UNIT–I : RELATIONS AND FUNCTIONS

CHAPTER 1

Term-I

RELATIONS AND FUNCTIONS

Syllabus

- ***Types of relations : Reflexive, Symmetric, Transitive and Equivalence relations. One-to-one and onto functions***

STAND ALONE MCQs (1 Mark each)

Q. 1. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to $b \ \forall \ a, b \in T$. Then R is

(A) reflexive but not transitive

(B) transitive but not symmetric

(C) equivalence relation

(D) None of these

Ans. Option (C) is correct.

> ***Explanation:*** Consider that aRb, if a is congruent to b, $\forall \ a, b \in T$.
> Then, $aRa \Rightarrow a \cong a$,
> Which is true for all $a \in T$
> So, R is reflexive, ...(i)
> Let $aRb \Rightarrow a \cong b$
> $\Rightarrow b \cong a$
> $\Rightarrow bRa$
> So, R is symmetric. ...(ii)
> Let aRb and bRc
> $\Rightarrow b \cong b$ and $b \cong a$
> $\Rightarrow a \cong c \Rightarrow aRc$
> So, R is transitive ...(iii)
> Hence, R is equivalence relation.

Q. 2. Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is

(A) symmetric but not transitive

(B) transitive but not symmetric

(C) neither symmetric nor transitive

(D) both symmetric and transitive

Ans. Option (B) is correct.

> ***Explanation:*** $aRb \Rightarrow a$ is brother of b.
> This does not mean b is also a brother of a as b can be a sister of a.
> Hence, R is not symmetric.
> $aRb \Rightarrow a$ is brother of b
> and $bRc \Rightarrow b$ is a brother of c.
> So, a is brother of c.
> Hence, R is transitive.

Q. 3. The maximum number of equivalence relations on the set A = {1, 2, 3} are

(A) 1 **(B)** 2

(C) 3 **(D)** 5

Ans. Option (D) is correct.

> ***Explanation:*** Given that, $A = \{1, 2, 3\}$
> Now, number of equivalence relations are as follows:
> $$R_1 = \{(1, 1), (2, 2), (3, 3)\}$$
> $$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$$

$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$
$R_4 = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$
$R_5 = \{(1, 2, 3) \Leftrightarrow A \times A = A^2\}$
$\therefore$ Maximum number of equivalence relations on the set $A = \{1, 2, 3\} = 5$

Q. 4. If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is

(A) reflexive **(B)** transitive
(C) symmetric **(D)** None of these

Ans. Option (B) is correct.

Explanation: R on the set $\{1, 2, 3\}$ is defined by $R = \{(1, 2)\}$
It is clear that R is transitive.

Q. 5. Let us define a relation R in R as aRb if $a \geq b$. Then R is

(A) an equivalence relation
(B) reflexive, transitive but not symmetric
(C) symmetric, transitive but not reflexive
(D) neither transitive nor reflexive but symmetric.

Ans. Option (B) is correct.

Explanation: Given that, aRb if $a \geq b$
$\Rightarrow$ $aRa \Rightarrow a \geq a$ which is true
Let aRb, $a \geq b$, then $b \geq a$ which is not true as R is not symmetric.
But aRb and bRc
$\Rightarrow$ $a \geq b$ and $b \geq c$
$\Rightarrow$ $a \geq c$
Hence, R is transitive.

Q. 6. Let $A = \{1, 2, 3\}$ and consider the relation $R = (1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$.
Then R is

(A) reflexive but not symmetric
(B) reflexive but not transitive
(C) symmetric and transitive
(D) neither symmetric, nor transitive

Ans. Option (A) is correct.

Explanation: Given that $A = \{1, 2, 3\}$
and $R = (1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$.
$\because$ $(1, 1), (2, 2), (3, 3) \in R$
Hence, R is reflexive.
$(1, 2) \in R$ but $(2, 1) \notin R$
Hence, R is not symmetric.
$(1, 2) \in R$ and $(2, 3) \in R$
$\Rightarrow$ $(1, 3) \in R$
Hence, R is transitive.

