Chapter
Electrostatic Potential and Capacitance

Topic-1: Electrostatic Potential and Equipotential Surfaces

1. MCQs with One Correct Answer

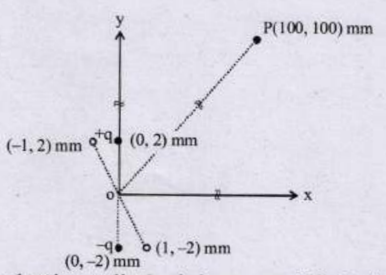
1. An electric dipole is formed by two charges +q and –q located in xy-plane at (0,2) mm and (0,–2) mm, respectively, as shown in the figure. The electric potential at point P (100,100) mm due to the dipole is V_0. The charges +q and –q are then moved to the points (–1,2) mm and (1,–2) mm, respectively. What is the value of electric potential at P due to the new dipole? **[Adv. 2023]**

(a) $V_0/4$

(b) $V_0/2$

(c) $V_0/\sqrt{2}$

(d) $3V_0/4$



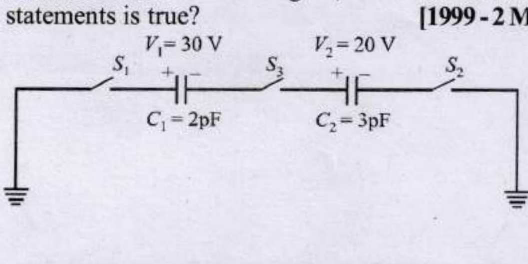
2. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral. **[2007]**

(a) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
(b) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
(c) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders
(d) No potential difference appears between the two cylinders when same charge density is given to both the cylinders.

3. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B and C satisfy: **[2001S]**

(a) $V_A < V_B$ (b) $V_A > V_B$ (c) $V_A < V_C$ (d) $V_A > V_C$

4. For the circuit shown in Figure, which of the following statements is true? **[1999 - 2 Marks]**



(a) With S_1 closed $V_1 = 15$ V, $V_2 = 20$ V
(b) With S_3 closed, $V_1 = V_2 = 25$ V
(c) With S_1 and S_2 closed, $V_1 = V_2 = 0$
(d) With S_1 and S_3 closed, $V_1 = 30V$, $V_2 = 20V$

5. A charge $+q$ is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$,..... $x = \infty$ on the x axis, and a charge $-q$ is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$,.... $x = \infty$. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/(4\pi\varepsilon_0 r)$. Then, the potential at the origin due to the above system of charges is **[1998S - 2 Marks]**

(a) 0 (b) $\dfrac{q}{8\pi\varepsilon_0 x_0 \ln 2}$

(c) ∞ (d) $\dfrac{q \ln 2}{4\pi\varepsilon_0 x_0}$

6. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is : **[Similar 8 April 2019 (I) 1989 - 2 Marks]**

(a) V (b) $2V$ (c) $4V$ (d) $-2V$

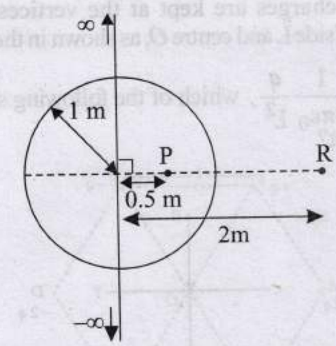
7. A hollow metal sphere of radius 5 cms is charged such that the potential on its surface is 10 volts. The potential at the centre of the sphere is **[1983 - 1 Mark]**

(a) zero (b) 10 volts
(c) same as at a point 5 cms away from the surface
(d) same as at a point 25 cms away from the surface

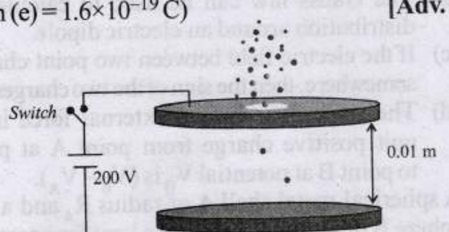
2. Integer Value Answer

8. An infinitely long thin wire, having a uniform charge density per unit length of 5 nC/m, is passing through a spherical shell of radius 1 m, as shown in the figure. A 10 nC charge is distributed uniformly over the spherical shell. If the configuration of the charges remains static, the magnitude of the potential difference between points P and R, in Volt, is ______.

[Given: In SI units $\dfrac{1}{4\pi\epsilon_0} = 9\times10^9$, ln 2 = 0.7. Ignore the area pierced by the wire.] **[Adv. 2024]**



9. Two large circular discs separated by a distance of 0.01 m are connected to a battery via a switch as shown in the figure. Charged oil drops of density 900 kg m^{-3} are released through a tiny hole at the center of the top disc. Once some oil drops achieve terminal velocity, the switch is closed to apply a voltage of 200 V across the discs. As a result, an oil drop of radius 8×10^{-7} m stops moving vertically and floats between the discs. The number of electrons present in this oil drop is ______ (neglect the buoyancy force, take acceleration due to gravity = 10 ms^{-2} and charge on an electron (e) = 1.6×10^{-19} C) **[Adv. 2020]**

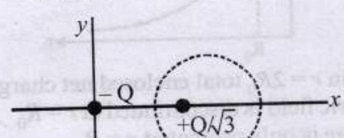


10. A particle, of mass 10^{-3} *kg* and charge 1.0 *C*, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t \hat{i}$, where E_0 = 1.0 NC^{-1} and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is ______. **[Adv. 2018]**

3 Numeric / New Stem Based Questions

Stem for Qs. No. 11-12

Two point charges $-Q$ and $+Q/\sqrt{3}$ are placed in the xy-plane at the origin (0, 0) and a point (2, 0), respectively, as shown in the figure. This results in an equipotential circle of radius R and potential $V = 0$ in the xy-plane with its center at $(b, 0)$. All lengths are measured in meters.



11. The value of R is ____ meter. **[Adv. 2021]**

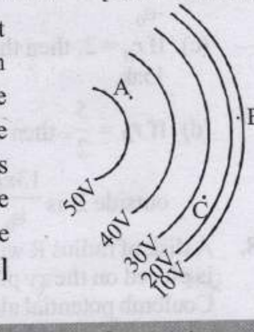
12. The value of b is ____ meter. **[Adv. 2021]**

4 Fill in the Blanks

13. 512 identical drops of mercury are charged to a potential of 2 V each. The drops are joined to form a single drop. The potential of this drop is ______ V. **[Feb. 25, 2021 (I)]**

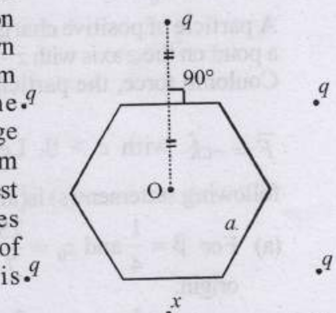
14. The electric potential V at any point x, y, z (all in metres) in space is given by $V = 4x^2$ volts. The electric field at the point (1m, 0, 2 m) is .... V/m. **[1992 - 1 Mark]**

15. Figure shows line of constant potential in a region in which an electric field is present. The values of the potential are written in brackets. Of the points A, B and C, the magnitude of the electric field is greatest at the point ... **[1984- 2 Marks]**



6 MCQs with One or More than One Correct Answer

16. Six charges are placed around a regular hexagon of side length a as shown in the figure. Five of them have charge q, and the remaining one has charge x. The perpendicular from each charge to the nearest hexagon side passes through the center O of the hexagon and is bisected by the side.



Which of the following statement(s) is(are) correct in SI units? **[Adv. 2022]**

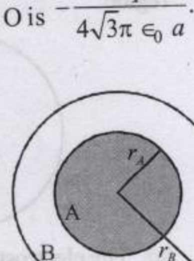
(a) When $x = q$, the magnitude of the electric field at O is zero.

(b) When $x = -q$, the magnitude of the electric field at O is $\dfrac{q}{6\pi \epsilon_0 a^2}$.

(c) When $x = 2q$, the potential at O is $\dfrac{7q}{4\sqrt{3}\pi \epsilon_0 a}$.

(d) When $x = -3q$, the potential at O is $-\dfrac{3q}{4\sqrt{3}\pi \epsilon_0 a}$.

17. In the figure, the inner (shaded) region A represents a sphere of radius $r_A = 1$, within which the electrostatic charge density varies with the radial distance r from the center as $\rho_A = kr$, where k is positive.
In the spherical shell B of outer radius r_B, the electrostatic charge density varies as $\rho_B = \dfrac{2k}{r}$.



Assume that dimensions are taken care of. All physical quantities are in their SI units. **[Adv. 2022]**

Which of the following statement(s) is (are) correct?

(a) If $r_B = \sqrt{\frac{3}{2}}$, then the electric field is zero everywhere outside B.

(b) If $r_B = \frac{3}{2}$, then the electric potential just outside B is $\frac{k}{\epsilon_0}$.

(c) If $r_B = 2$, then the total charge of the configuration is $15\pi k$.

(d) If $r_B = \frac{5}{2}$, then the magnitude of the electric field just outside B is $\frac{13\pi k}{\epsilon_0}$

18. A disk of radius R with uniform positive charge density σ is placed on the *xy* plane with its center at the origin. The Coulomb potential along the z-axis is

$$V(z) = \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2} - z\right).$$

A particle of positive charge q is placed initially at rest at a point on the z axis with $z = z_0$ and $z_0 > 0$. In addition to the Coulomb force, the particle experiences a vertical force $\vec{F} = -c\hat{k}$ with $c > 0$. Let $\beta = \frac{2c\epsilon_0}{q\sigma}$. Which of the following statement(s) is(are) correct? **[Adv. 2022]**

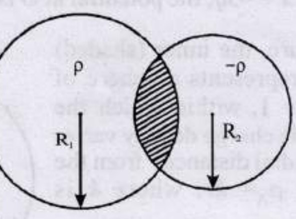
(a) For $\beta = \frac{1}{4}$ and $z_0 = \frac{25}{7}R$, the particle reaches the origin.

(b) For $\beta = \frac{1}{4}$ and $z_0 = \frac{3}{7}R$, the particle reaches the origin.

(c) For $\beta = \frac{1}{4}$ and $z_0 = \frac{R}{\sqrt{3}}$, the particle returns back to $z = z_0$.

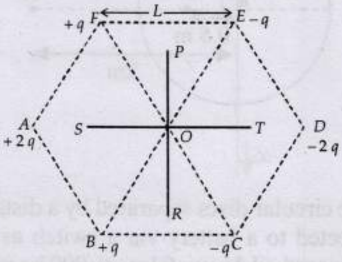
(d) For $\beta > 1$ and $z_0 > 0$, the particle always reaches the origin.

19. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region **[Adv. 2013]**



(a) The electrostatic field is zero
(b) The electrostatic potential is constant
(c) The electrostatic field is constant in magnitude
(d) The electrostatic field has same direction

20. Six point charges are kept at the vertices of a regular hexagon of side L and centre *O*, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is (are) correct? **[2012]**



(a) The electric field at *O* is 6K along *OD*
(b) The potential at *O* is zero
(c) The potential at all points on the line *PR* is same
(d) The potential at all points on the line *ST* is same

21. Which of the following statement(s) is/are correct? **[2011]**

(a) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.

(b) The Gauss law can be used to calculate the field distribution around an electric dipole.

(c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

(d) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

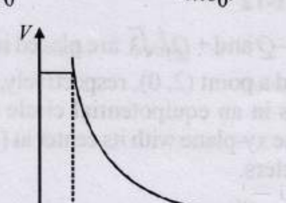
22. A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B (< R_A)$ are kept far apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then **[2011]**

(a) $E_A^{inside} = 0$ (b) $Q_A > Q_B$

(c) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (d) $E_A^{on\,surface} < E_B^{on\,surface}$

23. A spherical symmetric charge system is centered at origin. Given, Electric potential **[2006S - 5 Marks]**

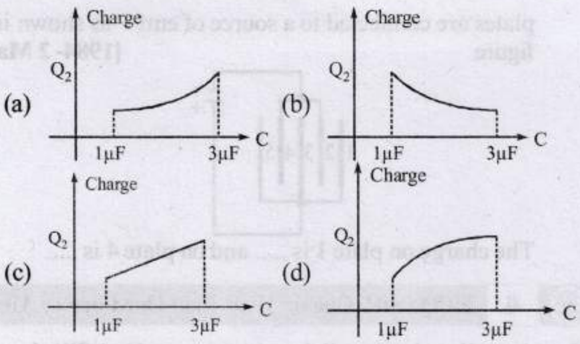
$$V = \frac{Q}{4\pi\varepsilon_0 R_0}\ (r \le R_0),\ V = \frac{Q}{4\pi\varepsilon_0 r}\ (r > R_0)$$



(a) Within $r = 2R_0$ total enclosed net charge is Q
(b) Electric field is discontinued at $r = R_0$
(c) Charge is only present at $r = R_0$
(d) Electrostatic energy is zero for $r < R_0$

9 Assertion and Reason Type Questions

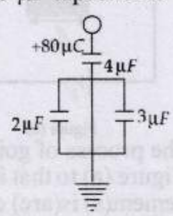
24. **STATEMENT-1 :** For practical purposes, the earth is used as a reference at zero potential in electrical circuits.
and



4. A parallel plate capacitor is made of two circular plates separated by a distance 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m the charge density of the positive plate will be close to: **[2014]**

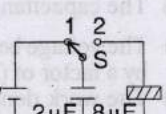
(a) 6×10^{-7} C/m^2 (b) 3×10^{-7} C/m^2
(c) 3×10^{4} C/m^2 (d) 6×10^{4} C/m^2

5. In the given circuit, a charge of $+80\ \mu C$ is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is **[2012]**



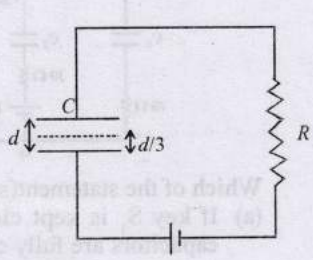
(a) $+32\ \mu C$
(b) $+40\ \mu C$
(c) $+48\ \mu C$
(d) $+80\ \mu C$

6. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is **[2011]**



(a) 0%
(b) 20%
(c) 75%
(d) 80%

7. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $d/3$ initially. Suppose the liquid level decreases at a constant speed v, the time constant as a function of time t is – **[2008]**



(a) $\frac{6\varepsilon_0 R}{5d+3vt}$

(b) $\frac{(15d+9vt)\,\varepsilon_0 R}{2d^2-3dvt-9v^2t^2}$

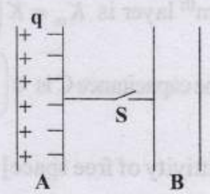
(c) $\frac{6\varepsilon_0 R}{5d-3vt}$

(d) $\frac{(15d-9vt)\,\varepsilon_0 R}{2d^2-3dvt-9v^2t^2}$

8. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is **[2002S]**

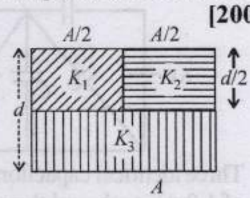
(a) $\frac{1}{4}C\left(V_1^2-V_2^2\right)$ (b) $\frac{1}{4}C\left(V_1^2+V_2^2\right)$
(c) $\frac{1}{4}C\left(V_1-V_2\right)^2$ (d) $\frac{1}{4}C\left(V_1+V_2\right)^2$

9. Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is **[2001S]**



(a) zero (b) $q/2$ (c) q (d) $2q$

10. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants k_1, k_2 and k_3 as shown. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant k is given by **[2000S]**



(a) $\frac{1}{K}=\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{2K_3}$

(b) $\frac{1}{K}=\frac{1}{K_1+K_2}+\frac{1}{2K_3}$

(c) $K=\frac{K_1K_2}{K_1+K_2}+2K_3$ (d) $K=K_1+K_2+2K_3$

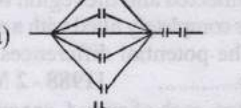
11. Two identical metal plates are given positive charges Q_1 and Q_2 ($<Q_1$) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is **[Similar July 29, 2022 (II) 1999 - 2 Marks]**

