# Chapter - Conic Sections

## Topic-1: Circles

### 1 MCQs with One Correct Answer

1. Consider a triangle $\Delta$ whose two sides lie on the $x$-axis and the line $x+y+1=0$. If the orthocenter of $\Delta$ is $(1, 1)$, then the equation of the circle passing through the vertices of the triangle $\Delta$ is **[Adv. 2021]**
   (a) $x^2+y^2-3x+y=0$ (b) $x^2+y^2+x+3y=0$
   (c) $x^2+y^2+2y-1=0$ (d) $x^2+y^2+x+y=0$

2. A line $y=mx+1$ intersectctrs the circle $(x-3)^2+(y+2)^2=25$ at the points P and Q. If the midpoint of the line segment PQ has x-coordinate $-\frac{3}{5}$, then which one of the following options is correct ? **[Adv. 2019]**
   (a) $2 \le m < 4$ (b) $-3 \le m < -1$
   (c) $4 \le m < 6$ (d) $6 \le m < 8$

3. The common tangents to the circle $x^2+y^2=2$ and the parabola $y^2=8x$ touch the circle at the points $P, Q$ and the parabola at the points $R, S$. Then the area of the quadrilateral $PQRS$ is **[Adv. 2014]**
   (a) 3 (b) 6 (c) 9 (d) 15

4. The circle passing through the point $(-1, 0)$ and touching the $y$-axis at $(0, 2)$ also passes through the point. **[2011]**
   (a) $\left(-\frac{3}{2},0\right)$ (b) $\left(-\frac{5}{2},2\right)$
   (c) $\left(-\frac{3}{2},\frac{5}{2}\right)$ (d) $(-4, 0)$

5. Tangents drawn from the point $P(1, 8)$ to the circle $x^2+y^2-6x-4y-11=0$ touch the circle at the points $A$ and $B$. The equation of the circumcircle of the triangle $PAB$ is **[2009]**
   (a) $x^2+y^2+4x-6y+19=0$
   (b) $x^2+y^2-4x-10y+19=0$
   (c) $x^2+y^2-2x+6y-29=0$
   (d) $x^2+y^2-6x-4y+19=0$

6. A circle is given by $x^2+(y-1)^2=1$, another circle $C$ touches it externally and also the x-axis, then the locus of its centre is **[2005S]**
   (a) $\{(x,y): x^2=4y\} \cup \{(x,y): y \le 0\}$
   (b) $\{(x,y): x^2+(y-1)^2=4\} \cup \{(x,y): y \le 0\}$
   (c) $\{(x,y): x^2=y\} \cup \{(0,y): y \le 0\}$
   (d) $\{(x,y): x^2=4y\} \cup \{(0,y): y \le 0\}$

7. If one of the diameters of the circle $x^2+y^2-2x-6y+6=0$ is a chord to the circle with centre $(2, 1)$, then the radius of the circle is **[2004S]**
   (a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 3 (d) 2

8. The centre of circle inscribed in square formed by the lines $x^2-8x+12=0$ and $y^2-14y+45=0$, is **[2003S]**
   (a) (4,7) (b) (7,4) (c) (9,4) (d) (4,9)

9 If the tangent at the point $P$ on the circle $x^2+y^2+6x+6y=2$ meets a straight line $5x-2y+6=0$ at a point $Q$ on the y-axis, then the length of $PQ$ is **[2002S]**
   (a) 4 (b) $2\sqrt{5}$ (c) 5 (d) $3\sqrt{5}$

10. Let $PQ$ and $RS$ be tangents at the extremities of the diameter $PR$ of a circle of radius $r$. If $PS$ and $RQ$ intersect at a point $X$ on the circumference of the circle, then $2r$ equals **[2001S]**
    (a) $\sqrt{PQ.RS}$ (b) $(PQ+RS)/2$
    (c) $2PQ.RS/(PQ+RS)$ (d) $\sqrt{(PQ^2+RS^2)/2}$

11. Let $AB$ be a chord of the circle $x^2+y^2=r^2$ subtending a right angle at the centre. Then the locus of the centroid of the triangle $PAB$ as $P$ moves on the circle is **[2001S]**
    (a) a parabola (b) a circle
    (c) an ellipse (d) a pair of straight lines

12. If the circles $x^2+y^2+2x+2ky+6=0$, $x^2+y^2+2ky+k=0$ intersect orthogonally, then $k$ is **[2000S]**
    (a) $2$ or $-\frac{3}{2}$ (b) $-2$ or $-\frac{3}{2}$
    (c) $2$ or $\frac{3}{2}$ (d) $-2$ or $\frac{3}{2}$

**13.** The triangle $PQR$ is inscribed in the circle $x^2+y^2=25$. If $Q$ and $R$ have co-ordinates (3,4) and (–4, 3) respectively, then $\angle QPR$ is equal to **[2000S]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

**14.** If two distinct chords, drawn from the point $(p, q)$ on the circle $x^2+y^2=px+qy=0$ (where $pq \neq 0$) are bisected by the x –axis, then **[1999 - 2 Marks]**

(a) $p^2=q^2$ (b) $p^2=8q^2$
(c) $P^2<8q^2$ (d) $p^2>8q^2$.

**15.** The angle between a pair of tangents drawn from a point $P$ to the circle $x^2+y^2+4x-6y+9\sin^2\alpha+13\cos^2\alpha=0$ is $2\alpha$. The equation of the locus of the point $P$ is **[1996 - 1 Mark]**

(a) $x^2+y^2+4x-6y+4=0$
(b) $x^2+y^2+4x-6y-9=0$
(c) $x^2+y^2+4x-6y-4=0$
(d) $x^2+y^2+4x-6y+9=0$

**16.** The circles $x^2+y^2-10x+16=0$ and $x^2+y^2=r^2$ intersect each other in two distinct points if **[1994]**

(a) $r<2$ (b) $r>8$ (c) $2<r<8$ (d) $2\le r\le 8$

**17.** The locus of the centre of a circle, which touches externally the circle $x^2+y^2-6x-6y+14=0$ and also touches the y-axis, is given by the equation: **[1993 - 1 Marks]**

(a) $x^2-6x-10y+14=0$ (b) $x^2-10x-6y+14=0$
(c) $y^2-6x-10y+14=0$ (d) $y^2-10x-6y+14=0$

**18.** The centre of a circle passing through the points (0, 0), (1, 0) and touching the circle $x^2+y^2=9$ is **[1992 - 2 Marks]**

(a) $\left(\frac{3}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},\frac{3}{2}\right)$ (c) $\left(\frac{1}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},-2^{\frac{1}{2}}\right)$

**19.** The lines $2x-3y=5$ and $3x-4y=7$ are diameters of a circle of area 154 sq. units. Then the equation of this circle is **[1989 - 2 Marks]**

(a) $x^2+y^2+2x-2y=62$ (b) $x^2+y^2+2x-2y=47$
(c) $x^2+y^2-2x+2y=47$ (d) $x^2+y^2-2x+2y=62$

**20.** If the two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct points, then **[1989 - 2 Marks]**

(a) $2<r<8$ (b) $r<2$ (c) $r=2$ (d) $r>2$

**21.** If a circle passes through the point $(a, b)$ and cuts the circle $x^2+y^2=k^2$ orthogonally, then the equation of the locus of its centre is **[1988 - 2 Marks]**

(a) $2ax+2by-(a^2+b^2+k^2)=0$
(b) $2ax+2by-(a^2-b^2+k^2)=0$
(c) $x^2+y^2-3ax-4by+(a^2+b^2-k^2)=0$
(d) $x^2+y^2-2ax-3by+(a^2-b^2-k^2)=0$

**22.** The locus of the mid-point of a chord of the circle $x^2+y^2=4$ which subtends a right angle at the origin is **[1984 - 2 Marks]**

(a) $x+y=2$ (b) $x^2+y^2=1$
(c) $x^2+y^2=2$ (d) $x+y=1$

**23.** The equation of the circle passing through (1, 1) and the points of intersection of $x^2+y^2+13x-3y=0$ and $2x^2+2y^2+4x-7y-25=0$ is **[1983 - 1 Mark]**

(a) $4x^2+4y^2-30x-10y-25=0$
(b) $4x^2+4y^2+30x-13y-25=0$
(c) $4x^2+4y^2-17x-10y+25=0$
(d) none of these

**24.** Two circles $x^2+y^2=6$ and $x^2+y^2-6x+8=0$ are given. Then the equation of the circle through their points of intersection and the point (1, 1) is **[1980]**

(a) $x^2+y^2-6x+4=0$ (b) $x^2+y^2-3x+1=0$
(c) $x^2+y^2-4y+2=0$ (d) none of these

**25.** A square is inscribed in the circle $x^2+y^2-2x+4y+3=0$. Its sides are parallel to the coordinate axes. The one vertex of the square is **[1980]**

(a) $(1+\sqrt{2},-2)$ (b) $(1-\sqrt{2},-2)$
(c) $(1,-2+\sqrt{2})$ (d) none of these

## 2 Integer Value Answer/ Non-Negative Integer

**26.** Let $A_1, A_2, A_3, \ldots\ldots A_8$ be the vertices of a regular octagon that lie on a circle of radius 2. Let P be a point on the circle and let $PA_i$ denote the distance between the points P and $A_i$ for i = 1,2.....8. If P varies over the circle, then the maximum value of the product $PA_1 . PA_2 \ldots\ldots PA_8$, is **[Adv. 2023]**

**27.** Let $C_1$ be the circle of radius 1 with center at the origin. Let $C_2$ be the circle of radius r with center at the point $A=(4,1)$, where $1<r<3$. Two distinct common tangents PQ and ST of $C_1$ and $C_2$ are drawn. The tangent PQ touches $C_1$ at P and $C_2$ at Q. The tangent ST touches $C_1$ at S and $C_2$ at T. Mid points of the line segments PQ and ST are joined to form a line which meets the x-axis at a point B . If AB $=\sqrt{5}$, then the value of $r^2$ is **[Adv. 2023]**

**28.** Let $G$ be a circle of radius $R>0$. Let $G_1, G_2, \ldots G_n$ be $n$ circles of equal radius $r>0$. Suppose each of the $n$ circles $G_1, G_2, G_2, \ldots, G_n$ touches the circle $G$ externally. Also, for $i=1,2,\ldots n-1$, the circle $G_i$ touches $G_{i+1}$ externally, and $G_n$ touches $G_1$ externally. Then, which of the following statements is/are TRUE ? **[Adv. 2022]**

(a) If $n=4$, then $\left(\sqrt{2}-1\right)r<R$
(b) If $n=5$, then $r<R$
(c) If $n=8$, then $\left(\sqrt{2}-1\right)r<R$
(d) If $n=12$, then $\sqrt{2}\left(\sqrt{3}+1\right)r>R$

**29.** Let $O$ be the centre of the circle $x^2+y^2=r^2$, where $r>\frac{\sqrt{5}}{2}$. Suppose $PQ$ is a chord of this circle and the equation of the line passing through $P$ and $Q$ is $2x+4y=5$. If the centre of the circumcircle of the triangle $OPQ$ lies on the line $x+2y=4$, then the value of $r$ is _____ **[Adv. 2020]**

**30.** Let the point B be the reflection of the point $A(2,3)$ with respect to the line $8x-6y-23=0$. Let $\Gamma_A$ and $\Gamma_B$ be circles of radii 2 and 1 with centers A and B respectively. Let $T$ be a common tangent to the circles $\Gamma_A$ and $\Gamma_B$ such that both the circles are on the same side of $T$. If $C$ is the point of intersection of $T$ and the line passing through $A$ and B, then the length of the line segment AC is _____. **[Adv. 2019]**

**31.** For how many values of p, the circle $x^2+y^2+2x+4y-p=0$ and the coordinate axes have exactly three common points? **[Adv. 2017]**

32. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts.
If $S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$ then the number of points (s) in S lying inside the smaller part is **[2011]**

33. The centres of two circles $C_1$ and $C_2$ each of unit radius are at a distance of 6 units from each other. Let $P$ be the mid point of the line segement joining the centres of $C_1$ and $C_2$ and $C$ be a circle touching circles $C_1$ and $C_2$ externally. If a common tangent to $C_1$ and $C$ passing through $P$ is also a common tangent to $C_2$ and $C$, then the radius of the circle $C$ is **[2009]**

## 3 Numeric/ New Stem Based Questions

34. Let $ABC$ be the triangle with $AB = 1$, $AC = 3$ and $\angle BAC = \frac{\pi}{2}$. If a circle of radius $r > 0$ touches the sides $AB$, $AC$ and also touches internally the circumcircle of the triangle $ABC$, then the value of $r$ is _________. **[Adv. 2022]**

## 4 Fill in the Blanks

35. The chords of contact of the pair of tangents drawn from each point on the line $2x+y=4$ to circle $x^2+y^2 = 1$ pass through the point .................... **[1997 - 2 Marks]**

36. For each natural number $k$, let $C_k$ denote the circle with radius $k$ centimetres and centre at the origin. On the circle $C_k$, α-particle moves $k$ centimetres in the counter-clockwise direction. After completing its motion on $C_k$, the particle moves to $C_{k+1}$ in the radial direction. The motion of the particle continues in this manner. The particle starts at (1, 0). If the particle crosses the positive direction of the x-axis for the first time on the circle $C_n$ then $n =$ ................ **[1997 - 2 Marks]**

37. The intercept on the line y = x by the circle $x^2 + y^2 - 2x = 0$ is $AB$. Equation of the circle with $AB$ as a diameter is ................... **[1996 - 1 Mark]**

38. The equation of the locus of the mid-points of a chord of the circle $4x^2 + 4y^2 - 12x + 4y + 1 = 0$ that subtend an angle of $2\pi/3$ at its centre is ................ **[1993 - 2 Marks]**

39. If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of $\lambda =$ ................ **[1991 - 2 Marks]**

40. The area of the triangle formed by the positive x-axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is, .................... **[1989 - 2 Marks]**

41. If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle $C_2$ of radius 5 in such a manner that common chord is of maximum length and has a slope equal to 3/4, then the coordinates of the centre of $C_2$ are .................... **[1988 - 2 Marks]**

42. The area of the triangle formed by the tangents from the point (4, 3) to the the circle $x^2 + y^2 = 9$ and the line joining their points of contact is .................... **[1987 - 2 Marks]**

43. From the point $A(0, 3)$ on the circle $x^2 + 4x + (y-3)^2 = 0$, a chord $AB$ is drawn and extended to a point $M$ such that $AM = 2AB$. The equation of the locus of $M$ is ................. **[1986 - 2 Marks]**

44 The equation of the line passing through the points of intersection of the circles $3x^2 + 3y^2 - 2x + 12y - 9 = 0$ and $x^2 + y^2 + 6x + 2y - 15 = 0$ is .................... **[1986 - 2 Marks]**

45. From the origin chords are drawn to the circle $(x-1)^2 + y^2 = 1$. The equation of the locus of the mid-points of these chords is .................... **[1985 - 2 Marks]**

46. Let $x^2 + y^2 - 4x - 2y - 11 = 0$ be a circle. A pair of tangents from the point (4, 5) with a pair of radii form a quadrilateral of area .................... **[1985 - 2 Marks]**

47. The lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the same circle. The radius of this circle is .................... **[1984 - 2 Marks]**

48. The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are .................... and .................... **[1983 - 2 Marks]**

49. If $A$ and $B$ are points in the plane such that $PA/PB = k$ (constant) for all $P$ on a given circle, then the value of $k$ cannot be equal to .................... **[1982 - 2 Marks]**

## 5 True / False

50. The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 - 6x + 2y = 0$. **[1989 - 1 Mark]**

51. No tangent can be drawn from the point (5/2, 1) to the circumcircle of the triangle with vertices $(1, \sqrt{3})$ $(1, -\sqrt{3})$, $(3, -\sqrt{3})$. **[1985 - 1 Mark]**

## 6 MCQs with One or More than One Correct Answer

52. For any complex number $w = c + id$, let $\arg(w) \in (-\pi, \pi]$, where $i = \sqrt{-1}$. Let α and β be real numbers such that for all complex numbers $z = x + iy$ satisfying $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, the ordered pair $(x, y)$ lies on the circle $x^2 + y^2 + 5x - 3y + 4 = 0$.
Then which of the following statements is (are) **TRUE**? **[Adv. 2021]**
(a) $\alpha = -1$
(b) $\alpha\beta = 4$
(c) $\alpha\beta = -4$
(d) $\beta = 4$

53. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) **[Adv. 2016]**

(a) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (b) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (d) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

**54.** A circle $S$ passes through the point $(0, 1)$ and is orthogonal to the circles $(x-1)^2+y^2=16$ and $x^2+y^2=1$. Then **[Adv. 2014]**

(a) radius of $S$ is 8 (b) radius of $S$ is 7

(c) centre of $S$ is $(-7, 1)$ (d) centre of $S$ is $(-8, 1)$

**55.** Circle(s) touching $x$-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on $y$-axis is (are) **[Adv. 2013]**

(a) $x^2+y^2-6x+8y+9=0$

(b) $x^2+y^2-6x+7y+9=0$

(c) $x^2+y^2-6x-8y+9=0$

(d) $x^2+y^2-6x-7y+9=0$

**56.** The number of common tangents to the circles $x^2+y^2=4$ and $x^2+y^2-6x-8y=24$ is **[1998 - 2 Marks]**

(a) 0 (b) 1 (c) 3 (d) 4

**57.** The equations of the tangents drawn from the origin to the circle $x^2+y^2-2rx-2hy+h^2=0$, are **[1988 - 2 Marks]**

(a) $x=0$ (b) $y=0$

(c) $(h^2-r^2)x-2rhy=0$ (d) $(h^2-r^2)x+2rhy=0$

## 7 Match the Following

**58.** Let the straight line $y=2x$ touch a circle with center $(0, \alpha)$, $\alpha > 0$, and radius $r$ at a point $A_1$. Let $B_1$ be the point on the circle such that the line segment $A_1B_1$ is a diameter of the circle. Let $\alpha + r = 5+\sqrt{5}$.

Match each entry in **List-I** to the correct entry in **List-II**.

| List-I | List-II |
|---|---|
| (P) $\alpha$ equals | (1) $(-2,4)$ |
| (Q) $r$ equals | (2) $\sqrt{5}$ |
| (R) $A_1$ equals | (3) $(-2,6)$ |
| (S) $B_1$ equals | (4) 5 |
| | (5) $(2,4)$ |

The correct option is **[Adv. 2024]**

(a) (P)$\to$(4) (Q)$\to$(2) (R)$\to$(1) (S)$\to$(3)

(b) (P)$\to$(2) (Q)$\to$(4) (R)$\to$(1) (S)$\to$(3)

(c) (P)$\to$(4) (Q)$\to$(2) (R)$\to$(5) (S)$\to$(3)

(d) (P)$\to$(2) (Q)$\to$(4) (R)$\to$(3) (S)$\to$(5)

**(For Q. 59 and 60)** Let the circles $C_1: x^2+y^2=9$ and $C_2: (x-3)^2+(y-4)^2=16$, intersect at the points $X$ and $Y$. Suppose that another circle $C_3: (x-h)^2+(y-k)^2=r^2$ satisfies the following conditions:

(i) Centre of $C_3$ is collinear with the centres of $C_1$ and $C_2$

(ii) $C_1$ and $C_2$ both lie inside $C_3$, and

(iii) $C_3$ touches $C_1$ at M and $C_2$ at N

Let the line through X and Y intersect $C_3$ at $Z$ and $W$, and let a common tangent of $C_1$ and $C_3$ be a tangent to the parabola $x^2=8\alpha y$.

There are some expressions given in the Column-I whose values are given in Column-II below **[Adv. 2019]**

| Column I | Column II |
|---|---|
| (A) $2h+k$ | (p) 6 |
| (B) $\frac{\text{Length of } ZW}{\text{Length of } XY}$ | (q) $\sqrt{6}$ |
| (C) $\frac{\text{Area of triangle } MZN}{\text{Area of triangle } ZMW}$ | (r) $\frac{5}{4}$ |
| (D) $\alpha$ | (s) $\frac{21}{5}$ |
| | (t) $2\sqrt{6}$ |
| | (u) $\frac{10}{3}$ |

**59.** Which of the following is the only CORRECT combination?

(a) (A), (u) (b) (A), (s)

(c) (B), (t) (d) (B), (q)

**60.** Which of the following is the only INCORRECT combination?

(a) (D), (s) (b) (A), (p)

(c) (C), (r) (d) (D), (u)

## 8 Comprehension/Passage Based Questions

### PASSAGE - 1

Let $M=\{(x, y) \in \mathbb{R}\times\mathbb{R}: x^2+y^2 \le r^2\}$ where $r>0$.

Consider the geometric progression $a_n=\frac{1}{2^{n-1}}, n=1, 2, 3, \ldots$. Let $S_0=0$ and, for $n\ge 1$, let $S_n$ denote the sum of the first $n$ terms of this progression. For $n \ge 1$, let $C_n$ denote the circle with center $(S_{n-1}, 0)$ and radius $a_n$, and $D_n$ denote the circle with center $(S_{n-1}, S_{n-1})$ and radius $a_n$.

**61.** Consider $M$ with $r=\frac{1025}{513}$. Let $k$ be the number of all those circles $C_n$ that are inside $M$. Let $l$ be the maximum possible number of circles among these $k$ circles such that no two circles intersect. Then **[Adv. 2021]**

(a) $k+2l=22$ (b) $2k+l=26$

(c) $2k+3l=34$ (d) $3k+2l=40$

**62.** Consider $M$ with $r=\frac{(2^{199}-1)\sqrt{2}}{2^{198}}$. The number of all those circles $D_n$ that are inside $M$ is **[Adv. 2021]**

(a) 198 (b) 199 (c) 200 (d) 201

PASSAGE - 2

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$. **[Adv. 2018]**

**63.** Let $E_1E_2$ and $F_1F_2$ be the chords of $S$ passing through the point $P_0(1, 1)$ and parallel to the $x$-axis and the $y$-axis, respectively. Let $G_1G_2$ be the chord of $S$ passing through $P_0$ and having slope $-1$. Let the tangents to $S$ at $E_1$ and $E_2$ meet at $E_3$, the tangents to $S$ at $F_1$ and $F_2$ meet at $F_3$, and the tangents to $S$ at $G_1$ and $G_2$ meet at $G_3$. Then, the points $E_3$, $F_3$, and $G_3$ lie on the curve

(a) $x+y=4$ (b) $(x-4)^2+(y-4)^2=16$

(c) $(x-4)(y-4)=4$ (d) $xy=4$

**64.** Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

(a) $(x+y)^2 = 3xy$ (b) $x^{2/3} + y^{2/3} = 2^{4/3}$

(c) $x^2 + y^2 = 2xy$ (d) $x^2 + y^2 = x^2y^2$

PASSAGE-3

A tangent $PT$ is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3},1)$. A straight line $L$, perpendicular to $PT$ is a tangent to the circle $(x-3)^2 + y^2 = 1$. **[2012]**

**65.** A possible equation of $L$ is

(a) $x-\sqrt{3}\,y=1$ (b) $x+\sqrt{3}y=1$

(c) $x-\sqrt{3}y=-1$ (d) $x+\sqrt{3}y=5$

**66.** A common tangent of the two circles is

(a) $x=4$ (b) $y=2$

(c) $x+\sqrt{3}y=4$ (d) $x+2\sqrt{2}y=6$

PASSAGE-4

$ABCD$ is a square of side length 2 units. $C_1$ is the circle touching all the sides of the square $ABCD$ and $C_2$ is the circumcircle of square $ABCD$. $L$ is a fixed line in the same plane. **[2006 - 5M, –2]**

**67.** If $P$ is any point of $C_1$ and $Q$ is another point on $C_2$, then

$$\frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}$$ is equal to

(a) 0.75 (b) 1.25 (c) 1 (d) 0.5

**68.** If a circle is such that it touches the line $L$ and the circle $C_1$ externally, such that both the circles are on the same side of the line, then the locus of centre of the circle is

(a) ellipse (b) hyperbola

(c) parabola (d) pair of straight line

**69.** A line $L'$ through $A$ is drawn parallel to $BD$. Point $S$ moves such that its distances from the line $BD$ and the vertex $A$ are equal. If locus of $S$ cuts $L'$ at $T_2$ and $T_3$ and $AC$ at $T_1$, then area of $\Delta T_1T_2T_3$ is

(a) $\frac{1}{2}$ sq. units (b) $\frac{2}{3}$ sq. units

(c) 1 sq. units (d) 2 sq. units

## 9 Assertion and Reason/Statements Type Questions

**70.** Consider $L_1: 2x+3y+p-3=0$
$L_2: 2x+3y+p+3=0$

where $p$ is a real number, and $C: x^2+y^2+6x-10y+30=0$

**STATEMENT - 1:** If line $L_1$ is a chord of circle $C$, then line $L_2$ is not always a diameter of circle $C$

and

**STATEMENT - 2:** If line $L_1$ is a diameter of circle C, then line $L_2$ is not a chord of circle $C$. **[2008]**

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is **NOT** a correct explanation for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

**71.** Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

**STATEMENT-1:** The tangents are mutually perpendicular.

**because**

**STATEMENT-2:** The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$. **[2007 -3 marks]**

(a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

## 10 Subjective Problems

**72.** Circles with radii 3, 4 and 5 touch each other externally. If $P$ is the point of intersection of tangents to these circles at their points of contact, find the distance of $P$ from the points of contact. **[2005 - 2 Marks]**

**73.** Find the equation of circle touching the line $2x+3y+1=0$ at $(1, -1)$ and cutting orthogonally the circle having line segment joining $(0, 3)$ and $(-2, -1)$ as diameter. **[2004 - 4 Marks]**

**74.** For the circle $x^2 + y^2 = r^2$, find the value of $r$ for which the area enclosed by the tangents drawn from the point $P(6, 8)$ to the circle and the chord of contact is maximum. **[2003 - 2 Marks]**

**75.** Let $C_1$ and $C_2$ be two circles with $C_2$ lying inside $C_1$. A circle $C$ lying inside $C_1$ touches $C_1$ internally and $C_2$ externally. Identify the locus of the centre of $C$. **[2001 - 5 Marks]**

**76.** Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin $O$ to a circle of radius 3 with centre in the first quadrant. If $A$ is one of the points of contact, find the length of $OA$. **[2001 - 5 Marks]**

**77.** $C_1$ and $C_2$ are two concentric circles, the radius of $C_2$ being twice that of $C_1$. From a point $P$ on $C_2$, tangents $PA$ and $PB$ are drawn to $C_1$. Prove that the centroid of the triangle $PAB$ lies on $C_1$. **[1998 - 8 Marks]**

**78.** Let $C$ be any circle with centre $(0, \sqrt{2})$. Prove that at the most two rational points can be there on $C$. (A rational point is a point both of whose coordinates are rational numbers.) **[1997 - 5 Marks]**

**79.** A circle passes through three points $A$, $B$ and $C$ with the line segment $AC$ as its diameter. A line passing through $A$ intersects the chord $BC$ at a point $D$ inside the circle. If angles $DAB$ and $CAB$ are $\alpha$ and $\beta$ respectively and the distance between the point $A$ and the mid point of the line segment $DC$ is $d$, prove that the area of the circle is

$$\frac{\pi d^2 \cos^2 \alpha}{\cos^2 \alpha + \cos^2 \beta + 2\cos\alpha\cos\beta\cos(\beta-\alpha)}$$

**[1996 - 5 Marks]**

**80.** Find the intervals of values of $a$ for which the line $y + x = 0$ bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ to the circle $2x^2 + 2y^2 - (1+\sqrt{2}a)x - (1-\sqrt{2}a)y = 0$. **[1996 - 5 Marks]**

**81.** Consider a family of circles passing through two fixed points $A(3,7)$ and $B(6,5)$. Show that the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinate of this point. **[1993 - 5 Marks]**

**82.** Let a circle be given by $2x(x - a) + y(2y - b) = 0$, $(a \neq 0, b \neq 0)$. Find the condition on $a$ and $b$ if two chords, each bisected by the x- axis, can be drawn to the circle from $\left(a, \frac{b}{2}\right)$. **[1992 - 6 Marks]**

**83.** Two circles, each of radius 5 units, touch each other at (1, 2). If the equation of their common tangent is $4x + 3y = 10$, find the equation of the circles. **[1991 - 4 Marks]**

**84.** A circle touches the line $y = x$ at a point $P$ such that $OP = 4\sqrt{2}$, where $O$ is the origin. The circle contains the point $(-10, 2)$ in its interior and the length of its chord on the line $x + y = 0$ is $6\sqrt{2}$. Determine the equation of the circle. **[1990 - 5 Marks]**

**85.** If $\left(m_i, \frac{1}{m_i}\right)$, $m_i > 0, i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1 m_2 m_3 m_4 = 1$ **[1989 - 2 Marks]**

**86.** The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k(x^2 + y^2)^{1/2} = 0$. Find $k$. **[1987 - 4 Marks]**

**87.** Let a given line $L_1$ intersects the $x$ and $y$ axes at $P$ and $Q$, respectively. Let another line $L_2$, perpendicular to $L_1$, cut the $x$ and $y$ axes at $R$ and $S$, respectively. Show that the locus of the point of intersection of the lines $PS$ and $QR$ is a circle passing through the origin. **[1987 - 3 Marks]**

**88.** Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle $C_1$ of diameter 6. If the centre of $C_1$ lies in the first quadrant, find the equation of the circle $C_2$ which is concentric with $C_1$ and cuts intercepts of length 8 on these lines. **[1986 - 5 Marks]**

**89.** The abscissa of the two points $A$ and $B$ are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$. Find the equation and the radius of the circle with $AB$ as diameter. **[1984 - 4 Marks]**

**90.** Through a fixed point $(h, k)$ secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the mid-points of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$. **[1983 - 5 Marks]**

**91.** Find the equations of the circle passing through $(-4, 3)$ and touching the lines $x + y = 2$ and $x - y = 2$. **[1982 - 3 Marks]**

**92.** Let $A$ be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points $B(1, 7)$ and $D(4, -2)$ on the circle meet at the point $C$. Find the area of the quadrilateral $ABCD$. **[1981 - 4 Marks]**

**93.** Find the equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at the point (5, 5). **[1978]**

## Topic-2: Parabola

### 1 MCQs with One Correct Answer

**1.** Let P be a point on the parabola $y^2 = 4ax$, where $a > 0$. The normal to the parabola at P meets the x-axis at a poinit Q. The area of the triangle PFQ, where F is the focus of the parabola, is 120. If the slope m of the normal and $a$ are both positive integers, then the pair (a, m) is **[Adv. 2023]**

(a) (2, 3) (b) (1, 3) (c) (2, 4) (d) (3, 4)

**2.** Let $(x, y)$ be any point on the parabola $y^2 = 4x$. Let $P$ be the point that divides the line segment from (0, 0) to $(x, y)$ in the ratio 1 : 3. Then the locus of $P$ is **[2011]**

(a) $x^2 = y$ (b) $y^2 = 2x$ (c) $y^2 = x$ (d) $x^2 = 2y$

**3.** The axis of a parabola is along the line $y = x$ and the distances of its vertex and focus from origin are $\sqrt{2}$ and $2\sqrt{2}$ respectively. If vertex and focus both lie in the first quadrant, then the equation of the parabola is **[2006 - 3M, –1]**

(a) $(x+y)^2 = (x-y-2)$ (b) $(x-y)^2 = (x+y-2)$
(c) $(x-y)^2 = 4(x+y-2)$ (d) $(x-y)^2 = 8(x+y-2)$

**4.** Tangent to the curve $y = x^2 + 6$ at a point (1, 7) touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point $Q$. Then the coordinates of $Q$ are **[2005S]**

(a) $(-6, -11)$ (b) $(-9, -13)$
(c) $(-10, -15)$ (d) $(-6, -7)$

**5.** The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is **[2004S]**

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

**6.** The focal chord to $y^2 = 16x$ is tangent to $(x-6)^2 + y^2 = 2$, then the possible values of the slope of this chord, are **[2003S]**

(a) $\{-1, 1\}$ (b) $\{-2, 2\}$
(c) $\{-2, -1/2\}$ (d) $\{2, -1/2\}$

**7.** The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix **[2002S]**

(a) $x = -a$ (b) $x = -a/2$ (c) $x = 0$ (d) $x = a/2$

**8.** The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is **[2001S]**

(a) $x = -1$ (b) $x = 1$ (c) $x = -3/2$ (d) $x = 3/2$

9. The equation of the common tangent touching the circle $(x-3)^2+y^2=9$ and the parabola $y^2=4x$ above the $x$-axis is [2001S]
(a) $\sqrt{3}y=3x+1$ (b) $\sqrt{3}y=-(x+3)$
(c) $\sqrt{3}y=x+3$ (d) $\sqrt{3}y=-(3x+1)$

10. If the line $x-1=0$ is the directrix of the parabola $y^2-kx+8=0$, then one of the values of $k$ is [2000S]
(a) 1/8 (b) 8 (c) 4 (d) 1/4

11. If $x+y=k$ is normal to $y^2=12x$, then $k$ is [2000S]
(a) 3 (b) 9 (c) –9 (d) –3

12. Consider a circle with its centre lying on the focus of the parabola $y^2=2px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle and parabola is [1995S]
(a) $\left(\frac{p}{2},p\right)$ or $\left(\frac{p}{2},-p\right)$ (b) $\left(\frac{p}{2},-\frac{p}{2}\right)$
(c) $\left(-\frac{p}{2},p\right)$ (d) $\left(-\frac{p}{2},-\frac{p}{2}\right)$

13. The centre of the circle passing through the point (0, 1) and touching the curve $y=x^2$ at (2, 4) is [1983 - 1 Mark]
(a) $\left(\frac{-16}{5},\frac{27}{10}\right)$ (b) $\left(\frac{-16}{7},\frac{53}{10}\right)$
(c) $\left(\frac{-16}{5},\frac{53}{10}\right)$ (d) none of these

## 2 Integer Value Answer/ Non-Negative Integer

14. A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0,-\alpha)$ to the parabola $x^2=-4ay$, where $a>0$. Let L be the line passing through $(0,-\alpha)$ and parallel to the directrix of the parabola.
Suppose that L intersects the parabola at two points A and B. Let $r$ denote the length of the latus rectum and $s$ denote the square of the length of the line segment AB. If r : s = 1 : 16, then the value of $24a$ is ________. [Adv. 2024]

15. Let the curve $C$ be the mirror image of the parabola $y^2=4x$ with respect to the line $x+y+4=0$. If $A$ and $B$ are the points of intersection of $C$ with the line $y=-5$, then the distance between $A$ and $B$ is [Adv. 2015]

16. If the normals of the parabola $y^2=4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2+(y+2)^2=r^2$, then the value of $r^2$ is [Adv. 2015]

17. Consider the parabola $y^2=8x$. Let $\Delta_1$ be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2},2\right)$ on the parabola and $\Delta_2$ be the area of the triangle formed by drawing tangents at $P$ and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is [2011]

## 4 Fill in the Blanks

18. The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2=4x$ is....... [1994 - 2 Marks]

## 6 MCQs with One or More than One Correct Answer

19. Let $A_1$, $B_1$, $C_1$ be three points in the $xy$-plane. Suppose that the lines $A_1C_1$ and $B_1C_1$ are tangents to the curve $y^2=8x$ at $A_1$ and $B_1$, respectively. If O = (0, 0) and $C_1$ = (–4, 0), then which of the following statements is (are) TRUE? [Adv. 2024]
(a) The length of the line segment $OA_1$ is $4\sqrt{3}$
(b) The length of the line segment $A_1B_1$ is 16
(c) The orthocenter of the triangle $A_1B_1C_1$ is (0, 0)
(d) The orthocenter of the triangle $A_1B_1C_1$ is (1, 0)

20 Consider the parabola $y^2=4x$. Let $S$ be the focus of the parabola. A pair of tangents drawn to the parabola from the point $P=(-2,1)$ meet the parabola at $P_1$ and $P_2$. Let $Q_1$ and $Q_2$ be points on the lines $SP_1$ and $SP_2$ respectively such that $PQ_1$ is perpendicular to $SP_1$ and $PQ_2$ is perpendicular to $SP_2$. Then, which of the following is/are TRUE? [Adv. 2022]
(a) $SQ_1=2$ (b) $Q_1Q_2=\frac{3\sqrt{10}}{5}$
(c) $PQ_1=3$ (d) $SQ_2=1$

21. Let $E$ denote the parabola $y^2=8x$. Let $P=(-2,4)$, and let $Q$ and $Q'$ be two distinct points on $E$ such that the lines $PQ$ and $PQ'$ are tangents to $E$. Let $F$ be the focus of $E$. Then which of the following statements is (are) **TRUE**? [Adv. 2021]
(a) The triangle $PFQ$ is a right-angled triangle
(b) The triangle $QPQ'$ is a right-angled triangle
(c) The distance between $P$ and $F$ is $5\sqrt{2}$
(d) $F$ lies on the line joining $Q$ and $Q'$

22. If a chord, which is not a tangent, of the parabola $y^2=16x$ has the equation $2x+y=p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k? [Adv. 2017]
(a) p = –2, h = 2, k = –4 (b) p = –1, h = 1, k = –3
(c) p = 2, h = 3, k = –4 (d) p = 5, h = 4, k = –3

23. Let $P$ be the point on the parabola $y^2=4x$ which is at the shortest distance from the center S of the circle $x^2+y^2-4x-16y+64=0$. Let Q be the point on the circle dividing the line segment SP internally. Then [Adv. 2016]
(a) $SP=2\sqrt{5}$
(b) $SQ:QP=\left(\sqrt{5}+1\right):2$
(c) the x-intercept of the normal to the parabola at P is 6
(d) the slope of the tangent to the circle at Q is $\frac{1}{2}$

