01

Algebraic Identities

1. Formulae for two index, two term.

1.1. $(a+b)^2 = a^2 + 2ab + b^2$
$= (a-b)^2 + 4ab$

1.2. $(a-b)^2 = a^2 - 2ab + b^2$
$= (a+b)^2 - 4ab$

1.3. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

1.4. $(a+b)^2 - (a-b)^2 = 4ab$

2. Formulae for three index, two term.

2.1. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$= a^3 + b^3 + 3ab(a+b)$

2.2. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$= a^3 - b^3 - 3ab(a-b)$

2.3. $(a+b)^3 + (a-b)^3 = 2(a^3 + 3ab^2) = 2a(a^2 + 3b^2)$

2.4. $(a+b)^3 - (a-b)^3 = 3a^2b + 2b^3 = 2b(3a^2 + b^2)$

3. Formulae for four and five index, two term.

3.1. $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

3.2. $(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

3.3. $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

3.4. $(a-b)^5 = a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$

4. Ordinary factors.

4.1. $a^2 - b^2 = (a-b)(a+b)$

4.2. $a^3 + b^3 = (a+b)(a^2 - ab + b^2) = (a+b)^3 - 3ab(a+b)$

4.3. $a^3 - b^3 = (a-b)(a^2 + ab + b^2) = (a-b)^3 + 3ab(a-b)$

4.4 .$a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a-b)(a+b)(a^2 + b^2)$
$= (a-b)(a^3 + a^2b + ab^2 + b^3)$

4.5. $x^2 + (a+b)x + ab = (x+a)(x+b)$

4.6. $x^2 - (a+b)x + ab = (x-a)(x-b)$

5. Factor for $a^n - b^n$.

5.1. $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots\ldots + ab^{n-2} + b^{n-1})$

How to Remember : *First factor of RHS is $(a-b)$. In the second factor, power of a in first term is $n-1$. In consecutive terms power of a is decreasing by 1 where as power of b is increasing by 1. Last term does not contain a and power of b (in last term) is $n-1$.*

If $n = 2$ then, $a^2 - b^2 = (a-b)(a+b)$

$n = 3$ then, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$n = 4$ then, $a^4 - b^4 = (a-b)(a^3 + a^2b + ab^2 + b^3)$

$n = 5$ then, $a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$ etc.

5.2. If $a = 1, b = x$ then,

$$(1 - x^n) = (1-x)(1 + x + x^2 + x^3 + \ldots\ldots + x^{n-1})$$

at, $n = 2$ $\quad 1 - x^2 = (1-x)(1+x)$

at, $n = 3$ $\quad 1 - x^3 = (1-x)(1+x+x^2)$

at, $n = 4$ $\quad 1 - x^4 = (1-x)(1+x+x^2+x^3)$

at, $n = 5$ $\quad 1 - x^5 = (1-x)(1+x+x^2+x^4)$ etc.

6. Special factors :

6.1. $a^4 + a^2b^2 + b^4 = (a^2+b^2)^2 - a^2b^2$

$= (a^2 - ab + b^2)(a^2 + ab + b^2)$

6.2. $(a+b+c)(bc+ca+ab) - abc = (b+c)(c+a)(a+b)$

6.3. $(a+b+c)^3 - a^3 - b^3 - c^3 = 3(b+c)(c+a)(a+b)$

7. 7.1. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

7.2. If $a + b + c = 0$, then, $a^3 + b^3 + c^3 = 3abc$

7.3. $\because \quad a^2+b^2+c^2-ab-bc-ca = \frac{1}{2}\{(a-b)^2+(b-c)^2+(c-a)^2\}$

$\therefore \quad a^3+b^3+c^3-3abc = \frac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}$

But, if $\{(a-b)^2+(b-c)^2+(c-a)^2\} = 0$

then, $a-b=0, b-c=0$, and $c-a=0$

or, $a=b, \ b=c, \ c=a$

or, $a=b=c$

Hence, $a^3+b^3+c^3-3abc=0$

$\Rightarrow a+b+c=0$ or, $a=b=c$

7.4. $(a-b)^2+(b-c)^2+(c-a)^2 \geq 0$

$\Rightarrow \quad 2(a^2+b^2+c^2-ab-bc-ca) \geq 0$

$\Rightarrow \quad a^2+b^2+c^2 \geq ab+bc+ca$

7.5. $(a + b + c)^2 \geq 0$

$\Rightarrow \quad a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 0$

$\Rightarrow \quad ab + bc + ca \geq \frac{-1}{2}(a^2 + b^2 + c^2)$

8. Cyclic factor

8.1. $a^2(b - c) + b^2(c - a) + c^2(a - b) = -(a - b)(b - c)(c - a)$

8.2. $bc(b - c) + ca(c - a) + ab(a - b) = -(a - b)(b - c)(c - a)$

8.3. $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) = (a - b)(b - c)(c - a)$

9. Formulae for two index, three and four terms.

9.1. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

9.2. $(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd)$

Note : $(a - b - c)^2 = a^2 + b^2 + c^2 + 2a(-b) + 2a(-c) + 2(-b)(-c)$

$= a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$ etc.

9.3. $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(b + c)(c + a)(a + b)$

10. Special formula :

10.1. If $x + \frac{1}{x} = a$ then $x^2 + \frac{1}{x^2} = a^2 - 2$

10.2. If $x + \frac{1}{x} = a$ then $x^3 + \frac{1}{x^3} = a^3 - 3a$

10.3. If $x + \frac{1}{x} = a$ then $x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$

10.4. If $x + \frac{1}{x} = a$ then $x^5 + \frac{1}{x^5} = a^5 - 5a^3 + 5a$

10.5. If $x + \frac{1}{x} = a$ then $x^6 + \frac{1}{x^6} = a^6 - 6a^4 + 9a^2 - 2$

11. 11.1. If $x - \frac{1}{x} = a$ then $x^2 + \frac{1}{x^2} = a^2 + 2$

11.2. If $x - \frac{1}{x} = a$ then $x^3 - \frac{1}{x^3} = a^3 + 3a$

11.3. If $x - \frac{1}{x} = a$ then $x^4 + \frac{1}{x^4} = a^4 + 4a^2 + 2$

11.4. If $x - \frac{1}{x} = a$ then $x^5 - \frac{1}{x^5} = a^5 + 5a^3 + 5a$

11.5. If $x - \frac{1}{x} = a$ then $x^6 + \frac{1}{x^6} = a^6 + 6a^4 + 9a^2 + 2$

Proof of some of the above identities are given in solved examples.

12. 12.1. If $x^2 + \frac{1}{x^2} = 1$ then $x^6 = -1$

12.2. If $x^2 + \frac{1}{x^2} = -1$ then $x^6 = 1$

Explanation : $x^2 + \frac{1}{x^2} = 1$

$\Rightarrow \quad x^4 - x^2 + 1 = 0]$

Multiplying both sides by $(x^2 + 1)$, $(x^2 + 1)(x^4 - x^2 + 1) = 0$

or, $\left(x^2\right)^3 + 1 = 0$

$\therefore \quad x^6 = -1$ etc.

Note : *Here x is an imaginary (complex) Number.*

13. 13.1. Componendo–dividendo

If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ or, $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

13.2. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$ *(by componendo)*

13.3. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$ *(by dividendo)*

14. Ratio–proportion

14.1. $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = \frac{a-c}{b-d} = \frac{\sqrt{ac}}{\sqrt{bd}} = \frac{\sqrt{a^2+c^2}}{\sqrt{b^2+d^2}}$

14.2. $\frac{a}{b} = \frac{c}{d} = \frac{ka+mb}{kb+md} = \frac{ka-mb}{kb-md}$

14.3. $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f} = \frac{a+c-e}{b+d-f} = \frac{\sqrt[3]{ace}}{\sqrt[3]{bdf}} = \frac{\sqrt{a^2+c^2+e^2}}{\sqrt{b^2+d^2+f^2}}$ etc.

15. Remainder Theorem and Factor Theorem :

15.1. Remainder theorem

When a polynomial $p(x)$ of one or more than one degree divided by $x - a$, the remainder is $p(a)$.

e.g., suppose $p(x) = 2x^3 + 3x^2 - x - 1$ is a polynomial.

When it is divided by $x - 3$,

Remainder $p(3) = 2 \cdot 3^3 + 3 \cdot 3^2 - 3 - 1 = 54 + 27 - 4 = 77$.

15.2. Factor theorem

When a polynomial $p(x)$ of one or more than one degree is divided by $(x - a)$ and remainder is zero. Then '$x - a$' is a factor of $p(x)$. e.g.,

In, $p(x) = x^3 - x^2 + x - 1$, $p(1) = 1 - 1 + 1 - 1 = 0$

Hence, $(x - 1)$ is a factor of $x^3 - x^2 + x - 1$.

16. Special cases for Remainder and Factor theorem.

16.1. When a polynomial $p(x)$ of one or more than one degree is divided by $(ax + b)$, the remainder is $p\left(\frac{-b}{a}\right)$.

16.2. When a polynomial $p(x)$ of one or more than one degree is divided by $(ax + b)$ and the remainder is zero then $ax + b$ is a factor of $p(x)$.

16.3. Suppose a polynomial $p(x)$ is divided by $x - a$ and $x - b$ and remainder are respectively m and n. When the polynomial is divided by $(x - a)(x - b)$, the remainder will be of the form $Ax + B$, where $p(a) = Aa + B = m$ and $p(b) = Ab + B = n$. Solve these two equations to get values of A and B. *(See solved example 22)*

Solved Example

1. If $a^2 + ab + b^2 = b^2 + bc + c^2$ where $a \neq b \neq c$ then find the value of $a + b + c$.

Solution : $a^2 + ab + b^2 = b^2 + bc + c^2$

$\Rightarrow \quad a^2 + ab = bc + c^2$

$\Rightarrow \quad a^2 - c^2 + ab - bc = 0$

$\Rightarrow \quad (a - c)(a + c) + b(a - c) = 0$

$\Rightarrow \quad (a - c)(a + c + b) = 0$

$\because \quad a \neq c \quad \therefore \; a + c + b = 0$

2. If $a + b = z$, $c + a = y$, $b + c = x$, $x^2 + y^2 + z^2 = 50$ and $xy + yz + zx = 47$ then find the value of $a^2 + b^2 + c^2 - ab - bc - ca$.

Solution : Here $x - y = b - a$, $y - z = c - b$ and $z - x = a - c$

Now, $a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\{(a - b)^2 + (b - c)^2 + (c - a)^2\}$

$= \frac{1}{2}\{(x - y)^2 + (y - z)^2 + (z - x)^2\} \qquad (\because (a - b)^2 = (b - a)^2)$

$= \frac{1}{2}\{2(x^2 + y^2 + z^2 - xy - yz - zx)\}$

$= x^2 + y^2 + z^2 - (xy + yz + zx) = 50 - 47 = 3$

3. If $ab + bc + ca = 0$ then prove that $a^2b^2 + b^2c^2 + c^2a^2 = -2abc(a + b + c)$

Solution : $(ab + bc + ca)^2 = (ab)^2 + (bc)^2 + (ca)^2 + 2(abbc + abca + bcca)$

or, $0 = a^2b^2 + b^2c^2 + c^2a^2 + 2abc(b + a + c)$

$\therefore \quad a^2b^2 + b^2c^2 + c^2a^2 = -2abc(b + a + c)$

4. If $x^3 - \frac{1}{x^3} = k^3 + 3k$ then $x - \frac{1}{x} = k$ Prove it.

Solution : $\left(x - \frac{1}{x}\right)^3 = x^3 - 3x + 3\frac{1}{x} - \frac{1}{x^3}$

$\Rightarrow \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$

$\Rightarrow \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right) = x^3 - \frac{1}{x^3}$ ... (i)

But it is given that, $k^3 + 3k = x^3 - \frac{1}{x^3}$. ... (ii)

Comparing (1) & (2), $x - \frac{1}{x} = k$

5. If $x + \frac{1}{x} = 4$ then find the value of $x^3 + \frac{1}{x^3}$.

Solution : $\left(x + \frac{1}{x}\right)^3 = x^3 + 3x + 3\,\frac{1}{x} + \frac{1}{x^3}$

$\Rightarrow 4^3 = x^3 + 3\left(x + \frac{1}{x}\right) + \frac{1}{x^3}$

$\Rightarrow 64 = x^3 + 3 \times 4 + \frac{1}{x^3}$

$\therefore x^3 + \frac{1}{x^3} = 64 - 12 = 52$

6. If $p^3 - \frac{1}{p^3} = 140$ then find the value of $p - \frac{1}{p}$

Solution : $\left(p - \frac{1}{p}\right)^3 = p^3 - 3p + \frac{3}{p} - \frac{1}{p^3}$

$\Rightarrow \left(p - \frac{1}{p}\right)^3 = p^3 - \frac{1}{p^3} - 3\left(p - \frac{1}{p}\right)$

$\Rightarrow \left(p - \frac{1}{p}\right)^3 + 3\left(p - \frac{1}{p}\right) = 140 = 125 + 15$ ... (i)

$\therefore \left(p - \frac{1}{p}\right)^3 + 3\left(p - \frac{1}{p}\right) = 5^3 + 3 \times 5$

Comparing $p - \frac{1}{p} = 5$

Second Method : Let $p - \frac{1}{p} = t$

$\therefore$ from (i) of above method,

$\Rightarrow t^3 + 3t - 140 = 0$

$\Rightarrow \quad t^3 - 125 + 3t - 15 = 0$

$\Rightarrow \quad (t-5)(t^2+5t+25) + 3(t-5) = 0$

$\Rightarrow \quad (t-5)(t^2+5t+25+3) = 0$

$\Rightarrow \quad t = 5,\ t^2 + 5t + 28 = 0$

$\Rightarrow \quad t = 5,\ t = \dfrac{-5 \pm \sqrt{25-112}}{2}$, which is imaginary.

