2 Chapter
Multiplications

Multiplications are the next calculation which we need to look at—these are obviously crucial because most questions in Mathematics do involve multiplications.

The fundamental view of multiplication is essentially that when we need to add a certain number consecutively—say we want to add the number 17 seven times:

i.e. 17 + 17 + 17 + 17 + 17 + 17 + 17 it can also be more conveniently done by using 17 × 7.

Normally in aptitude exams like the CAT, multiplications would be restricted to 2 digits multiplied by 2 digits, 2 digits multiplied by 3 digits and 3 digits multiplied by 3 digits.

So what are the short cuts that are available in Multiplications? Let us take a look at the various options you have in order to multiply.

1. The straight line method of multiplying two numbers (From Vedic Mathematics and also from the Trachtenberg System of Speed Mathematics)

Let us take an example to explain this process:

Suppose you were multiplying two 2 digit numbers like 43 × 78.

The multiplication would be done in the following manner:

$$\begin{array}{r} 4\ 3 \\ \times\ 7\ 8 \\ \hline \end{array}$$

Step 1: Finding the Unit's digit

The first objective would be to get the unit's digit. In order to do this we just need to multiply the units' digit of both the numbers. Thus, 3 × 8 would give us 24. Hence, we would write 4 in the units' digit of the answer and carry over the digit 2 to the tens place as follows:

$$\begin{array}{r} 4\ 3 \\ \times\ 7\ 8 \\ \hline 4 \end{array}$$

2 carry over to the tens place

At this point we know that the units' digit is 4 and also that there is a carry over of 2 to the tens place of the answer.

Step 2: Finding the tens' place digit

$$\begin{array}{r} 4\ 3 \\ \times\ 7\ 8 \\ \hline 5\ 4 \end{array}$$

5 carry over to
hundreds place

Thought Process:

4 × 8 + 3 × 7 = 32 + 21 = 53

53 + 2 (from carry over) = 55

Thus we write 5 in the tens place and carry over 5 to the hundreds place

In the above case, we have multiplied the units digit of the second number with the tens digit of the first number and added the multiplication of the units digit of the first number with the tens digit of the second number. Thus we would get:

8 (units digit of the second number) × 4 (tens digit of the first number) + 7 (tens digit of the second number) × 3 (units digit of the first number + 2 (carry over from the units' digit calculation)

= 32 + 21 + 2 = 55.

We write down 5 in the tens' digit of the answer and carry over 5 to the hundreds digit of the answer.

Step 3: Finding the hundreds' place digit

$$\begin{array}{r} 4\ 3 \\ \times\ 7\ 8 \\ \hline 3\ 3\ 5\ 4 \end{array}$$

Thought Process:

4 × 7 = 28

28 + 5 = 33

Since, 4 and 7 are the last digits on the left in both the numbers this is the last calculation in this problem and hence we can write 33 for the remaining 2 digits in the answer

Thus, the answer to the question is 3354.

With a little bit of practice you can do these kinds of calculations mentally without having to write the intermediate steps.

Let us consider another example where the number of digits is larger:

Suppose you were trying to find the product of 43578 × 6921

Step 1: Finding the units digit

Units digit: 1 × 8 = 8

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times\ 6\ 9\ 2\ 1 \\ \hline 8 \end{array}$$

Step 2: Finding the tens digit

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times\ 6\ 9\ 2\ 1 \\ \hline 3\ 8 \end{array}$$

Carry over 2

Thought Process:

Tens digit would come by multiplying tens with units and units with tens

7 × 1 + 2 × 8 = 7 + 16 = 23

In order to think about this, we can think of the first pair - by thinking about which number would multiply 1 (units digit of the second number) to make it into tens.

Once, you have spotted the first pair the next pair would get spotted by moving right on the upper number (43578) and moving left on the lower number (6921)

Step 3: Finding the hundreds digit

Let us look at the broken down thought process for this step:

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times\ 6\ 9\ 2\ 1 \\ \hline 3\ 8 \end{array}$$

Carry over 2

Thought Process:

Locate the first pair that would give you your hundreds digit.