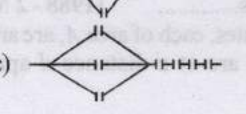
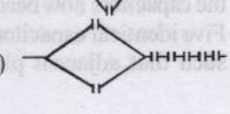
(a) $(Q_1+Q_2)/(2C)$ (b) $(Q_1+Q_2)/C$
(c) $(Q_1-Q_2)/C$ (d) $(Q_1-Q_2)/(2C)$

12. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is **[1995S]**

(a) zero (b) $\frac{3}{2}CV^2$ (c) $\frac{25}{6}CV^2$ (d) $\frac{9}{2}CV^2$

13. Seven capacitors each of capacitance $2\mu F$ are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{10}{11}\right)\mu F$. Which of the combination (s) shown in figure will achieve the desired result ? **[1990 - 2 Marks]**

(a)  (b) 

(c)  (d) 

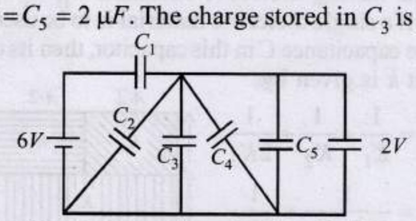
2 Integer Value Answer

14. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \dfrac{d}{N}$. The dielectric constant of the m[th] layer is $K_m = K\left(1+\dfrac{m}{N}\right)$. For a very large N(>$10^3$), the capacitance C is $\alpha\left(\dfrac{K\epsilon_0 A}{d\ln 2}\right)$. The value of α will be ___

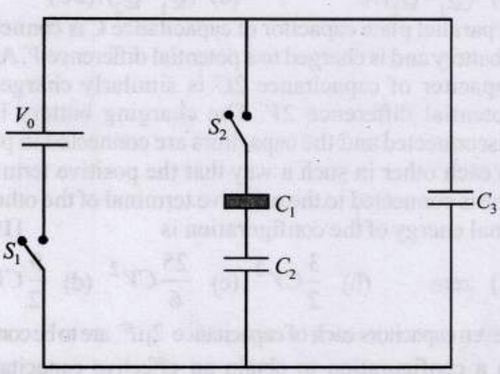
[ϵ_0 is the permittivity of free space] **[Adv. 2019]**

3 Numeric Answers

15. In the following circuit $C_1 = 12\,\mu F, C_2 = C_3 = 4\,\mu F$ and $C_4 = C_5 = 2\,\mu F$. The charge stored in C_3 is ____ μC. **[Adv. 2022]**



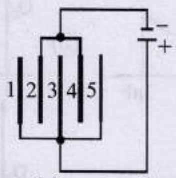
16. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5 μC. The value of ε_r = ________. **[Adv. 2018]**



4 Fill in the Blanks

17. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to a potential difference V. The battery is then disconnected and the region between the plates of the capacitor C is completely filled with a material of dielectric constant K. The potential differences across the capacitors now becomes............ **[1988 - 2 Marks]**

18. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are at a distance d apart, the plates are connected to a source of emf V as shown in the figure **[1984- 2 Marks]**



The charge on plate 1 is ..... and on plate 4 is .....

6 MCQs with One or More than One Correct Answer

19. A medium having dielectric constant $K > 1$ fills the space between the plates of a parallel plate capacitor. The plates have large area, and the distance between them is d. The capacitor is connected to a battery of voltage V, as shown in Figure (a). Now, both the plates are moved by a distance of $\dfrac{d}{2}$ from their original positions, as shown in Figure (b).

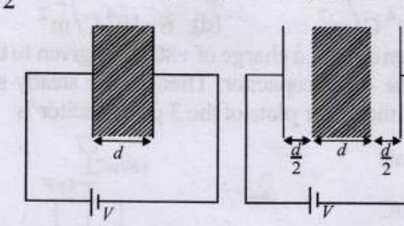


Figure (a) Figure (b)

In the process of going from the configuration depicted in Figure (a) to that in Figure (b), which of the following statement(s) is(are) correct? **[Adv. 2022]**

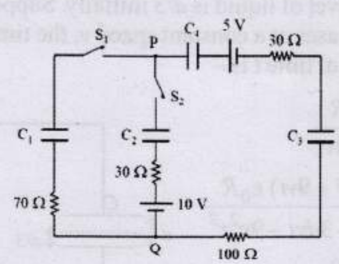
(a) The electric field inside the dielectric material is reduced by a factor of $2K$.

(b) The capacitance is decreased by a factor of $\dfrac{1}{K+1}$.

(c) The voltage between the capacitor plates is increased by a factor of $(K+1)$.

(d) The work done in the process **DOES NOT** depend on the presence of the dielectric material.

20. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\mu F$, $C_2 = 30\mu F$ and $C_3 = C_4 = 80\mu F$. **[Adv. 2019]**



Which of the statement(s) is/are correct?

(a) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10V.

(b) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30resistor (between points P and Q) will be 0.2A (round off to 1[st] decimal place)

(c) At time $t = 0$, the key S_1 is closed, the instantaneous current in the closed circuit will be 25mA.

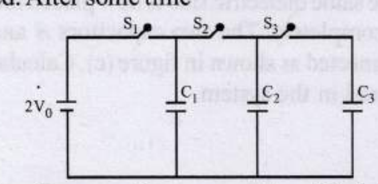
(d) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitors C_1 will be 4V.

21. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects. **[Adv. 2014]**

(a) $\dfrac{E_1}{E_2} = 1$ (b) $\dfrac{E_1}{E_2} = \dfrac{1}{K}$

(c) $\dfrac{Q_1}{Q_2} = \dfrac{3}{K}$ (d) $\dfrac{C}{C_1} = \dfrac{2+K}{K}$

22. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time **[Adv. 2013]**



(a) The charge on the upper plate of C_1 is $2CV_0$
(b) The charge on the upper plate of C_1 is CV_0
(c) The charge on the upper plate of C_2 is 0
(d) The charge on the upper plate of C_2 is $-CV_0$

23. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As one goes from 0 to $3d$, **[1998S - 2 Marks]**

(a) the magnitude of the electric field remains the same.
(b) the direction of the electric field remains the same.
(c) the electric potential increases continuously.
(d) the electric potential increases at first, then decreases and again increases.

24. A parallel plate capacitor of plate area A and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted), and work done on the system, in question, in the process of inserting the slab, then **[1991 - 2 Marks]**

(a) $Q = \dfrac{\varepsilon_0 AV}{d}$ (b) $Q = \dfrac{\varepsilon_0 KAV}{d}$

(c) $E = \dfrac{V}{Kd}$ (d) $W = \dfrac{\varepsilon_0 AV^2}{2d}\left[1 - \dfrac{1}{K}\right]$

25. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved farther apart by means of insulating handles: **[1987 - 2 Marks]**

(a) the charge on the capacitor increases.
(b) the voltage across the plates increases.
(c) the capacitance increases.
(d) the electrostatic energy stored in the capacitor increases

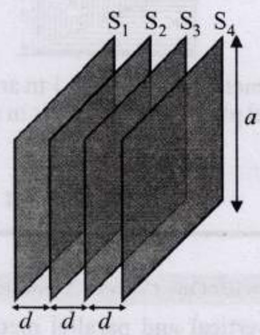
26. A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous one as **[1985 - 2 Marks]**

(a) $Q > Q_0$ (b) $V > V_0$ (c) $E > E_0$ (d) $U > U_0$

7 Match the Following

27. Four identical thin, square metal sheets, S_1, S_2, S_3 and S_4, each of side a are kept parallel to each other with equal distance d ($<< a$) between them, as shown in the figure.

Let $C = \dfrac{\varepsilon_0 a^2}{d}$, where ε_0 is the permittivity of free space.



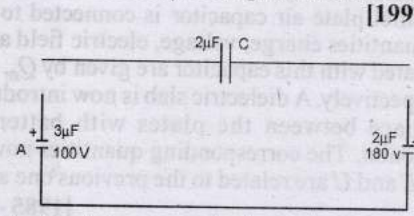
Match the quantities mentioned in List-I with their values in List-II and choose the correct option. **[Adv. 2024]**

List-I	List-II
(P) The capacitance between S_1 and S_4, with S_2 and S_3 not connected, is	(1) $3C_0$
(Q) The capacitance between S_1 and S_4, with S_2 shorted to S_3, is	(2) $\dfrac{C_0}{2}$
(R) The capacitance between S_1 and S_3, with S_2 shorted to S_4, is	(3) $\dfrac{C_0}{3}$
(S) The capacitance between S_1 and S_2, with S_3 shorted to S_1, and S_2 shorted to S_4, is	(4) $\dfrac{2C_0}{3}$
	(5) $2C_0$

(a) $P \to 3; Q \to 2; R \to 4; S \to 5$
(b) $P \to 2; Q \to 3; R \to 2; S \to 1$
(c) $P \to 3; Q \to 2; R \to 4; S \to 1$
(d) $P \to 3; Q \to 2; R \to 2; S \to 5$

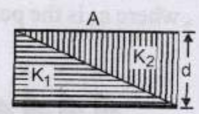
10 Subjective Problems

28. Two capacitors A and B with capacities $3\ \mu F$ and $2\ \mu F$ are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in the figure with one wire from each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged $2\ \mu F$ capacitor C with lead wires falls on the free ends to complete the circuit. Calculate **[1997 - 5 Marks]**



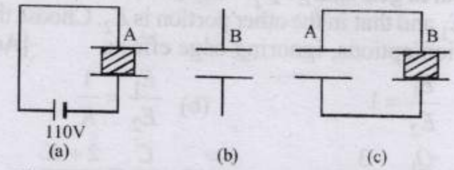
(i) the final charge on the three capacitors. and
(ii) the amount of electrostatic energy stored in the system before and after the completion of the circuit.

29. The capacitance of a parallel plate capacitor with plate area A and separation d is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2, respectively. Find the capacitance of the resulting capacitor. **[1996 - 2 Marks]**



30. Two square metal plates of side 1 m are kept 0.01 m apart like a parallel plate capacitor in air in such a way that one of their edges is perpendicular to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of emf 500 V. The plates are then lowered vertically into the oil at a speed of $0.001\ ms^{-1}$. Calculate the current drawn from the battery during the process. (Dielectric constant of oil = 11, $\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-1}$) **[1994 - 6 Marks]**

31. Two parallel plate capacitors A and B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate area of A and B are $0.04\ m^2$ and $0.02 m^2$ respectively. A slab of dielectric constant (relative permittivity) $K = 9$ has dimensions such that it can exactly fill the space between the plates of capacitor B. **[1993 - 2 + 3 + 2 Marks]**



(i) The dielectric slab is placed inside A as shown in figure (a). A is then charged to a potential difference of 110V. Calculate the capacitance of A and the energy stored in it.
(ii) The battery is disconnected and then the dielectric slab is moved from A. Find the work done by the external agency in removing the slab from A.
(iii) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in figure (c). Calculate the energy stored in the system.

Topic-4: Miscellaneous (Mixed Concepts) Problems

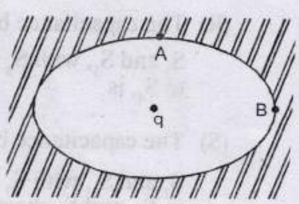
1 MCQs with One Correct Answer

1. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly **[2012]**

(a) $1 \times 10^{-5}\ V$ (b) $1 \times 10^{-7} V$
(c) $1 \times 10^{-9}\ V$ (d) $1 \times 10^{-10}\ V$

6 MCQs with One or More than One Correct Answer

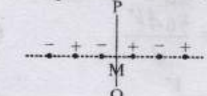
2. An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then **[1999S - 3 Marks]**

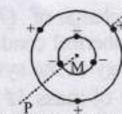


(a) electric field near A in the cavity = electric field near B in the cavity
(b) charge density at A = charge density at B
(c) potential at A = potential at B
(d) total electric field flux through the surface of the cavity is q/ε_0

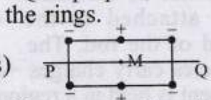
7 Match the Following

3. Six point charges, each of the same magnitude q, are arranged in different manners as shown in **Column-II**. In each case, a point M and line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to the equivalent to a steady current. **[2009]**

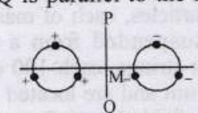
Column-I	Column-II
(A) $E = 0$	(p)  Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon.
(B) $V \neq 0$	(q)  Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges.

(C) $B = 0$ (r) 

Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.

(D) $\mu \neq 0$ (s) 

Charges are placed at the corners of a rectangle of sides a and $2a$ and at the mid points of the longer sides. M is at the centre of the rectangle. PQ is parallel to the longer sides.

(t) 

Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid-point between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings.

8 Comprehension/Passage Based Questions

Passage-1

Consider a simple RC circuit as shown in Figure 1.

Process 1: In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time T >> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2: In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time T >> RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in Figure 2.

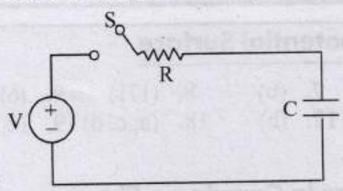


Figure 1

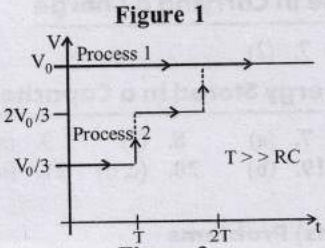


Figure 2

4. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by : **[Adv. 2017]**

(a) $E_C = E_D$ (b) $E_C = E_D \ln 2$

(c) $E_C = \frac{1}{2}E_D$ (d) $E_C = 2E_D$

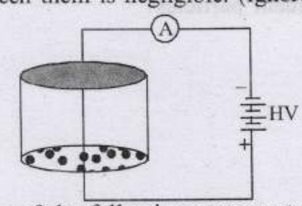
5. In Process 2, total energy dissipated across the resistance E_D is : **[Adv. 2017]**

(a) $E_D = \frac{1}{2}CV_0^2$ (b) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$

(c) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (d) $E_D = 3CV_0^2$

Passage-2

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius r <<h. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)



6. Which one of the following statements is correct? **[Adv. 2016]**

(a) The balls will stick to the top plate and remain there
(b) The balls will bounce back to the bottom plate carrying the same charge they went up with
(c) The balls will bounce back to the bottom plate carrying the opposite charge they went up with
(d) The balls will execute simple harmonic motion between the two plates

7. The average current in the steady state registered by the ammeter in the circuit will be **[Adv. 2016]**

(a) zero
(b) proportional to the potential V_0
(c) proportional to $V_0^{1/2}$
(d) proportional to V_0^2

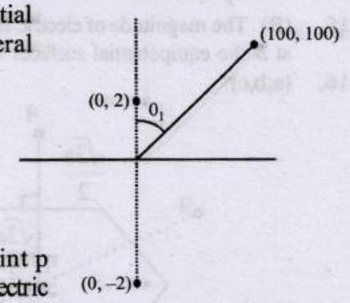
10 Subjective Problems

8. Four point charges $+8mC$, $-1mC$, $-1mC$, and $+8mC$ are fixed at the points $-\sqrt{\frac{27}{2}}$m, $-\sqrt{\frac{3}{2}}$m, $+\sqrt{\frac{3}{2}}$m and $+\sqrt{\frac{27}{2}}$m respectively on the y-axis. A particle of mass 6×10^{-4} kg and charge +0.1 μC moves along the -x direction. Its speed at $x = +\infty$ is V_0. Find the least value of V_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. Given $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\,Nm^2/C^2$. **[2000 - 10 Marks]**

Hints & Solutions

Topic-1: Electrostatic Potential and Equipotential Surfaces

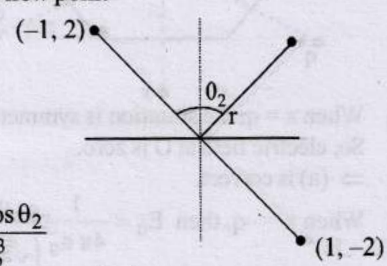
1. **(b)** Since electric potential due to a dipole at general point is given by



$$V = \frac{Kp\cos\theta}{r^3}$$

$$V_1 \propto \frac{p_1\cos\theta_1}{r_1^3}$$

Electric potential at point p (100, 100) $V_1 = V_0$ and electric potential at new point



$$V_2 \propto \frac{p_2\cos\theta_2}{r_2^3}$$

$$\frac{V_2}{V_1} = \frac{p_2\cos\theta_2}{p_1\cos\theta_1} = \frac{q(-2\hat{i}+4\hat{j})\cdot(\hat{i}+\hat{j})}{q(0\hat{i}+4\hat{j})\cdot(\hat{i}+\hat{j})} = \frac{1}{2}$$

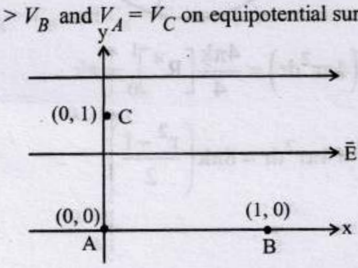
$$\therefore V_{new} = \frac{V_0}{2}$$

2. **(a)** According to Gauss's theorem, electric field between two cylinders $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. This electric field will produce a potential differences, $dV = -\vec{E}.d\vec{r}$. When a charge density is given to the inner cylinder, the potential developed at its surface is different from that on the outer cylinder. This is because the potential decreases with distance for a charged conducting cylinder when the point of consideration is outside the cylinder.

