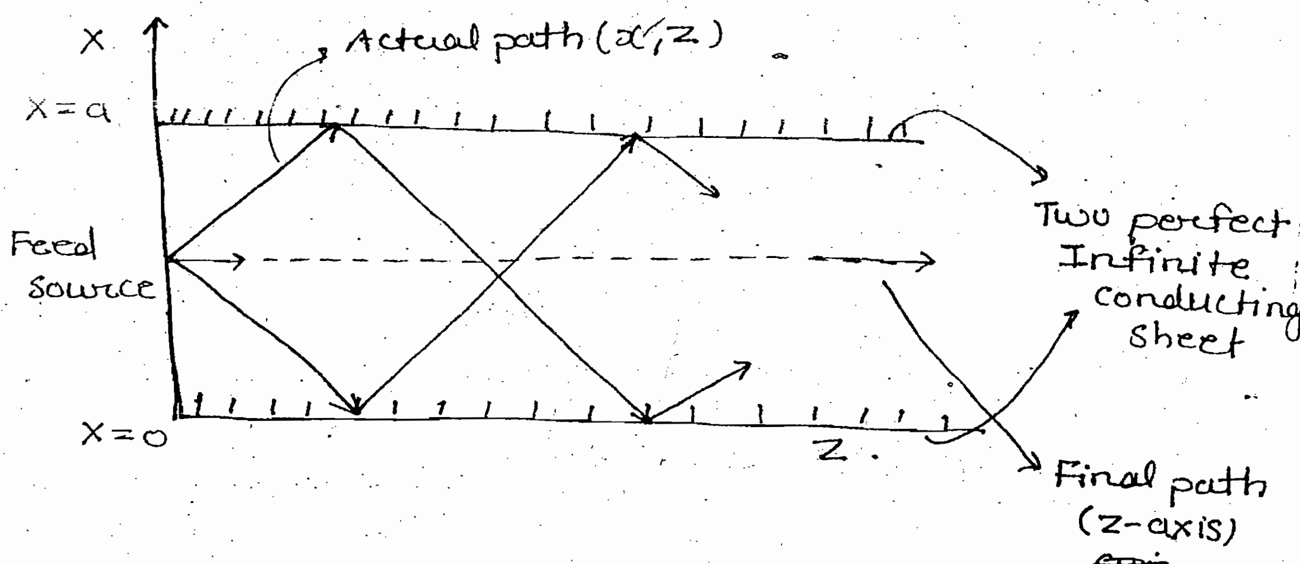
eg:- earth and ionosphere guiding

$E(x,y,z,t)$ }
$H(x,y,z,t)$ } 2 dimension restriction in $x$, 1 dimension propogation in $z$

↓

Rectangular Waveguide

Parallel Plane Waveguides :-




Boundary conditions at conductor walls :-

$E_{tang} = 0$ at $x=0$, $X=a$

$E(x)_{tang} = 0$ at $X=0$, $X=a$

$E(x)_y$ & $E(x)_z = 0$ at Guide walls

## Propogation along the Guide Axis:-

Using $\nabla^2 E = \gamma^2 E$ Helmoltz's Equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu \epsilon E$$

Let $E(z)$ be any natural harmonic $e^{-\bar{\gamma} z}$ as the z side is un-restricted

where $\bar{\gamma} = \gamma_z$ = Propogation consist in guide axis.

$$\frac{\partial^2 E}{\partial x^2} + \bar{\gamma}^2 E = -\omega^2 \mu \epsilon E$$

$$\frac{\partial^2 E}{\partial x^2} = -(\gamma^2 + \omega^2 \mu \epsilon) E$$

where $\bar{\gamma}^2 + \omega^2 \mu \epsilon = \gamma_x^2$

where $\gamma_x$ = Propogation constant in x side or restricted side

$$\frac{\partial^2 E}{\partial x^2} = -\gamma_x^2 E$$

The $E(x)$ solution is also harmonic

$$E(x) = c_1 \sin(\gamma_x x) + c_2 \cos(\gamma_x x)$$

The restricted side propogation has to be trignometric harmonic only

Applying the Boundary conditions

at $x = 0$

$$E(0)_{tang} = 0 + c_2 = 0$$

$$\Rightarrow c_2 = 0$$

**Note:-**

The tangential E field harmonic has to be a 'sin' only is the restricted side

at $x = a$

$$E(a)_{tang} = c_1 \sin(\gamma_x a) = 0$$

$$\boxed{\gamma_x = \frac{m\pi}{a}} \qquad m = 0, 1, 2, 3 \ldots$$

Note :-

The restricted side propogation constant can take only discrete solutions but not any continous value