For this first think of what you need to multiply the digit in the units place of the second number (digit 1) with to get the hundreds digit of the answer.

Since:

Units × Hundreds = Hundreds

We need to pair 1 with 5 in the upper number as shown in the figure.

Once you have identified 5 × 1 as the first pair of digits, to identify the next pair, move 1 to the right of the upper number and move 1 to the left of the lower number.

Thus, you should be able to get 7 × 2 as your next pair.

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times\ 6\ 9\ 2\ 1 \\ \hline 3\ 8 \end{array}$$

Carry over 2

For the last pair, you can again repeat the above thought- move to the right in the upper number and move to the left in the lower number.

Thus, the final thought for this situation would look like:

$$\begin{array}{r} 4\;3\;5\;7\;8 \\ \times\;6\;9\;2\;1 \\ \hline 3\;3\;8 \end{array}$$

9 carry over to thousands place

Thought Process:

First pair: 5 × 1

Second pair: 7 × 2 (move right on upper number and move left on the lower number)

Third pair: 8 × 9 (move right on upper number and move left on the lower number)

Thus, 5 × 1 + 7 × 2 + 8 × 9 = 91

91 + 2 (carry over) = 93

Hundreds place digit would be 3 and carry over to the thousands place would be 9

Step 4: Finding the thousands digit

We would follow the same process as above. For doing the same first identify the first pair as 3 × 1 (thousands from the first number multiplied by units from the second number) and then start moving right and moving left on both the numbers to find the other pairs.

$$\begin{array}{r} 4\;3\;5\;7\;8 \\ \times\;6\;9\;2\;1 \\ \hline 3\;3\;1\;8 \end{array}$$

13 carry over to
ten thousands place

Thought Process:

3 × 1 + 5 × 2 + 7 × 9 + 8 × 6 = 124

thousands × units + hundreds × tens + tens × hundreds + units × thousands

124 + 9 (from the carry over) = 133

put down 3 as the thousands place digit and carry over 13 to the ten thousands place

Step 5: Finding the ten thousands digit

4 × 1 would be the first pair here followed by 3 × 2; 5 × 9 and 7 × 6 as shown below:

$$\begin{array}{r} 4\;3\;5\;7\;8 \\ \times\;6\;9\;2\;1 \\ \hline 0\;3\;3\;3\;8 \end{array}$$

11 carry over to
the lacs place

Thought Process:

4 × 1 + 3 × 2 + 5 × 9 + 7 × 6 = 97

97 + 13 (from the carry over of the previous step) = 110.

Hence, 0 becomes the digit which would come into the answer and 11 would be carried over

Step 6: Finding the digit in the lakhs' place

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times\ 6\ 9\ 2\ 1 \\ \hline 6\ 0\ 3\ 3\ 3\ 8 \end{array}$$

7 carry over to
the ten lakhs place

Thought Process:

Since we have already used up 4 × 1 (the left most digit in the first number multiplied with the right most digit in the second number, it is no longer possible to use the digit 1 (units digit of the second number) to form a pair.

Use this as a signal to fix the digit 4 from the first number and start to write down the pairs by pairing this digit 4 (ten thousands' digit of the first number) with the corresponding digit of the second number to form the lacs digit.

It is evident that ten thousands × tens = lacs.

Thus, the first pair is 4 × 2.

Note that you could also think of the first pair as 4 × 2 by realising that since we have used 4 × 1 as the first pair for the previous digit we can use the number to the left of 1 to form 4 × 2.

Subsequent pairs would be 3 × 9 and 5 × 6.

Thus, 4 × 2 + 3 × 9 + 5 × 6 = 8 + 27 + 30 = 65

65 + 11 (carry over) = 76.

Thus, 6 becomes the lacs digit and we get a carry over of 7.

