Chapter 19. Statistics

Formulae

1. **Mean:**

(i) Mean (for ungrouped data) $= \frac{\Sigma x_i}{n}$,

where $x_1, x_2, x_3, \ldots, x_n$ are the observations and n is the total number of observations.

(ii) Mean (for grouped data) $= \frac{\Sigma f_i x_i}{\Sigma f_i}$,

where $x_1, x_2, x_3, \ldots, x_n$ are different variates with frequencies $f_1, f_2, f_3, \ldots, f_n$ respectively.

(iii) Mean for continuous distribution.

Let there be n continuous classes, and y_i be the class mark and f_i be the frequency of the ith class, then

$$\text{mean} = \frac{\Sigma f_i y_i}{\Sigma f_i} \quad \text{(Direct method)}$$

Let A be the assumed mean, then

$$\text{mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i},$$

where $d_i = y_i - A$ (Short cut method)

If the classes are of equal size, say c, then

$$\text{mean} = A + c \times \frac{\Sigma f_i u_i}{\Sigma f_i},$$

where $u_i = \frac{y_i - A}{c}$ (Step deviation method)

2. **Median:**

Let n be the total number of observations, then

$$\text{median} = \begin{cases} \frac{n+1}{2}\text{th observation, if } n \text{ is odd} \\ \frac{\frac{n}{2}\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2} \end{cases}$$

if n is even.

3. **Quartiles:**

Lower quartile (Q_1)

$$= \begin{cases} \frac{n+1}{4} \text{ th observation, if } n \text{ is odd} \\ \frac{n}{4} \text{ th observation, if } n \text{ is even} \end{cases}$$

Upper quartile (Q_3)

$$= \begin{cases} \frac{3(n+1)}{4} \text{ th observation, if } n \text{ is odd} \\ \frac{3n}{4} \text{ th observation, if } n \text{ is even} \end{cases}$$

Interquartile range $= Q_3 - Q_1$ and

semi interquartile range $= \frac{Q_3 - Q_1}{2}$.

Formulae Based Questions

Question 1. There are 45 students in a class, in which 15 are girls. The average weight of 15 girls is 45 kg and 30 boys is 52 kg. Find the mean weight in kg of the entire class.

Solution.

Here $n_1 = 15$, $n_2 = 30$, $\bar{X}_1 = 45$ kg and $\bar{X}_2 = 52$ kg.

$$\therefore \quad \bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2} = \frac{15 \times 45 + 30 \times 52}{15 + 30} \text{ kg.}$$

$$= \frac{2235}{45} \text{ kg}$$

$$= 49{\cdot}67 \text{ kg.}$$

Hence, the mean weight of the entire class is 49·67 kg. Ans.

Question 2. A school has 4 sections of Chemistry in class X having 40, 35, 45 and 42 students. The mean marks obtained in Chemistry test are 50, 60, 55 and 45 respectively for the 4 sections. Determine the overall average of marks per student.

Solution : Here $n_1 = 40$, $n_2 = 35$, $n_3 = 45$, $n_4 = 42$,
$\bar{X}_1 = 50$, $\bar{X}_2 = 60$, $\bar{X}_3 = 55$ and $\bar{X}_4 = 45$.

$$\therefore \quad \bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2 + n_3\bar{X}_3 + n_4\bar{X}_4}{n_1 + n_2 + n_3 + n_4}$$

$$= \frac{40 \times 50 + 35 \times 60 + 45 \times 55 + 42 \times 45}{40 + 35 + 45 + 42}$$

$$= \frac{2000 + 2100 + 2475 + 1890}{162}$$

$$= \frac{8465}{162}$$

$$= 52{\cdot}25$$

Hence, the overall average marks of per student is 52·25. Ans.

Question 3. Find the mean of 4, 7, 12, 8, 11, 9, 13, 15, 2, 7.

Solution :

Here $n = 10$

and $\Sigma x = 4 + 7 + 12 + 8 + 11 + 9 + 13 + 15 + 2 + 7$

$$\therefore \quad \text{Mean } \bar{X} = \frac{\Sigma x}{n}$$

$$= \frac{88}{10} = 8{\cdot}8. \quad \text{Ans.}$$

Question 4. Find the mean of first five natural numbers.

Solution : First five natural numbers are

1, 2, 3, 4 and 5

Hence,

$$\text{Mean } \bar{X} = \frac{\Sigma x}{n}$$

$$= \frac{1 + 2 + 3 + 4 + 5}{5} = \frac{15}{5} = 3$$

$$\bar{X} = 3.$$

Question 5. In X standard, there are three sections A, B and C with 25, 40 and 35 students respectively. The average marks of section A is 70%, section B is 65% and of section C is 50%.

Find the average marks of the entire X standard.

Solution : Here , $n_1 = 25, n_2 = 40, n_3 = 35, \overline{X}_1 = 70, \overline{X}_2 = 65$ and $\overline{X}_3 = 50$.

Let $\overline{X}$ denote the average marks of the entire X standard. Then,

$$\overline{X} = \frac{n_1\overline{X}_1 + n_2\overline{X}_2 + n_3\overline{X}_3}{n_1 + n_2 + n_3}$$

$$= \frac{25 \times 70 + 40 \times 65 + 35 \times 50}{25 + 40 + 35}$$

$$= \frac{1750 + 2600 + 1750}{100}$$

$$= \frac{6100}{100} = 61$$

Hence, the average marks of the entire X standard is 61%. Ans.

Question 6. The average score of boys in an examination of a school is 71 and of girls is 73. The averages score of school in that examination is 71.8. Find the ratio of the number of boys between number of girls appeared in the examination.

Solution . Let $\bar{X}_1$ and $\bar{X}_2$ be the average scores of boys and girls respectively and $\bar{X}$ be the average of both boys and girls. Then

$$\bar{X}_1 = 71, \ \bar{X}_2 = 73, \ \bar{X} = 71{\cdot}8.$$

$$\therefore \quad \bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

$$\Rightarrow \quad 71{\cdot}8 = \frac{n_1 \times 71 + n_2 \times 73}{n_1 + n_2}$$

$$\Rightarrow \quad 71{\cdot}8\,(n_1 + n_2) = 71n_1 + 73n_2$$

$$\Rightarrow \quad 0{\cdot}8n_1 = 1{\cdot}2n_2$$

$$\Rightarrow \quad 8n_1 = 12n_2$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{12}{8} = \frac{3}{2}$$

Hence $n_1 : n_2 = 3 : 2$. Ans.

Question 7. There are 50 students in a class in which 40 are boys and rest are girls. The average weight of the class is 44 kgs and the average weight of the girls is 40 kgs. Find the

average weight of the boys.

Solution : We have

n = No., of students in a class = 50

n_1 = No., of boys in a class = 40

n_2 = No., of girls in a class = 10

$\overline{X}_1$ = Average weight of boys = ?

$\overline{X}_2$ = Average weight of girls = 40 kgs.

$$\therefore \quad \overline{X} = \frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2}$$

$$\Rightarrow \quad 44 = \frac{40\overline{X}_1 + 10 \times 40}{40 + 10}$$

$$\Rightarrow \quad 50 \times 44 = 40\overline{X}_1 + 400$$

$$\Rightarrow \quad 2200 = 40\overline{X}_1 + 400$$

$$\Rightarrow \quad 40\overline{X}_1 = 1800$$

$$\Rightarrow \quad \overline{X}_1 = 45$$

Hence, the average weight of boys is 45 kgs.

Question 8. From the following numbers find the median:
10, 75, 3, 81, 17, 27, 4, 48, 12, 47, 9, 15.

Solution : On arranging in ascending order

3, 4, 9, 10, 12, 15, 17, 27, 47, 48, 75, 81

Here, n = 12 which is even

$$\text{Therefore, median} = \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2} + 1\right)^{th} \text{term}}{2}$$

$$= \frac{\left(\frac{12}{2}\right)^{th} \text{term} + \left(\frac{12}{2} + 1\right)^{th} \text{term}}{2}$$

$$= \frac{6^{th} \text{ term} + 7^{th} \text{ term}}{2}$$

$$= \frac{15 + 17}{2} = \frac{32}{2} = 16$$

Median = 16. Ans.

Question 9. The median of the following observation 11, 12, 14, 18, (x + 4), 30, 32, 35, 41 arranged in ascending order is 24. Find x.

Solution :

11, 12, 14, 18, (x + 4), 30, 32, 35, 41

No. of terms are odd (9)

$$\therefore \quad \text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{term}$$

$$= \left(\frac{9+1}{2}\right)^{th} \text{term}$$

$$= 5^{th} \text{ term}$$

$$\therefore \quad \text{Median} = x + 4$$

$$\text{and Given Median} = 24$$

$$\therefore \quad x + 4 = 24$$

$$x = 20$$

Question 10. The median of the following observations arranged in ascending order is 24. Find x:

11, 12, 14, 18, x + 2, x + 4, 30, 32, 35, 41.

Solution :

11, 12, 14, 18, x + 2, x + 4, 30, 32, 35

$n = 10$ (even), Median = 24

$$\therefore \text{ Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$$

$$= \frac{\left(\frac{10}{2}\right)^{th} \text{term} + \left(\frac{10}{2}+1\right)^{th} \text{term}}{2}$$

$$\text{Median} = \frac{5^{th} \text{ term} + 6^{th} \text{ term}}{2}$$

$$24 = \frac{x + 2 + x + 4}{2}$$

$$2x + 6 = 24 \times 2$$

$$\Rightarrow \quad 2x = 48 - 6$$

$$\Rightarrow \quad 2x = 42$$

$$\Rightarrow \quad x = 21. \qquad \text{Ans.}$$

Question 11. Find the mean, median and mode of the following distribution:
8,10, 7, 6,10,11, 6,13,10

Solution : Arranging the number of in ascending order 6, 6, 7, 8, 10, 10, 11, 13.

$$\text{Mean } \overline{X} = \frac{\Sigma x}{n} = \frac{81}{9} = 9. \quad \text{Ans.}$$

$$\text{Median} = \left[\frac{n+1}{2}\right]^{th} \text{term}$$

$$= \left(\frac{9+1}{2}\right)^{th} = 5^{th} \text{ term}$$

Median = 10. Ans.

Mode = 10 is repeating 3 times which is highest frequences.

So mode is 10. **Ans.**

Question 12. Find the median of the following values: 37, 31, 42, 43, 46, 25, 39, 45, 32.

Solution : Arranging the data in ascending order, we have

25, 31, 32, 37, 39, 42, 45, 46.

Here, the number of observations n = 9(odd)

$$\therefore \quad \text{Median} = \text{Value of } \left(\frac{9+1}{2}\right)^{th} \text{ observation}$$

$$= \text{Value of } 5^{th} \text{ observation}$$

$$= 39. \quad \text{Ans.}$$

Question 13. Find the mode from the following data: 110,120,130,120,110,140,130,120,140,120.

Solution : Arranging the data in the form of a frequency table, we have :

Value	Tally bars	Frequency
110	II	2
120	IIII	4
130	II	2
140	II	2

Since the value 120 occurs maximum number of times i.e., 4. Hence, the modal value is 120. Ans.

Question 14. Find the mode for the following series: 2.5, 2.3, 2.2, 2.2, 2.4, 2.7, 2.7, 2.5, 2.3, 2.2, 2.6, 2.2.

Solution : Arranging the data in the form of a frequency table, we have :

Value	Tally bars	Frequency
2·2	IIII	4
2·3	II	2
2·4	I	1
2·5	II	2
2·6	I	1
2·7	II	2

We see that the value 2·2 has the maximum frequency i.e., 4.

So, 2·2 is the mode for the given series. Ans.

Question 15. Find out the mode from the following data:

Wages (in ₹)	No. of persons
125	3
175	8
225	21
275	6
325	4
375	2

Solution : Clearly, the value 225 occurs maximum number of times So, the modal wage is ₹ 225. Ans.

Data Based Questions

Question 1. The contents of 100 match box were checked to determine the number of match sticks they contained.

Number of match sticks	Number of boxes
35	6
36	10
37	18
38	25
39	21
40	12
41	8

(i) Calculate correct to one decimal place, the mean number of match sticks per box.
(ii) Determine how many matchsticks would have to be added. To the total contents of the 100 boxes to bring the mean up exactly 39 match sticks.