7. If $x + \frac{1}{x} = a$ then $x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$ prove it.

Solution : $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow \quad a^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = a^2 - 2$

Squaring both sides, $x^4 + \frac{1}{x^4} + 2 = a^4 - 4a^2 + 4$

$\therefore \quad x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$

8. If $t + \frac{1}{t} = c$ then $t^5 + \frac{1}{t^5} = c^5 - 5c^3 + 5c$ prove it. *[SSC Tier-I 2014]*

Solution : $t + \frac{1}{t} = c \Rightarrow t^2 + \frac{1}{t^2} + 2 = c^2 \Rightarrow t^2 + \frac{1}{t^2} = c^2 - 2$

and $t + \frac{1}{t} = c$

$\Rightarrow \quad t^3 + 3t + \frac{3}{t} + \frac{1}{t^3} = c^3$

$\Rightarrow \quad t^3 + \frac{1}{t^3} + 3\left(t + \frac{1}{t}\right) = c^3$

$\Rightarrow \quad t^3 + \frac{1}{t^3} + 3c = c^3$

$\Rightarrow \quad t^3 + \frac{1}{t^3} = c^3 - 3c$

Now, $\left(t^2 + \frac{1}{t^2}\right)\left(t^3 + \frac{1}{t^3}\right) = t^5 + \frac{1}{t} + t + \frac{1}{t^5}$

$\Rightarrow \quad (c^2 - 2)(c^3 - 3c) = t^5 + c + \frac{1}{t^5}$

$\Rightarrow \quad c^5 - 3c^3 - 2c^3 + 6c = t^5 + \frac{1}{t^5} + c$

$\therefore \quad t^5 + \frac{1}{t^5} = c^5 - 5c^3 + 5c$

9. If $a^4 + \frac{1}{a^4} = 47$ then find the value of $a + \frac{1}{a}$.

Solution : We know that $a + \frac{1}{a} = t$

(As in question no. 7)

$\Rightarrow \quad a^4 + \frac{1}{a^4} = t^4 - 4t^2 + 2$

$t^4 - 4t^2 + 2 = 47$

$t^4 - 4t^2 - 45 = 0$

$(t^2 - 9)(t^2 + 5) = 0$

$\Rightarrow \quad t^2 = 9$

$\Rightarrow \quad t = \pm 3$

10. If $x - \frac{1}{x} = a$ then prove that $x^4 + \frac{1}{x^4} = a^4 + 4a^2 + 2$.

Solution : $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

$\Rightarrow \quad a^2 + 2 = x^2 + \frac{1}{x^2}$

Squaring, $a^4 + 4a^2 + 4 = x^4 + \frac{1}{x^4} + 2$

$\therefore \quad x^4 + \frac{1}{x^4} = a^4 + 4a^2 + 2$

11. If $x^4 + \frac{1}{x^4} = 14$ then find the value of $x - \frac{1}{x}$.

Solution : We know that if $x - \frac{1}{x} = a$ then $x^4 + \frac{1}{x^4} = a^4 + 4a^2 + 2$

According to question if $x - \frac{1}{x} = a$ then $a^4 + 4a^2 + 2 = 14$

$\Rightarrow \quad a^4 + 4a^2 - 12 = 0$

$\Rightarrow \quad (a^2 + 6)(a^2 - 2) = 0$

$\Rightarrow \quad a^2 = 2$

$\therefore \quad a = \pm \sqrt{2}$

12. If $x + \frac{1}{x} = a$ then prove that $x^6 + \frac{1}{x^6} = a^6 - 6a^4 + 9a^2 - 2$. [SSC Tier-I 2014]

Solution : $x^6 + \frac{1}{x^6} = (x^2)^3 + \left(\frac{1}{x^2}\right)^3$

$= \left(x^2 + \frac{1}{x^2}\right)\left(x^4 - x^2 \cdot \frac{1}{x^2} + \frac{1}{x^4}\right)$

$$= \left\{\left(x+\frac{1}{x}\right)^2 - 2\right\}\left\{\left(x^2+\frac{1}{x^2}\right)^2 - 2 - 1\right\}$$

$$= \left\{\left(x+\frac{1}{x}\right)^2 - 2\right\}\left\{\left(\left(x+\frac{1}{x}\right)^2 - 2\right)^2 - 3\right\}$$

$$= \left\{\left(a^2-2\right)\right\}\left\{\left(a^2-2\right)^2 - 3\right\} = (a^2-2)(a^4-4a^2+1)$$

$$= a^6 - 4a^4 + a^2 - 2a^4 + 8a^2 - 2 = a^6 - 6a^4 + 9a^2 - 2$$

13. If $x = b - c + a$, $y = c - a + b$, $z = a - b + c$, then prove that

$(b-a)x + (c-b)y + (a-c)z = 0$

Solution : 1[st] term $= (b-a)x = (b-a)(b-c+a) = (b-a)\{(b+a)-c\}$

$= (b-a)(b+a) - (b-a)c = b^2 - a^2 - bc + ac$ ... (i)

2[nd] term $= (c-b)y = (c-b)(c-a+b) = (c-b)\{(c+b)-a\}$

$= (c-b)(c+b) - (c-b)a = c^2 - b^2 - ca + ab$ ... (ii)

3[rd] term $= (a-c)z = (a-c)(a-b+c) = (a-c)\{(a+c)-b\}$

$= (a-c)(a+c) - (a-c)b = a^2 - c^2 - ab + bc$ ... (iii)

$\therefore$ from (i), (ii) and (iii), $(b-a)x + (c-b)y + (a-c)z$

$= b^2 - a^2 + c^2 - b^2 + a^2 - c^2 - bc + ac - ca + ab - ab + bc = 0$

14. If $2s = a + b + c$,

then prove that, $(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2 = a^2 + b^2 + c^2$.

Solution : LHS$= (s-a)^2 + (s-b)^2 + (s-c)^2 + s^2$

$= (s^2 - 2as + a^2) + (s^2 - 2bs + b^2) + (s^2 - 2cs + c^2) + s^2$

$= 4s^2 - 2s(a+b+c) + a^2 + b^2 + c^2$

$= 4s^2 - 2s \times 2s + a^2 + b^2 + c^2$ $(\because a + b + c = 2s)$

$= 4s^2 - 4s^2 + a^2 + b^2 + c^2 = a^2 + b^2 + c^2$

15. Prove that $2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4$

$= (a+b+c)(b+c-a)(c+a-b)(a+b-c)$

Solution : Given expression $= 4b^2c^2 - (a^4 + b^4 + c^4 + 2b^2c^2 - 2c^2a^2 - 2a^2b^2)$

$= (2bc)^2 - (a^2 - b^2 - c^2)^2$

$= \{2bc + (a^2 - b^2 - c^2)\}\{2bc - (a^2 - b^2 - c^2)\}$

$(\because x^2 - y^2 = (x+y)(x-y)$

$= \{a^2 - (b^2 - 2bc + c^2)\}\{(b^2 + 2bc + c^2) - a^2\}$

$= \{a^2 - (b-c)^2\}\{(b+c)^2 - a^2\}$

$= \{a + (b - c)\}\{a - (b - c)\}\{(b + c) + a\}\{(b + c) - a\}$

$= (a + b - c)(a - b + c)(b + c + a)(b + c - a)$

$= (a + b + c)(b + c - a)(c + a - b)(a + b - c)$

16. Prove that $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(b + c)(c + a)(a + b)$

Solution : $(a + b + c)^3 = \{(a + b) + c\}^3$

$= (a + b)^3 + c^3 + 3(a + b)c\{(a + b) + c\}$

$\because \quad (x + y)^3 = x^3 + y^3 + 3xy(x + y)$

Here, $x = a + b, y = c)$

$= \{a^3 + b^3 + 3ab(a + b)\} + c^3 + 3(a + b)c(a + b + c)$

$= a^3 + b^3 + c^3 + \{3ab(a + b) + 3(a + b)c(a + b + c)\}$

$= a^3 + b^3 + c^3 + 3(a + b)\{ab + c(a + b + c)\}$

$= a^3 + b^3 + c^3 + 3(a + b)\{c^2 + c(a + b) + ab\}$

$= a^3 + b^3 + c^3 + 3(a + b)(c + b)(c + a)$

$= a^3 + b^3 + c^3 + 3(b + c)(c + a)(a + b)$

17. If $a + b + c = 0$ then prove the following.

(i) $a^2 + b^2 + c^2 = -2(bc + ca + ab)$

(ii) $a^3 + b^3 + c^3 = 3abc$

(iii) $(bc + ca + ab)^2 = b^2c^2 + c^2a^2 + a^2b^2 = \frac{1}{4}(a^2 + b^2 + c^2)^2$

(iv) $a^4 + b^4 + c^4 = 2(b^2c^2 + c^2a^2 + a^2b^2) = \frac{1}{2}(a^2 + b^2 + c^2)^2$

(v) $a^5 + b^5 + c^5 = -5abc(bc + ca + ab)$

$= \frac{5}{2}abc(a^2 + b^2 + c^2)$

$= \frac{5}{6}(a^2 + b^2 + c^2)(a^3 + b^3 + c^3)$

Solution : (i) We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2bc + 2ca + 2ab$

$\therefore \quad 0^2 = a^2 + b^2 + c^2 + 2(bc + ca + ab)$

or, $a^2 + b^2 + c^2 = -2(bc + ca + ab)$

(ii) We know that $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - bc - ca - ab)$

$= 0 \times (a^2 + b^2 + c^2 - bc - ca - ab) = 0$

or, $a^3 + b^3 + c^3 = 3abc$

(iii) $(bc + ca + ab)^2 = b^2c^2 + c^2a^2 + a^2b^2 = \frac{1}{4}(a^2 + b^2 + c^2)^2$

$\because \quad (bc + ca + ab)^2 = b^2c^2 + c^2a^2 + a^2b^2 + 2abc(a + b + c)$

$\therefore \quad (bc + ca + ab)^2 = b^2c^2 + c^2a^2 + a^2b^2 + 2abc \times 0 = b^2c^2 + c^2a^2 + a^2b^2$

But from (i), $bc + ca + ab = -\frac{1}{2}(a^2 + b^2 + c^2)$

$\therefore \quad (bc + ca + ab)^2 = \frac{1}{4}(a^2 + b^2 + c^2)^2$

Hence, $(bc + ca + ab)^2 = b^2c^2 + c^2a^2 + a^2b^2 = \frac{1}{4}(a^2 + b^2 + c^2)^2$

(iv) We know from question no. (15) that,

$2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4$

$= (a + b + c)(b + c - a)(c + a - b)(a + b - c)$

$= 0 \times (b + c - a)(c + a - b)(a + b - c) = 0$

$\therefore \quad a^4 + b^4 + c^4 = 2b^2c^2 + 2c^2a^2 + 2a^2b^2 = 2(b^2c^2 + c^2a^2 + a^2b^2)$

$= \frac{1}{2}(a^2 + b^2 + c^2)^2$ (from (iii))

(v) $\because \quad a + b + c = 0 \Rightarrow a + b = -c$

$\therefore \quad (a + b)^5 = (-c)^5$

or, $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 = -c^5$

or, $a^5 + b^5 + c^5 = -5a^4b - 10a^3b^2 - 10a^2b^3 - 5ab^4$

$= -5ab(a^3 + 2a^2b + 2ab^2 + b^3)$

$= -5ab(a + b)(a^2 + ab + b^2)$

$= -5ab(-c)\{(a + b)^2 - ab\} = 5abc\{(a + b)(-c) - ab\}$

$= 5abc(-ac - bc - ab) = -5abc(bc + ca + ab)$

$= \frac{5abc}{2}(a^2 + b^2 + c^2) = \frac{5}{6}(a^2 + b^2 + c^2) \cdot 3abc$

$= \frac{5}{6}(a^2 + b^2 + c^2)(a^3 + b^3 + c^3)$ $[\because a^3 + b^3 + c^3 = 3abc]$

18. If $x^2 + y^2 + z^2 = a$,

then prove that value of $xy + yz + zx$ lies between $\frac{-a}{2}$ and a.

Solution : From, $(x + y + z)^2 \geq 0$

$x^2 + y^2 + z^2 + 2(xy + yz + zx) \geq 0$

or, $a + 2(xy + yz + zx) \geq 0$

or, $xy + yz + zx \geq \frac{-a}{2}$

or, $\frac{-a}{2} \leq xy + yz + zx$ ... (i)

Again, $(x - y)^2 + (y - z)^2 + (z - x)^2 \geq 0$

$x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + z^2 + x^2 - 2zx \geq 0$

or, $2(x^2 + y^2 + z^2 - (xy + yz + zx)) \geq 0$

or, $x^2+y^2+z^2 \geq xy+yz+zx$

or, $a \geq xy+yz+zx$

or, $xy+yz+zx \leq a$

from (i) & (ii), $\frac{-a}{2} \leq xy+yz+zx \leq a$ ... (ii)

19. If $\frac{(m+n)x-(a-b)}{(m-n)x-(a+b)}=\frac{(m+n)x+a+c}{(m-n)x+a-c}$ then find the value of x.

Solution : We know that,

$$\frac{A}{B}=\frac{C}{D} \Leftrightarrow \frac{A+B}{A-B}=\frac{C+D}{C-D}$$ *(by componendo and dividendo)*

Given relation is,

$$\frac{mx+nx-a+b}{mx-nx-a-b}=\frac{mx+nx+a+c}{mx-nx+a-c}$$

or, $\frac{(mx-a)+(nx+b)}{(mx-a)-(nx+b)}=\frac{(mx+a)+(nx+c)}{(mx+a)-(nx+c)}$

or, $\frac{mx-a}{nx+b}=\frac{mx+a}{nx+c}$ *(by componendo and dividendo)*

or, $(mx-a)(nx+c)=(mx+a)(nx+b)$

or, $mnx^2+cmx-anx-ac=mnx^2+mbx+anx+ab$

or, $cmx-anx-mbx-anx=ab+ac$

or, $x(cm-2an-mb)=a(b+c)$

or, $x=\frac{a(b+c)}{cm-2an-bm}$

20. If $\frac{2y+2z-x}{a}=\frac{2z+2x-y}{b}=\frac{2x+2y-z}{c}$ then prove that

$$\frac{x}{2b+2c-a}=\frac{y}{2c+2a-b}=\frac{z}{2a+2b-c}$$

Solution : We know that $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{ka+mc+ne}{kb+md+nf}$

Multiplying first ratio by –1, second by 2, and third by 2,

Each ratio $=\frac{-(2y+2z-x)+2(2z+2x-y)+2(2x+2y-z)}{-a+2b+2c}$

$=\frac{-2y-2z+x+4z+4x-2y+4x+4y-2z}{-a+2b+2c}$

$=\frac{9x}{-a+2b+2c}$ ... (i)

Similarly, Multiplying first ratio by 2, second by – 1 and third by 2,

Each ratio $=\frac{9y}{2a-b+2c}$ *(do yourself)* ... (ii)

Again, Multiplying first ratio by 2, second by 2 and third by –1,

Each ratio $= \frac{9z}{2a+2b-c}$ ... (iii)

From (i), (ii) and (iii), $\frac{9x}{-a+2b+2c} = \frac{9y}{2a-b+2c} = \frac{9z}{2a+b-2c}$

$\therefore \quad \frac{x}{2b+2c-a} = \frac{y}{2c+2a-b} = \frac{z}{2a+2b-c}$

21. If $(x-1)$ and $(x-2)$ are two factors of $x^3 - ax^2 + 14x + b$ then find the value of a and b.

Solution : $\because$ $x-1$ is a factor of $x^3 - ax^2 + 14x + b$.

$\therefore$ putting $x = 1$,

$1 - a + 14 + b = 0$

or, $a - b = 15$ ... (i)

$\because$ $x-2$ is also a factor of $x^3 - ax^2 + 14x + b$

$\therefore$ putting $x = 2$,

$8 - 4a + 28 + b = 0$

or, $4a - b = 36$ ... (ii)

Solving (i) & (ii) $a = 7$, $b = -8$

22. If a polynomial is divided by $x-2$ the remainder is 1 and when it is divided by $x-3$ the remainder is 2. What will be the remainder when the polynomial is divided by $x^2 - 5x + 6$.

Solution : Let polynomial be $p(x)$ then by remainder theorem $p(2) = 1$ and $p(3) = 2$

$\because$ $x^2 - 5x + 6 = (x-2)(x-3)$

Let $p(x) = h(x)(x-2)(x-3) + ax + b$

$\therefore$ $p(2) = 0 + 2a + b$

or, $1 = 2a + b$ ... (i)

Again, $p(3) = 0 + 3a + b$

or, $2 = 3a + b$ ... (ii)

Subtracting (i) from (ii),

$$\begin{array}{r} 3a + b = 2 \\ \underline{2a + b = 1} \\ a = 1 \end{array}$$

From (i), $2 \times 1 + b = 1$ or, $b = -1$

Hence, Required remainder $ax + b = x - 1$.

