Chapter

4

Circle and Systems of Circles

CONTENTS

G. Cramer

Cramer (1750 A.D.) made formal use of the two axes and gane the eqution of a circle as $(y-a)^2 + (b-x)^2 = r.r.$ He gave the best exposition of the analytic geometry of his time.

Kochanski gives an approximate method to find the length of the circumference of a circle. Jones introduces the Greek latter to represent the ratio of the circumference of a circle to its diameter in his Synopsis palmariorum matheseos (A new introduction to Mathematics).

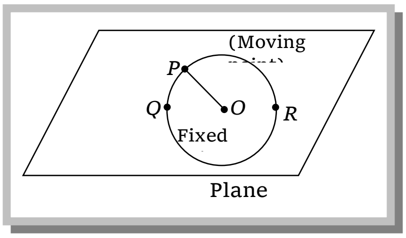
Feuerbach publishes his discoveries on the nine point circle of a triangle.

Nicholas of Cusa studies geometry and logic. He contributes to the study of infinity, the infinitely large and the infinitely small. He looks at the circle as the limit of regular polygons.

Circle and Systems of Circles

4.1 Definition

A circle is defined as the locus of a point which moves in a plane such that its distance from a fixed point in that plane always remains the same *i.e.* constant.



The fixed point is called the centre of the circle and the fixed distance is called the radius of the circle.

Note :
- If $r(r>0)$ is the radius of a circle, the diameter $d=2r$ is the maximum distance between any two points on the given circle.
- The length of the curve or perimeter (also called circumference) of circle $=2\pi r$ or πd.
- The area of circle $=\pi r^2$ or $\frac{\pi d^2}{4}$.
- Line joining any two points of a circle is called chord of circle.
- Curved section between any two points of a circle is called arc of circle.
- Angle subtended at the centre of a circle by any arc = arc/radius.
- Angle subtended at the centre of a circle by an arc is double of angle subtended at the circumference of a circle.

4.2 Standard forms of Equation of a Circle

(1) **General equation of a circle :** The general equation of a circle is $x^2+y^2+2gx+2fy+c=0$, where g, f, c are constant.

(i) Centre of the circle is $(-g, -f)$. *i.e.*, $(-\frac{1}{2}$ coefficient of x, $-\frac{1}{2}$ coefficient of $y)$

(ii) Radius of the circle is $\sqrt{g^2+f^2-c}$.

Note :
- The general equation of second degree $ax^2+by^2+2hxy+2gx+2fy+c=0$ represents a circle if $a=b\neq 0$ and $h=0$.
- Locus of a point P represent a circle if its distance from two points A and B is not equal *i.e.* $PA=kPB$ represent a circle if $k\neq 1$.
- Discussion on nature of the circle :
  - If $g^2+f^2-c>0$, then the radius of the circle will be real. Hence, in this case, it is possible to draw a circle on a plane.
  - If $g^2+f^2-c=0$, then the radius of the circle will be zero. Such a circle is known as point circle.

- If $g^2+f^2-c<0$, then the radius $\sqrt{g^2+f^2-c}$ of the circle will be an imaginary number. Hence, in this case, it is not possible to draw a circle.

❑ Special features of the general equation $x^2+y^2+2gx+2fy+c=0$ of the circle :

This equation has the following peculiarities :

- It is a quadratic equation in x and y.
- Here the co-efficient of x^2 = the co-efficient of y^2

In working out problems it is advisable to keep the co-efficient of x^2 and y^2 as unity.

- There is no term containing xy, *i.e.* the co-efficient of the term xy is zero.
- This equation contains three arbitrary constants. If we want to find the equation of a circle of which neither the centre nor the radius is known, we take the equation in the above form and determine the values of the constants g, f, c for the circle in question from the given geometrical conditions.

❑ Keeping in mind the above special features, we can say that the equation

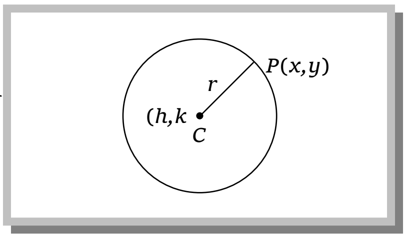
$ax^2+ay^2+2gx+2fy+c=0$ .....(i) also represents a circle.

This equation can also be written as $x^2+y^2+2\frac{g}{a}x+2\frac{f}{a}y+\frac{c}{a}=0$, dividing by $a\neq 0$.