24. The circle $C_1: x^2+y^2=3$, with centre at O, intersects the parabola $x^2=2y$ at the point P in the first quadrant. Let the tangent to the circle $C_1$, at P touches other two circles $C_2$ and $C_3$ at $R_2$ and $R_3$, respectively. Suppose $C_2$ and $C_3$ have equal radii $2\sqrt{3}$ and centres $Q_2$ and $Q_3$, respectively. If $Q_2$ and $Q_3$ lie on the y–axis, then [Adv. 2016]
(a) $Q_2Q_3=12$
(b) $R_2R_3=4\sqrt{6}$
(c) area of the triangle $OR_2R_3$ is $6\sqrt{2}$
(d) area of the triangle $PQ_2Q_3$ is $4\sqrt{2}$

**25.** Let $P$ and $Q$ be distinct points on the parabola $y^2 = 2x$ such that a circle with $PQ$ as diameter passes through the vertex $O$ of the parabola. If $P$ lies in the first quadrant and the area of the triangle $\Delta OPQ$ is $3\sqrt{2}$, then which of the following is (are) the coordinates of $P$? **[Adv. 2015]**

(a) $(4, 2\sqrt{2})$ (b) $(9, 3\sqrt{2})$

(c) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (d) $(1, \sqrt{2})$

**26.** Let $L$ be a normal to the parabola $y^2 = 4x$. If $L$ passes through the point (9, 6), then $L$ is given by **[2011]**

(a) $y - x + 3 = 0$ (b) $y + 3x - 33 = 0$

(c) $y + x - 15 = 0$ (d) $y - 2x + 12 = 0$

**27.** Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be **[2010]**

(a) $-\frac{1}{r}$ (b) $\frac{1}{r}$ (c) $\frac{2}{r}$ (d) $-\frac{2}{r}$

**28.** The tangent $PT$ and the normal $PN$ to the parabola $y^2 = 4ax$ at a point $P$ on it meet its axis at points $T$ and $N$, respectively. The locus of the centroid of the triangle $PTN$ is a parabola whose **[2009]**

(a) vertex is $\left(\frac{2a}{3}, 0\right)$ (b) directrix is $x = 0$

(c) latus rectum is $\frac{2a}{3}$ (d) focus is $(a, 0)$

**29.** The equations of the common tangents to the parabola $y = x^2$ and $y = -(x-2)^2$ is/are **[2006 - 5M, –1]**

(a) $y = 4(x-1)$ (b) $y = 0$

(c) $y = -4(x-1)$ (d) $y = -30x - 50$

## 7 Match the Following

**30.** Match the following : (3, 0) is the pt. from which three normals are drawn to the parabola $y^2 = 4x$ which meet the parabola in the points $P$, $Q$ and $R$. Then **[2006 - 6M]**

| Column I | Column II |
|---|---|
| (A) Area of $\Delta PQR$ | (p) 2 |
| (B) Radius of circumcircle of $\Delta PQR$ | (q) 5/2 |
| (C) Centroid of $\Delta PQR$ | (r) (5/2, 0) |
| (D) Circumcentre of $\Delta PQR$ | (s) (2/3, 0) |

## 8 Comprehension/Passage Based Questions

### PASSAGE-1

Let $a$, $r$, $s$, $t$ be nonzero real numbers. Let $P\ (at^2, 2at)$, $Q$, $R\ (ar^2, 2ar)$ and $S\ (as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that $PQ$ is the focal chord and lines $QR$ and $PK$ are parallel, where $K$ is the point $(2a, 0)$ **[Adv. 2014]**

**31.** The value of $r$ is

(a) $-\frac{1}{t}$ (b) $\frac{t^2+1}{t}$ (c) $\frac{1}{t}$ (d) $\frac{t^2-1}{t}$

**32.** If $st = 1$, then the tangent at $P$ and the normal at $S$ to the parabola meet at a point whose ordinate is

(a) $\frac{(t^2+1)^2}{2t^3}$ (b) $\frac{a(t^2+1)^2}{2t^3}$

(c) $\frac{a(t^2+1)^2}{t^3}$ (d) $\frac{a(t^2+2)^2}{t^3}$

### PASSAGE-2

Let $PQ$ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at $P$ and $Q$ meet at a point lying on the line $y = 2x + a, a > 0$. **[Adv. 2013]**

**33.** Length of chord $PQ$ is

(a) $7a$ (b) $5a$ (c) $2a$ (d) $3a$

**34.** If chord $PQ$ subtends an angle $\theta$ at the vertex of $y^2 = 4ax$, then $\tan\theta =$

(a) $\frac{2}{3}\sqrt{7}$ (b) $\frac{-2}{3}\sqrt{7}$ (c) $\frac{2}{3}\sqrt{5}$ (d) $\frac{-2}{3}\sqrt{5}$

### PASSAGE-3

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at $P$ and $Q$ in the first and the fourth quadrants, respectively. Tangents to the curcle at $P$ and $Q$ intersect the x-axis at $R$ and tangents to the parabola at $P$ and $Q$ intersect the x-axis at $S$. **[2007 -4 marks]**

**35.** The ratio of the areas of the triangles $PQS$ and $PQR$ is

(a) $1 : \sqrt{2}$ (b) $1 : 2$ (c) $1 : 4$ (d) $1 : 8$

**36.** The radius of the circumcircle of the triangle $PRS$ is

(a) 5 (b) $3\sqrt{3}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

**37.** The radius of the incircle of the triangle $PQR$ is

(a) 4 (b) 3 (c) $\frac{8}{3}$ (d) 2

## 9 Assertion and Reason/Statements Type Questions

**38.** **STATEMENT-1:** The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$. because

**STATEMENT-2:** A parabola is symmetric about its axis. **[2007 -3 marks]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

## 10 Subjective Problems

**39.** Normals are drawn from the point $P$ with slopes $m_1, m_2, m_3$ to the parabola $y^2 = 4x$. If locus of $P$ with $m_1 m_2 = \alpha$ is a part of the parabola itself then find $\alpha$. **[2003 - 4 Marks]**

**40.** Let $C_1$ and $C_2$ be respectively, the parabolas $x^2 = y - 1$ and $y^2 = x - 1$. Let $P$ be any point on $C_1$ and $Q$ be any point on $C_2$. Let $P_1$ and $Q_1$ be the reflections of $P$ and $Q$, respectively, with respect to the line $y = x$. Prove that $P_1$ lies on $C_2$, $Q_1$ lies on $C_1$ and $PQ \geq \min\{PP_1, QQ_1\}$. Hence or otherwise determine points $P_0$ and $Q_0$ on the parabolas $C_1$ and $C_2$ respectively such that $P_0Q_0 \leq PQ$ for all pairs of points $(P, Q)$ with $P$ on $C_1$ and $Q$ on $C_2$. **[2000 - 10 Marks]**

**41.** From a point $A$ common tangents are drawn to the circle $x^2 + y^2 = a^2/2$ and parabola $y^2 = 4ax$. Find the area of the quadrilateral formed by the common tangents, the chord of contact of the circle and the chord of contact of the parabola. **[1996 - 2 Marks]**

**42.** Suppose that the normals drawn at three different points on the parabola $y^2 = 4x$ pass through the point $(h, k)$. Show that $h > 2$. **[1981 - 4 Marks]**

# Topic-3: Ellipse

## 1 MCQs with One Correct Answer

1. Consider the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let $S(p,q)$ be a point in the first quadrant such that $\frac{p^2}{9}+\frac{q^2}{4}>1$. Two tangents are drawn from $S$ to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point $T$ in the fourth quadrant. Let $R$ be the vertex of the ellipse with positive $x$-coordinate and $O$ be the center of the ellipse. If the area of the triangle $\Delta ORT$ is $\frac{3}{2}$, then which of the following options is correct? **[Adv. 2024]**
   (a) $q=2, p=3\sqrt{3}$ (b) $q=2, p=4\sqrt{3}$
   (c) $q=1, p=5\sqrt{3}$ (d) $q=1, p=6\sqrt{3}$

2. The ellipse $E_1:\frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a rectangle $R$ whose sides are parallel to the coordinate axes. Another ellipse $E_2$ passing through the point (0, 4) circumscribes the rectangle $R$. The eccentricity of the ellipse $E_2$ is **[2012]**
   (a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

3. The normal at a point $P$ on the ellipse $x^2+4y^2=16$ meets the $x$-axis at $Q$. If $M$ is the mid point of the line segment $PQ$, then the locus of $M$ intersects the latus rectums of the given ellipse at the points **[2009]**
   (a) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (b) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\sqrt{\frac{19}{4}}\right)$
   (c) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (d) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

4. The line passing through the extremity $A$ of the major axis and extremity $B$ of the minor axis of the ellipse $x^2+9y^2=9$ meets its auxiliary circle at the point $M$. Then the area of the triangle with vertices at $A$, $M$ and the origin $O$ is **[2009]**
   (a) $\frac{31}{10}$ (b) $\frac{29}{10}$ (c) $\frac{21}{10}$ (d) $\frac{27}{10}$

5. The minimum area of triangle formed by the tangent to the $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & coordinate axes is **[2005S]**
   (a) $ab$ sq. units (b) $\frac{a^2+b^2}{2}$ sq. units
   (c) $\frac{(a+b)^2}{2}$ sq. units (d) $\frac{a^2+ab+b^2}{3}$ sq. units

6. If tangents are drawn to the ellipse $x^2+2y^2=2$, then the locus of the mid-point of the intercept made by the tangents between the coordinate axes is **[2004S]**
   (a) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (b) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$
   (c) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (d) $\frac{x^2}{4}+\frac{y^2}{2}=1$

7. The area of the quadrilateral formed by the tangents at the end points of latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is **[2003S]**
   (a) 27/4 sq. units (b) 9 sq. units
   (c) 27/2 sq. units (d) 27 sq. units

8. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having its centre at (0, 3) is **[1995S]**
   (a) 4 (b) 3 (c) $\sqrt{\frac{1}{2}}$ (d) $\frac{7}{2}$

9. Let $E$ be the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and $C$ be the circle $x^2+y^2=9$. Let $P$ and $Q$ be the points (1, 2) and (2, 1) respectively. Then **[1994]**
   (a) $Q$ lies inside $C$ but outside $E$
   (b) $Q$ lies outside both $C$ and $E$
   (c) $P$ lies inside both $C$ and $E$
   (d) $P$ lies inside $C$ but outside $E$

## 2 Integer Value Answer/ Non-Negative Integer

10. Suppose that the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1>0$ and $f_2<0$. Let $P_1$ and $P_2$ be two parabolas with a common vertex at (0, 0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$, respectively. Let $T_1$ be a tangent to $P_1$ which passes through $(2f_2, 0)$ and $T_2$ be a tangent to $P_2$ which passes through $(f_1, 0)$. If $m_1$ is the slope of $T_1$ and $m_2$ is the slope of $T_2$, then the value of $\left(\frac{1}{m_1^2}+m_2^2\right)$ is **[Adv. 2015]**

11. A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1=\max_{1/2\le h\le 1}\Delta(h)$ and $\Delta_2=\min_{1/2\le h\le 1}\Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1-8\Delta_2=$ **[Adv. 2013]**
   (a) $g(x)$ is continuous but not differentiable at a
   (b) $g(x)$ is differentiable on $R$
   (c) $g(x)$ is continuous but not differentiable at $b$
   (d) $g(x)$ is continuous and differentiable at either $(a)$ or $(b)$ but not both

## 3 Numeric/ New Stem Based Questions

**12.** Let $E$ be the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$. For any three distinct points $P, Q$ and $Q'$ on $E$, let $M(P, Q)$ be the mid-point of the line segment joining $P$ and $Q$, and $M(P, Q')$ be the mid-point of the line segment joining $P$ and $Q'$. Then the maximum possible value of the distance between $M(P, Q)$ and $M(P, Q')$, as $P, Q$ and $Q'$ vary on $E$, is ____. **[Adv. 2021]**

## 6 MCQs with One or More than One Correct Answer

**13.** Let $T_1$ and $T_2$ be two distinct common tangents to the ellipse $E: \frac{x^2}{6}+\frac{y^2}{3}=1$ and the parabola $P: y^2 = 12x$. Suppose that the tangent $T_1$ touches P and E at the points $A_1$ and $A_2$, respectively and the tangent $T_2$ touches P and E at the points $A_4$ and $A_3$, respectively. Then which of the following statements is(are) true? **[Adv. 2023]**

(a) The area of the quadrilateral $A_1A_2A_3A_4$ is 35 square units

(b) The area of the quadrilateral $A_1A_2A_3A_4$ is 36 square units

(c) The tangents $T_1$ and $T_2$ meet the x–axis at the point (–3, 0)

(d) The tangents $T_1$ and $T_2$ meet the x–axis at the point (–6, 0)

**14.** Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point $Q$. Consider the ellipse whose center is at the origin $O(0, 0)$ and whose semi-major axis is $OQ$. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE? **[Adv. 2018]**

(a) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(b) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(c) The area of the region bounded by the ellipse between the lines $x=\frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi-2)$

(d) The area of the region bounded by the ellipse between the lines $x=\frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi-2)$

**15.** Let $E_1$ and $E_2$ be two ellipses whose centers are at the origin. The major axes of $E_1$ and $E_2$ lie along the x-axis and the $y$-axis, respectively. Let S be the circle $x^2 + (y-1)^2 = 2$. The straight line $x + y = 3$ touches the curves $S$, $E_1$ and $E_2$ at $P$, $Q$ and R respectively. Suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$. If $e_1$ and $e_2$ are the eccentricities of $E_1$ and $E_2$, respectively, then the correct expression(s) is (are) **[Adv. 2015]**

(a) $e_1^2+e_2^2=\frac{43}{40}$ (b) $e_1e_2=\frac{\sqrt{7}}{2\sqrt{10}}$

(c) $\left|e_1^2-e_2^2\right|=\frac{5}{8}$ (d) $e_1e_2=\frac{\sqrt{3}}{4}$

**16.** In a triangle $ABC$ with fixed base $BC$, the vertex $A$ moves such that

$$\cos B+\cos C=4\sin^2\frac{A}{2}.$$

If $a$, $b$ and $c$ denote the lengths of the sides of the triangle opposite to the angles $A$, $B$ and $C$, respectively, then **[2009]**

(a) $b+c=4a$

(b) $b+c=2a$

(c) locus of point $A$ is an ellipse

(d) locus of point $A$ is a pair of straight lines

**17.** Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are **[2008]**

(a) $x^2+2\sqrt{3}\,y=3+\sqrt{3}$ (b) $x^2-2\sqrt{3}\,y=3+\sqrt{3}$

(c) $x^2+2\sqrt{3}\,y=3-\sqrt{3}$ (d) $x^2-2\sqrt{3}\,y=3-\sqrt{3}$

**18.** On the ellipse $4x^2+9y^2=1$, the points at which the tangents are parallel to the line $8x = 9y$ are **[1999 - 3 Marks]**

(a) $\left(\frac{2}{5},\frac{1}{5}\right)$ (b) $\left(-\frac{2}{5},\frac{1}{5}\right)$

(c) $\left(-\frac{2}{5},-\frac{1}{5}\right)$ (d) $\left(\frac{2}{5},-\frac{1}{5}\right)$

**19.** If $P = (x, y)$, $F_1 = (3, 0)$, $F_2 = (-3, 0)$ and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equals **[1998 - 2 Marks]**

(a) 8 (b) 6 (c) 10 (d) 12

**20.** The number of values of $c$ such that the straight line $y = 4x + c$ touches the curve $(x^2/4) + y^2 = 1$ is **[1998 - 2 Marks]**

(a) 0 (b) 1 (c) 2 (d) infinite.

## 7 Match the Following

**21.** Consider the ellipse

$$\frac{x^2}{4}+\frac{y^2}{3}=1.$$

Let $H(\alpha, 0)$, $0 < \alpha < 2$, be a point. A straight line drawn through $H$ parallel to the y-axis crosses the ellipse and its auxiliary circle at points $E$ and $F$ respectively, in the first quadrant. The tangent to the ellipse at the point $E$ intersects the positive $x$-axis at a point $G$ Suppose the straight line joining $F$ and the origin makes an angle $\phi$ with the positive $x$-axis. **[Adv. 2022]**

| List-I | List-II |
|---|---|
| (I) If $\phi=\frac{\pi}{4}$, then the area of the triangle $FGH$ is | (P) $\frac{\left(\sqrt{3}-1\right)^4}{8}$ |

(II) If $\phi = \frac{\pi}{3}$, then the area of the triangle $FGH$ is (Q) 1

(III) If $\phi = \frac{\pi}{6}$, then the area of the triangle $FGH$ is (R) $\frac{3}{4}$

(IV) If $\phi = \frac{\pi}{12}$, then the area of the triangle $FGH$ is (S) $\frac{1}{2\sqrt{3}}$

(T) $\frac{3\sqrt{3}}{2}$

The correct option is:
(a) (I) → (R); (II) → (S); (III) → (Q); (IV) → (P)
(b) (I) → (R); (II) → (T); (III) → (S); (IV) → (P)
(c) (I) → (Q); (II) → (T); (III) → (S); (IV) → (P)
(d) (I) → (Q); (II) → (S); (III) → (Q); (IV) → (P)

## 8 Comprehension/Passage Based Questions

### PASSAGE-1

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$ for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{8}=1$. Suppose a parabola having vertex at the origin and focus at $F_2$ intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant. **[Adv. 2016]**

**22.** The orthocentre of the triangle $F_1MN$ is
(a) $\left(-\frac{9}{10}, 0\right)$ (b) $\left(\frac{2}{3}, 0\right)$
(c) $\left(\frac{9}{10}, 0\right)$ (d) $\left(\frac{2}{3}, \sqrt{6}\right)$

**23.** If the tangents to the ellipse at $M$ and $N$ meet at $R$ and the normal to the parabola at $M$ meets the x-axis at Q, then the ratio of area of the triangle $MQR$ to area of the quadrilateral $MF_1NF_2$ is
(a) 3 : 4 (b) 4 : 5 (c) 5 : 8 (d) 2 : 3

### PASSAGE-2

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at points A and B. **[2010]**

**24.** The coordinates of A and B are
(a) (3, 0) and (0, 2)
(b) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$
(c) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2)
(d) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

**25.** The orthocenter of the triangle PAB is
(a) $\left(5, \frac{8}{7}\right)$ (b) $\left(\frac{7}{5}, \frac{25}{8}\right)$ (c) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (d) $\left(\frac{8}{25}, \frac{7}{5}\right)$

**26.** The equation of the locus of the point whose distances from the point P and the line AB are equal, is
(a) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$
(b) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$
(c) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$
(d) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

## 10 Subjective Problems

**27.** Find the equation of the common tangent in 1st quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$. Also find the length of the intercept of the tangent between the coordinate axes. **[2005 - 4 Marks]**

**28.** Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix. **[2002 - 5 Marks]**

**29.** Let $P$ be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $0 < b < a$. Let the line parallel to y–axis passing through $P$ meet the circle $x^2 + y^2 = a^2$ at the point $Q$ such that $P$ and $Q$ are on the same side of x–axis. For two positive real numbers $r$ and $s$, find the locus of the point $R$ on $PQ$ such that $PR : RQ = r : s$ as $P$ varies over the ellipse. **[2001 - 4 Marks]**

**30.** Let $ABC$ be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose perpendiculars from $A$, $B$, $C$ to the major axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a > b)$ meets the ellipse respectively, at $P$, $Q$, $R$. so that $P$, $Q$, $R$ lie on the same side of the major axis as $A$, $B$, $C$ respectively. Prove that the normals to the ellipse drawn at the points $P$, $Q$ and $R$ are concurrent. **[2000 - 7 Marks]**

**31.** Consider the family of circles $x^2 + y^2 = r^2$, $2 < r < 5$. If in the first quadrant, the common tangent to a circle of this family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at $A$ and $B$, then find the equation of the locus of the mid-point of $AB$. **[1999 - 10 Marks]**

**32.** A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at $P$ and $Q$. Prove that the tangents at $P$ and $Q$ of the ellipse $x^2 + 2y^2 = 6$ are at right angles. **[1997 - 5 Marks]**

**33.** Let '$d$' be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent drawn at a point $P$ on the ellipse. If $F_1$ and $F_2$ are the two foci of the ellipse, then show that $(PF_1 - PF_2)^2 = 4a^2\left(1-\frac{b^2}{d^2}\right)$. **[1995 - 5 Marks]**

# Topic-4: Hyperbola

## 1 MCQs with One Correct Answer

1. The locus of the orthocentre of the triangle formed by the lines
$$(1+p)x-py+p(1+p)=0,$$
$$(1+q)x-qy+q(1+q)=0,$$
and $y=0$, where $p \neq q$, is **[2009]**
(a) a hyperbola (b) a parabola
(c) an ellipse (d) a straight line

2. Consider a branch of the hyperbola
$$x^2-2y^2-2\sqrt{2}x-4\sqrt{2}y-6=0$$
with vertex at the point $A$. Let $B$ be one of the end points of its latus rectum. If $C$ is the focus of the hyperbola nearest to the point $A$, then the area of the triangle $ABC$ is **[2008]**
(a) $1-\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{3}{2}}-1$ (c) $1+\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{3}{2}}+1$

3. Let $a$ and $b$ be non-zero real numbers. Then, the equation $(ax^2+by^2+c)(x^2-5xy+6y^2)=0$ represents **[2008]**
(a) four straight lines, when $c=0$ and $a, b$ are of the same sign.
(b) two straight lines and a circle, when $a=b$, and $c$ is of sign opposite to that of $a$
(c) two straight lines and a hyperbola, when $a$ and $b$ are of the same sign and $c$ is of sign opposite to that of $a$
(d) a circle and an ellipse, when $a$ and $b$ are of the same sign and $c$ is of sign opposite to that of $a$

4. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2+4y^2=12$. Then its equation is **[2007 - 3 marks]**
(a) $x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$
(b) $x^2\sec^2\theta - y^2\text{cosec}^2\theta = 1$
(c) $x^2\sin^2\theta - y^2\cos^2\theta = 1$
(d) $x^2\cos^2\theta - y^2\sin^2\theta = 1$

5. If the line $2x+\sqrt{6}\,y=2$ touches the hyperbola $x^2-2y^2=4$, then the point of contact is **[2004S]**
(a) $(-2, \sqrt{6})$ (b) $(-5, 2\sqrt{6})$
(c) $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ (d) $\left(4, -\sqrt{6}\right)$

6. For hyperbola $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$ which of the following remains constant with change in 'α' **[2003S]**
(a) abscissae of vertices (b) abscissae of foci
(c) eccentricity (d) directrix

7. The equation of the common tangent to the curves $y^2=8x$ and $xy=-1$ is **[2002S]**
(a) $3y=9x+2$ (b) $y=2x+1$
(c) $2y=x+8$ (d) $y=x+2$

8. The curve described parametrically by $x=t^2+t+1$, $y=t^2-t+1$ represents **[1999 - 2 Marks]**
(a) a pair of straight lines (b) an ellipse
(c) a parabola (d) a hyperbola

9. If $x=9$ is the chord of contact of the hyperbola $x^2-y^2=9$, then the equation of the corresponding pair of tangents is **[1999 - 2 Marks]**
(a) $9x^2-8y^2+18x-9=0$ (b) $9x^2-8y^2-18x+9=0$
(c) $9x^2-8y^2-18x-9=0$ (d) $9x^2-8y^2+18x+9=0$

10. Let $P$ $(a\sec\theta, b\tan\theta)$ and $Q$ $(a\sec\phi, b\tan\phi)$, where $\theta+\phi=\pi/2$, be two points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If $(h, k)$ is the point of intersection of the normals at $P$ and $Q$, then $k$ is equal to **[1999 - 2 Marks]**
(a) $\frac{a^2+b^2}{a}$ (b) $-\left(\frac{a^2+b^2}{a}\right)$
(c) $\frac{a^2+b^2}{b}$ (d) $-\left(\frac{a^2+b^2}{b}\right)$

11. The equation $2x^2+3y^2-8x-18y+35=k$ represents **[1994]**
(a) no locus if $k>0$ (b) an ellipse if $k<0$
(c) a point if $k=0$ (d) a hyperbola if $k>0$

12. Each of the four inequalties given below defines a region in the $xy$ plane. One of these four regions does not have the following property. For any two points $(x_1, y_1)$ and $(x_2, y_2)$ in the region, the point $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is also in the region. The inequality defining this region is **[1981 - 2 Marks]**
(a) $x^2+2y^2 \le 1$ (b) Max $\{|x|, |y|\} \le 1$
(c) $x^2-y^2 \le 1$ (d) $y^2-x \le 0$

13. The equation $\frac{x^2}{1-r}-\frac{y^2}{1+r}=1,\ r>1$ represents **[1981 - 2 Marks]**
(a) an ellipse (b) a hyperbola
(c) a circle (d) none of these

## 2 Integer Value Answer/ Non-Negative Integer

14. Consider the hyperbola $\frac{x^2}{100}-\frac{y^2}{64}=1$ with foci at $S$ and $S_1$, where $S$ lies on the positive x-axis. Let P be a point on the hyperbola, in the first quadrant. Let $\angle SPS_1=\alpha$, with $\alpha < \frac{\pi}{2}$. The straight line passing through the point $S$ and having the same slope as that of the tangent at $P$ to the hyperbola, intersects the straight line $S_1P$ at $P_1$. Let $\delta$ be the distance of $P$ from the straight line $SP_1$, and $\beta=S_1P$. Then the greatest integer less than or equal to $\frac{\beta\delta}{9}\sin\frac{\alpha}{2}$ is ________. **[Adv. 2022]**

15. The line $2x+y=1$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is **[2010]**

## 4 Fill in the Blanks

16. An ellipse has eccentricity $\frac{1}{2}$ and one focus at the point $P\left(\frac{1}{2},1\right)$. Its one directrix is the common tangent, nearer to the point $P$, to the circle $x^2+y^2=1$ and the hyperbola $x^2-y^2=1$. The equation of the ellipse, in the standard form, is............ **[1996 - 2 Marks]**

## 6 MCQs with One or More than One Correct Answer

17. Let $a$ and $b$ be positive real numbers such that $a>1$ and $b<a$. Let $P$ be a point in the first quadrant that lies on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Suppose the tangent to the hyperbola at $P$ passes through the point (1, 0), and suppose the normal to the hyperbola at $P$ cuts off equal intercepts on the coordinate axes. Let $\Delta$ denote the area of the triangle formed by the tangent at $P$, the normal at $P$ and the $x$-axis. If $e$ denotes the eccentricity of the hyperbola, then which of the following statements is/are TRUE? **[Adv. 2020]**

(a) $1<e<\sqrt{2}$ (b) $\sqrt{2}<e<2$

(c) $\Delta=a^4$ (d) $\Delta=b^4$

18. If $2x-y+1=0$ is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{16}=1$, then which of the following cannot be sides of a right angled triangle? **[Adv. 2017]**

(a) a, 4, 1 (b) a, 4, 2

(c) 2a, 8, 1 (d) 2a, 4, 1

19. Consider the hyperbola $H: x^2-y^2=1$ and a circle S with center $N(x_2, 0)$. Suppose that $H$ and $S$ touch each other at a point $P(x_1, y_1)$ with $x_1>1$ and $y_1>0$. The common tangent to $H$ and $S$ at $P$ intersects the x-axis at point $M$. If $(l, m)$ is the centroid of the triangle $PMN$, then the correct expression(s) is(are) **[Adv. 2015]**

(a) $\frac{dl}{dx_1}=1-\frac{1}{3x_1^2}$ for $x_1>1$

(b) $\frac{dm}{dx_1}=\frac{x_1}{3\left(\sqrt{x_1^2-1}\right)}$ for $x_1>1$

(c) $\frac{dl}{dx_1}=1+\frac{1}{3x_1^2}$ for $x_1>1$

(d) $\frac{dm}{dy_1}=\frac{1}{3}$ for $y_1>0$

20. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line $2x-y=1$. The points of contact of the tangents on the hyperbola are **[2012]**

(a) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (b) $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

(c) $\left(3\sqrt{3},-2\sqrt{2}\right)$ (d) $\left(-3\sqrt{3},2\sqrt{2}\right)$

21. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2+4y^2=4$. If the hyperbola passes through a focus of the ellipse, then **[2011]**

(a) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$

(b) a focus of the hyperbola is (2, 0)

(c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(d) the equation of the hyperbola is $x^2-3y^2=3$

22. An ellipse intersects the hyperbola $2x^2-2y^2=1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then **[2009]**

(a) equation of ellipse is $x^2+2y^2=2$

(b) the foci of ellipse are $(\pm1, 0)$

(c) equation of ellipse is $x^2+2y^2=4$

(d) the foci of ellipse are $(\pm\sqrt{2}, 0)$

23. Let a hyperbola passes through the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then **[2006 - 5M, –1]**

(a) the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$

(b) the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$

(c) focus of hyperbola is (5, 0)

(d) vertex of hyperbola is $(5\sqrt{3}, 0)$

24. If the circle $x^2+y^2=a^2$ intersects the hyperbola $xy=c^2$ in four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, $S(x_4, y_4)$, then **[1998 - 2 Marks]**

(a) $x_1+x_2+x_3+x_4=0$ (b) $y_1+y_2+y_3+y_4=0$

(c) $x_1x_2x_3x_4=c^4$ (d) $y_1y_2y_3y_4=c^4$

## 7 Match the Following

**25.** Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a > b > 0$, be a hyperbola in the *xy*-plane whose conjugate axis *LM* subtends an angle of 60° at one of its vertices *N*. Let the area of the triangle *LMN* be $4\sqrt{3}$. **[Adv. 2018]**

| List I | List II |
|---|---|
| P. The length of the conjugate axis of *H* is | 1. 8 |
| Q. The eccentricity of *H* is | 2. $\frac{4}{\sqrt{3}}$ |
| R. The distance between the foci of *H* is | 3. $\frac{2}{\sqrt{3}}$ |
| S. The length of the latus rectum of *H* is | 4. 4 |

The correct option is:

(a) P → 4; Q → 2; R → 1; S → 3
(b) P → 4; Q → 3; R → 1; S → 2
(c) P → 4; Q → 1; R → 3; S → 2
(d) P → 3; Q → 4; R → 2; S → 1

**(Qs. 26-28) :** By appropriately matching the information given in the three columns of the following table. Column 1, 2, and 3 contain conics, equations of tangents to the conics and points of contact, respectively.

| Column 1 | Column 2 | Column 3 |
|---|---|---|
| (I) $x^2 + y^2 = a^2$ | (i) $my = m^2x + a$ | (P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ |
| (II) $x^2 + a^2y^2 = a^2$ | (ii) $y = mx + a\sqrt{m^2+1}$ | (Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$ |
| (III) $y^2 = 4ax$ | (iii) $y = mx + \sqrt{a^2m^2 - 1}$ | (R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$ |
| (IV) $x^2 - a^2y^2 = a^2$ | (iv) $y = mx + \sqrt{a^2m^2+1}$ | (S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$ |

**[Adv. 2017]**

**26.** For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (– 1, 1), then which of the following options is the only correct combination for obtaining its equation?

(a) (I) (i) (P)  (b) (I) (ii) (Q)  (c) (II) (ii) (Q)  (d) (III) (i) (P)

**27.** If a tangent to a suitable conic (column 1) is found to be $y = x + 8$ and its point of contact is (8, 16), then which of the following options is the only correct combination?

(a) (I) (ii) (Q)  (b) (II) (iv) (R)  (c) (III) (i) (P)  (d) (III) (ii) (Q)

**28.** The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only correct combination?

(a) (IV) (iii) (S)  (b) (IV) (iv) (S)  (c) (II) (iii) (R)  (d) (II) (iv) (R)

**29.** Match the conics in **Column I** with the statements/expressions in **Column II**. **[2009]**

| Column I | Column II |
|---|---|
| (A) Circle | (p) The locus of the point $(h,k)$ for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$ |
| (B) Parabola | (q) Points *z* in the complex plane satisfying $\lvert z+2\rvert - \lvert z-2\rvert = \pm 3$ |
| (C) Ellipse | (r) Points of the conic have parametric representation |

$$x=\sqrt{3}\left(\frac{1-t^2}{1+t^2}\right),\quad y=\frac{2t}{1+t^2}$$

(D) Hyperbola

(s) The eccentricity of the conic lies in the interval $1 \le x < \infty$

(t) Points z in the complex plane satisfying $\text{Re}\,(z+1)^2 = |z|^2 + 1$

30. Match the statements in Column I with the properties in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS. **[2007 -6 marks]**

| Column I | Column II |
|---|---|
| (A) Two intersecting circles | (p) have a common tangent |
| (B) Two mutually external circles | (q) have a common normal |
| (C) Two circles, one strictly inside the other | (r) do not have a common tangent |
| (D) Two branches of a hyperbola | (s) do not have a common normal |

### 8 Comprehension Passage Based Questions

PASSAGE

The circle $x^2+y^2-8x=0$ and hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ intersect at the points A and B. **[2010]**

31. Equation of the circle with AB as its diameter is
 (a) $x^2+y^2-12x+24=0$ (b) $x^2+y^2+12x+24=0$
 (c) $x^2+y^2+24x-12=0$ (d) $x^2+y^2-24x-12=0$

32. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is
 (a) $2x-\sqrt{5}y-20=0$ (b) $2x-\sqrt{5}y+4=0$
 (c) $3x-4y+8=0$ (d) $4x-3y+4=0$

### 10 Subjective Problems

33. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ to the circle $x^2+y^2=9$. Find the locus of mid-point of the chord of contact. **[2005 - 4 Marks]**

34. The angle between a pair of tangents drawn from a point $P$ to the parabola $y^2=4ax$ is 45°. Show that the locus of the point $P$ is a hyperbola. **[1998 - 8 Marks]**

# Answer Key

## Topic-1 : Circles

**1.** (b) **2.** (a) **3.** (d) **4.** (d) **5.** (b) **6.** (d) **7.** (c) **8.** (a) **9.** (c) **10.** (a)
**11.** (b) **12.** (a) **13.** (c) **14.** (d) **15.** (d) **16.** (c) **17.** (d) **18.** (d) **19.** (c) **20.** (a)
**21.** (a) **22.** (c) **23.** (b) **24.** (b) **25.** (d) **26.** (512) **27.** (2) **28.** (c, d) **29.** (2) **30.** (10)
**31.** (2) **32.** (2)

**33.** (8) **34.** (0.84) **35.** $\left(\frac{1}{2},\frac{1}{4}\right)$ **36.** (7) **37.** $x^2+y^2-x-y=0$ **38.** $16x^2+16y^2-48x+16y+31=0$ **39.** (2)

**40.** $2\sqrt{3}$ sq. units **41.** $\left(-\frac{9}{5},\frac{12}{5}\right)$ or $\left(\frac{9}{5},-\frac{12}{5}\right)$ **42.** $\frac{192}{25}$ **43.** $x^2+y^2+8x-6y+9=0$

**44.** $10x-3y-18=0$ **45.** $x^2+y^2-x=0$ **46.** 8 sq. units **47.** $\frac{3}{4}$ **48.** (4, 2), (–2, –6) **49.** (1)
**50.** (True) **51.** (True) **52.** (b, d) **53.** (a, c) **54.** (b, c) **55.** (a, c) **56.** (b) **57.** (a, c) **58.** (c) **59.** (d)
**60.** (a) **61.** (d) **62.** (b) **63.** (a) **64.** (d) **65.** (a) **66.** (d) **67.** (a) **68.** (c) **69.** (c)
**70.** (c) **71.** (a)

## Topic-2 : Parabola

**1.** (a) **2.** (c) **3.** (d) **4.** (d) **5.** (c) **6.** (a) **7.** (c) **8.** (d) **9.** (c) **10.** (c)
**11.** (b) **12.** (a) **13.** (c) **14** (12) **15.** (4) **16.** (2) **17.** (2) **18.** (–1, 1) **19.** (d, c)
**20.** (a, b, c, d) **21.** (a,b,d) **22.** (c) **23.** (a, c, d) **24.** (a, b, c) **25.** (a, d) **26.** (a, b, d) **27.** (c, d) **28.** (a, d)
**29.** (a, b) **30.** (A) → (p); (B) → (q); (C) → (s); (D) → (r) **31.** (d) **32.** (b) **33.** (b) **34.** (d) **35.** (c)
**36.** (b) **37.** (d) **38.** (a)

## Topic-3 : Ellipse

**1.** (a) **2.** (c) **3.** (c) **4.** (d) **5.** (a) **6.** (a) **7.** (d) **8.** (a) **9.** (d) **10.** (4)
**11.** (9) **12.** (4) **13.** (a, c) **14.** (a, c) **15.** (a, b) **16.** (b, c) **17.** (b, c) **18.** (b, d) **19.** (c) **20.** (c)
**21.** (c) **22.** (a) **23.** (c) **24.** (d) **25.** (c) **26.** (a)

## Topic-4 : Hyperbola

**1.** (d) **2.** (b) **3.** (b) **4.** (a) **5.** (d) **6.** (b) **7.** (d) **8.** (c) **9.** (b) **10.** (d)

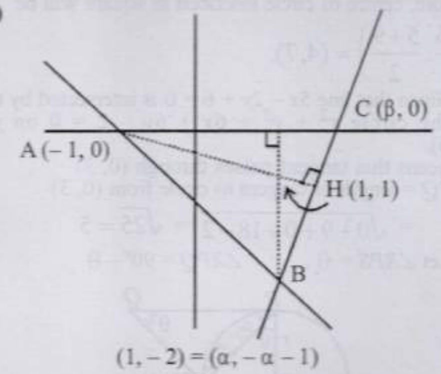
**11.** (c) **12.** (c) **13.** (d) **14.** (7) **15.** (2) **16.** $\frac{\left(x-\frac{1}{3}\right)^2}{\left(\frac{1}{3}\right)^2}+\frac{(y-1)^2}{\left(\frac{1}{2}\sqrt{3}\right)^2}=1$ **17.** (a, d)

**18.** (a, b, c) **19.** (a, b, d) **20.** (a, b) **21.** (b, d) **22.** (a, b) **23.** (a, c) **24.** (a, b, c, d) **25.** (b) **26.** (b)
**27.** (c) **28.** (d) **29.** A → (p); B → (s, t); C → (r); D → (q, s) **30.** A → (p, q); B → (p,q); C → (q, r); D → (q, r)
**31.** (a) **32.** (b)

# Hints & Solutions

## Topic-1: Circles

**1. (b)**




$(1, -2) = (\alpha, -\alpha - 1)$

$\Rightarrow \alpha = 1$

It is clear from question that one of the vertex of triangle is intersection of $x$-axis and $x + y + 1 = 0 \Rightarrow A(-1, 0)$

Let vertex B be $(\alpha, -\alpha - 1)$

Line AC $\perp$ BH so, $m_{AC} \cdot m_{BH} = -1$

$\Rightarrow \; O = -\dfrac{(1-\alpha)}{\alpha+2} \Rightarrow \alpha = 1 \Rightarrow B(1, -2)$

Let vertex C be $(\beta, 0)$

Line AH $\perp$ BC

$\therefore \; m_{AH} \cdot m_{BC} = -1$

$\Rightarrow \; \dfrac{1}{2} \cdot \dfrac{2}{\beta - 1} = -1 \Rightarrow \beta = 0$

Centroid of $\Delta ABC$ is $\left(0, -\dfrac{2}{3}\right)$

We know that G (centroid) divides line joining circumcentre (O) and orthocentre (H) in the ratio 1 : 2.