But when a charge density is given to the outer cylinder, it will change its potential by the same amount as that of the inner cylinder. Hence no potential difference will be produced between the cylinders.

3. **(b)** As we move along the direction of electric field the potential decreases.

$\therefore$ $V_A > V_B$ and $V_A = V_C$ on equipotential surface.



4. **(d)** The potential difference across capacitor plates remains unchanged when switch S_3 is closed. With the closing of switch S_3 and S_1 the negative charge on C_2 will attract the positive charge on C_2 thereby maintaining the negative charge on C_1. The negative charge on C_1 will attract the positive charge on C_1. No transfer of charge will take place. Therefore p.d across C_1 and C_2 will be 30 V and 20 V respectively.

5. **(d)** Potential at origin will be given by

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4x_0} + ...\right]$$

$$V = \frac{q}{4\pi\varepsilon_0 x_0}\ln(2)$$

6. **(a)** The potential inside the shell is equal to the potential on its surface. When we add $-3Q$ charge on the surface, the potential on the surface changes by the same amount as that inside. Therefore the potential difference remains the same (v).

7. **(b)** The electric potential at the surface of a hollow or conducting sphere is same as the potential at the centre of the sphere and any point inside the sphere.

Hence electric potential at surface of the sphere = potential at centre of the sphere = 10V

8. **(171)** Due to wire $dV = -E.\,dx$

$$\int_{V_P}^{V_R} dV = -\int_{0.5}^{2} \frac{2k\lambda}{x}dx$$

$$V_R - V_P = 2k\lambda \ln\frac{2}{0.5}$$

$$= -2\times 9\times 10^9 \times 3\times 10^{-9} \times 2\times 0.7 = -126\text{V}$$

Due to sphere

$$V_R - V_P = \frac{kQ}{2} - \frac{kQ}{1} = -\frac{kQ}{2} = \frac{-9\times 10^9 \times 10 + 10^{-9}}{2} = -45\text{V}$$

$$V_R - V_P = -126 - 45 = -171\text{ V}$$

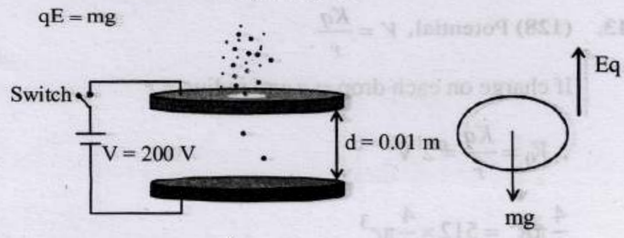
or, $V_P - V_R = 171$ V

9. **(6)** Let n no. of electrons present in the oil drop

Electric field, $E = \dfrac{V}{d} = \dfrac{200}{0.01} = 2\times 10^4$ V/m

When terminal velocity is achieved

$qE = mg$



As $q = ne$ and $m = \frac{4}{3}\pi R^3\rho$

$\therefore n \times 1.6 \times 10^{-19} \times 2 \times 10^4 = \frac{4\pi}{3}(8 \times 10^{-7})^3 \times 900 \times 10$

$\Rightarrow n = \frac{4}{3}\pi \times \frac{(8\times10^{-7})\times 900 \times 10}{2\times10^4 \times 1.6\times10^{-19}} \quad \therefore n = 6$

10. **(2)** $\because F = ma \therefore qE = m\frac{dv}{dt}$

$\Rightarrow dv = \frac{qEdt}{m} = \frac{q \sin 1000t\, \hat{i}}{m} dt$

($\because E = \sin 10^3 t\ \hat{i}$ given)

$\therefore \int_0^v dv = \frac{q}{m}\int_0^{\pi/\omega} \sin 1000t\, dt \left[\text{max. speed is at } \frac{T}{2} = \frac{2\pi}{\omega \times 2}\right]$

$\therefore \quad V = -\frac{q}{m}\left[\frac{\cos 1000t}{1000}\right]_0^{\pi/\omega} = -\frac{1}{10^{-3}} \times \frac{[\cos 1000t]_0^{\pi/\omega}}{1000}$

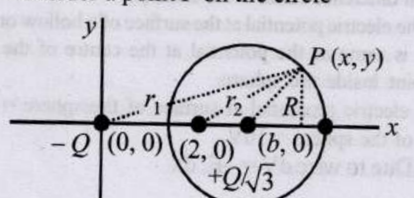
$\left(\begin{array}{l}\because m = 10^{-3} kg, q = 1C, E_0 = 1NC^{-1} \\ and\ w = 10^3 \text{ rads}^{-1} \text{ given}\end{array}\right)$

$\therefore \quad V = -\left[\cos 1000 \times \frac{\pi}{1000} - \cos 0\right] = -[-1-1] = 2 \text{ ms}^{-1}$

Hence maximum speed attained by the particle.

11. **(1.73)** **12.** **(3.00)**

Sol. Let us consider a point P on the circle



$$V_P = 0 = \frac{k(-Q)}{r_1} + \frac{kQ/\sqrt{3}}{r_2} \Rightarrow \frac{kQ}{r_1} = \frac{kQ/\sqrt{3}}{r_2}$$

$\Rightarrow \frac{1}{\sqrt{x^2+y^2}} - \frac{1}{\sqrt{3}\sqrt{(x-2)^2+y^2}}$

$\Rightarrow 3(x-2)^2 + 3y^2 = x^2 + y^2$

$\Rightarrow 3(x^2 + 4 - 4x) - x^2 + 2y^2 = 0$

$\Rightarrow 2x^2 + 12 - 12x + 2y^2 = 0$

$\Rightarrow x^2 + 6 - 6x + y^2 = 0$

$\Rightarrow (x-3)^2 + y^2 = (\sqrt{3})^2$

or $(x-b)^2 + y^2 = (\sqrt{3})^2 = R^2$

$\therefore R = \sqrt{3} = 1.73$ and $b = 3$

13. **(128)** Potential, $V = \frac{Kq}{r}$

If charge on each drop = q and radius = r

$\therefore V_0 = \frac{Kq}{r} = 2$ V

$\frac{4}{3}\pi R^3 = 512 \times \frac{4}{3}\pi r^3$

$\Rightarrow R = (512 \times r^3)^{1/3} = 8r$

Potential of bigger drop,

$$V = \frac{K(512)q}{8r} = \frac{512}{8}\frac{Kq}{r} = \frac{512}{8} \times 2 \text{ V} = 128 \text{ V}.$$

14. **(–8)** Given electric potential $V = 4x^2$ volts

Therefore, electric potential changes only along x-axis,

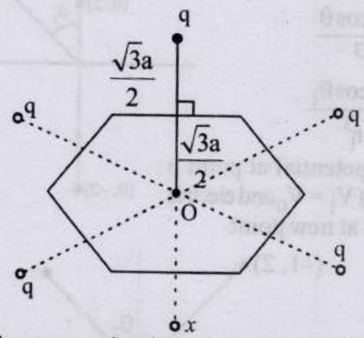
We know $E_x = \frac{-dV}{dx} \quad \Rightarrow E_x = -\frac{d(4x^2)}{dx} = -8x$

The electric field at point (1, 0, 2) (here $x = 1$)

$E_x = -8$ V/m

15. **(B)** The magnitude of electric field is greatest at point B because at B the equipotential surfaces are closest.

16. **(a,b,c)**



(a) When x = q, the situation is symmetric
So, electric field at O is zero.
$\Rightarrow$ (a) is correct.

(b) When x = –q, then $E_0 = \frac{1}{4\pi \epsilon_0}\frac{q}{(\sqrt{3}a)^2} \times 2$

$\Rightarrow E_0 = \frac{q}{6\pi \epsilon_0 a^2}$

$\Rightarrow$ (b) is correct.

(c) When x = 2q

$V_0 = 5 \times \frac{Kq}{\sqrt{3}a} + \frac{K(2q)}{\sqrt{3}a} \Rightarrow V_0 = \frac{7q}{4\sqrt{3}\pi \epsilon_0 a}$

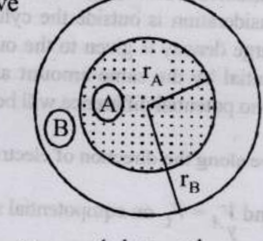
$\Rightarrow$ (c) is correct.

(d) When x = –3q

$V_0 = 5 \times \frac{Kq}{\sqrt{3}a} + \frac{K(-3q)}{\sqrt{3}a} = \frac{1}{4\pi \epsilon_0}\frac{2q}{\sqrt{3}a} = \frac{q}{2\sqrt{3}\pi \epsilon_0 a}$

$\Rightarrow$ (d) is incorrect.

17. **(b)** We have



$q_A = \int_0^1 kr(4\pi r^2 dr) = \frac{4\pi k}{4}\left[R^4\right]_0^1 = \pi k$

$q_B = \int_1^r \frac{2k}{r} \times 4\pi r^2 dr = 8\pi k\left(\frac{r^2 - 1}{2}\right)$

$= 4\pi k r^2 - 4\pi k$

So, $q_{net} = q_A + q_B = 4\pi k r^2 - 3\pi k$

$= \pi k\left(4r^2 - 3\right)$

(A) $E_{net} = 0 \Rightarrow q_{net} = 0 \Rightarrow 4r^2 - 3 = 0 \Rightarrow r = \frac{\sqrt{3}}{2}$

So (a) is incorrect

(B) $V = \frac{1}{4\pi \epsilon_0}\frac{q_{net}}{r} = \frac{1}{4\pi \epsilon_0}\frac{\pi k\left(4r^2 - 3\right)}{r}$

$\Rightarrow V = \frac{k}{4\epsilon_0}\left(4r - \frac{3}{r}\right) \Rightarrow V = \frac{k}{4\epsilon_0}\left(4 \times \frac{3}{2} - \frac{3}{3} \times 2\right)$

$\Rightarrow V = \frac{k}{\epsilon_0}$. So (b) is correct.

(C) If $r_B = 2$ i.e $r = 2$.
then, $q_{net} = \pi k(16 - 3)$
$= 13\ \pi k$. So (c) is incorrect.

(D) $E = \frac{k\, q_{net}}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\pi k\left(4r^2 - 3\right)}{r^2} = \frac{k}{4\epsilon_0}\left[4 - \frac{3}{r^2}\right]$

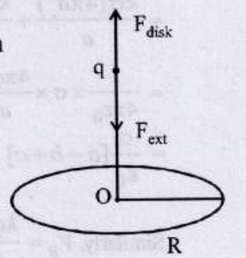
$E = \frac{k}{4\epsilon_0}\left[4 - \frac{3}{25} \times 4\right] = \frac{k}{4\epsilon_0}\left[\frac{88}{25}\right] = \frac{22k}{25\epsilon_0}$

So (d) is correct.

18. **(a, c, d)** Particle will reach the origin only if

$\Delta K \geq 0$

$\Rightarrow w_{el} + w_{ext} > 0$



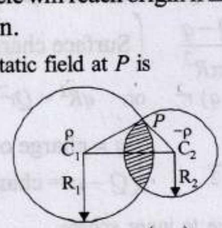
$\Rightarrow \frac{\sigma q}{2\epsilon_0}\left[\left(\sqrt{R^2 + 0^2} - 0\right) + \left(\sqrt{R^2 + Z_0^2} - Z_0\right)\right] + CZ_0 \geq 0$

$\Rightarrow \frac{\sigma q}{2\epsilon_0 C}\left(R - \sqrt{R^2 + Z_0^2} + Z_0\right) + \frac{CZ_0}{C} \geq 0$

$\Rightarrow \frac{1}{\beta}\left(R + Z_0 - \sqrt{R^2 + Z_0^2}\right) + Z_0 \geq 0$

Substitute Z_0 and β, and check the condition.
If $\Delta K > 0$, particle will reach origin if $\Delta K > 0$ otherwise if will not reach origin.

19. **(c, d)** Electrostatic field at *P* is



$\vec{E}_p = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}\overline{C_1P} + \frac{(-\rho)}{3\varepsilon_0}\overline{C_2P}$

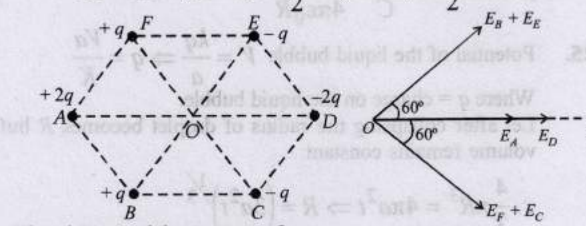
$= \frac{\rho}{3\varepsilon_0}\left(\overline{C_1P} - \overline{C_2P}\right) = \frac{\rho}{3\varepsilon_0}\left(\overline{C_1P} + \overline{PC_2}\right) = \frac{\rho}{3\varepsilon_0}\left(\overline{C_1C_2}\right)$

20. **(a, b, c)**

Here $\frac{|\overrightarrow{E_A}|}{2} = |\overrightarrow{E_B}| = |\overrightarrow{E_C}| = \frac{|\overrightarrow{E_D}|}{2} = |\overrightarrow{E_E}| = |\overrightarrow{E_F}| = K$

$\therefore\ E_O = E_A + E_D + (E_F + E_C)\cos 60° + (E_B + E_C)\cos 60°$

$= 2K + 2K + (K + K) \times \frac{1}{2} + (K + K) \times \frac{1}{2} = 6K$



Electric potential at center, *O*

$V_O = \frac{1}{4\pi\varepsilon_0 L}\left[2q + q + q - q - q - 2q\right] = 0$

Potential at all points on the line PR is same not on line ST. *PR* is perpendicular bisector (the equatorial line) for the electric dipoles *AB*, *FE* and *BC*. Therefore the electric potential will be zero at any point on *PR*.

21. **(c, d)** (a) Gauss's law is valid only when $E \propto r^{-2}$

(b) Gauss's law cannot be used to calculate the field distribution around an electric dipole.

(c) is correct as between two point charges we will get a point where the electric field due to the two point charges cancel out each other. If two point charges are of opposite sign then the two fields are along the same direction hence they cannot be zero.

(d) $W_{AB} = q\,(\Delta V) = 1\,(V_B - V_A)$ or, $W_{AB} = V_B - V_A$

22. **(a, b, c, d)** (a) Electric field inside a spherical metallic shell with charge on the surface is always zero. *i.e.*, E_A inside = 0

(b) When the shells are connected with a thin metal wire then electric potentials will be equal, *i.e.*, $V_A = V_B = V$.