Q. 7. Let R be the relation in the set $\{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Choose the correct answer:

(A) R is reflexive and symmetric but not transitive
(B) R is reflexive and transitive but not symmetric
(C) R is symmetric and transitive but not reflexive
(D) R is an equivalence relation

Ans. Option (B) is correct.

Explanation: Let R be the relation in the set $\{1, 2, 3, 4\}$ is given by:
$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$
(a) $(1, 1), (2, 2), (3, 3), (4, 4) \in R$
Therefore, R is reflexive.
(b) $(1, 2) \in R$ but $(2,1) \notin R$.
Therefore, R is not symmetric.
(c) If $(1, 3) \in R$ and $(3, 2) \notin R$ then $(1, 2) \in R$.
Therefore, R is transitive.

Q. 8. Let $A = \{1, 2, 3\}$. Then number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and symmetric but not transitive is

(A) 1 **(B)** 2
(C) 3 **(D)** 4

Ans. Option (A) is correct.

Explanation: The given set is $A = \{1, 2, 3\}$.
The smallest relation containing $(1, 2)$ and $(1, 3)$, which is reflexive and symmetric, but not transitive is given by:
$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (3, 1)\}$
This is because relation R is reflexive as $(1, 1), (2, 2), (3, 3) \in R$.
Relation R is symmetric since $(1, 2), (2, 1) \in R$ and $(1, 3), (3, 1) \in R$.
But relation R is not transitive as $(3, 1), (1, 2) \in R$, but $(3, 2) \notin R$.
Now, if we add any two pairs $(3, 2)$ and $(2, 3)$ (or both) to relation R, then relation R will become transitive.
Hence, the total number of desired relations is one.

Q. 9. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is

(A) 720 **(B)** 120
(C) 0 **(D)** None of these

Ans. Option (C) is correct.

Explanation: We know that, if A and B are two non-empty finite sets containing m and n elements, respectively, then the number of one-one and onto mapping from A to B is
$n!$ if $m = n$
0, if $m \neq n$
Given that, $m = 5$ and $n = 6$
$\therefore m \neq n$
Number of one-one and onto mapping = 0

Q. 10. Let $A = \{1, 2, 3, ...n\}$ and $B = \{a, b\}$. Then the number of surjections from A into B is

(A) ${}^nP_2$ **(B)** $2^n - 2$
(C) $2^n - 1$ **(D)** None of these

Ans. Option (B) is correct.

Explanation: Total number of functions from A to $B = 2^n$

Number of into functions = 2

Number of surjections from A to $B = 2^n - 2$

Q. 11. Let $f : R \rightarrow R$ be defined by $f(x) = \frac{1}{x}$, $\forall x \in R$. Then f is

(A) one-one **(B)** onto

(C) bijective **(D)** f is not defined

Ans. Option (D) is correct.

Explanation: We have, $f(x) = \frac{1}{x}$, $\forall x \in R$

For $x = 0$, $f(x)$ is not defined.

Hence, $f(x)$ is a not defined function.

Q. 12. Which of the following functions from Z into Z are bijections?

(A) $f(x) = x^3$ **(B)** $f(x) = x + 2$

(C) $f(x) = 2x + 1$ **(D)** $f(x) = x^2 + 1$

Ans. Option (B) is correct.

Explanation: For bijection on Z, $f(x)$ must be one-one and onto.

Function $f(x) = x^2 + 1$ is many-one as $f(1) = f(-1)$

Range of $f(x) = x^3$ is not Z for $x \in$ Z.

Also $f(x) = 2x + 1$ takes only values of type $= 2k + 1$ for $x \in k \in$ Z

But $f(x) = x + 2$ takes all integral values for $x \in$ Z

Hence $f(x) = x + 2$ is bijection of Z.

Q. 13. Let $f : R \rightarrow R$ be defined as $f(x) = x^4$. Choose the correct answer.

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto

Ans. Option (D) is correct.

Explanation: We know that $f : R \rightarrow R$ is defined as $f(x) = x^4$.

Let $x, y \in R$ such that $f(x) = f(y)$

$\Rightarrow \quad x^4 = y^4$

$\Rightarrow \quad x = \pm y$

$\therefore \quad f(x) = f(y)$

does not imply that $x = y$.