$\Rightarrow$ $(h, k)$ — $\left(0, -\dfrac{2}{3}\right)$ — $(1, 1)$ (O — 1 — G — 2 — H)

$2h + 1 = 0 \Rightarrow \dfrac{2k+1}{3} = -\dfrac{2}{3}$

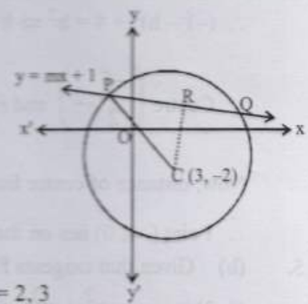
$\Rightarrow h = -\dfrac{1}{2} \Rightarrow k = -\dfrac{3}{2} \Rightarrow$ Circumcentre is $\left(-\dfrac{1}{2}, -\dfrac{3}{2}\right)$.

Equation of circum circle is (passing through C (0, 0)) is $x^2 + y^2 + x + 3y = 0$

**2. (a)** Given : Circle $(x-3)^2 + (y+2)^2 = 25$, with centre $C(3, -2)$ and radius 5 is intersected by a line $y = mx + 1$ at P & Q such that co-ordinates of mid point R of PQ is $-\dfrac{3}{5}$.

Since x-coordinates of point R is $-\dfrac{3}{5}$ and point R lies on the line $y = mx + 1$, therefore y-coordinate of R will be $\dfrac{3m}{5} + 1$.

$\therefore \; R\left(-\dfrac{3}{5}, \dfrac{-3m}{5} + 1\right)$

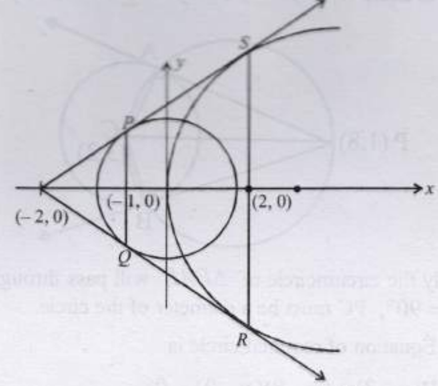



Since R is the mid point of PQ, therefore $CR \perp PQ$

$\Rightarrow \; \dfrac{\dfrac{-3m}{5} + 1 + 2}{-\dfrac{3}{5} - 3} \times m = -1$

$\Rightarrow \; m^2 - 5m + 6 = 0 \Rightarrow m = 2, 3$

**3. (d)**




Let the tangent to $y^2 = 8x$ be $y = mx + \dfrac{2}{m}$

If it is common tangent to parabola and circle $x^2 + y^2 = 2$, then distance of the tangent from the centre of the circle is equal to radius of the circle

$\therefore \; \left|\dfrac{\frac{2}{m}}{\sqrt{m^2+1}}\right| = \sqrt{2} \Rightarrow \dfrac{4}{m^2(1+m^2)} = 2$

$\Rightarrow m^4 + m^2 - 2 = 0 \Rightarrow (m^2 + 2)(m^2 - 1) = 0 \Rightarrow m = 1$ or $-1$

$\therefore$ Required tangents are $y = x + 2$ and $y = -x - 2$

Their common point is $(-2, 0)$

$\therefore$ Tangents are drawn from $(-2, 0)$

$\therefore$ Chord of contact $PQ$ to circle is

$x.(-2) + y.0 = 2 \Rightarrow x = -1$

and Chord of contact $RS$ to parabola is

$y.0 = 4(x - 2) \Rightarrow x = 2$

Hence coordinates of $P$ and $Q$ are $(-1, 1)$ and $(-1, -1)$ respectively.

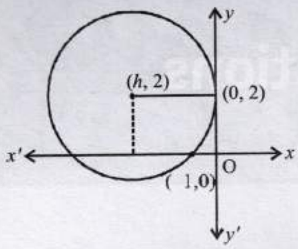
Also coordinates of $R$ and $S$ are $(2, -4)$ and $(2, 4)$ respectively.

$\therefore$ Area of trapezium $PQRS = \dfrac{1}{2}(2 + 8) \times 3 = 15$

**4. (d)** Let centre of the circle be (h, 2) then radius $= h$

$\therefore$ Equation of circle becomes $(x-h)^2 + (y-2)^2 = h^2$

Since the circle passes through $(-1, 0)$



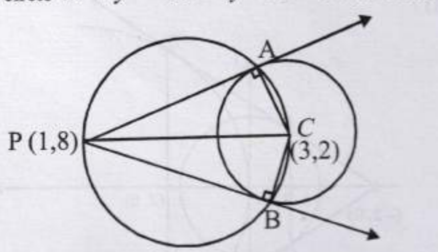

$\therefore\ (-1-h)^2+4=h^2 \Rightarrow h=\dfrac{-5}{2}$

$\therefore$ Centre $\left(\dfrac{-5}{2},2\right)$ and $r=\dfrac{5}{2}$

Now, distance of centre from (–4, 0) is $\dfrac{5}{2}$

$\therefore$ Point (–4, 0) lies on the circle.

**5.** **(b)** Given that tangents PA and PB are drawn from the point P (1, 3) to circle $x^2+y^2-6x-4y-11=0$ with centre C (3, 2).



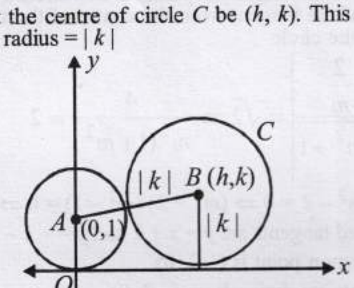

Clearly the circumcircle of $\Delta PAB$ will pass through $C$ and as $\angle A=90°$, PC must be a diameter of the circle.

$\therefore$ Equation of required circle is

$(x-1)(x-3)+(y-8)(y-2)=0$

$\Rightarrow\ x^2+y^2-4x-10y+19=0$

**6** **(d)** Let the centre of circle $C$ be $(h, k)$. This circle touches x-axis. $\therefore$ radius = $|k|$




Also it touches the given circle $x^2+(y-1)^2=1$, with centre (0, 1) and radius 1, externally

$\therefore$ Distance between centres = sum of radii

$\Rightarrow\ \sqrt{(h-0)^2+(k-1)^2}=1+|k|$

$\Rightarrow\ h^2+k^2-2k+1=1+2|k|+k^2$

$\Rightarrow\ h^2=2k+2|k|$

$\therefore$ Locus of $(h, k)$ is, $x^2=2y+2|y|$

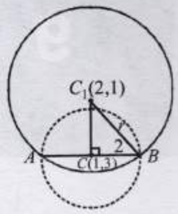
Now if $y>0$, it becomes $x^2=4y$

and if $y\le 0$, it becomes $x=0$

$\therefore$ Combining the two, the required locus is $\{(x,y): x^2=4y\}\cup\{(0,y): y\le 0\}$

**7.** **(c)** The given circle is $x^2+y^2-2x-6y+6=0$ with centre $C(1, 3)$ and radius

$=\sqrt{1+9-6}=2$. Let $AB$ be one of its diameter which is the chord of other circle with centre at $C_1$ (2, 1).




Then in $\Delta C_1CB$,

$$C_1B^2=CC_1^2+CB^2$$
$$=(2-1)^2+(1-3)^2+(2)^2$$
$$=1+4+4=9 \Rightarrow C_1B=3.$$

**8.** **(a)** $x^2-8x+12=0 \Rightarrow (x-6)(x-2)=0$

$y^2-14y+45=0 \quad \Rightarrow (y-5)(y-9)=0$

Hence, sides of square are

$x=2, x=6, y=5$ and $y=9$

Therefore, centre of circle inscribed in square will be

$\left(\dfrac{2+6}{2},\dfrac{5+9}{2}\right)=(4,7).$

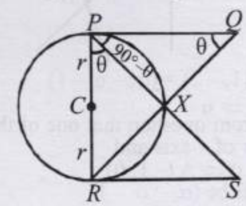
**9.** **(c)** Given that line $5x-2y+6=0$ is intersected by tangent at $P$ to the circle $x^2+y^2+6x+6y-2=0$ on $y$-axis at $Q$ (0, 3).

This means that tangent passes through (0, 3)

$\therefore$ $PQ$ = length of tangent to circle from (0, 3)

$=\sqrt{0+9+0+18-2}=\sqrt{25}=5$

**10.** **(a)** Let $\angle RPS=\theta$, $\therefore$ $\angle XPQ=90°-\theta$



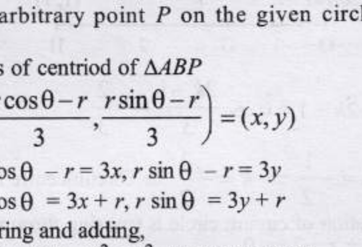

and $\angle PQX=\theta$ $\quad$ ($\because \angle PXQ=90°$)

$\therefore$ $\Delta PRS\sim\Delta QPR$ $\quad$ (By AA similarity)

$\therefore$ $\dfrac{PR}{QP}=\dfrac{RS}{PR} \Rightarrow PR^2=PQ\,.\,RS$

$\Rightarrow\ PR=\sqrt{PQ.RS} \Rightarrow 2r=\sqrt{PQ.RS}$

**11.** **(b)** Given a circle $x^2+y^2=r^2$ with centre at (0, 0) and radius $r$.




Let $A$ and $B$ be $(-r, 0)$ and $(0, -r)$, so that $\angle AOB=90°$ and an arbitrary point $P$ on the given circle be $(r\cos\theta, r\sin\theta)$.

For locus of centriod of $\Delta ABP$

$\left(\dfrac{r\cos\theta-r}{3},\dfrac{r\sin\theta-r}{3}\right)=(x,y)$

$\Rightarrow\ r\cos\theta-r=3x,\ r\sin\theta-r=3y$

$\Rightarrow\ r\cos\theta=3x+r,\ r\sin\theta=3y+r$

On squaring and adding,

$(3x+r)^2+(3y+r)^2=r^2$, which is a circle.

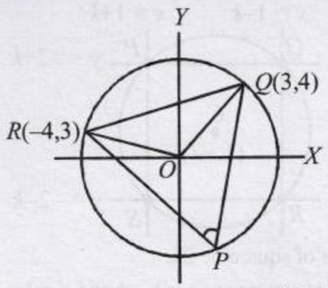
**12.** **(a)** Two circles intersects each other orthogonally iff $2g_1g_2+2f_1f_2=c_1+c_2$

Since the two given circles interseces each other orthogonally

$\therefore\ 2(1)(0)+2(k)(k)=6+k$

$\Rightarrow\ 2k^2-k-6=0 \Rightarrow k=-3/2, 2$

**13.** **(c)** $O$ is the point at centre and $P$ is the point at circumference. Therefore, angle $QOR$ is double the angle $QPR$.




So, it sufficient to find the angle $QOR$. Now slope of $OQ = 4/3 = m_1$ (let)

Slope of $OR = -3/4 = m_2$ (let); Now, $m_1 m_2 = -1$

Therefore, $\angle QOR = 90°$ $\therefore$ $\angle QPR = 45°$.

**14.** **(d)** Given : Equation of the circle is

$$x^2 + y^2 - px - qy = 0, pq \neq 0$$

Let the chord drawn from $(p, q)$ is bisected by $x$-axis at point $(x_1, 0)$, then equation of chord is

$$x x_1 - \frac{p}{2}(x + x_1) - \frac{q}{2}(y + 0) = x_1^2 - px_1 \quad (\text{using } T = S_1)$$

As it passes through $(p, q)$,

$$\therefore \quad p x_1 - \frac{p}{2}(p + x_1) - \frac{q^2}{2} = x_1^2 - px_1$$

$$\Rightarrow \quad x_1^2 - \frac{3}{2}px_1 + \frac{p^2}{2} + \frac{q^2}{2} = 0$$

$$\Rightarrow \quad 2x_1^2 - 3px_1 + p^2 + q^2 = 0$$

As through $(p,q)$ two distinct chords can be drawn.

$\therefore$ Roots of above equation be real and distinct.

$\therefore$ $D > 0$.

$\Rightarrow$ $9p^2 - 4 \times 2(p^2 + q^2) > 0$

$\Rightarrow$ $p^2 > 8q^2$

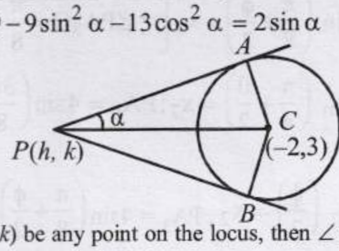
**15.** **(d)** Given : Circle

$x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$

its centre is $C(-2, 3)$ and its radius

$= \sqrt{2^2 + (-3)^2 - 9\sin^2\alpha - 13\cos^2\alpha}$

$= \sqrt{4 + 9 - 9\sin^2\alpha - 13\cos^2\alpha} = 2\sin\alpha$




Let $P(h, k)$ be any point on the locus, then $\angle APC = \alpha$

Also $\angle PAC = \frac{\pi}{2}$

Now, in right triangle $APC$,

$$\sin\alpha = \frac{AC}{PC} = \frac{2\sin\alpha}{\sqrt{(h+2)^2 + (k-3)^2}}$$

$\Rightarrow \sqrt{(h+2)^2 + (k-3)^2} = 2$

$\Rightarrow (h+2)^2 + (k-3)^2 = 4 \Rightarrow h^2 + k^2 + 4h - 6k + 9 = 0$

Thus required equation of the locus is

$$x^2 + y^2 + 4x - 6y + 9 = 0$$

**16.** **(c)** Centres and radii of two circles are $C_1(5, 0)$; $3 (= r_1)$ and $C_2(0, 0)$; $r (= r_2)$

As the circles intersect each other in two distinct points,

$\therefore$ $|r_1 - r_2| < C_1C_2 < r_1 + r_2$

$\Rightarrow$ $|r - 3| < 5 < r + 3 \Rightarrow 2 < r < 8$

**17.** **(d)** The given circle is $x^2 + y^2 - 6x - 6y + 14 = 0$, centre $(3, 3)$, radius $= 2$

Let $(h, k)$ be the centre of touching circle. Then radius of touching circle $= h$ [as it touches $y$-axis also]

$\therefore$ Distance between centres of two circles
$=$ sum of the radii of two circles

$\Rightarrow$ $\sqrt{(h-3)^2 + (k-3)^2} = 2 + h$

$\Rightarrow$ $(h-3)^2 + (k-3)^2 = (2+h)^2$

$\Rightarrow$ $k^2 - 10h - 6k + 14 = 0$

$\therefore$ locus of $(h, k)$ is $y^2 - 10x - 6y + 14 = 0$

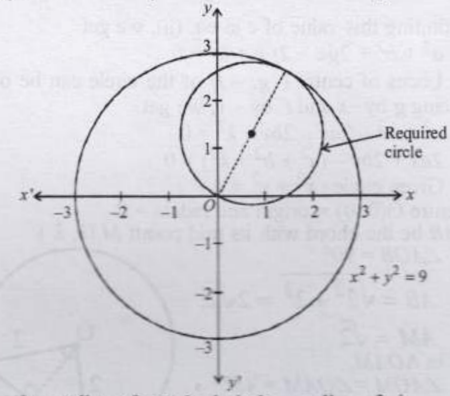
**18.** **(d)** Let the equation of the circle whose equation is to be find out be

$x^2 + y^2 + 2gx + 2fy + c = 0$.

As this circle passes through $(0, 0)$ and $(1, 0)$,

$\therefore$ $c = 0,\ g = -\frac{1}{2}$

Since the required circle touches the given circle $x^2 + y^2 = 9$ internally like as shown in the figure.




$\therefore$ 2 × radius of required circle = radius of given circle

$\Rightarrow$ $2\sqrt{g^2 + f^2} = 3 \Rightarrow g^2 + f^2 = \frac{9}{4}$

$\Rightarrow$ $\frac{1}{4} + f^2 = \frac{9}{4} \Rightarrow f = \pm\sqrt{2}$

$\therefore$ The centre is $\left(\frac{1}{2}, \sqrt{2}\right)$ or $\left(\frac{1}{2}, -\sqrt{2}\right)$.

**19.** **(c)** As $2x - 3y - 5 = 0$ and $3x - 4y - 7 = 0$ are diameters of circle, therefore centre of circle is the point of intersection of the two lines i.e., the solution of the two given equation of the lines

$$\frac{x}{21-20} = \frac{y}{-15+14} = \frac{1}{-8+9} \Rightarrow x = 1, y = -1$$

$\therefore$ centre of the required circle $= (1, -1)$

Also area of circle, $\pi r^2 = 154$

$\Rightarrow$ $r^2 = \frac{154}{22} \times 7 = 49 \Rightarrow r = 7$

$\therefore$ Equation of required circle is

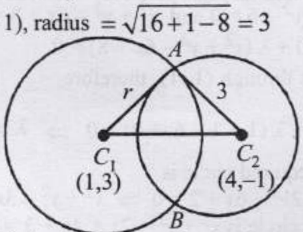
$(x-1)^2 + (y+1)^2 = 7^2 \Rightarrow x^2 + y^2 - 2x + 2y = 47$

**20.** **(a)** Given : Two circles $(x-1)^2 + (y-3)^2 = r^2$

Centre $(1, 3)$, radius $= r$

and $x^2 + y^2 - 8x + 2y + 8 = 0$

Centre $(4, -1)$, radius $= \sqrt{16 + 1 - 8} = 3$




As the two circles intersect each other in two distinct points we should have

$C_1C_2 < r_1 + r_2$ and $\quad C_1C_2 > |r_1 - r_2|$

$\Rightarrow\ C_1C_2 < r+3$ and $C_1C_2 > |r_1 - r_2|$

$\Rightarrow\ \sqrt{9+16} < r+3$ and $5 > |r-3|$

$\Rightarrow\ 5 < r+3$ and $|r-3| < 5$

$\Rightarrow\ r > 2$ and $-5 < r-3 < 5$

$\Rightarrow\ r > 2$ ... (i) and $-2 < r < 8$ ... (ii)

On combining (i) and (ii), we get

$2 < r < 8$

**21.** **(a)** Two circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ cuts each other orthogonally iff $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

Let the required circle be,

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

As it passes through (a, b),

$a^2 + b^2 + 2ag + 2bf + c = 0$ ... (ii)

Given circle : $x^2 + y^2 = k^2$ ... (iii)

Since circles (i) and (ii) cuts each other orthogonally, therefore $c = k^2$

Substituting this value of c in eq. (ii), we get

$a^2 + b^2 + 2ga + 2fb + k^2 = 0$

$\therefore$ Locus of centre $(-g, -f)$ of the circle can be obtained by replacing $g$ by $-x$ and $f$ by $-y$, we get

$a^2 + b^2 - 2ax - 2by + k^2 = 0$

$\Rightarrow\ 2ax + 2by - (a^2 + b^2 + k^2) = 0$

**22.** **(c)** Given circle : $x^2 + y^2 = 4$

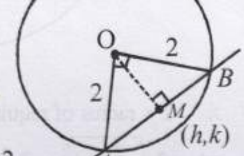
Its centre O(0, 0) = origin and radius = 2

Let $AB$ be the chord with its mid pointt $M(h, k)$.

$\because\ \angle AOB = 90°$

$\therefore\ AB = \sqrt{2^2 + 2^2} = 2\sqrt{2}.$

$\therefore\ AM = \sqrt{2}$




Now in $\Delta OAM$,

$\angle AOM = \angle OAM = 45°$

$AM = OM = MB$

$\therefore\ OM = \sqrt{2} \Rightarrow h^2 + k^2 = 2$

$\therefore$ locus of $(h, k)$ is $x^2 + y^2 = 2$

**23.** **(b)** Circle through point of intersection of two circles $S_1 = 0$ and $S_2 = 0$ is $S_1 + \lambda S_2 = 0$

$\therefore$ Req. circle is

$$(x^2 + y^2 + 13x - 3y) + \lambda\left(x^2 + y^2 + 2x - \frac{7}{2}y - \frac{25}{2}\right) = 0$$

$$\Rightarrow (1+\lambda)x^2 + (1+\lambda)y^2 + (13 + 2\lambda)x + \left(-3 - \frac{7}{2}\lambda\right)y - \frac{25\lambda}{2} = 0$$

As this circle passes through (1, 1),

$$\therefore\ 1 + \lambda + 1 + \lambda + 13 + 2\lambda - 3 - \frac{7\lambda}{2} - \frac{25\lambda}{2} = 0$$

$\Rightarrow\ -12\lambda + 12 = 0\ \Rightarrow\ \lambda = 1$

$\therefore$ Req. circle is

$$2x^2 + 2y^2 + 15x - \frac{13y}{2} - \frac{25}{2} = 0$$

$\Rightarrow\ 4x^2 + 4y^2 + 30x - 13y - 25 = 0$

**24.** **(b)** The circle through points of intersection of the two given circles $x^2 + y^2 - 6 = 0$ and $x^2 + y^2 - 6x + 8 = 0$ is

$(x^2 + y^2 - 6) + \lambda(x^2 + y^2 - 6x + 8) = 0$

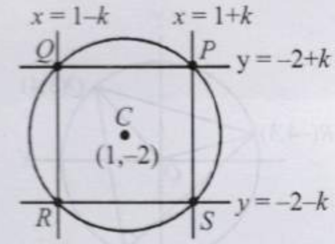
As it passes through (1, 1), therefore

$$(1 + 1 - 6) + \lambda(1 + 1 - 6 + 8) = 0 \Rightarrow \lambda = \frac{4}{4} = 1$$

$\therefore$ The required circle is

$2x^2 + 2y^2 - 6x + 2 = 0 \Rightarrow x^2 + y^2 - 3x + 1 = 0$

**25.** **(d)** Given circle is $x^2 + y^2 - 2x + 4y + 3 = 0$. Its centre $(1, -2)$. Lines through centre $(1, -2)$ and parallel to axes are $x = 1$ and $y = -2$.




Let the side of square be $2k$.

Then sides of square are $x = 1 - k$ and $x = 1 + k$ and $y = -2 - k$ and $y = -2 + k$

$\therefore$ Co-ordinates of $P, Q, R, S$ are $(1 + k, -2 + k)$, $(1 - k, -2 + k)$, $(1 - k, -2 - k)$, $(1 + k, -2 - k)$ respectively.

Also $P(1 + k, -2 + k)$ lies on the given circle

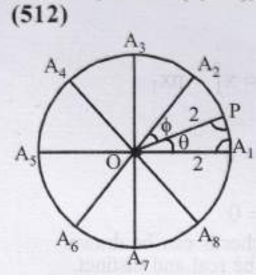
$\therefore\ (1 + k)^2 + (-2 + k)^2 - 2(1 + k) + 4(-2 + k) + 3 = 0$

$\Rightarrow\ 2k^2 = 2\ \Rightarrow\ k = 1$ or $-1$

If $k = 1$, then $P(2, -1)$, $Q(0, -1)$, $R(0, -3)$, $S(2, -3)$

If $k = -1$, then $P(0, -3)$, $Q(2, -3)$, $R(2, -1)$, $S(0, -1)$

**26.** **(512)**




$\theta + \phi = \frac{\pi}{4}$

In $\Delta A_1OP$, $\angle OA_1P = \angle OPA_1 = \frac{\pi}{2} - \frac{\theta}{2}$

$$\frac{PA_1}{2} = \frac{\sin\theta}{\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)} = 2\sin\frac{\theta}{2} \Rightarrow PA_1 = 4\sin\left(\frac{\theta}{2}\right) = x_1(\text{say})$$

$$PA_8 = 4\sin\left(\frac{\pi}{8} + \frac{\phi}{2}\right) = x_8\left[\because \angle PA_8O = \frac{\pi}{8} + \frac{\theta}{2}\right]$$

$$PA_7 = 4\sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = x_7; PA_6 = 4\sin\left(\frac{3\pi}{8} + \frac{\theta}{2}\right) = X_6$$

Similarly

$$PA_2 = 4\sin\left(\frac{\phi}{2}\right) = x_2;\ PA_3 = 4\sin\left(\frac{\pi}{8} + \frac{\phi}{2}\right) = x_3$$

$$PA_4 = 4\sin\left(\frac{\pi}{4} + \frac{\phi}{2}\right) = x_4;\ PA_5 = 4\sin\left(\frac{3\pi}{8} + \frac{\phi}{2}\right) = x_5$$

Now, $PA_1 . PA_2 . PA_3 \ldots PA_8 =$

$$P = 4^8\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\pi}{8} + \frac{\phi}{2}\right).\sin\left(\frac{\pi}{8} + \frac{\theta}{2}\right)\sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

$$\sin\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\sin\left(\frac{\pi}{8} + \frac{\phi}{2}\right).\sin\left(\frac{3\pi}{8} + \frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right)$$

$$= 4^8\left\{\sin\frac{\theta}{2}.\cos\frac{\theta}{2}.\sin\left(\frac{\pi}{8} + \frac{\theta}{2}\right)\cos\left(\frac{\pi}{8} + \frac{\theta}{2}\right).\sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right.$$

$$\left.\cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\sin\left(\frac{3\pi}{8} + \frac{\theta}{2}\right)\cos\left(\frac{3\pi}{8} + \frac{\theta}{2}\right)\right\}$$

$$= 4^8\left\{\frac{\sin\theta\sin\left(\frac{\pi}{4}+\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)\sin\left(\frac{3\pi}{4}+\theta\right)}{2^4}\right\}$$

$$= 4^6\left\{\sin\theta\cos\theta\sin\left(\frac{\pi}{4}+\theta\right)\cos\left(\frac{\pi}{4}+\theta\right)\right\}$$

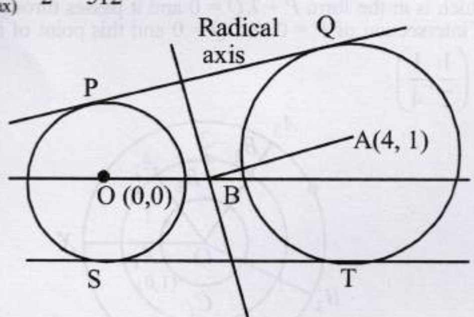
$$= 4^6\left\{\frac{\sin 2\theta\sin\left(\frac{\pi}{2}+2\theta\right)}{4}\right\}$$

$$= 4^5\,\frac{\sin(4\theta)}{2} = 2^9\sin 4\theta$$

P is maximum when $\sin 4\theta = 1 \Rightarrow \theta = \frac{\pi}{8}$

$P_{(max)} = 2^9 = 512$

**27. (2)**




$C_1 : x^2 + y^2 = 1$ ... (i)

Let $C_2 : (x-4)^2 + (y-1)^2 = r^2$ ... (ii)

radical axis $8x + 2y - 17 = 1 - r^2$ [from (i) and (ii)]

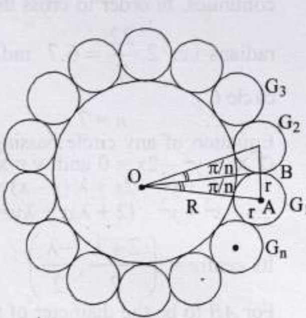
$8x + 2y = 18 - r^2$

$B\left(\frac{18-r^2}{8}, 0\right)$ and A (4, 1)

Given, $AB = \sqrt{5} \Rightarrow \sqrt{\left(\frac{18-r^2}{8}-4\right)^2+1} = \sqrt{5} \Rightarrow r^2 = 2$

**28. (c, d)** Refer to diagram,

In $\Delta AOB$




$\sin\left(\frac{\pi}{n}\right) = \frac{r}{R+r}$

$\Rightarrow \operatorname{cosec}\left(\frac{\pi}{n}\right) = \frac{R}{r} + 1$

$\Rightarrow R = r\left[\operatorname{cosec}\left(\frac{\pi}{n}\right) - 1\right]$

If n = 4 then $R = r(\sqrt{2}-1)$

If n = 5 then $R = r\left(\operatorname{cosec}\frac{\pi}{5} - 1\right)$

$\because \operatorname{cosec}\frac{\pi}{5} < \operatorname{cosec}\frac{\pi}{6}$

$\left(\operatorname{cosec}\frac{\pi}{5} - 1\right) < 2 - 1 = 1 \therefore R < r$

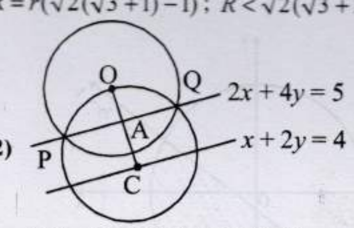
If n = 8 then $R = r\left(\operatorname{cosec}\frac{\pi}{8} - 1\right) \therefore \operatorname{cosec}\frac{\pi}{8} > \operatorname{cosec}\frac{\pi}{4}$

$\left(\operatorname{cosec}\frac{\pi}{8} - 1\right) > \sqrt{2} - 1 \Rightarrow R > r\left(\sqrt{2}-1\right)$

If n = 12, then $R = r\left(\operatorname{cosec}\frac{\pi}{12} - 1\right)$

$R = r(\sqrt{2}(\sqrt{3}+1)-1)$; $R < \sqrt{2}(\sqrt{3}+1)r$

**29. (2)**




$\because$ Centre of circle is O (0, 0).

OA = perpendicular distance from point O to line

$2x + 4y = 5 = \left|\frac{0+0-5}{\sqrt{4+16}}\right| = \frac{\sqrt{5}}{2}$

OC = perpendicular distance from point O to line $x + 2y = 4$

$= \left|\frac{0+0-4}{\sqrt{1+4}}\right| = \frac{4}{\sqrt{5}}$

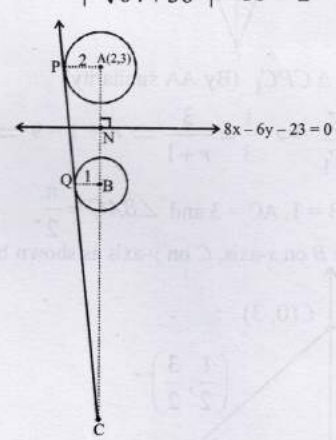
$\therefore CA = OC - OA = \frac{3}{2\sqrt{5}} \because CQ = OC = \frac{4}{\sqrt{5}}$ (radius)

Now $AQ^2 = CQ^2 - CA^2$ ($\because AC \perp PQ$) $= \frac{16}{5} - \frac{9}{20} = \frac{11}{4}$

$\therefore OQ = r = \sqrt{OA^2 + AQ^2}$

$\Rightarrow r = \sqrt{\frac{5}{4}+\frac{11}{4}} \Rightarrow r = \sqrt{4} = 2$

**30. (10)** $AN = \left|\frac{16-18-23}{\sqrt{64+36}}\right| = \frac{25}{10} = \frac{5}{2} = BN$




$\because$ $\Delta CPA \sim \Delta CQB$ (By AA similarity)

$\therefore \quad \frac{CA}{CB} = \frac{PA}{QB} \Rightarrow \frac{CA}{CA-5} = \frac{2}{1}$

$\Rightarrow \quad CA = 2CA - 10 \quad \Rightarrow CA = 10$

**31. (2)** Centre of the circle is (–1, –2)

Geometrically, circle will have exactly 3 common points with axes in the cases

(i) Passing through origin $\Rightarrow$ p = 0

(ii) Touching x-axis and intersecting y-axis at two points i.e. $f^2 > C$ and $g^2 = C$.

i.e. $4 > -p$ and $1 = -p \Rightarrow p > -4$ and $p = -1 \therefore p = -1$

(iii) Touching y-axis and intersecting x-axis at two points

i.e. $f^2 = c$ and $g^2 > C$

$\Rightarrow$ $4 = -p$ and $1 > -p$

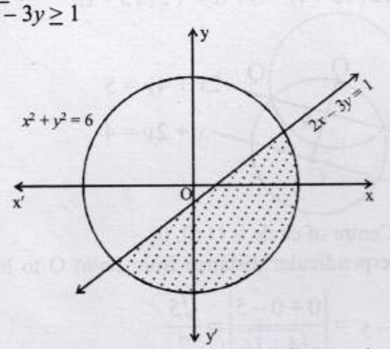
$\Rightarrow$ $p = -4$ and $p > -1$, which is not possible.

$\therefore$ only two values of p are possible.

**32.** **(2)** The smaller region of circle is the region given by

$x^2 + y^2 \le 6$ ...(i)

and $2x - 3y \ge 1$ ...(ii)




We observe that only two points $\left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$ satisfy both the inequations (i) and (ii)

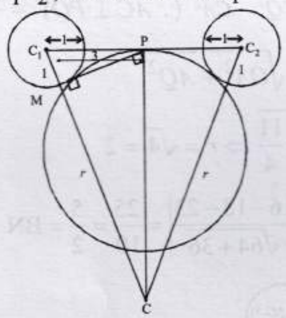
$\therefore$ 2 points in S lie inside the smaller part.

**33.** **(8)** Let $r$ be the radius of required circle.

Clearly, in $\Delta C_1CC_2$, $C_1C = C_2C = r + 1$

and $P$ is mid point of $C_1C_2$

$\therefore$ $CP \perp C_1C_2$, Also $PM \perp CC_1$

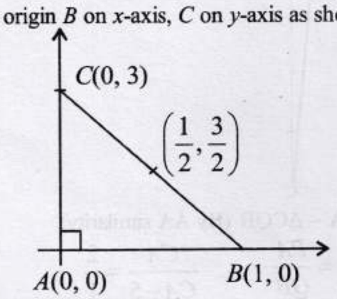



Now $\Delta PMC_1 \sim \Delta CPC_1$ (By AA similarity)

$\therefore$ $\dfrac{MC_1}{PC_1} = \dfrac{PC_1}{CC_1} \Rightarrow \dfrac{1}{3} = \dfrac{3}{r+1} \Rightarrow r + 1 = 9 \Rightarrow r = 8.$

**34.** **(0.84)** We have AB = 1, AC = 3 and $\angle BAC = \dfrac{\pi}{2}$

Let $A$ be the origin $B$ on $x$-axis, $C$ on $y$-axis as shown below




$\therefore$ Equation of circumcircle is

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \left(\sqrt{(1-0)^2 + (0-3)^2} \div 2\right)^2 = \frac{5}{2} \quad ...(i)$$

[$\because$ $r$ = Hypotenuse $\div$ 2]

Required circle touches $AB$ and $AC$, have radius r

$\therefore$ Equation be $(x - r)^2 + (y - r)^2 = r^2$ ...(ii)

If circle in equation (ii) touches circumcircle internally, we have $d_{c_1c_2} = |r_1 - r_2|$

$$\Rightarrow \left(\frac{1}{2} - r\right)^2 + \left(\frac{3}{2} - r\right)^2 = \left(\left|\sqrt{\frac{5}{2}} - r\right|\right)^2$$

$$\Rightarrow \frac{1}{4} + r^2 - r + \frac{9}{4} + r^2 - 3r = \left(\sqrt{\frac{5}{2}} - r\right)^2 \text{ or } \left(r - \sqrt{\frac{5}{2}}\right)^2$$

$$\Rightarrow 2r^2 - 4r + \frac{5}{2} = \frac{5}{2} + r^2 - \sqrt{10}\, r$$

$\Rightarrow$ $r = 0$ or $4 - \sqrt{10}$

$\Rightarrow$ r = 0.837 = 0.84 (on rounding off)

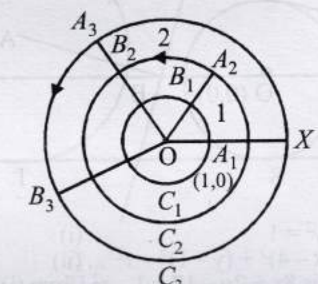
**35.** Let $(h, k)$ be any point on the given line.

$\therefore$ $2h + k = 4$ and chord of contact is $hx + ky = 1$

$\Rightarrow$ $hx + (4 - 2h)y = 1$ $\Rightarrow$ $(4y - 1) + h(x - 2y) = 0$

which is in the form $P + \lambda Q = 0$ and it passes through the point of intersection of $P = 0$ and $Q = 0$ and this point of intersection is $\left(\frac{1}{2}, \frac{1}{4}\right)$.

**36.**




The radius of circle $C_1$ is 1 cm, $C_2$ is 2 cm and so on.

It starts from $A_1$ (1, 0) on $C_1$, moves a distance of 1 cm on $C_1$ to come to $B_1$. The angle subtended by $A_1B_1$ at the centre will be $\dfrac{\ell}{r} = \dfrac{1}{1} = 1$ radian.