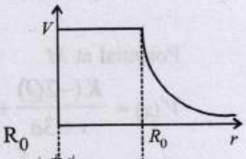
$\therefore\ \frac{1}{4\pi\epsilon_0}\frac{Q_A}{R_A} = \frac{1}{4\pi\epsilon_0}\frac{Q_B}{R_B} = V \quad \because R_A > R_B \therefore Q_A > A_B.$

(c) As $\frac{\sigma_A}{\sigma_B} = \frac{\frac{Q_A}{4\pi R_A^2}}{\frac{Q_B}{4\pi R_B^2}} = \frac{R_B^2}{R_A^2} \times \frac{Q_A}{Q_B} = \frac{R_B^2}{R_A^2} \times \frac{4\pi\epsilon_0\, R_A V}{4\pi\epsilon_0\, R_B V} = \frac{R_B}{R_A}$

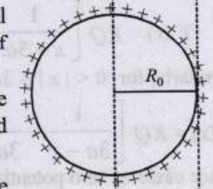
(d) $E_A = \frac{\sigma_A}{\epsilon_0}$ and $E_B = \frac{\sigma_B}{\epsilon_0}$

$\frac{E_A}{E_B} = \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A} < 1 \quad \therefore E_A < E_B$

23. **(a, b, d)** The given *V*-*r* graph is of charged conducting sphere of radius R_0



(a) The whole charge *Q* will be enclosed in a sphere of diameter $2R_0$.

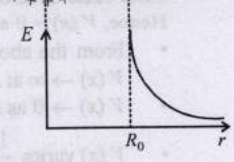


(b) Electric field *E* = zero inside the sphere. Hence electric field is discontinued at $r = R_0$.

(c) Changes in V and *E* are continuously present for $r > R_0$.

Option (c) is incorrect.

(d) For $r < R_0$, the potential V is constant and the electric intensity



is zero. Obviously, the electrostatic energy is zero for $r < R_0$.

24. **(b)** The earth is used as a refrance at zero potential in electrical circuits for practical purposes.
For spherical capacitor, capacitance $C = 4\pi\varepsilon_0 R$ and electrical potential $V = \frac{Q}{C} = \frac{Q}{4\pi\varepsilon_0 R}$

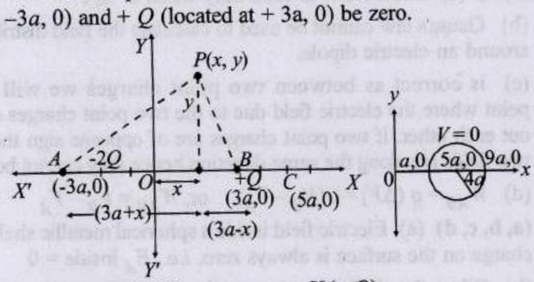
25. Potential of the liquid bubble $V = \frac{kq}{a} \Rightarrow q = \frac{Va}{K}$
Where q = charge on the liquid bubble.
Let after collapsing the radius of droplet becomes R but here volume remains constant

$$\therefore \frac{4}{3}\pi R^3 = 4\pi a^2 t \Rightarrow R = \left(3a^2 t\right)^{1/3}$$

Potential of the droplet

$$V^1 = \frac{Kq}{R} = \frac{K\left(\frac{Va}{k}\right)}{\left(3a^2 t\right)^{1/3}} = V\left(\frac{a}{3t}\right)^{1/3}$$

26. (a) Let P be a point in the X-Y plane with coordinates (x, y) at which the potential due to charges $-2Q$ (located at $-3a$, 0) and $+Q$ (located at $+3a$, 0) be zero.



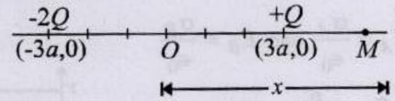
$$\therefore \frac{K(2Q)}{\sqrt{(3a+x)^2 + y^2}} = \frac{K(+Q)}{\sqrt{(3a-x)^2 + y^2}}$$

$$\Rightarrow 2\sqrt{(3a-x)^2 + y^2} = \sqrt{(3a+x)^2 + y^2}$$

$$\Rightarrow (x - 5a)^2 + (y)^2 = (4a)^2$$

This is the equation of a circle with centre at $(5a, 0)$ and radius $4a$.

(b) **For $x > 3a$**
Let us consider a point (arbitrary) M at a distance x from the origin on x-axis.



Potential at M

$$V(x) = \frac{K(-2Q)}{x + 3a} + \frac{K(+Q)}{(x - 3a)} \text{ where } k = \frac{1}{4\pi\varepsilon_0}$$

$$\therefore V(x) = KQ\left[\frac{1}{x - 3a} - \frac{2}{x + 3a}\right] \text{ for } |x| > 3a$$

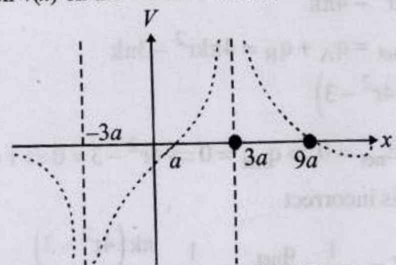
Similarly, for $0 < |x| < 3a$

$$V(x) = KQ\left[\frac{1}{3a - x} - \frac{2}{3a + x}\right]$$

Since circle of zero potential cuts the x-axis at $(a, 0)$ and $(9a, 0)$
Hence, $V(x) = 0$ at $x = a$, at $x = 9a$

- From the above expressions
  $V(x) \to \infty$ at $x \to 3a$ and $V(x) \to -\infty$ at $x \to -3a$
- $V(x) \to 0$ as $x \to \pm\infty$
- $V(x)$ varies $\frac{1}{x}$ in general.

Function v(x) on the whole x–axis as shown below.



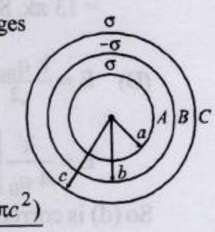
(c) From energy conservation principle
Let v be the speed of particle

$$(\text{K.E.} + \text{P.E.})_{\text{centre}} = (\text{K.E.} + \text{P.E.})_{\text{circumference}}$$

$$0 + K\left[\frac{Qq}{2a} - \frac{2Qq}{8a}\right] = \frac{1}{2}mv^2 + K\left[\frac{Qq}{6a} - \frac{2Qq}{12a}\right]$$

$$\frac{1}{2}mv^2 = \frac{KQq}{4a}, \quad \therefore v = \sqrt{\frac{KQq}{2ma}} = \sqrt{\frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{2ma}\right)}$$

27. (a) Potential at any shell is due to charges at shell A, B and C.



Potential of shell A

$$V_A = \frac{kq_A}{a} + \frac{kq_B}{b} + \frac{kq_C}{c}$$

$$= \frac{k\sigma(4\pi a^2)}{a} + \frac{k(-\sigma)(4\pi b^2)}{b} + \frac{k\sigma(4\pi c^2)}{c}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \sigma \times \frac{4\pi a^2}{a} - \frac{1}{4\pi\varepsilon_0}\sigma\frac{(4\pi b^2)}{b} + \frac{1}{4\pi\varepsilon_0} \times \sigma\frac{(4\pi c^2)}{c}$$

$$= \frac{\sigma}{\varepsilon_0}[a - b + c]$$

Similarly, $V_B = \frac{kq_A}{b} + \frac{kq_B}{b} + \frac{kq_C}{c}$ or, $V_B = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$

And $V_C = \frac{kq_A}{c} + \frac{kq_B}{c} + \frac{kq_C}{c}$ or, $V_C = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2 + c^2}{c}\right]$

(b) $\because V_A = V_C$ (given)

$$\frac{\sigma}{\varepsilon_0}(a - b + c) = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2 + c^2}{c}\right]$$

or $ac - bc + c^2 = a^2 - b^2 + c^2$ or $c = a + b$

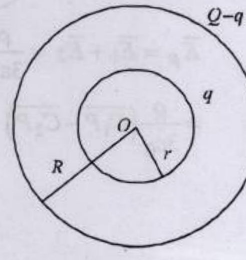
28. Given surface charge densities are equal.

$$\therefore \frac{q}{4\pi r^2} = \frac{Q - q}{4\pi R^2} \quad \left(\text{Surface charge density}, \sigma = \frac{q}{A}\right)$$

or, $qR^2 = (Q - q)r^2$ or, $qR^2 = Qr^2 - qr^2$

$$\therefore q = \frac{Qr^2}{R^2 + r^2} \quad \left(\begin{array}{l} q = \text{charge on inner sphere} \\ Q - q = \text{charge on outer sphere} \end{array}\right)$$

Potential at O due to inner sphere



$$V_i = K\frac{q}{r} = \frac{K}{r}\left(\frac{Qr^2}{R^2 + r^2}\right)$$

$$V_i = K\frac{Qr}{R^2 + r^2}$$

Potential at O due to outer sphere

$$V_0 = K\frac{(Q-q)}{R} = \frac{K}{R}\left[Q - \frac{Qr^2}{R^2+r^2}\right]$$

$$= \frac{K\left[QR^2 + Qr^2 - Qr^2\right]}{R\,(R^2+r^2)} = \frac{K(QR)^2}{R\left(R^2+r^2\right)} = \frac{KQR}{\left(R^2+r^2\right)}$$

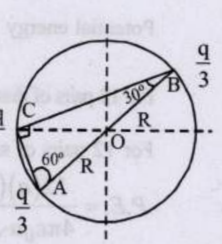
$\therefore$ Total potential at the common centre 'O'

$$V = V_i + V_0 = \frac{KQr}{R^2+r^2} + \frac{KQR}{R^2+r^2} = \frac{KQ(R+r)}{R^2+r^2}$$

Topic-2: Electric Potential Energy and Work Done in Carrying a Charge

1. **(c)** (a) The electric field due to charge $\frac{q}{3}$ at A and charge $\frac{q}{3}$ at B at O will get cancelled. The electric field at O due to charge $\left(\frac{-2q}{3}\right)$

$$E = \frac{1}{4\pi \epsilon_o}\frac{2q/3}{R^2} = \frac{q}{6\pi \epsilon_o R^2}.$$



In Δ ABC $\frac{AC}{AB} = \sin 30°$

$$\Rightarrow AC = \frac{AB}{2} = R$$

Also $\frac{BC}{AB} = \sin 60° \Rightarrow BC = \frac{\sqrt{3}AB}{2} = \sqrt{3}\,R$

(b) Potential energy of the system K

$$\left[\frac{(q/3)(2/3)}{2R}\right] + K\left[\frac{(q/3)(-2q/3)}{R}\right] + K\left[\frac{(q/3)(-2q/3)}{\sqrt{3}R}\right]$$

$$= \frac{kq^2}{9R}\left[\frac{1}{2} - 2 - \frac{2}{\sqrt{3}}\right] \neq 0$$

(c) Magnitude of force between B and C

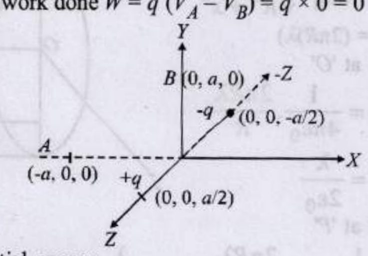
$$F = \frac{1}{4\pi \epsilon_o}\frac{(2q/3)(q/3)}{\left(\sqrt{3}R\right)^2} = \frac{q^2}{54\pi \epsilon_o R^2}$$

(d) Potential $V_O = K\left[\frac{+q/3}{R}\right] + K\left[\frac{+q/3}{R}\right] + K\left[\frac{-2q/3}{R}\right] = 0$

2. **(c)** Two charges make an electric dipole. A and B points lie on the equatorial plane of the dipole.

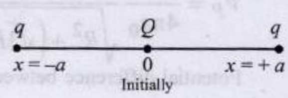
$\therefore$ Potential at A, V_A = potential at B, $V_B = 0$

Hence, work done $W = q\,(V_A - V_B) = q \times 0 = 0$

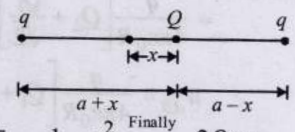


3. **(b)** Initial energy

$$U_i = \frac{2Qq}{4\pi\varepsilon_0 a} + \frac{1}{4\pi\varepsilon_0}\frac{a^2}{2a}$$



Final energy



$$U_f = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{a+x} + \frac{1}{a-x}\right] + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a} = \frac{2Qqa}{4\pi\varepsilon_0\left(a^2-x^2\right)}$$

$$\therefore\ U_i - U_f = \frac{2Qq}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{a}{\left(a^2-x^2\right)}\right]$$

$$= \frac{2Qq}{4\pi\varepsilon_0}\left[\frac{a^2-x^2-a^2}{a\left(a^2-x^2\right)}\right] = \frac{-2Qqx^2}{4\pi\varepsilon_0 a^3}$$

when $x << a$ then x^2 is neglected in denominator.

$$U_i - U_f = \left(\frac{-Qq}{2\pi\varepsilon_0 a^3}\right)x^2 \ \therefore\ U_i - U_f \propto x^2$$

4. **(b)** Since net electrostatic energy $\Sigma u = 0$

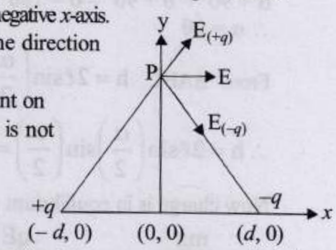
$$\therefore\ \frac{Qq}{a} + \frac{q^2}{a} + \frac{Qq}{a\sqrt{2}} = 0$$

or $Q\sqrt{2} + q\sqrt{2} + Q = 0 \ \therefore\ Q = -\frac{q\sqrt{2}}{\sqrt{2}+1} = -\frac{2q}{2+\sqrt{2}}$

5. **(b)** (a) For $-d \leq x \leq d$, the electric field is along $+x$-axis. For all other points, E is along negative x-axis. Hence, E will not have same direction along entire, x-axis.

(b) Electric field at P, a point on y-axis is parallel to x-axis. It is not along x-axis itself.

(c) Electric potential at origin = zero

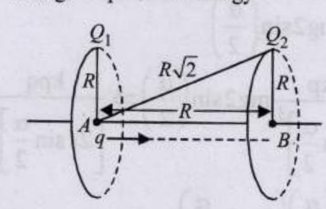


No work has to be done in bringing a test charge from infinity to the origin.

(d) The dipole moment is directed from $-q$ charge to $+q$ charge, along negative direction, of x-axis.

$\therefore$ Dipole moment $= -2\,qd$ along x-axis.

6. **(b)** The work done in moving a charge q from centre of one ring to other = change in potential energy



$W_{AB} = \mu$

$$\mu_A = \left[\left(\frac{Q_1}{4\pi\varepsilon_0 R}\right)\times q + \left(\frac{Q_2}{4\pi\varepsilon_0\sqrt{R^2+R^2}}\right)q\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}}\right]$$

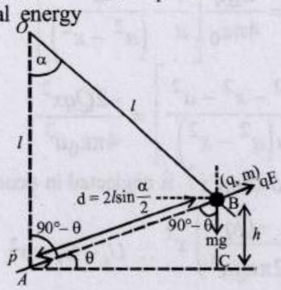
$$\mu_B = \left[\left(\frac{Q_2}{4\pi\varepsilon_0 R}\right)q + \left(\frac{Q_1}{4\pi\varepsilon_0\sqrt{R^2+R^2}}\right)q\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_2 + \frac{Q_1}{\sqrt{2}}\right]$$

$$\therefore\ W_{AB} = \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}} - Q_2 - \frac{Q_1}{\sqrt{2}}\right]$$

$$= \frac{q(Q_1 - Q_2)}{4\pi\varepsilon_0 R}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

7. (2) Initial potential energy, $U_i = 0$
Final potential energy



$$U_f = qV + mgh = \frac{qKp}{d^2} + mgh$$

$$\text{or, } U_f = \frac{kqP}{\left(2\ell\sin\frac{\alpha}{2}\right)^2} + mgh \quad ...(i)$$

From ΔOAB
$\alpha + 90° - \theta + 90° - \theta = 180°$
$\therefore\ \alpha = 2\theta$

From ΔABC, $h = 2\ell\sin\left(\frac{\alpha}{2}\right)\sin\theta$

$$\therefore h = 2\ell\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\alpha}{2}\right) = 2\ell\sin^2\left(\frac{\alpha}{2}\right)$$

Now charge is in equilibrium at point B, therefore from sine rule

$$\frac{mg}{\sin\left[90° + \frac{\alpha}{2}\right]} = \frac{qE}{\sin[180° - 2\theta]} \Rightarrow \frac{mg}{\cos\frac{\alpha}{2}} = \frac{qE}{\sin 2\theta}$$

$$\Rightarrow \frac{mg}{\cos\frac{\alpha}{2}} = \frac{qE}{\sin\alpha} \Rightarrow \frac{mg}{\cos\frac{\alpha}{2}} = \frac{qE}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$$

$$\Rightarrow qE = mg2\sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \frac{q2kp}{\left[2\ell\sin\frac{\alpha}{2}\right]^3} = mg2\sin\left(\frac{\alpha}{2}\right) \Rightarrow \frac{kpq}{\left[2\ell\sin\frac{\alpha}{2}\right]^2}$$

$$= mg\sin\left(\frac{\alpha}{2}\right)\left(2\ell\sin\frac{\alpha}{2}\right)$$

$$\Rightarrow \frac{kpq}{\left[2\ell\sin\frac{\alpha}{2}\right]^2} = mgh$$ Substituting this value in equation (i)

we get,
$U_f = 2mgh$
Work done in bringing the dipole
$W = U_f - U_i = 2mgh - 0 = 2mgh$
$W = \Delta U = 2mgh = Nmgh\ \therefore\ N = 2$

8. (a) $U_i = 0$ (since the charge is far away)

$$U_f = -Q \times \frac{1}{4\pi\varepsilon_0}\frac{p}{d^2}$$

$$\therefore\ \text{K.E.} = |U_f - U_i| = \frac{1}{4\pi\varepsilon_0}\frac{pQ}{d^2}$$

(b) Electric field at origin due to dipole

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{d^3}\hat{i}$$

Hence, force on charge Q

$$\vec{F} = Q\vec{E} = \frac{2pQ}{4\pi\varepsilon_0 d^3}\hat{i}$$

9. Here total charge-pairs $^8C_2 = \frac{8\times 7}{1\times 2} = 28$
12 pairs of dissimilar charges at a separation of a.
4 pairs of dissimilar charges at a separation of $\sqrt{3}a$.
12 pairs of similar charges at a separation of $\sqrt{2}a$.