Exercise–1A

1. If $x+y+z=0$, then $(x+y)(y+z)(z+x)$ is equal to which of the following?
   (a) $-xyz$
   (b) $x^2+y^2+z^3$
   (c) $x^3+y^3+z^3+3xyz$
   (d) xyz

2. What is the LCM of $(6x^3+60x^2+150x)$ and $(3x^4+12x^3-15x^2)$?
   (a) $6x^2(x+5)^2(x-1)$
   (b) $3x^2(x+5)^2(x-1)$
   (c) $6x^2(x+5)^2(x-1)^2$
   (d) $3x^2(x+5)(x-1)^2$

3. If HCF of (x^2+x-12) and $(2x^2-kx-9)$ is $(x-k)$, then value of k is
   (a) -3 (b) 3 (c) -4 (d) 4

4. What is the HCF of $(x^2+bx-x-b)$ and $[x^2+x(a-1)-a]$?
   (a) $x+b$ (b) $x+a$ (c) $x+1$ (d) $x-1$

5. If $(3x^3-2x^2y-13xy^2+10y^3)$ is divided by $(x-2y)$, then what is the remainder?
   (a) 0 (b) $y+5$ (c) $y+1$ (d) y^2+3

6. If (x^3+5x^2+10k) is divided by (x^2+2) then remainder is $-2x$, value of k is
   (a) -2 (b) -1 (c) 1 (d) 2

7. If $(5x^2+14x+2)^2-(4x^2-5x+7)^2$ is divided by x^2+x+1, then what is the remainder ?
   (a) -1 (b) 0 (c) 1 (d) 2

8. What are the components of $(x^{29}-x^{24}+x^{13}-1)$?
   (a) $(x-1)$ only
   (b) $(x+1)$ only
   (c) both $(x-1)$ and $(x+1)$
   (d) Neither $(x-1)$ nor $(x+1)$

9. If $x^2-4x+1=0$, then what is the value of $x^3+\frac{1}{x^3}$?
   (a) 44 (b) 48 (c) 52 (d) 64

If $x+y+z=6$ and $xy+yz+zx=11$ then what is the value of $x^3+y^3+z^3-3xyz$?
   (a) 18 (b) 36 (c) 54 (d) 66

11. If a is a rational number such that $(x-a)$ is a factor of x^3-3x^2-3x+9, then
   (a) a may be any integer
   (b) a is an integer divisible by 9
   (c) a cannot be integer
   (d) a can have three values

12. If $x^2-11x+a$ and $x^2-14x+2a$ have a common factor then what are the values of a ?
   (a) 0, 7 (b) 5, 20 (c) 0, 24 (d) 1, 3

13. For what value of k, HCF of $2x^2 + kx - 12$ and $x^2 + x - 2k - 2$ is $(x + 4)$.
(a) 5 (b) 7 (c) 10 (d) -4

14. $x(y-z)(y+z) + y(z-x)(z+x) + z(x-y)(x+y)$ equals
(a) $(x+y)(y+z)(z+x)$ (b) $(x-y)(x-z)(z-y)$
(c) $(x+y)(z-y)(x-z)$ (d) $(y-x)(z-y)(x-z)$

15. If $\left(\frac{x}{y}\right) = \left(\frac{z}{w}\right)$ then $(xy + zw)^2$ equals
(a) $(x^2+z^2)(y^2+w^2)$ (b) $x^2y^2 + z^2w^2$
(c) $x^2w^2 + y^2z^2$ (d) $(x^2+w^2)(y^2+z^2)$

16. If $\frac{1}{x+1} + \frac{2}{y+2} + \frac{1009}{z+1009} = 1$, then what is the value of $\frac{x}{x+1} + \frac{y}{y+2} + \frac{z}{z+1009}$?
(a) 0 (b) 2 (c) 3 (d) 4

17. If $a = 258$, $b = 260$ and $c = 262$ then value of $a^3 + b^3 + c^3 - 3abc$ is
(a) 9360 (b) 6240 (c) 7040 (d) 10560

18. If HCF of $x^3 - 27$ and $x^3 + 4x^2 + 12x + k$ is a quadratic polynomial then, the value of k is
(a) 27 (b) 9 (c) 3 (d) -3

19. When $x^{40} + 2$ is divided by $x^4 + 1$, then what is the remainder ?
(a) 1 (b) 2 (c) 3 (d) 4

20. If $a^x = b^y = c^z$ and $abc = 1$ then $xy + yz + zx$ is equal to which of the following ?
(a) xyz (b) $x + y + z$ (c) 0 (d) 1

21. If $a = \frac{1+x}{2-x}$ then $\frac{1}{a+1} + \frac{2a+1}{a^2-1}$ equals
(a) $\frac{(1+x)(2+x)}{2x-1}$ (b) $\frac{(1-x)(2-x)}{x-1}$ (c) $\frac{(1+x)(2-x)}{2x-1}$ (d) $\frac{(1-x)(2-x)}{2x+1}$

22. If $pq + qr + rp = 0$ then what is the value of $\frac{p^2}{p^2-qr} + \frac{q^2}{q^2-rp} + \frac{r^2}{r^2-pq}$?
(a) 0 (b) 1 (c) -1 (d) 3

23. If $x + y + z = 0$ then what is the value of

$$\frac{1}{x^2+y^2-z^2} + \frac{1}{y^2+z^2-x^2} + \frac{1}{z^2+x^2-y^2} \ ?$$

(a) $\frac{1}{x^2+y^2+z^2}$ (b) 1 (c) -1 (d) 0

24. $\frac{(x-y)^3 + (y-z)^3 + (z-x)^3}{4(x-y)(y-z)(z-x)}$ equals
(a) $-\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) 0

25. If $a+b+c=6$, $a^2+b^2+c^2=14$ and $a^3+b^3+c^3=36$ then value of abc is equal to

(a) 0 (b) 4 (c) 1 (d) 6

26. If $x(x+y+z)=9$, $y(x+y+z)=16$ and $z(x+y+z)=144$ then what is the value of x?

(a) $\frac{9}{5}$ (b) $\frac{9}{7}$ (c) $\frac{9}{13}$ (d) $\frac{16}{3}$

27. If u, v, w are real numbers such that $u^3-8v^3-27w^3=18uvw$, which one of the following is true?

(a) $u-v+w=0$ (b) $u=-v=-w$ (c) $u-2v=3w$ (d) $u+2v=-3w$

28. If $a+b+c=0$ then what is the value of $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}$?

(a) -3 (b) 0 (c) 1 (d) 3

29. If $(x^4+x^{-4})=322$ then which one of the following is the value of $x-x^{-1}$?

(a) 18 (b) 16 (c) 8 (d) 4

30. If x varies as m^{th} power of y, y varies as n^{th} power of z and x varies as p^{th} power of z then which one of the following is true?

(a) $p=m+n$
(b) $p=m-n$
(c) $p=mn$
(d) None of these

31. If $x=(b-c)(a-d)$, $y=(c-a)(b-d)$, $z=(a-b)(c-d)$ then which one is equal to $x^3+y^3+z^3$?

(a) xyz (b) $2xyz$ (c) $3xyz$ (d) $-3xyz$

32. If $a+b+c=6$, $a^2+b^2+c^2=26$, then $ab+bc+ca$ is equal to

(a) 0 (b) 2 (c) 4 (d) 5

33. If $3x^3-2x^2y-13xy^2+10y^3$ is divided by $x-2y$, then what will be the remainder?

(a) 0 (b) x (c) $y+5$ (d) $x-3$

34. If $\left(a+\frac{1}{a}\right)^2=3$ then what is the value of

$$1+a^6+a^{12}+a^{18}+a^{84}+a^{90}+a^{200}+a^{206}?$$

(a) 1 (b) 0 (c) 8 (d) $2a^2$

35. If $x+y+z=0$, then $\frac{xyz}{(x+y)(y+z)(z+x)}$ equals $[x\neq -y, y\neq -z, z\neq -x]$

(a) -1
(b) 1
(c) $xy+yz+zx$
(d) None of these

36. If $\frac{5x-7y+10}{1}=\frac{3x+2y+1}{8}=\frac{11x+4y-10}{9}$, then $x+y$ equals

(a) 1 (b) 2 (c) 3 (d) -3

37. If $\frac{2x-3y+1}{2}=\frac{x+4y+8}{3}=\frac{4x-7y+2}{5}$, then $x+y$ is equal to which of the following ?

(a) 3 (b) 2 (c) 0 (d) -2

38. If $x=a(b-c)$, $y=b(c-a)$ and $z=c(a-b)$ then $\left(\frac{x}{a}\right)^3+\left(\frac{y}{b}\right)^3+\left(\frac{z}{c}\right)^3=?$

(a) $\frac{xyz}{3abc}$ (b) $3xyzabc$ (c) $\frac{3xyz}{abc}$ (d) $\frac{xyz}{abc}$

39. If $x^2+2=2x$, then value of $x^4-x^3+x^2+2$ is

(a) 0 (b) 1 (c) -1 (d) $\sqrt{2}$

40. If $2^x=3^y=6^{-z}$, then $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ is equal to

(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

41. If $\frac{1}{x+y}=\frac{1}{x}+\frac{1}{y}$ $(x\neq 0, y\neq 0, x\neq y)$ then value of x^3-y^3 is

(a) 0 (b) 1 (c) -1 (d) 2

42. For real a, b, c if $a^2+b^2+c^2=ab+bc+ca$ then value of $\frac{a+c}{b}$ is

(a) 1 (b) 2 (c) 3 (d) 0

43. If $x+\frac{1}{x}=5$ then $\frac{2x}{3x^2-5x+3}$ is

(a) 5 (b) $\frac{1}{5}$ (c) 3 (d) $\frac{1}{3}$

44. If $x^4+\frac{1}{x^4}=119$ and $x>1$ then what is the value of $x^3-\frac{1}{x^3}$?

(a) 54 (b) 18 (c) 72 (d) 36

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45. If $(x+y-z)^2+(y+z-x)^2+(z+x-y)^2=0$

then what is the value of $x+y+z$?

(a) $\sqrt{3}$ (b) $3\sqrt{3}$ (c) 3 (d) 0

46. If $x-y=\frac{x+y}{7}=\frac{xy}{4}$ then what is the value of xy ?

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

47. If $x+y+z=0$ then what is the value of $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}$?

(a) $(xyz)^2$ (b) $x^2+y^2+z^2$ (c) 9 (d) 3

48. If $x+y=a$ and $xy=b^2$, then the value of $x^3-x^2y-y^3$ in terms of a, b is

(a) $(a^2+4b^2)a$ (b) a^3-3b^2

(c) a^3-4b^2a (d) a^3+3b^2

49. If $(3a+1)^2+(b-1)^2+(2c-3)^2=0$, then value of $(3a+b+2c)$ is
(a) 3 (b) -1 (c) 2 (d) 5

50. If $xy(x+y)=1$, then value of $\frac{1}{x^3y^3}-x^3-y^3$ is
(a) 0 (b) 1 (c) 3 (d) -2

51. If $\frac{x}{2x^2+5x+2}=\frac{1}{6}$ then value of $\left(x+\frac{1}{x}\right)$ is
(a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) -2

52. Value of the expression $\frac{(a-b)^2}{(b-c)(c-a)}+\frac{(b-c)^2}{(a-b)(c-a)}+\frac{(c-a)^2}{(a-b)(b-c)}$ is
(a) 0 (b) 3 (c) $\frac{1}{3}$ (d) 2

53. If $3x+\frac{1}{2x}=5$ then what is the value of $8x^3+\frac{1}{27x^3}$?
(a) $118\frac{1}{2}$ (b) $30\frac{10}{27}$ (c) 0 (d) 1

54. If $(a-3)^2+(b-4)^2+(c-9)^2=0$ then what is the value of $\sqrt{a+b+c}$?
(a) -4 (b) 4 (c) ± 4 (d) ± 2

55. If average of x and $\frac{1}{x}$ $(x \neq 0)$ is M then what is the average of x^2 and $\frac{1}{x^2}$?
(a) $1-M^2$ (b) $1-2M$ (c) $2M^2-1$ (d) $2M^2+1$

56. If a, b, c are real and $a^2+b^2+c^2=2(a-b-c)-3$, then what is the value of $2a-3b+4c$?
(a) -1 (b) 0 (c) 1 (d) 2

57. If $a+b+c=0$ then what is the value of $\frac{1}{(a+b)(b+c)}+\frac{1}{(a+c)(b+a)}+\frac{1}{(c+a)(c+b)}$?
(a) 1 (b) 0 (c) -1 (d) -2

58. If $5x^2-4xy+y^2-2x+1=0$ then value of $x+y$ is
(a) 1 (b) 0 (c) -3 (d) 3

59. If $x=b+c-2a, y=c+a-2b, z=a+b-2c$ then the value of $x^2+y^2-z^2+2xy$ is
(a) 0 (b) $a+b+c$ (c) $a-b+c$ (d) $a+b-c$

60. $(y-z)^3+(z-x)^3+(x-y)^3$ is equal to ?
(a) $3(y-z)(z+x)(y-x)$
(b) $(x-y)(y+z)(x-z)$
(c) $3(y-z)(z-x)(x-y)$
(d) $(y-z)(z-x)(x-y)$

61. If $x=2-2^{1/3}+2^{2/3}$, then value of $x^3-6x^2+18x+18$ is
(a) 22 (b) 33 (c) 40 (d) 45

62. If $a^3 - b^3 - c^3 - 3abc = 0$, then

(a) $a = b = c$ (b) $a + b + c = 0$ (c) $a + c = b$ (d) $a = b + c$

63. If $x^2 - 3x + 1 = 0$ then what is the value of $x^3 + \frac{1}{x^3}$?

(a) 9 (b) 18 (c) 27 (d) 1

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64. If $2x + \frac{1}{3x} = 5$ then find the value of $\frac{5x}{6x^2 + 20x + 1}$

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{1}{7}$

65. If $m + \frac{1}{m-2} = 4$ then find the value of $(m-2)^2 + \frac{1}{(m-2)^2}$ is

(a) -2 (b) 0 (c) 2 (d) 4

66. If $a^2 = b + c$, $b^2 = c + a$ and $c^2 = a + b$ then what is the value of $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c}$?

(a) abc (b) $a^2b^2c^2$ (c) 1 (d) 0

67. If $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$ then what is the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$?

(a) 1 (b) 2 (c) 3 (d) 4

68. If $a^2 + b^2 + 2b + 4a + 5 = 0$ then what is the value of $\frac{a-b}{a+c}$?

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

69. If $\frac{x}{(b-c)(b+c-2a)} = \frac{y}{(c-a)(c+a-2b)} = \frac{z}{(a-b)(a+b-2c)}$ then the value of $x + y + z$ is

(a) $a + b + c$ (b) $a^2 + b^2 + c^2$

(c) 0 (d) $(a + b + c)^2$

70. If $\frac{x^3 + 3x}{3x^2 + 1} = \frac{189}{61}$ then value of x is

(a) 9 (b) 11 (c) 7 (d) 13

71. If $(x^2 + y^2)(p^2 + q^2) = (xp + yq)^2$ then

(a) $xy = pq$ (b) $px = yq$ (c) $xq = yp$ (d) None of these

72. If $\frac{x}{2x + y + z} = \frac{y}{x + 2y + z} = \frac{z}{x + y + 2z}$ then each terms is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) None of these

$\frac{a^2 + b^2 + c^2}{}$ is

74. If $x^2 + 8y^2 + 9z^2 - 4xy - 12xz = 0$ then,

(a) $x = y = z$
(b) $3x = 2y = z$
(c) $x = 2y = 3z$
(d) $x + 2\sqrt{2}\, y + 3z = 0$

75. If $p^2 (a^2 + b^2 + c^2) - 2p\,(ab + bc + cd) + (b^2 + c^2 + d^2) \leq 0$ then which of the following statement is true ?

(a) $p = \frac{b}{a} + \frac{c}{a} + \frac{d}{c}$
(b) $p\,(a + b + c) = 0$
(c) $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$
(d) $ab = bc = cd$

76. If $b + c = 10$, $c + a = 20$, $a + b = 30$ then value of $a^3 + b^3 + c^3$ is

(a) 9000 (b) 12000 (c) 18000 (d) 27000

77. If $b + c = 2x$, $c + a = 2y$ and $a + b = 2z$ then value of $a^3 + b^3 + c^3$ is

(a) $(x + y + z)^3$
(b) $(x + y + z)^3 + 24xyz$
(c) $(x + y + z)^3 - 24xyz$
(d) $24xyz$

78. If $a + b + c = 0$ then value of $a^3 + b^3 + c^3$ is

(a) $3a\,(a + b)\,(b + c)$
(b) $3a\,(a + b)(c + a)$
(c) $3a\,(b + c)(c + a)$
(d) $3(a + b)(b + c)(c + a)$

79. Value of $(x - y)^3 + (y - z)^3 + (z - x)^3$ is

(a) $3xyz$
(b) $3(x + y)(y + z)(z + x)$
(c) $3(x - y)(y - z)(z - x)$
(d) 0

80. If $a + b + c = p$, $abc = q$ and $ab + bc + ca = 0$ then what is the value of $a^2b^2 + b^2c^2 + c^2a^2$?

(a) $2pq$ (b) $-2pq$ (c) $3pq$ (d) $-3pq$

81. If $a = 89$, $b = -69$, $c = 8$ then the value of $9(a + b)^2 + 49c^2 - 42(a + b)c$ is

(a) 2 (b) 4 (c) 16 (d) 0

82. If $x = q + r + s$, $y = r + s - p$ and $z = p + q + r$ then the value of $x^2 + y^2 + z^2 - 2xy - 2xz + 2yz$ is

(a) p^2 (b) q^2 (c) r^2 (d) s^2

83. If $x + y + z = 10$, $x^2 + y^2 + z^2 = 60$ then value of $xy + yz + zx$ is

(a) 40 (b) 80 (c) 160 (d) 20

84. If $a^2 + b^2 = 1$, $c^2 + d^2 = 2$ then $(ac - bd)^2 + (ad + bc)^2$ is

(a) 2 (b) 0 (c) 1 (d) 4

85. $(x - a)(x - b)(a - b) + (x - b)(x - c)(b - c) + (x - c)(x - a)(c - a)$ is equal to which of the following ?

