

10.3. Some Partial Differential Equations

1181. The Laplace Equation

$$\frac{\partial^2 \mathbf{u}}{\partial x^2} + \frac{\partial^2 \mathbf{u}}{\partial y^2} = 0$$

applies to potential energy function $u(x,y)$ for a conservative force field in the xy -plane. Partial differential equations of this type are called **elliptic**.

1182. The Heat Equation

$$\frac{\partial^2 \mathbf{u}}{\partial x^2} + \frac{\partial^2 \mathbf{u}}{\partial y^2} = \frac{\partial \mathbf{u}}{\partial t}$$

applies to the temperature distribution $u(x,y)$ in the xy -plane when heat is allowed to flow from warm areas to cool ones. The equations of this type are called **parabolic**.

1183. The Wave Equation

$$\frac{\partial^2 \mathbf{u}}{\partial x^2} + \frac{\partial^2 \mathbf{u}}{\partial y^2} = \frac{\partial^2 \mathbf{u}}{\partial t^2}$$

applies to the displacement $u(x,y)$ of vibrating membranes and other wave functions. The equations of this type are called **hyperbolic**.