Potential energy between two charges = $\frac{Q_1Q_2}{4\pi\varepsilon_0 r}$

For 12 pairs of dissimilar charges, $P.E. = \frac{12(q)(-q)}{4\pi\varepsilon_0 a} = -\frac{12q^2}{4\pi\varepsilon_0 a}$

For 12 pairs of similar charges,

$$P.E. = \frac{12(q)(q)}{4\pi\varepsilon_0 a\sqrt{2}a} = -\frac{12q^2}{4\pi\varepsilon_0 a}\times 0.707$$

For 4 pairs of dissimilar charges

$$P.E. = \frac{4(q)(1-q)}{4\pi\varepsilon_0\sqrt{3}a} = -\frac{4q^2}{4\pi\varepsilon_0 a}\times 0.577$$

Total Potential energy of system

$$U = \frac{-12q^2}{4\pi\varepsilon_0 a} + \frac{12q^2}{4\pi\varepsilon_0 a}\times 0.707 + \left(\frac{-4q^2}{4\pi\varepsilon_0 a}\times 0.577\right)$$

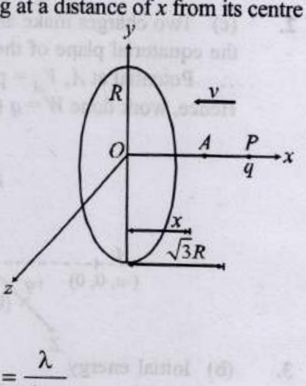
$$= -5.824\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$$

$\therefore$ Binding energy of system = $5.824\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$

$\therefore$ Work done to separate the charges, $W = 5.824\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$

10. We know, potential due to a ring at a distance of x from its centre on its axis.

$$V(x) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2 + x^2}}$$



(Here $Q = (2\pi R)\lambda$)
Potential at 'O'

$$V_O = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\pi R\lambda}{R}$$

$$\text{or}\quad V_O = \frac{\lambda}{2\varepsilon_0}$$

Potential at 'P'

$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{2\pi R\lambda}{\sqrt{R^2 + \left(\sqrt{3}R\right)^2}} = \frac{\lambda}{4\varepsilon_0}$$

Potential difference between points O and P

$$V = V_O - V_P = \frac{\lambda}{2\varepsilon_0} - \frac{\lambda}{4\varepsilon_0} = \frac{\lambda}{4\varepsilon_0}$$

Kinetic energy of the charged particle is converted into its potential energy at O.

$\therefore$ Potential energy of charge $(q) = qV$

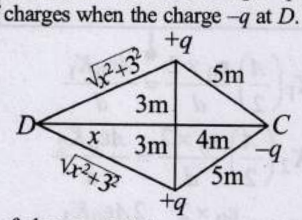
Kinetic energy of charged particle $= \frac{1}{2}mv^2$

For minimum speed of particle so that it does not return to P,

$$\frac{1}{2}mv^2 = qV \Rightarrow v^2 = \frac{2qV}{m} = \frac{2q \times \lambda}{m \times 4\varepsilon_0}$$

or $v = \sqrt{\frac{q\lambda}{2\varepsilon_0 m}}$

11. According to the principle of conservation of energy, total energy of system of charges when the charge $-q$ at 'c' = total energy of system of charges when the charge $-q$ at D.



P.E. + K.E. of charge $-q$ at c

$$= \left[\frac{Kq \times q}{6} + \frac{K(q)(-q)}{5} + \frac{Kq(-q)}{5}\right] + 4$$

= P.E. + K.E., when $-q$ is at D and momentarily at rest.

$$= \left[\frac{Kq \times q}{6} + \frac{Kq(-q)}{\sqrt{x^2+3^2}} + \frac{Kq(-q)}{\sqrt{x^2+3^2}}\right] + 0$$

$$= \frac{Kq \times q}{6} + \frac{2Kq(-q)}{\sqrt{x^2+3^2}}$$

$$\frac{kq \times q}{6} + \frac{2kq(-q)}{5} + 4 = \frac{kq \times q}{6} + \frac{2kq(-q)}{\sqrt{x^2+3^2}}$$

$$\Rightarrow \; 2 = kq^2\left[\frac{1}{5} - \frac{1}{\sqrt{x^2+3^2}}\right]$$

or, $x^2 + 9 = 81 \quad \therefore x = 8.48$ m

Topic-3: Capacitors, Grouping of Capacitors and Energy Stored in a Capacitor

1. (b) In t = 10 s volume of liquid

V = 250 × 10 = 2500 cc

$$\therefore h = \frac{V}{A} = \frac{2500}{50 \times 5} = 10\text{cm}$$

Capacitance of the capacitor when filled with liquid of dielectric constant K = 3

$$C_d = \frac{A_d \varepsilon_0 k}{d} = \frac{50 \times 10^{-2} \times 10 \times 10^{-2}\,\varepsilon_0 \times 3}{5 \times 10^{-2}} = 3\varepsilon_0$$

Capacitance of the air filled capacitor

$$C_a = \frac{A_a \varepsilon_0}{d} = \frac{50 \times 10^{-2} \times 40 \times 10^{-2}\,\varepsilon_0}{5 \times 10^{-2}} = 4\varepsilon_0$$

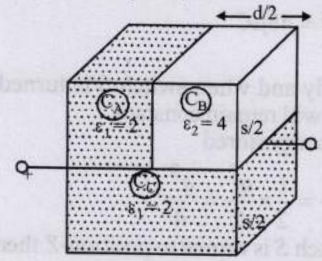
Equivalent Capacitance, $C = C_a + C_d = 7\varepsilon_0$
$= 7 \times 9 \times 10^{-12} = 63$ pF

2. (d) As we know, the capacitance of a parallel, plate capacitor

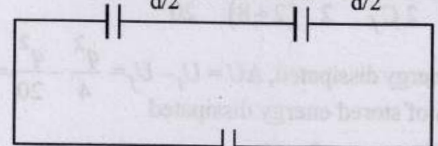
$$C = \frac{\varepsilon_0 A}{d}$$

Initialy, capacitance, $C_1 = \frac{\varepsilon_0 s}{d}$

When two dielectrics of permittivity $\varepsilon_1 = 2$ and $\varepsilon_2 = 4$ are introduced between the plates, then



$$C_A = \frac{2\,\varepsilon_0\, s/2}{d/2} \qquad C_B = \frac{4\,\varepsilon_0\, s/2}{d/2}$$



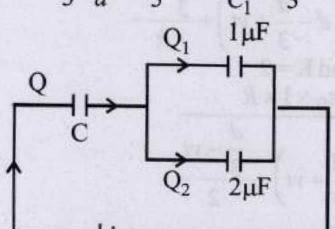
$$C_C = \frac{2\,\varepsilon_0\, s/2}{d} = \frac{\varepsilon_0 s}{d}$$

$$C_2 = \frac{C_A \times C_B}{C_A + C_B} + C_C = \frac{\frac{2\varepsilon_0 s}{d} \times \frac{4\,\varepsilon_0 s}{d}}{\frac{6\varepsilon_0 s}{d}} + \frac{\varepsilon_0 s}{d}$$

$$= \frac{4}{3}\frac{\varepsilon_0 s}{d} + \frac{\varepsilon_0 s}{d}$$

$$\therefore \quad C_2 = \frac{7}{3}\frac{\varepsilon_0 s}{d} = \frac{7}{3}C_1 = \frac{C_2}{C_1} = \frac{7}{3} \qquad \left[\because C_1 = \frac{\varepsilon_0 s}{d}\right]$$

3. (d)

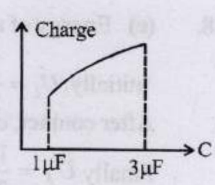


From figure, $Q_2 = \frac{2}{2+1}Q = \frac{2}{3}Q$

$$Q = E\left(\frac{C \times 3}{C+3}\right)$$

Therefore graph d correctly dipicts.

$$\therefore Q_2 = \frac{2}{3}\left(\frac{3CE}{C+3}\right) = \frac{2CE}{C+3}$$



4. (a) Electric field in presence of dielectric between the two plates of a parallel plate capacitor is given by, $E = \frac{\sigma}{K\varepsilon_0}$

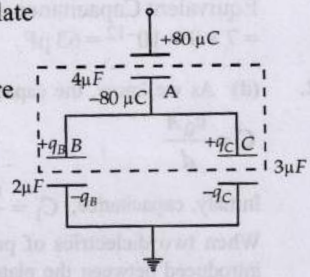
Then, charge density, $\sigma = K\varepsilon_0 E$
$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4 \approx 6 \times 10^{-7}$ C/m^2

5. (c) The total charge on plate A will be 80 μC.

$2\mu F$ and $3\mu F$ capacitors are in parallel. Therefore,

$C_{eq} = 2 + 3 = 5\ \mu F$

Charge on capacitor of 3μF capacitance



$$q = \frac{3}{5} \times 80 = 48\mu C$$

6. (d) Initially and when switch S is turned to position –2 charge (q) will remain constant.

Initially energy stored

$$U_i = \frac{1}{2}\frac{q^2}{C_i} = \frac{1}{2} \times \frac{q^2}{2} = \frac{q^2}{4}$$

When switch S is turned to position-Z then energy stored.

$$U_f = \frac{1}{2}\frac{q^2}{C_f} = \frac{1}{2} \times \frac{q^2}{(2+8)} = \frac{q^2}{20}$$

$\therefore$ Energy dissipated, $\Delta U = U_i - U_f = \frac{q^2}{4} - \frac{q^2}{20} = \frac{q^2}{5}$

$\therefore$ % of stored energy dissipated

$$= \frac{\Delta U}{U_i} \times 100 = \frac{q2}{5} \times \frac{4}{q^2} \times 100 = 80\%$$

7. (a) Time constant as a function of time $T_C = CR$

Capacitance $C = \dfrac{\varepsilon_0 A}{d - T + \dfrac{T}{k}}$

and, T (= thickness) of liquid after time t, $= \dfrac{d}{3} - vt$

$$\therefore T_C = CR = \frac{\varepsilon_0 AR}{\left(d - \frac{d}{3} + vt\right) + \frac{\frac{d}{3} - vt}{k}}$$

Given: $A = 1$ and $K = 2$

$$\therefore T_C = \frac{\varepsilon_0 \times 1 \times R}{\left(d - \frac{d}{3} + vt\right) + \frac{\frac{d}{3} - vt}{2}}$$

or, $T_C = \dfrac{6\varepsilon_0 R}{5d + 3vt}$

8. (c) Energy of a capacitor $U = \frac{1}{2}CV^2$

Initially, $U_i = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2 = \frac{1}{2}C\left(V_1^2 + V_2^2\right)$

After contact, common potential = V

Finally $U_f = \frac{1}{2}(2C)\left(\frac{V_1 + V_2}{2}\right)^2$

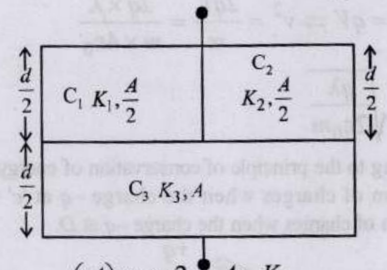
$$\therefore \Delta U = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2 - \frac{C}{4}(V_1 + V_2)^2$$

$$= \frac{C}{4}\left[2V_1^2 + 2V_2^2 - V_1^2 - V_2^2 - 2V_1V_2\right]$$

$$= \frac{C}{4}\left[V_1^2 + V_2^2 - 2V_1V_2\right] = \frac{C}{4}(V_1 - V_2)^2.$$

9. (a) When switch S is closed, there will be no shifting of negative charge from plate A to B as the charge $-q$ is held by the charge $+q$ as unlike charges attract each other. Neither there will be any shifting of charge from B to A. Hence charge on capacitor $B = 0$

10. (b) Let C_1, C_2 and C_3 are the Capacitance of capacitor with dielectric constant, K_1, K_2 and K_3 respectively



$$\therefore\ C_1 = K_1\left(\frac{A}{2}\right)\frac{\varepsilon_0 \times 2}{d} = \frac{A\varepsilon_0 K_1}{d}$$

$$\therefore\ C_2 = K_2\left(\frac{A}{2}\right)\frac{\varepsilon_0 \times 2}{d} = \frac{A\varepsilon_0 K_2}{d}$$

$$\therefore\ C_3 = K_3(A)\frac{\varepsilon_0 \times 2}{d} = \frac{2A\varepsilon_0 K_3}{d}$$

C_1 and C_2 are in parallel

$$\therefore\ C_{eq} = \frac{A\varepsilon_0}{d}(K_1 + K_2)$$

Again this C_{eq} and C_3 are in series

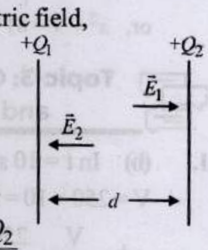
$$\therefore\ \frac{1}{C} = \frac{d}{A\varepsilon_0(K_1 + K_2)} + \frac{d}{2A\varepsilon_0 K_3}$$

But $C = \dfrac{KA\varepsilon_0}{d}$ for single equivalent capacitor

$$\therefore\ \frac{d}{KA\varepsilon_0} = \frac{d}{A\varepsilon_0(K_1 + K_2)} + \frac{d}{2A\varepsilon_0 K_3}$$

or $\dfrac{1}{K} = \dfrac{1}{K_1 + K_2} + \dfrac{1}{2K_3}$.

11. (d) Within the capacitor plates, electric field,



$E_1 = \dfrac{Q_1}{2\varepsilon_0 A}$; $E_2 = \dfrac{Q_2}{2\varepsilon_0 A}$;

$E = E_1 - E_2 = \dfrac{1}{2\varepsilon_0 A}(Q_1 - Q_2)$

Hence, potential differences $V = Ed$

Lor, $V = \dfrac{1}{2}\dfrac{d}{\varepsilon_0 A}(Q_1 - Q_2) = \dfrac{Q_1 - Q_2}{2C}$

12. (b) When capacitors of capacitances C and $2C$ are connected in parallel to each other.