(a) $(a - b)(b - c)(c - a)$
(b) $(x - a)(x - b)(x - c)$
(c) $-(a - b)(b - c)(c - a)$
(d) $-(x - a)(x - b)(x - c)$

86. If $x + \frac{1}{x} = 2a$, $y + \frac{1}{y} = 2c$, $x - \frac{1}{x} = 2b$ and $y - \frac{1}{y} = 2d$ then the value of $xy + \frac{1}{xy}$ is

(a) $ac + bd$ (b) $ac - bd$ (c) $2(ac - bd)$ (d) $2(ac + bd)$

87. Which one of the following is not a factor of $x^8 + x^4 + 1$?

(a) $x^2 - \sqrt{x} + 1$ (b) $x^2 - x + 1$ (c) $x^4 - x^2 + 1$ (d) $x^2 - 2x + 1$

88. A factor of $a^4 - 11a^2b^2 + b^4$ is

(a) $(a^2 - b^2 - 3ab)$ (b) $a^2 + b^2 - 3ab$

(c) $(a^2 + b^2 + 3ab)$ (d) $(a^2 - b^2 + 4ab)$

89. If $a^2 + b^2 = x$, $ab = y$ then the value of $\frac{a^4 + b^4}{a^2 - ab\sqrt{2} + b^2}$ is

(a) $x + 2y$ (b) $x + \sqrt{2}y$ (c) $\sqrt{2}x + y$ (d) $2x + y$

90. If $(a + b)x = a$ and $(a + b)y = b$ then the value of $\frac{x^2 + y^2}{x^2 - y^2}$ is

(a) $\frac{a^2 - b^2}{a^2 + b^2}$ (b) $\frac{a^2}{a^2 + b^2}$ (c) $\frac{b^2}{a^2 + b^2}$ (d) $\frac{a^2 + b^2}{a^2 - b^2}$

91. If $x = \frac{p+q}{p-q}$ and $y = \frac{p-q}{p+q}$ then the value of $\frac{x-y}{x+y}$ is

(a) $\frac{p^2 + q^2}{2pq}$ (b) $\frac{2pq}{p^2 + q^2}$ (c) $\frac{2pq}{p^2 - q^2}$ (d) $\frac{2(p^2 - q^2)}{pq}$

92. If $x + \frac{a}{x} = 1$ then the value of $\frac{x^3 - x^2}{x^2 + x + a}$ in terms of a is

(a) $\frac{a}{2}$ (b) $\frac{-a}{2}$ (c) $2a$ (d) a

93. If $a = \frac{xy}{x+y}$, $b = \frac{xz}{x+z}$ and $c = \frac{yz}{y+z}$ where a, b, c are non zero then x is

(a) $\frac{2abc}{ac + bc - ab}$ (b) $\frac{2abc}{ab - ac + bc}$ (c) $\frac{2abc}{ab + bc + ac}$ (d) $\frac{2abc}{ab + ac - bc}$

94. HCF and LCM of two algebraic expressions are respectively $(a + 1)$ and $(a^3 + a^2 - a - 1)$. If one of the expression is $a^2 - 1$, then what is the second expression ?

(a) $(a + 1)$ (b) $(a - 1)^2$ (c) $(a + 1)^2$ (d) $(a + 1)(a - 1)$

95. If $\left(x^2 + \frac{1}{x^2}\right) = p$, then what is the value of $\left(x^3 + \frac{1}{x^3}\right)$?

(a) $p^{3/2}$ (b) $(p + 1)\sqrt{p+2}$ (c) $(p - 1)\sqrt{p+2}$ (d) $(p + 1)\sqrt{p-2}$

96. If $a + b + c = 0$, then what is the value of $\frac{a^2 + b^2 + c^2}{(a-b)^2 + (b-c)^2 + (c-a)^2}$?

(a) 1 (b) 3 (c) $\frac{1}{3}$ (d) 0

97. If $y = \left(x + \frac{1}{x}\right)$, then the expression $x^4 + x^3 - 4x^2 + x + 1 = 0$ can be simplified in terms of y as
(a) $y^2 + y - 2 = 0$ (b) $y^2 + y - 4 = 0$ (c) $y^2 + y - 6 = 0$ (d) $y^2 + y + 6 = 0$

98. Value of $(2 + 1)(2^2 + 1)(2^4 + 1)(2^8 + 1)(2^{16} + 1)(2^{32} + 1)(2^{64} + 1)$ is
(a) $2^{256} - 1$ (b) $2^{256} + 1$ (c) $2^{128} - 1$ (d) $2^{128} + 1$

99. HCF of polynomials $x^3 + 3x^2y + 2xy^2$ and $x^4 + 6x^3y + 8x^2y^2$ is
(a) $x(x + 2y)$
(b) $x(x + 3y)$
(c) $x + 2y$
(d) None of these

100. If $pqr = 1$, then what is value of the expression $\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$?
(a) 1 (b) -1 (c) 0 (d) $\frac{1}{3}$

101. If $x + y + z = 2s$, then $(s - x)^3 + (s - y)^3 + 3(s - x)(s - y)z$ equals
(a) z^3 (b) $-z^3$ (c) x^3 (d) y^3

102. If $x^2 = y + z, y^2 = z + x, z^2 = x + y$, then what is the value of $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$?
(a) 1 (b) 0 (c) -1 (d) 2

103. Suppose p, q, r are such that $p + q = r$ and $pqr = 30$, then what is the value of $p^3 + q^3 - r^3$?
(a) 0 (b) 90 (c) -90
(d) cannot be determined with given data

104. What is the square root of $\left(\frac{x^5 - 1}{x - 1}\right) + (x^3 + 2x^2 + x)$?
(a) $x^2 + x + 1$ (b) $x^2 - x + 1$ (c) $x^2 - x - 1$ (d) $x^2 + x - 1$

105. $\frac{x^8 + 4}{x^4 + 2x^2 + 2}$ on simplification, equals
(a) $x^4 + 2x^2 - 2$
(b) $x^4 - 2x^2 + 2$
(c) $x^4 - 2x^2 - 2$
(d) cannot be simplified

106. If $x + \left(\frac{1}{x}\right) = p$, then $x^6 + \left(\frac{1}{x^6}\right)$ equals
(a) $p^6 + 6p$
(b) $p^6 - 6p$
(c) $p^6 + 6p^4 + 9p^2 + 2$
(d) $p^6 - 6p^4 + 9p^2 - 2$ *[SSC Tier-I 2014]*

107. If $x + y + z = 0$, then $[(y - z - x)/2]^3 + [(z - x - y)/2]^3 + [(x - y - z)/2]^3$ equals
(a) $24\,xyz$ (b) $-24\,xyz$ (c) $3\,xyz$ (d) xyz

Answer-1A

1. (a)	2. (a)	3. (b)	4. (d)	5. (a)	6. (c)	7. (b)	8. (a)
9. (c)	10. (a)	11. (d)	12. (c)	13. (a)	14. (b)	15. (a)	16. (b)
17. (a)	18. (b)	19. (c)	20. (c)	21. (c)	22. (b)	23. (d)	24. (c)
25. (d)	26. (c)	27. (c)	28. (d)	29. (d)	30. (c)	31. (c)	32. (d)
33. (a)	34. (b)	35. (a)	36. (c)	37. (d)	38. (c)	39. (a)	40. (a)
41. (a)	42. (b)	43. (b)	44. (d)	45. (d)	46. (a)	47. (d)	48. (c)
49. (a)	50. (c)	51. (b)	52. (b)	53. (b)	54. (b)	55. (c)	56. (c)
57. (b)	58. (d)	59. (a)	60. (c)	61. (c)	62. (d)	63. (b)	64. (d)
65. (c)	66. (c)	67. (d)	68. (c)	69. (c)	70. (a)	71. (c)	72. (a)
73. (d)	74. (c)	75. (c)	76. (a)	77. (c)	78. (b)	79. (c)	80. (b)
81. (c)	82. (c)	83. (d)	84. (a)	85. (c)	86. (d)	87. (d)	88. (a)
89. (b)	90. (d)	91. (b)	92. (a)	93. (a)	94. (c)	95. (c)	96. (c)
97. (c)	98. (c)	99. (a)	100. (a)	101. (a)	102. (a)	103. (b)	104. (a)
105. (b)	106. (d)	107. (c)					

Explanation

1. (a) $\because \quad x+y+z=0 \Rightarrow x+y=-z,\ y+z=-x,\ x+z=-y$

$\therefore \quad (x+y)(y+z)(z+x)=(-z)(-x)(-y)=-xyz$

2. (a) First polynomial $= 6x^3+60x^2+150x$

$= 6x(x^2+10x+25) = 3 \times 2 \times x \times (x+5)^2$

Second polynomial $= 3x^4+12x^3-15x^2$

$= 3x^2(x^2+4x-5) = 3x^2(x^2+5x-x-5) = 3x^2(x+5)(x-1)$

$\therefore$ Required LCM $= 3 \times 2 \times x^2 \times (x+5)^2(x-1) = 6x^2(x+5)^2(x-1)$

3. (b) $\because x-k$ is a factor of $2x^2-kx-9$

$\therefore \quad 2k^2-k^2-9=0$

$\Rightarrow \quad k^2-9=0$

$\therefore \quad k=\pm 3$

But factor of x^2+x-12 are $(x+4),(x-3)$

Hence value of k is 3.

4. (d) First polynomial $= x^2+bx-x-b = x(x+b)-1(x+b)$

$= (x-1)(x+b)$

Second polynomial $= x^2+xa-x-a = x(x+a)-1(x+a)$

$= (x+a)(x-1)$

$\therefore$ Required LCM $= x-1$

5. (a) Required remainder $= 3(2y)^3 - 2(2y)^2 y - 13(2y) y^2 + 10y^3$

(using factor theorem)

$$= 24y^3 - 8y^3 - 26y^3 + 10y^3 = 34y^3 - 34y^3 = 0$$

6. (c) $x^2 + 2) \; x^3 + 5x^2 + 10k \; (x + 5$

$$\begin{array}{r} \underline{x^3 + 2x} \\ 5x^2 - 2x + 10k \\ \underline{5x^2 + 10} \\ -2x + 10k - 10 \end{array}$$

But remainder is $-2x$,

$$\Rightarrow -2x = -2x + 10k - 10$$

$$\Rightarrow 10k - 10 = 0$$

$$\therefore \quad k = 1$$

7. (b) $\because (5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2$

$$= (5x^2 + 14x + 2 - 4x^2 + 5x - 7) \times (5x^2 + 14x + 2 + 4x^2 - 5x + 7)$$

$$= (x^2 + 19x - 5)(9x^2 + 9x + 9) = 9(x^2 + 19x - 5)(x^2 + x + 1)$$

Clearly, $(x^2 + x + 1)$is a factor of $\{(5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2\}$.

Hence remainder is zero.

8. (a) At $x = 1$

Polynomial $= (1)^{29} - (1)^{24} + (1)^{13} - 1 = 1 - 1 + 1 - 1 = 0$

$\therefore$ $(x - 1)$ is a factor of $x^{29} - x^{24} + x^{13} - 1$

at, $x = -1$

Polynomial $= (-1)^{29} - (-1)^{24} + (-1)^{13} - 1 = -1 - 1 - 1 - 1 = -4$

$\therefore$ $(x + 1)$ is not a factor of $x^{29} - x^{24} + x^{13} - 1$

Hence option (a) is correct.

9. (c) $x^2 - 4x + 1 = 0$

$\Rightarrow$ $x^2 + 1 = 4x$

Dividing by x, $x + \frac{1}{x} = 4$

$$\therefore \quad x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)\left(x \cdot \frac{1}{x}\right) = 4^3 - 3 \times 4 \times 1 = 64 - 12 = 52$$

10. (a) Given $x + y + z = 6$ and $xy + yz + zx = 11$

$$\therefore \quad x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= (x + y + z)[(x + y + z)^2 - 3(xy + yz + zx)]$$

$$= 6[6^2 - 3(11)] = 6 \times 3 = 18$$

$0y^3$

on the ...

$4y^3 - 0$

Here factors are $\sqrt{3}, -\sqrt{3}, 3$. Hence option (d) is correct.

12. (c) Let $x - \alpha$ is a common factor.

$\therefore \quad \alpha^2 - 11\alpha + a = 0$ and $\alpha^2 - 14\alpha + 2a = 0$

Solving, $\alpha^2 - 11\alpha + a = \alpha^2 - 14\alpha + 2a$

$\Rightarrow \quad 3\alpha = a$

$\Rightarrow \quad \alpha = \frac{a}{3}$

Putting $\alpha = \frac{a}{3}$, in the first equation (i) $\left(\frac{a}{3}\right)^2 - 11\frac{a}{3} + a = 0$

$\Rightarrow \quad a^2 - 33a + 9a = 0$

$a^2 - 24a = 0$

$\Rightarrow \quad a = 0, 24$

$5x + 7)$

$1)$

$^2)$.

13. (a) Since $x + 4$ is HCF, therefore it divides both the expression and each expression becomes zero at $x = -4$.

$\therefore \quad 2(-4)^2 + k(-4) - 12 = 0$ and $(-4)^2 + (-4) - 2k - 2 = 0$

$\Rightarrow \quad 32 - 12 = 4k$ and $16 - 6 = 2k$

$\Rightarrow \quad k = 5$ and $k = 5$

4

14. (b) $x(y - z)(y + z) + y(z - x)(z + x) + z(x - y)(x + y)$

$= x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$

$= x(y^2 - z^2) + yz^2 - yx^2 + zx^2 - zy^2$

$= x(y - z)(y + z) + x^2(z - y) + yz(z - y)$

$= (y - z)[xy + xz - x^2 - yz] = (y - z)[y(x - z) + x(z - x)]$

$= (y - z)(x - z)(y - x) = (x - y)(x - z)(z - y)$

15. (a) Given, $\frac{x}{y} = \frac{z}{w} \Rightarrow xw = yz$

Now, $(xy + zw)^2 = x^2y^2 + z^2w^2 + 2(xyzw) = x^2y^2 + z^2w^2 + 2[yz \cdot yz]$

$= x^2y^2 + y^2z^2 + z^2w^2 + y^2z^2 = y^2(x^2 + z^2) + z^2w^2 + x^2w^2$

$= y^2(x^2 + z^2) + w^2(x^2 + z^2) = (x^2 + z^2)(y^2 + w^2)$

16. (b) Given, $\frac{1}{x+1} + \frac{2}{y+2} + \frac{1009}{z+1009} = 1$

$\Rightarrow \quad \frac{1}{x+1} - 1 + \frac{2}{y+2} - 1 + \frac{1009}{z+1009} - 1 = 1 - 3$

$\Rightarrow \quad -\frac{x}{x+1} - \frac{y}{y+2} - \frac{z}{z+1009} = -2$

17. (a) $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$$=(a+b+c)\frac{1}{2}\left((a-b)^2+(b-c)^2+(c-a)^2\right)$$

$$=(258+260+262)\frac{1}{2}\left\{(-2)^2+(-2)^2+4^2\right\}$$

$$=(780)\frac{1}{2}\times 24=780\times 12=9360$$

18. (b) We have, $x^3-27=(x-3)(x^2+9+3x)$

Now, $x^2+9+3x\overline{)\,x^3+4x^2+12x+k\,}(x+1$

$$\underline{\underset{-}{x^3}+\underset{-}{3x^2}+\underset{-}{9x}}$$

$$x^2+3x+k$$

$$\underline{\underset{-}{x^2}+\underset{-}{3x}+\underset{-}{9}}$$

$$k-9$$

$\therefore$ Value of k should be 9.