For example, $f(1) = f(-1) = 1$

$\therefore$ f is not one-one.

Consider an element 2 in co-domain R. It is clear that there does not exist any x in domain R such that $f(x) = 2$.

$\therefore$ f is not onto.

Hence, function f is neither one-one nor onto.

Q. 14. Let $f : R \rightarrow R$ be defined as $f(x) = 3x$. Choose the correct answer.

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto

Ans. Option (A) is correct.

Explanation: $f : R \rightarrow R$ is defined as $f(x) = 3x$.

Let $x, y \in R$ such that $f(x) = f(y)$

$\Rightarrow \quad 3x = 3y$

$\Rightarrow \quad x = y$

$\therefore$ f is one-one.

Also, for any real number y in co-domain R, there exists $\frac{y}{3}$ in R such that $f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$.

$\therefore$ f is onto.

Hence, function f is one-one and onto.

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Let W be the set of words in the English dictionary. A relation R is defined on W as

$R = \{(x, y) \in W \times W$ such that x and y have at least one letter in common$\}$.

Assertion (A): R is reflexive.

Reason (R): R is symmetric.

Ans. Option (B) is correct.

Explanation: For any word $x \in W$

x and x have atleast one (all) letter in common

$\therefore \quad (x, x) \in R, \forall x \in W \therefore R$ is reflexive

Symmetric: Let $(x, y) \in R, x, y \in W$

$\Rightarrow$ x and y have atleast one letter in common

$\Rightarrow$ y and x have atleast one letter in common

$\Rightarrow$ $(y, x) \in R \therefore R$ is symmetric

Hence A is true, R is true; R is not a correct explanation for A.

Q. 2. Let R be the relation in the set of integers Z given by $R = \{(a, b) : 2 \text{ divides } a - b\}$.

Assertion (A): R is a reflexive relation.

Reason (R): A relation is said to be reflexive if xRx, $\forall x \in Z$.

Ans. Option (A) is correct.

Explanation: By definition, a relation in Z is said to be reflexive if xRx, $\forall x \in Z$. So R is true.

$a - a = 0 \Rightarrow 2$ divides $a - a \Rightarrow aRa$.

Hence R is reflexive and A is true.

R is the correct explanation for A.

Q. 3. Consider the set $A = \{1, 3, 5\}$.

Assertion (A): The number of reflexive relations on set A is 2^9.

Reason (R): A relation is said to be reflexive if xRx, $\forall x \in A$.

Ans. Option (D) is correct.

Explanation: By definition, a relation in A is said to be reflexive if xRx, $\forall\, x \in A$. So R is true.

The number of reflexive relations on a set containing n elements is 2^{n^2-n}.

Here $n = 3$.

The number of reflexive relations on a set $A = 2^{9-3} = 2^6$.

Hence A is false.

Q. 4. Consider the function $f : R \to R$ defined as $f(x) = x^3$

Assertion (A): $f(x)$ is a one-one function.

Reason (R): $f(x)$ is a one-one function if co-domain = range.

Ans. Option (C) is correct.

Explanation: $f(x)$ is a one-one function if

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2.$$

Hence R is false.

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$

$\Rightarrow (x_1)^3 = (x_2)^3$

$\Rightarrow x_1 = x_2$

Hence $f(x)$ is one-one.

Hence A is true.

Q. 5. If $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B.

Assertion (A): $f(x)$ is a one-one function.

Reason (R): $f(x)$ is an onto function.

Ans. Option (C) is correct.

Given, $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f : A \to B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$ *i.e.*, $f(1) = 4$, $f(2) = 5$ and $f(3) = 6$.

It can be seen that the images of distinct elements of A under f are distinct. So, f is one-one.

So, A is true.

Range of $f = \{4, 5, 6\}$.

Co-domain $= \{4, 5, 6, 7\}$.

Since co-domain $\neq$ range, $f(x)$ is not an onto function. Hence R is false.

Q. 6. Consider the function $f : R \to R$ defined as

$$f(x) = \frac{x}{x^2+1}.$$

Assertion (A): $f(x)$ is not one-one.