Solution:

Number of match sticks (x_i)	Number of boxes (f_i)	$f_i x_i$
35	6	210
36	10	360
37	18	666
38	25	950
39	21	819
40	12	480
41	8	328
	$\Sigma f_i = 100$	$\Sigma f_i x_i = 3813$

$$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{3813}{100}$$

$$= 38{\cdot}13 \sim 38{\cdot}1 \qquad \text{Ans.}$$

(ii) Now the number of extra sticks to be added.

$$= 39 \times 100 - 38{\cdot}13 \times 100$$

$$= 3900 - 3813 = 87 \qquad \text{Ans.}$$

Question 2. Find the mean of the following distribution:

x	4	6	9	10	15
f	5	10	10	7	8

Solution : Calculation of Arithmetic Mean :

x_i	f_i	$f_i x_i$
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
	$N = \Sigma f_i = 40$	$\Sigma f_i x_i = 360$

$$\therefore \quad \text{Mean} = \bar{X} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{360}{40} = 9.$$

Question 3. The mean of the following distribution is 6. Find the value at P:

x	2	4	6	10	P + 5
f	3	2	3	1	2

Solution : Calculation of Arithmetic Mean :

x_i	f_i	f_ix_i
2	3	6
4	2	8
6	3	18
10	1	10
$P+5$	2	$2P+10$
	$\Sigma f_i = 11$	$\Sigma f_ix_i = 2P+52$

We have,

$$\Sigma f_i = 11$$

$$\Sigma f_ix_i = 2P + 52$$

$$\therefore \quad \text{Mean} = \frac{\Sigma f_ix_i}{\Sigma f_i}$$

$$6 = \frac{2P+52}{11}$$

$$\Rightarrow \quad 66 = 2P + 52$$

$$\Rightarrow \quad 2P = 66 - 52$$

$$\Rightarrow \quad 2P = 14$$

$$\Rightarrow \quad P = 7. \qquad \text{Ans.}$$

Question 4. If the mean of the following distribution is 7.5, find the missing frequency 'f':

Variable : 5 6 7 8 9 10 11 12

Frequency : 20 17 f 10 8 6 7 6

Solution :

Variable	Frequency (f)	fx
5	20	100
6	17	102
7	f	$7f$
8	10	80
9	8	72
10	6	60
11	7	77
12	6	72
	$\Sigma f = 74 + f$	$563 + 7f$

$$\because \quad M = \frac{\Sigma fx}{\Sigma f}$$

$$\therefore \quad 7.5 = \frac{563+7f}{74+f}$$

$$\therefore \quad 555 + 7.5f = 563 + 7f$$

$$\therefore \quad 0.5f = 8$$

$$\therefore \quad f = 16 \qquad \text{Ans.}$$

Question 5. Marks obtained by 40 students in a short assessment is given below; where a and b are two missing data.

Marks	No. of Students
5	6
6	a
7	16
8	13
9	b

If the mean of the distribution is 7·2 find a & b.

Solution :

Marks (x)	No. of Students (f)	fx
5	6	30
6	a	$6a$
7	16	112
8	13	104
9	b	$9b$
Total	$\Sigma f = 35 + a + b$	$\Sigma fx = 246 + 6a + 9b$

Now, $\Sigma f = 40$

$$35 + a + b = 40$$

$$a + b = 5 \quad \ldots(1)$$

and $$\bar{X} = \frac{\Sigma fx}{\Sigma f}$$

$$7{\cdot}2 = \frac{246 + 6a + 9b}{40}$$

$$\Rightarrow \quad 6a + 9b + 246 = 288$$

$$\Rightarrow \quad 6a + 9b = 42$$

$$\Rightarrow \quad 2a + 3b = 14 \quad \ldots(2)$$

From (1) and (2), $a = 1, b = 4$ Ans.

Question 6. Find the mean of the following distribution:

x	10	30	50	70	89
f	7	8	10	15	10

Solution : Calculation of Mean :

x_i	f_i	f_ix_i
10	7	70
30	8	240
50	10	500
70	15	1050
89	10	890
	$\Sigma f_i = N = 50$	$\Sigma f_ix_i = 2750$

$$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{N}$$

$$= \frac{2750}{50} = 55.$$

Question 7. Find the mean of the following distribution:

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	10	6	8	12	5

Solution :

Class Interval	Frequency (f)	Mid value x	fx
0 – 10	10	5	50
10 – 20	6	15	90
20 – 30	8	25	200
30 – 40	12	35	420
40 – 50	5	45	225
	$\Sigma f = 41$		$\Sigma fx = 985$

$$\therefore \quad \text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{985}{41}$$

$$= 24{\cdot}02. \qquad \text{Ans.}$$

Question 8. Find the mean of the following frequency distribution:

Class Interval	Frequency
0 — 50	4
50 — 100	8
100 — 150	16
150 — 200	13
200 — 250	6
250 — 300	3

Solution :

Class - Interval	(x)	(f)	(fx)
0 — 50	25	4	100
50 — 100	75	8	600
100 — 150	125	16	2,000
150 — 200	175	13	2,275
200 — 250	225	6	1,350
250 — 300	275	3	825
		$\Sigma f = 50$	$\Sigma fx = 7{,}150$

$$\therefore \quad \text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{7150}{50}$$

$$= 143 \quad \text{Ans.}$$

Question 9. Find the Median of the following data:

(i) 12,17, 3,14, 6, 9,8,15,20

(ii) 2,10,9,9,5,2,3,7,11,15.

Solution : (i) Arranging the data in ascending order, we get

3, 6, 8, 9, 12, 14, 15, 17, 20

Here, $n = 9$ (odd)

Hence,

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{item}$$

$$= \left(\frac{9+1}{2}\right)^{\text{th}} \text{item}$$

$$= 5^{\text{th}} \text{ item} = 12. \quad \text{Ans.}$$

(ii) Arranging the data in ascending order, we get

2, 2, 3, 5, 7, 9, 9, 10, 11, 15

Here $n = 10$ (Even)

So, Median

$$= \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{term} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{term}}{2}$$

$$= \frac{\left(\frac{10}{2}\right)^{\text{th}} \text{term} + \left(\frac{10}{2}+1\right)^{\text{th}} \text{term}}{2}$$

$$= \frac{5^{\text{th}} \text{ term} + 6^{\text{th}} \text{ term}}{2}$$

$$= \frac{7+9}{2} = \frac{16}{2} = 8. \quad \textbf{Ans.}$$

Question 10. Find the Median of the following distribution:

x	3	5	10	12	8	15
f	2	4	6	10	8	7

Solution : Arranging the terms in ascending order and preparing the cumulative frequency table :

x	f	$c.f.$
3	2	2
5	4	6
8	8	14
10	6	20
12	10	30
15	7	37

Here, $n = 37$ which is odd.

So $$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{term}$$

$$= \left(\frac{37+1}{2}\right)^{\text{th}} \text{term}$$

$$= \left(\frac{38}{2}\right)^{\text{th}}$$

$$= 19^{\text{th}} \text{ term}$$

Hence, Median is the value of the 19^{th} term = 10. Ans.

Question 11. Find the mode and median of the following frequency distribution:

x	10	11	12	13	14	15
f	1	4	7	5	9	3

Solution :

x	f	$c.f.$
10	1	1
11	4	5
12	7	12
13	5	17
14	9	26
15	3	29

$\Rightarrow$ Mode = 14

(Since 14 has highest frequency)

Now, $n = 29$ (odd)

$\therefore$ $$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{value}$$

$$= \left(\frac{29+1}{2}\right)^{\text{th}} \text{value}$$

$$= 15\text{th value} = 13$$ Ans.

Question 12. Calculate the median of the following distribution:

Weight (in nearest kg.)	No. of students
46	7
48	5
50	8
52	12
53	10
54	2
55	1

Solution: The given variates (weights of students) are already in ascending order. We construct the cumulative frequency table as under:

Variate (weight)	Frequency (No. of Students)	Cumulative frequency
46	7	7
48	5	12
50	8	20
52	12	32
53	10	42
54	2	44
55	1	45

Here, $n = 45$, which is odd.

$\therefore$ Median $= \frac{n+1}{2}$ th observation

$= 23^{rd}$ observation $= 52$.

($\because$ All observation from 21^{st} to 32^{nd} are equal, each = 52). Ans.

Question 13. Obtain the median for the following frequency distribution:

x:	1	2	3	4	5	6	7	8	9
f:	8	10	11	16	20	25	15	9	6

Solution : Calculation of Median

x	f	$c.f.$
1	8	8
2	10	18
3	11	29
4	16	45
5	20	65
6	25	90
7	15	105
8	9	114
9	6	120
		$N = 120$

Here, $N = 120$, so, $\frac{N}{2} = 60$.

The cumulative frequency just greater than $\frac{N}{2}$ *i.e.*, 60 is 65. The value of the variate corresponding to 65 is 5. Hence, median = 5. Ans.

Question 14. Calculate the median of the following distribution:

No. of goals	0	1	2	3	4	5
No. of matches	2	4	7	6	8	3

Solution : The given variates (no. of goals) are already in ascending order. We construct the cumulative frequency table as under :

Variate (No. of goals)	Frequency (No. of matches)	Cumulative frequency
0	2	2
1	4	6
2	7	13
3	6	19
4	8	27
5	3	30

Here, $n = 30$, which is even.

$\therefore$ Median

$$= \frac{\frac{n}{2}\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{15\text{th observation} + 16\text{th observation}}{2}$$

$$= \frac{3+3}{2} = 3. \qquad \text{Ans.}$$

($\because$ All observation form 14^{th} to 19^{th} are equal, each = 3).

Question 15. The following table gives the wages of worker in a factory:

Wages in ₹	45–50	50–55	55–60	60–65	65–70	70–75	75–80
No. of Worker's	5	8	30	25	14	12	6

Calculate the mean by the short cut method.

Solution :

Class Interval	Frequency f_i	Observation (mid value) x_i	$d_i = x_i - A$	$f_i d_i$
45—50	5	47·5	– 15	– 75
50—55	8	52·5	– 10	– 80
55—60	30	57·5	– 5	– 150
60—65	25	62·5 = A	0	0
65—70	14	67·5	5	70
70—75	12	72·5	10	120
75—80	6	77·5	15	90
	$\Sigma f_i = 100$			$\Sigma f_i d_i = -25$

$$\text{Mean } \bar{X} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 62{\cdot}5 + \frac{-25}{100} = 62{\cdot}25. \qquad \text{Ans.}$$

Question 16. The following table shows the weight of 12 students:

Weight in kg.	67	70	72	73	75
Number of students	4	3	2	2	1

Find the Mean weight.

Solution : Let the assumed mean be $A = 72$. Calculation of Mean :

Weight in kgs.	Number of students (f_i)	$d_i = x_i - A = x_i - 72$	$f_i d_i$
67	4	-5	-20
70	3	-2	-6
72	2	0	0
73	2	1	2
75	1	3	3
	$N = \Sigma f_i = 12$		$\Sigma f_i d_i = -21$

We have, $N = 12, E = f_i d_i = -21, A = 72$

$\therefore$ Mean $= A + \frac{1}{N}(\Sigma f_i d_i)$

$= 72 + \left(\frac{-21}{12}\right) = 72 - \frac{7}{4} = \frac{288-7}{4} = \frac{281}{4} = 70{\cdot}25$ kgs.

$\therefore$ Mean weight = 70·25 kgs. Ans.

Question 17. Find the mean wage of a worker from the following data:

Wages (In ₹)	1400	1450	1500	1550	1600	1650	1700
Number of workers	15	20	18	27	15	3	2

Solution. Let the assumed mean be $A = 1550$. Thus

Wages (In ₹) x_i	Number of workers (f_i)	$d_i = x_i - 1550$	$f_i d_i$
1400	15	-150	-2250
1450	20	-100	-2000
1500	18	-50	-900
$A = 1550$	27	0	0
1600	15	50	750
1650	3	100	300
1700	2	150	300
	$\Sigma f_i = 100$		$\Sigma f_i d_i = -3800$

$\therefore$ Mean Wages $= A + \frac{\Sigma f_i d_i}{\Sigma f_i} = \left(1{,}550 - \frac{3{,}800}{100}\right) = 1{,}550 - 38 =$ ₹ 1,512. Ans.

Question 18. The marks obtained by a set of students in an examination all given below:

Marks	5	10	15	20	25	30
Number of students	6	4	6	12	x	4

Given that the mean marks of the set of students is 18, Calculate the numerical value of x.