19. (c) Let $f(x)=x^{40}+2$

putting $x^4=-1$

$f(x)=(-1)^{10}+2=3$

20. (c) Given, $a^x=b^y=c^z=k$

$\Rightarrow$ $a=k^{1/x}$, $b=k^{1/y}$ and $c=k^{1/z}$

$\therefore$ $abc=k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$

$\Rightarrow$ $1=k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$

$\Rightarrow$ $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

$\Rightarrow$ $xy+yz+zx=0$

21. (c) $a=\frac{1+x}{2-x}$

$$\therefore\quad \frac{1}{a+1}+\frac{2a+1}{a^2-1}=\frac{3a}{a^2-1}=\frac{3\left(\frac{1+x}{2-x}\right)}{\left(\frac{1+x}{2-x}\right)^2-1}$$

$$=\frac{3(1+x)(2-x)}{1+x^2+2x-(4+x^2-4x)}$$

$$=\frac{3(1+x)(2-x)}{6x-3}$$

$$=\frac{(1+x)(2-x)}{2x-1}$$

22. (b) Given, $pq + qr + rp = 0$

$\Rightarrow \quad -qr = pq + rp$ etc.

$$\therefore \quad \frac{p^2}{p^2-qr}+\frac{q^2}{q^2-rp}+\frac{r^2}{r^2-pq}$$

$$=\frac{p^2}{p^2+rp+pq}+\frac{q^2}{q^2+pq+qr}+\frac{r^2}{r^2+qr+rp}$$

$$=\frac{p}{p+q+r}+\frac{q}{p+q+r}+\frac{r}{p+q+r}=\frac{p+q+r}{p+q+r}=1$$

23. (d) $x + y + z = 0$

$\Rightarrow \quad (x+y)^2 = (-z)^2$

$\Rightarrow \quad x^2 + y^2 - z^2 = -2xy$ etc.

$$\therefore \quad \frac{1}{x^2+y^2-z^2}+\frac{1}{y^2+z^2-x^2}+\left(\frac{1}{z^2+x^2-y^2}\right)$$

$$=\frac{1}{-2xy}+\frac{1}{-2yz}+\frac{1}{-2zx}=-\frac{1}{2}\left\{\frac{z+x+y}{xyz}\right\}=0 \qquad (\because x+y+z=0)$$

24. (c) We know that, $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Here, $x - y + y - z + z - x = 0$

$$\therefore \quad \frac{(x-y)^3+(y-z)^3+(z-x)^3}{4(x-y)(y-z)(z-x)}=\frac{3(x-y)(y-z)(z-x)}{4(x-y)(y-z)(z-x)}=\frac{3}{4}$$

25. (d) $\because \quad (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\therefore \quad 6^2 = 14 + 2(ab + bc + ca)$

$\Rightarrow \quad ab + bc + ca = \dfrac{36-14}{2} = 11$

Now, $a^3 + b^3 + c^3 - 3abc = (a + b + c)$ from, $(a^2 + b^2 + c^2 - ab - bc - ca)$

$36 - 3abc = 6(14 - 11) = 18$

or, $3abc = 36 - 18 = 18$

$\Rightarrow \quad abc = 6$

Second method :

By inspection (trial) $a = 1, b = 2, c = 3$

26. (c) Given, $x(x + y + z) = 9$, $y(x + y + z) = 16$ and $z(x + y + z) = 144$

Adding all the equations, $(x + y + z)(x + y + z) = 9 + 16 + 144$

$\Rightarrow \quad (x + y + z)^2 = 169$

$\Rightarrow \quad x + y + z = 13$

$\because \quad x(x + y + z) = 9$

$\Rightarrow \quad x(13) = 9 \qquad \therefore \quad x = \dfrac{9}{13}$

27. (c) Given, $(u)^3 + (-2v)^3 + (-3w)^3 = 3 \times (-2)(-3)\,uvw$

$\therefore \quad u + (-2v) + (-3w) = 0$

$\Rightarrow \quad u - 2v - 3w = 0$

$\Rightarrow \quad u - 2v = 3w$

28. (d) $\because \quad a + b + c = 0$

$\Rightarrow \quad a^3 + b^3 + c^3 = 3abc$

$\Rightarrow \quad \frac{a^3}{abc} + \frac{b^3}{abc} + \frac{c^3}{abc} = 3$

$\Rightarrow \quad \frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} = 3$

29. (d) $\because \quad x^4 + \frac{1}{x^4} = 322$

$\Rightarrow \quad \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 322$

$\Rightarrow \quad \left(x^2 + \frac{1}{x^2}\right)^2 = 324 = 18^2$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 18$

$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 + 2 = 18$

$\therefore \quad x - \frac{1}{x} = 4$

30. (c) $\because \quad x \propto y^m, \qquad y \propto z^n, \qquad x \propto z^p$

$\Rightarrow \quad z \propto x^{\frac{1}{p}}$

$\Rightarrow \quad y \propto y^{\frac{n}{p}}$

$\Rightarrow \quad x \propto x^{\frac{mn}{p}}$

$\Rightarrow \quad 1 = \frac{mn}{p}$

$\Rightarrow \quad p = mn$

31. (c) $\because \quad x = (b-c)(a-d),\ y = (c-a)(b-d)$ and $z = (a-b)(c-d)$

$\therefore \quad x + y + z = (b-c)(a-d) + (c-a)(b-d) + (a-b)(c-d) = 0$

Hence, $x^3 + y^3 + z^3 = 3xyz$

32. (d) Given, $\quad a + b + c = 6$ ... (i

and $\quad a^2 + b^2 + c^2 = 26$ (ii

We know that, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\Rightarrow (a+b+c)^2 = (a^2+b^2+c^2)+2(ab+bc+ca)$

$\Rightarrow (ab+bc+ca) = \frac{1}{2}\{(a+b+c)^2-(a^2+b^2+c^2)\}$

$= \frac{1}{2}\{(6)^2-26\} = \frac{1}{2}\{36-26\} = \frac{1}{2}\times 10 = 5$

33. **(a)** Given expression $= 3x^3-2x^2y-13xy^2+10y^3$.

$\because$ $x-2y$ is factor of given expression, the expression will be zero when $x-2y=0$

Putting, $x = 2y$ in the given expression

$= 3(2y)^3-2(2y)^2y-13(2y)y^2+10y^3$

$= 3(8y^3)-2(4y^2)y-13(2y^3)+10y^3$

$= 24y^3-8y^3-26y^3+10y^3 = 34y^3-34y^3$

$= 0$, which is remainder.

34. **(b)** $a^2+\frac{1}{a^2}+2=3 \Rightarrow a^2+\frac{1}{a^2}=1$

$\Rightarrow \frac{a^4+1}{a^2}=1$

$\Rightarrow a^4-a^2+1=0$

Multiplying both sides by a^2+1, $(a^2+1)(a^4-a^2+1) = 0\times(a^2+1)$

$\Rightarrow (a^2)^3+1^3=0$ $\quad (\because (x+y)(x^2-xy+y^2)=x^3+y^3)$

$\Rightarrow a^6=-1$

$\therefore$ $a^{12}=1, a^{18}=-1, a^{84}=1, a^{90}=-1$

$a^{200}=a^{198}\cdot a^2=(a^6)^{33}a^2=-a^2$ and $a^{206}=a^{204}\cdot a^2=(a^6)^{34}a^2=+a^2$

Required sum $= 1-1+1-1+1-1-a^2+a^2=0$

35. **(a)** Given, $x+y+z=0$ ... (i)

Expression $= \frac{xyz}{(x+y)(y+z)(z+x)}$

$= \frac{xyz}{\{(x+y+z)-z\}\{(x+y+z)-x\}\{(x+y+z)-y\}}$

$= \frac{xyz}{(0-z)(0-x)(0-y)}$ [from (i)]

$= \frac{xyz}{-xyz} = -1$

... (i)

(ii)

36. (c) From first two ratio,

$8(5x-7y+10) = 1\,(3x+2y+1)$

or, $37x - 58y = -79$ ... (i)

From first and third ratio, $9(5x-7y+10) = 1\,(11x+4y-10)$

or, $45x - 63y + 90 = 11x + 4y - 10$

or, $34x - 67y = -100$ ... (ii)

from, (i) – (ii), $3x + 9y = 21$ ... (iii)

or, $x + 3y = 7$

putting, $x = 7 - 3y$ in (i),

$37(7-3y) - 58y = -79$

or, $259 - 111y - 58y = -79$

or, $338 = 169y$

or, $y = 2$

Again from (iii), $x + 6 = 7$

$\Rightarrow \; x = 1$

$\therefore \; x + y = 1 + 2 = 3$

37. (d) From I and II term,

$$\frac{2x-3y+1}{2} = \frac{x+4y+8}{3}$$

$\Rightarrow \; 6x - 9y + 3 = 2x + 8y + 16$

$\Rightarrow \; 4x - 17y = 13$ ... (

From I and III term,

$$\Rightarrow \; \frac{x+4y+8}{3} = \frac{4x-7y+2}{5}$$

$\Rightarrow \; 5x + 20y + 40 = 12x - 21y + 6$

$7x - 41y = 34$ ... (i

Solving equation, (i) & (ii), $x = -1$, $y = -1$

$\therefore \; x + y = -1 - 1 = -2$

38. (c) $\frac{x}{a} = b - c$, $\frac{y}{b} = c - a$, $\frac{z}{c} = a - b$

Again, $b - c + c - a + a - b = 0$

$$\therefore \; \left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$$

$$= (b-c)^3 + (c-a)^3 + (a-b)^3$$

$$= 3(b-c)(c-a)(a-b) = \frac{3xyz}{abc}$$

39. (a) $x^4 - x^3 + x^2 + 2 = x^2 \cdot x^2 - x \cdot x^2 + (x^2 + 2)$

$= (2x-2)^2 - x(2x-2) + 2x$ $\quad (\because x^2 + 2 = 2x)$

… (i)

$= 4x^2 - 8x + 4 - 2x^2 + 2x + 2x$

$= 2x^2 - 4x + 4 = 2(x^2+2) - 4x$

$= 2(2x) - 4x = 0$

… (ii)

40. (a) $2^x = 3^y = 6^{-z} = k$

… (iii)

$\Rightarrow 2 = k^{\frac{1}{x}}; \quad 3 = k^{\frac{1}{y}}; \quad 6 = k^{\frac{-1}{z}} \quad \because 2 \times 3 = 6$

$\Rightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{y}}$

$\Rightarrow k^{\frac{1}{x}+\frac{1}{y}} = k^{\frac{1}{z}}$

$\Rightarrow \frac{1}{x} + \frac{1}{y} = -\frac{1}{z}$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

41. (a) $\because \quad \frac{1}{x+y} = \frac{1}{x} + \frac{1}{y} = \frac{y+x}{xy}$

$\Rightarrow (x+y)^2 = xy$

$\Rightarrow x^2 + 2xy + y^2 = xy$

$\Rightarrow x^2 + xy + y^2 = 0$

$\therefore x^3 - y^3 = (x-y)(x^2 + xy + y^2) = 0$

… (i)

42. (b) $\because \quad a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow \frac{1}{2}\left\{(a-b)^2 + (b-c)^2 + (c-a)^2\right\} = 0$

$\Rightarrow a = b = c$

… (ii)

$\therefore \quad \frac{a+c}{b} = \frac{a+a}{a} = 2$

43. (b) $\frac{2x}{3x^2 - 5x + 3} = \frac{2x}{x\left(3x - 5 + \frac{3}{x}\right)} = \frac{2}{3\left(x + \frac{1}{x}\right) - 5} = \frac{2}{3 \cdot 5 - 5} = \frac{2}{10} = \frac{1}{5}$

44. (d) $\because \quad x^4 + \frac{1}{x^4} = 119$

$x^2 + \frac{1}{x^2} = \sqrt{119+2} = 11$

$\therefore \quad x - \frac{1}{x} = \sqrt{11-2} = 3$

Hence, $x^3 - \frac{1}{x^3} = (3)^3 + 3 \times 3 = 27 + 9 = 36$

45. **(d)** $\because$ $(x+y-z)^2+(y+z-x)^2+(z+x-y)^2=0$, It is possible only

When, $x+y-z=0$, $y+z-x=0$ and $z+x-y=0$

Adding, $(x+y+z)=0$

46. **(a)** From ratio proportion, $\frac{x-y}{1}=\frac{x+y}{7}=\frac{xy}{4}$

$$=\frac{(x-y)+(x+y)}{1+7}$$

$$=\frac{(x-y)-(x+y)}{1-7}$$

$\Rightarrow \frac{x-y}{1}=\frac{x+y}{7}=\frac{xy}{4}=\frac{x}{4}=\frac{y}{3}$ *(Multiplication rule of ratio)*

Again, $\frac{xy}{4}=\frac{x}{4}=\frac{y}{3}=\frac{\sqrt{xy}}{\sqrt{4\times 3}}$

or, $\frac{xy}{4}=\frac{\sqrt{xy}}{\sqrt{12}}$

or, $\frac{(xy)^2}{16}=\frac{xy}{12}$

$\Rightarrow xy=\frac{16}{12}=\frac{4}{3}$

47. **(d)** We know that when $x+y+z=0$, then, $x^3+y^3+z^3=3xyz$,

$\therefore \frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=\frac{x^3+y^3+z^3}{xyz}$

$\Rightarrow \frac{3xyz}{xyz}=3$

48. **(c)** $x^3-x^2y-xy^2+y^3$

$=x^3+y^3-x^2y-xy^2=(x+y)^3-3xy(x+y)-xy(x+y)$

$=(x+y)^3-4xy(x+y)=a^3-4b^2a$

49. **(a)** $\because$ $(3a+1)^2+(b-1)^2+(2c-3)^2=0$

$\Rightarrow 3a+1=0$

$\Rightarrow 3a=-1$

$b-1=0$

$\Rightarrow b=1$

$2c-3=0$

$\Rightarrow 2c=3$

$\therefore 3a+b+2c=-1+1+3=3$

50. ...

$\Rightarrow \quad x + y = \frac{1}{xy}$

On cubing both sides, $x^3 + y^3 + 3xy(x + y) = \frac{1}{x^3y^3}$

$\Rightarrow \quad \frac{1}{x^3y^3} - x^3 - y^3 = 3xy(x + y) = 3 \times 1 = 3$

51. **(b)** $\frac{x}{2x^2+5x+2} = \frac{x}{x\left(2x+5+\frac{2}{x}\right)} = \frac{1}{2\left(x+\frac{1}{x}\right)+5}$

$= \frac{1}{2t+5}$ (Let $x + \frac{1}{x} = t$)

of ratio)

According to question, $\frac{1}{2t+5} = \frac{1}{6}$

$\Rightarrow \quad 6 = 2t + 5$

$\Rightarrow \quad t = \frac{1}{2}$

$\therefore \quad x + \frac{1}{x} = \frac{1}{2}$

52. **(b)** $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)}$

$= \frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)} = \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$

[Since $(a - b) + (b - c) + (c - a) = a - b + b - c + c - a = 0$

$\therefore \quad (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$]

53. **(b)** $3x + \frac{1}{2x} = 5$

or, $\frac{3}{2}\left(2x+\frac{1}{3x}\right) = 5$

or, $2x + \frac{1}{3x} = \frac{10}{3}$

Cubing both sides, $8x^3 + \frac{1}{27x^3} + 3\left(2x+\frac{1}{3x}\right)\left(2x\cdot\frac{1}{3x}\right) = \frac{1000}{27}$

or, $8x^3 + \frac{1}{27x^3} + 3\cdot\frac{10}{3}\cdot\frac{2}{3} = \frac{1000}{27}$

or, $8x^3 + \frac{1}{27x^3} = \frac{1000}{27} - \frac{20}{3}$

$= \frac{1000-180}{27} = \frac{820}{27} = 30\frac{10}{27}$

54. **(b)** $(a-3)^2+(b-4)^2+(c-9)^2=0$

is possible only when each of $(a-3)^2$, $(b-4)^2$ and $(c-9)^2$ is zero.