From $B_1$ it moves along radius, $OB_1$ and comes to $A_2$ on circle $C_2$ of radius 2. From $A_2$ it moves on $C_2$ a distance 2 cm and comes to $B_2$. The angle subtended by $A_2B_2$ is again as before 1 radian. The total angle subtended at the centre is 2 radians. The process continues. In order to cross the $x$-axis again, it must describe $2\pi$ radians i.e. $2.\dfrac{22}{7} = 6.7$ radians. Hence it must be moving on circle $C_7$.

$\therefore$ $n = 7$

**37.** Equation of any circle passing through the point of intersection of $x^2 + y^2 - 2x = 0$ and $y = x$ is

$x^2 + y^2 - 2x + \lambda(y - x) = 0$

$\Rightarrow$ $x^2 + y^2 - (2 + \lambda)x + \lambda y = 0$ .... (i)

Its centre = $\left(\dfrac{2+\lambda}{2}, \dfrac{-\lambda}{2}\right)$

For $AB$ to be the diameter of the required circle, the centre must lie on $AB$. i.e., on line (i)

$\therefore$ $\dfrac{2+\lambda}{2} = -\dfrac{\lambda}{2} \Rightarrow \lambda = -1$

$\therefore$ Equation of required circle is

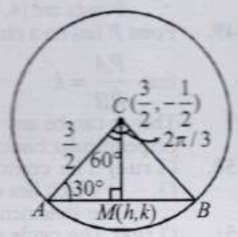
$x^2 + y^2 - x - y = 0$

**38.** Given circle is $4x^2 + 4y^2 - 12x + 4y + 1 = 0$

$\Rightarrow$ $x^2 + y^2 - 3x + y + \dfrac{1}{4} = 0$ with centre $\left(\dfrac{3}{2}, -\dfrac{1}{2}\right)$

and radius $= \sqrt{\frac{9}{4}+\frac{1}{4}-\frac{1}{4}} = \frac{3}{2}$

Let $M(h, k)$ be the mid pt. of the chord $AB$ of the given circle, then $CM \perp AB$ and $\angle ACB = 120°$.
In $\Delta ACM$,




$$\angle ACM = \frac{1}{2}\angle ACB = 60°$$

and $\angle A = 30°$

$\therefore \quad \sin A = \frac{CM}{AC}$

$$\Rightarrow \quad \sin 30° = \frac{\sqrt{(h-3/2)^2+(k+1/2)^2}}{3/2}$$

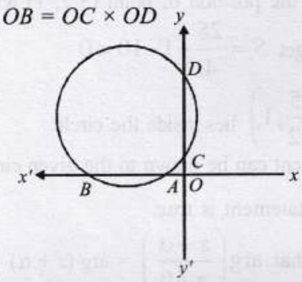
$$\Rightarrow \quad \left(\frac{3}{4}\right)^2 = \left(h-\frac{3}{2}\right)^2 + \left(k+\frac{1}{2}\right)^2$$

$$\Rightarrow \quad 16h^2 + 16k^2 - 48h + 16k + 31 = 0$$

$\therefore$ locus of $(h, k)$ is $16x^2 + 16y^2 - 48x + 16y + 31 = 0$

**39.** The given lines are $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$ which meet x-axis at $A\left(-\frac{1}{\lambda}, 0\right)$ and $B(-3, 0)$ and y-axis at $C(0, 1)$ and $D\left(0, \frac{3}{2}\right)$ respectively.

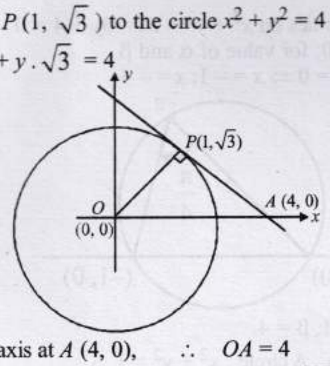
$\therefore \quad OA \times OB = OC \times OD$




$$\Rightarrow \left(-\frac{1}{\lambda}\right)(-3) = 1 \times \frac{3}{2} \Rightarrow \lambda = 2$$

**40.** Tangent at $P(1, \sqrt{3})$ to the circle $x^2 + y^2 = 4$ is

$x \cdot 1 + y \cdot \sqrt{3} = 4$



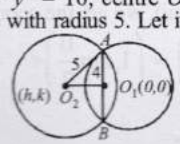

It meets x-axis at $A(4, 0)$, $\therefore \quad OA = 4$

Also $OP$ = radius of circle = 2, $\therefore PA = \sqrt{4^2 - 2^2} = \sqrt{12}$

$\therefore$ Area of $\Delta OPA = \frac{1}{2} \times OP \times PA = \frac{1}{2} \times 2 \times \sqrt{12}$

$= 2\sqrt{3}$ sq. units

**41.** We have $C_1 : x^2 + y^2 = 16$, centre $O_1(0, 0)$ and radius $= 4$. $C_2$ is another circle with radius 5. Let its centre $O_2$ be $(h, k)$.




Now the common chord of circles $C_1$ and $C_2$ is of maximum length when chord is diameter of smaller circle $C_1$. In this case, the common chord passes through centre $O_1$ of circle $C_1$. Given that slope of this chord is 3/4.

$\therefore$ Equation of $AB$ is,

$$y = \frac{3}{4}x \Rightarrow 3x - 4y = 0 \quad \ldots \text{(i)}$$

In right $\Delta AO_1O_2$,

$$O_1O_2 = \sqrt{5^2 - 4^2} = 3$$

Also $O_1O_2$ = perpendicular distance from $(h, k)$ to circle (i).

$$\therefore \quad 3 = \left|\frac{3h-4k}{\sqrt{3^2+4^2}}\right| \quad \Rightarrow \pm 3 = \frac{3h-4k}{5}$$

$$\Rightarrow 3h - 4k \pm 15 = 0 \quad \ldots \text{(ii)}$$

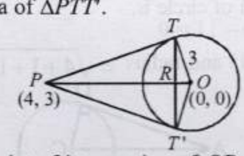
Now, $AB \perp O_1O_2 \Rightarrow m_{AB} \times m_{O_1O_2} = -1$

$$\Rightarrow \frac{3}{4} \times \frac{k}{h} = -1 \Rightarrow 4h + 3k = 0 \quad \ldots \text{(iii)}$$

From (ii) and (iii),
$h = -9/5, k = 12/5$ or $h = 9/5, k = -12/5$

Thus the required centre is $\left(\frac{-9}{5}, \frac{12}{5}\right)$ or $\left(\frac{9}{5}, \frac{-12}{5}\right)$.

**42.** From $P(4, 3)$ two tangents $PT$ and $PT'$ are drawn to the circle $x^2 + y^2 = 9$ with O (0, 0) as centre and radius = 3.
To find the area of $\Delta PTT'$.




Let $R$ be the point of intersection of $OP$ and $TT'$.
Clearly $OP$ is the perpendicular bisector of $TT'$.
Equation of chord of contact $TT'$ is $4x + 3y = 9$
Now, $OR$ = length of the perpendicular from $O$ to $TT'$

$$= \left|\frac{4 \times 0 + 3 \times 0 - 9}{\sqrt{4^2+3^2}}\right| = \frac{9}{5}$$

$OT$ = radius of circle = 3

$$\therefore \quad TR = \sqrt{OT^2 - OR^2} = \sqrt{9 - \frac{81}{25}} = \frac{12}{5}$$

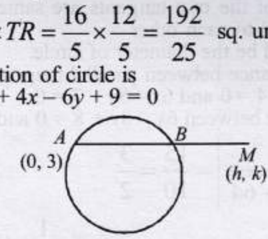
Now $OP = \sqrt{(4-0)^2 + (3-0)^2} = 5$

$$\therefore \quad PR = OP - OR = 5 - \frac{9}{5} = \frac{16}{5}$$

Area of the triangle $PTT'$

$$= PR \times TR = \frac{16}{5} \times \frac{12}{5} = \frac{192}{25} \text{ sq. units.}$$

**43.** Given ; Equation of circle is
$$x^2 + y^2 + 4x - 6y + 9 = 0 \quad \ldots \text{(i)}$$




Now, $AM = 2AB$, $\therefore AB = BM$
Let the co-ordinates of $M$ be $(h, k)$

$$\therefore \quad B\left(\frac{0+h}{2}, \frac{3+k}{2}\right) = \left(\frac{h}{2}, \frac{k+3}{2}\right)$$

As $B$ lies on circle (i),

$$\therefore \quad \left(\frac{h}{2}\right)^2 + \left(\frac{k+3}{2}\right)^2 + 4 \times \frac{h}{2} - 6\left(\frac{k+3}{2}\right) + 9 = 0$$

$\Rightarrow\ h^2 + k^2 + 8h - 6k + 9 = 0$

$\therefore$ locus of (h, k) is, $x^2 + y^2 + 8x - 6y + 9 = 0$

**44.** Given : Equation of two circles are

$$x^2 + y^2 - \frac{2}{3}x + 4y - 3 = 0 \quad \dots (i)$$

and $x^2 + y^2 + 6x + 2y - 15 = 0$ ... (ii)

Now, we know that equation of common chord of two circles $S_1 = 0$ and $S_2 = 0$ is $S_1 - S_2 = 0$

Hence, equation of common chord of two given circles is

$$\Rightarrow\ 6x + \frac{2}{3}x + 2y - 4y - 15 + 3 = 0$$

$$\Rightarrow\ \frac{20x}{3} - 2y - 12 = 0 \Rightarrow 10x - 3y - 18 = 0$$

**45.** Given circle :

$(x-1)^2 + y^2 = 1$

$\Rightarrow\ x^2 + y^2 - 2x = 0$ ... (i)

We know that equation of chord of curve $S = 0$, whose mid point is $(x_1, y_1)$ is given by $T = S_1$, where $T$ is tangent to curve $S = 0$ at $(x_1, y_1)$.

$\therefore$ If $(x_1, y_1)$ is the mid point of chord of given circle (i), then equation of chord is

$$xx_1 + yy_1 - (x + x_1) = x_1^2 + y_1^2 - 2x_1$$

$$\Rightarrow\ (x_1 - 1)x + y_1 y + x_1 - x_1^2 - y_1^2 = 0$$

As it passes through origin, we get

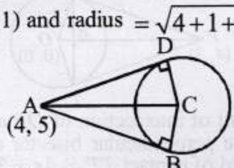
$$x_1 - x_1^2 - y_1^2 = 0 \text{ or } x_1^2 + y_1^2 - x_1 = 0$$

$\therefore$ locus of $(x_1, y_1)$ is $x^2 + y^2 - x = 0$

**46.** Given : Equation of circle is,

$x^2 + y^2 - 4x - 2y - 11 = 0$

It's centre is (2, 1) and radius $= \sqrt{4+1+11} = 4 = BC$




Length of tangent from a point $(x_1, y_1)$ to a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by

$$\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

$\therefore$ Length of tangent from the point (4, 5) to the given circle

$= \sqrt{16 + 25 - 16 - 10 - 11} = \sqrt{4} = 2 = AB$

$\therefore$ Area of quadrilateral $ABCD$

$= 2$ (Area of $\Delta ABC$) $= 2 \times \frac{1}{2} \times AB \times BC$

$= 2 \times \frac{1}{2} \times 2 \times 4 = 8$ sq. units.

**47.** Let $3x - 4y + 4 = 0$ be the tangent at point $A$ and $6x - 8y - 7 = 0$ be the tangent of point $B$ of the circle.

As the slopes of the two tangents are same, therefore the two tangents parallel to each other

$\therefore$ $AB$ should be the diameter of circle.

$\therefore$ $AB$ = distance between parallel tangents.

$3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$

= distance between $6x - 8y + 8 = 0$ and $6x - 8y - 7 = 0$

$$= \left|\frac{8+7}{\sqrt{36+64}}\right| = \frac{15}{10} = \frac{3}{2}$$

$\therefore$ Radius of the circle $= \frac{1}{2}(AB) = \frac{3}{4}$ units.

**48.** Equation of given line

$4x - 3y - 10 = 0$ ... (i)

and equation of given circle

$x^2 + y^2 - 2x + 4y - 20 = 0$ ... (ii)

From (i) and (ii), we get

$$\left(\frac{3y+10}{4}\right)^2 + y^2 - 2\left(\frac{3y+10}{4}\right) + 4y - 20 = 0$$

$\Rightarrow\ y^2 + 4y - 12 = 0 \Rightarrow y = 2, -6 \Rightarrow x = 4, -2$

$\therefore$ Points are (4, 2) and (– 2, – 6).

**49.** Point $P$ lies on a circle and $A$ and $B$ are two points in a plane such that $\frac{PA}{PB} = k$

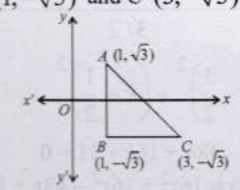
Then k can be any real number except 1, otherwise $P$ will lie on perpendicular bisector of $AB$ which is a line.

**50.** **(True)** The centre of the circle $x^2 + y^2 - 6x + 2y = 0$ is $(3, -1)$ which lies on the line $x + 3y = 0$

$\therefore$ The statement is true.

**51.** **(True)** The circle passes through the points

$A(1, \sqrt{3}), B(1, -\sqrt{3})$ and $C(3, -\sqrt{3})$.




Here line $AB$ is parallel to $y$-axis and $BC$ is parallel to $x$-axis, $\therefore$ $\angle ABC = 90°$

$\therefore$ $AC$ is a diameter of circumcircle.

$\therefore$ Equation of the circumcircle is

$(x-1)(x-3) + (y - \sqrt{3})(y + \sqrt{3}) = 0$

$\Rightarrow\ x^2 + y^2 - 4x = 0$ ... (i)

Let us check the position of point (5/2, 1) with respect to the circle (i), we get $S = \frac{25}{4} + 1 - 10 < 0$

$\therefore$ Point $\left(\frac{5}{2}, 1\right)$ lies inside the circle.

$\therefore$ No tangent can be drawn to the given circle from point (5/2, 1).

$\therefore$ Given statement is true.

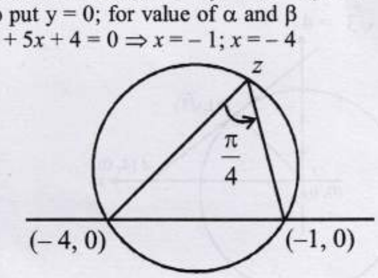
**52.** **(b, d)** Given that $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \arg(z+\alpha) - \arg(z+\beta)$

$= \frac{\pi}{4}$ implies $z$ is on arc and $(-\alpha, 0)$ & $(-\beta, 0)$ subtend $\frac{\pi}{4}$ on $z$.

Given that $z$ lies on $x^2 + y^2 + 5x - 3y + 4 = 0$

So put y = 0; for value of $\alpha$ and $\beta$

$x^2 + 5x + 4 = 0 \Rightarrow x = -1; x = -4$




Hence $\alpha = 1, \beta = 4$.

**53.** **(a, c)** Given : A circle : $x^2 + y^2 = 1$

Let coordinates of P = $(\cos\theta, \sin\theta)$

$\therefore$ Equation of tangent at $P(\cos\theta, \sin\theta)$ is

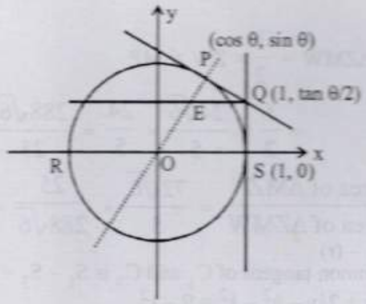
$x\cos\theta + y\sin\theta = 1$ ...(i)

Equation of normal at P is $y = x\tan\theta$ ...(ii)

Now, equation of tangent at S is $x = 1$ ... (iii)

On solving (i) and (iii), we get the coordinates of Q as

$$\left(1, \frac{1-\cos\theta}{\sin\theta}\right) = \left(1, \tan\frac{\theta}{2}\right)$$




$\therefore$ Equation of line through Q and parallel to RS is

$y = \tan\frac{\theta}{2}$ ... (iv)

Intersection point E of normal (ii) and line (iv) can be find out by solving (ii) and (iv).

Now from (ii) and (iv),

$$\tan\frac{\theta}{2} = x\tan\theta \Rightarrow x = \frac{1-\tan^2\theta/2}{2}$$

$$\therefore \text{Locus of E is } x = \frac{1-y^2}{2} \Rightarrow y^2 = 1-2x$$

It is satisfied by the points $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{3}, \frac{-1}{\sqrt{3}}\right)$.

**54.** **(b, c)** Let the equation of circles be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

It passes through (0, 1)

$\therefore 1 + 2f + c = 0$ ...(ii)

Since circle (i) is orthogonal to circle $(x-1)^2 + y^2 = 16$

*i.e.* $x^2 + y^2 - 2x - 15 = 0$

and $x^2 + y^2 - 1 = 0$

$\therefore \quad 2g \times (-1) + 2f \times 0 = c - 15$

$\Rightarrow \quad 2g + c - 15 = 0$ ...(iii)

and $2g \times 0 + 2f \times 0 = c - 1$

$\Rightarrow \quad c = 1$ ...(iv)

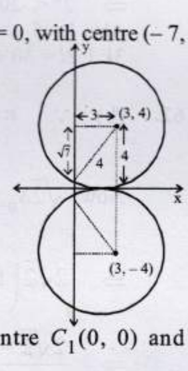
Solving (ii), (iii) and (iv), we get

$c = 1, g = 7, f = -1$

$\therefore$ Required circle is $x^2 + y^2 + 14x - 2y + 1 = 0$, with centre (– 7, 1) and radius = 7

$\therefore$ (b) and (c) are correct options.

**55.** **(a, c)** Here, there are two possibilites for the given circle as shown in the figure.




$\therefore$ The equations of circles can be

$(x-3)^2 + (y-4)^2 = 4^2$

or $(x-3)^2 + (y+4)^2 = 4^2$

$\Rightarrow \quad x^2 + y^2 - 6x - 8y + 9 = 0$

or $x^2 + y^2 - 6x + 8y + 9 = 0$

**56.** **(b)** Given Circle : $x^2 + y^2 = 4$ with centre $C_1(0, 0)$ and $R_1 = 2$.

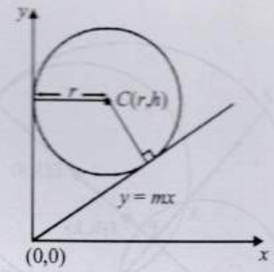
And circle $x^2 + y^2 - 6x - 8y - 24 = 0$ with centre $C_2(3, 4)$ and $R_2 = 7$.

Now $C_1 C_2 = 5 = R_2 - R_1$

Therefore, the given circles touch internally and hence they can have just one common tangent at the point of contact.

**57.** **(a, c)** Given : A circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ with centre (r, h) and radius = r.

Clearly circle touches y-axis so one of its tangent is $x = 0$.




Let y = mx be the other tangent through origin.

Then length of perpendicular from C (r, h) to y = mx should be equal to r.

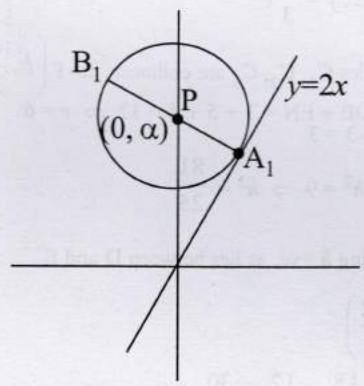
$$\therefore \quad \left|\frac{mr-h}{\sqrt{m^2+1}}\right| = r$$

$$\Rightarrow \quad m^2r^2 - 2mrh + h^2 = m^2r^2 + r^2 \Rightarrow m = \frac{h^2-r^2}{2rh}$$

$\therefore \quad$ Other tangent is $y = \frac{h^2-r^2}{2rh}x$

$\Rightarrow \quad (h^2 - r^2)x - 2rhy = 0$

**58.** **(c)**




Consider centre as P $(0, \alpha)$, $\alpha > 0$

Distance of $A_1P = \left|\frac{2(0)-\alpha}{\sqrt{5}}\right| = r$

$|-\alpha| = \sqrt{5}r \Rightarrow \alpha = \sqrt{5}r$

$\therefore \alpha + r = 5 + \sqrt{5}$

$\sqrt{5}r + r = \sqrt{5}(\sqrt{5}+1) \Rightarrow r = \sqrt{5}, \alpha = 5$

$\therefore$ P(0, 5)

Foot of perpendicular from P to line $2x - y = 0$

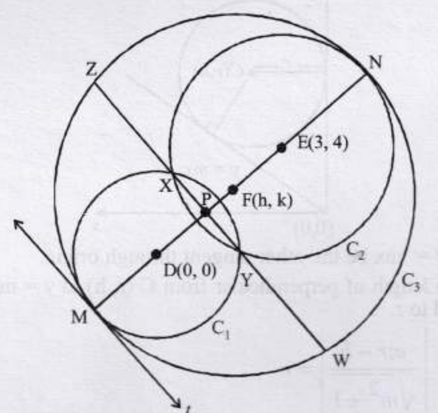
$$\frac{x-0}{2} = \frac{y-5}{-1} = \frac{-(2(0)-5)}{5} = 1$$

$x = 2, y = 4$ $A_1(2, 4)$

Let B $(x_1, y_1)$ $\therefore \frac{x_1+2}{2} = 0, \frac{y_1+4}{2} = 5$

$\therefore x_1 = -2, y_1 = 6$ B(–2, 6)

**For Questions 59 and 60**




Given three circles are
$C_1 : x^2 + y^2 = 9$
$C_2 : (x-3)^2 + (y-4)^2 = 16$
$C_3 : (x-h)^2 + (y-k)^2 = r^2$
Centres of circles $C_1$, $C_2$, $C_3$ are D(0, 0), E(3, 4), F(h, k) respectively
and radii of circles $C_1 : C_2 : C_3$ are 3, 4, $r$ respectively.

Equation of DE : $y = \frac{4}{3}x$

Centres of circles $C_1$, $C_2$, $C_3$ are collinear $\Rightarrow F\left(h, \frac{4}{3}h\right)$

MN = MD + DE + EN = 3 + 5 + 4 = 12 $\Rightarrow r = 6$
$\therefore$ DE = 6 − 3 = 3

$\Rightarrow h^2 + \frac{16}{9}h^2 = 9 \Rightarrow h^2 = \frac{81}{25}$

$\Rightarrow h = \frac{9}{5}$ taking $h$ +ve, as lies between D and E

$\therefore F\left(\frac{9}{5}, \frac{12}{5}\right)$

$\therefore 2h + k = \frac{18}{5} + \frac{12}{5} = \frac{30}{5} = 6$

$\therefore$ (A) − (p)
DE is common chord of circles $C_1$ and $C_2$
$\therefore$ Equation of XY : $S_1 - S_2 = 0$
$\Rightarrow 6x + 8y - 18 = 0 \Rightarrow 3x + 4y - 9 = 0$

Length of $\perp$ from D to XY $= \frac{9}{5}$ = DP

Also DX = 3, $\therefore$ PX $= \sqrt{9 - \frac{81}{25}} = \sqrt{\frac{225-81}{25}} = \frac{12}{5}$

$\therefore$ XY = 2PX $= \frac{24}{5}$

ZW is chord of $C_3$.

FP = MF − MP $= 6 - \left(3 + \frac{9}{5}\right) = 6 - \frac{24}{5} = \frac{6}{5}$

$\therefore$ ZP $= \sqrt{6^2 - \left(\frac{6}{5}\right)^2} = \frac{6\sqrt{24}}{5} = \frac{12\sqrt{6}}{5}$ $\therefore$ ZW $= \frac{24\sqrt{6}}{5}$

Hence, $\frac{\text{Length of ZW}}{\text{Length of XY}} = \frac{24\sqrt{6}/5}{24/5} = \sqrt{6}$

$\therefore$ (B) − (q)

Area of $\Delta$MZN $= \frac{1}{2}$ MN $\times$ ZP $= \frac{1}{2} \times 12 \times \frac{12\sqrt{6}}{5} = \frac{72\sqrt{6}}{5}$

Area of $\Delta$ZMW $= \frac{1}{2} \times$ ZW $\times$ MP

$= \frac{1}{2} \times \frac{24\sqrt{6}}{5} \times \frac{24}{5} = \frac{288\sqrt{6}}{25}$

$\therefore \frac{\text{Area of } \Delta MZN}{\text{Area of } \Delta ZMW} = \frac{72\sqrt{6}}{5} \times \frac{25}{288\sqrt{6}} = \frac{5}{4}$

$\therefore$ (C) − (r)

Now common tangent of $C_1$ and $C_3$ is $S_1 - S_3 = 0$

$\Rightarrow 2hx + 2ky - h^2 - k^2 = 9 - r^2$

or $\frac{18}{5}x + \frac{24}{5}y - \frac{81}{25} - \frac{144}{25} = 9 - 36$

$\Rightarrow 3x + 4y + 15 = 0$

It is tangent to $x^2 = 8\alpha y$

Putting value of $y$ from common tangent in parabola, we get

$$x^2 = -8\alpha\left(\frac{3x+15}{4}\right) \Rightarrow x^2 + 6\alpha x + 30\alpha = 0$$

It should have equal roots

$\therefore 36\alpha^2 - 4 \times 30\alpha = 0 \Rightarrow \alpha = \frac{10}{3}$ $\therefore$ (D) − (u)

Thus (B) − (q) is the only correct combination
and (D) − (s) is the only incorrect combination.

**59.** **(d)** Option (d) is correct.

**60.** **(a)** Option (a) is incorrect.

**61.** **(d)** $\because a_n = \frac{1}{2^{n-1}}$

$$S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{n-1}} = 2\left(1 - \frac{1}{2^n}\right) = 2 - \frac{1}{2^{n-1}}$$

For circles $C_n$ to inside $M$

$S_{n-1} + a_n < \frac{1025}{513} \Rightarrow 2 - \frac{1}{2^{n-2}} + \frac{1}{2^{n-1}} < \frac{1025}{513}$

$\Rightarrow 1 - \frac{1}{2^n} < \frac{1025}{1026} = 1 - \frac{1}{1026}$

$\Rightarrow 2^n < 2026 \Rightarrow n \le 10 \Rightarrow k = 10$

Also $l = 5$

$3k + 2l = 30 + 10 = 40$

**62.** **(b)** $\because r = \frac{(2^{199} - 1)\sqrt{2}}{2^{198}}$

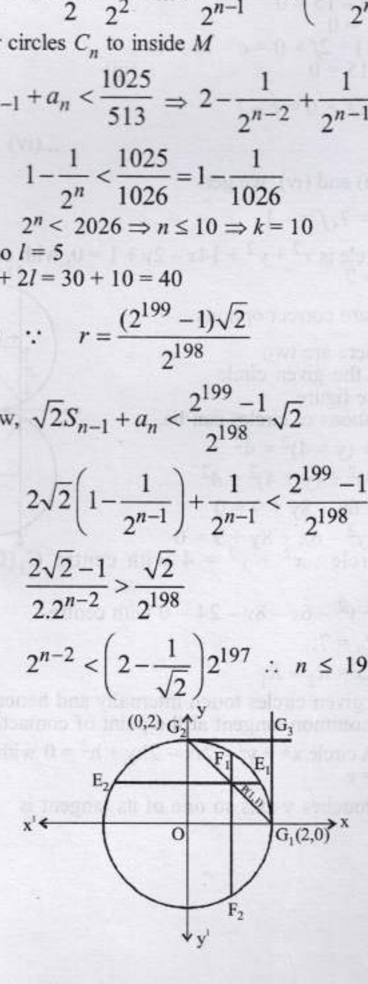
Now, $\sqrt{2}S_{n-1} + a_n < \frac{2^{199} - 1}{2^{198}}\sqrt{2}$

$\Rightarrow 2\sqrt{2}\left(1 - \frac{1}{2^{n-1}}\right) + \frac{1}{2^{n-1}} < \frac{2^{199} - 1}{2^{198}}$

$\Rightarrow \frac{2\sqrt{2} - 1}{2.2^{n-2}} > \frac{\sqrt{2}}{2^{198}}$

$\Rightarrow 2^{n-2} < \left(2 - \frac{1}{\sqrt{2}}\right)2^{197}$ $\therefore n \le 199 \Rightarrow n = 199$

**63.** **(a)**



Equation of $E_1 E_2$ is $y = 1$
Equation of $F_1 F_2$ is $x = 1$
Equation of $G_1 G_2$ is $x + y = 2$
By symmetry, tangents at $E_1$ and $E_2$ will meet on y-axis and tangents at $F_1$ and $F_2$ will meet on x-axis

$E_1 \equiv (\sqrt{3}, 1)$ and $F_1 \equiv (1, \sqrt{3})$

Equation of tangent at $E_1$ is $\sqrt{3}x + y = 4$

Equation of tangent at $F_1$ is $x + \sqrt{3}y = 4$

$\therefore$ Points $E_3$ (0, 4) and $F_3$(4, 0)

Tangents at $G_1$ and $G_2$ are $x = 2$ and $y = 2$ respectively intersecting each other at $G_3(2, 2)$.

Clearly $E_3$, $F_3$ and $G_3$ lie on the curve $x + y = 4$.

**64.** **(d)** Let point P be $(2 \cos \theta, 2 \sin \theta)$

Tangent at P is $x \cos \theta + y \sin \theta = 2$

$$\therefore \quad M\left(\frac{2}{\cos\theta}, 0\right) \text{ and } N\left(0, \frac{2}{\sin\theta}\right)$$

$$\therefore \quad \text{Mid point of MN} = \left(\frac{1}{\cos\theta}, \frac{1}{\sin\theta}\right)$$

For locus of mid point (x, y) of MN,

$$x = \frac{1}{\cos\theta}, \; y = \frac{1}{\sin\theta}$$

$$\Rightarrow \quad \frac{1}{x^2} + \frac{1}{y^2} = 1 \Rightarrow x^2 + y^2 = x^2y^2$$

**65.** **(a)** Equation of tangent *PT* to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$ is $x\sqrt{3} + y = 4$

Let the line *L*, perpendicular to tangent *PT* be

$$x - y\sqrt{3} + \lambda = 0$$

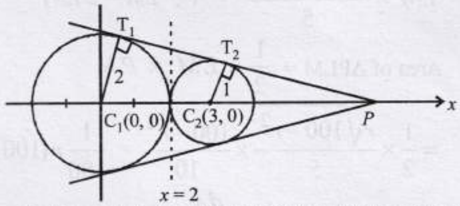
As it is tangent to the circle $(x - 3)^2 + y^2 = 1$

$\therefore$ Length of perpendicular from centre of circle to the Tangent = radius of circle.

$$\Rightarrow \left|\frac{3+\lambda}{2}\right| = 1 \Rightarrow \lambda = -1 \text{ or } -5$$

$\therefore$ Equation of *L* can be $x - \sqrt{3}y = 1$ or $x - \sqrt{3}y = 5$

**66.** **(d)**




From the figure it is clear that the intersection point of two direct common tangents lies on *x*-axis.

Also $\Delta PT_1C_1 \sim \Delta PT_2C_2$ $\therefore$ $PC_1 : PC_2 = 2 : 1$

or $P$ divides $C_1C_2$ in the ratio 2 : 1 externally

$\therefore$ Coordinates of *P* are (6, 0).

Let the equation of tangent through *P* be

$$y = m(x - 6)$$

As it touches $x^2 + y^2 = 4$

$$\therefore \left|\frac{6m}{\sqrt{m^2+1}}\right| = 2 \Rightarrow 36m^2 = 4(m^2+1) \therefore \quad m = \pm\frac{1}{2\sqrt{2}}$$

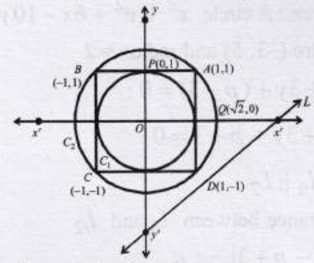
$\therefore$ Equations of common tangents are

$$y = \pm\frac{1}{2\sqrt{2}}(x - 6)$$

Also $x = 2$ is the common tangent to the two circles.

**67.** **(a)** According to the given question, we can assume the square *ABCD* with its vertices *A* (1, 1), *B* (–1, 1), *C* (–1, –1), *D* (1, –1).

*P* be the point (0, 1) and *Q* be the point $(\sqrt{2}, 0)$.



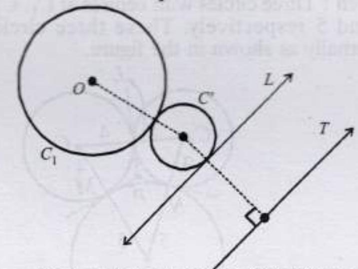

$$\text{Then, } \frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$$

$$= \frac{1+1+5+5}{2[(\sqrt{2}-1)^2+1] + 2[(\sqrt{2}+1)^2+1]} = \frac{12}{16} = 0.75$$

**68.** **(c)** Let $C'$ be the said circle touching $C_1$ and *L*, so that $C_1$ and $C'$ are on the same side of *L*. Let us draw a line *T* parallel to *L* at a distance equal to the radius of circle $C_1$, on opposite side of *L*.

Then the centre of $C'$ is equidistant from the centre of $C_1$ and from line *T*.

$\Rightarrow$ Locus of centre of $C'$ is a parabola.



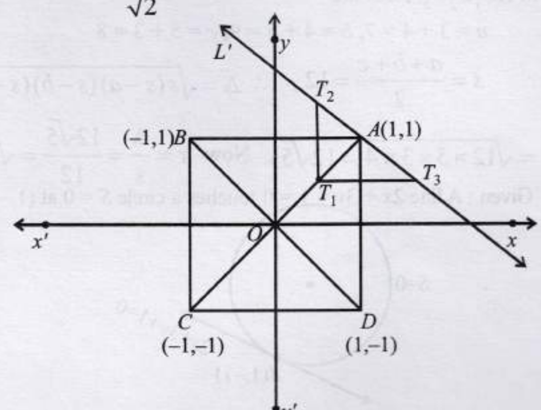

**69.** **(c)** Since *S* is equidistant form *A* and line *BD*, it traces a parabola.

Clearly, *AC* is the axis, *A* (1, 1) is the focus and $T_1\left(\frac{1}{2}, \frac{1}{2}\right)$ is the vertex of the parabola.

$$AT_1 = \frac{1}{\sqrt{2}}.$$

$T_2T_3$ = latus rectum of parabola

$$= 4 \times \frac{1}{\sqrt{2}} = 2\sqrt{2}$$



$\therefore$ Area $(\Delta T_1T_2T_3) = \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}} \times 2\sqrt{2} = 1$ sq. units

**70.** (c) Given : A circle $x^2 + y^2 + 6x - 10y + 30 = 0$

with centre (–3, 5) and radius = 2

$L_1 : 2x + 3y + (p-3) = 0$;

$L_2 : 2x + 3y + p + 3 = 0$

Clearly $L_1 \parallel L_2$

$\therefore$ Distance between $L_1$ and $L_2$

$$= \left|\frac{p+3-p+3}{\sqrt{2^2+3^2}}\right| = \frac{6}{\sqrt{13}} < 2$$

$\Rightarrow$ If one line is a chord of the given circle, other line may or may not the diameter of the circle.

$\therefore$ Statement 1 is true and statement 2 is false.

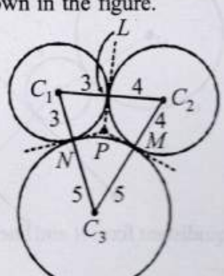
**71.** (a) Equation of director circle of the given circle $x^2 + y^2 = 169$ is $x^2 + y^2 = 2 \times 169 = 338$.

We know from every point on director circle, the tangents drawn to given circle are perpendicular to each other.

Since (17, 7) lies on director circle.

$\therefore$ The tangent from (17, 7) to given circle are mutually perpendicular.

**72.** **Given :** Three circles with centres at $C_1, C_2, C_3$ and with radii 3, 4 and 5 respectively. These three circles touch each other externally as shown in the figure.




$P$ is the point of intersection of the three tangents drawn at the points of contacts $L, M$ and $N$. Since lengths of tangents to a circle from a point are equal,

$\therefore$ $PL = PM = PN$

Also $PL \perp C_1C_2, PM \perp C_2C_3, PN \perp C_1C_3$

Clearly $P$ is the incentre of $\Delta C_1C_2C_3$ and its distance from point of contact i.e., $PL$ is the radius of incircle of $\Delta C_1C_2C_3$.

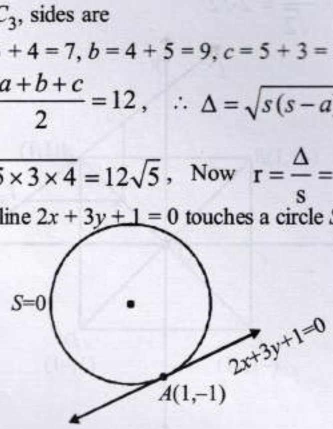
In $\Delta C_1C_2C_3$, sides are

$a = 3 + 4 = 7, b = 4 + 5 = 9, c = 5 + 3 = 8$

$\therefore$ $s = \dfrac{a+b+c}{2} = 12$, $\therefore$ $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12 \times 5 \times 3 \times 4} = 12\sqrt{5}$, Now $r = \dfrac{\Delta}{s} = \dfrac{12\sqrt{5}}{12} = \sqrt{5}$

**73.** Given : A line $2x + 3y + 1 = 0$ touches a circle $S = 0$ at (1, – 1).




$\therefore$ Equation of the circle can be
$(x-1)^2 + (y+1)^2 + \lambda(2x + 3y + 1) = 0$.

$\Rightarrow$ $x^2 + y^2 + 2x(\lambda - 1) + y(3\lambda + 2) + (\lambda + 2) = 0$ ...(i)

But given that this circle is orthogonal to the circle, the extremities of whose diameter are (0, 3) and (– 2, – 1) i.e.