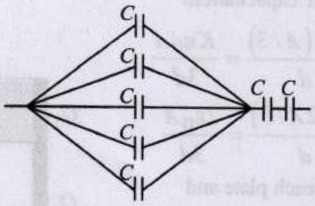
Resultant capacity $C_R = (2C + C) = 3C$

Net potential $V_R = 2V - V = V$

$\therefore$ Final energy, $E = \frac{1}{2}C_R(V_R)^2 = \frac{1}{2}(3C)(V)^2 = \frac{3}{2}CV^2$

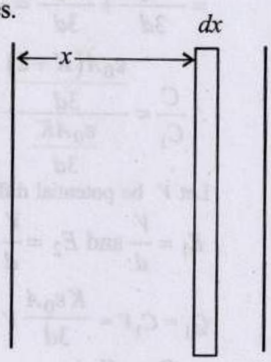
13. (a) The equivalent capacitance

$$\frac{1}{C_{eq}}=\frac{1}{2}+\frac{1}{2}+\frac{1}{2\times 5}=\frac{11}{10}\Rightarrow C_{eq}=\frac{10}{11}\mu F$$



14. **(1)** Let the region between the plates is filled with N dielectric layers.
m = number of dielectric layers within x
d = distance between the plates.

Here, $\frac{x}{m}=\frac{d}{N}$



$$dc=\frac{k\left(1+\frac{m}{N}\in_0 A\right)}{dx}$$

$$\therefore \frac{1}{dc}=\frac{dx}{k\left(1+\frac{m}{N}\right)\in_0 A}$$

$$\frac{1}{C}=\int dc=\int_0^d \frac{dx}{k\left(1+\frac{m}{N}\right)\in_0 A}$$

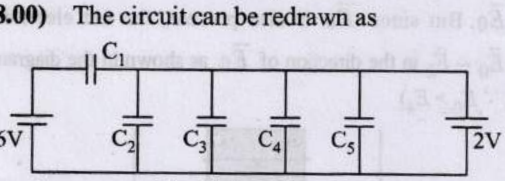
$$=\int_0^d \frac{dx}{k\left(1+\frac{x}{d}\right)\in_0 A}=\frac{d}{k\in_0 A}\int_0^d \frac{dx}{d+x}$$

or, $\frac{1}{C}=\frac{d}{k\in_0 A}ln_\alpha \Rightarrow C=\frac{k\in_0 A}{d\,ln^\alpha}$

Comparing this equivalent capacitance

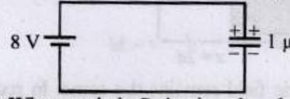
$C=\frac{k\in_0 A}{d ln_\alpha}$ with $\alpha\left(\frac{k\in_0 A}{d ln_\alpha}\right)$ we get, $\alpha = 1.00$

15. **(8.00)** The circuit can be redrawn as



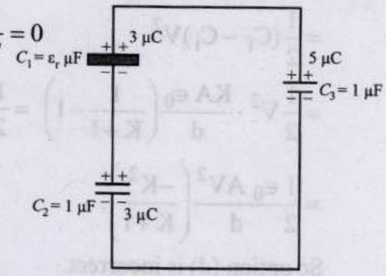
So, charge stored in C_3 is given as
$Q_3 = C_3 \times 2V = 4\mu F \times 2V = 8\mu C$

16. **(1.50)** When switch S_1 is closed and S_2 is opened, capacitor C_3 becomes fully charged.
$\therefore$ Charge on capacitor C_3, $q_3 = C_3V = 1\times 8\ \mu C = 8\ \mu C$



When switch S_2 is closed and S_1 is opened,
When all the capacitors reach equilibrium charge on C_3 is found to be 5 μC therefore charges on C_1 and C_2 are 3 μC each
Applying Kirchhoff loop rule

$$\frac{CV_0-q}{C}-\frac{q}{\varepsilon_r C}-\frac{q}{C}=0$$

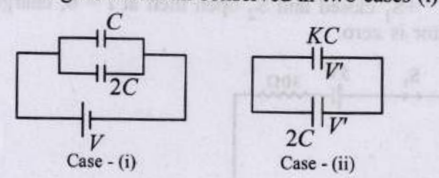


$$\Rightarrow \frac{5}{1}-\frac{3}{\varepsilon_r}-\frac{3}{1}=0$$

$$\therefore 5=3\left[1+\frac{1}{\varepsilon_r}\right]$$

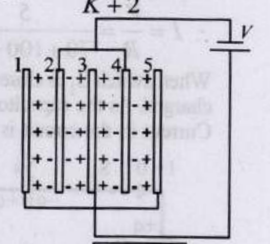
$$\therefore \frac{1}{\varepsilon_r}=\frac{5}{3}-1=\frac{2}{3}\quad \therefore \varepsilon_r=1.5$$

17. Total charge will remain conserved in two cases (i) & (ii)



$\therefore \quad CV+2CV=KCV'+2CV'$ or, $V'=\frac{3V}{K+2}$

18. Plate 1 is connected to +ve terminal so +ve charge on it

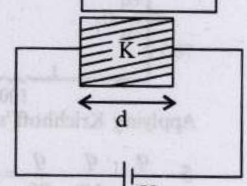


$\therefore\ q=CV=\frac{\varepsilon_0 A}{d}\times V$

Plate 4 is connected to –ve terminal so charge,

$-2q=\frac{-2\varepsilon_0 A}{d}\times V$

19. **(b)** **Case a:**

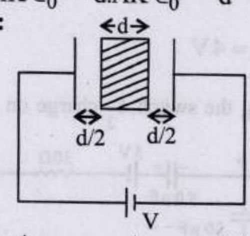


$$C_a=\frac{KA\in_0}{d}$$

$$q_a=C_aV=\frac{KA\in_0}{d}V$$

$$E_a=\frac{q_a}{AK\in_0}=\frac{KA\in_0 V}{d.AK\in_0}=\frac{V}{d}$$

Case b:



$$C_b=\frac{A\in_0}{2d+\frac{d}{k}-d}=\frac{A\in_0}{d+\frac{d}{k}}=\frac{KA\in_0}{d(K+1)}$$

$$q_b=\frac{\in_0 AKV}{d(K+1)}$$

$$(E_b)_{dielectric}=\frac{(E_{air})_b}{K}=\frac{q_b}{KA\in_0}=\frac{\in_0 AKV}{d(K+1)KA\in_0}$$

$$=\frac{V}{d(K+1)}$$

So, capacitance decreases by factor of $\frac{1}{K+1}$ and same is true for electric field. So (a) is incorrect and (b) is correct work done in process, $U_f - U_i$

$$=\frac{1}{2}(C_f - C_i)V^2$$

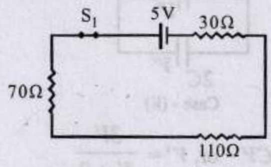
$$=\frac{1}{2}V^2 \cdot \frac{KA\in_0}{d}\left(\frac{1}{K+1}-1\right)=\frac{1}{2}V^2\frac{KA\in_0}{d}\left(\frac{-K}{K+1}\right)$$

$$=\frac{1}{2}\frac{\in_0 AV^2}{d}\left(\frac{-K^2}{K+1}\right).$$

So option (d) is incorrect.

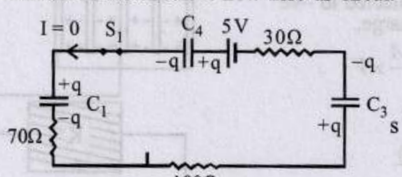
As voltage supply remains constant, so there will be no change in voltage across capacitor. So option (c) is incorrect.

20. **(c, d)** $+S_1$ closed and S_2 open then at $t = 0$, charge on each capacitor is zero.



$$\therefore\ I=\frac{V}{R}=\frac{5}{70+100+30}=0.025\text{ A}=25\text{ mA}.$$

When switch S_1 is closed for a long time all the capacitors are fully charged. As the capacitors are in series these carry equal charge q. Current in the circuit is now zero as circuit is in steady state.



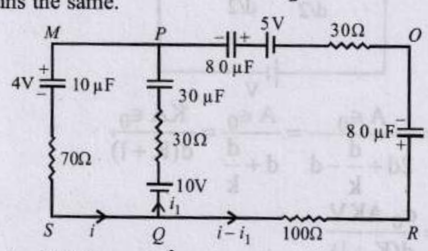
Applying Krichhoff's voltage law

$$5-\frac{q}{80}-\frac{q}{10}-\frac{q}{80}=0 \qquad \therefore\ q = 40\ \mu c$$

Potential difference across C_1

$$V=\frac{q}{C_1}=\frac{40\times10^{-6}}{10\times10^{-6}}=4\text{V}$$

Now just after closing the switch S_2 charge on each capacitor remains the same.



$V_P - 4 - 70 \times 25 \times 10^{-3} = V_Q$

$\therefore\ V_P - V_Q = 4 + 1.75 = 5.75$ V

In loop MPQS

$+10 - 30\,i_1 - 4 - 70\,i = 0$

$70\,i + 30\,i_1 = 6$ ..... (i)

In loop QROPQ,

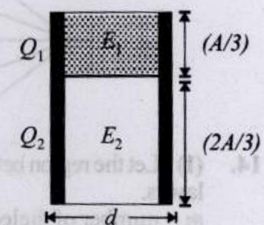
$$+10-30\,i_1+\frac{40}{80}-5+(i-i_1)\times130+\frac{40}{80}=0$$

$130\,i - 160\,i_1 = -6$ ..... (ii)

Solving (i) & (ii), we get $i = 0.05$ A $\quad \therefore\ i_1 = 0.077$ A

21. **(a, d)** The given capacitor is equivalent to two capacitors in parallel with capacitances

$$C_1=\frac{K\varepsilon_0(A/3)}{d}=\frac{K\varepsilon_0 A}{3d}$$



$$C_2=\frac{\varepsilon_0(2A/3)}{d}=\frac{2\varepsilon_0 A}{3d}$$

A = area of each plate and

d = distance between the plates

$\therefore\ C = C_1 + C_2$

$$=\frac{K\varepsilon_0 A}{3d}+\frac{2\varepsilon_0 A}{3d}=\frac{\varepsilon_0 A}{3d}(K+2)$$

$$\therefore\ \frac{C}{C_1}=\frac{\frac{\varepsilon_0 A(K+2)}{3d}}{\frac{\varepsilon_0 AK}{3d}}=\frac{K+2}{K}$$

Let V be potential difference between the plates

$$E_1=\frac{V}{d}\text{ and }E_2=\frac{V}{d} \qquad \therefore\ \frac{E_1}{E_2}=1$$

$$Q_1=C_1V=\frac{K\varepsilon_0 A}{3d}V\text{ and }Q_2=C_2V=\frac{2\varepsilon_0 A}{3d}V$$

$$\therefore\ \frac{Q_1}{Q_2}=\frac{K}{2}$$

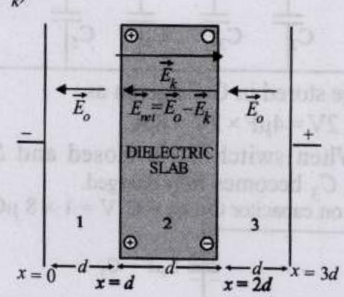
22. **(b, d)** **When switch S_1 is pressed,** the capacitor C_1 gets charged such that its upper plate acquires a positive charge $+2\,CV_0$ and lower plate $-2\,CV_0$.

When switch S_2 is pressed and S_1 is release. As $C_1 = C_2$ the charge gets distributed equal. The upper plates of C_1 and C_2 now take charge $+CV_0$ each and lower plate $-CV_0$ each.

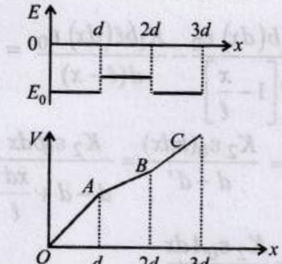
When S_2 is released and S_3 is pressed, the charge on the upper plate of C_1 is CV_0 and charge on upper plate of C_2 is $-CV_0$.

23. **(b, c)** (a) In region 1 and 3, there will be electric field $\vec{E}_0$ directed from positive to negative. In region 2, due to orientation of dipoles, there is an electric field $\vec{E}_k$ present in opposite direction of $\vec{E}_0$. But since $\vec{E}_0$ is also present, the net electric field $\vec{E}_0-\vec{E}_k$ in the direction of $\vec{E}_0$ as shown in the diagram.

$(\because E_0 > E_k)$



(b) The direction of electric field remains the same. In region-2 electric field is less an compared to region 1 and 3.

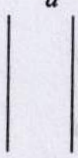


(c) When one moves opposite to the direction of electric field, the potential always increases. The stronger the electric field, the more is the potential increase. Since in region 2, the electric field is less as compared to 1 and 3 therefore the increase in potential will be less but there has to be increase in potential in all the regions from $x = 0$ to $x = 3d$.

Hence option (c) is correct and (d) is incorrect.

24. **(a, c, d)** Initially,

As $C = \frac{\varepsilon_0}{d} A$



$Q = CV$

$= \frac{\varepsilon_0 A}{d} \times V$

When dielectric slab is inserted

$C' = \frac{K\varepsilon_0 A}{d}$

$V' = \frac{V}{K}$

$Q = \frac{\varepsilon_0 AV}{d} = C'V'$

Q will remain same as no charge is leaving or entering the plates during the process of slab insertion because battery is removed.

Now, $Q = C'V' = C'E'd$

$$\Rightarrow E' = \frac{Q}{C'd} = \frac{\frac{\varepsilon_0 AV}{d}}{\frac{K\varepsilon_0 A}{d}} \times \frac{1}{d} = \frac{V}{Kd}$$

Work done is the change in energy stored

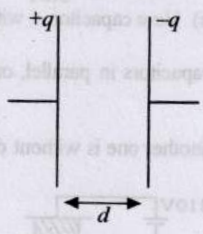
$$\therefore W = \frac{1}{2}CV^2 - \frac{1}{2}C'V'^2$$

$$= \frac{1}{2}\frac{\varepsilon_0 AV^2}{d} - \frac{1}{2}\frac{K\varepsilon_0 A}{d} \times \left(\frac{V}{K}\right)^2 \quad \left[\because V' = E'd = \frac{V}{K}\right]$$

$$\text{or, } W = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2\left[1 - \frac{1}{K}\right]$$

25. **(b, d)** Charge on plate is q

Capacitances, $C = \frac{\varepsilon_0 A}{d}$



$q = CV \Rightarrow V = \frac{q}{C}$,

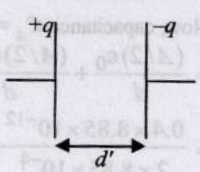
$U = \frac{1}{2}q \times V$

If the plates of the capacitor are moved farther a part so $d' > d$

$C' = \frac{\varepsilon_0 A}{d'} \quad \therefore C' < C$,

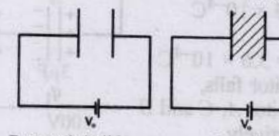
$V' = \frac{q}{C'} \quad \therefore V' > V$

$U' = \frac{1}{2}qV' \Rightarrow U' = U$



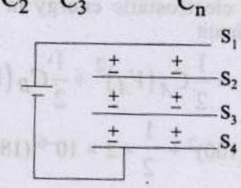
26. **(a, d)**

Capacitor without dielectric	Capacitor with dielectric
(i) Potential difference = V_0 Capacitance = C	Potential difference = V_0 Capacitance = KC [K is the dielectric constant of slab $K > 1$]
(ii) Charge $Q_0 = CV_0$	New charge = KCV_0
(iii) Potential Energy $= \frac{1}{2}CV_0^2$	New potential energy $= \frac{1}{2}KCV_0^2$
(iv) Electric field $E = \frac{V_0}{d}$	$E = \frac{V_0}{d}$



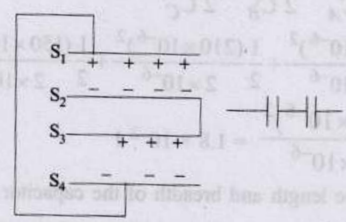
27. **(c)** For P, the capacitance between S_1 and S_4, with S_2 and S_3 not connected, all are in series and in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots + \frac{1}{C_n}$$



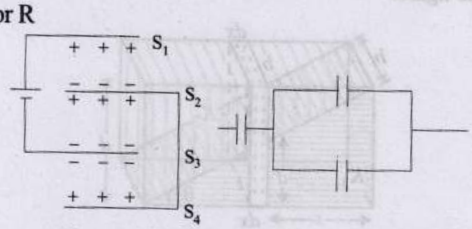
$$\therefore \frac{1}{C_{eq}} = \frac{1}{C_0} + \frac{1}{C_0} + \frac{1}{C_0} \therefore C_{eq} = \frac{C_0}{3}$$

For Q



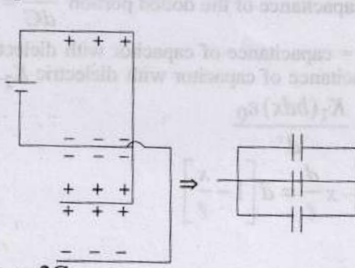
$$C_{eq} = \frac{C_0}{2}$$

For R



$$C_{eq} = \frac{2C}{3}$$

For S



$$\Rightarrow C_{eq} = 3C_0$$

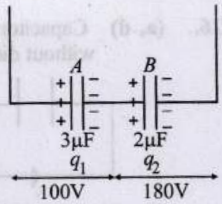
28. **(i)** Before uncharged capacitor falls

Charge on capacitor A

$q_A = 3 \times 10^{-6} \times 100 = 3 \times 10^{-4}$C

Charge on capacitor B

$q_B = 2 \times 10^{-6} \times 180 = 3.6 \times 10^{-4}$C



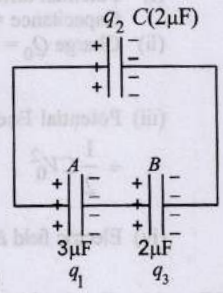
When uncharged capacitor falls,

Let the charge on capacitor A, C and B be q_1, q_2 and q_3 respectively.