Solution : Calculation of Mean :

Marks (x)	Number of students (f)	fx
5	6	30
10	4	40
15	6	90
20	12	240
25	x	$25x$
30	4	120
	$\Sigma f = 32 + x$	$\Sigma fx = 520 + 25x$

$$\therefore \quad \text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\therefore \quad 18 = \frac{520 + 25x}{32 + x}$$

$$\Rightarrow \quad 576 + 18x = 520 + 25x$$

$$\Rightarrow \quad 7x = 56$$

$$\Rightarrow \quad x = 8.$$

Question 19. Find the mean of the following distribution by step deviation method:

Class interval	20—30	30—40	40—50	50—60	60—70	70—80
Frequency	10	6	8	12	5	9

Solution :

C.I.	f	'X' mid values	$\mu = \frac{x - A}{h}$	$f.\mu.$
20—30	10	25	−3	−30
30—40	6	35	−2	−12
40—50	8	45	−1	−8
50—60	12	$55 = A$	0	0
60—70	5	65	1	5
70—80	9	75	2	18
	$\Sigma f = 50$			$\Sigma f\mu = -27$

Here, A = Assumed mean = 55

$h = 10$

$$\therefore \quad \text{Mean } (\overline{X}) = A + \frac{\Sigma f\mu}{\Sigma f} \times h$$

$$= 55 + \frac{(-27)}{50} \times 10$$

$$= 55 - 5{\cdot}4 = 49{\cdot}6$$

Ans.

Question 20. Helping the step deviation method find the arithmetic mean of the distribution:

Variate (x)	5	10	15	20	25	30	35	40	45	50
Frequency (f)	20	43	75	67	72	45	39	9	8	6

Solution : Let the assumed Mean be $A = 25$ and $h = 5$.

x_i	Frequencies f_i	Deviation $d_i = x_i - 25$	$u_i = \frac{x_i - 25}{5}$	$f_i u_i$
5	20	−20	−4	−80
10	43	−15	−3	−129
15	75	−10	−2	−150
20	67	−5	−1	−67
$A = 25$	72	0	0	0
30	45	5	1	45
35	39	10	2	78
40	9	15	3	27
45	8	20	4	32
50	6	25	5	30
	$N = \Sigma f_i = 384$			$\Sigma f_i u_i = -214$

We have, $N = 384$, $A = 25$, $h = 5$ and $\Sigma f_i u_i = -214$

$$\text{Mean } (\overline{X}) = A + h\left(\frac{1}{N}\Sigma f_i u_i\right)$$

$$= 25 + 5 \times \left(\frac{-214}{384}\right)$$

$$= 25 - 2{\cdot}786$$

$$= 22{\cdot}214.$$

Question 21. The weights of 50 apples were recorded as given below. Calculate the mean weight, to the nearest gram. by the Step Deviation Method.

Weights in grams	No. of apples
80—85	5
85—90	8
90—95	10
95—100	12
100—105	8
105—110	4
110—115	3

Solution:

Weight in gms.	No. of apples	x_i	$x_i - A$	$u_i = \frac{x_i - A}{5}$	$f_i u_i$
80—85	5	82·5	– 15	– 3	– 15
85—90	8	87·5	– 10	– 2	– 16
90—95	10	92·5	– 5	– 1	– 10
95—100	12	(97·5)A	0	0	0
100—105	8	102·5	5	1	8
105—110	4	107·5	10	2	8
110—115	3	112·5	15	3	9
	$\Sigma f_i = 50$				$\Sigma f_i u_i = -16$

$A = 97{\cdot}5,\ \Sigma f_i = 50,\ \Sigma f_i u_i = -16,\ h = 5.$

$$\therefore \quad \text{Mean } (\bar{X}) = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 90{\cdot}5 + \frac{-16}{50} \times 5 = 95{\cdot}9 \quad \text{Ans.}$$

Question 22. A frequency distribution of the life times of 400 T.V., picture tubes leased in tube company is given below. Find the average life of tube:

Life time (in hrs)	Number of tubes
300 – 399	14
400 – 499	46
500 – 599	58
600 – 699	76
700 – 799	68
800 – 899	62
900 – 999	48
1000 – 1099	22
1100 – 1199	6

Solution: Here, the class-intervals are formed by exclusive method. If we make the series an inclusive one the mid-values remain same. So, there is no need to convert the series.

Let the assumed mean be $A = 749{\cdot}5$ and $h = 100$.

Calculation of Mean

Life time (in hrs)	Frequency (f_i)	Mid-values (x_i)	$d_i = x_i - A$ $= x_i - 749{\cdot}5$	$u_i = \frac{x_i - A}{h} = \frac{x_i - 749{\cdot}5}{100}$	$f_i u_i$
300 – 399	14	349·5	– 400	– 4	– 56
400 – 499	46	449·5	– 300	– 3	– 138
500 – 599	58	549·5	– 200	– 2	– 116
600 – 699	76	649·5	– 100	– 1	– 76
700 – 799	68	$A = 749{\cdot}5$	0	0	0
800 – 899	62	849·5	100	1	62
900 – 999	48	949·5	200	2	96
1000 – 1099	22	1049·5	300	3	66
1100 – 1199	6	1149·5	400	4	24
	$N = \Sigma f_i = 400$				$\Sigma f_i u_i = -138$

Here, $N = 400$, $A = 749{\cdot}5$, $h = 100$ and $\Sigma f_i u_i = -138$

$\therefore \quad \overline{X} = A + \frac{h}{N} \Sigma f_i u_i$

$\Rightarrow \quad \overline{X} = 749{\cdot}5 + 100 \times \left(\frac{-138}{400}\right) = 749{\cdot}5 - \frac{138}{4} = 749{\cdot}5 - 34{\cdot}5 = 715.$

Hence, the average life time of a tube is 715 hours. Ans.

Question 23. (i) Using step-deviation method, calculate the mean marks of the following distribution, (ii) State the modal class.

Class interval	50–55	55–60	60–65	65–70	70–75	75–80	80–85	85–90
Frequency	5	20	10	10	9	6	12	8

Solution : (i)

C.I.	f	x		$u = \frac{x - A}{i}$	$f \cdot u$
50–55	5	52·5		– 3	– 15
55–60	20	57·5		– 2	– 40
60–65	10	62·5		– 1	– 10
65–70	10	67·5	$A = 67{\cdot}5$	0	0
70–75	9	72·5		1	9
75–80	6	77·5		2	12
80–85	12	82·5		3	36
85–90	8	87·5		4	32
	$\Sigma f = 80$				$\Sigma f \cdot u = 24$

$\therefore \quad$ Mean $(\overline{X}) = A + \frac{\Sigma f \cdot u}{\Sigma f} \times i$ [i = length of C·I]

$= 67{\cdot}5 + \frac{24}{80} \times 5 = 67{\cdot}5 + 1{\cdot}5 = 69$ Ans.

(ii) Modal class = 55 – 60 Ans.

Question 24. Calculate the mean of the distribution given below using the short cut method.

Marks	11–20	21–30	31–40	41–50	51–60	61–70	71–80
No. of students	2	6	10	12	9	7	4

Solution :

Class Interval (Inclusive form)	Class Interval (Exclusive form)	No. of Students (f_i)	x_i	$A_i = 45{\cdot}5$ $d_1 = x - 45{\cdot}5$	$f_i d_i$	
11—20	10·5 – 20·5	2	15·5	– 30	– 60	
21—30	20·5 – 30·5	6	25·5	– 20	– 120	– 280
31—40	30·5 – 40·5	10	35·5	– 10	– 100	
41—50	40·5 – 50·5	12	45·5	—	—	
51—60	50·5 – 60·5	9	55·5	10	90	
61—70	60·5 – 70·5	7	65·5	20	140	350
71—80	70·5 – 80·5	4	75·5	30	120	
		$\Sigma f_i = 50$			$\Sigma f_i d_i = 70$	

Assumed mean (Ai) = 45·5

$\Sigma f_i = 50$, $\Sigma f_i d_i = 70$.

$$\text{Mean} = Ai + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 45{\cdot}5 + \frac{70}{50}$$

$$= 45{\cdot}5 + 1{\cdot}4 = 46{\cdot}9.$$ Ans.

Question 25. A study of the yield of 150 tomato plants, resulted in the record:

Tomatoes per Plant	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25
Number of Plants	20	50	46	22	12

(i) Calculate the mean of the number of tomatoes per plant.

(ii) Name the modal class.

(iii) What is the frequency of the class preceding the modal class ?

Solution : (i)

Tomatoes per Plant	Mid-Point (x)	Number of Plants (f)	fx
1 – 5	3	20	60
6 – 10	8	50	400
11 – 15	13	46	598
16 – 20	18	22	396
21 – 25	23	12	276
Total		150	1730

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{1730}{150} = \frac{173}{15} = 11{\cdot}53.$$ Ans.

(ii) The modal class is 6 – 10. Ans.

(iii) The frequency of the class preceding the modal class is 20. Ans.

Question 26. For the following frequency distribution find:
(i) Lower quartile
(ii) Upper quartile
(iii) Inter quartile range
(iv) Semi-inter quartile range.

x	1	2	3	4	5	6	7	8
f	3	5	9	15	20	16	10	2

Solution :

x	f	Cumulative frequency
1	3	3
2	5	8
3	9	17
4	15	32
5	20	52
6	16	68
7	10	78
8	2	80
		$n = 80$

(i) Lower quartile

$$(Q_1) = \text{The value of } \left(\frac{n}{4}\right)^{th} \text{ observation}$$

$$= \text{The value of } \left(\frac{80}{4}\right)^{th} \text{ observation}$$

$$= \text{The value of } 20^{th} \text{ observation}$$

$Q_1 = 4.$ Ans.

(ii) Upper quartile

$$(Q_3) = \text{The value of } \left(\frac{3n}{4}\right)^{th} \text{ observation}$$

$$= \text{The value of } \left(\frac{3 \times 80}{4}\right)^{th} \text{ observation}$$

$$= \text{The value of } 60^{th} \text{ observation}$$

$\therefore \quad Q_3 = 6.$ Ans.

(iii) Inter quartile range

$$= Q_3 - Q_1$$
$$= 6 - 4$$
$$= 2.$$ Ans.

(iv) Semi-quartile range

$$= \frac{Q_3 - Q_1}{2}$$

$$= \frac{2}{2} = 1.$$ Ans.

Prove the Following

Question 1. If the mean of n observations ax_1, ax_2, ax_3,...,ax_n is $a\overline{X}$, show that

$$(ax_1 - a\overline{X}) + (ax_2 - a\overline{X}) + \ldots (ax_n - a\overline{X}) = 0.$$

Solution : We have

$$a\overline{X} = \frac{ax_1 + ax_2 + \ldots + ax_n}{n}$$

$$\Rightarrow \quad ax_1 + ax_2 + \ldots + ax_n = n(a\overline{X}) \qquad \ldots(i)$$

Now $(ax_1 - a\overline{X}) + (ax_2 - a\overline{X}) + \ldots + (ax_n - a\overline{X})$

$= (ax_1 + ax_2 + \ldots + ax_n) - (a\overline{X} + a\overline{X} + \ldots + a\overline{X}$ n -times)

$= n(a\overline{X} - n(a\overline{X}) = 0.$ [Using (i)]

Hence proved.

Question 2. The Mean of n observations x_1, x_2,..., x_n is $\overline{X}$. If (a – b) is added to each of the observation, show that the mean of the new set of observations is $\overline{X} + (a - b)$.

Solution : We have

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n} \qquad \ldots(i)$$

Let $\overline{X}$ be the mean of $x_1 + (a - b)$, $x_2 + (a - b)$,..., $x_n + (a - b)$. Then

$$\overline{X} = \frac{[x_1 + (a-b)] + [x_2 + (a-b)] + \ldots + [x_n + (a-b)]}{n}$$

$$= \frac{x_1 + x_2 + \ldots + x_n + n(a-b)}{n}$$

$$= \frac{x_1 + x_2 + \ldots + x_n}{n} + \frac{n(a-b)}{n}$$

$= \overline{X} + (a - b)$. [Using (i)] Hence proved.