Hence, the centre $=\left(\frac{-g}{a},\frac{-f}{a}\right)$ and radius $=\sqrt{\frac{g^2}{a^2}+\frac{f^2}{a^2}-\frac{c}{a}}$

(2) **Central form of equation of a circle :** The equation of a circle having centre (h, k) and radius r is

$$(x-h)^2+(y-k)^2=r^2$$



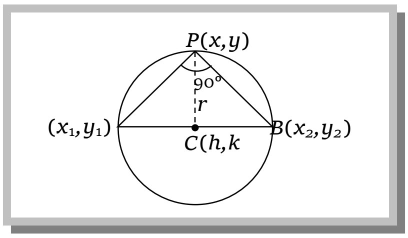
Note : ❑ If the centre is origin, then the equation of the circle i

$$x^2+y^2=r^2$$

❑ If r = 0, then circle is called point circle and its equation is $(x-h)^2+(y-k)^2=0$

(3) **Concentric circle :** Two circles having the same centre C (h, k) but different radii r_1 and r_2 respectively are called concentric circles. Thus the circles $(x-h)^2+(y-k)^2=r_1^2$ and $(x-h)^2+(y-k)^2=r_2^2$, $r_1\neq r_2$ are concentric circles. Therefore, the equations of concentric circles differ only in constant terms.

(4) **Circle on a given diameter :** The equation of the circle drawn on the straight line joining two given points (x_1,y_1) and (x_2,y_2) as diameter is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.



Note : ❑ If the coordinates of the end points of a diameter of a circle are given, we can also find the equation of the circle by finding the coordinates of the centre and radius. The centre is the mid-point of the diameter and radius is half of the length of the diameter.

(5) **Parametric coordinates**

(i) The parametric coordinates of any point on the circle $(x-h)^2+(y-k)^2=r^2$ are given by $(h+r\cos\theta, k+r\sin\theta)$, $(0\le\theta<2\pi)$

In particular, co-ordinates of any point on the circle $x^2+y^2=r^2$ are $(r\cos\theta, r\sin\theta)$, $(0\le\theta<2\pi)$

(ii) The parametric co-ordinates of any point on the circle $x^2+y^2+2gx+2fy+c=0$ are

$$x=-g+\sqrt{(g^2+f^2-c)}\cos\theta \text{ and } y=-f+\sqrt{(g^2+f^2-c)}\sin\theta, \quad (0\le\theta<2\pi)$$

(6) **Equation of a circle under given conditions:** The general equation of circle, *i.e.*, $x^2+y^2+2gx+2fy+c=0$ contains three independent constants *g, f* and *c*. Hence for determining the equation of a circle, three conditions are required.

(i) The equation of the circle through three non-collinear points $A(x_1,y_1), B(x_2,y_2), C(x_3,y_3)$:

Let the equation of circle be $x^2+y^2+2gx+2fy+c=0$ .....(i)

If three points (x_1,y_1), (x_2,y_2), (x_3,y_3) lie on the circle (i), their co-ordinates must satisfy its equation. Hence solving equations $x_1^2+y_1^2+2gx_1+2fy_1+c=0$ .....(ii)

$$x_2^2+y_2^2+2gx_2+2fy_2+c=0 \quad .....(iii)$$

$$x_3^2+y_3^2+2gx_3+2fy_3+c=0 \quad .....(iv)$$

g, f, c are obtained from (ii), (iii) and (iv). Then to find the circle (i).

Alternative method

(1) The equation of the circle through three non-collinear points $A(x_1,y_1), B(x_2,y_2), C(x_3,y_3)$ is

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix}=0$$

(2) From given three points taking any two as extremities of diameter of a circle *S* = 0 and equation of straight line passing through these two points is *L* = 0. Then required equation of circle is $S+\lambda L=0$, where λ is a parameter which can be found out by putting third point in the equation.

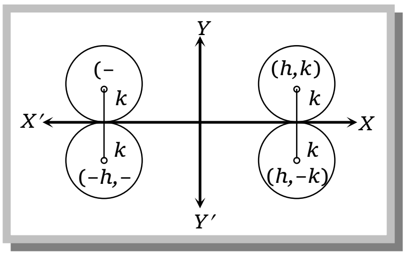
Note : ❑ **Cyclic quadrilateral :** If all the four vertices of a quadrilateral lie on a circle, then the quadrilateral is called a cyclic quadrilateral. The four vertices are said to be concylic.

