8. Polynomials

Questions Pg-147

1. Question

In rectangles with one side 1 centimetre shorter than the other, take the length of the shorter side as x centimetres.

i) Taking their perimeters as p(x) centimetres, write the relation between p(x) and x as an equation.

ii) Taking their area as a(x) square centimetres, write the relation between a(x) and x as an equation.

iii) Calculate p(1), p(2), p(3), p(4), p(5). Do you see any pattern?

iv) Calculate a(1), a(2), a(3), a(4), a(5). Do you see any pattern?

Answer

Given one side is smaller than the other by 1 cm.

∴ The two adjacent sides of the triangle are x, x + 1.

(i) perimeter $p(x) = 2 \times [(x) + (x + 1)]$

⇒ $p(x) = 4x + 2$ – (1)

(ii) Area $a(x) = (x) \times (x + 1)$

⇒ $a(x) = x^2 + x$ – (2)

(iii) by (1)

$p(1) = 4 \times 1 + 2 = 6$

$p(2) = 4 \times 2 + 2 = 10$

$p(3) = 4 \times 3 + 2 = 14$

$p(4) = 4 \times 4 + 2 = 18$

$p(5) = 4 \times 5 + 2 = 22$

Here the difference between a number and its successor is always 4.

We can see that they are in Arithmetic Progression (AP) with a common difference of 4.

(iv) by (2)

$a(1) = 1^2 + 1 = 2$

$a(2) = 2^2 + 2 = 6$

$a(3) = 3^2 + 3 = 12$

$a(4) = 4^2 + 4 = 20$

$a(5) = 5^2 + 5 = 30$

Here the difference between a number and its previous number is increasing by 2.

$a(2) – a(1) = 4$

$a(3) – a(2) = 6$

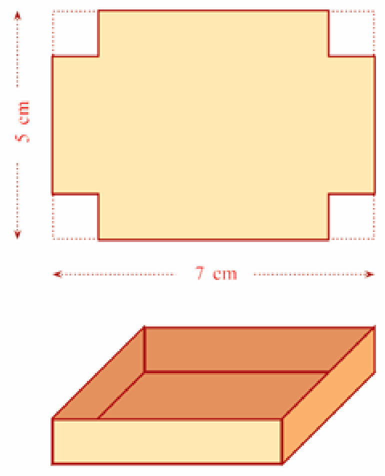
$a(4) – a(3) = 8$

$a(5) – a(4) = 10$

∴ Their common difference is in AP.

2. Question

From the four corners of a rectangle, small squares are cut off and the sides are folded up to make a box, as shown below:



i) Taking side of the square as x centimetres, write the dimensions of the box in terms of x.

ii) Taking the volume of the box as v(x) cubic centimetres, write the relation between v(x) and x as an equation.

iii) Calculate $v\left(\frac{1}{2}\right)$, $v(1)$, $v\left(1\frac{1}{2}\right)$.

Answer

(i) The side of the square = x

⇒ The dimensions of the box are

L = 7 – 2x

B = 5 – 2x

H = x

(ii) v(x) = L × B × H

⇒ v(x) = (7 - 2x) × (5 – 2x) × (x)

⇒ v(x) = (35 + 4x^2 -24x) × (x)

⇒ v(x) = 4x^3 – 24x^2 + 35x – (1)

(iii) by (1)

$$V\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 24\left(\frac{1}{2}\right)^2 + 35\left(\frac{1}{2}\right)$$

$$\Rightarrow V\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 24\left(\frac{1}{2}\right)^2 + 35\left(\frac{1}{2}\right)$$

$$\Rightarrow V\left(\frac{1}{2}\right) = 12$$

Similarly,

v(1) = 4$(1)^3$ – 24$(1)^2$ + 35(1)

⇒ v(1) = 15

And

$$V\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^3 - 24\left(\frac{3}{2}\right)^2 + 35\left(\frac{3}{2}\right)$$

$\Rightarrow V\left(\frac{3}{2}\right) = 12$

3. Question

Consider all rectangles that can be made with a 1 metre long rope. Take one of its sides as x centimetres and the area enclosed as a(x) square centimetres.

i) Write the relation between a(x) and x as an equation.

ii) Why are the numbers a(10) and a(40) equal?

iii) To get the same numbers as a(x), for two different numbers as x, what must be the relation between the numbers?