Graphical Depiction

Question 1. Marks obtained by 200 students in an examination are given below :

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	5	11	10	20	28	37	40	29	14	6

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine:

(i) The median marks

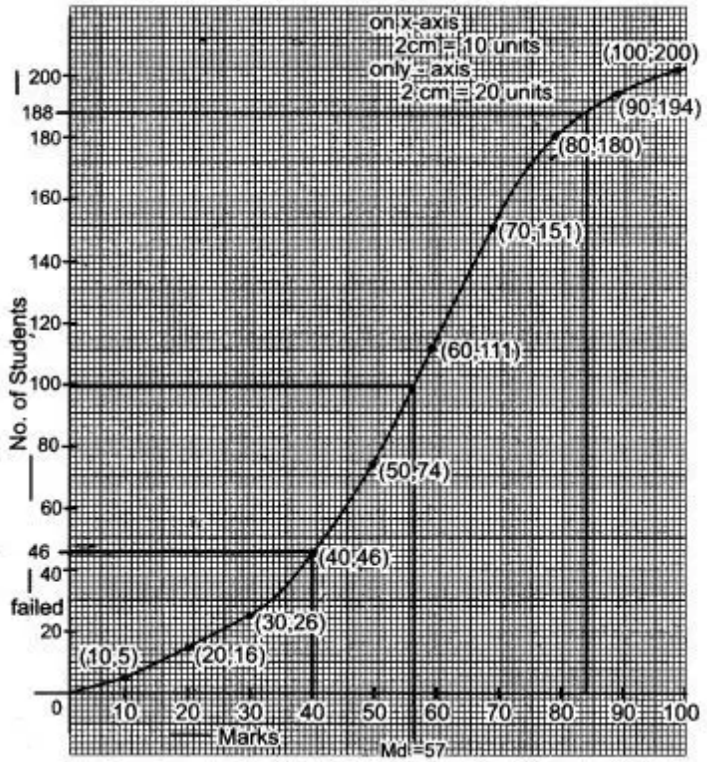
(ii) The number of students who failed if minimum marks required to pass is 40.

(iii) If scoring 85 and more marks is considered as grade one, find the number of students who secured grade one in the examination.

Solution :

On graph

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
f	5	11	10	20	28	37	40	29	14	6
$c.f.$	5	16	26	46	74	111	151	180	194	200



(i) $$\text{Median} = \left(\frac{n}{2}\right)^{\text{th}} \text{obv.} = \left(\frac{200}{2}\right)^{\text{th}} \text{obv.}$$

$$= 100^{\text{th}} \text{ obv.}$$

$$= 57$$

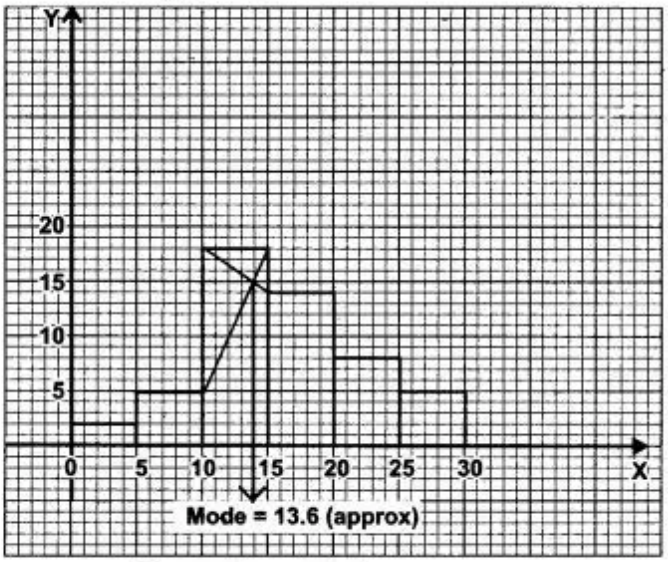
(ii) No. of students who failed = 46

(iii) No. of students who secured grade one = 200 – 188 = 12

Question 2. Draw a histogram from the following frequency distribution and find the mode from the graph:

Class	0–5	5–10	10–15	15–20	20–25	25–30
Frequency	2	5	18	14	8	5

Solution :



Mode = 13·6

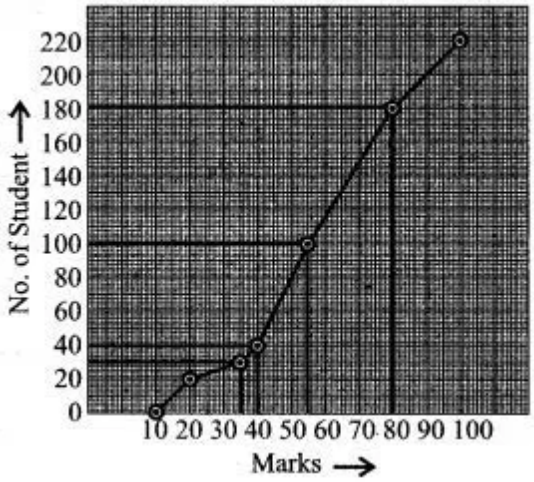
Question 3. The marks obtained by 200 students in an examination are given below:

Marks	Number of students
0 – 10	5
10 – 20	10
20 – 30	11
30 – 40	20
40 – 50	27
50 – 60	38
60 – 70	40
70 – 80	29
80 – 90	14
90 – 100	6

Using a graph paper, draw an Ogive for the above distribution. Use your Ogive to estimate:
(i) the median; (ii) the lower quartile;
(iii) the number of students who obtained more than 80% marks in the examination and
(iv) the number of students who did not pass, if the pass percentage was 35.
Use the scale as 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis.

Solution :

Less than	C. F.	Points
10	5	(10, 5)
20	15	(20, 15)
30	26	(30, 26)
40	46	(40, 46)
50	73	(50, 73)
60	111	(60, 111)
70	151	(70, 151)
80	180	(80, 180)
90	194	(90, 194)
100	200	(100, 200)



(i) Median $= \frac{N}{2} = \frac{200}{2}$

$= 100 \rightarrow$ On seing the corresponding value.

Median = 57

(ii) $Q_1 = \frac{N}{4} = \frac{200}{4}$

$= 50 \rightarrow$ On seing the corresponding value. = 38

(iii) 200 – 180 = 20 students

(iv) 38 students did not pass.

Question 4. The following table give the marks scored by students in an examination:

Marks	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
No. of students	3	7	15	24	16	8	5	2

(i) Find the modal group.

(ii) Which group has the least frequency ?

Solution: (i) 15 – 20 is the modal group.
(ii) The group 35 – 40 has the least frequency.

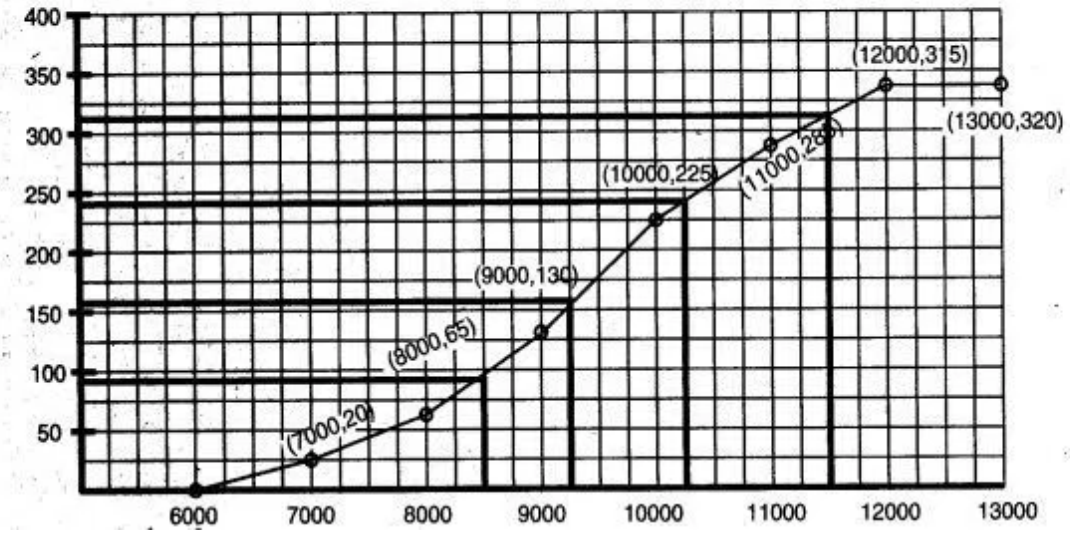
Questions 5. The monthly income of a group of 320 employees in a company is given below:

Monthly Income	No. of Employees
6000–7000	20
7000–8000	45
8000–9000	65
9000–10000	95
10000–11000	60
11000–12000	30
12000–13000	5

Draw an ogive of the given distribution on a graph sheet taking 2 cm = Rs. 1000 on one axis and 2 cm = 50 employees on the other axis. From the graph determine:
(i) the median wage
(ii) the number of employees whose income is below Rs. 8,500.
(iii) If the salary of a senior employee is above Rs. 11,500, find the number of senior employees in the company.
(iv) the upper quartile.

Solution :

Monthly	No. of Employees	*c.f.*
6000–7000	20	20
7000–8000	45	65
8000–9000	65	130
9000–10000	95	225
10000–11000	60	285
11000–12000	30	315
12000–13000	5	320



(i) From the graph, the median wage = 160·5 (appro.)
(ii) The number of employees whose income is below ₹ 8500 = 90 (appro.)
(iii) The number of senior employees whose salary is above ₹ 11500 = 20 (appro.)
(iv) The upper quartile Q_3 = 240 (appro.)

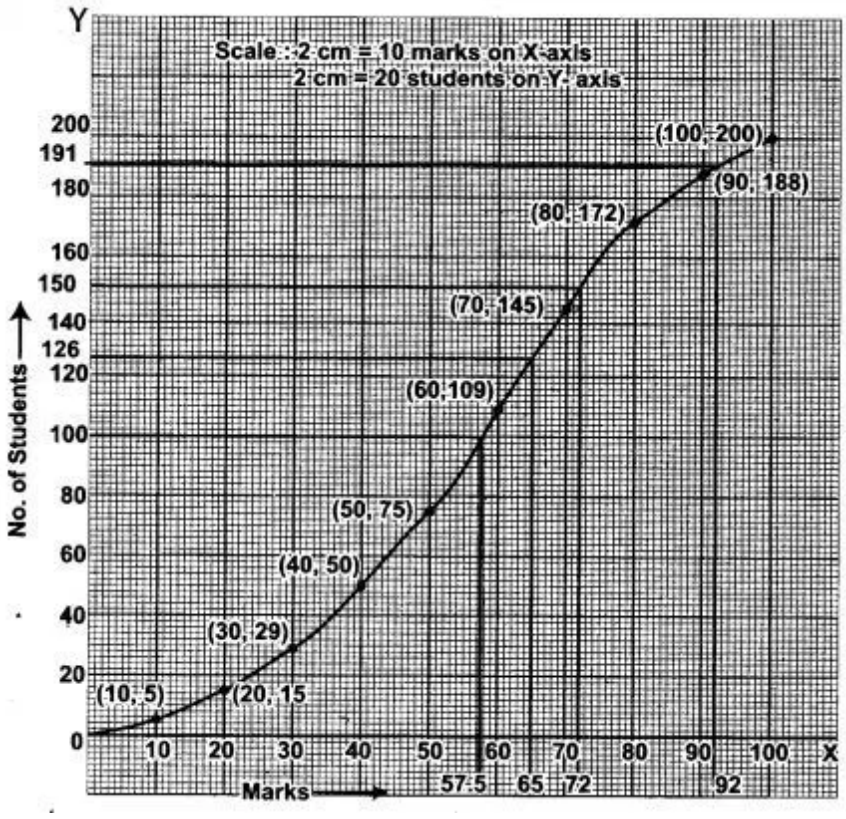
Question 6. Attempt this question on graph paper. Marks obtained by 200 students in examination are given below:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
No. of students	5	10	14	21	25	34	36	27	16	12

Draw an ogive for the given distribution taking 2 cm = 10 makrs on one axis and 2 cm = 20 students on the other axis.
From the graph find:
(i) the median
(ii) the upper quartile
(iii) number of student scoring above 65 marks.
(iv) If to students qualify for merit scholarship, find the minimum marks required to qualify.

Solution :

Marks	No. of Students	c.f.	Points
0–10	5	5	(10, 5)
10–20	10	15	(20, 15)
20–30	14	29	(30, 29)
30–40	21	50	(40, 50)
40–50	25	75	(50, 75)
50–60	34	109	(60, 109)
60–70	36	145	(70, 145)
70–80	27	172	(80, 172)
80–90	16	188	(90, 188)
90–100	12	200	(100, 200)
	n = 200		



(i) Let A be the point on y – axis representing frequency

Here, n (no. of students) = 200 (even)

$$\text{Median} = \left(\frac{n}{2}\right)^{\text{th}} \text{ term}$$

$$= \left(\frac{200}{2}\right)^{\text{th}} \text{ term}$$

$$= 100^{\text{th}} \text{ term}$$

From the graph 100^{th} term = 57·5

(ii) Upper quartile $= \frac{3n}{4}$

$$= \frac{3 \times 200^{\text{th}}}{4} \text{ term}$$

$$= \frac{600}{4} = 150^{\text{th}} \text{ term}$$

From graph 150th term = 72

The upper quartile = 72

(iii) No. of students scoring above 65 marks

⇒ Total No. of students – No. of students scoring ≤ 65 marks

⇒ 200 – 126

⇒ 74 (approx.)