4.3 Equation of a Circle in Some special cases

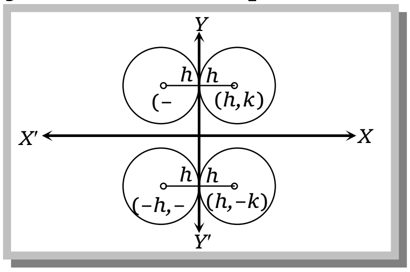
(1) If centre of the circle is (h,k) and it passes through origin then its equation is

$$(x-h)^2+(y-k)^2=h^2+k^2 \Rightarrow x^2+y^2-2hx-2ky=0$$

(2) If the circle touches x axis then its equation is (Four cases) $(x \pm h)^2 + (y \pm k)^2 = k^2$

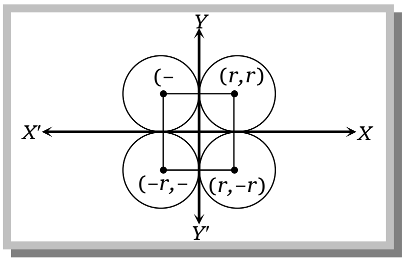


(3) If the circle touches y axis then its equation is (Four cases) $(x \pm h)^2 + (y \pm k)^2 = h^2$



(4) If the circle touches both the axes then its equation is (Four

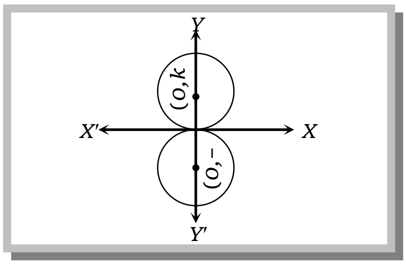
$(x \pm r)^2 + (y \pm r)^2 = r^2$



(5) If the circle touches x- axis at origin (Two cases)

$x^2 + (y \pm k)^2 = k^2$

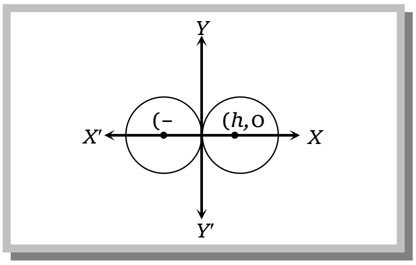
$\Rightarrow x^2 + y^2 \pm 2ky = 0$



(6) If the circle touches y-axis at origin (Two cases)

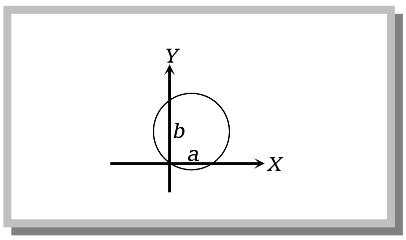
$(x \pm h)^2 + y^2 = h^2$

$\Rightarrow x^2 + y^2 \pm 2xh = 0$



(7) If the circle passes through origin and cut intercepts of a and b on axes, the equation of circle is (Four cases)

$x^2 + y^2 - ax - by = 0$ and centre is $(a/2, b/2)$



Note : ❑ **Circumcircle of a triangle :** If we are given sides of a triangle, then first we should find vertices then we can find the equation of the circle using general form.

Alternate : If equation of the sides are $L_1 = 0, L_2 = 0$ and $L_3 = 0$, then equation of circle is $(L_1.L_2) + \lambda(L_2.L_3) + \mu(L_3.L_1) = 0$, where λ and μ are the constant which can be found out by the conditions, coefficient of $x^2 =$ coefficient of y^2 and coefficient of $xy = 0$

❑ If the triangle is right angled then its hypotenuse is the diameter of the circle. So using diameter form we can find the equation.

❑ **Circumcircle of a square or a rectangle** : Diagonals of the square and rectangle will be diameters of the circumcircle. Hence finding the vertices of a diagonal, we can easily determine the required equation.

Alternate : If sides of a quadrilateral are $L_1 = 0, L_2 = 0, L_3 = 0$ and $L_4 = 0$. Then equation of circle is $L_1L_3 + \lambda L_2L_4 = 0,$ where λ is a constant which can be obtained by the condition of circle.