Step 7: Finding the ten lakh's digit

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times\ 6\ 9\ 2\ 1 \\ \hline 1\ 6\ 0\ 3\ 3\ 3\ 8 \end{array}$$

6 carry over to the next place

Thought Process:

First pair: 4 × 9

Next pair: 3 × 6

Thus, 4 × 9 + 3 × 6 = 54

54 + 7 (from the carry over) = 61

1 becomes the next digit in the answer and we carry over 6.

Step 8: Finding the next digit

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times\ 6\ 9\ 2\ 1 \\ \hline 3\ 0\ 1\ 6\ 0\ 3\ 3\ 3\ 8 \end{array}$$

Thought Process:

4 × 6 = 24

24 + 6 (from the carry over = 30)

This is the last step because we have multiplied the left most two digits in the number.

The above process of multiplication- although it looks extremely attractive and magical – especially for larger numbers, it's actual usage in the examination context might actually be quite low. This is because there are better ways of doing multiplication of 2 to 3 digits and larger multiplications might not be required to be executed in an exam like the CAT.

However, in order to solve questions where you might be asked to find the hundreds or even the thousands' digit of a big multiplication like the one showed above, this might be your only option.

Let us look at a few more alternative approaches in order to calculate multiplication problems.

2. Using squares to multiply two numbers

In this approach the usage of the mathematical result $a^2 - b^2 = (a - b)(a + b)$ helps us to find the result of a multiplication.

For instance 18 × 22 can be done using $20^2 - 2^2 = 400 - 4 = 396$, taking $a = 20$ and $b = 2$

Similarly, $22 \times 28 = 25^2 - 3^2 = 625 - 9 = 616$

$35 \times 47 = 41^2 - 6^2 = 1681 - 36 = 1645$

In case the difference between the two numbers is not even, we can still use this process by modifying it thus:

$24 \times 33 = 24 \times 32 + 24 = 28^2 - 4^2 + 24$

$= 784 - 16 + 24 = 792$

However, obviously this process might be ineffective in the following cases:

(i) If the values of the squares required to calculate an multiplication are difficult to ascertain (For two digit numbers, we can bypass this by knowing the short cut to calculate the squares of 2 digit numbers- **You may want to look at the methods given in Chapter 4 of this part to find out the squares of 2 digit numbers in order to be more effective with these kinds of calculations.)**

(ii) When one is trying to multiply two numbers which are very far from each other, there might be other processes for multiplying them that might be better than this process. For instance, if you are trying to multiply 24 × 92 trying to do it as $58^2 - 34^2$ obviously would not be a very convenient process.

(iii) Also, in case one moves into trying to multiply larger numbers, obviously this process would fail. For instance 283 × 305 would definitely not be a convenient calculation if we use this process.

3. Multiplying numbers close to 100 and 1000

A specific method exists for multiplying two numbers which are both close to 100 or 1000 or 10000.

For us, the most important would be to multiply 2 numbers which are close to 100.

The following example will detail this process for you:

Let us say you are trying to multiply 94 × 96.

Step 1: Calculate the difference from 100 for both numbers and write them down (or visualize them) as follows:

Difference

from 100

$$\begin{array}{rr} 94 & -6 \\ \times\ 96 & -4 \\ \hline \end{array}$$

Step 2: The answer would be calculated in two steps-

(a) The last two digits of the answer would be calculated by multiplying –6 × –4 to get 24.

$$\begin{array}{rr} 94 & -6 \\ & \times \\ \times\ 96 & -4 \\ \hline \_\_\_\_ & \_\_\ \_\_ \end{array}$$

initial digits of the multiplication | last 2 digits would be got by multiplying –6 × –4

Note here that we divide the answer into two parts:

Last 2 digits and Initial digits

(In case the numbers were close to 1000 we would divide the calculation into the last three digits and the initial digits)

When we multiply –6 × –4 we get 24 and hence we would write that as our last 2 digits in the answer.