$x(x+2) + (y-3)(y+1) = 0$

$\Rightarrow$ $x^2 + y^2 + 2x - 2y - 3 = 0$ ... (ii)

On applying the condition of orthogonality for circles (i) and (ii),

$$2(\lambda-1).1 + 2\left(\frac{3\lambda+2}{2}\right).(-1) = \lambda + 2 + (-3)$$

$(\because 2g_1g_2 + 2f_1f_2 = c_1 + c_2)$

$\Rightarrow$ $2\lambda - 2 - 3\lambda - 2 = \lambda - 1 \Rightarrow 2\lambda = -3 \Rightarrow \lambda = \dfrac{-3}{2}$

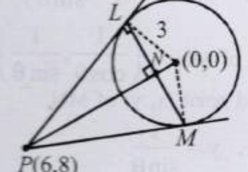
Substituting this value of $\lambda$ in equation (i), we get the required circle as

$$x^2 + y^2 - 5x - \frac{5}{2}y + \frac{1}{2} = 0$$

$\Rightarrow$ $2x^2 + 2y^2 - 10x - 5y + 1 = 0$

**74.** The given circle is $x^2 + y^2 = r^2$

From point (6, 8) tangents are drawn to this circle.




Then length of tangent,

$$PL = \sqrt{6^2 + 8^2 - r^2} = \sqrt{100 - r^2}$$

Also equation of chord of contact $LM$ is

$6x + 8y - r^2 = 0$

$PN$ = length of perpendicular from $P$ to $LM$

$$= \frac{36 + 64 - r^2}{\sqrt{36+64}} = \frac{100 - r^2}{10}$$

Now in right angled $\Delta PLN$, $LN^2 = PL^2 - PN^2$

$$= (100 - r^2) - \frac{(100-r^2)^2}{100} = \frac{(100-r^2)r^2}{100}$$

$$\Rightarrow \quad LN = \frac{r\sqrt{100-r^2}}{10}$$

$$\therefore \quad LM = \frac{r\sqrt{100-r^2}}{5} \qquad (\because LM = 2\,LN)$$

$\therefore$ Area of $\Delta PLM = \dfrac{1}{2} \times LM \times PN$

$$= \frac{1}{2} \times \frac{r\sqrt{100-r^2}}{5} \times \frac{100-r^2}{10} = \frac{1}{100} r(100-r^2)^{\frac{3}{2}}$$

For maximum value of area, $\dfrac{dA}{dr} = 0$

$$\Rightarrow \quad \frac{1}{100}\left[(100-r^2)^{\frac{3}{2}} + r.\frac{3}{2}(100-r^2)^{\frac{1}{2}}(-2r)\right] = 0$$

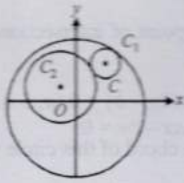
$\Rightarrow$ $(100-r^2)^{\frac{1}{2}}[100 - r^2 - 3r^2] = 0 \Rightarrow r = 10$ or $r = 5$

But $r = 10$ gives length of tangent $PL = 0$

$\therefore$ $r \neq 10$ and hence, $r = 5$

**75.** Let equation of $C_1$ be $x^2 + y^2 = r_1^2$ and of $C_2$ be

$(x-a)^2 + (y-b)^2 = r_2^2$




Let centre of $C$ be $(h, k)$ and radius be $r$, then by the given conditions.

$$\sqrt{(h-a)^2+(k-b)^2} = r + r_2 \text{ and } \sqrt{h^2+k^2} = r_1 - r$$

$$\Rightarrow \sqrt{(h-a)^2+(k-b)^2} + \sqrt{h^2+k^2} = r_1 + r_2$$

Equation of required locus is

$$\sqrt{(x-a)^2+(y-b)^2} + \sqrt{x^2+y^2} = r_1 + r_2,$$

which represents an ellipse whose foci are at $(a, b)$ and $(0, 0)$.

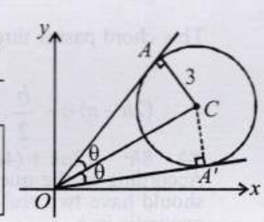
[$\because PS + PS' =$ constant $\Rightarrow$ locus of $P$ is an ellipse with foci at $S$ and $S'$]

**76.** The equation $2x^2 - 3xy + y^2 = 0$ represents pair of tangents $OA$ and $OA'$.

Let angle between these to tangents be $2\theta$.

$$\text{Then, } \tan 2\theta = \frac{2\sqrt{\left(\frac{-3}{2}\right)^2 - 2\times 1}}{2+1}$$

$$\left[\because \tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}\right]$$




$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{1}{3} \Rightarrow \tan^2\theta + 6\tan\theta - 1 = 0$$

$$\tan\theta = \frac{-6\pm\sqrt{36+4}}{2} = -3\pm\sqrt{10}$$

Since $\theta$ is acute, $\therefore \tan\theta = \sqrt{10} - 3$

Now we know that line joining the point through which tangents are drawn to the centre bisects the angle between the tangents,

$$\therefore \angle AOC = \angle A'OAC = \theta$$

In $\Delta AOC$, $\tan\theta = \dfrac{3}{OA}$

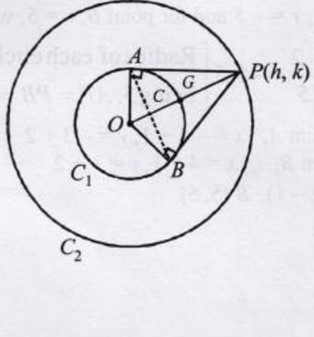
$$\Rightarrow \text{OA} = \frac{3}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3}, \therefore OA = 3(3+\sqrt{10}).$$

**77.** Let $P(h, k)$ be on $C_2$

$\therefore \quad h^2 + k^2 = 4r^2$ ... (i)

Chord of contact of $P$ w.r.t. $C_1$ is $hx + ky = r^2$

It intersects $C_1$, $x^2 + y^2 = a^2$ in $A$ and $B$.




Eliminating $y$, we get

$$x^2 + \left(\frac{r^2 - hx}{k}\right)^2 = r^2$$

$$\Rightarrow \quad x^2(h^2+k^2) - 2r^2hx + r^4 - r^2k^2 = 0$$

$$\Rightarrow \quad x^2.4r^2 - 2r^2hx + r^2(r^2 - k^2) = 0$$

$$\therefore \quad x_1 + x_2 = \frac{2r^2h}{4r^2} = \frac{h}{2}, y_1 + y_2 = \frac{k}{2}$$

If $(x, y)$ be the centroid of $\Delta PAB$, then

$$3x = x_1 + x_2 + h = \frac{h}{2} + h = \frac{3h}{2}$$

$$\therefore \quad x = \frac{h}{2} \text{ or } h = 2x \text{ and similarly } k = 2y$$

Putting the value of $h$ and $k$ in (i), we get

$4x^2 + 4y^2 = 4r^2$

$\therefore$ Locus is $x^2 + y^2 = r^2$

**78.** Given $C$ is the circle with centre at $(0, \sqrt{2})$ and radius $r$ (say), then $C \equiv x^2 + (y-\sqrt{2})^2 = r^2$

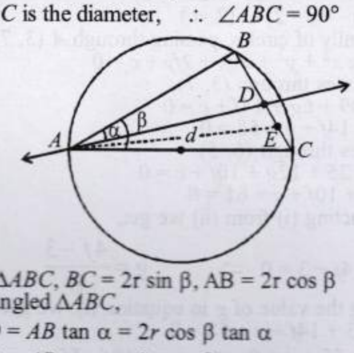
$$\Rightarrow \quad (y-\sqrt{2})^2 = (r^2 - x^2) \Rightarrow y - \sqrt{2} = \pm\sqrt{r^2 - x^2}$$

$$\Rightarrow \quad y = \sqrt{2} \pm \sqrt{r^2 - x^2} \qquad \text{... (i)}$$

The only rational value which y can have is 0. Suppose the possible value of $x$ for which y is 0 is $x_1$. Certainly $-x_1$ will also give the value of $y$ as 0 (from (i)). Thus, at the most, there are two rational points which satisfy the equation of C.

**79.** Let $r$ be the radius of circle, then $AC = 2r$

Since, $AC$ is the diameter, $\therefore \angle ABC = 90°$




$\therefore$ In $\Delta ABC$, $BC = 2r \sin\beta$, $AB = 2r\cos\beta$

In right angled $\Delta ABC$,

$BD = AB\tan\alpha = 2r\cos\beta\tan\alpha$

$AD = AB\sec\alpha = 2r\cos\beta\sec\alpha$

$\therefore \quad DC = BC - BD = 2r\sin\beta - 2r\cos\beta\tan\alpha$

Since $E$ is the mid point of $DC$,

$$\therefore \quad DE = \frac{DC}{2} = \frac{2r\sin\beta - 2r\cos\beta\tan\alpha}{2}$$

$\Rightarrow DE = r\sin\beta - r\cos\beta\tan\alpha$

Now in $\Delta ADC$, $AE$ is the median.

$\therefore \quad 2(AE^2 + DE^2) = AD^2 + AC^2$

$\Rightarrow \quad 2[d^2 + r^2(\sin\beta - \cos\beta\tan\alpha)^2]$

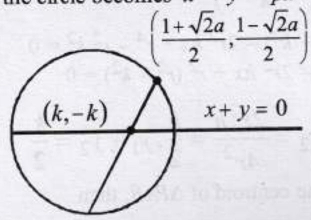
$= 4r^2\cos^2\beta\sec^2\alpha + 4r^2$

$$\Rightarrow \quad r^2 = \frac{d^2\cos^2\alpha}{\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta\cos(\beta-\alpha)}$$

$\Rightarrow$ Area of circle $= \pi r^2$

$$= \frac{\pi d^2\cos^2\alpha}{\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta\cos(\beta-\alpha)}$$

**80.** Let the given point be $(p, \bar{p}) = \left(\frac{1+\sqrt{2}\,a}{2}, \frac{1-\sqrt{2}\,a}{2}\right)$, then the equation of the circle becomes $x^2 + y^2 - px - \bar{p}y = 0$




Since the chord is bisected by the line $x + y = 0$, its mid-point can be chosen as $(k, -k)$. Hence the equation of the chord represented by $T = S_1$ is

$$kx - ky - \frac{p}{2}(x+k) - \frac{\bar{p}}{2}(y - k) = k^2 + k^2 - pk + \bar{p}k$$

Since, it passes through $A\ (p, \bar{p})$,

$$\therefore \quad kp - k\bar{p} - \frac{p}{2}(p+k) - \frac{\bar{p}}{2}(\bar{p} - k) = 2k^2 - pk + \bar{p}k$$

or $\quad 3k(p - \bar{p}) = 4k^2 + (p^2 + \bar{p}^2) \quad \ldots$ (i)

Put $p - \bar{p} = a\sqrt{2}$ and $p^2 + \bar{p}^2 = 2.\frac{(1+2a^2)}{4} = \frac{1+2a^2}{2}$ … (ii)

Hence, from (i) using (ii), we get

$$4k^2 - 3\sqrt{2}\,ak + \frac{1}{2}(1 + 2a^2) = 0 \quad \ldots \text{(iii)}$$

Since, there are two chords which are bisected by $x + y = 0$, we must have two real values of $k$ from (iii)

$\therefore \quad 18a^2 - 8(1 + 2a^2) > 0$

$\Rightarrow \quad a^2 - 4 > 0 \Rightarrow (a+2)(a-2) > 0 \Rightarrow a < -2$ or $> 2$

$\therefore \quad a \in (-\infty, -2) \cup (2, \infty)$

**81.** Let the family of circles, passing through $A\ (3, 7)$ and $B\ (6, 5)$, be $x^2 + y^2 + 2gx + 2fy + c = 0$

Since it passes through (3, 7),

$\therefore \quad 9 + 49 + 6g + 14f + c = 0$

$\Rightarrow \quad 6g + 14f + c + 58 = 0 \quad \ldots$ (i)

As it passes through (6, 5)

$\therefore \quad 36 + 25 + 12g + 10f + c = 0$

$\quad 12g + 10f + c + 61 = 0 \quad \ldots$ (ii)

On substracting (i) from (ii) we get,

$$6g - 4f + 3 = 0 \;\Rightarrow\; g = \frac{4f-3}{6}$$

On putting the value of $g$ in equation (i), we get

$\quad 4f - 3 + 14f + c + 58 = 0$

$\Rightarrow \quad 18f + 55 + c = 0 \;\Rightarrow\; c = -18f - 55$

Thus the family of circles is

$$x^2 + y^2 + \left(\frac{4f-3}{3}\right)x + 2fy - (18f + 55) = 0$$

Since, members of this family are cut by the circle $x^2 + y^2 - 4x - 6y - 3 = 0$

$\therefore$ Equation of family of chords of intersection of above two circles is $S_1 - S_2 = 0$

$\Rightarrow \quad \left(\frac{4f-3}{3} + 4\right)x + (2f + 6)y - 18f + 52) = 0$, which can be written as

$$(3x + 6y - 52) + f\left(\frac{4}{3}x + 2y - 18\right) = 0,$$

which represents the family of lines passing through the point of intersection of the lines

$\quad 3x + 6y - 52 = 0$ and $4x + 6y - 54 = 0$

On solving of these equations, we get $x = 2$ and $y = 23/3$.

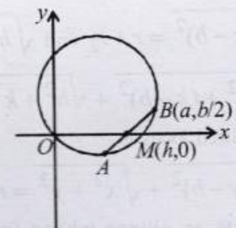
Thus the required point of intersection is $\left(2, \frac{23}{3}\right)$.

**82.** Given : A circle

$\quad 2x(x - a) + y(2y - b) = 0 \ (a, b \neq 0)$

$\Rightarrow \quad 2x^2 + 2y^2 - 2ax - by = 0 \quad \ldots$(i)

Let us consider the chord of this circle which passes through the point $\left(a, \frac{b}{2}\right)$ and whose mid point lies on $x$-axis.




Let $(h, 0)$ be the mid point of the chord, then equation of chord can be obtained by $T = S_1$

i.e., $\quad 2xh + 2y.0 - a(x + h) - \frac{b}{2}(y + 0) = 2h^2 - 2ah$

$$\Rightarrow \quad (2h - a)x - \frac{b}{2}y + ah - 2h^2 = 0$$

This chord passes through $\left(a, \frac{b}{2}\right)$,

$$\therefore \quad (2h - a)a - \frac{b}{2}.\frac{b}{2} + ah - 2h^2 = 0$$

$\Rightarrow \quad 8h^2 - 12ah + (4a^2 + b^2) = 0$

According to the question, two such chords are there, so we should have two real and distinct values of $h$ from the above quadratic in $h$.

$\therefore \quad D > 0$

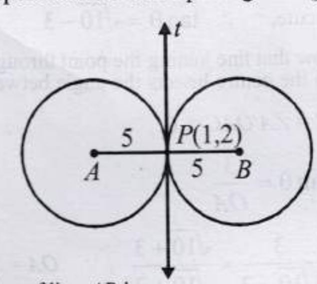
$\Rightarrow \quad (12a)^2 - 4 \times 8 \times (4a^2 + b^2) > 0 \;\Rightarrow\; a^2 > 2b^2$

**83.** Let $t$ be the common tangent given by

$\quad 4x + 3y = 10 \quad \ldots$ (i)

Common point of contact being $P\ (1, 2)$

Let $A$ and $B$ be the centres of the required circles. Clearly, $AB$ is the line perpendicular to $t$ and passing through $P\ (1, 2)$.




$\therefore$ Equation of line $AB$ is

$$\frac{x-1}{4/5} = \frac{x-2}{3/5} = r \left[\begin{array}{l}\text{As slope of } t \text{ is} = -4/3 \\ \therefore \text{slope of } AB \text{ is} = 3/4 = \tan\theta \\ \therefore \cos\theta = 4/5; \sin\theta = 3/5\end{array}\right]$$

For point $A$, $r = -5$ and for point $B$, $r = 5$, we get

$$\frac{x-1}{4/5} = \frac{y-2}{3/5} = -5, 5 \left[\begin{array}{l}\text{Radius of each circle} \\ \text{being } 5, AP = PB = 5\end{array}\right]$$

$\Rightarrow$ For point $A$, $\quad x = -4 + 1, y = -3 + 2$

and For point $B$, $\quad x = 4 + 1, y = 3 + 2$

$\therefore \quad A(-3, -1),\ B(5, 5)$.

$\therefore$ Equation of required circles are

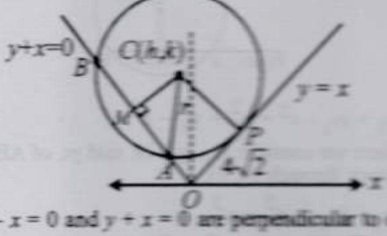
$(x+3)^2+(y+1)^2=5^2$

and $(x-5)^2+(y-5)^2=5^2$

$\Rightarrow \quad x^2+y^2+6x+2y-15=0$

and $x^2+y^2-10x-10y+25=0$

**84.** Let $AB$ be the length of chord intercepted by circle on $y+x=0$
and $CM$ be perpendicular to $AB$ from centre $C\,(h,k)$.




Also $y-x=0$ and $y+x=0$ are perpendicular to each other.

$\therefore$ $OPCM$ is rectangle. $\therefore$ $CM=OP=4\sqrt{2}$.

Let $r$ be the radius of cirlce.

Also $AM=\dfrac{1}{2}AB=\dfrac{1}{2}\times 6\sqrt{2}=3\sqrt{2}$

$\therefore$ In $\Delta CAM$, $AC^2=AM^2+MC^2$

$\Rightarrow r^2=(3\sqrt{2})^2+(4\sqrt{2})^2 \Rightarrow r^2=(5\sqrt{2})^2 \Rightarrow r=5\sqrt{2}$

Since $y=x$ is tangent to the circle at $P$

$\therefore$ $CP=r$

$\Rightarrow \left|\dfrac{h-k}{\sqrt{2}}\right|=5\sqrt{2} \Rightarrow h-k=\pm 10$ — (i)

Now $CM=4\sqrt{2}$

$\therefore \left|\dfrac{h+k}{\sqrt{2}}\right|=4\sqrt{2} \Rightarrow h+k=\pm 8$ ... (ii)

On solving (i) and (ii), we get the possible centres as $(9,-1), (1,-9), (-1,9), (-9,1)$

Hence, possible circles are

$(x-9)^2+(y+1)^2-50=0$

$(x-1)^2+(y+9)^2-50=0$

$(x+1)^2+(y-9)^2-50=0$

and $(x+9)^2+(y-1)^2-50=0$

But the point $(-10, 2)$ lies inside the circle.

$\therefore$ $S_1<0$ which is satisfied only for $(x+9)^2+(y-1)^2-50=0$

$\therefore$ The required equation of circle is $x^2+y^2+18x-2y+32=0$.

**85.** Given : $\left(m_i, \dfrac{1}{m_i}\right)$, $m_i>0$, i = 1, 2, 3, 4 are four distinct points on a circle.

Let the equation of the circle be $x^2+y^2+2gx+2fy+c=0$

As the point $\left(m, \dfrac{1}{m}\right)$ lies on it.

$\therefore \quad m^2+\dfrac{1}{m^2}+2gm+\dfrac{2f}{m}+c=0$

$\Rightarrow \quad m^4+2gm^3+cm^2+2fm+1=0$

$\because$ $m_1, m_2, m_3, m_4$ are roots of this equation, hence $m_1m_2m_3m_4=1$

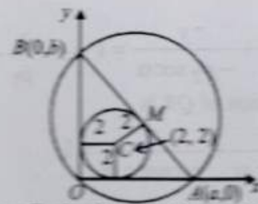
**86.** Given circle :

$x^2+y^2-4x-4y+4=0.$

$\Rightarrow \quad (x-2)^2+(y-2)^2=4,$

which has centre $C\,(2,2)$ and radius 2.

Let the equation of third side $AB$ of $\Delta OAB$ is $\dfrac{x}{a}+\dfrac{y}{b}=1$ such that $A\,(a,0)$ and $B\,(0,b)$




Length of perpendicular form $(2,2)$ on $AB$ = radius = $CM=2$

$\therefore \quad \dfrac{\left|\dfrac{2}{a}+\dfrac{2}{b}-1\right|}{\sqrt{\dfrac{1}{a^2}+\dfrac{1}{b^2}}}=2$

Since $(2,2)$ and origin lie on same side of $AB$

$\therefore \quad \dfrac{-\left(\dfrac{2}{a}+\dfrac{2}{b}-1\right)}{\sqrt{\dfrac{1}{a^2}+\dfrac{1}{b^2}}}=2 \Rightarrow \dfrac{2}{a}+\dfrac{2}{b}-1=-2\sqrt{\dfrac{1}{a^2}+\dfrac{1}{b^2}}$ ... (i)

$\because \quad \angle AOB=\pi/2$

Therefore, $AB$ is the diameter of the circle passing through the vertices of the $\Delta OAB$. Hence centre of the circle is the mid-point $\left(\dfrac{a}{2}, \dfrac{b}{2}\right)$ of the circle.

Let centre be $(h,k)\equiv\left(\dfrac{a}{2}, \dfrac{b}{2}\right)$

then $a=2h$, $b=2k$.

On putting the values of $a$ and $b$ in (i), we get

$\dfrac{2}{2h}+\dfrac{2}{2k}-1=-2\sqrt{\dfrac{1}{4h^2}+\dfrac{1}{4k^2}}$

$\Rightarrow \dfrac{1}{h}+\dfrac{1}{k}-1=-\sqrt{\dfrac{1}{h^2}+\dfrac{1}{k^2}}$

$\Rightarrow h+k-hk+\sqrt{h^2+k^2}=0$

$\therefore$ Locus of $M\,(h,k)$ is,

$x+y-xy+\sqrt{x^2+y^2}=0$ ... (ii)

Comparing it with given equation of locus of circumcentre of the triangle i.e.

$x+y-xy+k\sqrt{x^2+y^2}=0$ ...(iii)

We get, $k=1$

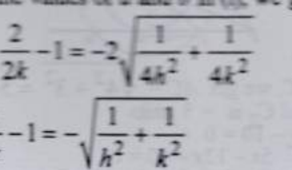
**87.** Let the equation of $L_1$ be $x\cos\alpha+y\sin\alpha=p_1$

Then any line perpendicular to $L_1$ is

$x\sin\alpha-y\cos\alpha=p_2$, where $p_2$ is a variable.

Then $L_1$ meets $x$-axis at $P\,(p_1\sec\alpha, 0)$ and $y$-axis at $Q\,(0, p_1\operatorname{cosec}\alpha)$.

Similarly $L_2$ meets $x$-axis at $R\,(p_2\operatorname{cosec}\alpha, 0)$ and $y$-axis at $S\,(0, -p_2\sec\alpha)$.




Now equation of $PS$ is

$$\frac{x}{p_1 \sec\alpha} + \frac{y}{-p_2 \sec\alpha} = 1 \Rightarrow \frac{x}{p_1} - \frac{y}{p_2} = \sec\alpha \quad \ldots \text{(i)}$$

Similarly, equation of $QR$ is

$$\Rightarrow \frac{x}{p_2 \text{cosec}\,\alpha} + \frac{y}{p_1 \text{cosec}\,\alpha} = 1$$

$$\Rightarrow \frac{x}{p_2} + \frac{y}{p_1} = \text{cosec}\,\alpha \quad \ldots \text{(ii)}$$

Locus of point of intersection of $PS$ and $QR$ can be obtained by eliminating the variable $p_2$ from (i) and (ii)

i.e. $\left(\frac{x}{p_1} - \sec\alpha\right)\frac{x}{y} + \frac{y}{p_1} = \text{cosec}\,\alpha$

[On substituting the value of $\frac{1}{p_2}$ from (i) in (ii)]

$\Rightarrow (x - p_1 \sec\alpha)\, x + y^2 = p_1\, y \,\text{cosec}\,\alpha$

$\Rightarrow x^2 + y^2 - p_1\, x \sec\alpha - p_1\, y\,\text{cosec}\,\alpha = 0$

which is a circle through origin.

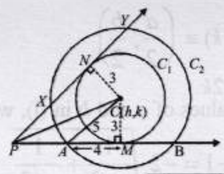
**88.** Let equation of tangent $PAB$ be $5x + 12y - 10 = 0$ and that of $PXY$ be $5x - 12y - 40 = 0$

Now let centre of circles $C_1$ and $C_2$ be $C(h, k)$.

Let $CM \perp PAB$, then $CM$ = radius of $C_1 = 3$

Also $C_2$ makes an intercept of length 8 units on $PAB$

$\Rightarrow AM = 4$




Now in $\Delta AMC$, we get $AC = \sqrt{4^2 + 3^2} = 5$

$\therefore$ Radius of $C_2$ is = 5 units

Since $5x + 12y - 10 = 0$ ... (i)

and $5x - 12y - 40 = 0$ ... (ii)

are tangents to $C_1$, therefore length of perpendicular from $C$ to $AB$ = 3 units

$$\therefore \frac{5h + 12k - 10}{13} = \pm 3$$

$\Rightarrow 5h + 12k - 49 = 0$ ... (i)

or $5h + 12k + 29 = 0$ ... (ii)

Similarly $\frac{5h - 12k - 40}{13} = \pm 3$

$\Rightarrow 5h - 12k - 79 = 0$ ... (iii)

or $5h - 12k - 1 = 0$ ... (iv)

Since $C$ lies in first quadrant,

$\therefore$ $h, k$ are + ve

$\therefore$ Equation (ii) is not possible.

On solving (i) and (iii), we get

$h = 64/5, k = -5/4$

This is also not possible.

Now solving (i) and (iv), we get $h = 5, k = 2$.

Thus centre of $C_2$ is (5, 2) and radius 5.

$\therefore$ Equation of $C_2$ is $(x - 5)^2 + (y - 2)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 10x - 4y + 4 = 0$

**89.** Let the two points be $A = (\alpha_1, \beta_1)$ and $B = (\alpha_2, \beta_2)$

Thus $\alpha_1, \alpha_2$ are roots of

$x^2 + 2ax - b^2 = 0$

$\therefore \alpha_1 + \alpha_2 = -2a$ ... (i)

and $\alpha_1 \alpha_2 = -b^2$ ... (ii)

$\beta_1, \beta_2$ are roots of $x^2 + 2px - q^2 = 0$

$\therefore \beta_1 + \beta_2 = -2p$ ... (iii)

and $\beta_1\beta_2 = -q^2$ ... (iv)

Now equation of circle with $AB$ as diameter is

$(x - \alpha_1)(x - \alpha_2) + (y - \beta_1)(y - \beta_2) = 0$

$\Rightarrow x^2 - (\alpha_1 + \alpha_2)x + \alpha_1\alpha_2 + y^2 - (\beta_1 + \beta_2)y + \beta_1\beta_2 = 0$

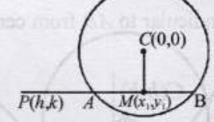
$\Rightarrow x^2 + 2ax - b^2 + y^2 + 2py - q^2 = 0$

[using equations (i), (ii), (iii) and (iv)]

$\Rightarrow x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0,$

which is the equation of required circle, with its centre $(-a, -p)$ and radius $= \sqrt{a^2 + p^2 + b^2 + q^2}$.

**90.** Equation of chord whose mid point is given is $T = S_1$




$\Rightarrow xx_1 + yy_1 - r^2 = x_1^2 + y_1^2 - r^2$

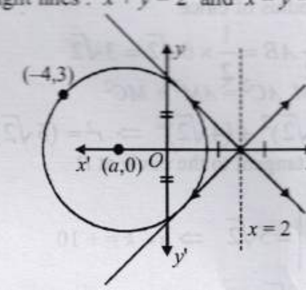
[Here we consider $(x_1, y_1)$ be mid pt. of AB]

As it passes through $(h, k)$,

$\therefore hx_1 + ky_1 = x_1^2 + y_1^2$

$\Rightarrow$ Locus of $(x_1, y_1)$ is $x^2 + y^2 = hx + ky$

**91.** Given straight lines : $x + y = 2$ and $x - y = 2$




As centre lies on angle bisector of given lines which are the lines $y = 0$ and $x = 2$.

$\therefore$ Centre lies on $x$ axis or $x = 2$.

But as it passes through $(-4, 3)$, i.e., II quadrant.

$\therefore$ Centre must lie on $x$-axis.

Let it be $(a, 0)$, then distance between $(a, 0)$ and $(-4, 3)$ = length of perpendicular distance from $(a, 0)$ to $x + y - 2 = 0$

$$\therefore (a + 4)^2 + (0 - 3)^2 = \left(\frac{a - 2}{\sqrt{2}}\right)^2$$

$\Rightarrow a^2 + 20a + 46 = 0 \Rightarrow a = -10 \pm \sqrt{54}$

$\therefore$ Equation of circle is

$[x + (10 \pm \sqrt{54}\,)]^2 + y^2 = [-(10 \pm \sqrt{54}\,) + 4]^2 + 3^2$

$\Rightarrow x^2 + y^2 + 2\,(10 \pm \sqrt{54}\,)\,x + 8\,(10 \pm \sqrt{54}\,) - 25 = 0$

$\Rightarrow x^2 + y^2 + 2\,(10 \pm \sqrt{54}\,)\,x + 55 \pm \sqrt{54} = 0.$

**92.** Equation of circle is

$x^2 + y^2 - 2x - 4y - 20 = 0$

with centre (1, 2) and radius $= \sqrt{1 + 4 + 20} = 5$

Using equation of tangent at $(x_1, y_1)$ of

$x^2 + y^2 + 2gx_1 + 2fy_1 + c = 0$ is

$xx_1 + yy_1 + g\,(x + x_1) f\,(y + y_1) + c = 0$

Equation of tangent at (1, 7) is

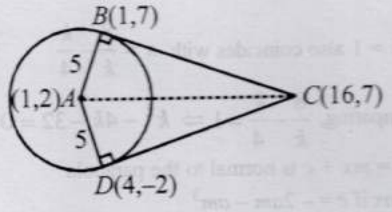
$x \,.\, 1 + y \,.\, 7 - (x + 1) - 2\,(y + 7) - 20 = 0$

$\Rightarrow y - 7 = 0$ ... (i)

Similarly, equation of tangent at (4, − 2) is

$4x - 2y - (x + 4) - 2\,(y - 2) - 20 = 0$

$\Rightarrow 3x - 4y - 20 = 0$ ... (ii)




For point $C$, solving (i) and (ii), we get
$x = 16, y = 7 \quad \therefore \quad C\,(16, 7).$
Clearly $ar$ (quad $ABCD$) = 2 $ar$ (rt $\Delta ABC$)

$= 2\times\frac{1}{2}\times AB\times BC = AB\times BC$, where

$AB$ = radius of the circle = 5
and $BC$ = length of tangent from $C$ to the circle

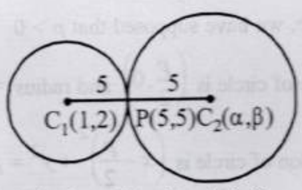
$= \sqrt{16^2+7^2-32-28-20} = \sqrt{225} = 15$

$\therefore$ $ar$ (quad $ABCD$) = 5 × 15 = 75 sq. units.

93. Given : A circle
$x^2+y^2-2x-4y-20=0$
with centre (1, 2) and radius = 5

Radius of required circle is also 5.

Let its centre be $C_2\,(\alpha, \beta)$. Both the circles touch each other at $P$ (5, 5).




Clearly $P$ (5, 5) is the mid-point of $C_1C_2$.

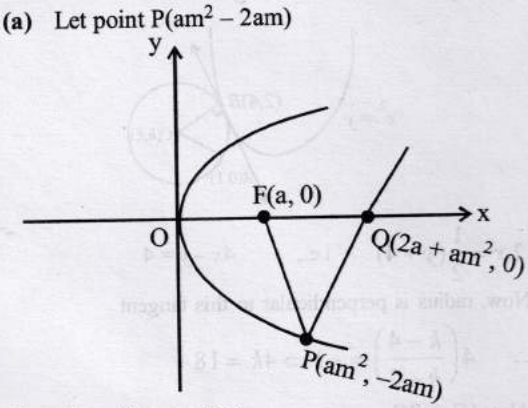
$\therefore \quad \frac{1+\alpha}{2}=5$ and $\frac{2+\beta}{2}=5 \Rightarrow \alpha=9$ and $\beta=8$

$\therefore$ Centre of required circle is (9, 8) and equation of required circle is $(x-9)^2+(y-8)^2=5^2$

$\Rightarrow \quad x^2+y^2-18x-16y+120=0$

## Topic-2: Parabola

1. (a) Let point P($am^2$ – 2am)

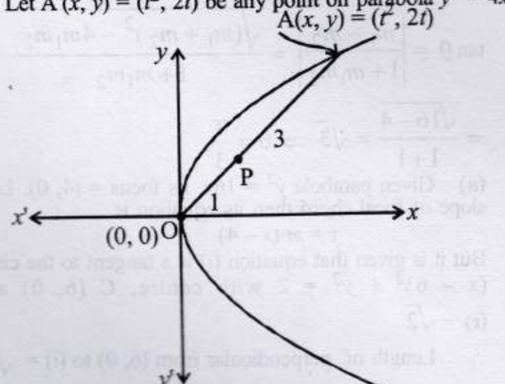



Equation of normal at
P($am^2$, – 2am) is
$y = mx - 2am - a\,m^3$
Area of ΔPFQ

$=\frac{1}{2}\,(a+am^2)\times 2am = 120$

$\Rightarrow \quad a^2m(1+m^2)=120$
pair (a, m) = (2, 3) satisfies above equation.

2. (c) Let A $(x, y) = (t^2, 2t)$ be any point on parabola $y^2 = 4x$.

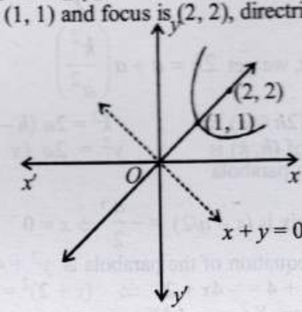



Let P $(h, k)$ divides OA in the ratio 1 : 3

$\therefore\ (h, k) = \left(\frac{t^2}{4}, \frac{2t}{4}\right) \Rightarrow h = \frac{t^2}{4}$ and $k = \frac{t}{2} \Rightarrow h = k^2$

$\therefore$ Locus of P $(h, k)$ is $x = y^2$.

3. (d) Since, distance of vertex and focus of the parabola from origin is $\sqrt{2}$ and $2\sqrt{2}$.
$\therefore$ Vertex is (1, 1) and focus is (2, 2), directrix $x + y = 0$




$\therefore$ Equation of parabola is

$(x-2)^2+(y-2)^2 = \left(\frac{x+y}{\sqrt{2}}\right)^2$

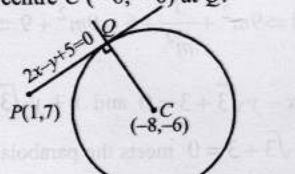
$\Rightarrow 2(x^2-4x+4)+2(y^2-4y+4) = x^2+y^2+2xy$

$\Rightarrow \quad x^2+y^2-2xy = 8\,(x+y-2) \Rightarrow (x-y)^2 = 8\,(x+y-2)$

4. (d) The given curve is $y = x^2 + 6$
Equation of tangent at (1, 7) is

$\frac{1}{2}(y+7) = x.1+6 \Rightarrow 2x-y+5=0 \quad$ ...(i)

Since tangent (i) touches the circle
$x^2+y^2+16x+12y+c=0$
with centre $C$ (– 8, – 6) at $Q$.




$\therefore$ Equation of $CQ$ which is perpendicular to (i) is

$y+6 = -\frac{1}{2}(x+8) \Rightarrow x+2y+20=0 \quad$ ...(ii)

On solving equation (i) and (ii) we get the co-ordinate of $Q$ as
$x = -6, y = -7$
$\therefore$ Co-ordinate of $Q$ is (– 6, – 7).

5. (c) If $m$ be the slope of the tangent to the parabola, then its equation is $\quad y = mx + 1/m$
Since the tangent passes through (1, 4)
$\therefore \quad 4 = m + 1/m \Rightarrow m^2 - 4m + 1 = 0$
If angle between two tangents to the parabola be θ, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2}$$

$$= \frac{\sqrt{16-4}}{1+1} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$$

**6.** **(a)** Given parabola $y^2 = 16x$, its focus = (4, 0). Let m be the slope of focal chord then its equation is

$$y = m(x - 4) \quad \text{...(i)}$$

But it is given that equation (i) is a tangent to the circle $(x - 6)^2 + y^2 = 2$ with centre, $C$ (6, 0) and radius (r) $= \sqrt{2}$

$\therefore$ Length of perpendicular from (6, 0) to (i) $= \sqrt{2}$

$$\Rightarrow \frac{6m - 4m}{\sqrt{m^2+1}} = \sqrt{2} \Rightarrow \quad 2m = \sqrt{2(m^2+1)}$$

$\Rightarrow$ $2m^2 = m^2 + 1 \Rightarrow m = \pm 1$

**7.** **(c)** If $(h, k)$ is the mid point of line joining focus $(a, 0)$ and $Q$ $(at^2, 2at)$ on parabola then $h = \dfrac{a + at^2}{2}, k = at$

Eliminating $t$, we get $2h = a + a\left(\dfrac{k^2}{a^2}\right)$

$\Rightarrow$ $k^2 = a(2h - a) \Rightarrow$ $k^2 = 2a(h - a/2)$

$\therefore$ Locus of $(h, k)$ is $y^2 = 2a(x - a/2)$, which is equation of a parabola

whose directrix is $(x - a/2) = -\dfrac{a}{2} \Rightarrow x = 0$

**8.** **(d)** Given equation of the parabola is $y^2 + 4y + 4x + 2 = 0$

$\Rightarrow$ $y^2 + 4y + 4 = -4x + 2 \Rightarrow (y + 2)^2 = -4(x - 1/2)$

It is of the form $Y^2 = -4AX$.