(iv) From the above diagram, we observe the students from 191 to 200 qualify for merit scholarship.

∴ The student who qualifies for merit scholarship scores more than 91 marks.

∴ The minimum marks required to qualify for merit scholarship = 92 (approx.) Ans.

Question 7. The marks of 200 students in a test were recorded as follows :

Marks %	No. of students
10 – 19	7
20 – 29	11
30 – 39	20
40 – 49	46
50 – 59	57
60 – 69	37
70 – 79	15
80 – 89	7

Draw the cumulative frequency table.

Draw an ogive and use it to find :

(i) The median

(ii) The number of students who scored more than 35% marks.

Solution : The given frequency distribution is discontinuous, to convert it into continuous distribution.

Adjustment factor $= \frac{20-19}{2} = 0{\cdot}5$.

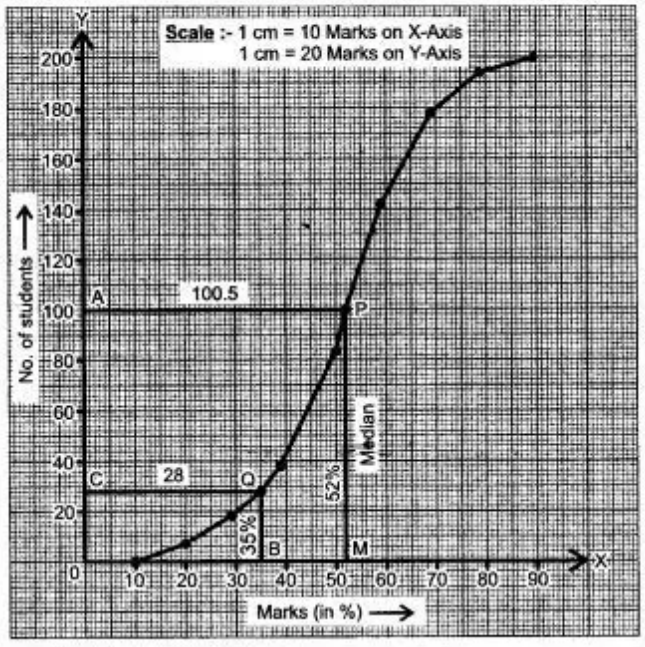
Cumulative (continuous) frequency table for the given data is :

Marks % (Classes before adjustment)	Marks % (Classes after adjustment)	Frequency	Cumulative frequency
10 – 19	9·5 – 19·5	7	7
20 – 29	19·5 – 29·5	11	18
30 – 39	29·5 – 39·5	20	38
40 – 49	39·5 – 49·5	46	84
50 – 59	49·5 – 59·5	57	141
60 – 69	59·5 – 69·5	37	178
70 – 79	69·5 – 79·5	15	193
80 – 89	79·5 – 89·5	7	200

Take 1 cm along X-axis = 10% marks and 1 cm along Y-axis = 25 students.

Plot the points (19·5, 7), (29·5–18), (39·5–38), (49·5–141), (69·5–178), (9·5–193), (89·5–200) and (9·5–0) join these points by a free hand drawing.

The required ogive is drawn in the figure given below :



(i) To find the median : Let A be a point on Y-axis representing frequency

$$= \frac{1}{2}\left[\left(\frac{n^{th}}{2} \text{ term}\right) + \left(\frac{n}{2}+1\right)^{th} \text{ term}\right]$$

$$= \frac{1}{2}(100 + 101)$$

$$= 100{\cdot}5.$$

Through A draw a horizontal line to meet the ogive at P. Through P draw a vertical line to meet X-axis at M. The abscissae of point M represents 52%.

∴ The required median = 52%. Ans.

(ii) Let the point B on X-axis represent 35% marks. Through B draw a vertical line to meet the ogive at Q. Through Q draw a horizontal line to meet Y-axis at C. The ordinate of the point C represents 28 students on Y-axis.

∴ The number of students who scored more than 35% marks = total no. of students – no. of students who scored ≤ 35%

= 200 – 28

= 172. Ans.

Question 8. Use graph paper for this question.
The table given below shows the monthly wages of some factory workers.

(i) Using the table, calculate the cumulative frequency of workers.

(ii) Draw the cumulative frequency curve.

Use 2 cm = ₹ 500, starting the origin at ₹ 6,500 on X-axis, and 2 cm = 100 worker at they Y-axis.

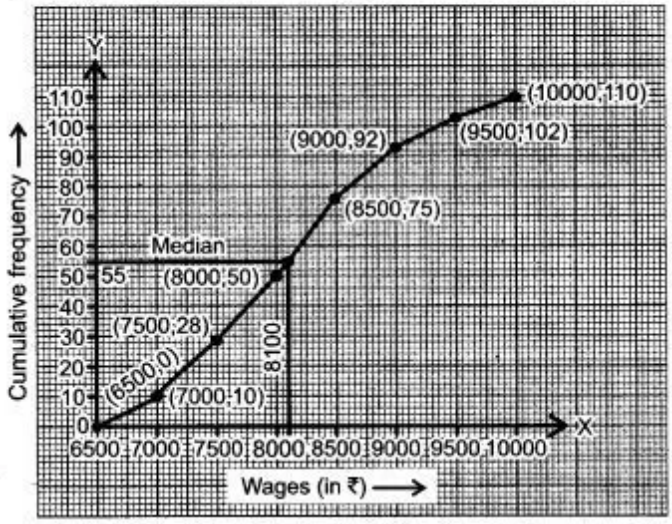
(iii) Use your graph to write down the median wages in ₹.

Wages in ₹ (Class)	No. of workers (frequency)	Cumulative frequency f (x)
6500—7000	10	—
7000—7500	18	—
7500—8000	22	—
8000—8500	25	—
8500—9000	17	—
9000—9500	10	—
9500—10000	8	—

Solution : (i)

Wages in ₹ (Class)	No. of workers (frequency)	Cumulative frequency f (x)
6500—7000	10	10
7000—7500	18	28
7500—8000	22	50
8000—8500	25	75
8500—9000	17	92
9000—9500	10	102
9500—10000	8	110

(ii) Plot the points (6500, 0), (7000, 10), (7500, 28), (8000, 50), (8500, 75), (9000, 92), (9500, 102), (10000, 110) and join them by a free hand curve.



(iii) Here, N = 110

To find the median we shall construct a horizontal line at cumulative frequency

$$= \frac{N}{2} = \frac{110}{2} = 55,$$

intersecting the ogive at (8100, 55)

Hence, median wages = ₹ 8100. Ans.

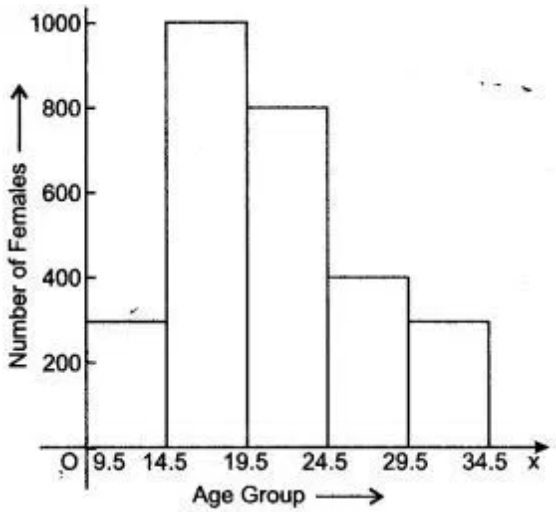
Question 9. Following table present educational level (middle stage) of females in Arunachal pradesh according to 1981 census:

Age group	Number of females (to the nearest ten)
10 – 14	300
15 – 19	980
20 – 24	800
25 – 29	380
30 – 34	290

Draw a histogram to represent the above data.

Solution : Let us convert the given class intervals into continuous class intervals. Then the given frequency distribution takes the form :

Age group	Number of females (to the nearest ten)
9·5 – 14·5	300
14·5 – 19·5	980
19·5 – 24·5	800
24·5 – 29·5	380
29·5 – 34·5	290

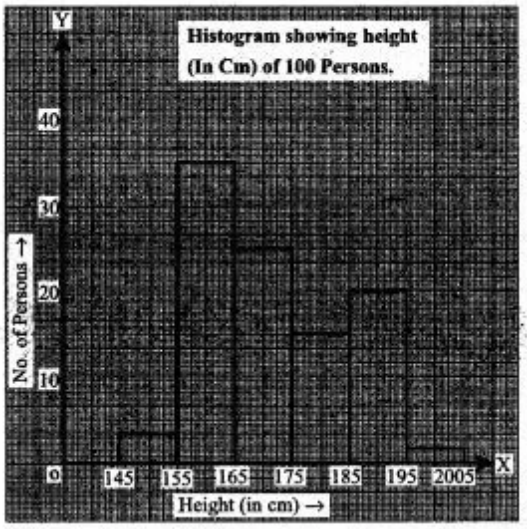


Question 10. Distribution of height in cm of 100 people is given below:

Class interval (cm)	Frequency
145 – 155	3
155 – 165	35
165 – 175	25
175 – 185	15
185 – 195	20
195 – 205	2

Draw a histogram to represent the above data.

Solution :



Question 11. The time taken, in seconds, to solve a problem for each of 25 persons is as follows:

16	20	26	27	28
30	33	37	38	40
42	43	46	46	47
48	49	50	53	58
59	60	64	52	20

(i) Construct a frequency distribution for these data using a class internal of 10 seconds.

(ii) In a school the weekly pocket money of 50 students is as follow's :

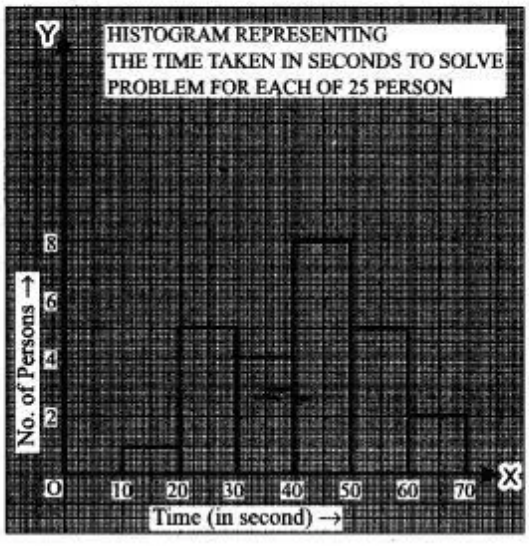
Weekly pocket money (₹)	No. of student
40 – 50	2
59 – 60	8
60 – 70	12
70 – 80	14
80 – 90	8
90 – 100	6

Draw a histogram and a frequency polygon on the same graph. Find mode from the graph.

Solution : (i) Frequency table

Time (in seconds)	Tally marks	Frequency
10 – 20	I	1
20 – 30	𝍸	5
30 – 40	𝍸 I	4
40 – 50	𝍸 III	8
50 – 60	𝍸	5
60 – 70	II	2

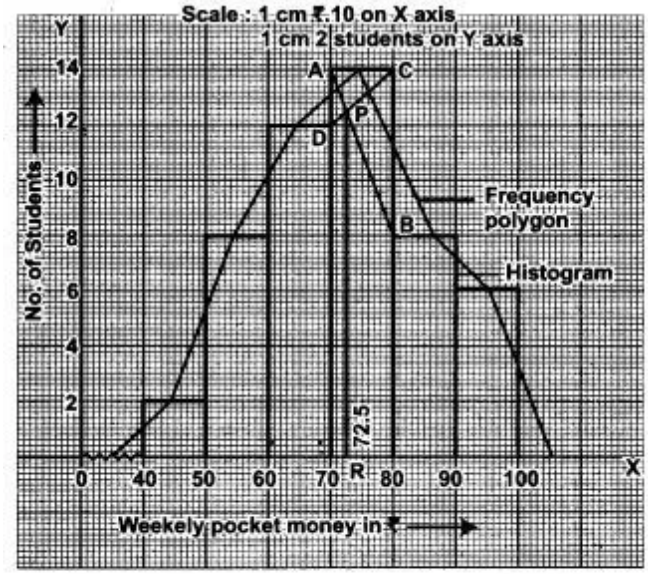
Histogram representing the time taken in seconds, to solve. A problem for each of 25 persons.