Reason (R): $f(x)$ is not onto.

Ans. Option (B) is correct.

Explanation: Given, $f : R \to R$;

$$f(x) = \frac{x}{1+x^2}$$

$$\text{Taking } x_1 = 4,\ x_2 = \frac{1}{4} \in R$$

$$f(x_1) = f(4) = \frac{4}{17}$$

$$f(x_2) = f\left(\frac{1}{4}\right) = \frac{4}{17} \qquad (x_1 \neq x_2)$$

$\therefore$ f is not one-one.

A is true.

Let $y \in R$ (co-domain)

$f(x) = y$

$$\Rightarrow \frac{x}{1+x^2} = y$$

$$\Rightarrow y.(1+x^2) = x$$

$$\Rightarrow yx^2 + y - x = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

since, $x \in R$,

$$\therefore 1 - 4y^2 \geq 0$$

$$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$$

So Range $(f) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Range $(f) \neq R$ (Co-domain)

$\therefore f$ is not onto.

R is true.

R is not the correct explanation for A.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever

Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

ONE – NATION
ONE – ELECTION
FESTIVAL OF DEMOCRACY
GENERAL ELECTION - 2019

$R = \{(V_1, V_2) : V_1, V_2 \in$ I and both use their voting right in general election – 2019$\}$

[CBSE QB 2021]

Q. 1. Two neighbours X and $Y \in I$. X exercised his voting right while Y did not cast her vote in general election - 2019. Which of the following is true?

(A) $(X, Y) \in R$ **(B)** $(Y, X) \in R$
(C) $(X, X) \notin R$ **(D)** $(X, Y) \notin R$

Ans. Option (D) is correct.

Explanation: $(X, Y) \notin$ R.
$\because$ X exercised his voting right while, Y did not cast her vote in general election-2019
And $R = \{(V_1, V_2) : V_1 V_2 \in$ I and both use their voting right in general election-2019$\}$

Q. 2. Mr. 'X' and his wife 'W' both exercised their voting right in general election -2019, Which of the following is true?

(A) both (X, W) and $(W, X) \in R$
(B) $(X, W) \subset R$ but $(W, X) \not\subset R$
(C) both (X, W) and $(W, X) \notin R$
(D) $(W, X) \in R$ but $(X, W) \notin R$

Ans. Option (A) is correct.

Q. 3. Three friends F_1, F_2 and F_3 exercised their voting right in general election-2019, then which of the following is true?

(A) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$
(B) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \notin R$
(C) $(F_1, F_2) \in R$, $(F_2, F_2) \in R$ but $(F_3, F_3) \notin R$
(D) $(F_1, F_2) \notin R$, $(F_2, F_3) \notin R$ and $(F_1, F_3) \notin R$

Ans. Option (A) is correct.

Q. 4. The above defined relation R is __________

(A) Symmetric and transitive but not reflexive
(B) Universal relation
(C) Equivalence relation
(D) Reflexive but not symmetric and transitive

Ans. Option (C) is correct.

Explanation: R is reflexive, since every person is friend or itself.
i.e., $(F_1, F_2) \in R$
Further, $(F_1, F_2) \in R$
$\Rightarrow F_1$ is friend of F_2
$\Rightarrow F_2$ is friend of F_1
$\Rightarrow (F_2, F_1) \in R$
$\Rightarrow R$ is symmetric
Moreover, $(F_1, F_2), (F_2, F_3) \in R$
$\Rightarrow F_1$ is friend of F_2 and F_2 is friend of F_3.
$\Rightarrow F_1$ is a friend of F_3.
$\Rightarrow (F_1, F_3) \in R$
Therefore, R is an equivalence relation.

Q. 5. Mr. Shyam exercised his voting right in General Election - 2019, then Mr. Shyam is related to which of the following?

(A) All those eligible voters who cast their votes
(B) Family members of Mr. Shyam
(C) All citizens of India
(D) Eligible voters of India

Ans. Option (A) is correct.