Equation of whose directrix is given by $X = A$

$\therefore$ Equation of required directrix is $x - 1/2 = 1 \Rightarrow x = 3/2$.

**9.** **(c)** Let the equation of tangent to the parabola $y^2 = 4x$ be $y = mx + \dfrac{1}{m}$, where $m$ is the slope of the tangent.

If $y = mx + \dfrac{1}{m}$ is also tangent to the circle $(x - 3)^2 + y^2 = 9$, then length of perpendicular to the tangent from centre (3, 0) should be equal to the radius 3.

ie, $$\frac{3m + \frac{1}{m}}{\sqrt{m^2+1}} = 3 \Rightarrow 9m^2 + \frac{1}{m^2} + 6 = 9m^2 + 9 \Rightarrow m = \pm\frac{1}{\sqrt{3}}$$

$\therefore$ Tangents are $x - y\sqrt{3} + 3 = 0$ and $x + y\sqrt{3} + 3 = 0$

out of which $x - y\sqrt{3} + 3 = 0$ meets the parabola at

$(3, 2\sqrt{3})$ i.e., above $x$-axis.

**10.** **(c)** The directrix of the parabola $y^2 = 4a(x - x_1)$ is given by $x = x_1 - a$.

Now given parabola is

$$y^2 = kx - 8 \Rightarrow y^2 = k\left(x - \frac{8}{k}\right)$$

$\therefore$ Directrix of parabola is $x = \dfrac{8}{k} - \dfrac{k}{4}$;

Now, $x = 1$ also coincides with $x = \dfrac{8}{k} - \dfrac{k}{4}$

On comparing, $\dfrac{8}{k} - \dfrac{k}{4} = 1 \Rightarrow k^2 - 4k - 32 = 0 \therefore k = 4$

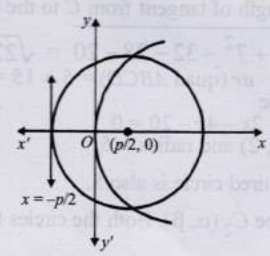
**11.** **(b)** $y = mx + c$ is normal to the parabola $y^2 = 4ax$ if $c = -2am - am^3$

Now given line $x + y = k$ normal to $y^2 = 12x$

$\therefore$ $m = -1, c = k$ and $a = 3$

$\Rightarrow$ $c = k = -2(3)(-1) - 3(-1)^3 = 9$

**12.** **(a)** The focus of parabola $y^2 = 2px$ is $\left(\dfrac{p}{2}, 0\right)$ and directrix $x = -p/2$




In the figure, we have supposed that $p > 0$

$\therefore$ Centre of circle is $\left(\dfrac{p}{2}, 0\right)$ and radius $= \dfrac{p}{2} + \dfrac{p}{2} = p$

$\therefore$ Equation of circle is $\left(x - \dfrac{p}{2}\right)^2 + y^2 = p^2$

For points of intersection of $y^2 = 2px$ ...(i)

and $4x^2 + 4y^2 - 4px - 3p^2 = 0$ ...(ii)

can be obtained by solving (i) and (ii) as follows

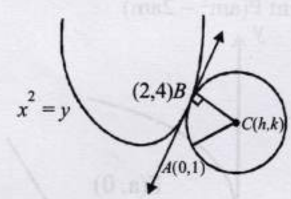
$$4x^2 + 8px - 4px - 3p^2 = 0 \Rightarrow (2x + 3p)(2x - p) = 0$$

$\Rightarrow$ $x = \dfrac{-3p}{2}, \dfrac{p}{2}$

$\Rightarrow$ $y^2 = -3p^2$ (not possible), $p^2 \Rightarrow y = \pm p$

$\therefore$ Required points are $(p/2, p)$ and $(p/2, -p)$

**13.** **(c)** Let $C(h, k)$ be the centre of circle touching $x^2 = y$ at $B$ (2, 4). Then equation of common tangent at $B$ is




$2.x = \dfrac{1}{2}(y + 4)$ i.e., $4x - y = 4$

Now, radius is perpendicular to this tangent

$\therefore$ $4\left(\dfrac{k-4}{h-2}\right) = -1 \Rightarrow 4k = 18$ ... (i)

Also $AC = BC$

$\therefore$ $h^2 + (k - 1)^2 = (h - 2)^2 + (k - 4)^2$

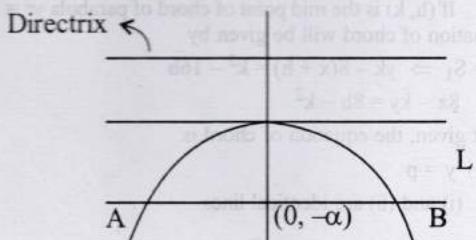
$\Rightarrow$ $4h + 6k = 19$ ... (ii)

On solving (i) and (ii), we get the centre as $\left(-\dfrac{16}{5}, \dfrac{53}{10}\right)$.

**14.** **(12)** $x^2 = -4ay$

Equation of normal

$$y = mx - 2a - \frac{a}{m^2}$$




$$-\alpha = -2a - \frac{a}{\frac{1}{6}} = -8a$$

$\Rightarrow \alpha = 8a$

Equation of required line passing through $(0, -\alpha)$ and parallel to directrix is

$y = -\alpha \Rightarrow y = -8a$, Solving with $x^2 = -4ay$

$$\Rightarrow x^2 = 32a^2 \Rightarrow x = \pm 4\sqrt{2}a = \pm\frac{\alpha}{\sqrt{2}}$$

$$A\left(\frac{\alpha}{\sqrt{2}}, -\alpha\right), B\left(\frac{-\alpha}{\sqrt{2}}, -\alpha\right) \Rightarrow AB = \sqrt{2}\alpha$$

$$\Rightarrow \frac{r}{s} = \frac{4a}{2\alpha^2} = \frac{1}{16} \Rightarrow \frac{4a}{2\times 64a^2} = \frac{1}{16}$$

$$\Rightarrow a = \frac{1}{2} \Rightarrow 24a = 12$$

**15.** **(4)** Let $(t^2, 2t)$ be any point on $y^2 = 4x$ and $(h, k)$ be the image of $(t^2, 2t)$ in the line $x + y + 4 = 0$.

$$\therefore \quad \frac{h - t^2}{1} = \frac{k - 2t}{1} = \frac{-2(t^2 + 2t + 4)}{2}$$

$\Rightarrow \quad h = -(2t + 4)$ and $k = -(t^2 + 4)$

For its intersection with, $y = -5$, we get

$-(t^2 + 4) = -5 \quad \Rightarrow t = \pm 1$

$\therefore \quad A(-6, -5)$ and $B(-2, -5)$, $\quad \therefore AB = 4$.

**16.** **(2)** End points of latus rectum of $y^2 = 4x$ are $(1, \pm 2)$

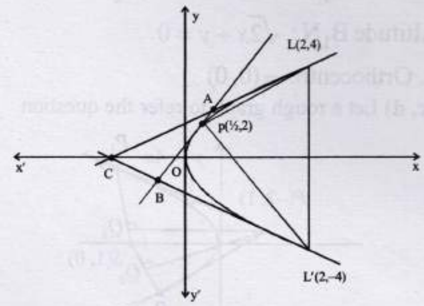
Equation of normal to $y^2 = 4x$ at $(1, 2)$ is

$y - 2 = -1(x - 1) \Rightarrow x + y - 3 = 0$

As it is tangent to circle $(x - 3)^2 + (y + 2)^2 = r^2$

$$\therefore \quad \left|\frac{3 + (-2) - 3}{\sqrt{2}}\right| = r \Rightarrow r^2 = 2$$

**17.** **(2)** $\Delta_1 = \text{area}(\Delta PLL') = \frac{1}{2}\times 8 \times \frac{3}{2} = 6$




Equation of AB is $y = 2x + 1$; equation of AC is $y = x + 2$ and equation of BC is $-y = x + 2$.

On solving the above equations pair wise, we get

A (1, 3), B (−1, −1) and C (−2, 0)

$$\therefore \quad \Delta_2 = \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ -1 & -1 & 1 \\ -2 & 0 & 1 \end{vmatrix} = 3, \qquad \therefore \frac{\Delta_1}{\Delta_2} = 2$$

**18.** **(−1, 1)** Given parabola is $y^2 = 4x$; $a = 1$

Tangent to ends of latus rectum are $(1, 2)$ and $(1, -2)$.

$y^2 = 4x$ at $(1, 2)$ is $y.2 = 2(x + 1) \Rightarrow y = x + 1$ ...(i)

Similarly tangent at $(1, -2)$ is, $y = -x - 1$ ...(ii)

Point of intersection of these tangents can be obtained by solving (i) and (ii), which is $(-1, 0)$.

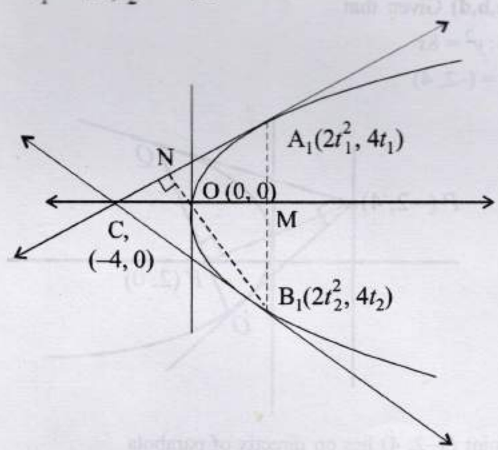
**19.** **(d, c)** Let parametric coordinates of $A_1$ and $B_1$ are

$A_1 = (2t_1^2, 4t_1)$ and $B_1 = (2t_2^2, 4t_2)$

$C \equiv (-4, 0) \equiv (2t_1t_2, 2(t_1 + t_2))$

$\Rightarrow t_2 = -t_1$ and $t_1(-t_1) = -2$

$t_1 = \sqrt{2}, t_2 = -\sqrt{2}$




$$A_1 \equiv (4, 4\sqrt{2}), B_1 \equiv (4, -4\sqrt{2})$$

$$\therefore OA_1 = \sqrt{4^2 + (4\sqrt{2})^2} = 4\sqrt{3}$$

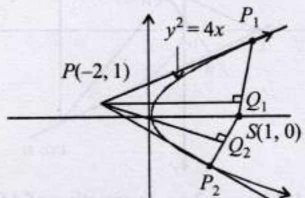
Length of line segment $A_1B_1 = 8\sqrt{2}$

Altitude $C_1M : y = 0$ ...(i)

Altitude $B_1N : \sqrt{2}x + y = 0$ ...(ii)

$\therefore$ Orthocentre $\equiv (0, 0)$

**20.** **(a, b, c, d)** Let a rough graph to refer the question




Let parametric point at $P_1$ $(t^2, 2t)$ then tangent at $P_1$

$ty = x + t^2$

Since it passes through (–2, 1)

$\therefore \quad t^2 - t - 2 = 0$

$\therefore \quad t = 2, -1$

$\therefore \quad P_1(4, 4)$ and $P_2(1, -2)$

$\therefore \quad SP_1 : 4x - 3y - 4 = 0$ and $SP_2 : x - 1 = 0$

and for $Q_1 : \dfrac{x_1 + 2}{4} = \dfrac{y_1 - 1}{-3} = \dfrac{-(-8-3-4)}{25} = \dfrac{3}{5}$

$\therefore \quad x_1 = \dfrac{2}{5}, y_1 = \dfrac{-4}{5}$ and $Q_2 = (1, 1)$

So, $SQ_1 = \sqrt{\left(1 - \dfrac{2}{5}\right)^2 + \left(\dfrac{4}{5}\right)^2} = 1$

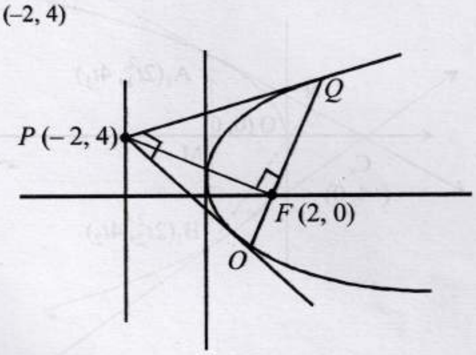
$Q_1Q_2 = \sqrt{\dfrac{9}{25} + \dfrac{81}{25}} = \sqrt{\dfrac{90}{25}} = \dfrac{3\sqrt{10}}{5}$

$PQ_1 = \sqrt{\dfrac{144}{25} + \dfrac{81}{25}} = \dfrac{15}{5} = 3\,;\, SQ_2 = 1$

**21.** **(a,b,d)** Given that

$E : y^2 = 8x$

$P = (-2, 4)$




Point $P(-2, 4)$ lies on directrix of parabola

So, $\angle QPQ' = \dfrac{\pi}{2}$ and chord $QQ'$ is a focal chord and segment $PQ$ subtends right angle at the focus. So, $\angle PFQ = \dfrac{\pi}{2}$

Slope of $QQ' = \dfrac{2}{t_1 + t_2} = 1$

Slope of $PF = -1 \therefore QQ' \perp PF$

$PF = 4\sqrt{2}$

**22.** **(c)** If (h, k) is the mid point of chord of parabola $y^2 = 16x$, then equation of chord will be given by

$T = S_1 \Rightarrow yk - 8(x + h) = k^2 - 16h$

$\Rightarrow \quad 8x - ky = 8h - k^2$ ...(i)

But given, the equation of chord is

$2x + y = p$ ...(ii)

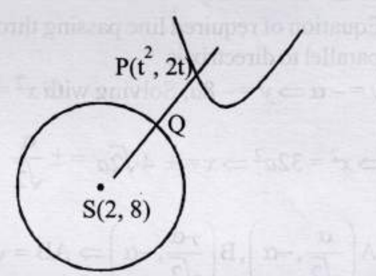
$\therefore$ (i) and (ii) are identical lines

$\Rightarrow \quad \dfrac{8}{2} = \dfrac{-k}{1} = \dfrac{8h - k^2}{p} \Rightarrow k = -4$ and $p = 2h - 4$

which are satisfied by option (c).

**23.** **(a, c, d)** Let point P on parabola $y^2 = 4x$ be $(t^2, 2t)$

$\because$ PS is shortest distance, therefore PS should be the normal to parabola.




Equation of normal to $y^2 = 4x$ at P $(t^2, 2t)$ is

$y - 2t = -t(x - t^2)$

It passes through S(2, 8)

$\therefore \quad 8 - 2t = -t(2 - t^2)$

$\Rightarrow \quad t^3 = 8$ or $t = 2$, $\therefore$ P(4, 4)

Also slope of tangent to circle at $Q = \dfrac{-1}{\text{Slope of PS}} = \dfrac{1}{2}$

Equation of normal at t = 2 is 2x + y = 12

Clearly x-intercept = 6, Now SP = $2\sqrt{5}$ and SQ = r = 2

$\therefore$ Q divides SP in the ratio SP : PQ

$= 2 : 2\left(\sqrt{5} - 1\right)$ or $\left(\sqrt{5} + 1\right) : 4$

**24.** **(a, b, c)** Given circle, $C_1 : x^2 + y^2 = 3$ ..(i)

and parabola : $x^2 = 2y$ ...(ii)

Intersection point of (i) and (ii) in first quadrant

$y^2 + 2y - 3 = 0 \Rightarrow y = 1 \quad (\because y \neq -3)$

$\therefore \; x = \sqrt{2} \Rightarrow P(\sqrt{2}, 1)$

Equation of tangent to circle $C_1$ at P is $\sqrt{2}x + y - 3 = 0$

Let centre of circle $C_2$ be (0, k); r = $2\sqrt{3}$

$\therefore \left|\frac{k-3}{\sqrt{3}}\right| = 2\sqrt{3} \Rightarrow k = 9 \text{ or } -3$

$\therefore Q_2(0, 9), Q_3(0, -3)$

(a) $Q_2Q_3 = 12$

(b) $R_2R_3$ = length of transverse common tangent

$= \sqrt{(Q_2Q_3)^2 - (r_1 + r_2)^2} = \sqrt{(12)^2 - (4\sqrt{3})^2} = 4\sqrt{6}$

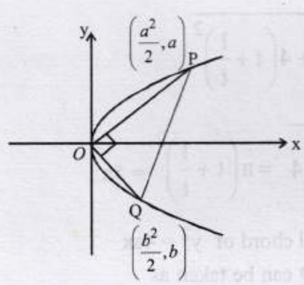
(c) Area $(\Delta OR_2R_3) = \frac{1}{2} \times R_2R_3 \times$ length of $\perp$ from origin to

tangent $= \frac{1}{2} \times 4\sqrt{6} \times \sqrt{3} = 6\sqrt{2}$

(d) $ar(\Delta PQ_2Q_3) = \frac{1}{2} \times Q_2Q_3 \perp$ distance of P from y-axis

$= \frac{1}{2} \times 12 \times \sqrt{2} = 6\sqrt{2}$

**25.** **(a, d)** Let point P be in first quadrant and lying on parabola $y^2 = 2x$ be $\left(\frac{a^2}{2}, a\right)$. Let $Q$ be the point $\left(\frac{b^2}{2}, b\right)$. Clearly $a > 0$.




$\because$ $PQ$ is the diameter of circle through $P$, $O$ and $Q$

$\therefore \angle POQ = 90° \Rightarrow \frac{a}{a^2/2} \times \frac{b}{b^2/2} = -1 \Rightarrow ab = -4$

$\Rightarrow$ $b$ is negative. ($\because a > 0$) Given area $(\Delta POQ) = 3\sqrt{2}$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{a^2}{2} & a & 1 \\ \frac{b^2}{2} & b & 1 \end{vmatrix} = 3\sqrt{2}$$

$\Rightarrow \frac{1}{4}ab(a-b) = \pm 3\sqrt{2} \Rightarrow a - b = \pm 3\sqrt{2}$

As a is positive and $b$ is negative, we have $a - b = 3\sqrt{2}$

$\Rightarrow a + \frac{4}{a} = 3\sqrt{2}$ ($\because ab = -4$)

$\Rightarrow a^2 - 3\sqrt{2}\,a + 4 = 0 \Rightarrow (a - 2\sqrt{2})(a - \sqrt{2}) = 0$

$\therefore a = 2\sqrt{2}, \sqrt{2}$

$\therefore$ Point $P$ can be $\left(\frac{(2\sqrt{2})^2}{2}, 2\sqrt{2}\right)$ or $\left(\frac{(\sqrt{2})^2}{2}, \sqrt{2}\right)$

i.e. $(4, 2\sqrt{2})$ or $(1, \sqrt{2})$

**26.** **(a, b, d)** The equation of normal to $y^2 = 4x$ is $y = mx - 2m - m^3$

Since the normal passes through (9, 6), $\therefore 6 = 9m - 2m - m^3$

$\Rightarrow m^3 - 7m + 6 = 0 \Rightarrow (m-1)(m^2 + m - 6) = 0$

$\Rightarrow (m-1)(m+3)(m-2) = 0 \Rightarrow m = 1, 2, -3$

$\therefore$ Normal is $y - x + 3 = 0$ or $y - 2x + 12 = 0$ or $y + 3x - 33 = 0$

**27.** **(c, d)** Given equation of parabola is $y^2 = 4x$

Its axis is x-axis.

Let $A(t_1^2, 2t_1)$ and $B(t_2^2, 2t_2)$

Then centre of circle drawn with AB as diameter is

$\left(\frac{t_1^2 + t_2^2}{2}, t_1 + t_2\right)$

As circle touches axis of parabola i.e., x-axis

$\therefore r = |t_1 + t_2| \Rightarrow t_1 + t_2 = \pm r$

$\therefore$ Slope of AB $= \frac{2(t_2 - t_1)}{t_2^2 - t_1^2} = \frac{2}{t_2 + t_1} = \pm\frac{2}{r}$

**28.** **(a, d)** Let $P(at^2, 2at)$ be any point on the parabola $y^2 = 4ax$.

$\therefore$ Tangent to the parabola at $P$ is $y = \frac{x}{t} + at$,

which meets the axis of parabola *i.e x*-axis *at* $T(-at^2, 0)$.

Also normal to parabola at $P$ is $tx + y = 2at + at^3$

which meets the axis of parabola at $N(2a + at^2, 0)$

Let $G(x, y)$ be the centriod of $\Delta PTN$, then

$x = \frac{at^2 - at^2 + 2a + at^2}{3}$ and $y = \frac{2at}{3}$

$\Rightarrow x = \frac{2a + at^2}{3}$ ... (i) and $y = \frac{2at}{3}$ ... (ii)

Eliminating $t$ from (i) and (ii), we get the locus of centriod $G$ as

$3x = 2a + a\left(\frac{3y}{2a}\right)^2 \Rightarrow y^2 = \frac{4a}{3}\left(x - \frac{2}{3}a\right)$,

which is a parabola with vertex $\left(\frac{2a}{3}, 0\right)$, directrix as

$x - \frac{2a}{3} = -\frac{a}{3} \Rightarrow x = \frac{a}{3}$, latus rectum as $\frac{4a}{3}$ and focus as $(a, 0)$.

**29.** **(a, b)** If $y = mx + c$ is tangent to $y = x^2$ then $x^2 - mx - c = 0$ has equal roots

$\Rightarrow \quad m^2 + 4c = 0 \Rightarrow c = -\dfrac{m^2}{4}$

$\therefore \quad y = mx - \dfrac{m^2}{4}$ is tangent to $y = x^2$

$\therefore$ This is also tangent to $y = -(x-2)^2$

$\Rightarrow \quad mx - \dfrac{m^2}{4} = -x^2 + 4x - 4$

$\Rightarrow \quad x^2 + (m-4)x + \left(4 - \dfrac{m^2}{4}\right) = 0$ has equal roots

$\therefore \quad m^2 - 8m + 16 = -m^2 + 16 \Rightarrow m = 0, 4$

$\therefore \quad y = 0$ or $y = 4x - 4$ are the tangents.

**30.** **(A) → (p); (B) → (q); (C) → (s); (D) → (r)**

Let $y = mx - 2m - m^3$ be the equation of normal to $y^2 = 4x$.

Since it passes through (3, 0), $\therefore \ m = 0, 1, -1$

Hence three points on parabola are given by $(m^2, -2m)$ for $m = 0, 1, -1$

$\therefore \quad P(0, 0)$, Q (1, 2) and $R(1, -2)$

$\therefore \quad$ Area $(\Delta PQR) = \dfrac{1}{2} \times \begin{vmatrix} 0 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 2$

Radius of circum-circle, $R = \dfrac{abc}{4\Delta} = \dfrac{\sqrt{5} \times \sqrt{5} \times 4}{4 \times 2} = \dfrac{5}{2}$

(where, $a, b, c$ are the sides of $\Delta PQR$)

Centroid of $\Delta PQR = \left(\dfrac{2}{3}, 0\right)$ ; Circumcentre $= \left(\dfrac{5}{2}, 0\right)$

**31.** **(d)** $\because PQ$ is a focal chord, $Q\left(\dfrac{a}{t^2}, \dfrac{-2a}{t}\right)$

Also $QR \parallel PK$, $\therefore \quad m_{QR} = m_{PK}$

$$\Rightarrow \frac{-\frac{2a}{t} - 2ar}{\frac{a}{t^2} - ar^2} = \frac{0 - 2at}{2a - at^2}$$

$$\Rightarrow \frac{-2a\left(\frac{1}{t} + r\right)}{a\left(\frac{1}{t} + r\right)\left(\frac{1}{t} - r\right)} = \frac{-2at}{a\left(2 - t^2\right)} \Rightarrow 2 - t^2 = t\left(\frac{1}{t} - r\right)$$

$\left[\because r \neq -\dfrac{1}{t}\right.$ otherwise $Q$ will coincide with $\left.R\right]$

$\Rightarrow 2 - t^2 = 1 - tr \Rightarrow r = \dfrac{t^2 - 1}{t}$

**32.** **(b)** Tangent at $P$ is $ty = x + at^2$ ....(i)

Normal at S is $sx + y = 2as + as^3$ ....(ii)

Given $st = 1 \Rightarrow s = \dfrac{1}{t}$

$\therefore \dfrac{x}{t} + y = \dfrac{2a}{t} + \dfrac{a}{t^3} \Rightarrow xt^2 = yt^3 = 2at^2 + a$

Now putting the value of $x$ from equation (i) in above equation, we get

$\Rightarrow t^2\left(ty - at^2\right) + yt^3 = 2at^2 + a$

$\Rightarrow 2t^3 y = a\left(t^4 + 2t^2 + 1\right)$

$\therefore \ y = \dfrac{a\left(t^4 + 2t^2 + 1\right)}{2t^3} = \dfrac{a\left(t^2 + 1\right)^2}{2t^3}$

**33.** **(b)** $PQ = \sqrt{\left(at^2 - \dfrac{a}{t^2}\right)^2 + \left(2at + \dfrac{2a}{t}\right)^2}$

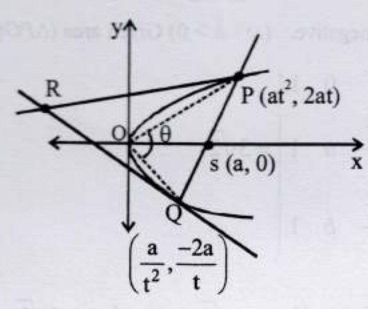
$= a\sqrt{\left(t + \dfrac{1}{t}\right)^2\left(t - \dfrac{1}{t}\right)^2 + 4\left(t + \dfrac{1}{t}\right)^2}$

$= a\left(t + \dfrac{1}{t}\right)\sqrt{\left(t - \dfrac{1}{t}\right)^2 + 4} = a\left(t + \dfrac{1}{t}\right)^2 = 5a$

**34.** **(d)** Since PQ is the focal chord of $y^2 = 4ax$

$\therefore$ Coordinates of P and Q can be taken as

$P(at^2, 2at)$ and $Q\left(\dfrac{a}{t^2}, \dfrac{-2a}{t}\right)$




Equation of tangents at P and Q are

$y = \dfrac{x}{t} + at$ and $y = -xt - \dfrac{a}{t}$,

which intersect each other at $R\left(-a, a\left(t - \dfrac{1}{t}\right)\right)$

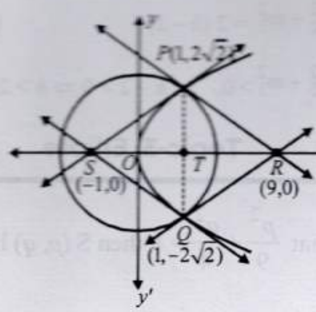
As R lies on the $y = 2x + a$, $a > 0$

$\therefore a\left(t - \dfrac{1}{t}\right) = -2a + a \Rightarrow t - \dfrac{1}{t} = -1 \Rightarrow t + \dfrac{1}{t} = \sqrt{5}$

Now, $m_{OP} = \frac{2}{t}$ and $m_{OQ} = -2t$

$$\therefore \tan\theta = \frac{\frac{2}{t}+2t}{1-4} = \frac{2\left(t+\frac{1}{t}\right)}{-3} = \frac{2\sqrt{5}}{-3}$$

**For Qs. 35-37 :**




**35.** **(c)** $\frac{\text{area }(\Delta PQS)}{\text{area }(\Delta PQR)} = \frac{\frac{1}{2}PQ\times ST}{\frac{1}{2}PQ\times TR} = \frac{ST}{TR} = \frac{2}{8} = \frac{1}{4}$

**36.** **(b)** For $\Delta PRS$,

area $(\Delta PRS) = \Delta = \frac{1}{2}\times SR\times PT = \frac{1}{2}\times 10\times 2\sqrt{2} = 10\sqrt{2}$,

$a = PS = 2\sqrt{3}$, $b = PR = 6\sqrt{2}$, $c = SR = 10$

$\therefore$ Radius of circumference of $\Delta PRS$,

$$R = \frac{abc}{4\Delta} = \frac{2\sqrt{3}\times 6\sqrt{2}\times 10}{4\times 10\sqrt{2}} = 3\sqrt{3}$$

**37.** **(d)** Radius of incircle

$$= \frac{\Delta}{s} = \frac{\text{area }(\Delta PQR)}{\text{semi perimeter of }\Delta PQR}$$

Here $a = PR = 6\sqrt{2}$, $b = QP = PR = 6\sqrt{2}$

and $c = PQ = 4\sqrt{2}$

and area $(\Delta PQR) = \frac{1}{2}\times PQ\times TR = 16\sqrt{2}$

$\therefore$ Perimeter of $\Delta PQR = \frac{6\sqrt{2}+6\sqrt{2}+4\sqrt{2}}{2} = 8\sqrt{2}$,

$$\therefore \quad r = \frac{16\sqrt{2}}{8\sqrt{2}} = 2$$

**38.** **(a)** Given curve is $y = -\frac{x^2}{2} + x + 1$

$\Rightarrow$ $(x-1)^2 = -2(y-3/2)$, which is a parabola. It is symmetric with respect to its axis $x - 1 = 0$

$\therefore$ Both the statements are true and statement-2 is a correct explanation for statement-1.

**39.** Let $P$ be the point $(h, k)$. Then equation of normal to parabola $y^2 = 4x$ from point $(h, k)$, if m is the slope of normal, is $y = mx - 2m - m^3$

Since, it passes through $(h, k)$,

$\therefore$ $mh - k - 2m - m^3 = 0 \Rightarrow m^3 + (2-h)m + k = 0$ ...(i)

which gives three values of $m$ say $m_1$, $m_2$ and $m_3$.

$\therefore$ $m_1 m_2 m_3 = -k$. Now given $m_1 m_2 = \alpha$

$\therefore$ $m_3 = -\frac{k}{\alpha}$, which must satisfy equation (i)

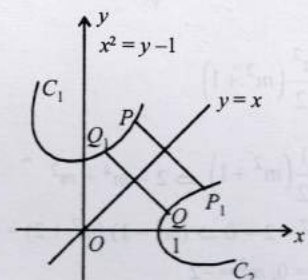
$$\therefore \quad \frac{-k^3}{\alpha^3} + (2-h)\left(\frac{-k}{\alpha}\right) + k = 0$$

$\Rightarrow$ $k^2 - 2\alpha^2 - h\alpha^2 - \alpha^3 = 0$

$\therefore$ Locus of $P$ $(h, k)$ is $y^2 = \alpha^2 x + (\alpha^3 - 2\alpha^2)$

Since locus of $P$ is a part of parabola $y^2 = 4x$, hence comparing the two, we get $\alpha^2 = 4$ and $\alpha^3 - 2\alpha^2 = 0 \Rightarrow \alpha = 2$

**40.** Reflection of A(x,y) in y = x in B(y, x). Let coordinates of P be $(t, t^2 + 1)$. Reflection of p in y = x is p, $(t^2 + 1, t)$




which clearly lies on
$y^2 = x - 1$.

Similarly let co-ordinates of Q be $(s^2 + 1, s)$.

It is reflection in $y = x$ is $Q$ $(s, s^2 + 1)$ which lies on $x^2 = y - 1$. We have.

$$PQ_1^2 = (t-s)^2 + (t^2 - s^2)^2 = P_1Q^2$$

$\Rightarrow PQ_1 = P_1Q$

Also, $PP_1 \| QQ_1$ (both are perpendicular to $y = x$)

Thus, $PP_1$ $QQ_1$ is an isosceles trapezium.

Also, $P$ lies in $PQ_1$ and $Q$ lies $P_1Q$, we have

$PQ \geq \min\{PP_1, QQ_1\}$

Let us take $\min\{PP_1, QQ_1\} = PP_1$

$PQ^2 \geq PP_1{}^2 = (t^2 + 1 - t)^2 + (t^2 + 1 - t)^2$

$= 2(t^2 + 1 - t)^2 = f(t)$ (say)

we have $f'(t) = 4(t^2 + 1 - t)(2t - 1)$

$$= 4\left[\left(t - \frac{1}{2}\right)^2 + \frac{3}{4}\right](2t-1)$$

Now $f'(t) = 0 \Rightarrow t = 1/2$

Also, $f'(t) < 0$ for $t < 1/2$ and $f'(t) > 0$ for $t > 1/2$ thus $f(t)$ is least when $t = 1/2$

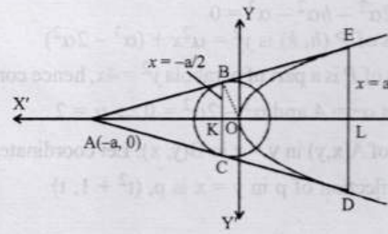
Corresponding to $t = 1/2$, point $P_0$ on $C_1$ is (1/2, 5/4) and $Q_0$ on $C_2$ are (5/4, 1/2).

Note that $P_0Q_0 \le PQ$ for all points of $(P, Q)$ with $P$ on $C_1$ and $Q$ on $C_2$.

**41.** Equation of any tangent to the parabola.

$$y = mx + \frac{a}{m}.$$

This line will touch the circle $x^2 + y^2 = \frac{a^2}{2}$




If $\left(\frac{a}{m}\right)^2 = \frac{a^2}{2}\left(m^2+1\right)$

$\Rightarrow \quad \frac{1}{m^2} = \frac{1}{2}\left(m^2+1\right) \Rightarrow 2 = m^4 + m^2$

$\Rightarrow \quad m^4 + m^2 - 2 = 0 \Rightarrow (m^2 - 1)(m^2 + 2) = 0$

$\Rightarrow \quad m^2 - 1 = 0, m^2 = -2$

$\Rightarrow \quad m = \pm 1 \quad [m^2 = -2$ is not possible]

Therefore, two common tangents are

$y = x + a \quad$ and $\quad y = -x - a$

These two intersect at $A(-a, 0)$.

The chord of contact of A $(-a, 0)$ for the parabola $y^2 = 4ax$ is 0.

$y = 2a(x - a) \Rightarrow x = a$

Again, length of $BC = 2\,BK$

$$= 2\sqrt{OB^2 - OK^2} = 2\sqrt{\frac{a^2}{2} - \frac{a^2}{4}} = 2\sqrt{\frac{a^2}{4}} = a$$

and we know that, DE is the latusrectum of the parabola, so its length is 4a.

Thus, area of the quadrilateral BCDE

$$= \frac{1}{2}(BC + DE)(KL) = \frac{1}{2}(a + 4a)\left(\frac{3a}{2}\right) = \frac{15a^2}{4}$$

**42.** The equation of a normal to the parabola $y^2 = 4ax$ in its slope form is given by $y = mx - 2am - am^3$

$\therefore$ Eq. of normal to $y^2 = 4x$, is $y = mx - 2m - m^3$ ...(i)

Since the normal drawn at three different points on the parabola pass through $(h, k)$, it must satisfy the equation (i)

$\therefore \quad k = mh - 2m - m^3 \Rightarrow m^3 - (h - 2)m + k = 0$

It has three different roots say $m_1, m_2, m_3$

$\therefore \quad m_1 + m_2 + m_3 = 0$ ...(ii)

$m_1m_2 + m_2m_3 + m_3m_1 = -(h - 2)$ ...(iii)

On squaring (ii), we get

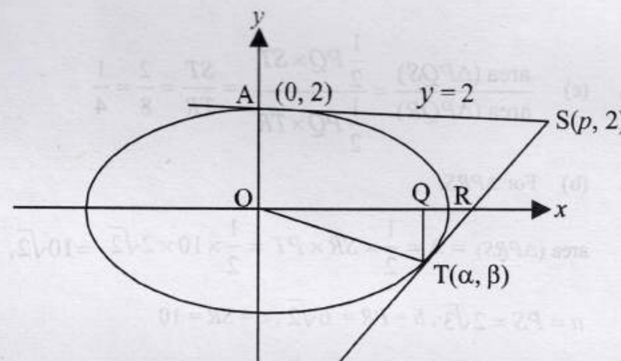
$$m_1^2 + m_2^2 + m_3^2 = -2(m_1m_2 + m_2m_3 + m_3m_1)$$

$\Rightarrow \quad m_1^2 + m_2^2 + m_3^2 = 2(h - 2)$ [using (iii)]

$\because \quad m_1^2 + m_2^2 + m_3^2 > 0, \ \therefore \ h - 2 > 0 \Rightarrow h > 2$

## Topic-3: Ellipse

**1.** **(a)** Given that $\frac{p^2}{9} + \frac{q^2}{4} > 1$ then S $(p, q)$ lies outside the ellipse




SA is tangent

$\therefore q = 2$

Area of $\Delta$ ORT $= \frac{3}{2}$

$\Rightarrow \left|\frac{1}{2} \times OR \times QT\right| = \frac{3}{2}$

$\Rightarrow \left|\frac{1}{2} \times 3 \times \beta\right| = \frac{3}{2} \Rightarrow \beta = -1$

$\therefore \frac{\alpha^2}{9} + \frac{\beta^2}{4} = 1 \Rightarrow \frac{\alpha^2}{9} = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \alpha^2 = \frac{27}{4} \Rightarrow \alpha = \frac{3\sqrt{3}}{2}$

Tangent at T

T = 0

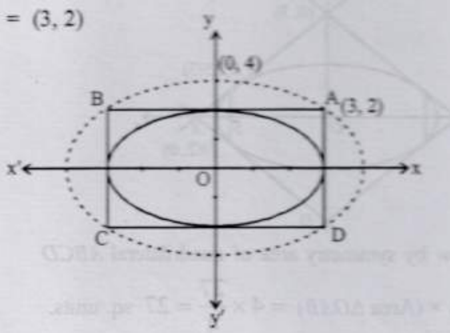
$$\left.\frac{x.\frac{3\sqrt{3}}{2}}{9} + \frac{y(-1)}{4} = 1\right|_{(p,2)}$$

$$\Rightarrow \frac{p\sqrt{3}}{6}-\frac{1}{2}=1 \Rightarrow \frac{p\sqrt{3}}{6}=\frac{3}{2} \Rightarrow p=3\sqrt{3}$$

$$\therefore p=3\sqrt{3}, q=2$$

2. (c) As rectangle $ABCD$ circumscribed the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$,

$\therefore A = (3, 2)$




Let the ellipse circumscribing the rectangle $ABCD$ is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ... (i)

Given that ellipse (i) passes through $(a, 4)$ $\therefore b^2 = 16$
Also ellipse (i) passes through $A$ (3, 2)

$$\therefore \quad \frac{9}{a^2}+\frac{4}{16}=1 \Rightarrow a^2=12 \therefore e=\sqrt{1-\frac{12}{16}}=\sqrt{\frac{1}{4}}=\frac{1}{2}$$

3. (c) Given ellipse is $\frac{x^2}{4^2}+\frac{y^2}{2^2}=1$

$$\therefore \quad a^2 = 16 \text{ and } b^2 = 4 \therefore e^2=1-\frac{4}{16}=\frac{3}{4} \Rightarrow e=\frac{\sqrt{3}}{2}$$

Let $P(4\cos\theta, 2\sin\theta)$ be any point on the ellipse, then equation of normal at $P$ is

$$4x\sin\theta-2y\cos\theta=12\sin\theta\cos\theta$$

$$\Rightarrow \frac{x}{3\cos\theta}-\frac{y}{6\sin\theta}=1$$

$\therefore$ $Q$, the point where normal at $P$ meets $x$-axis, has coordinates $(3\cos\theta, 0)$

$\therefore$ Mid point of $PQ$ is $M\left(\frac{7\cos\theta}{2}, \sin\theta\right)$

For locus of point $M$ we consider

$$x=\frac{7\cos\theta}{2} \text{ and } y=\sin\theta \Rightarrow \cos\theta=\frac{2x}{7} \text{ and } \sin\theta=y$$

Since $\sin^2\theta + \cos^2\theta = 1$

$$\therefore \quad \frac{4x^2}{49}+y^2=1 \qquad ...(i)$$

Also the latus rectum of given ellipse is

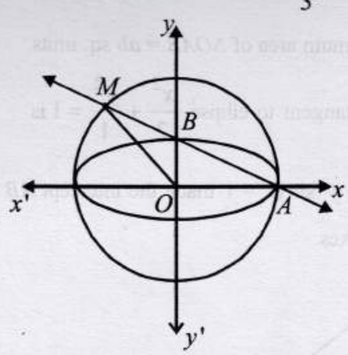
$$x=\pm ae=\pm 4\times\frac{\sqrt{3}}{2} \Rightarrow x=\pm 2\sqrt{3} \qquad ...(ii)$$

On solving equations (i) and (ii), we get

$$\frac{4\times 12}{49}+y^2=1 \Rightarrow y^2=\frac{1}{49} \text{ or } y=\pm\frac{1}{7}$$

$\therefore$ The required points are $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$.