By charge conservation.

The sum of charge on +ve plate of capacitor A and $C = q_A$

$\therefore \quad q_1 + q_2 = 3 \times 10^{-4}$C ... (i)



Similarly the sum of charge on –ve plates of capacitor C and $B = q_B$

$\therefore \quad -q_2 - q_3 = -3.6 \times 10^{-4}$C

or, $q_2 + q_3 = 3.6 \times 10^{-4}$C ... (ii)

Applying Kirchhoff's law in the closed loop,

$$\frac{q_1}{3 \times 10^{-6}} - \frac{q_2}{2 \times 10^{-6}} + \frac{q_3}{2 \times 10^{-6}} = 0$$

or, $2q_1 - 3q_2 + 3q_3 = 0$ ....(iii)

Solving (i), (ii) and (iii), we get

$q_1 = 90 \times 10^{-6}$ C, $q_2 = 210 \times 10^{-6}$ C, and $q_3 = 150 \times 10^{-6}$ C,

(ii) Amount of electrostatic energy in the system before completing the circuit

$$U_i = U_A + U_B = \frac{1}{2}C_A(V_A)^2 + \frac{1}{2}C_B(V_B)^2$$

$$= \frac{1}{2} \times 3 \times 10^{-6}(100)^2 + \frac{1}{2} \times 2 \times 10^{-6}(180)^2$$

$= 4.74 \times 10^{-2}$J

Amount of electrostatic energy stored after completing the circuit

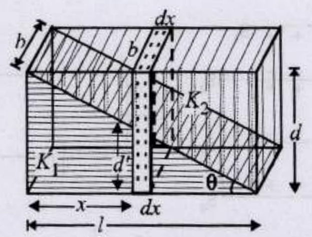
$$U_f = \frac{1}{2}\frac{q_1^2}{C_A} + \frac{1}{2}\frac{q_2^2}{C_B} + \frac{1}{2}\frac{q_3^2}{C_C}$$

$$= \frac{1}{2}\frac{(90 \times 10^{-6})^2}{3 \times 10^{-6}} + \frac{1}{2}\frac{(210 \times 10^{-6})^2}{2 \times 10^{-6}} + \frac{1}{2}\frac{(150 \times 10^{-6})^2}{2 \times 10^{-6}}$$

$$+ \frac{1}{2}\frac{(150 \times 10^{-6})^2}{2 \times 10^{-6}} = 1.8 \times 10^{-2} \text{ J}$$

29. Let ℓ, b the length and breadth of the capacitor plate.

Therefore $\ell \times b = A$. d be the distance between the plates. Let us consider a small dotted element of thickness dx as shown in the figure.



The small capacitance of the dotted portion $\frac{1}{dC} = \frac{1}{dC_1} + \frac{1}{dC_2}$

where dC_1 = capacitance of capacitor with dielectric K_1

dC_2 = capacitance of capacitor with dielectric K_2.

$$dC_1 = \frac{K_1(bdx)\varepsilon_0}{d'}$$

$$d' = d - x\frac{d}{\ell} = d\left[1 - \frac{x}{\ell}\right]$$

$$\therefore \quad dC_1 = \frac{K_1 b(dx)\varepsilon_0}{d\left[1 - \frac{x}{\ell}\right]} = \frac{K_1 b\ell(dx)\varepsilon_0}{d(\ell - x)} = \frac{K_1 A\varepsilon_0(dx)}{d(\ell - x)}$$

Similarly, $dC_2 = \dfrac{K_2\varepsilon_0(bdx)}{d - d'} = \dfrac{K_2\varepsilon_0 bdx}{d - d + \dfrac{xd}{\ell}}$

$$\frac{K_2\varepsilon_0 b.\ell.dx}{xd} = \frac{K_2\varepsilon_0 A dx}{xd}$$

$$\therefore \frac{1}{dC} = \frac{d(\ell - x)}{K_1 A\varepsilon_0(dx)} + \frac{xd}{K_2 A\varepsilon_0(dx)}$$

$$\Rightarrow \quad \frac{K_1 K_2 A\varepsilon_0 dx}{K_2\ell d + d(K_1 - K_2)x} = dC$$

Hence capacitance of the whole capacitor,

$$C = \int_0^\ell \frac{K_1 K_2 A\varepsilon_0 dx}{K_2\ell d + d(K_1 - K_2)x}$$

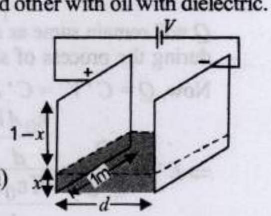
$$= K_1 K_2 A\varepsilon_0 \int_0^\ell \frac{dx}{K_2\ell d + d(K_1 - K_2)x}$$

$$= K_1 K_2 A\varepsilon_0 \left[\frac{\log[K_2\ell d + d(K_1 - K_2)x]}{d(K_1 - K_2)}\right]_0^\ell$$

or, $C = \dfrac{K_1 K_2 A\varepsilon_0}{d(K_1 - K_2)}\log\dfrac{K_1}{K_2}$

30. Here the system as shown in the figures, is equivalent to two capacitors in parallel one with air and other with oil with dielectric.

$C = C_1 + C_2$



$$= \frac{k\varepsilon_0(x \times 1)}{d} + \frac{\varepsilon_0[(1 - x) \times 1)}{d}$$

$$C = \frac{\varepsilon_0}{d}[kx + 1 - x] \quad \text{... (i)}$$

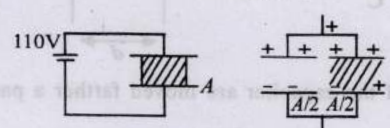
x = side of the dielectric = $1m$ and $d = 0.01m$

$$\therefore q = CV = \frac{\varepsilon_0 V}{0.01}\left[1 + x(K - 1)\right]$$

$$\therefore \text{Current, } I = \frac{dq}{dt} = \frac{\varepsilon_0 V}{0.01}[K - 1]\frac{dx}{dt}$$

$$\text{So, } I = \frac{8.85 \times 10^{-12} \times 500}{0.01}[11 - 1] \times 0.001 = 4.425 \times 10^{-9} A$$

31. (i) Here capacitor A with dielectric slab can be considered as two capacitors in parallel, one having dielectric slab and area $\frac{A}{2}$ and another one is without dielectric slab and area $\frac{A}{2}$.



Now, capacitance $C_A = C_1 + C_2$

$$= \frac{(A/2)\varepsilon_0}{d} + \frac{(A/2)\varepsilon_0\varepsilon_r}{d} = \frac{A}{2}\frac{\varepsilon_0}{d}[1 + \varepsilon_r]$$

$$= \frac{0.4 \times 8.85 \times 10^{-12}}{2 \times 8.85 \times 10^{-4}}[1 + 9] = 2 \times 10^{-9} F$$

Energy stored in this capacitor when connected to $V = 110$ volt battery.

$$U_A = \frac{1}{2}C_A V^2 = \frac{1}{2} \times 2 \times 10^{-9} \times (110)^2$$
$= 1.21 \times 10^{-5}$ J

(ii) While taking out the dielectric, the charge on the capacitor plate remains the same but capacitance of A will get changed, hence energy will change. This change in energy is equal to work done.

$$W = \frac{q^2}{2C_A'} - \frac{q^2}{2C_A}$$

$$C_A' = \frac{A\varepsilon_0}{d} = \frac{0.04 \times 8.85 \times 10^{-14}}{8.85 \times 10^{-4}} = 0.4 \times 10^{-9}\ F$$
$q = C_A V = 2 \times 10^{-9} \times 110 = 2.2 \times 10^{-7}\ C$

$$\therefore\quad W = \frac{(2.2 \times 10^{-7})^2}{2}\left[\frac{1}{0.4 \times 10^{-9}} - \frac{1}{2 \times 10^{-9}}\right]$$
or, $W = 4.84 \times 10^{-5}$ J

(iii) Capacitance of $B = \dfrac{\varepsilon_0\, \varepsilon_r A_B}{d}$

$$= \frac{8.85 \times 10^{-12} \times 9 \times 0.02}{8.85 \times 10^{-4}} = 1.8 \times 10^{-9}\ F$$

Charge on A, $q_A = 2.2 \times 10^{-7}\ C$ gets distributed into two parts.
$\therefore\quad q_1 + q_2 = 2.2 \times 10^{-7}\ C$
Also the potential difference across A = p.d. across B

$$\frac{q_1}{C'_A} = \frac{q_2}{C_B} \Rightarrow q_1 = \frac{C_A}{C_B}\, q_2 = \frac{0.4 \times 10^{-9}}{1.8 \times 10^{-9}}\, q_2 = 0.22\, q_2$$
$\therefore\quad 0.22\, q_2 + q_2 = 2.2 \times 10^{-7}$

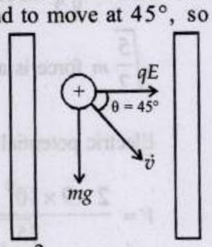
$$\Rightarrow q_2 = \frac{2.2}{1.22} \times 10^{-7} = 1.8 \times 10^{-7}\ C$$
$\Rightarrow q_1 = 0.4 \times 10^{-7}\ C$

$$\text{Total energy stored} = \frac{q_1^2}{2C'_A} + \frac{q_2^2}{2C_B}$$

$$= \frac{0.4 \times 0.4 \times 10^{-14}}{2 \times 0.4 \times 10^{-9}} + \frac{1.8 \times 1.8 \times 10^{-14}}{2 \times 1.8 \times 10^{-8}}$$
$= 0.2 \times 10^{-5} + 0.9 \times 10^{-5} = 1.1 \times 10^{-5}$ J

Topic-4: Miscellaneous (Mixed Concepts) Problems

1. **(c)** According to the question, a proton is released at rest mid-way between the two plates and is found to move at 45°, so net force is at 45° from vertical and two forces acting on the proton just after the release are as shown in the figure.



$qE = mg$

$$\therefore\quad q\left(\frac{V}{d}\right) = mg$$

$$\therefore\quad V = \frac{mgd}{q} = \frac{1.67 \times 10^{-27} \times 10 \times 10^{-2}}{1.6 \times 10^{-19}} = 10^{-9}\ \text{V}$$

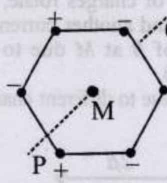
Hence X is nearly 1×10^{-9} V

2. **(c, d)** The potential of all points lie on the conductor is same. Thus potential at A = Potential at B

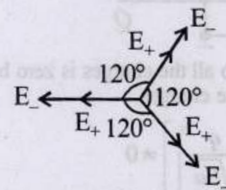
Total electric flux through cavity $= \dfrac{q}{\epsilon_0}$ this according to Gauss's theorem.

Option (a) and (b) are dependent on the curvature which is different at points A and B because cavity is elliptical.

3. (P) Here PQ is perpendicular to the plane of the hexagon.



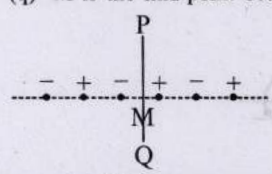
Clearly electric field at the centre M of the hexagon due to the charges at the corners of regular hexagon is zero.



Here $|E_+| = |E_-|$.

i.e., $E = 0$ at M. (By symmetry)
The electric potential due to all the charges at M is zero.
i.e., $V = 0$ at M
When the system of charges is rotated about line PM, the net current will be zero.
Hence magnetic field $B = 0$ at M.
When $B = 0$ then $\mu = 0$

(q) M is the mid-point between the two innermost charges.



Electric field due to the inner most positive and negative charges at M $E_1 = 2\left[k\dfrac{q}{r^2}\right]$ towards left.

Electic field due to the next positive and negative charges at M $E_2 = 2\left[k\dfrac{q}{(2r)^2}\right]$ towards right. Electric field due to the outermost positive and negative charges at M

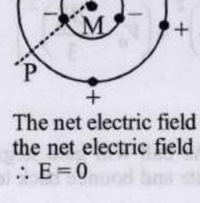
$E_3 = 2\left[k\dfrac{q}{(3r)^2}\right]$ towards left. i.e., $\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \vec{d}0$

The electric potential due to the charges at M

$$V = k\left[\frac{+q}{r} - \frac{q}{r} + \frac{q}{2r} - \frac{q}{2r} + \frac{q}{3r} - \frac{q}{3r}\right] = 0$$

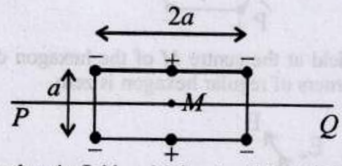
Net current due to the innermost positive and negative charges is zero. Similarly the net current due to other charges in pairs is zero. Therefore, $B = 0\ \therefore \mu = 0$

(r)



The net electric field due to negative charges $E_{inner} = 0$. Similarly the net electric field due to positive charges $E_{outer} = 0$.
$\therefore E = 0$

The electric potential due to negative charges at M is different from the electric potential due to positive charges at M.
$\therefore V \neq 0$ at M.
When the system of charges rotate, we get a current I_1 due to negative charges and another current I due to positive charges. The magnitude of B at M due to the currents is different.
$\therefore B \neq 0$ and $\mu \neq 0$.
Here electric field due to different charges cancel out in pairs.
$\therefore E = 0$
(s)

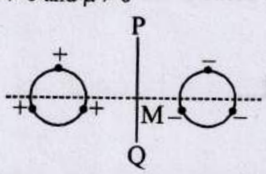


The electric field at M due to all the charges is zero because
The potential at M due to the charges

$$V = k\left[\frac{+q}{a/2} + \frac{q}{a/2} - 4\left(\frac{q}{\frac{\sqrt{5a}}{4}}\right)\right] \neq 0$$

When the whole system is set into rotation with a constant angular velocity about the line PQ we get three loops in which current is flowing.
The magnetic field due to these currents produce a resultant magneic field at M which is not equal to zero. Therefore a net magnetic dipole moment will be produced.
$\therefore B \neq 0$ and $\mu \neq 0$

(t)



Here $E \neq 0$ as there will be a net electric field due to the arrangement of charges at M towards the right side.
Electic potential $V = 0$ at M due to symmetrical arrangement of positive and negative charges.
Net current is zero when the system of charges rotates about PQ, due to symmetrical arrangement of charges. $\therefore B = 0$ and $\mu = 0$

4. **(a)** In process 1, work done by battery, $w = q \times V = CV_o \times V_o = CV_o^2$

Energy stored in the battery $E_C = \frac{1}{2}CV_0^2$

Heat dissipated across resistance,

$$E_D = W - E_C = CV_0^2 - \frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2$$

$\therefore \quad E_C = E_D$

5. **(c)** Let V_i and V_f be the initial and final voltage in each step of process 2. Then
Energy dissipated = $W_{battery} - \Delta U$

$$= C(V_f - V_i)V_f - \frac{1}{2}C(V_f - V_i)^2$$

$$= \frac{1}{2}C(V_f - V_i)^2$$

$\therefore$ Total energy dissipated across the resistance,

$$E_D = \frac{1}{2}C\left[\left(\frac{V_o}{3} - 0\right)^2 + \left(\frac{2V_o}{3} - \frac{V_o}{3}\right)^2 + \left(V_o - \frac{2V_o}{3}\right)^2\right]$$

or, $E_d = \frac{1}{6}CV_o^2$

6. **(c)** After colliding the top plate, the ball will gain negative charge and get repelled by the top plate and bounce back to the bottom plate. But ball do not execute simple harmonic motion as force on it $\propto x$

7. **(d)** Average current, $I_{av} \propto \frac{Q}{t}$ ...(i)

Here $Q \propto V_0$ ...(ii)

From $S = ut + \frac{1}{2}at^2$

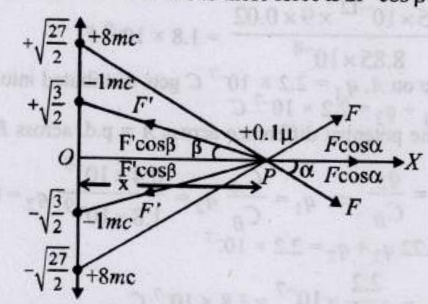
$$h = \frac{1}{2}\frac{QE}{m}t^2 = \frac{1}{2}\left(\frac{Q \times 2V_0}{mh}\right) \times t^2 \left(\because a = \frac{F}{m} = \frac{qE}{m}\right)$$

$\therefore \quad t \propto \frac{1}{V_0}$ – (iii) $[\because Q \propto V_0]$

From eq. (i), (ii) and (iii)

$$I_{av} \propto \frac{V_0}{1/V_0} = I_{av} \propto V_0^2$$

8. Let P be any point at a distance x from the origin. As shown in the figure, there are two forces of repulsion acting due to two charges of + 8 mC. The net force is $2F \cos \alpha$ towards right. Similarly there are two forces of attraction due to two charges of – 1 mC. The net force due to these force is $2F' \cos \beta$ towards left.