❑ If a circle is passing through origin then constant term is absent *i.e.* $x^2 + y^2 + 2gx + 2fy = 0$

❑ If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ touches X-axis, then $-f = \sqrt{g^2 + f^2 - c}$ or $g^2 = c$

❑ If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ touches Y-axis, then $-g = \sqrt{g^2 + f^2 - c}$ or $f^2 = c$

❑ If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ touches both axes, then $-g = -f = \sqrt{g^2 + f^2 - c}$ or $g^2 = f^2 = c$

Example: 1 A point P moves in such a way that the ratio of its distances from two coplanar points is always fixed number $(\neq 1)$. Then its locus is **[IIT 1970]**

(a) Straight line (b) Circle (c) Parabola (d) A pair of straight lines

Solution: (b) Let two coplanar points are (0, 0) and (a, 0) and coordinates of point P is (x, y).

Under given conditions, we get $\dfrac{\sqrt{x^2 + y^2}}{\sqrt{(x-a)^2 + y^2}} = \lambda$ (where λ is any number and $\lambda \neq 1$)

$\Rightarrow x^2 + y^2 = \lambda^2[(x-a)^2 + y^2] \Rightarrow x^2 + y^2 + \left(\dfrac{\lambda^2}{\lambda^2 - 1}\right)(a^2 - 2ax) = 0$, which is equation of a circle.

Example : 2 The lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 square units. The equation of the circle is

[IIT 1989; AIEEE 2003; DCE 2001]

(a) $x^2 + y^2 + 2x - 2y = 62$ (b) $x^2 + y^2 - 2x + 2y = 47$

(c) $x^2 + y^2 + 2x - 2y = 47$ (d) $x^2 + y^2 - 2x + 2y = 62$

Solution : (b) Centre of circle = Point of intersection of diameters,

On solving equations, $2x - 3y = 5$ and $3x - 4y = 7$, we get, $(x, y) = (1, -1)$

$\therefore$ Centre of circle = $(1, -1)$. Now area of circle = 154 $\Rightarrow \pi r^2 = 154 \Rightarrow r = 7$

Hence, the equation of required circle is $(x-1)^2 + (y+1)^2 = (7)^2 \Rightarrow x^2 + y^2 - 2x + 2y = 47$.

Example : 3 The equation of a circle with origin as centre passing through the vertices of an equilateral triangle whose median is of length $3a$ is **[BIT Ranchi 1992; AIEEE 2002]**

(a) $x^2+y^2=9a^2$ (b) $x^2+y^2=16a^2$ (c) $x^2+y^2=a^2$ (d) None of these

Solution : (d) Since the triangle is equilateral, therefore centroid of the triangle is the same as the circumcentre and radius of the circum-circle $=\frac{2}{3}$ (median) $=\frac{2}{3}(3a)=2a$ [$\because$ Centroid divides median in ratio of 2 : 1]

Hence, the equation of the circum-circle whose centre is (0, 0) and radius $2a$ is $x^2+y^2=(2a)^2$

$\Rightarrow x^2+y^2=4a^2$

Example : 4 A circle of radius 5 units touches both the axes and lies in first quadrant. If the circle makes one complete roll on x-axis along the positive direction of x-axis, then its equation in the new position is

(a) $x^2+y^2+20\pi x-10y+100\pi^2=0$ (b) $x^2+y^2+20\pi x+10y+100\pi^2=0$

(c) $x^2+y^2-20\pi x-10y+100\pi^2=0$ (d) None of these

Solution : (d) The x-coordinate of the new position of the circle is 5 + circumferrence of the first circle $=5+10\pi$

The y-coordinate is 5 and the radius is also 5.

Hence, the equation of the circle in the new position is $(x-5-10\pi)^2+(y-5)^2=(5)^2$

$\Rightarrow x^2+25+100\pi^2-10x+100\pi-20\pi x+y^2+25-10y=25$

$\Rightarrow x^2+y^2-20\pi x-10x-10y+100\pi^2+100\pi+25=0$

Example : 5 The abscissae of A and B are the roots of the equation $x^2+2ax-b^2=0$ and their ordinates are the roots of the equation $y^2+2by-q^2=0$. The equation of the circle with AB as diameter is **[IIT 1984]**

(a) $x^2+y^2+2ax+2by-b^2-q^2=0$ (b) $x^2+y^2+2ax+by-b^2-q^2=0$

(c) $x^2+y^2+2ax+2by+b^2+q^2=0$ (d) None of these

Solution : (a) Let x_1, x_2 and y_1, y_2 be roots of $x^2+2ax-b^2=0$ and $y^2+2by-q^2=0$ respectively.