(ii) Frequency distribution table is

Weekly pocket money (in ₹)	Class Marks	No. of Students
40–50	45	2
50–60	55	8
60–70	65	12
70–80	75	14
80–90	85	8
90–100	95	6

Draw the histogram and frequency polygon on the graph.



Now, in the highest rectangle, draw two straight line AB and CD from the corners of the rectangle on either sides of the highest rectangle to opposite corners of the highest rectangle. They intersect P. Draw PR X-axis, then abscissa of the point prepresents ₹ 72·5.

Hence, the required mode is ₹ 72·5. Ans.

Question 12. Using a graph paper, drawn an Ogive for the following distribution which shows a record of the weight in kilograms of 200 students.

Weight	Frequency
40 – 45	5
45 – 50	17
50 – 55	22
55 – 60	45
60 – 65	51
65 – 70	31
70 – 75	20
75 – 80	9

Use your ogive to estimate the following :

(i) The percentage of students weighing 55 kg or more.

(ii) The weight above which the heaviest 30% of the students fall.

(iii) The number of students who are :

(1) under-weight and

(2) over-weight, if 55·70 kg is considered as standard weight.

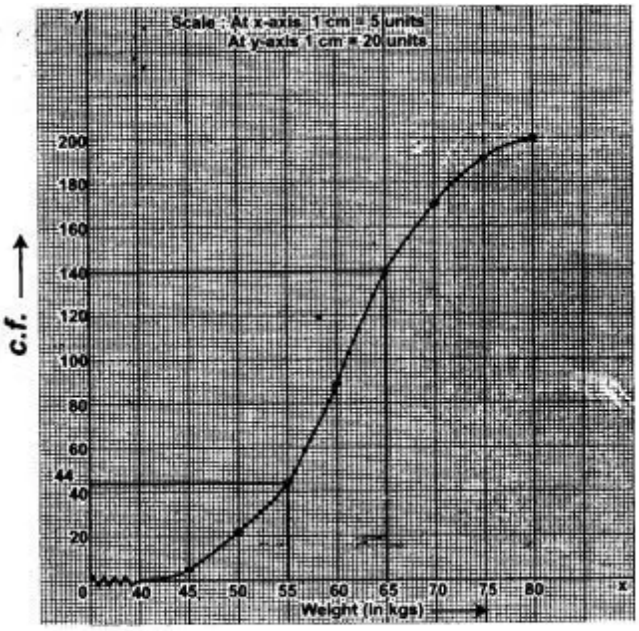
Solution :

Weight	Frequency	*c.f.*
40–45	5	5
45–50	17	22
50–55	22	44
55–60	45	89
60–65	51	140
65–70	31	171
70–75	20	191
75–80	9	200

(i) Number of student weighing 55 kg or more = 200 – 44 = 156

$$\therefore \text{Percentage} = \frac{156 \times 100}{200}$$

$$= 78\%$$ Ans.



(ii) 30% of 200 = 60

∴ Heaviest wt. (least) = wt. of 200 – 60

= 140th student

= 65 kg or more Ans.

(iii) From ogive c.f. against 55.70 kg

= 45

∴ (1) number of under wt. students

= 44 Ans.

(2) number of over wt. students

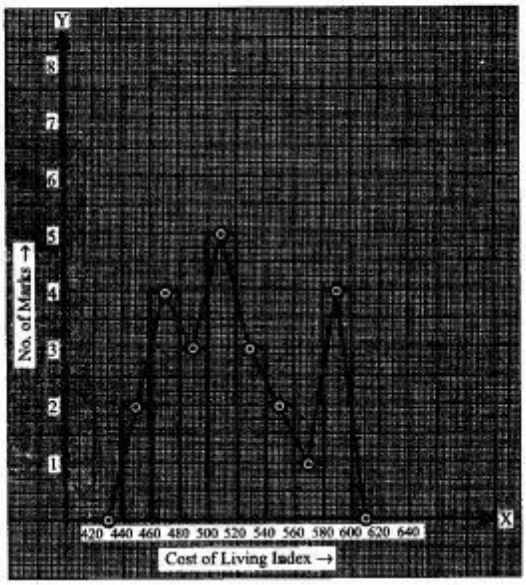
= 200 – 44

= 156 Ans.

Question 13. Draw a histogram and frequency polygon to represent the following data (on the same scale) which shows the monthly cost of living index of a city in a period of 2 years:

Cost of living Index	Number of months
440 – 460	2
460 – 480	4
480 – 500	3
500 – 520	5
520 – 540	3
540 – 560	2
560 – 580	1
580 – 600	4
Total	24

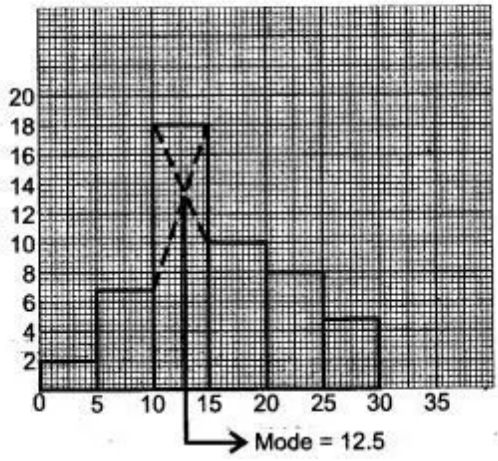
Solution : Histogram and frequency polygon representing the cost of living index of city in a period of 2 years :



Question 14. Draw the histogram for the following frequency distribution and hence estimate the mode for the distribution.

Class	Frequency
0 – 5	2
5 – 10	7
10 – 15	18
15 – 20	10
20 – 25	8
25 – 30	5
Total	24

Solution :



Question. 15. The frequency distribution of scores obtained by 230 candidates in a medical entrance test is as ahead:

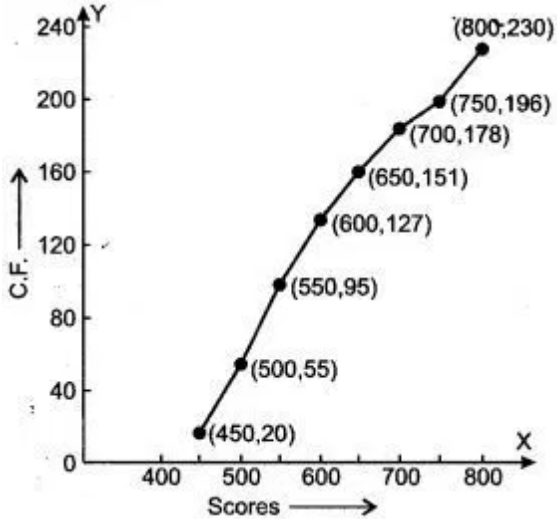
Cost of living Index	Number of Months
400 – 450	20
450 – 500	35
500 – 550	40
550 – 600	32
600 – 650	24
650 – 700	27
700 – 750	18
750 – 800	34
Total	230

Draw a cummulative polygon (ogive) to represent the above data.

Solution : The cummulative frequency table for the given frequency table as given below :

Interval (score)	Frequency	Cummulative Frequency
400 – 450	20	20
450 – 500	35	55
500 – 550	40	95
550 – 600	32	127
600 – 650	24	151
650 – 700	27	178
700 – 750	18	196
750 – 800	34	230

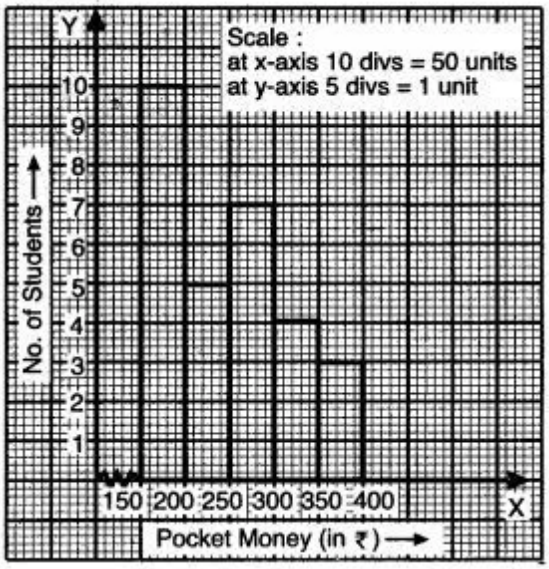
Ogive representing the scores obtained by 230 candidates in a medical entrance test.



Question 16. Draw a histogram to represent the following data:

Pocket money in ₹	No. of Students
150 – 200	10
200 – 250	5
250 – 300	7
300 – 350	4
350 – 400	3

Solution :



Question 17. Use graph paper for this question. The following table shows the weights in gm of a sample of 100 potatoes taken from a large consignment:

Weight (gms)	Frequency
50 – 60	8
60 – 70	10
70 – 80	12
80 – 90	16
90 – 100	18
100 – 110	14
110 – 120	12
120 – 130	10

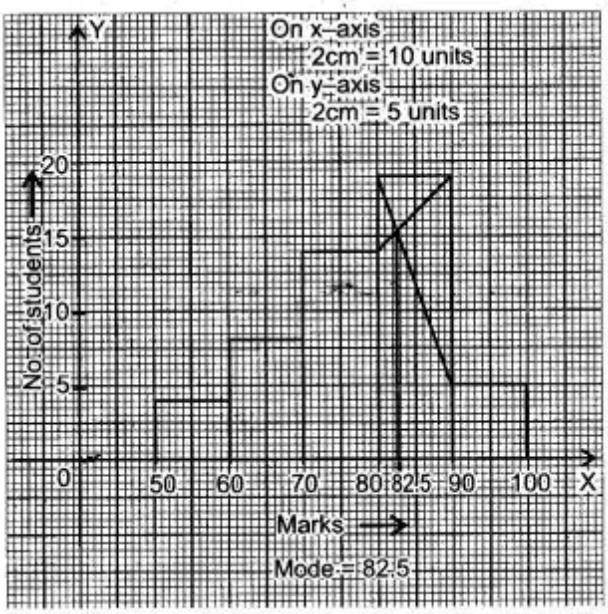
(i) Calculate the cumulative frequencies.

(ii) Draw the cummulative frequency curve and from it determine the median weights of the potatoes.

Solution : (i) The cummulative frequencies table is :

Weight (gm)	Number of potatoes (f)	Cummulative Frequency
50 – 60	8	8
60 – 70	10	18
70 – 80	12	30
80 – 90	16	46
90 – 100	18	64
100 – 110	14	78
110 – 120	12	90
120 – 130	10	100

(ii) Plotting the points (60, 8), (70, 18), (80, 30), (90, 46), (100, 64), (110, 78), (120, 90), (130, 100) and joining them by a free hand we get cummulative frequency curve as shown the figure. To complete it, we join the curve to the point (lower limit of the lowest class, 50) *i.e.*, (50, 0).



The positive of median is given by $\frac{n}{2} = \frac{100}{2} = 50$.

On vertical axis form the mark of 50. Draw the horizontal line cutting the curve at a point for which the abscissa is 92 gms. Which is the value of the median. Ans.

Question 18. Attempt this question on a graph paper. The table shows the distribution of marks gained by a group of 400 students in an examination:

Marks less than	No. of Students
10	5
20	10
30	30
40	60
50	105
60	180
70	270
80	355
90	390
100	400

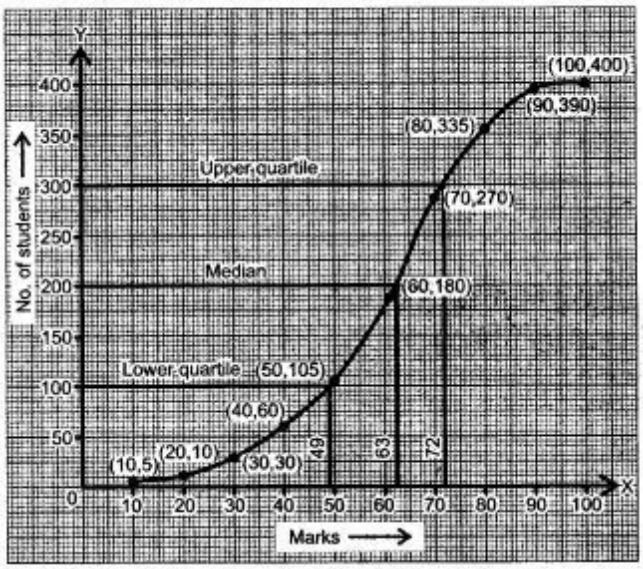
Using a scale of 2 cm to represents 10 marks and 2 cm to represent 50 students, plot these values and draw a smooth curve through the points.

Estimate for the graph (i) the median mark, (ii) the quartile marks.