Answer

According to the question the perimeter of the rectangle must be always equal to 100 cm.

Now, let one side be x cm and other adjacent side be y cm.

$\Rightarrow P = 2 \times [x + y]$

$\Rightarrow 100 = 2x + 2y$

$\Rightarrow y = 50 – x$ – (1)

(i) $a(x) = (50-x) \times (x)$ – (by (1))

$a(x) = 50x – x^2$

(ii) $a(10) = 50 \times (10) – (10)^2 = 400$

$a(40) = 50 \times (40) – (40)^2 = 400$

Now if a side a is 10 cm, then the other side

$P = (10 + y) \times 2$

$P = 100$

$\Rightarrow 50 – 10 = y$

$\Rightarrow y = 40$ cm

∴ the in case of x = 10 cm y = 40 cm, and in case of x = 40 cm y = 10 cm. So, in both cases we have same rectangle hence the area is same.

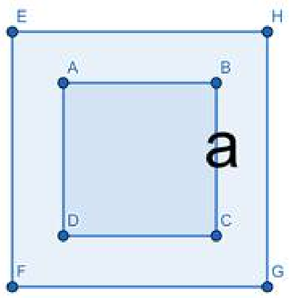
Questions Pg-150

1 A. Question

Write each of the relations below in algebra and see if it gives a polynomial. Give reasons for your conclusion also.

An 1 metre wide path goes around a square ground. The relation between the length of a side of the ground and the area of the path.

Answer



Let the side of the ground be a.

The width of the path is 1 m.

⇒ Length of one side of the path is a + 2 m.

⇒ Area of the path = $(a + 2)^2 - (a)^2$

Now, Area of the path is a is defined p(a)

⇒ $p(a) = (a + 2)^2 - (a)^2$

⇒ $p(a) = 4 + 4 \times a$

1 B. Question

Write each of the relations below in algebra and see if it gives a polynomial. Give reasons for your conclusion also.

A liquid contains 7 litres of water and 18 litres of acid. More acid is added to it. The relation between the amount of acid added and the change in the percentage of acid in the liquid.

Answer

A liquid is 7 litres water and 18 litres acid.

Let x litres acid is added to this liquid.

Now,

The liquid is 7 litres water and (18 + x) litres acid.

Now,

Initial percentage of acid $= \frac{18}{18+7} \times 100$

⇒ Initial percentage of acid = 72% -- (1)

Final percentage of acid $= \frac{18+x}{18+x+7} \times 100$ – (2)

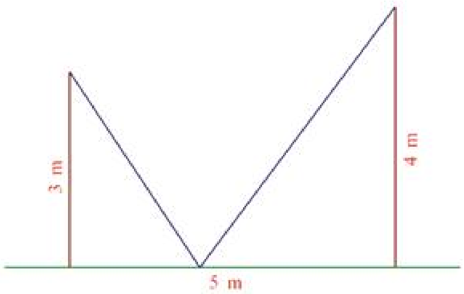
Let change in percentage of the acid be R

⇒ $R = \left(\frac{18+x}{18+x+7} \times 100\right) - 72$

1 C. Question

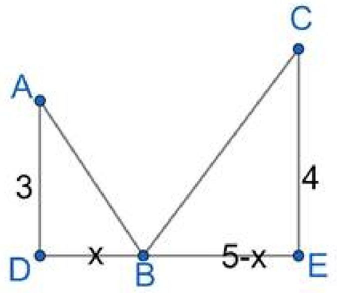
Write each of the relations below in algebra and see if it gives a polynomial. Give reasons for your conclusion also.

Two poles of heights 3 metres and 4 metres are erected upright on the ground, 5 metres apart. A rope is to be stretched from the top of one pole to some point on the ground and from there to the top of the other pole:



The relation between the distance of the point on the ground from the foot of a pole and the total length of the rope.

Answer



Let the distance of the point on ground be x m from the 3 m pole.