II. Read the following text and answer the following questions on the basis of the same:

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set $\{1, 2, 3, 4, 5, 6\}$. Let A be the set of players while B be the set of all possible outcomes.

$A = \{S, D\}, B = \{1, 2, 3, 4, 5, 6\}$ **[CBSE QB 2021]**

Q. 1. Let R : B $\rightarrow$ B be defined by R $= \{(x, y) : y$ is divisible by $x\}$ is

(A) Reflexive and transitive but not symmetric
(B) Reflexive and symmetric but not transitive
(C) Not reflexive but symmetric and transitive
(D) Equivalence

Ans. Option (A) is correct.

Explanation: R is reflexive, since every element of B i.e.,
$B = \{1, 2, 3, 4, 5, 6\}$ is divisible by itself.
i.e., $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \in R$
further, $(1, 2) \in R$
but $(2, 1) \notin R$
Moreover,
$(1, 2), (2, 4) \in R$
$\Rightarrow (1, 4) \in R$
$\Rightarrow R$ is transitive.
Therefore, R is reflexive and transitive but not symmetric.

Q. 2. Raji wants to know the number of functions from A to B. How many number of functions are possible?

(A) 6^2 **(B)** 2^6
(C) $6!$ **(D)** 2^{12}

Ans. Option (A) is correct.

Q. 3. Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Then R is

(A) Symmetric
(B) Reflexive
(C) Transitive
(D) None of these

Ans. Option (D) is correct.

> ***Explanation:*** $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$
> R is not reflexive.
> Since, $(1, 1), (3, 3), (4, 4), (6, 6) \in R$
> R is not symmetric.
> Because, for $(1, 2) \in R$ there does not exist $(2, 1) \in R$.
> R is not transitive.
> Because for all element of B there does not exist,
> $(a, b)\ (b, c) \in R$ and $(a, c) \in R$.

Q. 4. Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible?

(A) 6^2 (B) 2^6
(C) $6!$ (D) 2^{12}

Ans. Option (D) is correct.

Q. 5. Let $R : B \rightarrow B$ be defined by $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$, then R is

(A) Symmetric
(B) Reflexive and Transitive
(C) Transitive and symmetric
(D) Equivalence

Ans. Option (B) is correct.

III. Read the following text and answer the following questions on the basis of the same:

An organization conducted bike race under 2 different categories–boys and girls. Totally there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ $G = \{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race. **[CBSE QB 2021]**

Ravi decides to explore these sets for various types of relations and functions

Q. 1. Ravi wishes to form all the relations possible from B to G. How many such relations are possible?

(A) 2^6 (B) 2^5
(C) 0 (D) 2^3

Ans. Option (A) is correct.

Q. 2. Let $R : B \rightarrow B$ be defined by $R = \{(x, y) : x$ and y are students of same sex$\}$, Then this relation R is_______

(A) Equivalence
(B) Reflexive only
(C) Reflexive and symmetric but not transitive
(D) Reflexive and transitive but not symmetric

Ans. Option (A) is correct.

> ***Explanation:***
> $R : B \rightarrow B$ be defined by $R = \{(x, y) : x$ and y are students of same sex$\}$
> R is reflexive, since, $(x, x) \in R$
> R is symmetric, since, $(x, y) \in R$ and $(y, x) \in R$
> R is transitive. For $a, b, c \in B$
> $\exists\ (a, b)\ (b, c) \in R$
> and $(a, c) \in R$.
> Therefore R is equivalence relation.

Q. 3. Ravi wants to know among those relations, how many functions can be formed from B to G?

(A) 2^2 (B) 2^{12}
(C) 3^2 (D) 2^3

Ans. Option (D) is correct.

Q. 4. Let $R : B \rightarrow G$ be defined by $R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$, then R is__________

(A) Injective
(B) Surjective
(C) Neither Surjective nor Injective
(D) Surjective and Injective

Ans. Option (B) is correct.

> ***Explanation:***
> $R : B \rightarrow G$ be defined by $R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$
> R is surjective, since, every element of G is the image of some element of B under R, i.e., For $g_1, g_2 \in G$,
> there exists an elements $b_1, b_2, b_3 \in B$,
> $(b_1\ g_1)\ (b_2, g_2), (b_3, g_1) \in R$.