Solution : By plotting the points (10, 5), (20, 10), (30, 30), (40, 60), (50, 105), (60, 180), (70, 270), (80, 355), (90, 390) and (100, 400), we get the ogive for the given frequency table, as shown in the figure.

Scale : 2 cm to represents 10 marks

2 cm to represent 50 students.



(i) To find the median, we shall draw a horizontal line at $c.f. = \frac{N}{2} = \frac{400}{2} = 200$. Intersecting the ogive at the point (200, 63).

Hence the median is 63. Ans.

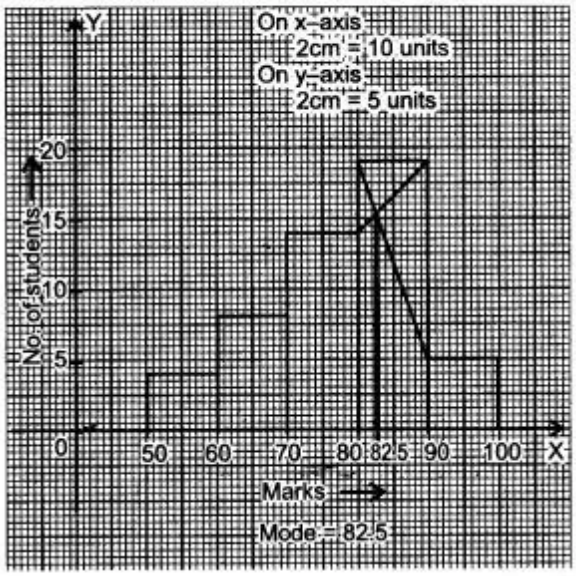
(ii) To find the lower quartile, we shall construct a horizontal line at $c.f\ \frac{N}{4} = \frac{400}{4} = 100$, intersecting the ogive at the point (49, 100). Hence, 49 is the lower line at $c.f. \frac{3N}{4} = \frac{3 \times 400}{4} = 300$. Intersecting the ogive at the point (300, 72). Hence, the upper quartile mark is 72. Ans.

Question 19. A Mathematics aptitude test of 50 students was recorded as follows:

Marks	No. of Students
50–60	4
60–70	8
70–80	14
80–90	19
90-100	5

Draw a histogram for the above data using a graph paper and locate the mode.

Solution : From the graph the mode of given data = 82·5



Question 20. The daily wages of 160 workers in a building project are given below:

Wages in ₹	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of Workers	12	20	30	38	24	16	12	8

Using a graph paper, draw an Ogive for the above distribution.

Use your Ogive to estimate :

(i) the median wage of the workers.

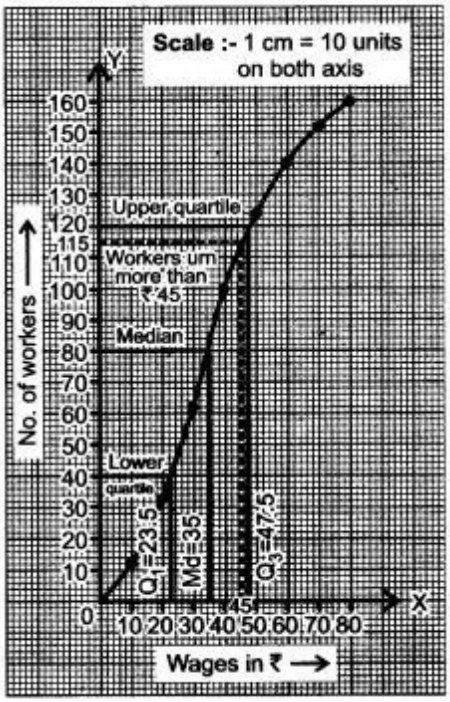
(ii) the upper quartile wage of the workers

(iii) the lower quartile wages of the workers

(iv) the percentage of workers who earn more than ₹ 45 a day.

Solution :

Wages (in ₹)	No. of Workers	Cumulative frequency
0–10	12	12
10–20	20	32
20–30	30	62
30–40	38	100
40–50	24	124
50–60	16	140
60–70	12	152
70–80	8	160



(i) $\text{Median} = \left(\frac{n}{2}\right)^{th} \text{observation} = \left(\frac{160}{2}\right)^{th} \text{observation}$

$= 80^{th} \text{ observation}$

Now the 80^{th} position in the ogive diagram represent the median wage of workers by the graph = 35. Ans.

(ii) $\text{Upper quartile} = \left(\frac{3n}{4}\right)^{th} \text{observation}$

$= \left(\frac{3 \times 160}{4}\right)^{th} \text{observation}$

$= 120^{th} \text{ observation}$

Now the 120^{th} observation in the ogive diagram represents wage of the workers $(Q_3) = 47{\cdot}5$ Ans.

(iii) $\text{Lower quartile} = \left(\frac{n}{4}\right)^{th} \text{observation} = \left(\frac{160}{4}\right)^{th} \text{observation}$

40^{th} observation

The 40^{th} observation in the ogive diagram represents wage of the workers $Q_1 = 23{\cdot}5$ Ans.

(iv) The percentage of workers earmore than ₹45

$= \frac{160 - 115}{100} \times 100 = \frac{45 \times 10}{16} = 28{\cdot}125\%$ Ans.

Question 21. The marks obtained by 120 students in a test are given below:

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of Students	5	9	16	22	26	18	11	6	4	3

Draw an ogive for the given distribution on a graph sheet.

Use suitable scale for ogive to estimate the following :

(i) The median.

(ii) The number of students who obtained more than 75% marks in the test.

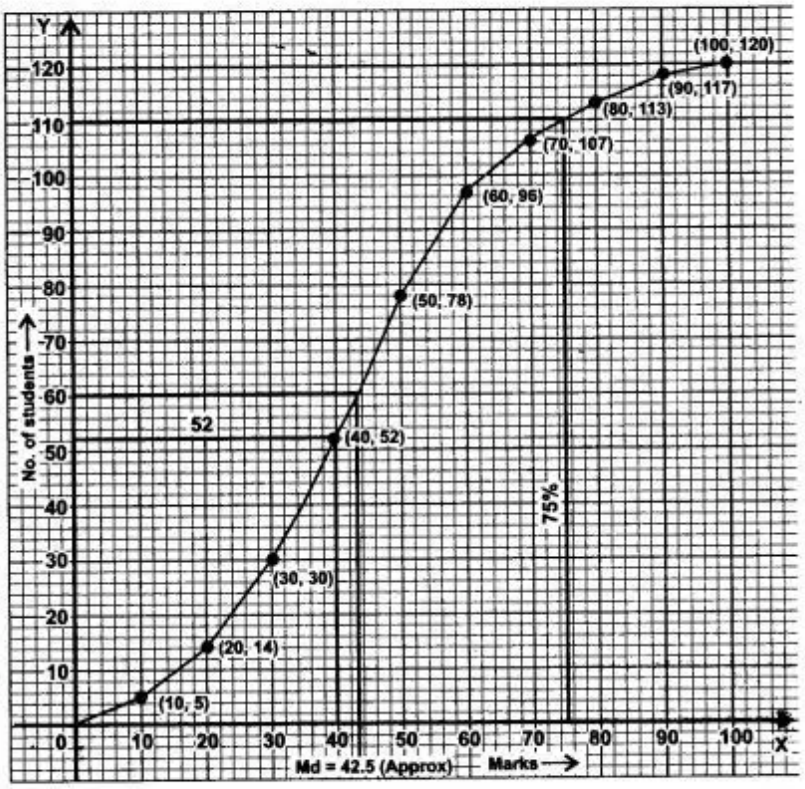
(iii) The number of students who did not pass the test if minimum marks required to pass is 40.

Solution :

Marks C.I.	No. of Students f	*C.f.*
0—10	5	5
10—20	9	14
20—30	16	30
30—40	22	52
40—50	26	78
50—60	18	96
60—70	11	107
70—80	6	113
80—90	4	117
90—100	3	120

(i) Here $n = 120$, even

$\therefore$ Median $= \left(\frac{120}{2}\right)^{th}$ observation $= 60^{th}$ observation $= 42{\cdot}5$ (approx)



(ii) Number of students who obtained more than 75% marks in the test
$= 120 - 110 = 10$

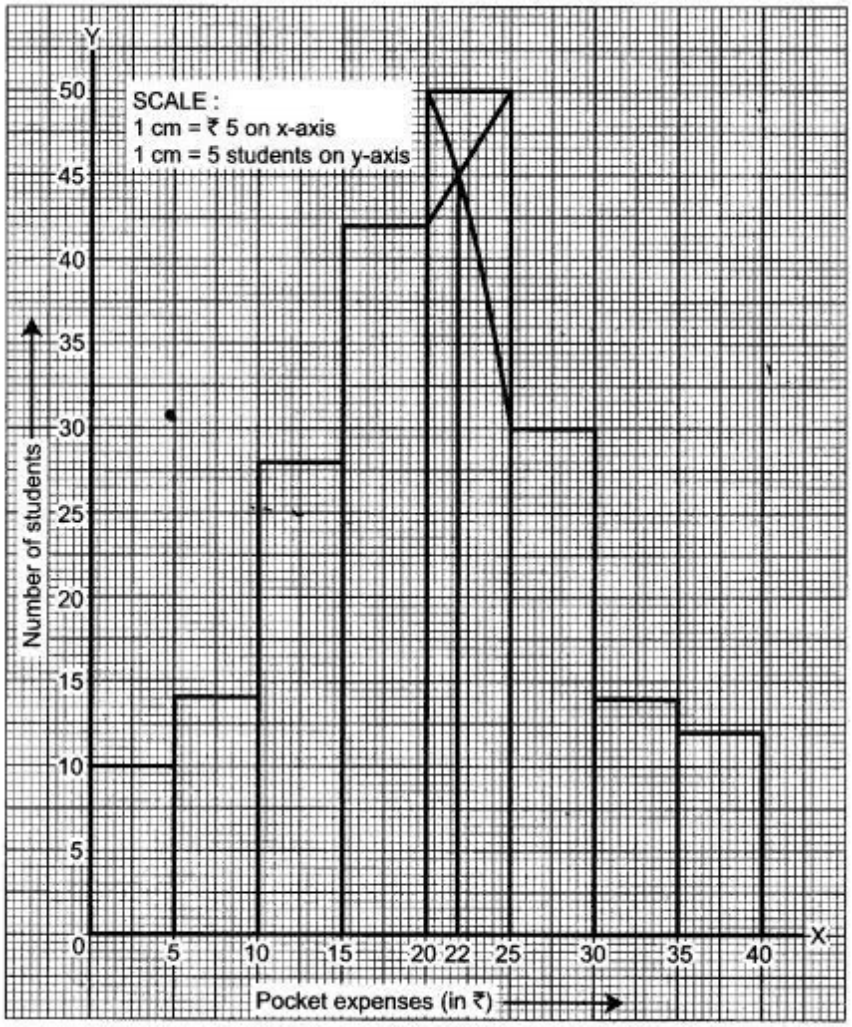
(iii) Number of students who did not pass the test if minimum marks required to pass is 40 = 52.

Question 22. (Use a graph paper for this question.) The daily pocket expenses of 200 students in a school are given below:

Pocket expenses (in ₹)	Number of students (frequency)
0—5	10
5—10	14
10—15	28
15—20	42
20—25	50
25—30	30
30—35	14
35—40	12

Draw a histogram representing the above distribution and estimate the mode from the graph.

Solution: Histogram on the graph paper.



Mode = 22

Question 23. The marks obtained by 100 students in a Mathematics test are given below:

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of Students	3	7	12	17	23	14	9	6	5	4

Draw an ogive for the given distribution on a graph sheet.
(Use a scale of 2 cm = 10 units on both axis).
use the ogive to estimate the :
(i) median.