For net force to become zero.
$2F \cos \alpha = 2F' \cos \beta$

$$\frac{K \times 8 \times 10^{-6} \times 0.1 \times 10^{-6}}{\left(\sqrt{x^2 + \frac{27}{2}}\right)^2} \times \frac{x}{\sqrt{x^2 + \frac{27}{2}}}$$

$$= \frac{K \times 1 \times 10^{-6} \times 0.1 \times 10^{-6}}{\left(\sqrt{x^2 + \frac{3}{2}}\right)^2} \times \frac{x}{\sqrt{x^2 + \frac{3}{2}}} \Rightarrow x = \pm\sqrt{\frac{5}{2}}$$

At $x > \sqrt{\frac{5}{2}}$ m force is repulsive towards + ve x-axis and for $x < \sqrt{\frac{5}{2}}$ m force is attractive towards negative x-axis.

Electric potential of the four charges at $x = \sqrt{\frac{5}{2}}$

$$V = \frac{2 \times 9 \times 10^9 \times 8 \times 10^{-6}}{\sqrt{\frac{5}{2} + \frac{27}{2}}} - \frac{2 \times 9 \times 10^9 \times 10^{-6}}{\sqrt{\frac{5}{2} + \frac{3}{2}}}$$

$$= 2 \times 9 \times 10^9 \times 10^{-6}\left[\frac{8}{4} - \frac{1}{2}\right] = 2.7 \times 10^4 \text{ V}$$

Kinetic energy is required to overcome the force of repulsion from $\propto$ to $x = \sqrt{\frac{5}{2}}$.

The work done in this process $W = q\,(V)$
$\therefore \quad W = 0.1 \times 10^{-6} \times 2.7 \times 10^4 = 2.7 \times 10^{-3}$ J

By energy conservation $\frac{1}{2}mV_0^2 = 2.7 \times 10^{-3}$

$$\Rightarrow \frac{1}{2} \times 6 \times 10^{-4} V_0^2 = 2.7 \times 10^{-3} \Rightarrow V_0 = 3 \text{ m/s}$$

K.E. at the origin

Potential at origin

$$V_{x=0} = \frac{2 \times 9 \times 10^9 \times 8 \times 10^{-6}}{\sqrt{\frac{27}{2}}} - \frac{2 \times 9 \times 10^9 \times 10^{-6}}{\sqrt{\frac{3}{2}}}$$

$= 2.4 \times 10^4$

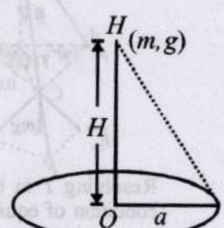
Again by energy conservation

$$\text{K.E.} = q\left[V_{x=\frac{\sqrt{5}}{2}} - V_{x=0}\right]$$

$\therefore$ K.E. $= 0.1 \times 10^{-6}\,[2.7 \times 10^4 - 2.4 \times 10^4]$
$= 0.1 \times 10^{-6} \times 0.3 \times 10^4 = 3 \times 10^{-4}$ J

9. (a) Given a = radius of disc, σ = surface charge density, $q/m = 4\varepsilon_0 g/\sigma$

Potential due to a charged disc at any axial point situated at a distance x from O



$$V(x) = \frac{\sigma}{2\varepsilon_0}[\sqrt{a^2 + x^2} - x]$$

At $x = H$

$$V(H) = \frac{\sigma}{2\varepsilon_0}[\sqrt{a^2 + H^2} - H]$$

And $V(O) = \frac{\sigma a}{2\varepsilon_0}$

When the particle is dropped along the axis of disc, energy is conserved. Loss of gravitational potential energy = gain in electric potential energy

$mgH = q\Delta V = q\,[V(0) - V(H)]$

$$mgH = q\frac{\sigma}{2\varepsilon_0}[a - \{\sqrt{(a^2 + H^2)} - H\}] \quad \text{... (i)}$$

From the given relation $\frac{2}{m} = \frac{4\varepsilon_0 g}{\sigma} \Rightarrow \frac{\sigma q}{2\varepsilon_0} = 2mg$

Putting this value in equation (i),

$$mgH = 2mg\,[a - \{\sqrt{(a^2 + H^2)} - H\}] \text{ or } H = \frac{4a}{3}$$

(b) Total potential energy of the particle at height H

$U(x) = mgx + qV(x)$

$$= mgx + \frac{q\sigma}{2\varepsilon_0}(\sqrt{a^2 + x^2} - x)$$

$$= mgx + 2mg\,[\sqrt{(a^2 + x^2)} - x] \quad \text{....(ii)}$$

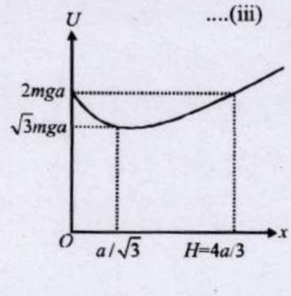
$$U_{(A)} = mgH + 2mg\left[\sqrt{a^2 + H^2} - H\right]$$

$$= mg\left[2\sqrt{a^2 + H^2} - H^2\right] \quad \text{....(iii)}$$

For equilibrium : $\frac{dU}{dH} = 0$

at $H = \frac{a}{\sqrt{3}}$

$\therefore U_{\min} = \sqrt{3}$ mga



From equation (ii), graph between $U(x)$ and x a parabola, is as shown below.

Hence $H = \frac{a}{\sqrt{3}}$ is stable equilibrium position.

10. (a) Let S_1 contain charge Q before contact with S_2. On contact, let a charge q_1 shift from S_1 to S_2.

$$\frac{k(Q - q_1)}{r} = \frac{kq_1}{R} \text{ where } k = \frac{1}{4\pi\varepsilon_0} \text{ or } q_1 = Q\left(\frac{R}{R+r}\right)$$

On second contact, similarly,

$$\frac{k\left[Q - (q_2 - q_1)\right]}{r} = \frac{kq_2}{R} \text{ or } rq_2 = RQ - Rq_2 + Rq_1$$

$$\text{or } rq_2 = RQ - Rq_2 + RQ\left(\frac{R}{R+r}\right)$$

$$\text{or } q_2 = Q\left[\left(\frac{R}{R+r}\right) + \left(\frac{R}{R+r}\right)^2\right]$$

On third contact,

$$q_3 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \left(\frac{R}{R+r}\right)^3\right]$$

On n[th] cotact,

$$q_n = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \ldots\ldots + \left(\frac{R}{R+r}\right)^n\right]$$

In G. P., $S_n = \frac{a(1 - r_1^n)}{(1 - r_1)}$ where r_1 is common ratio.

$$\text{or } q_n = \frac{QR}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right]$$

Electrostatic energy of S_2 after n such contacts

$$U_n = \frac{q_n^2}{2C} = \frac{q_n^2}{2(4\pi\varepsilon_0 R)}$$

$$\text{or } U_n = \frac{1}{2} \times \frac{1}{4\pi\varepsilon_0 R} \times \left[\frac{QR}{r}\left\{1 - \left(\frac{R}{R+r}\right)^n\right\}\right]^2$$

(b) Limiting value of energy as $n \to \infty$.

Let us calculate q_n when n tends to ∞.

For G.P., $S_\infty = \frac{a}{1 - r_1}$ where r_1 = common ratio

$$\therefore q_\infty = \frac{QR}{R+r}\left[\frac{1}{1 - \frac{R}{R+r}}\right] \text{ or } q_\infty = \frac{QR}{r}$$

$$\therefore U_\infty = \frac{q_\infty^2}{2C} = \left(\frac{QR}{r}\right)^2 \times \frac{1}{2 \times (4\pi\varepsilon_0) \times (R)}$$

$$\text{or } U_\infty = \frac{Q^2R^2}{r^2 \times 2 \times 4\pi\varepsilon_0 R} \text{ or } U_\infty = \frac{Q^2R}{2(4\pi\varepsilon_0)r^2}$$

11. (a) A charge is uniformly distributed over a non-conducting spherical volume of radius R. Let us consider a shell of the thickness dx at a distance x from the centre of a sphere

$$\text{Volume of the shell} = \frac{4}{3}\pi\left[(x + dx)^3 - \frac{4}{3}\pi x^3\right]$$

$= \frac{4}{3}\pi\left[(x+dx)^3 - x^3\right]$

$= \frac{4}{3}\pi x^3\left[\left(1+\frac{dx}{x}\right)^3 - 1\right]$

$= \frac{4}{3}\pi x^3\left[1+\frac{3dx}{x} - 1\right]$

$= \frac{4}{3}\pi x^3 \times \frac{3dx}{x} = 4\pi x^2 dx$

Let ρ be the charge per unit volume of the sphere

∴ Charge of the shell $= dq = 4\pi x^2 \rho dx$ ... (i)

Potential at the surface of the sphere of radius x

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{\rho \times \frac{4}{3}\pi x^3}{x} \qquad \left[\because V = k\frac{q}{r}\right]$$

∴ Potential at the surface of the sphere of radius $x = \frac{\rho x^2}{3\varepsilon_0}$

Work done in bringing the charge dq on the sphere of radius x dW

$$= \frac{\rho x^2}{3\varepsilon_0} \times dq \Rightarrow dW = \frac{\rho x^2}{3\varepsilon_0} \times 4\pi x^2 \rho\, dx$$

∴ Work done in accumulating the charge Q over a spherical volume of radius R

$$W = \int_0^R \frac{4\pi\rho^2}{3\varepsilon_0} x^4 dx = \frac{4\pi\rho^2}{3\varepsilon_0}\left[\frac{x^5}{5}\right]_0^R = \frac{4\pi\rho^2}{3\varepsilon_0}\frac{R^5}{5}$$

$$= \frac{4\pi}{3\varepsilon_0}\left(\frac{Q}{4/3\pi R^3}\right)^2 \frac{R^5}{5} = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

This work done is equal to the energy stored in the system.

(b) The corresponding energy needed to completely disassemble the planet earth against the gravitational pull.

$$E = \frac{3Q^2}{5\times(4\pi\varepsilon_0)R} = \frac{3KQ^2}{5R} \quad \text{where } K = \frac{1}{4\pi\varepsilon_0}$$

Replacing $\frac{1}{4\pi\varepsilon_0}$ or K by G and Q^2 by M^2. ∴ $E = \frac{3GM^2}{5R}$

$$g = \frac{GM}{R^2} \Rightarrow G = \frac{gR^2}{M}$$

$$\therefore \quad E = \frac{3}{5}\frac{gR^2}{M}\frac{M^2}{R} = \frac{3}{5}gMR = \frac{3}{5}\times 10 \times 2.5 \times 10^{31}$$

$= 1.5 \times 10^{32}$ J

(c) If the same charge of Q conlomb is given to a spherical conductor of the same radius, then $C = 4\pi\varepsilon_0 R$ and energy

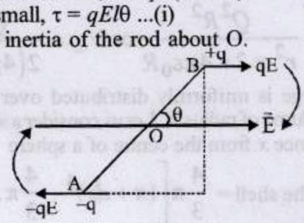
$$E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q2}{4\pi\varepsilon_0 R}$$

12. As shown in Fig, AB = l

Tonque on the rod AB, $\tau = qE(l \sin\theta)$

When θ is small, $\tau = qEl\theta$ ...(i)

Moment of inertia of the rod about O.



$$I = m(l/2)^2 + m(l/2)^2 = \frac{ml^2}{2}$$

As $\tau = I\alpha$ $\quad \therefore \alpha = \frac{\tau}{I} = \frac{qEl\theta}{ml^2/2} = \frac{2qE\theta}{ml} = \omega^2\theta$

Clearly, α is directly proportional to θ, and torque is trying to decrease θ.

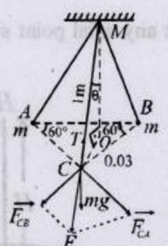
∴ The motion of the rod is S.H.M., with

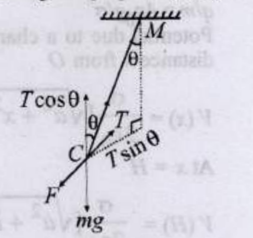
$$\omega^2 = \frac{2qE}{ml}, \omega = \sqrt{\frac{2qE}{ml}} = \frac{2\pi}{T} \therefore T = 2\pi\sqrt{\frac{ml}{2qE}}$$

The rod will become parallel to $\vec{E}$ from a position, q = 90° in a time

$$t = \frac{T}{4} = \frac{2\pi}{4}\sqrt{\frac{ml}{2qE}} = \frac{\pi}{2}\sqrt{\frac{ml}{2qE}}$$

13. Each particle will be in equilibrium under the act of three force tension of string T, weighting, resultant electrostatic force F of the two other charges. Force F and mg are perpendicular.





Resolving T in the direction of mg and F and applying the condition of equilibrium,

$T\cos\theta = mg$; $\qquad T\sin\theta = F$

$$\therefore \quad \tan\theta = \frac{F}{mg} \quad \text{... (i)}$$

$$F = \sqrt{F_{CA}^2 + F_{CB}^2 + 2F_{CA}F_{CB}\cos\alpha}$$

$$\therefore \quad F = \sqrt{F_{CA}^2 + F_{CA}^2 + 2F_{CA}^2 \times \frac{1}{2}}$$

$$F = \sqrt{3}F_{CA} = \sqrt{3} \times \frac{kq^2}{(CA)^2} \quad \text{... (ii)}$$

$[\,|\vec{F}_{CB}| = |\vec{F}_{CA}|$ and $\alpha = 60°\,]$

Let T make an angle θ with the vertical

$$OC = \frac{2}{3}\sqrt{(0.03)^2 - (0.015)^2} = 0.0173 \text{ m}$$

∴ $OM = 0.9997$

$$\therefore \tan\theta = \frac{OC}{OM} = \frac{0.0173}{0.9997} \quad \text{.... (iii)}$$

From eq. (i), (ii) and (iii)

$$\frac{0.0173}{0.9997} = \frac{\sqrt{3}\times 9 \times 10^9 \times q^2}{(0.03)^2 \times 10^{-3} \times 9.8}$$

∴ $q = 3.16 \times 10^{-9}$ C.