Then, $x_1+x_2=-2a,\ x_1x_2=-b^2$ and $y_1+y_2=-2b,\ y_1y_2=-q^2$

The equation of the circle with $A(x_1,y_1)$ and $B(x_2,y_2)$ as the end points of diameter is

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

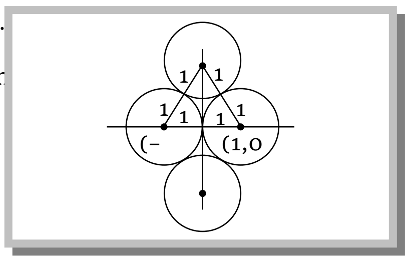
$x^2+y^2-x(x_1+x_2)-y(y_1+y_2)+x_1x_2+y_1y_2=0$; $x^2+y^2+2ax+2by-b^2-q^2=0$

Example : 6 The equation of a circle of radius 1 touching the circles $x^2+y^2-2|x|=0$ is

(a) $x^2+y^2+2\sqrt{3}x-2=0$ (b) $x^2+y^2-2\sqrt{3}y+2=0$

(c) $x^2+y^2+2\sqrt{3}y+2=0$ (d) $x^2+y^2+2\sqrt{3}x+2=0$

Solution : (b,c) The given circles are $x^2+y^2-2x=0,\ x>0,$ and $x^2+y^2+2x=0,\ x<0.$



From the figure, the centres of the required circles will be $(0,\sqrt{3})$ ar

$\therefore$ The equations of the circles are $(x-0)^2+(y\mp\sqrt{3})^2=1^2$.

$\Rightarrow x^2+y^2+3\mp 2\sqrt{3}y=1$

$\Rightarrow x^2+y^2\mp 2\sqrt{3}y+2=0$

Example : 7 If the line $x+2by+7=0$ is a diameter of the circle $x^2+y^2-6x+2y=0$, then $b=$ **[MP PET 1991]**

(a) 3 (b) – 5 (c) – 1 (d) 5

Solution : (d) Here the centre of circle (3, –1) must lie on the line $x+2by+7=0$. Therefore, $3-2b+7=0 \Rightarrow b=5$

Example : 8 The centre of the circle $r^2=2-4r\cos\theta+6r\sin\theta$ is

(a) (2, 3) (b) (– 2, 3) (c) (– 2, – 3) (d) (2, – 3)

Solution : (b) Let $r\cos\theta = x$ and $r\sin\theta = y$

Squaring and adding, we get $r^2=x^2+y^2$.

Putting these values in given equation, $x^2+y^2=2-4x+6y \Rightarrow x^2+y^2+4x-6y-2=0$

Hence, centre of the circle = (–2, 3)

Example : 9 The number of integral values of λ for which $x^2+y^2+\lambda x+(1-\lambda)y+5=0$ is the equation of a circle whose radius cannot exceed 5, is

(a) 14 (b) 18 (c) 16 (d) None of these

Solution : (c) Centre of circle $=\left(-\frac{\lambda}{2},-\frac{(1-\lambda)}{2}\right)$; Radius of circle $=\sqrt{\left(\frac{\lambda}{2}\right)^2+\left(\frac{1-\lambda}{2}\right)^2-5}\le 5$

$\Rightarrow 2\lambda^2-2\lambda-119\le 0$, $\therefore \frac{1-\sqrt{239}}{2}\le\lambda\le\frac{1+\sqrt{239}}{2}$

$\Rightarrow -7.2 \le \lambda \le 8.2$ (Nearly). $\therefore \lambda=-7,-6,$ ....,7, 8. Hence number of integral values of λ is 16.

Example : 10 Let $f(x,y)=0$ be the equation of a circle. If $f(0,\lambda)=0$ has equal roots $\lambda=2,2$ and $f(\lambda,0)=0$ has roots $\lambda=\frac{4}{5},5,$ then the centre of the circle is

(a) $\left(2,\frac{29}{10}\right)$ (b) $\left(\frac{29}{10},2\right)$ (c) $\left(-2,\frac{29}{10}\right)$ (d) None of these

Solution : (b) $\because f(x,y)\equiv x^2+y^2+2gx+2fy+c=0$

Now, $f(0,\lambda)\equiv\lambda^2+2f\lambda+c=0$ and its roots are 2, 2. $\therefore 2+2=-2f,\ 2\times 2=c,$ *i.e.* $f=-2,\ c=4$

$f(\lambda,0)\equiv\lambda^2+2g\lambda+c=0,$ and its roots are $\frac{4}{5}$, 5.

$\therefore \frac{4}{5}+5=-2g,\ \frac{4}{5}\times 5=c,$ *i.e.,* $g=\frac{-29}{10},\ c=4$. Hence, centre of the circle $=(-g,-f)=\left(\frac{29}{10},2\right)$.