$\therefore \quad (a-3)^2=0, (b-4)^2=0, (c-9)^2=0$

$\Rightarrow \quad a-3=0, b-4=0, c-9=0$

$\therefore \quad a=3, b=4$ and $c=9$

Hence, $\sqrt{a+b+c} = \sqrt{3+4+9} = \sqrt{16} = 4$

55. **(c)** Average of x and $\frac{1}{x} = M$

$\therefore \quad \frac{x+\frac{1}{x}}{2} = M$

$\Rightarrow \quad x+\frac{1}{x} = 2M$

Squaring both sides, $\left(x+\frac{1}{x}\right)^2 = (2M)^2$

$\Rightarrow \quad x^2+\frac{1}{x^2}+2\cdot x\cdot\frac{1}{x} = 4M^2$

$\Rightarrow \quad x^2+\frac{1}{x^2}+2 = 4M^2$

$\Rightarrow \quad x^2+\frac{1}{x^2} = 4M^2-2$ ...(i)

Average of x^2 and $\frac{1}{x^2} = \frac{x^2+\frac{1}{x^2}}{2} = \frac{4M^2-2}{2}$ [from (i)]

$= \frac{2(2M^2-1)}{2} = 2M^2-1$

56. **(c)** $a^2+b^2+c^2 = 2(a-b-c)-3$

or, $a^2-2a+1+b^2+2b+1+c^2-2c+1=0$

or, $(a-1)^2+(b+1)^2+(c+1)^2=0$

It is possible only when, $a=1, b=-1, c=-1$

Hence, $2a-3b+4c = 2+3-4=1$

57. **(b)** $\because \quad a+b+c=0$

then, $\frac{1}{(a+b)(b+c)}+\frac{1}{(a+c)(b+a)}+\frac{1}{(c+a)(c+b)}$

$= \frac{(c+a)+(b+c)+(a+b)}{(a+b)(b+c)(c+a)}$

$= \frac{c+a+b+c+a+b}{(a+b)(b+c)(c+a)}$

$$= \frac{2a+2b+2c}{(a+b)(b+c)(c+a)}$$

$$= \frac{2(a+b+c)}{(a+b)(b+c)(c+a)}$$

$$= \frac{2\times 0}{(a+b)(b+c)(c+a)} = 0$$

58. **(d)** Given expression $= 5x^2 - 4xy + y^2 - 2x + 1 = 0$

$\Rightarrow \quad 4x^2 - 4xy + y^2 + x^2 - 2x + 1$

$\Rightarrow \quad (2x-y)^2 + (x-1)^2 = 0$

It is possible only when $2x - y = 0$ and $x - 1 = 0$

$\therefore \quad x = 1$ and $y = 2$

$x + y = 1 + 2 = 3$

59. **(a)** $x + y + z = b + c - 2a + c + a - 2b + a + b - 2c$

$\Rightarrow \quad x + y + z = 0$

$\Rightarrow \quad x + y = -z$

Now, $x^2 + y^2 - z^2 + 2xy = (x+y)^2 - z^2 = (-z)^2 - z^2 = z^2 - z^2 = 0$

60. **(c)** $(y-z) + (z-x) + (x-y) = 0$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Hence, $(y-z)^3 + (z-x)^3 + (x-y)^3 = 3(y-z)(z-x)(x-y)$

61. **(c)** $\because \quad x = 2 - 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

$\Rightarrow \quad x - 2 = 2^{\frac{2}{3}} - 2^{\frac{1}{3}}$

Cubing both sides,

$$x^3 - 3x^2 \times 2 + 3x \times 4 - 8 = \left(2^{\frac{2}{3}}\right)^3 - \left(2^{\frac{1}{3}}\right)^3 - 3\cdot 2^{\frac{2}{3}}\cdot 2^{\frac{1}{3}}\left(2^{\frac{2}{3}} - 2^{\frac{1}{3}}\right)$$

$\Rightarrow \quad x^3 - 6x^2 + 12x - 8 = 4 - 2 - 6(x-2)$

$\Rightarrow \quad x^3 - 6x^2 + 12x - 8 = 2 - 6x + 12$

$\Rightarrow \quad x^3 - 6x^2 + 18x + 18 = 2 + 12 + 8 + 18 = 40$

62. **(d)** $a^3 + b^3 + c^3 - 3abc = 0$, if $a + b + c = 0$

$\therefore \quad a^3 - b^3 - c^3 - 3abc = 0$

$\Rightarrow \quad a - b - c = 0$

$\Rightarrow \quad a = b + c$

63. **(b)** $\because \quad x^2 - 3x + 1 = 0$ ... (i)

$\Rightarrow \quad x^2 + 1 = 3x$

Squaring both sides, $(x^2+1)^2 = (3x)^2$

$\Rightarrow \quad x^4 + 1 + 2x^2 = 9x^2$

$\Rightarrow \quad x^4 + 1 = 7x^2$ ...(i

Now, $x^3 + \frac{1}{x^3} = x^3 + \left(\frac{1}{x}\right)^3 = \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$

[$\because a^3 + b^3 = (a+b)(a^2+b^2-ab)$

$= \left(\frac{x^2+1}{x}\right)\left(\frac{x^4+1}{x^2}-1\right) = \left(\frac{3x}{x}\right)\left(\frac{7x^2}{x^2}-1\right)$ [From equation (i) & (ii)

$= 3(7-1) = 3 \times 6 = 18$

64. (d) $\because \quad 2x + \frac{1}{3x} = 5$

$\Rightarrow \quad \frac{6x^2+1}{3x} = 5 \qquad \therefore \quad 6x^2 + 1 = 15x$ ...(i)

Now, $\frac{5x}{6x^2+20x+1} = \frac{5x}{(6x^2+1)+20x}$

$= \frac{5x}{15x+20x} = \frac{5x}{35x} = \frac{1}{7}$ (from (i))

65. (c) $\because \quad m + \frac{1}{m-2} = 4$

$\Rightarrow \quad (m-2) + \frac{1}{m-2} = 4-2 = 2$

Squaring both sides, $(m-2)^2 + \frac{1}{(m-2)^2} + 2(m-2)\left(\frac{1}{m-2}\right) = 4$

$\Rightarrow \quad (m-2)^2 + \frac{1}{(m-2)^2} = 4-2 = 2$

66. (c) $\because \quad a^2 = b + c$

$\Rightarrow \quad a^2 + a = a + b + c$

$\Rightarrow \quad a(a+1) = a + b + c$

$\Rightarrow \quad (a+1) = \frac{a+b+c}{a}$

$\Rightarrow \quad \frac{1}{(a+1)} = \frac{a}{a+b+c}$

Similarly, $b^2 = c + a$

$\Rightarrow \quad \frac{1}{b+1} = \frac{b}{a+b+c}$ and $c^2 = a + b$

$\Rightarrow \quad \frac{1}{c+1} = \frac{c}{a+b+c}$

$\therefore \quad \frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1} = \frac{a}{a+b+c} + \frac{b}{a+b+c} + \frac{c}{a+b+c} = \frac{a+b+c}{a+b+c} = 1$

67. (

$$\Rightarrow \left(\frac{a}{1-a}+1\right)+\left(\frac{b}{1-b}+1\right)+\left(\frac{c}{1-c}+1\right)=3+1=4$$

$$\Rightarrow \frac{a+1-a}{1-a}+\frac{b+1-b}{1-b}+\frac{c+1-c}{1-c}=4$$

$$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=4$$

68. (c) $\because \quad a^2+b^2+2b+4a+5=0$

$$\Rightarrow a^2+4a+b^2+2b+5=0$$

$$\Rightarrow a^2+4a+4+b^2+2b+1=0$$

$$\Rightarrow (a+2)^2+(b+1)^2=0$$

It is possible only when, $a+2=0$

$$\Rightarrow a=-2$$

and $b+1=0$

$$\Rightarrow b=-1$$

$$\therefore \quad \frac{a-b}{a+b}=\frac{-2+1}{-2-1}=\frac{-1}{-3}=\frac{1}{3}$$

69. (c) Let each ratio $= k$ then

$$x=k(b-c)(b+c-2a)=k(b^2-c^2)-k\,2a(b-c)$$

$$y=k(c-a)(c+a-2b)=k(c^2-a^2)-k\,2b(c-a)$$

$$z=k(a-b)(a+b-2c)=k(a^2-b^2)-k\,2c(a-b)$$

Adding, $x+y+z=0$

70. (a) From componendo and dividendo

$$\frac{(x^3+3x)+(3x^2+1)}{(x^3+3x)-(3x^2+1)}=\frac{189+61}{189-61}$$

or, $$\frac{(x+1)^3}{(x-1)^3}=\frac{250}{128}=\frac{125}{64}.$$

or, $$\left(\frac{x+1}{x-1}\right)^3=\left(\frac{5}{4}\right)^3$$

or, $$\frac{x+1}{x-1}=\frac{5}{4}$$

$\therefore$ Solving $x=9$

$\Rightarrow x^2p^2 + x^2q^2 + y^2p^2 + y^2q^2 = x^2p^2 + y^2q^2 + 2xpyq$

$\Rightarrow x^2q^2 + y^2p^2 - 2xpyq = 0$

$\Rightarrow (xq - yp)^2 = 0$

$\therefore xq = yp$

72. (a) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$ *(ratio proportion law)*

Each ratio $= \frac{x+y+z}{(2x+y+z)+(x+2y+z)+(x+y+2z)}$

$= \frac{x+y+z}{4(x+y+z)} = \frac{1}{4}$

73. (d) $a + b + c = 0$

$\Rightarrow a + b = -c$

Squaring, $a^2 + b^2 + 2ab = c^2$

or, $a^2 + b^2 = c^2 - 2ab$ ... (i)

Given expression $= \frac{a^2+b^2+c^2}{c^2-ab} = \frac{(c^2-2ab)+c^2}{c^2-ab}$ *[from equation (i)]*

$= \frac{2(c^2-ab)}{(c^2-ab)} = 2$

74. (c) $x^2 + 8y^2 + 9z^2 - 4xy - 12xz = 0$

or, $x^2 + 4y^2 - 4xy + 4y^2 + 9z^2 - 12xz = 0$

or, $(x - 2y)^2 + (2y - 3z)^2 = 0$

$\therefore x - 2y = 0$

$\Rightarrow 2y - 3z = 0$

$\Rightarrow x = 2y = 3z$

75. (c) Given expression

$= p^2a^2 + p^2b^2 + p^2c^2 - 2pab - 2pbc - 2pcd + b^2 + c^2 + d^2 \le 0$

or, $(p^2a^2 - 2pab + b^2) + (p^2b^2 - 2pbc + c^2) + (p^2c^2 - 2pcd + d^2) \le 0$

or, $(pa - b)^2 + (pb - c)^2 + (pc - d)^2 \le 0$

It is possible only when, $pa - b = 0$, $pb - c = 0$ and $pc - d = 0$.

or, $p = \frac{b}{a}$, $p = \frac{c}{b}$ and $p = \frac{d}{c}$

$\therefore \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

`'49

76. **(a)** We know that,

$(a+b+c)^3 = a^3+b^3+c^3+3(b+c)(c+a)(a+b)$

$\therefore \quad (30)^3 = a^3+b^3+c^3+3\times 10\times 20\times 30$

$(\because b+c+c+a+a+b = 10+20+30 \Rightarrow a+b+c=30)$

$a^3+b^3+c^3 = 27000 - 3\times 10\times 20\times 30$

$= 27000 - 18000 = 9000$

77. **(c)** We know that $(a+b+c)^3 = a^3+b^3+c^3+3(b+c)(c+a)(a+b)$

$\because \quad 2x+2y+2z = b+c+c+a+a+b \Rightarrow x+y+z = a+b+c$

$\therefore \quad (x+y+z)^3 = a^3+b^3+c^3+3(2x)(2y)(2z)$

$\therefore \quad a^3+b^3+c^3 = (x+y+z)^3 - 24xyz$

78. **(b)** We know that, $a+b+c=0$ ... (i)

$\Rightarrow \quad a^3+b^3+c^3 = 3abc$

but, $a+b+c=0$, $b=-(a+c)$, $c=-(a+b)$

$\therefore$ from (i), $a^3+b^3+c^3 = 3a(-(a+c))(-(a+b))$

$= 3a(a+b)(c+a)$

79. **(c)** If $a = x-y$, $b = y-z$ and $c = z-x$ then

$a+b+c = x-y+y-z+z-x = 0$

$\therefore \quad a^3+b^3+c^3 = 3abc = 3(x-y)(y-z)(z-x)$

80. **(b)** $(ab+bc+ca)^2 = a^2b^2+b^2c^2+c^2a^2+2abbc+2abca+2bcca$

or, $0 = a^2b^2+b^2c^2+c^2a^2+2abc(b+a+c)$

$0 = a^2b^2+b^2c^2+c^2a^2+2pq$

$\therefore \quad a^2b^2+b^2c^2+c^2a^2 = -2pq$

81. **(c)** $49c^2+9(a+b)^2-42(a+b)c = (7c-3(a+b))^2$

$= (7\times 8 - 3(89-69))^2$

$= (56-60)^2 = (-4)^2 = 16$

82. **(c)** $x^2+y^2+z^2-2xy-2xz+2yz = (x-y-z)^2$

$= (q+r+s-r-s+p-p-q-r)^2$

$= (-r)^2 = r^2$

83. **(d)** $(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$

or, $10^2 = 60+2(xy+yz+zx)$

$\therefore \quad \frac{100-60}{2} = xy+yz+zx$

or, $xy+yz+zx = 20$

84. **(a)** $(ac-bd)^2+(ad+bc)^2 = a^2c^2+b^2d^2-2acbd+a^2d^2+b^2c^2+2adbc$

$$= a^2c^2+a^2d^2+b^2d^2+b^2c^2$$

$$= a^2(c^2+d^2)+b^2(d^2+c^2)$$

$$= a^2\cdot 2+b^2\cdot 2 = 2(a^2+b^2) \qquad (\because c^2+d^2=2)$$

$$= 2\times 1 = 2$$

85. **(c)** Let $x-a=p$, $x-b=q$ and $x-c=r$

then, $q-p=a-b$, $r-q=b-c$, $p-r=c-a$

Putting these values,

Given expression $= pq(q-p)+qr(r-p)+rp(p-r)$

$$= -(q-p)(r-p)(p-r)$$

[See result 7 of theory part]

$$= -(a-b)(b-c)(c-a)$$

$$(\because a-b=q-p,\ b-c=r-q \text{ and } c-a=p-r)$$

86. **(d)** $\left(x+\frac{1}{x}\right)\left(y+\frac{1}{y}\right)+\left(x-\frac{1}{x}\right)\left(y-\frac{1}{y}\right) = xy+\frac{x}{y}+\frac{y}{x}+\frac{1}{xy}+xy-\frac{x}{y}-\frac{y}{x}+\frac{1}{xy}$

$$= 2\left(xy+\frac{1}{xy}\right)$$

$$\therefore\quad xy+\frac{1}{xy} = \frac{1}{2}\left\{\left(x+\frac{1}{x}\right)\left(y+\frac{1}{y}\right)+\left(x-\frac{1}{x}\right)\left(y-\frac{1}{y}\right)\right\}$$

$$= \frac{1}{2}\{2a\cdot 2c+2b\cdot 2d\}$$

$$= 2(ac+bd)$$

87. **(d)** $x^8+x^4+1 = x^8+2x^4+1-x^4$

$$= (x^4+1)^2-x^4 = (x^4+1+x^2)(x^4+1-x^2)$$

$$= (x^4+2x^2+1-x^2)(x^4-x^2+1)$$

$$= ((x^2+1)^2-x^2)(x^4-x^2+1)$$

$$= (x^2+1-x)(x^2+1+x)(x^4-x^2+1)$$

$$= (x^2-x+1)(x^2+2x+1-x)(x^4-x^2+1)$$

$$= (x^2-x+1)((x+1)^2-x)(x^4-x^2+1)$$

$$= (x^2-x+1)(x+1+\sqrt{x})(x+1-\sqrt{x})(x^4-x^2+1)$$

Among given options x^2-2x+1 is not a factor.