⇒ This distance of the point on ground from 4 m pole is (5-x) m. (∵ the distance between the poles is 5 m.)

Now the length of the rope is AB + BC.

By Pythagoras Theorem

$AB^2 = AD^2 + DB^2$

⇒ $AB^2 = 3^2 + x^2$

⇒ $AB = \sqrt{9 + x^2}$

Similarly,

$BC = \sqrt{16 + x^2}$

Now, let the length of rope be a R

⇒ R = AB + BC

⇒ $R = \sqrt{9 + x^2} + \sqrt{16 + x^2}$

2. Question

Write each of the operations below as an algebraic expression, find out which are polynomials and explain why.

i) Sum of a number and its reciprocal.

ii) Sum of a number and its square root.

iii)Product of the sum of difference of a number and its square root.

Answer

(i) Let the number be x.

So its reciprocal is $\frac{1}{x}$

$\text{Sum} = x + \frac{1}{x}$

⇒ $\text{Sum} = x + x^{-1}$

It is not a polynomial as it has a negative power of x, but for a polynomial all the powers should be positive integers.

(ii) Let the number be x.

So its square root is $\sqrt{x}$

Sum = $x + \sqrt{x}$

It is not a polynomial as it has square root which is fractional exponent, but to be a polynomial it should only have positive integer exponents.

(iii) Let sum of number and its square root be a.

$\Rightarrow a = x + \sqrt{x}$

Let difference be b.

$\Rightarrow b = x - \sqrt{x}$

Product of a and b.

$\Rightarrow a \times b = (x + \sqrt{x}) \times (x - \sqrt{x})$

$\Rightarrow a \times b = x^2 – x$

It’s a Polynomial as all the exponents of x are positive integer.

3 A. Question

Find polynomials p(x) satisfying each set of conditions below.

First degree polynomial with p(1) = 1 and p(2) = 3.

Answer

let the polynomial be p(x)

p(x) = ax + b (where a and b are constants)

p(1) = a + b = 1 – (1) (given in the question)

p(2) = 2a + b = 3 – (2) (given in the question)

by (1) and (2)

a = 2, b = -1

$\Rightarrow$ p(x) = 2x – 1

3 B. Question

Find polynomials p(x) satisfying each set of conditions below.

First degree polynomial with p(1) = –1 and p(–2) = 3.

Answer

Let the polynomial be p(x)

$\Rightarrow$ p(x) = ax + b

$\Rightarrow$ p(-1) = -a + b = -1 – (1)

$\Rightarrow$ p(-2) = -2a + b = 3 – (2)

By (1) and (2)

a = -4, b = -5

$\Rightarrow$ p(x) = -4x -5

3 C. Question

Find polynomials p(x) satisfying each set of conditions below.

Second degree polynomial with p(0) = 0, p(1) = 2 and p(2) = 6.

Answer

Let the polynomial be p(x)

$p(x) = ax^2 + bx + c$ (a,b,c are constants)

$\Rightarrow$ p(0) = c = 0

$\Rightarrow$ p(1) = a + b = 2 – (1)($\because$ c = 0)

⇒ p(2) = 4a + 2b = 6 – (2)

By (1) and (2)

a = 1, b = 1

⇒ p(x) = x^2 + x

3 D. Question

Find polynomials p(x) satisfying each set of conditions below.

Three different second degree polynomials with p(0) = 0 and p(1) = 2.

Answer

Let an arbitrary polynomial be p(x)

p(x) = ax^2 + bx + c

⇒ p(0) = c = 0

⇒ p(1) = a + b = 1

⇒ p(x) = ax^2 + bx (where a + b = 1)

There could be infinite number of such polynomials but we have to find only 3.

$$p(x) = \frac{1}{2}x^2 + \frac{1}{2}x$$

$$p(x) = \frac{3}{4}x^2 + \frac{1}{4}x$$

$$p(x) = \frac{2}{3}x^2 + \frac{1}{3}x$$

They are any three possible polynomials.

Questions Pg-153

1. Question

Taking p(x) = $2x^2$ + 3x + 5, q(x) = x^2 + 4x + 1 and s(x) = p(x) + q(x), calculate p(10), q(10), s(10), p(10) + q(10).