Finally $\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon}$

Concept 1 :-

($f_c$) cut off frequency of the guide

If $\left(\frac{m\pi}{a}\right)^2 > \omega^2 \mu \epsilon$

then $\bar{\gamma} = \bar{\alpha} + j0$ No propogation along the guide axis

If $\omega^2 \mu \epsilon > \left(\frac{m\pi}{a}\right)^2$

$$\omega > \frac{m\pi}{a\sqrt{\mu\epsilon}}$$

then $\bar{\gamma} = 0 + j\bar{\beta}$

The wave travels along the guide axis without attenuation.

$\omega \geq \frac{m\pi c}{a}$

Note :-

Every waveguide has a minimum cut-off frequency below which these cannot be propogation i.e. there is a max. wavelength above which there cannot be propogation and this wavelength is comparable to the guide dimensions

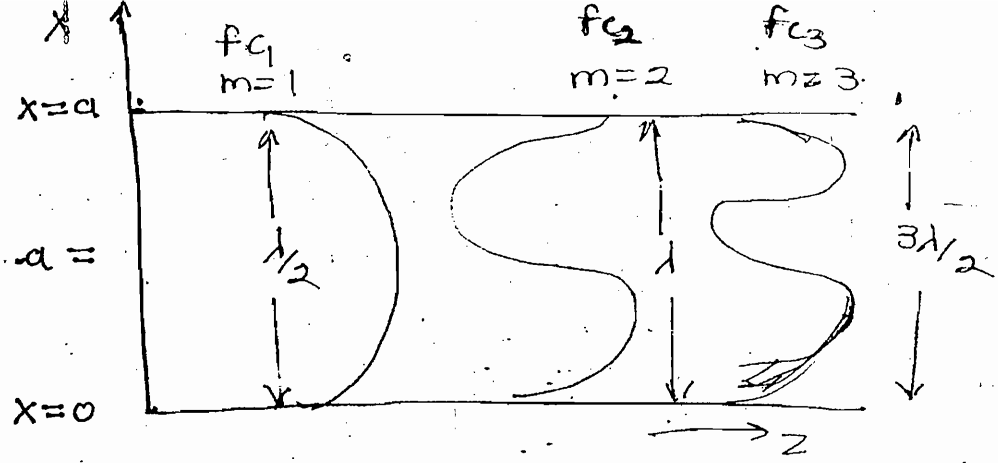
Hence $\omega_c > \frac{m\pi c}{a}$, $f_c > \frac{mc}{2a}$, $\lambda_c > \frac{2a}{m}$

$$\Rightarrow \omega_c = \frac{m\pi c}{a}$$

$$f_c = \frac{mc}{2a}$$

$$\lambda_c = \frac{2a}{m}$$

9p




At exact cut-off frequency where $\bar{\gamma} = 0$, the wave resonats b/w guide walls and oscillates b/w the walls for 'm' such frequencies.

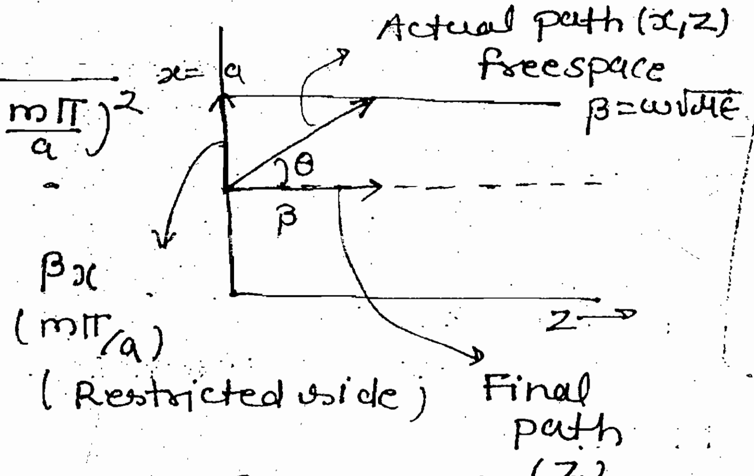
Concept 2 :-

Wave Angle or Tilt Angle :-

$$\bar{\beta} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon}$$

$$= j\sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$




$$\beta^2 = \beta_x^2 + \bar{\beta}^2$$

$$\bar{\beta} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$

Phase velocity along the guide axis, $\bar{V}_p = \dfrac{\omega}{\bar{\beta}} = \dfrac{\omega}{\sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}}$

$$\bar{V}_p = \frac{1}{\sqrt{\mu \epsilon - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$= \frac{1/\sqrt{\mu \epsilon}}{\sqrt{1 - \left(\frac{m\pi}{a\omega\sqrt{\mu\epsilon}}\right)^2}}$$

$$\boxed{\bar{V}_p = \frac{c}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}}$$