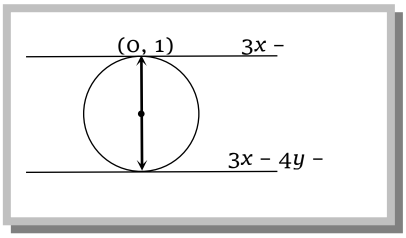
Example : 11 If the lines $3x-4y+4=0$ and $6x-8y-7=0$ are tangents to a circle, then the radius of the circle is

[IIT 1984; MP PET 1994, 2002; Rajasthan PET 1995, 97; Kurukshetra CEE 1998]

(a) 3/2 (b) 3/4 (c) 1/10 (d) 1/20

Solution : (b) Since both tangents are parallel to each other. The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x-4y+4=0$ and $3x-4y-\frac{7}{2}=0$ and so it is equal to $\frac{4}{\sqrt{9+16}}+\frac{7/2}{\sqrt{9+16}}=\frac{3}{2}$.

Hence radius of circle is $\frac{3}{4}$.



Alternative method : Perpendicular distance $=\frac{3(0)-4(1)-7/2}{5}=\frac{3}{2}$,

i.e., Diameter $=\frac{3}{2}$

Hence radius of circle is $\frac{3}{4}$.

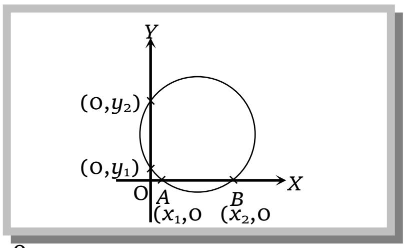
4.4 Intercepts on the Axes

The lengths of intercepts made by the circle $x^2+y^2+2gx+2fy+c=0$ with X and Y axes are $2\sqrt{g^2-c}$ and $2\sqrt{f^2-c}$ respectively.

Let the equation of circle be $x^2+y^2+2gx+2fy+c=0$ ......(i)

Length of intercepts on x-axis and y-axis are $|AB|=|x_2-x_1|$ and $|CD|=|y_2-y_1|$ respectively.

The circle intersects the x-axis, when y = 0, then $x^2+2gx+c=0$



Since the circle intersects the x-axis at $A(x_1,0)$ and $B(x_2,0)$.

Then $x_1+x_2=-2g,\ x_1x_2=c$

$\therefore\ |AB|=|x_2-x_1|=\sqrt{(x_2+x_1)^2-4x_1x_2}=2\sqrt{(g^2-c)}$

As the circle intersects the y-axis, when x = 0, then $y^2+2fy+c=0$

Since the circle intersects the y-axis at C (0, y_1) and D (0, y_2), then $y_1+y_2=-2f, y_1y_2=c$

$\therefore |CD|=|y_2-y_1|=\sqrt{(y_2+y_1)^2-4y_2y_1}=2\sqrt{(f^2-c)}$.

Note :

- If $g^2>c,$ then the roots of the equation $x^2+2gx+c=0$ are real and distinct, so the circle $x^2+y^2+2gx+2fy+c=0$ meets the x-axis in two real and distinct points and the length of the intercept on x-axis is $2\sqrt{g^2-c}.$
- If $g^2=c,$ then the roots of the equation $x^2+2gx+c=0$ are real and equal, so the circle touches x-axis and the intercept on x-axis is zero.
- If $g^2<c,$ then the roots of the equation $x^2+2gx+c=0$ are imaginary, so the circle $x^2+y^2+2gx+2fy+c=0$ does not meet x-axis in real points.
- Similarly, the circle $x^2+y^2+2gx+2fy+c=0$ cuts the y-axis in real and distinct points, touches or does not meet in real points according as $f^2 >,=$ or $<c$.

4.5 Position of a point with respect to a Circle

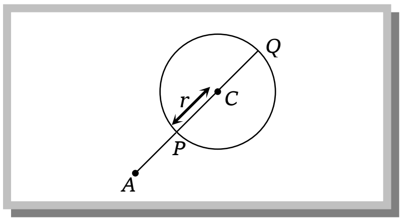
A point (x_1,y_1) lies outside, on or inside a circle $S\equiv x^2+y^2+2gx+2fy+c=0$ according as $S_1\equiv x_1^2+y_1^2+2gx_1+2fy_1+c$ is positive, zero or negative *i.e.,*

$S_1>0\Rightarrow$ Point is outside the circle.