Answer

(i) p(10)

p(x) = $2x^2$ + 3x + 5

p(10) = $2(10)^2$ + 3(10) + 5

p(10) = 235 – (1)

(ii) q(10)

q(x) = x^2 + 4x + 1

q(10) = $(10)^2$ + 4(10) + 1

q(10) = 141 – (1)

(iii) s(x) = p(x) + q(x)

s(x) = $2x^2$ + 3x + 5 + x^2 + 4x + 1

s(x) = $3x^2$ + 7x + 6

s(10) = $3(10)^2$ + 7(10) + 6

$s(10) = 376$

(iv) $p(10) + q(10)$

by (1) and (2)

$p(10) + q(10) = 235 + 141$

$p(10) + q(10) = 376$

This is the same result as (iii)

⇒ Polynomials have Commutative Property of Addition.

2. Question

What polynomial added to $x^2 + 4x – 5$ gives $2x^2 – 3x + 1$?

Answer

Let the required polynomial p(x)

Now, according to the question

$x^2 + 4x – 5 + p(x) = 2x^2 – 3x + 1$

$\Rightarrow p(x) = 2x^2 – 3x + 1 - x^2 - 4x + 5$

$\Rightarrow p(x) = x^2 - 7x + 6$

3. Question

What polynomial subtracted from $x^2 + 4x – 5$ gives $2x^2 – 3x + 1$?

Answer

Let the required polynomial p(x)

Now, according to the question

$x^2 + 4x – 5 – p(x) = 2x^2 – 3x + 1$

$\Rightarrow p(x) = x^2 + 4x – 5 - 2x^2 + 3x – 1$

$\Rightarrow p(x) = -x^2 + 7x – 6$

4. Question

Find polynomials p(x) and q(x) such that $p(x) + q(x) = x^2 – 4x + 1$ and $p(x) – q(x) = x^2 + 5x –2$.

Answer

$p(x) + q(x) = x^2 – 4x + 1$ – (1)

$p(x) – q(x) = x^2 + 5x –2$ – (2)

by adding (1) and (2)

$2 \times p(x) = 2x^2 + x –1$

$\Rightarrow p(x) = x^2 + \frac{1}{2}x - \frac{1}{2}$

by (1)

$q(x) = x^2 – 4x + 1 – p(x)$

$\Rightarrow q(x) = x^2 - 4x + 1 - x^2 - \frac{1}{2}x + \frac{1}{2}$

$\Rightarrow q(x) = -\frac{9}{2}x + \frac{3}{2}$

5. Question

Taking $p(x) = 3x^2 – 2x + 4$, write the following as polynomials:

i) $(x + 1) p(x) + (x – 1) p(x)$

ii) $(x + 1) p(x) – (x – 1) p(x)$

iii) $\frac{1}{2}(x+1)p(x) - \frac{1}{2}(x-1)p(x)$

Answer

(i) $p(x) = 3x^2 – 2x + 4$

Now,

$(x + 1) p(x) + (x – 1) p(x)$

taking p(x) common

$\Rightarrow p(x)[x + 1 + x-1]$

$\Rightarrow 2x \times p(x)$

$\Rightarrow 2x \times (3x^2 – 2x + 4)$

$\Rightarrow 6x^3 – 4x^2 + 8x$

(ii) $p(x) = 3x^2 – 2x + 4$

Now,

$(x + 1) p(x) - (x – 1) p(x)$

taking p(x) common

$\Rightarrow p(x)[x + 1-x + 1]$

$\Rightarrow 2 \times p(x)$

$\Rightarrow 2 \times (3x^2 – 2x + 4)$

$\Rightarrow 6x^2 – 4x + 8$

(iii) $p(x) = 3x^2 – 2x + 4$

Now,

$\frac{1}{2}(x + 1)p(x) - \frac{1}{2}(x – 1) p(x)$

taking p(x) common

$\Rightarrow p(x)[\frac{x}{2} + \frac{1}{2} - \frac{x}{2} + \frac{1}{2}]$

$\Rightarrow p(x)$

$\Rightarrow 3x^2 – 2x + 4$