88. (a) $a^4 - 11a^2b^2 + b^4 = a^4 - 2a^2b^2 + b^4 - 9a^2b^2$

$$= (a^2 - b^2)^2 - (3ab)^2$$

$$= (a^2 - b^2 - 3ab)(a^2 - b^2 + 3ab)$$

89. (b) $a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2$

$$a^4 + b^4 = (a^2 + b^2 - \sqrt{2}\,ab)(a^2 + b^2 + \sqrt{2}\,ab)$$

or, $\dfrac{a^4 + b^4}{a^2 + b^2 - \sqrt{2}\,ab} = a^2 + b^2 + \sqrt{2}\,ab = x + \sqrt{2}\,y$

90. (d) Given, $x = \dfrac{a}{a+b}$

$$\Rightarrow x^2 = \frac{a^2}{(a+b)^2}$$

$$y = \frac{b}{a+b}$$

$$\Rightarrow y^2 = \frac{b^2}{(a+b)^2}$$

$$\therefore \frac{x^2}{y^2} = \frac{a^2}{b^2}$$

By componendo and dividendo, $\dfrac{x^2 + y^2}{x^2 - y^2} = \dfrac{a^2 + b^2}{a^2 - b^2}$

91. (b) $\dfrac{x}{y} = \dfrac{(p+q)^2}{(p-q)^2}$

By componendo and dividendo,

$$\frac{x-y}{x+y} = \frac{(p+q)^2 - (p-q)^2}{(p+q)^2 + (p-q)^2} = \frac{4pq}{2(p^2+q^2)} = \frac{2pq}{p^2+q^2}$$

92. (a) $x + \dfrac{a}{x} = 1 \Rightarrow x - 1 = \dfrac{a}{x}$

Now, $\dfrac{x^3 - x^2}{x^2 + x + a} = \dfrac{x^2(x-1)}{x\left(x + \frac{a}{x}\right) + x} = \dfrac{x^2 \cdot \frac{a}{x}}{x \cdot 1 + x}$

putting $x - 1 = \dfrac{a}{x}$ and $x + \dfrac{a}{x} = 1$ $= \dfrac{ax}{2x} = \dfrac{a}{2}$

93. (a) $\because \quad a = \dfrac{xy}{x+y}, b = \dfrac{xz}{x+z}$ and $c = \dfrac{yz}{y+z}$

$$\therefore \quad \frac{x+y}{xy} = \frac{1}{a}, \frac{x+z}{xz} = \frac{1}{b}, \frac{y+z}{yz} = \frac{1}{c}$$

or, $\frac{1}{y}+\frac{1}{x}=\frac{1}{a}, \frac{1}{z}+\frac{1}{x}=\frac{1}{b}, \frac{1}{z}+\frac{1}{y}=\frac{1}{c}$

$\therefore \left(\frac{1}{y}+\frac{1}{x}\right)+\left(\frac{1}{z}+\frac{1}{x}\right)-\left(\frac{1}{z}+\frac{1}{y}\right)=\frac{1}{a}+\frac{1}{b}-\frac{1}{c}$

or, $\frac{2}{x}=\frac{bc+ca-ab}{abc}$

or, $x=\frac{2abc}{bc+ca-ab}$

94. **(c)** HCF of expression $= (a + 1)$

LCM of expression $= a^3 + a^2 - a - 1 = (a + 1)(a + 1)(a - 1)$

First expression $= a^2 - 1 = (a - 1)(a + 1)$

We know that,

$$\text{Second expression} = \frac{\text{HCF} \times \text{LCM}}{\text{First expression}}$$

$$= \frac{(a+1)(a+1)(a+1)(a-1)}{(a-1)(a+1)} = (a+1)^2$$

95. **(c)** $\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = p+2$

$\Rightarrow x+\frac{1}{x}=\sqrt{p+2}$

Now, $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)\left(x\cdot\frac{1}{x}\right)$

$= (\sqrt{p+2})^3 - 3(\sqrt{p+2})$

$= (\sqrt{p+2})((\sqrt{p+2})^2-3)$

$= (\sqrt{p+2})(p+2-3)$

$= (p-1)\sqrt{p+2}$

96. **(c)** We know that, when $a + b + c = 0$ then

$\Rightarrow a^2 + b^2 + c^2 = -(2ab + 2bc + 2ca)$ ... (i)

$\therefore$ Given expression, $\frac{a^2+b^2+c^2}{(a-b)^2+(b-c)^2+(c-a)^2}$

$= \frac{a^2+b^2+c^2}{2(a^2+b^2+c^2)-(2ab+2bc+2ca)}$

$= \frac{a^2+b^2+c^2}{2(a^2+b^2+c^2)+(a^2+b^2+c^2)}$ *[from equation (i)]*

$= \frac{a^2+b^2+c^2}{3(a^2+b^2+c^2)} = \frac{1}{3}$

97. (c) Given $x^4 + x^3 - 4x^2 + x + 1 = 0$ dividing by x^2

$\Rightarrow \quad x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0$

$\Rightarrow \quad x^2 + \frac{1}{x^2} + x + \frac{1}{x} - 4 = 0$

$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2 + x + \frac{1}{x} - 6 = 0$

$\Rightarrow \quad \left(x + \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right) - 6 = 0$

$\Rightarrow \quad y^2 + y - 6 = 0 \qquad (\because y = x + \frac{1}{x})$

Which is required expression.

98. (c) Given expression,

$= (2 + 1)(2^2 + 1)(2^4 + 1) \ldots (2^{64} + 1)$

$= \frac{(2-1)(2+1)(2^2+1)(2^4+1)\ldots(2^{64}+1)}{(2-1)}$

$= (2^2 - 1)(2^2 + 1)(2^4 + 1) \ldots (2^{64} + 1)(2^4 - 1)(2^4 + 1) \ldots (2^{64} + 1)$

= ....

$= (2^{64} - 1)(2^{64} + 1) = 2^{128} - 1$

99. (a) First polynomial

$p(x) = x^3 + 3x^2y + 2xy^2$

$= x(x^2 + 3xy + 2y^2) = x(x^2 + 2xy + xy + 2y^2)$

$= x[x(x + 2y) + y(x + 2y)] = x(x + y)(x + 2y)$

Second polynomial

$= x^4 + 6x^3y + 8x^2y^2$

$= x^2(x^2 + 6xy + 8y^2) = x^2(x^2 + 2xy + 4xy + 8y^2)$

$= x^2[x(x + 2y) + 4y(x + 2y)] = x^2(x + 2y)(x + 4y)$

$\therefore$ HCF $= x(x + 2y)$

00. (a) $\because pqr = 1 \qquad \therefore \quad r = p^{-1}q^{-1}$ and $r^{-1} = pq$

Eliminating r from given expression,

Given expression $= \frac{1}{1+p+q^{-1}} + \frac{1}{1+q+pq} + \frac{1}{1+p^{-1}q^{-1}+p^{-1}}$

$= \frac{q}{q+pq+1} + \frac{1}{1+q+pq} + \frac{pq}{pq+1+q} = \frac{q+1+pq}{q+pq+1} = 1$

101.(a) $\because \quad x + y + z = 2s$

and $(s - x) + (s - y) - z = 2s - (x + y + z) = 2s - 2s = 0$

$\therefore \quad (s - x)^3 + (s - y)^3 - z^3 + 3(s - x)(s - y)(z) = 0$

$\Rightarrow \quad (s - x)^3 + (s - y)^3 + 3(s - x)(s - y)(z) = z^3$

102.(a) $x^2 = y + z \Rightarrow x^2 + x = x + y + z$

$\Rightarrow \quad \frac{x}{x+y+z} = \frac{1}{x+1}$

Similarly $\frac{1}{y+1} = \frac{y}{x+y+z}$ and $\frac{1}{z+1} = \frac{z}{x+y+z}$

$\therefore \quad \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = \frac{x}{x+y+z} + \frac{y}{x+y+z} + \frac{z}{x+y+z} = \frac{x+y+z}{x+y+z} = 1.$

103.(b) $\because \quad p + q + (-r) = 0$

$\therefore \quad p^3 + q^3 + (-r)^3 = 3pq(-r) = 3 \times (-30) = -90$

(Recall that $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$)

104.(a) $\frac{x^5-1}{x-1} + x^3 + 2x^2 + x$

$= \frac{x^5 - 1 + x^4 + 2x^3 + x^2 - x^3 - 2x^2 - x}{x-1} = \frac{x^5 + x^4 + x^3 - x^2 - x - 1}{x-1}$

$= \frac{x^3(x^2+x+1) - 1(x^2+x+1)}{(x-1)} = \frac{(x^2+x+1)(x^3-1)}{x-1}$

$= \frac{(x^2+x+1)(x-1)(x^2+x+1)}{(x-1)} = (x^2 + x + 1)^2$

$\therefore$ Required square root $= \sqrt{(x^2+x+1)^2} = x^2 + x + 1$

Second method,

$\because \quad \frac{x^5-1}{x-1} = x^4 + x^3 + x^2 + x + 1$

$\therefore$ Given expression, $= x^4 + x^3 + x^2 + x + 1 + x^3 + 2x^2 + x$

$= x^4 + x^2 + 2 \cdot x^3 + 2x^2 + x = (x^2 + x + 1)^2$

105.(b) $\frac{x^8+4}{x^4+2x^2+2} = \frac{x^8 + 4x^4 + 4 - 4x^4}{x^4+2x^2+2}$

$= \frac{(x^4+2)^2 - (2x^2)^2}{x^4+2x^2+2}$

$= \frac{(x^4+2x^2+2)(x^4-2x^2+2)}{(x^4+2x^2+2)}$

$= x^4 - 2x^2 + 2$

106.(d) Given, $x+\frac{1}{x}=p$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2=p^2$$

$$\Rightarrow x^2+\frac{1}{x^2}+2=p^2$$

$$\Rightarrow x^2+\frac{1}{x^2}=p^2-2$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^3=(p^2-2)^3$$

$$\Rightarrow x^6+\frac{1}{x^6}+3\left(x^2+\frac{1}{x^2}\right)=p^6-8-6p^2(p^2-2)$$

$$\Rightarrow x^6+\frac{1}{x^6}+3(p^2-2)=p^6-8-6p^4+12p^2$$

$$\Rightarrow x^6+\frac{1}{x^6}=p^6-6p^4+9p^2-2$$

107.(c) Given, $\left(\frac{y-z-x}{2}\right)^3+\left(\frac{z-x-y}{2}\right)^3+\left(\frac{x-y-z}{2}\right)^3$

$$=\left(\frac{y-(z+x)}{2}\right)^3+\left(\frac{z-(x+y)}{2}\right)^3+\left(\frac{x-(y+z)}{2}\right)^3$$

$$=\left(\frac{y-(-y)}{2}\right)^3+\left(\frac{z-(-z)}{2}\right)^3+\left(\frac{x-(-x)}{2}\right)^3$$

($\because x+y+z=0, \Rightarrow x+z=-y$ etc)

$$=\left(\frac{2y}{2}\right)^3+\left(\frac{2z}{2}\right)^3+\left(\frac{2x}{2}\right)^3=y^3+z^3+x^3$$

$=3xyz$ ($\because$ if $a+b+c=0$, then $a^3+b^3+c^3=3abc$)

Exercise–1B

1. If $a+b+1=0$ then what is the value of $(a^3+b^3+1-3ab)$?

   (a) –1 (b) 1 (c) 3 (d) 0

   [SSC Tier-I 2012]

2. If $x=(0.08)^2$, $y=\frac{1}{(0.08)^2}$ and $z=(1-0.08)^2-1$ then which of the following relation is true ?

   (a) $y<z<x$ (b) $z<x<y$

   (c) $y<x$ and $x=z$ (d) $x<y$ and $x=z$ *[SSC Tier-I 2012]*

3. If $x^4 + \frac{1}{x^4} = 23$ then what is the value of $\left(x - \frac{1}{x}\right)^2$?

(a) -3 (b) 3 (c) 7 (d) -7

[SSC Tier-I 2012]

4. If $x + \frac{1}{x} = 3$ then what is the value of $x^5 + \frac{1}{x^5}$?

(a) 113 (b) 129 (c) 123 (d) 126

[SSC Tier-I 2012]

5. If $a + b = 6$, $a - b = 2$ then what is the value of $2(a^2 + b^2)$?

(a) 20 (b) 30 (c) 40 (d) 10

[SSC Tier-I 2012]

6. If $2a - \frac{2}{a} + 3 = 0$, then value of $\left(a^3 - \frac{1}{a^3} + 2\right)$ is—

(a) 5 (b) $-\frac{35}{8}$ (c) $-\frac{40}{7}$ (d) $\frac{-47}{8}$

[SSC Tier-I 2012]

7. If factors of $x^3 + (a+1)x^2 - (b-2)x - 6$ are $(x+1)$ and $(x-2)$ then values of a and b are respectively is—

(a) 2 and 8 (b) 1 and 7 (c) 5 and 3 (d) 3 and 7

[SSC Tier-I 2012]

8. If x is real and $x^4 + \frac{1}{x^4} = 119$, then value of $\left(x - \frac{1}{x}\right)$ is

(a) ± 4 (b) ± 9 (c) ± 3 (d) ± 2

[SSC Tier-I 2012]

9. If $x^3 + y^3 = 35$ and $x + y = 5$, then value of $\left(\frac{1}{x} + \frac{1}{y}\right)$ is

(a) $\frac{4}{7}$ (b) $\frac{3}{8}$ (c) $\frac{5}{6}$ (d) $\frac{3}{5}$

[SSC Tier-I 2012]

10. If $\frac{x^2}{by+cz} = \frac{y^2}{cz+ax} = \frac{z^2}{ax+by} = 1$, then value of $\frac{a}{a+x} + \frac{b}{b+y} + \frac{c}{c+z}$ is

(a) -1 (b) 2 (c) 1 (d) -2

[SSC Tier-I 2012]

11. Value of a and b $(a > 0, b < 0)$ for which $8x^3 - ax^2 + 54x + b$ is a perfect cube is

(a) $a = 12, b = -9$ (b) $a = 36, b = -27$

(c) $a = 18, b = -27$ (d) $a = 16, b = -9$

[SSC Tier-I 2012]

12. If $x = \frac{4ab}{a+b}$, then value of $\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$ is

(a) a (b) b (c) 0 (d) 2

[SSC Tier-I 2012]

13. If $a + b + c = 8$, then value of $(a-4)^3 + (b-3)^3 + (c-1)^3 - 3(a-4)(b-3)(c-1)$ is
(a) 2 (b) 4 (c) 1 (d) 0
[SSC Tier-I 2012]

14. If $x = \sqrt{a} + \frac{1}{\sqrt{a}}$, $y = \sqrt{a} - \frac{1}{\sqrt{a}}$, then value of $x^4 + y^4 - 2x^2y^2$ is
(a) 16 (b) 20 (c) 10 (d) 5
[SSC Tier-I 2012]

15. If $5a + \frac{1}{3a} = 5$, then value of $9a^2 + \frac{1}{25a^2}$ is
(a) $\frac{51}{5}$ (b) $\frac{29}{5}$ (c) $\frac{52}{5}$ (d) $\frac{39}{5}$
[SSC Tier-I 2012]

16. If $a + b + c = 0$, then value of $\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right)$ is
(a) 2 (b) 3 (c) 4 (d) 5
[SSC Tier-I 2012]

17. If a, b, c are real, $a^3 + b^3 + c^3 = 3abc$ and $a + b + c \neq 0$, then relation between a, b, c will be
(a) $a + b = c$ (b) $a + c = b$ (c) $a = b = c$ (d) $b + c = a$
[SSC Tier-I 2012]

18. If $a^2 + \frac{1}{a^2} = 98$ $(a > 0)$ then what is the value of $a^3 + \frac{1}{a^3}$?
(a) 535 (b) 1030 (c) 790 (d) 970
[SSC Tier-I 2012]

19. If $x + \frac{1}{x} = 5$ then what is the value of $\frac{x^4 + \frac{1}{x^2}}{x^2 - 3x + 1}$?
(a) 70 (b) 50 (c) 110 (d) 55
[SSC Tier-I 2012]