$S_1=0\Rightarrow$ Point is on the circle.

$S_1<0\Rightarrow$ Point is inside the circle.

(1) **The least and greatest distance of a point from a circle :** Let S = 0 be a circle and $A(x_1,y_1)$ be a point. If the diameter of the circle is passing through the circle at P



$AP = AC - r =$ least distance ; $AQ = AC + r =$ greatest distance

where 'r' is the radius and C is the centre of the circle.

Example : 12 The number of points with integral coordinates that are interior to the circle $x^2 + y^2 = 16$ is

(a) 43 (b) 49 (c) 45 (d) 51

Solution : (c) The number of points is equal to the number of integral solutions (x, y) such that $x^2 + y^2 < 16$.

So, x, y are integers such that $-3 \le x \le 3$, $-3 \le y \le 3$ satisfying the inequation $x^2 + y^2 < 16$. The number of selections of values of x is 7, namely −3, −2, −1, 0, 1, 2, 3. The same is true for y. So the number of ordered pairs (x, y) is 7×7. But (3, 3), (3, −3), (−3, 3), (−3, −3) are rejected because they do not satisfy the inequation $x^2 + y^2 < 16$.

So the number of points is 45.

Example : 13 The range of values of a for which the point $(a, 4)$ is outside the circles $x^2 + y^2 + 10x = 0$ and $x^2 + y^2 - 12x + 20 = 0$ is

(a) $(-\infty, -8) \cup (-2, 6) \cup (6, +\infty)$ (b) $(-8, -2)$

(c) $(-\infty, -8) \cup (-2, +\infty)$ (d) None of these

Solution : (a) For circle, $x^2 + y^2 + 10x = 0$;

$a^2 + (4)^2 + 10a > 0 \Rightarrow a^2 + 10a + 16 > 0 \Rightarrow (a+8)(a+2) > 0 \Rightarrow a < -8$ or $a > -2$ .....(i)

For circle, $x^2 + y^2 - 12x + 20 = 0$; $a^2 + (4)^2 - 12a + 20 > 0 \Rightarrow a^2 - 12a + 36 > 0$

$\Rightarrow (a-6)^2 > 0 \Rightarrow a \in R \sim \{6\}$ .....(ii)

Taking common values from (i) and (ii), $a \in (-\infty, -8) \cup (-2, 6) \cup (6, +\infty)$.

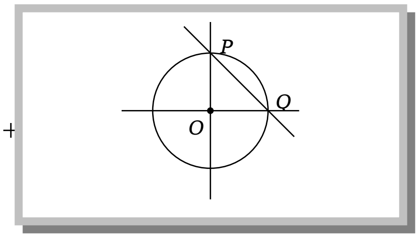
4.6 Intersection of a Line and a Circle

Let the equation of the circle be $x^2 + y^2 = a^2$ .....(i)

and the equation of the line be $y = mx + c$ .....(ii)

From (i) and (ii), $x^2 + (mx + c)^2 = a^2$ or $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ .....(iii)

Case I: When points of intersection are real and distinct. In this case (iii) has two distinct roots.



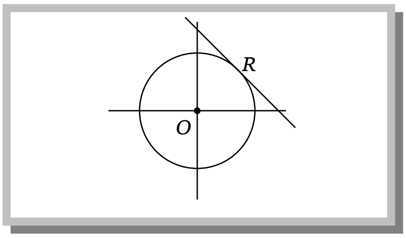
$\therefore$ $B^2 - 4AC > 0$ or $4m^2c^2 - 4(1 + m^2)(c^2 - a^2) > 0$ or $a^2 > \dfrac{c^2}{1 + m^2}$

or $a > \dfrac{|c|}{\sqrt{(1 + m^2)}}$ = length of perpendicular from (0, 0) to $y = mx +$

$\Rightarrow$ a > length of perpendicular from (0, 0) to $y = mx + c$

Thus, a line intersects a given circle at two distinct points if radius of circle is greater than the length of perpendicular from centre of the circle to the line.

Case II: When the points of intersection are coincident in this case (iii) has two equal roots.



$\therefore$ $B^2 - 4AC = 0 \Rightarrow 4m^2c^2 - 4(1 + m^2)(c^2 - a^2) = 0$

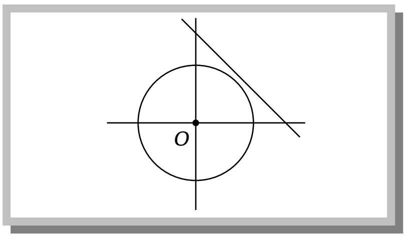
$\therefore$ $a^2 = \dfrac{c^2}{1 + m^2}$ or $a = \dfrac{|c|}{\sqrt{(1 + m^2)}}$

a = length of perpendicular from the point (0, 0) to $y = mx + c$.