We would then reach the following stage of the multiplication:

$$\begin{array}{r|r} 94 & -6 \\ & \times \\ 96 & -4 \\ \hline \_\_\_\_ & 24 \end{array}$$

initial digits of the multiplication

The next task is to find the initial digits of the answer:

This can be done by cross adding 94 + (–4) or 96 + (–6) to get the digits as 90 as shown in the figure below:

Difference from 100

$$\begin{array}{r|r} 94 & -6 \\ & \times \\ \times\ 96 & -4 \\ \hline 90 & 24 \end{array}$$

Thought Process:

Add along the diagonal connecting line shown in the figure to get the value of the initial digits of the answer:

Thus we would get 94 + (–4) = 90

or

96 + (–6) = 90

Thus, our initial digits of the answer are 90.

Let us take another example to illustrate a few more points which might arise in such a calculation:

Let us say, you were doing 102 × 103.

1 0 2	+2
1 0 3	+3
1 0 5	0 6
Initial digits	Last 2 digits

Thought Process:

In this case, the second part of the answer (the last two digits) turns out to be 2 × 3 = 6. In such a case, since we know that the second part of the answer has to be compulsorily in 2 digits, we would naturally need to take it as 06.

The initial digits of the answer would be got by cross addition:

102 + 3 = 103 + 2 = 105

Consider: 84 × 88

8 4	–16
8 8	–12
7 3	9 2
Initial digits	Last 2 digits

7392 is the answer

Thought Process:

In this case, the difference from 100 for the two numbers are –12 and –16 respectively.

We multiply them to get the last 2 digits of the number. However, –12 × –16 = 192 which is a 3 digit number.

Hence, retain 92 as the last 2 digits and carry over 1 to the initial digits.

Then while finding the initial digits you would need to add this carry over when you are writing the answer.

Thus, initial digits:

84 + (–12) + 1 (from the carry over) = 72 + 1 = 73

Alternately:

88 + (–16) + 1 (from the carry over) = 72 + 1 = 73

Now consider the situation where one number is above 100 and the other below 100.

For instance:

92 × 104

The following figure would show you what to do in this case:

$$
\begin{array}{rr|ll}
 & 1\ 0\ 4 & & +4 \\
 & \times\ 9\ 2 & & -8 \\
\hline
 & 9\ 6 & 0\ 0 & \\
 & - & 3\ 2 & \\
\hline
 & 9\ 5 & 6\ 8 & \\
\hline
 & \text{Initial digits} & \text{Last 2 digits} &
\end{array}
$$

Thought Process:

The problem in this calculation is that +4 × (–8) = –32 and hence cannot be directly written as the last two digits of the answer.

In this case, first leave the last 2 digits as 00 and find the initial digits of the answer.

Initial digits:

104 + (–8) = 96 = 92 + 4

When you write 96 having kept the last 2 digits of the number as 00, the meaning of the number's value would be 9600.

Now, from this subtract = 4 × (–8) = –32 to get the answer as 9600 – 32 = 9568.

For a multiplication like 994 × 996 the only adjustment you would need to do would be to look at the second part of the answer as a 3- digit number:

The following would make the process clear to you for such cases:

$$
\begin{array}{r|l}
9\ 9\ 4 & \quad -6 \\
\times\ 9\ 9\ 6 & \quad -4 \\
\hline
9\ 9\ 0 & 0\ 2\ 4 \\
\hline
\text{Initial digits} & \text{Last 3 digits}
\end{array}
$$

Thought Process:

Last 3 digits = –6 × –4 = 24 Æ Hence, we write it as 024

Initial digits: Cross addition of 994 + (–4) = 990

Alternately, 996 + (–6) = 990

Thus, the answer would be 990024.

Note:

(i) The above process for multiplication is extremely good in cases when the two numbers are close to any power of 10 (like 100,1000,10000 etc)

However, when the numbers are far away from a power of 10, the process becomes infeasible.