20. If $a^2 + b^2 + c^2 = 2(a - b - c) - 3$, then what is the value of $2a - 3b + 4c$?
(a) 3 (b) 1 (c) 2 (d) 4
[SSC Tier-I 2012]

21. If $2x - \frac{1}{2x} = 6$ then what is the value of $x^2 + \frac{1}{16x^2}$?
(a) $\frac{19}{2}$ (b) $\frac{17}{2}$ (c) $\frac{18}{3}$ (d) $\frac{15}{2}$
[SSC Tier-I 2012]

22. If $(5x^2 - 3y^2) : xy = 11 : 2$ then what is the positive value of $\frac{x}{y}$?
(a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{3}$ (d) $\frac{7}{2}$
[SSC Tier-I 2012]

[SSC Tier-I 2012]

23. If $ax + by = 6$, $bx - ay = 2$ and $x^2 + y^2 = 4$ then what is $(a^2 + b^2)$?
(a) 2 (b) 4 (c) 5 (d) 10
[SSC Tier-I 2012]

2
[SSC Tier-I 2012]

24. If $a + \frac{1}{a+2} = 0$ then what is the value of $(a+2)^{3}$ ... $(a+2)$

(a) 6 (b) 4 (c) 3 (d) 2

[SSC Tier-I 2012]

25. If $a^3 - b^3 = 56$ and $a - b = 2$ then what is the value of $(a^2 + b^2)$?

(a) -12 (b) 20 (c) 18 (d) -10

[SSC Tier-I 2012]

26. If $a + \frac{1}{a} = 1$ then what is the value of a^3 ?

(a) -2 (b) 2 (c) -1 (d) 4

[SSC Tier-I 2012]

27. If $(a - b) = 3$, $(b - c) = 5$ and $(c - a) = 1$ then what is the value of $\frac{a^3+b^3+c^3-3abc}{a+b+c}$?

(a) 17.5 (b) 20.5 (c) 10.5 (d) 15.5

[SSC Tier-I 2012]

28. If $x = 2t$ and $y = \frac{2t-1}{3}$, then for what value of $t, x = y$ is correct ?

(a) $\frac{1}{3}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{4}$ (d) $-\frac{1}{2}$

[SSC Tier-I 2012]

29. If x and y are two real numbers and $x + y = 8$, then maximum value of xy is

(a) 16 (b) 18 (c) 12 (d) 15

[SSC Tier-I 2012]

Answer–1B

1. (d)	2. (b)	3. (b)	4. (a)	5. (c)	6. (d)	7. (b)	8. (c)
9. (c)	10. (c)	11. (b)	12. (d)	13. (d)	14. (a)	15. (d)	16. (b)
17. (c)	18. (d)	19. (d)	20. (b)	21. (a)	22. (b)	23. (d)	24. (d)
25. (b)	26. (c)	27. (★)	28. (b)	29. (a)			

Explanation

1. (d) $\because a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 - 3abc = 0$, here $c = 1$.

2. (b) $z = (1 - 0.08)^2 - 1 = (1 - 0.08 - 1)\ (1 - 0.08 + 1)$

$= (-0.08)\ (1.92) =$ a negative quantity.

$\because\ x = (0.08)^2 \qquad \because 0 < x < 1$

$\because\ y = \frac{1}{(0.08)^2} \qquad \therefore\ y > 1$

Hence, $z < x < y$

3. (b) Trick, if $x - \frac{1}{x} = a$ then $x^4 + \frac{1}{x^4} = a^4 + 4a^2 + 2$

According to question, $a^4 + 4a^2 + 2 = 23$

or, $a^4 + 4a^2 - 21 = 0$

or, $a^4 - 3a^2 + 7a^2 - 21 = 0$

or, $(a^2 - 3)(a^2 + 7) = 0$

or, $a^2 = 3$ $(\because a^2 + 7 \neq 0)$

$\therefore \left(x - \frac{1}{x}\right)^2 = 3$

4. (a) Trick, if $x + \frac{1}{x} = a$ then $x^5 + \frac{1}{x^5} = a^5 - 5a^3 + 5a$

Here, $x + \frac{1}{x} = 3$

$\therefore x^5 + \frac{1}{x^5} = 3^5 - 5 \times 3^3 + 5 \times 3$

$= 243 - 145 + 15$

$= 243 - 130 = 113$

5. (c) $(a + b)^2 + (a - b)^2 = 6^2 + 2^2$

$2(a^2 + b^2) = 40$

$a^2 + b^2 = 20$

6. (d) $2\left(a - \frac{1}{a}\right) + 3 = 0$

$\Rightarrow a - \frac{1}{a} = \frac{-3}{2}$

Cubing both side, $a^3 - \frac{1}{a^3} - 3a^2\frac{1}{a} + 3a\frac{1}{a^2} = \frac{-27}{8}$

or, $a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right) = \frac{-27}{8}$

or, $a^3 - \frac{1}{a^3} - 3\left(-\frac{3}{2}\right) = \frac{-27}{8}$

or, $a^3 - \frac{1}{a^3} = \frac{-9}{2} - \frac{27}{8} = \frac{-63}{8}$

$\therefore a^3 - \frac{1}{a^3} + 2 = \frac{-63}{8} + 2 = \frac{-63 + 16}{8} = \frac{-47}{8}$

7. (b) Given expression $= x^3 + (a + 1)x^2 - (b - 2)x - 6$

$\because$ $x + 1$ and $x - 2$ are its factor, then $x = -1$ and $x = 2$ are its zeros.

putting $x = -1$, $-1 + (a + 1) + b - 2 - 6 = 0$ ... (i)

or, $a + b = 8$

putting $x = 2$, $8 + 4(a+1) - 2(b-2) - 6 = 0$ ... (ii)

or, $4a - 2b = -10$

Solving (1) and (2), $a = 1, b = 7$

9. (c) $x^3 + y^3 = 35$

$\Rightarrow (x+y)(x^2 - xy + y^2) = 35$

$\Rightarrow 5(x^2 - xy + y^2) = 35$

$\Rightarrow x^2 - xy + y^2 = 7$

or, $(x+y)^2 - 3xy = 7$ $(\because x + y = 5)$

or, $25 - 3xy = 7$

or, $3xy = 18$

or, $xy = 6$

$\therefore \frac{1}{x} + \frac{1}{y} = \frac{y+x}{xy} = \frac{5}{6}$

10. (c) From $\frac{x^2}{by+cz} = \frac{y^2}{cz+ax} = \frac{z^2}{ax+by} = 1$

$x^2 = by + cz,\ y^2 = cz + ax,\ z^2 = ax + by$ ... (i)

Given expression $= \frac{a}{a+x} + \frac{b}{b+y} + \frac{c}{c+z}$

$= \frac{a}{a+x} \times \frac{x}{x} + \frac{b}{b+y} \times \frac{y}{y} + \frac{c}{c+z} \times \frac{z}{z}$

$= \frac{ax}{ax+x^2} + \frac{by}{by+y^2} + \frac{cz}{cz+z^2}$

$= \frac{ax}{ac+by+cz} + \frac{by}{by+cz+ax} + \frac{cz}{cz+ax+by}$

Putting value of x^2, y^2, z^2 from (1) $\frac{ax+by+cz}{ax+by+cz} = 1$

11. (b) $8x^3 - ax^2 + 54x + b = (2x)^3 - 3\cdot(2x)^2\left(\frac{a}{12}\right) + 3(2x)(3)^2 + b$

Clearly $\frac{a}{12} = 3$ and $b = -3^3$

or, $a = 36, b = -27$

12. (d) $x = \frac{4ab}{a+b}$

$\Rightarrow \frac{x}{2a} = \frac{2b}{a+b}$

$\Rightarrow \frac{x+2a}{x-2a} = \frac{2b+a+b}{2b-(a+b)}$ *(by componendo and dividendo)*

$\Rightarrow \frac{x+2a}{x-2a} = \frac{3b+a}{b-a}$ ... (i)

Again from $x = \frac{4ab}{a+b}$, $\frac{x}{2b} = \frac{2a}{a+b}$

or, $\frac{x+2b}{x-2b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b}$ ... (ii)

Adding (1) and (2),

$$\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = \frac{(3b+a)-(3a+b)}{b-a} = \frac{2(b-a)}{b-a} = 2$$

tricky approach,

putting $a = 1$, $b = 3$, $x = \frac{4 \times 1 \times 3}{1+3} = 3$

$$\therefore \quad \frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = \frac{3+2}{3-2} + \frac{3+6}{3-6} = 5 - 3 = 2$$

13. (d) $a + b + c = 8$

$\Rightarrow \quad (a - 4) + (b - 3) + (c - 1) = 8 - 4 - 3 - 1 = 0$

$\because \quad x + y + z = 0$

$\Rightarrow \quad x^3 + y^3 + z^3 - 3xyz = 0$

putting, $x = a - 4$, $y = b - 3$, $z = c - 1$,

$$(a-4)^3 + (b-3)^3 + (c-1)^3 - 3(a-4)(b-3)(c-1) = 0$$

14. (a) $x^4 + y^4 - 2x^2y^2 = (x^2 - y^2)^2 = ((x+y)(x-y))^2$

$$= \left(2\sqrt{a} \cdot \frac{2}{\sqrt{a}}\right)^2 = 16 \qquad (\because x + y = 2\sqrt{a} \text{ and } x - y = \frac{2}{\sqrt{a}})$$

15. (d) $5a + \frac{1}{3a} = 5$

$\Rightarrow \quad \frac{5}{3}\left(3a + \frac{1}{5a}\right) = 5$

$\Rightarrow \quad 3a + \frac{1}{5a} = 3$ now squaring both sides.

16. (b) $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc} = \frac{3abc}{abc} = 3$

$(\because \; a + b + c = 0, a^3 + b^3 + c^3 = 3abc)$

17. (c) $a^3 + b^3 + c^3 - 3abc = (a + b + c)\,\frac{1}{2}\,\{(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

$\because \quad a + b + c \neq 0$

$\therefore \quad \frac{1}{2}\left\{(a-b)^2 + (b-c)^2 = (c-a)^2\right\} = 0$

It is possible only when $(a - b) = 0$, $b - c = 0$ and $c - a = 0$

$\therefore \quad a = b, b = c, c = a$ or, $a = b = c$

18. (d) $a^2 + \frac{1}{a^2} = 98$

$$\Rightarrow \left(a+\frac{1}{a}\right)^2 - 2a\cdot\frac{1}{a} = 98$$

$$\Rightarrow \left(a+\frac{1}{a}\right)^2 = 100$$

$$\Rightarrow a + \frac{1}{a} = 10$$

Now, $a^3 + \frac{1}{a^3} = \left(a+\frac{1}{a}\right)^3 - 3a\cdot\frac{1}{a}\left(a+\frac{1}{a}\right)$

$$= 10^3 - 3\cdot 10 = 1000 - 30 = 970$$

19. (d) $\frac{x^4 + \frac{1}{x^2}}{x^2 - 3x + 1} = \frac{x\left(x^3 + \frac{1}{x^3}\right)}{x\left(x - 3 + \frac{1}{x}\right)} = \frac{x^3 + \frac{1}{x^3}}{\left(x + \frac{1}{x} - 3\right)}$

$$= \frac{\left(x+\frac{1}{x}\right)^3 - 3x\cdot\frac{1}{x}\left(x+\frac{1}{x}\right)}{\left(x+\frac{1}{x}\right) - 3}$$

$$= \frac{5^3 - 3\cdot 1\cdot 5}{5-3} = \frac{110}{2} = 55$$

20. (b) Given, $a^2 + b^2 + c^2 - 2a + 2b + 2c + 3 = 0$

or, $(a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$

or, $(a-1)^2 + (b+1)^2 + (c+1)^2 = 0$

It is possible only when, $a - 1 = 0$, $b + 1 = 0$ and $c + 1 = 0$

$$\therefore\ a = 1, b = -1, c = -1$$

Required value= $2a - 3b + 4c = 2 + 3 - 4 = 1$

21. (a) $2x - \frac{1}{2x} = 6$

$$\Rightarrow 2\left(x - \frac{1}{4x}\right) = 6$$

$$\Rightarrow x - \frac{1}{4x} = 3$$

Squaring, $x^2 + \frac{1}{16x^2} - 2\cdot x\cdot\frac{1}{4x} = 9$

or, $x^2 + \frac{1}{16x^2} = 9 + \frac{1}{2} = \frac{19}{2}$

22. (b) $\frac{5x^2 - 3y^2}{xy} = \frac{11}{2}$

$5\frac{x}{y} - \frac{3y}{x} = \frac{11}{2}$

If $\frac{x}{y} = t$ then $5t - \frac{3}{t} = \frac{11}{2}$

or, $\frac{5t^2 - 3}{t} = \frac{11}{2}$

or, $10t^2 - 6 = 11t$

or, $10t^2 - 11t - 6 = 0$

or, $10t^2 - 15t + 4t - 6 = 0$

or, $5t(2t - 3) + 2(2t - 3) = 0$

$\therefore$ $t = \frac{3}{2}, \frac{-2}{5}$

Since $\frac{x}{y}$ is positive, $\therefore$ $t = \frac{x}{y} = \frac{3}{2}$

23. **(d)** $ax + by = 6$ and $bx - ay = 2$

Squaring and adding,

$a^2x^2 + b^2y^2 + 2axby + b^2x^2 + a^2y^2 - 2bxay = 36 + 4$

or, $a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2 = 40$

or, $(a^2 + b^2)x^2 + (b^2 + a^2)y^2 = 40$

or, $(a^2 + b^2)(x^2 + y^2) = 40$

or, $(a^2 + b^2) \times 4 = 40$

or, $a^2 + b^2 = 10$

24. **(d)** Adding 2 both sides of the equation $a + \frac{1}{a+2} = 0$

$a + 2 + \frac{1}{a+2} = 2$

Cubing, $(a + 2)^3 + 3(a + 2)^2\left(\frac{1}{a+2}\right) + 3(a+2)\left(\frac{1}{a+2}\right)^2 + \left(\frac{1}{a+2}\right)^3 = 8$

or, $(a+2)^3 + 3\left\{(a+2) + \frac{1}{a+2}\right\} + \frac{1}{(a+2)^3} = 8$

or, $(a+2)^3 + 3 \times 2 + \frac{1}{(a+2)^3} = 8$

$\therefore$ $(a + 2)^3 + \frac{1}{(a+2)^3} = 8 - 6 = 2$

25. (b) $\because\ a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

$\therefore\ 56 = 2(a^2 + b^2 + ab)$

$\Rightarrow\ 28 = a^2 + b^2 + ab = (a-b)^2 + 3ab$

$\Rightarrow\ 28 = 4 + 3ab$

$\Rightarrow\ ab = 8$

Now, $a^2 + b^2 = (a-b)^2 + 2ab = 2^2 + 2 \times 8 = 20$

26. (c) $a + \frac{1}{a} = 1$ Cubing, $a^3 + 3a + 3\cdot\frac{1}{a} + \frac{1}{a^3} = 1$

or, $a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = 1$

or, $a^3 + \frac{1}{a^3} + 3 \times 1 = 1$

or, $t + \frac{1}{t} = 1 - 3 = -2$ (where $t = a^3$)

or, $t^2 + 1 + 2t = 0$

$\Rightarrow\ (t+1)^2 = 0$

$\Rightarrow\ t = -1$

or, $a^3 = -1$

27. (★) $(a-b) + (b-c) = 3 + 5 = 8$

or, $a - c = 8$ or, $c - a = -8$

but in next relation $c - a = 1$

$\therefore$ question is wrong

28. (b) $x = y \Rightarrow 2t = \frac{2t-1}{3}$

$\Rightarrow 6t = 2t - 1$

$\Rightarrow t = \frac{-1}{4}$

29. (a) We know that, $(x-y)^2 \geq 0$

or, $x^2 + y^2 - 2xy \geq 0$

or, $(x+y)^2 - 4xy \geq 0$

or, $64 - 4xy \geq 0$

or, $xy \leq 16$

★★★