Q. 5. Ravi wants to find the number of injective functions from B to G. How many numbers of injective functions are possible?

(A) 0 (B) 2!
(C) 3! (D) 0!

Ans. Option (A) is correct.

IV. Read the following text and answer the following questions on the basis of the same:

Students of Grade 9, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L.

[CBSE QB 2021]

Q. 1. Let relation R be defined by $R = \{(L_1, L_2) : L_1 \parallel L_2$ where $L_1, L_2 \in L\}$ then R is______ relation

(A) Equivalence
(B) Only reflexive
(C) Not reflexive
(D) Symmetric but not transitive

Ans. Option (A) is correct.

Explanation: Let relation R be defined by
$R = \{(L_1, L_2) : L_1 \parallel L_2$ where $L_1, L_2 \in L\}$.
R is reflexive, since every line is parallel to itself.
Further, $(L_1, L_2) \in R$
$\Rightarrow L_1$ is parallel to L_2
$\Rightarrow L_2$ is parallel to L_1
$\Rightarrow (L_2, L_1) \in R$
Hence, R is symmetric.
Moreover, $(L_1, L_2), (L_2, L_3) \in R$
$\Rightarrow L_1$ is parallel to L_2 and L_2 is parallel to L_3
$\Rightarrow L_1$ is parallel to L_3
$\Rightarrow (L_1, L_3) \in R$
Therefore, R is an equivalence relation

Q. 2. Let $R = \{(L_1, L_2) : L_1 \perp L_2$ where $L_1, L_2 \in L\}$ which of the following is true?

(A) R is Symmetric but neither reflexive nor transitive
(B) R is Reflexive and transitive but not symmetric
(C) R is Reflexive but neither symmetric nor transitive
(D) R is an Equivalence relation

Ans. Option (A) is correct.

Explanation: R is not reflexive, as a line L_1 can not be perpendicular to itself, i.e., $(L_1, L_1) \notin R$.
R is symmetric as $(L_1, L_2) \in R$
As, L_1 is perpendicular to L_2
and L_2 is perpendicular to L_1

$$(L_2, L_1) \in R$$

R is not transitive. Indeed, it L_1 is perpendicular to L_2 and L_2 is perpendicular to L_3, then L_1 can never be perpendicular to L_3.
In fact L_1 is parallel to L_3,
i.e., $(L_1, L_2) \in R$, $(L_2, L_3) \in R$ but $(L_1, L_3) \notin R$
i.e., symmetric but neither reflexive nor transitive.

Q. 3. The function $f : R \rightarrow R$ defined by $f(x) = x - 4$ is__________

(A) Bijective
(B) Surjective but not injective
(C) Injective but not Surjective
(D) Neither Surjective nor Injective

Ans. Option (A) is correct.

Explanation:
The function f is one-one,
for $f(x_1) = f(x_2)$
$\Rightarrow x_1 - 4 = x_2 - 4$
$\Rightarrow x_1 = x_2$
Also, given any real number y in R, there exists $y + 4$ in R
Such that $f(y + 4) = y + 4 - 4 = y$
Hence, f is onto
Hence, function is both one-one and onto, *i.e.*, bijective.

Q. 4. Let $f : R \rightarrow R$ be defined by $f(x) = x - 4$. Then the range of $f(x)$ is ________

(A) R **(B)** Z
(C) W **(D)** Q

Ans. Option (A) is correct.

Explanation: Range of $f(x)$ is R

Q. 5. Let $R = \{(L_1, L_2) : L_1 \parallel L_2$ and $L_1 : y = x - 4\}$ then which of the following can be taken as L_2?

(A) $2x - 2y + 5 = 0$ **(B)** $2x + y = 5$
(C) $2x + 2y + 7 = 0$ **(D)** $x + y = 7$

Ans. Option (A) is correct.

Explanation: Since, $L_1 \parallel L_2$
then slope of both the lines should be same.