Thus, this process would not be effective at all in the case of 62 × 34.

(ii) For finding squares of numbers between 80 to 120, this process is extremely good and hence you should use this whenever you are faced with the task of finding the square of a number in this range.

For instance, if you are multiplying 91 × 91 you can easily see the answer as 8281.

4. Using additions to multiply

Consider the following view of an option for multiplying

Let us say we are trying to multiply 83 × 32

This can be converted most conveniently into 80 × 30 + 3 × 30 + 2 × 83 = 2400 + 90 + 166 = 2656

This could also have been done as: 83 × 30 + 2 × 83 = 2490 + 166 = 2656

However in the case of 77 × 48 the second conversion shown above might not be so easy to execute- while the first one would be much easier:

70 × 40 + 7 × 40 + 8 × 77 = 70 × 40 + 7 × 40 + 8 × 70 + 8 × 7 = 2800 + 280 + 560 + 56 = 3696.

The advantage of this type of conversion is that at no point of time in the above calculation are you doing anything more than single digit × single digit multiplication.

5. Use of percentages to multiply

Another option that you have can be explained as below:

Let us say you are trying to find 43 × 78.

In order to calculate 43 × 78 first calculate 43% of 78 as follows:

43% of 78 = 10% of 78 + 10% of 78 +10% of 78 +10% of 78 + 1% of 78 + 1% of 78 + 1% of 78 = 7.8 + 7.8 + 7.8 + 7.8 + 0.78 + 0.78 + 0.78 = 33.34

This can be done using: 7 × 4 =28 as the integer part.

For adding the decimals, consider all the decimals as two digit numbers. In the addition if your total is a 2 digit number, write that down in the decimals place of the answer. If the number is a 3 digit number, carry over the hundreds' digit to the integer part of the answer.

Thus, in this case you would get:

80 + 80 + 80 + 80 + 78 + 78 + 78 = 554. This 554 actually means 5.54 in the context that we have written down 0.80 as 80.

Thus, the total is 33.54.

We have found that 43% of 78 is 33.54 and our entire addition has been done in single and 2 digits. We of course realize that 43% of 78 being the same as 0.43 × 78 the digits for 43 × 78 would be the same as the digits for what we have calculated.

Now, the only thing that remains is to put the decimals back where they belong.

There are many ways to think about this- perhaps the easiest being that 43 × 78 should have 4 as it's units digit and hence the correct answer is 3354.

Of course, this could also have been done by calculating 78% of 43 as 21.5 + 10.75 + 0.43 + 0.43 + 0.43 = 31 + 2.54 = 33.54 Æ Hence, the answer is 3354.

You can even handle 2 digit × 3 digit multiplication through the same process:

Suppose you were multiplying 324 × 82, instead of doing the multiplication as given, find 324 % of 82.

The question converts to: 3.24 × 82 = 82 × 3 + 8.2 + 8.2 + 0.82 + 0.82 + 0.82 + 0.82 = 246 + 16 + 3.68 = 265.68.

Hence, the answer is 26568.

We would encourage you to try to multiply 2 digits × 2 digits and 2 digits × 3 digits and 3 digits × 3 digits by the methods you find most suitable amongst those given above.

In my view, the use of percentages to multiply is the most powerful tool for carrying out the kinds of multiplications you would come across in Aptitude exams. Once you can master how to think about the decimals digits in these calculations, it has the potential to give you a significant time saving in your examination.

Obviously, when you convert a multiplication into an addition using any of the two processes given above, the speed and efficiency of your calculation would depend largely on your ability to add well. If your 2 digit additions are good (or if you have made your additions of 2 digit numbers good by using the process given in the chapter on additions) you would find the addition processes given here the best.

The simple reason is because this process has the advantage of being the most versatile- in the sense that it is not dependant on particular types of numbers.

Besides, after enough practice you would be able to do 2 digits × 2 digits and 2 digits × 3 digits and 3 digits × 3 digits orally.