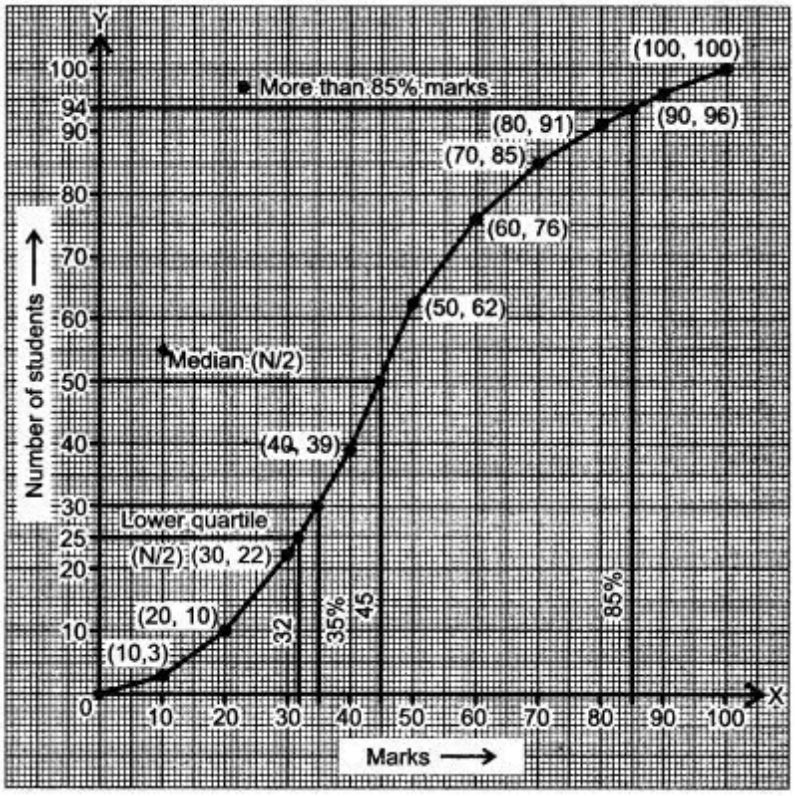
(ii) lower quartile.

(iii) number of students who obtained more than 85% marks in the test.

(iv) number of students who did not pass in the test if the pass percentage was 35.

Solution :

Marks	C.F.	Points
Less than 10	3	(10,3)
Less than 20	10	(20, 10)
Less than 30	22	(30, 22)
Less than 40	39	(40, 39)
Less than 50	62	(50, 62)
Less than 60	76	(60, 76)
Less than 70	85	(70, 85)
Less than 80	91	(80, 91)
Less than 90	96	(90, 96)
Less than 100	100	(100, 100)



(i) Median $= \left(\frac{n}{2}\right)^{th}$ observation $= \left(\frac{100}{2}\right)^{th}$ observation $= 50^{th}$ observation $= 45$

(ii) Lower Quartile $(Q_1) = \left(\frac{N}{4}\right)^{th}$ observation $= \left(\frac{100}{4}\right)^{th}$ observation $= 25^{th}$ observation $= 32$

(iii) Number of students who obtained more than 85% marks

$= (100 - 94) = 6.$

(iv) Number of students who did not pass if passing % of marks is 35

$= 30.$

Concept Based Questions

Question 1. The median of the following observations 11, 12, 14, (x – 2), (x + 4), (x + 9), 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean.

Solution : 11, 12, 14, $(x-2)$, $(x+4)$, $(x+9)$, 32, 38, 47

$$n = 9, \text{ odd}$$

$$\therefore \quad \text{Median} = \left(\frac{9+1}{2}\right)^{\text{th}}$$

$$24 = 5^{\text{th}} \text{ observation} = (x+4)$$

$$24 = x+4 \quad (\text{as Median} = 24)$$

$$24-4 = x$$

$$\Rightarrow \quad x = 20$$

$\therefore$ Observation are 11, 12, 14, $(20-2)$, $(20+4)$, $(20+9)$, 32, 38, 47

or 11, 12, 14, 18, 24, 29, 32, 38, 47

$$\text{Mean} = \overline{X}$$

$$= \frac{11+12+14+18+24+29+32+38+47}{9}$$

$$= \frac{225}{9} = 25 \qquad \text{Ans.}$$

Question 2. The mean of 16 numbers is 8. If 2 is added to every number, what will be the new mean ?

Solution : Let $x_1, x_2, x_3, \ldots, x_{16}$ be 16 numbers with their mean equal to 8. Then

$$\overline{X} = \frac{1}{n}\left(\sum_{i=1}^{16} x_i\right)$$

$$\Rightarrow \quad 8 = \frac{x_1 + x_2 + \ldots + x_{16}}{16}$$

$$\Rightarrow \quad x_1 + x_2 + \ldots + x_{16} = 16 \times 8 = 128 \qquad \ldots(i)$$

New numbers are $x_1 + 2, x_2 + 2, x_3 + 2, \ldots, x_{16} + 2$.

Let $\overline{X}$ be the mean of new numbers. Then,

$$\overline{X} = \frac{(x_1+2)+(x_2+2)+\ldots+(x_{16}+2)}{16}$$

$$= \frac{(x_1+x_2+\ldots+x_{16})+2\times 16}{16}$$

$$= \frac{128+32}{16}, \qquad [\text{Using (i)}]$$

$$= \frac{160}{16} = 10. \qquad \text{Ans.}$$

Question 3. The mean monthly salary of 10 members of a group is Rs.1,445, one more member whose monthly salary is Rs.1,500 has joined the group. Find the mean monthly salary of 11 members of the group.

Solution : Here, $n = 10$, $\overline{X} = 1445$.

So $$\overline{X} = \frac{1}{n}(\Sigma x_i)$$

$$\Rightarrow \quad n\overline{X} = \Sigma x_i$$

$$\Rightarrow \quad 10 \times 1445 = \Sigma x_i$$

$$\Rightarrow \quad \Sigma x_i = 14450.$$

So, total monthly salary of 10 persons is of ₹ 14450.

Monthly salary of one more person who joined the group is ₹ 1,500.

∴ Total monthly salary of 11 persons

= ₹ 14,450 + Rs. 1,500

= ₹ 15,950

So, average monthly salary of 11 persons

$$= \frac{\text{Total monthly salary}}{11}$$

$$= \frac{15950}{11} = ₹\ 1{,}450.$$ Ans.

Question 4. The mean of 40 observations was 160. It was detected on rechecking that the value of 165 was wrongly copied as 125 for computation of mean. Find the correct mean.

Solution : Here, $n = 40$, $\overline{X} = 160$.

So, $$\overline{X} = \frac{1}{n}(\Sigma x_i)$$

$$\Rightarrow \quad 160 = \frac{1}{40}(\Sigma x_i)$$

$$\Rightarrow \quad \Sigma x_i = 160 \times 40 = 6400$$

∴ Incorrect value of $\Sigma x_i = 6400$.

Now, correct value of

Σx_i = Incorrect value of Σx_i – Incorrect item + Correct item

= 6400 – 125 + 165 = 6440

∴ Correct mean

$$= \frac{\text{Correct value of } \Sigma x_i}{n}$$

$$= \frac{6440}{40} = 161.$$

Question 5. The mean of 100 items was found to be 30. If at the time of calculation two items were wrongly taken as 32 and 12 instead of 23 and 11, find the correct mean.

Solution : Here,

$$n = 100, X = 30.$$

So, $\overline{X} = \frac{1}{n}(\Sigma x_i)$

$\Rightarrow \quad \Sigma x_i = n\overline{X}$

$\Rightarrow \quad \Sigma x_i = 100 \times 30 = 3000$

$\therefore$ Incorrect value of $\Sigma x_i = 3000$

Now, correct value of

Σx_i = Incorrect value of Σx_i
– (Sum of incorrect values)
+ (Sum of correct values)

$= 3000 - (32 + 12) + (23 + 11)$

$= 2990.$

$\therefore$ Correct mean

$$= \frac{\text{Correct value of } \Sigma x_i}{n}$$

$$= \frac{2990}{100} = 29.9.$$ Ans.

Question 6. If $\overline{X}$ is the mean of n observations $x_1, x_2, x_3, \ldots, x_n$ then the mean of $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \ldots, \frac{x_n}{a}$ is $\frac{\overline{X}}{a}$, where a is an non-zero number.

i.e., if each observation is divided by a non-zero number, then the mean is also divided by it.

Solution : We have

$$\overline{X} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) \qquad \ldots(i)$$

Let $\overline{X}$ be the item of $\frac{x_1}{a}, \frac{x_2}{a}, \ldots, \frac{x_n}{n}$. Then

$$\overline{X} = \frac{1}{n}\left(\frac{x_1}{a} + \frac{x_2}{a} + \ldots + \frac{x_n}{n}\right)$$

$$= \frac{1}{n}\left(\frac{x_1 + x_2 + \ldots + x_n}{a}\right)$$

$$= \frac{1}{a}\left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right)$$

$$= \frac{1}{a}\left[\frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\right]$$

$$= \frac{1}{a}(\overline{X}), \qquad \text{[Using (i)]}$$

$$= \frac{\overline{X}}{a}.$$ Ans.

Question 7. The average score of girls in class X examination in school is 67 and that of boys is 63. The average score for the whole class is 64.5. Find the percentage of girls and boys in the class.

Solution : Let the number of girls and boys be n_1 and n_2 respectively.

We have

$\overline{X}_1$ = Average score of girls = 67

$\overline{X}_2$ = Average score of boys = 63

$\overline{X}$ = Average score of the whole class = 64·5

$$\therefore \quad \overline{X} = \frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2}$$

$$\Rightarrow \quad 64{\cdot}5 = \frac{67n_1 + 63n_2}{n_1 + n_2}$$

$$\Rightarrow \quad 64{\cdot}5n_1 + 64{\cdot}5n_2 = 67n_1 + 63n_2$$

$$\Rightarrow \quad 2{\cdot}5n_1 = 1{\cdot}5n_2$$

$$\Rightarrow \quad 25n_1 = 15n_2$$

$$\Rightarrow \quad 5n_1 = 3n_2$$

Total number of students in the class = $n_1 + n_2$.

$$\therefore \text{Percentage of girls} = \frac{n_1}{n_1 + n_2} \times 100$$

$$= \frac{n_1}{n_1 + \frac{5n_1}{3}} \times 100, \quad [\because 5n_1 = 3n_2]$$

$$= \frac{3n_1}{3n_1 + 5n_1} \times 100$$

$$= \frac{3}{8} \times 100 = 37{\cdot}5\%$$

$$\text{and Percentage of boys} = \frac{n_2}{n_1 + n_2} \times 100$$

$$= \frac{n_2}{\frac{3n_2}{5} + n_2} \times 100$$

$$= \frac{5n_2}{3n_2 + 5n_2} \times 100$$

$$= 62{\cdot}5\%$$

Hence, there are 37·5% girls and 62·5% boys in the class. Ans.

Question 8. The mean weight of 150 students in a certain class is 60 kgs. The mean weight of boys in the class is 70 kg and that of girls is 55 kgs. Find the number of boys and the number of girls in the class.

Solution : Let the number of boys and girls in the class be n_1 and n_2 respectively. Then

$$n_1 + n_2 = 150 \quad \ldots(i)$$

We have

$\overline{X}_1$ = Mean weight of boy = 70 kgs.

$\overline{X}_2$ = Mean weight of girls = 55 kgs.

$\overline{X}$ = Mean weight of all students = 60 kgs.

$$\therefore \quad \overline{X} = \frac{n_1\overline{X}_1 + n\overline{X}_2}{n_1 + n_2}$$

$$\Rightarrow \quad 60 = \frac{n_1 \times 70 + n_2 \times 55}{n_1 + n_2}$$

$$\Rightarrow \quad 60(n_1 + n_2) = 70n_1 + 55n_2$$

$$\Rightarrow \quad 60n_1 + 60n_2 = 70n_1 + 55n_2$$

$$\Rightarrow \quad 10n_1 = 5n_2$$

$$\Rightarrow \quad 2n_1 = n_2 \quad \ldots(ii)$$

Putting $n_2 = 2n_1$ from (ii) in (i), we get

$$n_1 + 2n_1 = 150$$

$$\Rightarrow \quad 3n_1 = 150$$

$$\Rightarrow \quad n_1 = 50.$$

Putting $n_1 = 50$ in $n_2 = 2n_1$, we get

$$n_2 = 100.$$

Hence, there are 50 boys and 100 girls in the class.

Question 9. The numbers 6, 8, 10, 12, 13, and x are arranged in an ascending order. If the mean of the observations is equal to the median, find the value of x.

Solution : The number's are 6, 8, 10, 12, 13 and x

$$n = 6$$

$$\text{Mean} = \frac{6 + 8 + 10 + 12 + 13 + x}{6}$$

$$\text{Mean} = \frac{49 + x}{6} \quad \ldots(1)$$

For Median $n = 6$ (even)

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{term} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{term}}{2}$$

$$\text{Median} = \frac{3\text{rd term} + 4\text{th term}}{2}$$

$$= \frac{10 + 12}{2}$$

$$= \frac{22}{2} = 11 \quad \ldots(2)$$

From (1) and (2) (From question)

$$\text{Median} = \text{Mean}$$

$$11 = \frac{49 + x}{6}$$

$$\Rightarrow \quad x = 66 - 49$$

$$\Rightarrow \quad x = 17.$$