4. (d) Given ellipse is $x^2+9y^2=9 \Rightarrow \frac{x^2}{3^2}+\frac{y^2}{1^2}=1$




$\therefore$ Co-ordinates of $A$ and $B$ are (3, 0) and (0, 1) respectively

$\therefore$ Equation of $AB$ is $\frac{x}{3}+\frac{y}{1}=1$

$\Rightarrow x+3y-3=0$ (i)
and equation of auxillary circle of given ellipse is

$x^2+y^2=9$ (ii)

On solving equations (i) and (ii), we get the point M where line $AB$ meets the auxillary circle.
Putting $x = 3 - 3y$ from (i) in (ii), we get

$$(3-3y)^2+y^2=9$$

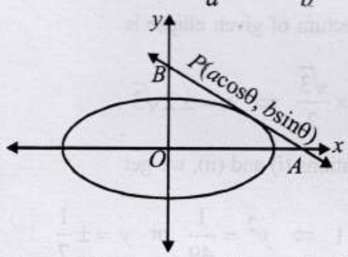
$$\Rightarrow 9-18y+9y^2+y^2=9 \Rightarrow 10y^2-18y=0$$

$$\Rightarrow y=0, \frac{9}{5} \Rightarrow x=3, \frac{-12}{5}$$

Clearly M $\left(\frac{-12}{5}, \frac{9}{5}\right)$

$$\therefore \quad \text{Area of } \Delta OAM=\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3 & 0 & 1 \\ \frac{-12}{5} & \frac{9}{5} & 1 \end{vmatrix}=\frac{27}{10}$$

**5.** **(a)** Any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $P(a\cos\theta, b\sin\theta)$ is $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$




It meets co-ordinate axes at $A$ ($a\sec\theta$, 0) and $B$ (0, $b\operatorname{cosec}\theta$)

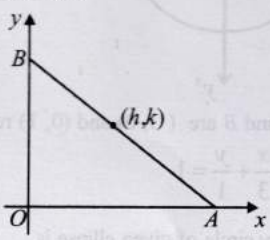
$\therefore$ Area of $\Delta OAB=\frac{1}{2}\times a\sec\theta\times b\operatorname{cosec}\theta=\frac{ab}{\sin 2\theta}$

For area of $\Delta OAB$ to be minimum, $\sin 2\theta$ should be maximum i.e., 1.

$\therefore$ Maximum area of $\Delta OAB = ab$ sq. units.

**6.** **(a)** Any tangent to ellipse $\frac{x^2}{2}+\frac{y^2}{1}=1$ is

$\frac{x\cos\theta}{\sqrt{2}}+y\sin\theta=1$ made the intercept $AB$ between the co-ordinate axes.




$\therefore$ $A(\sqrt{2}\sec\theta, 0)$ ; $B$ (0, $\operatorname{cosec}\theta$)

If ($h$, $k$) be the mid-point of $AB$, then

$2h=\sqrt{2}\sec\theta$ and $2k=\operatorname{cosec}\theta$

$\Rightarrow \cos\theta=\frac{1}{\sqrt{2}h}$ and $\sin\theta=\frac{1}{2k}$

Now $\cos^2\theta+\sin^2\theta=1$

$\Rightarrow \left(\frac{1}{\sqrt{2}h}\right)^2+\left(\frac{1}{2k}\right)^2=1 \Rightarrow \frac{1}{2h^2}+\frac{1}{4k^2}=1$

$\therefore$ Required locus is $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

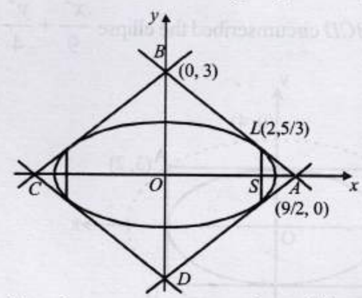
**7.** **(d)** Given equation of ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$

$\therefore$ $a^2=9, b^2=5 \Rightarrow$ $e=\sqrt{1-\frac{5}{9}}=\frac{2}{3}$

$\therefore$ End point of latus rectum in first quadrant is $L$ (2, 5/3)

Equation of tangent at $L$ is $\frac{2x}{9}+\frac{y}{3}=1$, which meets x-axis at $A$ (9/2, 0) and y-axis at $B$ (0, 3).

$\therefore$ Area of $\Delta$OAB $=\frac{1}{2}\times\frac{9}{2}\times 3=\frac{27}{4}$




Now by symmetry area of quadrilateral $ABCD$

$=4\times$ (Area $\Delta OAB$) $=4\times\frac{27}{4}=27$ sq. units.

**8.** **(a)** For ellipse $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$; $a=4$, $b=3$

$\Rightarrow e=\sqrt{1-\left(\frac{3}{4}\right)^2}=\frac{\sqrt{7}}{4}$

$\therefore$ Foci are $\left(\sqrt{7},0\right)$ and $\left(-\sqrt{7},0\right)$

Centre of circle is at (0, 3) and it passes through $\left(\pm\sqrt{7},0\right)$, therefore radius of circle $=\sqrt{\left(\sqrt{7}\right)^2+(3)^2}=4$

**9.** **(d)** Since $1^2+2^2=5<9$ and $2^2+1^1=5<9$ both $P$ and $Q$ lie inside $C$.

Also $\frac{1^2}{9}+\frac{2^2}{4}=\frac{1}{9}+1>1$ and $\frac{2^2}{9}+\frac{1}{4}=\frac{25}{36}<1$, $P$ lies outside $E$ and $Q$ lies inside $E$. Hence $P$ lies inside $C$ but outside $E$.

**10.** **(4)** Given : Ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$

$\Rightarrow a=3, b=\sqrt{5}$ and $e=\frac{2}{3}$ $\therefore f_1=2$ and $f_2=-2$

$P_1: y^2=8x$ and $P_2: y^2=-16x$

$T_1: y=m_1x+\frac{2}{m_1}$

It passes through (–4, 0), $\therefore 0=-4m_1+\frac{2}{m_1} \Rightarrow m_1^2=\frac{1}{2}$

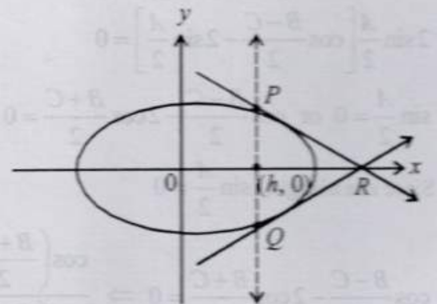
$T_2: y=m_2x-\frac{4}{m_2}$; It passes through (2, 0).

$\therefore 0=2m_2-\frac{4}{m_2} \Rightarrow m_2^2=2 \therefore \frac{1}{m_1^2}+m_2^2=2+2=4$

**11.** **(9)** Vertical line $x=h$, meets the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at

$P\left(h,\frac{\sqrt{3}}{2}\sqrt{4-h^2}\right)$ and $Q\left(h,\frac{-\sqrt{3}}{2}\sqrt{4-h^2}\right)$

By symmetry, tangents at $P$ and $Q$ will meet each other at $x$-axis.




Tangent at $P$ is $\frac{xh}{4}+\frac{y\sqrt{3}}{6}\sqrt{4-h^2}=1$, which meets $x$-axis at $R\left(\frac{4}{h},0\right)$

area $(\Delta PQR)=\frac{1}{2}\times\sqrt{3}\sqrt{4-h^2}\times\left(\frac{4}{h}-h\right)$

Let $\Delta(h)=\frac{\sqrt{3}}{2}\frac{(4-h^2)^{3/2}}{h}$

$\Rightarrow\ \frac{d\Delta}{dh}=-\sqrt{3}\left[\frac{\sqrt{4-h^2}\,(h^2+2)}{h^2}\right]<0$

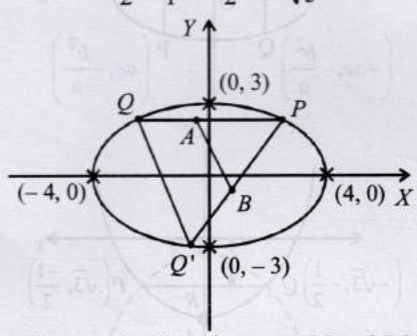
$\therefore$ $\Delta(h)$ is a decreasing function.

$\therefore\ \frac{1}{2}\le h\le 1 \Rightarrow \Delta_{\max}=\Delta\left(\frac{1}{2}\right)$ and $\Delta_{\min}=\Delta(1)$

$\therefore\ \Delta_1=\Delta_{\max}=\frac{\sqrt{3}}{2}\frac{\left(4-\frac{1}{4}\right)^{3/2}}{\frac{1}{2}}=\frac{45}{8}\sqrt{5}$

and $\Delta_2=\Delta_{\min}=\frac{\sqrt{3}}{2}\frac{3\sqrt{3}}{1}=\frac{9}{2}\quad\therefore\ \frac{8}{\sqrt{5}}\Delta_1-8\Delta_2=45-36=9$

**12.** **(4)**




Let $A$ and $B$ be midpoints of segment $PQ$ and $PQ'$ respectively

By midpoint theorem

$AB \parallel QQ'$ and $AB=\frac{1}{2}QQ'$

$\therefore$ Distance between $M(P, Q)$ and $M(P, Q)=\frac{1}{2}.QQ'$

Since, $Q, Q'$ must be on $E$, so, maximum of $QQ'=8$

$\therefore$ Maximum of $AB=\frac{8}{2}=4$

**13.** **(a, c)** Given equation of ellipse E : $\frac{x^2}{6}+\frac{y^2}{3}=1$.

Tangent : $y=m_1x\pm\sqrt{6m_1^2+3}$

Equation of parabola P : $y^2=12x$

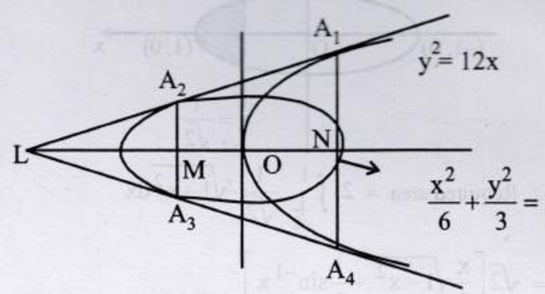
Tangent : $y=m_2x+\frac{3}{m_2}$

For common tangent

$m_1=m_2=m$ (say) and $\pm\sqrt{6m_1^2+3}=\frac{3}{m_2}$

$\Rightarrow\ m=\pm1.$

Equation of common tangents are




$T_1: y=x+3$ and

$T_2: y=-x-3.$

$\because$ point of contact for parabola is $\left(\frac{a}{m^2},\frac{2a}{m}\right)$

$\Rightarrow\ A_1(3, 6)$ and $A_4(3, -6)$

on solving $y=x+3$ or $y=-x-3$ and equation of ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ we get $A_2(-2, 1)$ and $A_3(-2, -1)$.

Area of quadrilateral $A_1A_2A_3A_4=\frac{1}{2}(12+2)\times5$

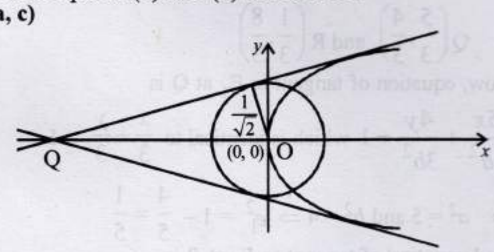
($\because A_1A_4=12, A_2A_3=2, MN=5$)

= 35 sq. units.

Put = y = 0 in $T_2$ and $T_2$ we get point of intersection with x-axis is $(-3, 0)$.

Hence option (a) and (c) are correct.

**14.** **(a, c)**




Let the equation of common tangent is $y=mx+\frac{1}{m}$

$\therefore\ \left|\frac{0+0+\frac{1}{m}}{\sqrt{1+m^2}}\right|=\frac{1}{\sqrt{2}}$

$\Rightarrow\ m^4+m^2-2=0\ \Rightarrow\ m=\pm1$

$\therefore$ Equation of common tangents are

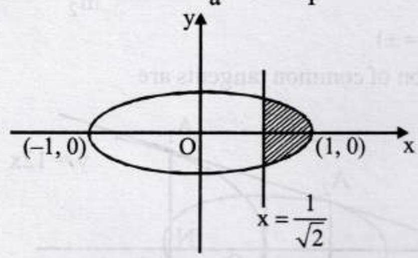
$y=x+1$ and $y=-x-1\ \therefore\ Q\equiv(-1, 0)$

$\therefore$ Equation of ellipse is $\frac{x^2}{1}+\frac{y^2}{1/2}=1$

(a) $e=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$

and latus rectum $=\frac{2b^2}{a}=\frac{2\left(\frac{1}{\sqrt{2}}\right)^2}{1}=1$

(c)




$\therefore$ Required area $=2\cdot\int_{\frac{1}{\sqrt{2}}}^{1}\frac{1}{\sqrt{2}}\cdot\sqrt{1-x^2}\,dx$

$=\sqrt{2}\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{1/\sqrt{2}}^{1}$

$=\sqrt{2}\left[\frac{\pi}{4}-\left(\frac{1}{4}+\frac{\pi}{8}\right)\right]=\sqrt{2}\left(\frac{\pi}{8}-\frac{1}{4}\right)=\frac{\pi-2}{4\sqrt{2}}$

**15.** **(a, b)** Let $E_1:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a>b$ ... (i)

and $E_2:\frac{x^2}{c^2}+\frac{y^2}{d^2}=1$, where $c<d$ ... (ii)

Also $S: x^2+(y-1)^2=2$ ... (iii)

Tangent at $P(x_1, y_1)$ to (iii) is $x+y=3$ ... (iv)

On solving (iii) and (iv), we get the point of contact P(1, 2)

Now, equation of tangent in parametric form,

$\frac{x-1}{\frac{-1}{\sqrt{2}}}=\frac{y-2}{\frac{1}{\sqrt{2}}}=\pm\frac{2\sqrt{2}}{3}\Rightarrow x=\frac{1}{3}$ or $\frac{5}{3}$ and $y=\frac{8}{3}$ or $\frac{4}{3}$

$\therefore$ $Q\left(\frac{5}{3},\frac{4}{3}\right)$ and $R\left(\frac{1}{3},\frac{8}{3}\right)$

Now, equation of tangent to $E_1$ at Q is

$\frac{5x}{3a^2}+\frac{4y}{3b^2}=1$ which is identical to $\frac{x}{3}+\frac{y}{3}=1$

$\Rightarrow$ $a^2=5$ and $b^2=4\Rightarrow e_1^2=1-\frac{4}{5}=\frac{1}{5}$

And equation of tangent to $E_2$ at $R$ is

$\frac{x}{3c^2}+\frac{8y}{3d^2}=1$, which is identical to $\frac{x}{3}+\frac{y}{3}=1$

$\Rightarrow$ $c^2=1, d^2=8\Rightarrow e_2^2=1-\frac{1}{8}=\frac{7}{8}$

$\therefore$ $e_1^2+e_2^2=\frac{43}{40}$, $e_1e_2=\frac{\sqrt{7}}{2\sqrt{10}}$, $\left|e_1^2-e_2^2\right|=\frac{27}{40}$

**16.** **(b,c)** In $\Delta ABC$, $\cos B+\cos C=4\sin^2\frac{A}{2}$ (Given)

$\Rightarrow$ $2\cos\frac{B+C}{2}\cos\frac{B-C}{2}-4\sin^2\frac{A}{2}=0$

$\Rightarrow$ $2\sin\frac{A}{2}\left[\cos\frac{B-C}{2}-2\sin\frac{A}{2}\right]=0$

$\Rightarrow$ $\sin\frac{A}{2}=0$ or $\cos\frac{B-C}{2}-2\cos\frac{B+C}{2}=0$

Since in a triangle, $\sin\frac{A}{2}\neq 0$

$\therefore$ $\cos\frac{B-C}{2}-2\cos\frac{B+C}{2}=0\Rightarrow\frac{\cos\left(\frac{B+C}{2}\right)}{\cos\left(\frac{B-C}{2}\right)}=\frac{1}{2}$

By componendo and dividendo, we get

$\frac{\cos\left(\frac{B+C}{2}\right)+\cos\left(\frac{B-C}{2}\right)}{\cos\left(\frac{B+C}{2}\right)-\cos\left(\frac{B-C}{2}\right)}=\frac{1+2}{1-2}=-3$

$\Rightarrow$ $\frac{2\cos\frac{B}{2}\cos\frac{C}{2}}{-2\sin\frac{B}{2}\sin\frac{C}{2}}=-3\Rightarrow\tan\frac{B}{2}\tan\frac{C}{2}=\frac{1}{3}$

$\Rightarrow$ $\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}=\frac{1}{3}$

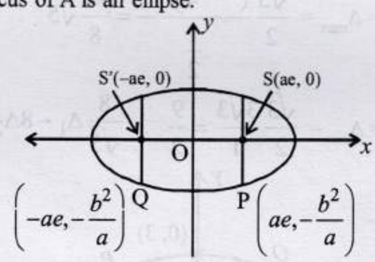
$\Rightarrow$ $\frac{s-a}{s}=\frac{1}{3}\Rightarrow 2s=3a\Rightarrow a+b+c=3a\Rightarrow b+c=2a$

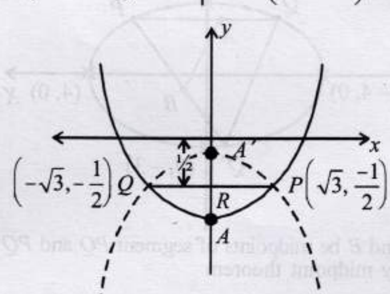
*i.e.* $AC+AB$ = constant

($\because$ Base $BC=a$ is given to be constant)

$\therefore$ Locus of A is an ellipse.

**17.** **(b,c)**




Given ellipse is $x^2+4y^2=4$

or $\frac{x^2}{2^2}+\frac{y^2}{1}=1\Rightarrow a=2, b=1$

$\therefore$ $e=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$ $\therefore ae=\sqrt{3}$

As per question $P\equiv(ae,-b^2/a)=\left(\sqrt{3},-\frac{1}{2}\right)$

$$Q \equiv (-ae, -b^2/a) = \left(-\sqrt{3}, -\frac{1}{2}\right) \therefore PQ = 2\sqrt{3}$$

Now if $PQ$ is the length of latus rectum of the parabola whose equation is to be found, then

$$PQ = 4a = 2\sqrt{3} \Rightarrow a = \frac{\sqrt{3}}{2}$$

Also as $PQ$ is horizontal, parabola with $PQ$ as latus rectum can be upward parabola (with vertex at $A$) or down ward parabola (with vertex at $A'$) as shown in the figure.
For upward parabola,

$$AR = a = \frac{\sqrt{3}}{2}, \therefore \text{ Coordinates of } A = \left(0, -\left(\frac{\sqrt{3}+1}{2}\right)\right)$$

$\therefore$ Equation of upward parabola is

$$x^2 = 2\sqrt{3}\left(y + \frac{\sqrt{3}+1}{2}\right) \Rightarrow x^2 - 2\sqrt{3}y = 3 + \sqrt{3} \quad ...(i)$$

For downward parabola $\quad A'R = a = \dfrac{\sqrt{3}}{2}$

$\therefore$ Coordinates of $A' = \left(0, -\left(\dfrac{1-\sqrt{3}}{2}\right)\right)$

$\therefore$ Equation of downward parabola is given by

$$x^2 = -2\sqrt{3}\left(y + \frac{1-\sqrt{3}}{2}\right) \Rightarrow x^2 + 2\sqrt{3}y = 3 - \sqrt{3} \quad ...(ii)$$

$\therefore$ Equation of required parabola is given by equation (i) or (ii).

**18.** **(b, d)** Let $y = \dfrac{8}{9}x + c$ be the tangent to $\dfrac{x^2}{1/4} + \dfrac{y^2}{1/9} = 1$

where $c = \pm\sqrt{a^2m^2 + b^2} = \pm\sqrt{\dfrac{1}{4} \times \dfrac{64}{81} + \dfrac{1}{9}} = \pm\dfrac{5}{9}$

and points of contact are $\left(\dfrac{-a^2m}{c}, \dfrac{b^2}{c}\right)$

$= \left(\dfrac{2}{5}, \dfrac{-1}{5}\right)$ or $\left(\dfrac{-2}{5}, \dfrac{1}{5}\right)$

**19.** **(c)** Given equation of curve can be written as

$\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ (ellipse)

Here $a^2 = 25$, $b^2 = 16$, but $b^2 = a^2(1 - e^2)$

$\Rightarrow 16/25 = 1 - e^2 \Rightarrow e = 3/5$

Foci of the ellipse are $(\pm ae, 0) = (\pm 3, 0)$, i.e., $F_1$ and $F_2$

Now $PF_1 + PF_2 = 2a = 10$ for every point $P$ on the ellipse.

**20.** **(c)** The given curve is $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$ (an ellipse) and given line is $y = 4x + c$.

We know that $y = mx + c$ touches the ellipse

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ if $c = \pm\sqrt{a^2m^2 + b^2}$

Hence the given line touches the given ellipse if

$c = \pm\sqrt{4 \times 16 + 1} = \pm\sqrt{65}$ $\therefore$ There are two values of $c$ exist.

**For (Q. 21 and 22)** Given : Ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{8} = 1$

$$e = \sqrt{1 - \frac{8}{9}} = \frac{1}{3} \quad ... (i)$$

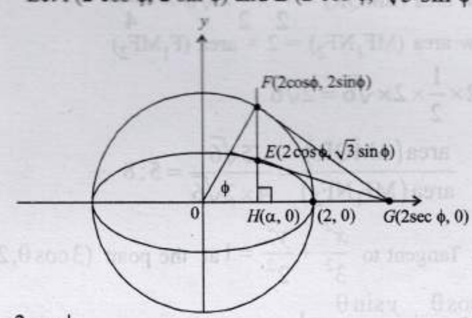
$\therefore$ $F_1(-1, 0)$ and $F_2(1, 0)$

Parabola with vertex at (0, 0) and focus at $F_2(1, 0)$ is

$y^2 = 4x \quad ... (ii)$

$\therefore$ On solving (i) and (ii), we get the intersection points of ellipse and parabola as $M\left(\dfrac{3}{2}, \sqrt{6}\right)$ and $N\left(\dfrac{3}{2}, -\sqrt{6}\right)$

**21.** **(c)** Let $F(2\cos\phi, 2\sin\phi)$ and $E(2\cos\phi, \sqrt{3}\sin\phi)$




$\alpha \equiv 2\cos\phi$

Tangent at $E\left(2\cos\phi, \sqrt{3}\sin\phi\right)$ to ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$

i.e. $\dfrac{x\cos\phi}{2} + \dfrac{y\sin\phi}{\sqrt{3}} = 1$ intersect $x$-axis at $G(2\sec\phi, 0)$

Area of triangle $FGH = \dfrac{1}{2} HG \times FH$

$= \dfrac{1}{2}(2\sec\phi - 2\cos\phi)\, 2\sin\phi$ ; $\Delta = 2\sin^2\phi \cdot \tan\phi$

$\Delta = (1 - \cos 2\phi) \cdot \tan\phi$

I. If $\phi = \dfrac{\pi}{4}, \Delta = 1 \to (Q)$

II. If $\phi = \dfrac{\pi}{3}, \Delta = 2.\left(\dfrac{\sqrt{3}}{2}\right)^2 . \sqrt{3} = \dfrac{3\sqrt{3}}{2} \to (T)$

III. If $\phi = \dfrac{\pi}{6}, \Delta = 2 \cdot \left(\dfrac{1}{2}\right)^2 \cdot \dfrac{1}{\sqrt{3}} = \dfrac{1}{2\sqrt{3}} \to (S)$

IV. If $\phi = \dfrac{\pi}{12}, \Delta = \left(1 - \dfrac{\sqrt{3}}{2}\right).\left(2 - \sqrt{3}\right) = \dfrac{\left(2-\sqrt{3}\right)^2}{2} \to (P)$

**22.** **(a)** One altitude of $\Delta F_1MN$ is x-axis i.e. y = 0 and altitude from M to $F_1N$ is

$$y - \sqrt{6} = \frac{5}{2\sqrt{6}}\left(x - \frac{3}{2}\right)$$

Putting y = 0 in above equation, we get $x = -\dfrac{9}{10}$

$\therefore$ Orthocentre $\left(-\dfrac{9}{10}, 0\right)$

**23.** **(c)** Tangents to ellipse at M and N are

$$\frac{x}{6} + \frac{y\sqrt{6}}{8} = 1 \quad ... (i)$$

and $$\frac{x}{6} - \frac{y\sqrt{6}}{8} = 1 \quad ... (ii)$$

On solving (i) and (ii), we get their intersection point R (6, 0).

Now equation of normal to parabola at $M\left(\dfrac{3}{2}, \sqrt{6}\right)$ is

$y-\sqrt{6}=-\frac{\sqrt{6}}{2}\left(x-\frac{3}{2}\right)$

Its intersection with x-axis is $Q\left(\frac{7}{2},0\right)$

Now area $(\Delta MQR)=\frac{1}{2}\times\frac{5}{2}\times\sqrt{6}=\frac{5\sqrt{6}}{4}$

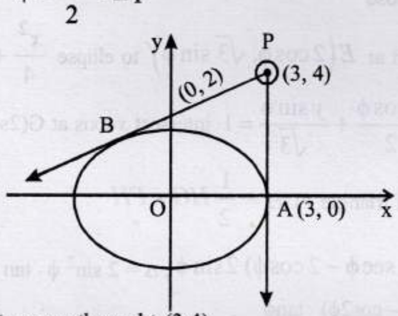
Now area $(MF_1NF_2)=2\times$ area $(F_1MF_2)$

$=2\times\frac{1}{2}\times2\times\sqrt{6}=2\sqrt{6}$

$\therefore\quad \frac{\text{area}(\Delta MQR)}{\text{area}(MF_1NF_2)}=\frac{5\sqrt{6}}{4\times2\sqrt{6}}=5:8$

**24.** **(d)** Tangent to $\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$ at the point $(3\cos\theta,2\sin\theta)$ is

$\frac{x\cos\theta}{3}+\frac{y\sin\theta}{2}=1$




Since it passes throught (3,4),

$\therefore\quad \cos\theta+2\sin\theta=1$

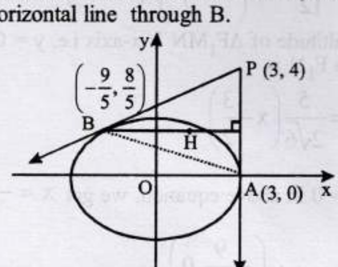
$\Rightarrow\ 4\sin^2\theta=1+\cos^2\theta-2\cos\theta$

$\Rightarrow\ 5\cos^2\theta-2\cos\theta-3=0$

$\Rightarrow\ \cos\theta=1,-\frac{3}{5}\ \Rightarrow\ \sin\theta=0,\frac{4}{5}$

$\therefore$ Required points are A (3,0) and $B\left(-\frac{9}{5},\frac{8}{5}\right)$

**25.** **(c)** Let H be the orthocentre of $\Delta PAB$, then as $BH\perp AP$, BH is a horizontal line through B.




$\therefore$ y- cordinate of B = 8/5

Let H has coordinater $(\alpha,8/5)$.

$\therefore$ Slope of $PH=\frac{\frac{8}{5}-4}{\alpha-3}=\frac{-12}{5(\alpha-3)}$

and slope of $AB=\frac{\frac{8}{5}-0}{\frac{-9}{5}-3}=\frac{8}{-24}=\frac{-1}{3}$

But $PH\perp AB$, $\therefore\ \frac{-12}{5(\alpha-3)}\times\left(\frac{-1}{3}\right)=-1$

$\Rightarrow\ 4=-5\alpha+15$ or $\alpha=11/5\ \therefore\ H\left(\frac{11}{5},\frac{8}{5}\right)$.

**26.** **(a)** Clearly the moving point traces a parabola with focus at $P(3,4)$ and directrix as

$AB:\frac{y-0}{x-3}=-\frac{1}{3}\Rightarrow x+3y-3=0$

$\therefore$ Equation of parabola is

$(x-3)^2+(y-4)^2=\frac{(x+3y-3)^2}{10}$

$\Rightarrow\ 9x^2+y^2-6xy-54x-62y+241=0$

**27.** Let the common tangent to circle $x^2+y^2=16$ and ellipse $x^2/25+y^2/4=1$ be $y=mx+\sqrt{25m^2+4}$ ..(i)

Since it is tangent to circle $x^2+y^2=16$.

$\therefore\quad \frac{\sqrt{25m^2+4}}{\sqrt{m^2+1}}=4$

[Since length of perpendicular from centre of the circle to the tangent is equal to the radius of the circle.]

$\Rightarrow\ 25m^2+4=16m^2+16\ \Rightarrow\ 9m^2=12\ \therefore\ m=\frac{-2}{\sqrt{3}}$

[Since, the slope of any tangent to the given circle at any point in the 1st quadrant will be positive.]

$\therefore$ Equation of common tangent is

$y=-\frac{2}{\sqrt{3}}x+\sqrt{25.\frac{4}{3}+4}\ \Rightarrow y=-\frac{2}{\sqrt{3}}x+4\sqrt{\frac{7}{3}}$

This tangent meets the axes at $A(2\sqrt{7},0)$ and $B\left(0,4\sqrt{\frac{7}{3}}\right)$

$\therefore$ Length of intercepted portion of tangent between the axes

$=AB=\sqrt{(2\sqrt{7})+\left(4\sqrt{\frac{7}{3}}\right)^2}=14/\sqrt{3}$

**28.** Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $O$ be the centre.

Tangent at $P(x_1,y_1)$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=0$, whose slope is $-\frac{b^2x_1}{a^2y_1}$ and focus is $S(ae,0)$.

Equation of the line perpendicular to tangent at $P$ is

$y=\frac{a^2y_1}{b^2x_1}(x-ae)$ ....(i)

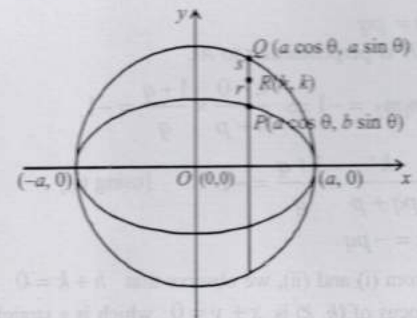
Equation of $OP$ is $y=\frac{y_1}{x_1}x$ ...(ii)

(i) and (ii) intersect $\Rightarrow \frac{y_1}{x_1}x=\frac{a^2y_1}{b^2x_1}(x-ae)$

$\Rightarrow\ x(a^2-b^2)=a^3e\Rightarrow x.\ a^2e^2=a^3e$

$\Rightarrow\ x=a/e$, which is the corresponding directrix.

**29.** Let the co-ordinates of $P$ be $(a\cos\theta, b\sin\theta)$ then co-ordinates of $Q$ are $(a\cos\theta, a\sin\theta)$




Since, $R$ $(h, k)$ divides $PQ$ in the ratio $r : s$,

$$\therefore \quad h=\frac{s.(a\cos\theta)+r(a\cos\theta)}{(r+s)}=a\cos\theta \Rightarrow \cos\theta=\frac{h}{a}$$

$$k=\frac{s(b\sin\theta)+r(a\sin\theta)}{(r+s)}=\frac{\sin\theta(bs+ar)}{(r+s)}$$

$$\Rightarrow \quad \sin\theta=\frac{k(r+s)}{(bs+ar)}$$

$$\because \quad \cos^2\theta+\sin^2\theta=1, \quad \therefore \frac{h^2}{a^2}+\frac{k^2(r+s)^2}{(bs+ar)^2}=1.$$

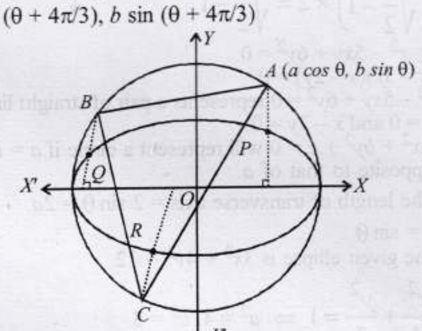
$\therefore$ Locus of $R$ is $\frac{x^2}{a^2}+\frac{y^2(r+s)^2}{(bs+ar)^2}=1$, which is equation of an ellipse.

**30.** Let the coordinates of $A \equiv (\alpha\cos\theta, \beta\sin\theta)$, so that the coordinates of

$B=\{\alpha\cos(\theta+2\pi/3), \alpha\sin(\theta+2\pi/3)\}$

and $C=\{\alpha\cos(\theta+4\pi/3), \alpha\sin(\theta+4\pi/3)\}$

According to the given condition, coordinates of $P$ are $(a\cos\theta\ b\sin\theta)$ and that of Q are $\{a\cos(\theta+2\pi/3), b\sin(\theta+2\pi/3)\}$ and that of R are $a\cos(\theta+4\pi/3), b\sin(\theta+4\pi/3)$




[$\because$ it is given that P, Q, R are on the same side of X-axis as A, B and C]

Equation of the normal to the ellipse at P is

$$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$$

or $ax\sin\theta - by\cos\theta=\frac{1}{2}(a^2-b^2)\sin 2\theta$ ...(i)

Equation of normal to the ellipse at Q is

$$ax\sin\left(\theta+\frac{2\pi}{3}\right)-by\cos\left(\theta+\frac{2\pi}{3}\right)$$

$$=\frac{1}{2}\left(a^2-b^2\right)\sin\left(2\theta+\frac{4\pi}{3}\right) \quad ...(ii)$$

Equation of normal to the ellipse at R is

$ax\sin(\theta+4\pi/3)-by\cos(\theta+4\pi/3)$

$$=\frac{1}{2}\left(a^2-b^2\right)\sin\left(2\theta+8\pi/3\right) \quad ...(iii)$$

But $\sin(\theta+4\pi/3)=\sin(2\pi+\theta-2\pi/3)$

$=\sin(\theta-2\pi/3)$

and $\cos(\theta+4\pi/3)=\cos(2\pi+\theta-2\pi/3)=\cos(\theta-2\pi/3)$

and $\sin(2\theta+8\pi/3)=\sin(4\pi+2\theta-4\pi/3)=\sin(2\theta-4\pi/3)$

Now, Eq. (iii) can be written as

$ax\sin(\theta-2\pi/3)-by\cos(\theta-2\pi/3)$

$$=\frac{1}{2}\left(a^2-b^2\right)\sin\left(2\theta-4\pi/3\right) \quad ...(iv)$$

For the lines (i), (ii) and (iv) to be concurrent, we must have the determinant

$$\Delta_1=\begin{vmatrix} a\sin\theta & -b\cos\theta & \frac{1}{2}\left(a^2-b^2\right)\sin 2\theta \\ a\sin\left(\theta+\frac{2\pi}{3}\right) & -b\cos\left(\theta+\frac{2\pi}{3}\right) & \frac{1}{2}\left(a^2-b^2\right)\sin\left(2\theta+4\pi/3\right) \\ a\sin\left(\theta-\frac{2\pi}{3}\right) & -b\cos\left(\theta-\frac{2\pi}{3}\right) & \frac{1}{2}\left(a^2-b^2\right)\sin\left(2\theta-4\pi/3\right) \end{vmatrix}=0$$

Thus, lines (i), (ii) and (iv) are concurrent.