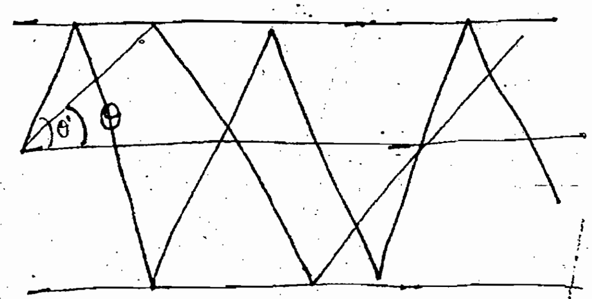
$$\bar{\beta} = \beta\cos\theta$$

$$\frac{2\pi}{\bar{\lambda}} = \frac{2\pi}{\lambda}\cos\theta$$

$$\Rightarrow \bar{\lambda} = \frac{\lambda}{\cos\theta}$$

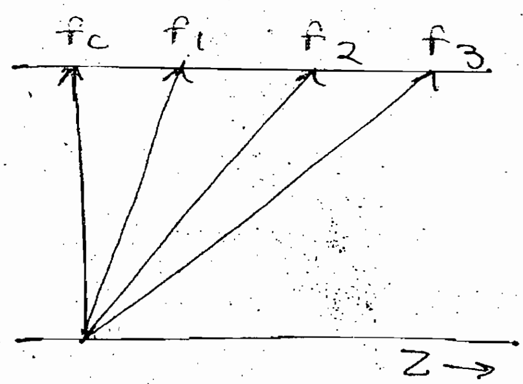
$$\Rightarrow \bar{\lambda} f = \frac{\lambda f}{\cos\theta}$$

$$\boxed{\bar{V}_p = \frac{c}{\cos\theta}}$$




By comparison

$$\boxed{\sin\theta = \frac{f_c}{f}}$$




Note:-

Every frequency has a unique tilt angle and unique velocity inside the guide so that two different frequencies never overlap to each other

$f_3 > f_2 > f_1 > f_c$

Concept 3:-

Group Velocity ($\bar{V}_g$):-

$$\bar{V}_p = \frac{c}{\cos\theta} \Rightarrow \bar{V}_p > c \rightarrow \text{Always}$$

In linear conditions, $\beta \propto \omega$

velocity is phase ~~vel~~ velocity $V_p = \frac{\omega}{\beta}$

eg!- lossless EM Waves in free space

lossless VI Waves in transmission line

$$\beta = \omega\sqrt{\mu\epsilon}$$
$$\beta = \omega\sqrt{LC}$$

But in dispersive condition $\beta$ is not linear with $\omega$

eg:- $V_g = \frac{d\omega}{d\bar{\beta}}$ = Group velocity

$$\bar{\beta} = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2} \Big] \rightarrow \text{dispersive conditions}$$

$\beta$ is not linear with $\omega$

$$\frac{d\bar{\beta}}{d\omega} = \frac{1}{\cancel{2}\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}} \cdot \cancel{2}\omega\mu\epsilon$$

$$= \frac{\mu\epsilon}{\sqrt{\mu\epsilon - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$= \frac{\sqrt{\mu\epsilon}}{\sqrt{1 - \left(\frac{m\pi}{a\omega\sqrt{\mu\epsilon}}\right)^2}}$$

$$= \frac{1}{c\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\bar{V_g} = c\cos\theta$$

$\bar{V_g} < c \longrightarrow$ Always

$\boxed{\bar{V_g} \cdot \bar{V_p} = c^2} \longrightarrow$ Always

Concept 4:-

Modes of operation

Concept 4 :-

Modes of Operation :-

Note :-

Mode stands for physical connection of field which can be axial or longitudinal feed

→ The no. of field connection decides the integer m of that mode and this m decides the cut-off freq of the mode

→ The mode can be identified from field such that m stands for no. of half cycles b/w the guide walls ~~and~~ or no. of maxima b/w the guide walls.