Slope of $L_1 = 1$
$\Rightarrow$ Slope of $L_2 = 1$
And $2x - 2y + 5 = 0$

$$-2y = -2x - 5$$
$$y = x + \frac{5}{2}$$

Slope of $2x - 2y + 5 = 0$ is 1
So, $2x - 2y + 5 = 0$ can be taken as L_2.

V. Read the following text and answer the following questions n the basis of the same:

Raji visited the Exhibition along with her family. The Exhibition had a huge swing, which attracted many children. Raji found that the swing traced the path of a Parabola as given by $y = x^2$.

[CBSE QB-2021]

Q. 1. Let $f : R \rightarrow R$ be defined by $f(x) = x^2$ is________

(A) Neither Surjective nor Injective

(B) Surjective

(C) Injective

(D) Bijective

Ans. Option (A) is correct.

> ***Explanation:***
> $f : R \rightarrow R$ be defined by $f(x) = x^2$
> $\because f(-1) = f(1) = 1$, but $-1 \neq 1$
> $\therefore f$ is not injective
> Now, $-2 \in R$. But, there does not exist any element $x \in R$ such that $f(x) = -2$ or $x^2 = -2$
> $\therefore f$ is not surjective.
> Hence, function f is neither injective nor surjective.

Q. 2. Let $f : N \rightarrow N$ be defined by $f(x) = x^2$ is ________

(A) Surjective but not Injective

(B) Surjective

(C) Injective

(D) Bijective

Ans. Option (C) is correct.

> ***Explanation:*** $f : N \rightarrow N$ be defined by $f(x) = x^2$
> for $x, y \in N, f(x) = f(y)$
> $\Rightarrow \quad x^2 = y^2$
> $\Rightarrow \quad x = y$
> $\therefore \quad f$ is injective
> Now; $2 \in N$, But, there does not exist any x in n such that $f(x) = x^2 = 2$
> $\therefore \quad f$ is not surjective
> Hence, function is injective but not surjective.

Q. 3. Let $f : \{1, 2, 3, ...\} \rightarrow \{1, 4, 9, ...\}$ be defined by $f(x) = x^2$ is _________

(A) Bijective

(B) Surjective but not Injective

(C) Injective but Surjective

(D) Neither Surjective nor Injective

Ans. Option (A) is correct.

> ***Explanation:***
> $f : \{1, 2, 3, ...\} \rightarrow \{1, 4, 9, ...\}$ be defined by $f(x) = x^2$
> $x_1 \in \{1, 2, 3, ...\}$ and $x_2 \in \{1, 2, 9, ...\}$
> $f(x_1) = f(x_2)$
> $\Rightarrow \quad x_1^2 = x_2^2$
> $\Rightarrow \quad x_1 = x_2$
> $\therefore \quad f$ is injective
> Now, $4 \in \{1, 4, 9...\}$, there exist 2 in $\{1, 2, 3 ...\}$ such that $f(x) = 2^2 = 4$, Hence, f is surjective
> Therefore f is bijective.

Q. 4. Let : $N \rightarrow R$ be defined by $f(x) = x^2$. Range of the function among the following is _________

(A) $\{1, 4, 9, 16,...\}$

(B) $\{1, 4, 8, 9, 10,...\}$

(C) $\{1, 4, 9, 15, 16,...\}$

(D) $\{1, 4, 8, 16,...\}$

Ans. Option (A) is correct.

> ***Explanation:***
> Range of $f = \{1, 4, 9, 16, ...\}$
> $\because \quad N = \{1, 2, 3, ...\}$

Q. 5. The function $f : Z \rightarrow Z$ defined by $f(x) = x^2$ is ______

(A) Neither Injective nor Surjective

(B) Injective

(C) Surjective

(D) Bijective

Ans. Option (A) is correct.

> ***Explanation:*** $f : z \rightarrow z$ defined by $f(x) = x^2$
> So, $\quad f(-1) = f(1)$, but $1 \neq -1$
> $\therefore \quad f$ is not injective
> Now, $-2 \in Z$, but, there does not exist any element $x \in z$ such that
> $f(x) = -2$
> or $\quad x^2 = -2$
> $\therefore \quad f$ is not surjective
> Hence, f is neither injective nor surjective.