**31.** Let any point $P$ on ellipse $4x^2+25y^2=100$ be $(5\cos\theta, 2\sin\theta)$. Hence equation of tangent to the ellipse at $P$ will be

$$\frac{x\cos\theta}{5}+\frac{y\sin\theta}{2}=1$$

Tangent (1) also touches the circle $x^2+y^2=r^2$, so its distance from origin must be $r$.

Tangent (2) intersects the coordinate axes at $A\left(\frac{5}{\cos\theta}, 0\right)$ and $B\left(0, \frac{2}{\sin\theta}\right)$ respectively. Let $M$ $(h, k)$ be the midpoint of line segment $AB$. Then by mid point formula

$$h=\frac{5}{2\cos\theta}, k=\frac{1}{\sin\theta} \Rightarrow \cos\theta=\frac{5}{2h}, \sin\theta=\frac{1}{k}$$

$$\Rightarrow \cos^2\theta+\sin^2\theta=\frac{25}{4h^2}+\frac{1}{k^2}$$

Hence locus of $M$ $(h, k)$ is $\frac{25}{x^2}+\frac{4}{y^2}=4$

Locus is independent of $r$.

**32.** The given ellipses are

$$\frac{x^2}{4}+\frac{y^2}{1}=1 \quad ...(i)$$

and $\frac{x^2}{6}+\frac{y^2}{3}=1$ ...(ii)

Equation of tangent to (i) at any point $T$ $(2\cos\theta, \sin\theta)$ is

$$\frac{x.2\cos\theta}{4}+\frac{y.\sin\theta}{1}=1 \Rightarrow \frac{x\cos\theta}{2}+y\sin\theta=1 \quad ...(iii)$$

Let this tangent meet the ellipse (ii) at $P$ and $Q$.

Let the tangents drawn to ellipse (ii) at $P$ and $Q$ meet each other at $R$ $(x_1, y_1)$.

$\therefore$ $PQ$ is chord of contact of ellipse (ii) with respect to the point $R$ $(x_1, y_1)$ and is given by $\frac{xx_1}{6}+\frac{yy_1}{3}=1$ ...(iv)

Clearly equations (iii) and (iv) represent the same lines and hence should be identical. Therefore on comparing the cofficients, we get

$$\frac{\frac{\cos\theta}{2}}{\frac{x_1}{6}} = \frac{\sin\theta}{\frac{y_1}{3}} = \frac{1}{1}$$

$\Rightarrow$ $x_1 = 3\cos\theta, y_1 = 3\sin\theta \Rightarrow x_1^2 + y_1^2 = 9$

$\therefore$ Locus of $(x_1, y_1)$ is $x^2 + y^2 = 9$,

which is the director circle of the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ and thus tangents at $P$ and $Q$ are at right angled.

[$\because$ Director circle is the locus of intersection point of the tangents which are at right angled]

33. Equation to the tangent at the point $P(a\cos\theta, b\sin\theta)$ on $x^2/a^2 + y^2/b^2 = 1$ is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ ...(i)

$\therefore$ $d$ = perpendicular distance of (i) from the centre (0, 0) of the ellipse

$$= \frac{1}{\sqrt{\frac{1}{a^2}\cos^2\theta + \frac{1}{b^2}\sin^2\theta}} = \frac{(ab)}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}$$

$$\therefore 4a^2\left(1 - \frac{b^2}{d^2}\right) = 4a^2\left\{1 - \frac{b^2\cos^2\theta + a^2\sin^2\theta}{a^2}\right\}$$

$= 4(a^2 - b^2)\cos^2\theta = 4a^2e^2\cos^2\theta$ ....(ii)

The coordinates of focii $F_1$ and $F_2$ are $F_1 = (ae, 0)$ and $F_2 = (-ae, 0)$

$$\therefore PF_1 = \sqrt{[(a\cos\theta - ae)^2 + (b\sin\theta)^2]}$$

$$= \sqrt{[(a^2(\cos\theta - e)^2 + (b\sin\theta)^2]}$$

$$= \sqrt{[(a^2(\cos\theta - e)^2 + a^2(1-e^2)\sin^2\theta)]}$$

[$\because b^2 = a^2(1 - e^2)$]

$$= a\sqrt{[1 + e^2(1 - \sin^2\theta) - 2e\cos\theta]}$$

$= a(1 - e\cos\theta)$

Similarly, $PF_2 = a(1 + e\cos\theta)$

$\therefore$ $(PF_1 - PF_2)^2 = 4a^2e^2\cos^2\theta$ ...(iii)

From (ii) and (iii), we have

$$(PF_1 - PF_2)^2 = 4a^2\left(1 - \frac{b^2}{a^2}\right)$$

## Topic-4: Hyperbola

1. (d) The triangle is formed by the lines

$AB: (1+p)x - py + p(1+p) = 0$

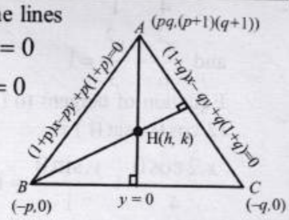
$AC: (1+q)x - qy + q(1+q) = 0$

$BC: y = 0$

So that the vertices of $\Delta ABC$ are

$A(pq, (p+1)(q+1))$,

$B(-p, 0)$ and $C(-q, 0)$




Let $H(h, k)$ be the orthocentre of $\Delta ABC$. Then as $AH \perp BC$ and passes through $A(pq, (p+1)(q+1))$

$\therefore$ Equation of $AH$ is $x = pq$

$\therefore$ $h = pq$ ...(i)

$\because$ $BH$ is perpendicular to $AC$

$$\therefore m_1m_2 = -1 \Rightarrow \frac{k-0}{h+p} \times \frac{1+q}{q} = -1$$

$$\Rightarrow \frac{k}{pq+p} \times \frac{1+q}{q} = -1 \quad \text{[using (i)]}$$

$\therefore$ $k = -pq$ ...(ii)

From (i) and (ii), we observe that $h + k = 0$

$\therefore$ Locus of $(h, k)$ is $x + y = 0$, which is a straight line.

2. (b) Given hyperbola is

$$x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$$

$$\Rightarrow (x^2 - 2\sqrt{2}x + 2) - 2(y^2 + 2\sqrt{2}y + 2) = 6 + 2 - 4$$

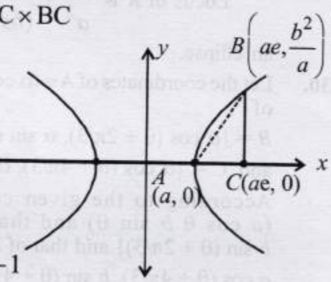
$$\Rightarrow (x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 = 4$$

$$\Rightarrow \frac{(x-\sqrt{2})^2}{2^2} - \frac{(y+\sqrt{2})^2}{(\sqrt{2})^2} = 1$$

$$\therefore a = 2, b = \sqrt{2} \Rightarrow e = \sqrt{1 + \frac{2}{4}} = \sqrt{\frac{3}{2}}$$

Clearly $\Delta ABC$ is a right triangle.

$\therefore$ Area $(\Delta ABC) = \frac{1}{2} \times AC \times BC$




$$= \frac{1}{2}(ae - a) \times \frac{b^2}{a}$$

$$= \frac{1}{2}(e - 1) \times b^2$$

$$= \frac{1}{2}\left(\sqrt{\frac{3}{2}} - 1\right) \times 2 = \sqrt{\frac{3}{2}} - 1$$

3. (b) $\because x^2 - 5xy + 6y^2 = 0$

$\Rightarrow$ $(x - 3y)(x - 2y) = 0$

$\therefore$ $x^2 - 5xy + 6y^2 = 0$ represents a pair of straight lines given by $x - 3y = 0$ and $x - 2y = 0$.

Also $ax^2 + by^2 + c = 0$ will represent a circle if $a = b$ and $c$ is of sign opposite to that of $a$.

4. (a) The length of transverse axis = $2\sin\theta = 2a$

$\Rightarrow$ $a = \sin\theta$

Also the given ellipse is $3x^2 + 4y^2 = 12$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1 \Rightarrow a^2 = 4, b^2 = 3$$

$$\therefore e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$$

$\therefore$ Focus of ellipse $= \left(2 \times \frac{1}{2}, 0\right) \Rightarrow (1, 0)$

Since, hyperbola is confocal with ellipse, therefore focus of hyperbola = (1, 0) $\Rightarrow ae = 1 \Rightarrow \sin\theta \times e = 1$

$\Rightarrow$ $e = \operatorname{cosec}\theta$

$\therefore$ $b^2 = a^2(e^2 - 1) = \sin^2\theta(\operatorname{cosec}^2\theta - 1) = \cos^2\theta$

$\therefore$ Equation of hyperbola is

$$\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$$

$\Rightarrow$ $x^2\operatorname{cosec}^2\theta - y^2\sec^2\theta = 1$

**5.** **(d)** Equation of tangent to hyperbola $x^2 - 2y^2 = 4$ at any point $(x_1, y_1)$ is $xx_1 - 2yy_1 = 4$ ... (i)

But the given tangent is $2x + \sqrt{6}y = 2$

On comparing equation (i) with $2x + \sqrt{6}\,y = 2$ i.e., $4x + 2\sqrt{6}\,y = 4$, we get

$x_1 = 4$ and $-2y_1 = 2\sqrt{6} \Rightarrow (4, -\sqrt{6})$ is the required point of contact.

**6.** **(b)** Given equation of hyperbola is

$$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1 \Rightarrow a = \cos\alpha,\ b = \sin\alpha$$

$$\Rightarrow \quad e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \tan^2\alpha} = \sec\alpha$$

$\Rightarrow$ $ae = \cos\alpha.\sec\alpha = 1$ $\therefore$ Foci $(\pm 1, 0)$

Hence, foci remain constant with respect to $\alpha$.

**7.** **(d)** Given equation of curves are

$y^2 = 8x$ ...(i)

and $xy = -1$ ...(ii)

If m is the slope of tangent to (i), then equation of tangent is $y = mx + 2/m$

If this tangent is also a tangent to (ii), then putting value of $y$ in curve (ii)

$$x\left(mx + \frac{2}{m}\right) = -1 \Rightarrow mx^2 + \frac{2}{m}x + 1 = 0 \Rightarrow m^2x^2 + 2x + m = 0$$

We should get repeated roots for the equation (condition of tangency)

$\Rightarrow$ $D = 0$, $\therefore$ $(2)^2 - 4m^2.m = 0 \Rightarrow m^3 = 1 \Rightarrow m = 1$

Hence equation of required tangent is $y = x + 2$

**8.** **(c)** The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a parabola if $\Delta \neq 0$ and $h^2 = ab$

where $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

Now we have $x = t^2 + t + 1$ and $y = t^2 - t + 1$

$$\therefore \quad \frac{x+y}{2} = t^2 + 1 \text{ and } \frac{x-y}{2} = t$$

On eliminating $t$, we get $2(x + y) = (x - y)^2 + 4$

$\Rightarrow$ $x^2 - 2xy + y^2 - 2x - 2y + 4 = 0$

Here, $a = 1, h = -1, b = 1, g = -1, f = -1, c = 4$

$\therefore$ $\Delta \neq 0$. and $h^2 = ab$

Hence the given curve represents a parabola.

**9.** **(b)** Chord $x = 9$ meets $x^2 - y^2 = 9$ at $(9, 6\sqrt{2})$ and $(9, -6\sqrt{2})$ at which tangents are

$9x - 6\sqrt{2}y = 9$ and $9x + 6\sqrt{2}y = 9$

$\Rightarrow$ $3x - 2\sqrt{2}y - 3 = 0$ and $3x + 2\sqrt{2}y - 3 = 0$

$\therefore$ Combined equation of tangents is

$(3x - 2\sqrt{2}y - 3)\ (3x + 2\sqrt{2}y - 3) = 0$

$\Rightarrow$ $9x^2 - 8y^2 - 18x + 9 = 0$

**10.** **(d)** Equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a\sec\alpha, b\tan\alpha)$ is given by

$ax\cos\alpha + by\cot\alpha = a^2 + b^2$

$\therefore$ Normals at P and Q are $ax\cos\theta + by\cot\theta = a^2 + b^2$ and $ax\cos\theta + by\cot\phi = a^2 + b^2$ respectively

where $\phi = \frac{\pi}{2} - \theta$

since the normals at P and Q pass through $(h, k)$,

$\therefore$ $ah\cos\theta + bk\cot\theta = a^2 + b^2$

and $ah\sin\theta + bk\tan\theta = a^2 + b^2$

On eliminating $h$, we get $bk(\cot\theta\ \sin\theta\ - \tan\ \cos\theta)$

$= (a^2 + b^2)(\sin\theta - \cos\theta) \Rightarrow k = -(a^2 + b^2)/b$

**11.** **(c)** $2x^2 + 3y^2 - 8x - 18y + 35 = k$

$\Rightarrow$ $2(x - 2)^2 + 3(y - 3)^2 = k$

For $k = 0$, we get $2(x - 2)^2 + 3(y - 3)^2 = 0$, which represents the point (2, 3).

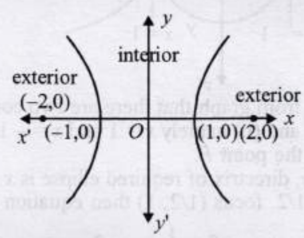
**12.** **(c)** (a) $x^2 + 2y^2 \leq 1$ represents interior region of an ellipse where on taking any two points the mid point of that segment will also lie inside that ellipse.

(b) Max $\{|x|, |y|\} \leq 1$

$\Rightarrow |x| \leq 1, |y| \leq 1 \Rightarrow -1 \leq x \leq 1$ and $-1 \leq y \leq 1$

which represents the interior region of a square with its sides $x = \pm 1$ and $y = \pm 1$ in which for any two points, their mid point also lies inside the region.

(c) $x^2 - y^2 \geq 1$ represents the exterior region of hyperbola in which if we take two points (2, 0) and (– 2, 0) then their mid point (0, 0) does not lie in the same region (as shown in the figure).




(d) $y^2 \leq x$ represents interior region of parabola in which for any two points and their mid point also lie inside the region.

**13.** **(d)** Given equation $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1,\ r > 1$

As $r > 1$

$\therefore$ $1 - r < 0$ and $1 + r > 0$ $\therefore$ Let $1 - r = -a^2,\ 1 + r = b^2$

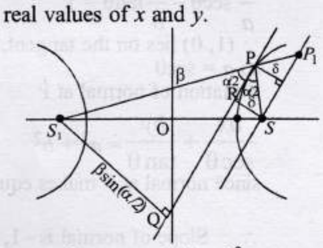
$$\therefore \quad \frac{x^2}{-a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = -1,$$

which is not possible for any real values of $x$ and $y$.

**14.** **(7)**

In $\Delta$ $S_1QP$, $\sin\frac{\alpha}{2} = \frac{S_1Q}{\beta}$

$\Rightarrow S_1Q = \beta\sin\frac{\alpha}{2}$




Product of distances of any tangent from two foci $= b^2$

$$\delta.S_1Q = \delta \times \beta\sin\frac{\alpha}{2} = b^2 \text{ So, } \frac{\beta\delta\sin\frac{\alpha}{2}}{9} = \frac{b^2}{9} = \frac{64}{9}$$

$$\therefore \quad \left[\frac{64}{9}\right] = 7_{ss}$$

**15.** **(2)** Intersection point of nearest directrix $x = \frac{a}{e}$ and x-axis is $\left(\frac{a}{e}, 0\right)$

Since $2x + y = 1$ passes through $\left(\frac{a}{e}, 0\right)$

$$\therefore \frac{2a}{e} = 1 \Rightarrow a = \frac{e}{2}$$

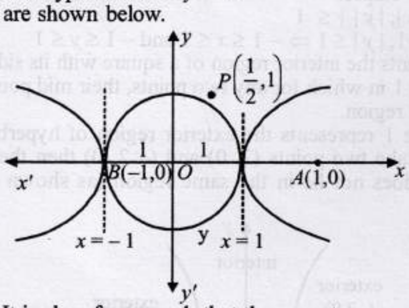
Also $y = -2x + 1$ is a tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore 1 = a^2(-2)^2 - b^2 \Rightarrow 4a^2 - b^2 = 1$

$\Rightarrow 4a^2 - a^2(e^2-1) = 1 \Rightarrow 4\times\dfrac{e^2}{4} - \dfrac{e^2}{4}(e^2-1) = 1$

$\Rightarrow e^2 = 4$ *as* $e > 1$ for hyperbola. $\Rightarrow e = 2$

**16.** Rough diagram of circle $x^2 + y^2 = 1$ ...(i)
and hyperbola $x^2 - y^2 = 1$ ...(ii)
are shown below.

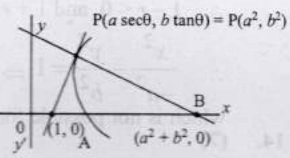



It is clear from graph that there are two common tangents to the curves (i) and (ii) namely $x = 1$ and $x = -1$ out of which $x = 1$ is nearer to the point $P$.
Therefore, directrix of required ellipse is $x - 1 = 0$
Also $e = 1/2$, focus $(1/2, 1)$ then equation of ellipse is given by

$(x - 1/2)^2 + (y-1)^2 = \dfrac{1}{4}(x-1)^2$

$\Rightarrow \dfrac{(x-1/3)^2}{(1/3)^2} + \dfrac{(y-1)^2}{(1/2\sqrt{3})^2} = 1$, which is the standard equation of the ellipse.

**17.** **(a, d)**
Let P $(a\sec\theta, b\tan\theta)$
Equation of tangent at P




$\dfrac{x}{a}\sec\theta - \dfrac{y}{b}\tan\theta = 1$

$\because$ (1, 0) lies on the tangent,
so $a = \sec\theta$
Equation of normal at P

$\dfrac{ax}{\sec\theta} + \dfrac{by}{\tan\theta} = a^2 + b^2$

since normal at P makes equal intercept on co-ordinate axes

$\therefore$ Slope of normal is –1, so $-\dfrac{a}{b}\sin\theta = -1$

$\Rightarrow b = \tan\theta$, hence, $a^2 - b^2 = 1$. ...(i)

$\because e = \sqrt{1+\dfrac{b^2}{a^2}} = \sqrt{1+\dfrac{a^2-1}{a^2}} = \sqrt{2-\dfrac{1}{a^2}}$ (from (i))

Since $a > 1$, so $e \in (1, \sqrt{2})$

Hence, option (a) is true.

Area of $\Delta PAB = \dfrac{1}{2}$ AP.PB

$= \dfrac{1}{2}\sqrt{(a^2-1)^2 + (b^2)^2} \times \sqrt{2b^4}$

$= \dfrac{1}{2}\sqrt{2b^4}\sqrt{2b^4} = b^4$ [from (i)]

Hence, option (d) is true.

**18.** **(a, b, c)** Given $2x - y + 1 = 0$ i.e. $y = 2x + 1$ is a tangent to hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{16} = 1$, $\therefore c^2 = a^2m^2 - b^2$

$\Rightarrow 1^2 = a^2 \times 2^2 - 16$

$\Rightarrow a^2 = \dfrac{17}{4} \Rightarrow a = \dfrac{\sqrt{17}}{2}$

$\therefore$ a, 4, 1; a, 4, 2; 2a, 8, 1 i.e. $\dfrac{\sqrt{17}}{2}, 4, 1; \dfrac{\sqrt{17}}{2}, 4, 2; \sqrt{17}, 8, 1$

cannot be the sides of a right triangle.

**19.** **(a, b, d)** H : $x^2 - y^2 = 1$ is a hyperbola and S : Circle with centre $N(x_2, 0)$. Common tangent to H and S at $P(x_1, y_1)$ is

$xx_1 - yy_1 = 1 \Rightarrow m_1 = \dfrac{x_1}{y_1}$

Now, radius of circle S with centre $N(x_2, 0)$ through the point of contact $(x_1, y_1)$ is perpendicular to the tangent

$\therefore m_1m_2 = -1 \Rightarrow \dfrac{x_1}{y_1} \times \dfrac{0-y_1}{x_2-x_1} = -1$

$\Rightarrow x_2 = 2x_1$

$\because$ *M* is the point of intersection of tangent at *P* and *x*-axis

$\therefore M\left(\dfrac{1}{x_1}, 0\right)$ $\because$ Centroid of $\Delta PMN$ is $(\ell, m)$

$\therefore x_1 + \dfrac{1}{x_1} + x_2 = 3\ell$ and $y_1 = 3m$

$\Rightarrow \dfrac{1}{3}\left(3x_1 + \dfrac{1}{x_1}\right) = l$ and $\dfrac{y_1}{3} = m$ $[\because x_2 = 2x_1]$

$\therefore \dfrac{dl}{dx_1} = 1 - \dfrac{1}{3x_1^2}, \dfrac{dm}{dy_1} = \dfrac{1}{3}$

Also $(x_1, y_1)$ lies on H, $\therefore x_1^2 - y_1^2 = 1 \Rightarrow y_1 = \sqrt{x_1^2 - 1}$

$\therefore m = \dfrac{1}{3}\sqrt{x_1^2-1}$ $\therefore \dfrac{dm}{dx_1} = \dfrac{x_1}{3\sqrt{x_1^2-1}}$

**20.** **(a, b)** If slope of tangents to hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is $m$, then equations of tangent to the hyperbola is

$y = mx \pm \sqrt{a^2m^2 - b^2}$ with the points of contact

$\left(\dfrac{\pm a^2m}{\sqrt{a^2m^2-b^2}}, \dfrac{\pm b^2}{\sqrt{a^2m^2-b^2}}\right)$

$\because$ Tangent to hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ is parallel to $2x - y = 1$,

$\therefore$ Slope of tangent = 2

$\therefore$ Points of contact are $\left(\dfrac{\pm 9\times 2}{\sqrt{9\times 4 - 4}}, \dfrac{\pm 4}{\sqrt{9\times 4-4}}\right)$

i.e. $\left(\dfrac{9}{2\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ and $\left(\dfrac{-9}{2\sqrt{2}}, \dfrac{-1}{\sqrt{2}}\right)$

**21.** **(b, d)** Given ellipse $x^2 + 4y^2 = 4 \Rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$

Its focus is $(\pm\sqrt{3}, 0)$ and eccentricity, $e = \sqrt{1 - \dfrac{1}{4}} = \dfrac{\sqrt{3}}{2}$

Given hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Its eccentricity $= \sqrt{1+\frac{b^2}{a^2}}$

According to the question, $\sqrt{1+\frac{b^2}{a^2}} = \frac{2}{\sqrt{3}} \Rightarrow \frac{b}{a} = \frac{1}{\sqrt{3}}$

As hyperbola passes through the eccentricity of the ellipse ($\pm\sqrt{3}$, 0)

$\therefore \frac{3}{a^2} = 1$ or $a = \sqrt{3}$ $\therefore b = 1$ and focus of hyperbola ($\pm$ 2, 0)

$\therefore$ Equation of hyperbola is $\frac{x^2}{3} - \frac{y^2}{1} = 1 \Rightarrow x^2 - 3y^2 = 3$

**22. (a, b)** The given hyperbola is

$x^2 - y^2 = \frac{1}{2}$ ...(i)

which is a rectangular hyperbola ($\because$ $a = b$) $\therefore$ $e = \sqrt{2}$.

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Its eccentricity $= \frac{1}{\sqrt{2}}$

$\therefore$ $b^2 = a^2\left(1 - \frac{1}{2}\right) \Rightarrow b^2 = \frac{a^2}{2}$

Hence, the equation of ellipse becomes

$x^2 + 2y^2 = a^2$ ...(ii)

Let the hyperbola (i) and ellipse (ii) intersect each other at $P(x_1, y_1)$.

Then slope of hyperbola (i) at $P$ is given by

$$m_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{x_1}{y_1}$$

and that of ellipse (ii) at $P$ is

$$m_2 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{x_1}{2y_1}$$

As the two curves intersect orthogonally,

$\therefore$ $m_1 m_2 = -1$

$\Rightarrow \frac{x_1}{y_1}\cdot\left(-\frac{x_1}{2y_1}\right) = -1 \Rightarrow x_1^2 = 2y_1^2$ ...(iii)

Also $P(x_1, y_1)$ lies on $x^2 - y^2 = \frac{1}{2}$

$\therefore$ $x_1^2 - y_1^2 = \frac{1}{2}$ ...(iv)

On solving (iii) and (iv), we get $y_1^2 = \frac{1}{2}$ and $x_1^2 = 1$

Also $P(x_1, y_1)$ lies on ellipse $x^2 + 2y^2 = a^2$

$\therefore$ $x_1^2 + 2y_1^2 = a^2 \Rightarrow 1 + 1 = a^2$ or $a^2 = 2$

$\therefore$ Equation of required ellipse is $x^2 + 2y^2 = 2$, whose foci are $(\pm ae, 0) = \left(\pm\sqrt{2} \times \frac{1}{\sqrt{2}}, 0\right) = (\pm 1, 0)$

**23. (a,c)** For the given ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1 \Rightarrow e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

$\Rightarrow$ Eccentricity of hyperbola $= \frac{5}{3}$

Let the hyperbola be $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ then

$B^2 = A^2\left(\frac{25}{9} - 1\right) = \frac{16}{9}A^2$ $\therefore$ $\frac{x^2}{A^2} - \frac{9y^2}{16A^2} = 1$

As it passes through focus of ellipse i.e. (3, 0)

$\therefore$ we get $A^2 = 9 \Rightarrow B^2 = 16$

$\therefore$ Equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

Its focus is (5, 0) and vertex is (3, 0).

**24. (a, b, c, d)** Given : Hyperbola $xy = c^2$ .... (i)

and circle $x^2 + y^2 = a^2$ .... (ii)

From (i) and (ii), we get the equation in term of $x$ as $x^2 + c^4/x^2 = a^2$

$\Rightarrow$ $x^4 - a^2x^2 + c^4 = 0$ ... (iii)

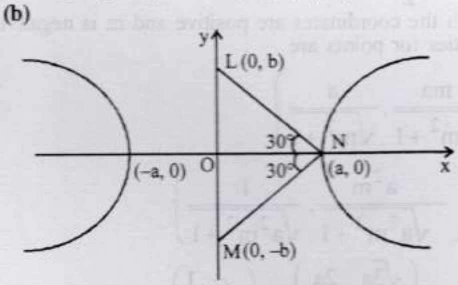
As $x_1, x_2, x_3$ and $x_4$ are roots of (iii),

$\Rightarrow$ $x_1 + x_2 + x_3 + x_4 = 0$ and $x_1 x_2 x_3 x_4 = c^4$

Similarly, forming equation in term of $y$, we get

$y_1 + y_2 + y_3 + y_4 = 0$ and $y_1 y_2 y_3 y_4 = c^4$.

**25. (b)**




Area of $\Delta$LMN $= 4\sqrt{3}$ (given)

$\Rightarrow \frac{1}{2} \times \text{LM} \times \text{ON} = 4\sqrt{3} \Rightarrow \frac{1}{2}(2b)(\sqrt{3}b) = 4\sqrt{3}$

$\therefore$ $b^2 = 4 \Rightarrow b = 2$

So, length of the conjugate axis of hyperbola = 2b = 4

Now $\tan 30° = \frac{\text{OL}}{\text{ON}} = \frac{b}{a} \Rightarrow a = \sqrt{3}b \Rightarrow a = 2\sqrt{3}$

$\therefore$ $b^2 = a^2(e^2 - 1) \Rightarrow 4 = 12(e^2 - 1) \Rightarrow e^2 = 1 + \frac{1}{3} = \frac{4}{3}$

$\therefore$ The eccentricity of hyperbola $= e = \frac{2}{\sqrt{3}}$ and

The distance between the foci of hyperbola = 2ae

$= 2 \times 2\sqrt{3} \times \frac{2}{\sqrt{3}} = 8$

And length of latus ractum of hyperbola

$= \frac{2b^2}{a} = \frac{2 \times 4}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$

26. (b) For a = $\sqrt{2}$ and point of contact (–1, 1).
Equation of circle is satisfied

$x^2 + y^2 = 2$

then eqn. of tangent is

$-x + y = 2 \Rightarrow$ m = 1 and point of contact

$$\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right) = \left(\frac{-\sqrt{2}}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{2}}\right) = (-1, 1)$$

∴ (I) (ii), (Q) is the correct combination.

27. (c) Tangent y = x + 8 $\Rightarrow$ m = 1 Point (8, 16)
∴ Both the coordinates as well as m, are positive, The only possibility of point is $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = (8, 16) \therefore a = 8$

Also it satisfies the equation of curve $y^2 = 4ax$ for the point (8, 16)

And equation of tangent $my = m^2x + a$ is satisfied by m = 1 and a = 8

∴ (III), (i), (P) is the correct combination.

28. (d) Point of contact $\left(\sqrt{3}, \frac{1}{2}\right)$ and tangent $\sqrt{3}x + 2y = 4$.

∴ $m = -\frac{\sqrt{3}}{2}$

∴ Both the coordinates are positive and m is negative. The possibilities for points are

$$Q\left(-\frac{ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$$

Or $$R\left(-\frac{a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$$

For point Q $\left(\frac{\sqrt{3}a}{\sqrt{7}}, \frac{2a}{\sqrt{7}}\right) = \left(\sqrt{3}, \frac{1}{2}\right)$

We get a = $\sqrt{7}$ and a = $\frac{\sqrt{7}}{4}$, which is not possible.

For point R $\left(\frac{a^2\sqrt{3}}{\sqrt{3a^2+4}}, \frac{2}{\sqrt{3a^2+4}}\right) = \left(\sqrt{3}, \frac{1}{2}\right)$

$\Rightarrow \frac{a^2}{\sqrt{3a^2+4}} = 1$ and $\frac{2}{\sqrt{3a^2+4}} = \frac{1}{2}$

$\Rightarrow a^4 - 3a^2 - 4 = 0$ and $3a^2 = 12$

∴ $a^2 = 4$

Also for $a^2 = 4$, equation of ellipse

$x^2 + a^2y^2 = a^2$ is satisfied for the point $\left(\sqrt{3}, \frac{1}{2}\right)$

∴ II, (iv), R is the correct combination.

29. **$A \to (p); B \to (s,t); C \to (r); D \to (q,s)$**

(p) As the line $hx + ky = 1$, touches the circle $x^2 + y^2 = 4$

∴ Length of perpendicular from centre (0 , 0) of circle to the line = radius of the circle

$$\Rightarrow \frac{1}{\sqrt{h^2+k^2}} = 2 \Rightarrow h^2 + k^2 = \frac{1}{4}$$

∴ Locus of (*h, k*) is $x^2 + y^2 = \frac{1}{4}$, which is a circle.

(q) We know that if $|z - z_1| - |z - z_2| = k$, where $|k| < |z_1 - z_2|$, then $z$ traces a hyperbola.

Here $|z+2| - |z-2| = \pm 3$

∴ Locus of $z$ is a hyperbola.

(r) Given : $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right), y = \frac{2t}{1+t^2}$

$\Rightarrow \frac{x}{\sqrt{3}} = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$

On squaring and adding, we get

$$\frac{x^2}{3} + y^2 = \frac{(1-t^2)^2 + 4t^2}{(1+t^2)^2} = 1 \Rightarrow \frac{x^2}{3} + \frac{y^2}{1} = 1$$

which is the equation of an ellipse.

(s) We know, eccentricity of a parabola = 1
eccentricity of an ellipse < 1
and eccentricity of a hyperbola > 1

Hence, the conics whose eccentricity lies in $1 \le x < \infty$ are parabola and hyperbola.

(t) Let $z = x + iy$

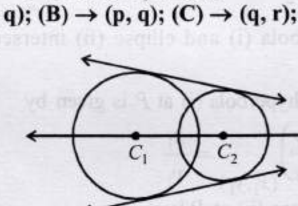
∴ $\text{Re}\,[(x+1) + iy]^2 = x^2 + y^2 + 1$

$\Rightarrow (x+1)^2 - y^2 = x^2 + y^2 + 1$

∴ $y^2 = x$, which is a parabola.

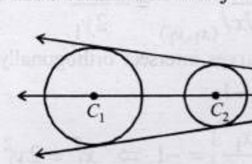
30. **(A) → (p, q); (B) → (p, q); (C) → (q, r); (D) → (q, r)**

(A) - p, q

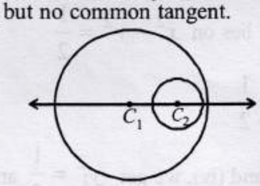



It is clear from the figure that two intersecting circles have a common tangent and a common normal joining the centres

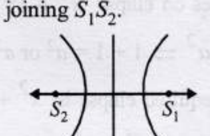
(B) - p, q




(C) – q, r
Two circle when one is strictly inside the other have a common normal $C_1C_2$ but no common tangent.




(D) – q, r
Two branches of hyperbola have no common tangent but have a common normal joining $S_1S_2$.



**31.** **(a)** Given a circle

$x^2 + y^2 - 8x = 0$ ...(i)

and a hyperbola $4x^2 - 9y^2 - 36 = 0$ ...(ii)

To find their point of intersection, substitute the value of $y^2$ from equation (i) in equation (ii), we get

$$4x^2 - 9(8x - x^2) = 36 \Rightarrow 13x^2 - 72x - 36 = 0$$

$$\Rightarrow x = 6, \frac{-6}{13} \Rightarrow y^2 = 12, \frac{-48}{13} - \frac{36}{169} \text{ (not possible)}$$

$\therefore \left(6, 2\sqrt{3}\right)$ and $\left(6, -2\sqrt{3}\right)$ are points of intersection.

$\therefore$ Equation of required circle is

$$(x-6)(x-6) + \left(y - 2\sqrt{3}\right)\left(y + 2\sqrt{3}\right) = 0$$

$$\Rightarrow x^2 + y^2 - 12x + 24 = 0$$

**32.** **(b)** Any tangent to $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is $\frac{x \sec\alpha}{3} - \frac{y \tan\alpha}{2} = 1$

It touches circle with center (4,0) and radius = 4

$$\therefore \frac{\frac{4\sec\alpha - 3}{3}}{\sqrt{\frac{\sec^2\alpha}{9} + \frac{\tan^2\alpha}{4}}} = 4$$

$$\Rightarrow 16\sec^2\alpha - 24\sec\alpha + 9 = 144\left(\frac{\sec^2\alpha}{9} + \frac{\tan^2\alpha}{4}\right)$$

$$\Rightarrow 12\sec^2\alpha + 8\sec - 15 = 0 \Rightarrow \sec\alpha = \frac{5}{6} \text{ or } \frac{-3}{2}$$

since $\sec\alpha = \frac{5}{6} < 1$ is not possible.

$$\therefore \sec\alpha = -3/2 \Rightarrow \tan\alpha = \pm\frac{\sqrt{5}}{2}$$

$$\therefore \text{Slope of tangent} = \frac{2\sec\alpha}{3\tan\alpha} = \frac{2(-3/2)}{3\left(-\sqrt{5}/2\right)} = \frac{2}{\sqrt{5}}$$

(for +ve value of tan $\alpha$)

$\therefore$ Equation of tangent is $\frac{-x}{2} + \frac{y\sqrt{5}}{4} = 1$

$$\Rightarrow 2x - \sqrt{5}y + 4 = 0$$

**33.** Any point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is $(3 \text{ sce}\,\theta, 2\tan\theta)$

$\therefore$ Equation of chord of contact to the circle $x^2 + y^2 = 9$ w.r.t. the point $(3\sec\theta, 2\tan\theta)$ is

$(3\sec\theta)x + (2\tan\theta)y = 9$ ...(i)

If $(h, k)$ be the mid point of chord of contact then equation of chord of contact will be

$hx + ky - 9 = h^2 + k^2 - 9$ $(\because T = S_1)$

$\Rightarrow hx + ky = h^2 + k^2$ ...(ii)

But equations (i) and (ii) represent the same straight line, therefore they should be identical and hence

$$\frac{3\sec\theta}{h} = \frac{2\tan\theta}{k} = \frac{9}{h^2 + k^2}$$

$$\Rightarrow \sec\theta = \frac{3h}{h^2 + k^2}, \tan\theta = \frac{9k}{2(h^2 + k^2)}$$

Now $\sec^2\theta - \tan^2\theta = 1$, $\therefore \frac{9h^2}{(h^2 + k^2)^2} - \frac{81k^2}{4(h^2 + k^2)^2} = 1$

$$\Rightarrow 4h^2 - 9k^2 = \frac{4}{9}(h^2 + k^2)^2$$

$$\Rightarrow \frac{h^2}{9} - \frac{k^2}{4} = \left(\frac{h^2 + k^2}{9}\right)^2$$

Hence, locus of $(h, k)$ is $\frac{x^2}{9} - \frac{y^2}{4} = \left(\frac{x^2 + y^2}{9}\right)^2$

**34.** Let $P(e, f)$ be any point on the locus. Equation of pair of tangents from $P(e, f)$ to the parabola $y^2 = 4ax$ is

$[fy - 2a(x + e)]^2 = (f^2 - 4ae)(y^2 - 4ax)$ $[\because T^2 = SS_1]$

Since angle between the two tangents is 45°,

$$\therefore 1 = \tan 45° = \frac{2\sqrt{h^2 - ab}}{a + b}$$

$\Rightarrow (a + b)^2 = 4(h^2 - ab)$

Here, $a$ = coefficient of $x^2 = 4a^2$

$2h$ = coefficient of $xy = -4af$

and $b$ = coefficient of $y^2 = f^2 - (f^2 - 4ae) = 4ae$

$\therefore (4a^2 + 4ae)^2 = 4[4a^2f^2 - (4a^2)(4ae)]$

$\Rightarrow (a + e)^2 = f^2 - 4ae$ or $e^2 + 6ae + a^2 - f^2 = 0$

$\Rightarrow (e + 3a)^2 - f^2 = 8a^2$

Therefore, the required locus is $(x + 3a)^2 - y^2 = 8a^2$, which is a hyperbola.