Thus, a line touches the circle if radius of circle is equal to the length of perpendicular from centre of the circle to the line.

Case III: When the points of intersection are imaginary. In this case (iii) has imaginary roots.

$\therefore \quad B^2 - 4AC < 0 \Rightarrow 4m^2c^2 - 4(1+m^2)(c^2 - a^2) < 0, \quad \therefore \ a^2 < \dfrac{c^2}{1+m^2}$



or $\quad a < \dfrac{|c|}{\sqrt{(1+m^2)}}$ = length of perpendicular from (0, 0) to $y = mx$

$\Rightarrow$ a < length of perpendicular from (0, 0) to $y = mx + c$

Thus, a line does not intersect a circle if the radius of circle is less than the length of perpendicular from centre of the circle to the line.

(1) **The length of the intercept cut off from a line by a circle :** The length of the intercept cut off from the line $y = mx + c$ by the circle $x^2 + y^2 = a^2$ is $2\sqrt{\dfrac{a^2(1+m^2) - c^2}{1+m^2}}$

(2) **Condition of tangency :** A line L = 0 touches the circle S = 0, if length of perpendicular drawn from the centre of the circle to the line is equal to radius of the circle *i.e.* $p = r$. This is the condition of tangency for the line L = 0.

Circle $x^2 + y^2 = a^2$ will touch the line $y = mx + c$ if $c = \pm a\sqrt{1+m^2}$

Again, (i) If $a^2(1+m^2) - c^2 > 0$ line will meet the circle at real and different points.

(ii) If $c^2 = a^2(1+m^2)$ line will touch the circle.

(iii) If $a^2(1+m^2) - c^2 < 0$ line will meet circle at two imaginary points.

Example : 14 If the straight line $y = mx$ is outside the circle $x^2 + y^2 - 20y + 90 = 0$, then
[Roorkee 1999]

(a) $m > 3$ (b) $m < 3$ (c) $|m| > 3$ (d) $|m| < 3$

Solution : (d) If the straight line $y = mx$ is outside the given circle then perpendicular distance of line from centre of circle > radius of circle

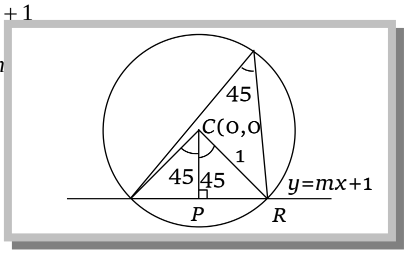
$\dfrac{10}{\sqrt{1+m^2}} > \sqrt{10} \quad \Rightarrow \quad (1+m^2) < 10 \Rightarrow m^2 < 9 \quad \Rightarrow |m| < 3$

Example : 15 If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45° at the major segment of the circle then value of m is
[AIEEE 2002]

(a) 2 (b) – 2 (c) ± 1 (d) None of these

Solution : (c) Given circle is $x^2 + y^2 = 1$, C (0,0) and radius = 1 and chord is $y = mx + 1$

$\cos 45^o = \dfrac{CP}{CR}$; CP = Perpendicular distance from (0, 0) to chord $y = m$

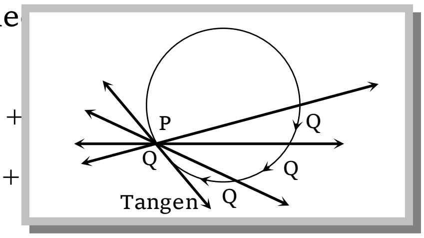


$CP = \dfrac{1}{\sqrt{m^2+1}}$ $\quad$ (CR = radius =1)

$\cos 45^o = \dfrac{\frac{1}{\sqrt{m^2+1}}}{1} \Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{m^2+1}} \quad \Rightarrow \ m^2 + 1 = 2 \Rightarrow m = \pm 1.$

4.7 Tangent to a Circle at a given Point

The limiting position of the line *PQ*, when *Q* moves towards *P* and ultimately coincides with *P*, is called the tangent to the circle at the point *P*. The point *P* is calle



(1) **Point form**

(i) The equation of tangent at (x_1, y_1) to circle $x^2+y^2=a^2$ is xx_1+

(ii) The equation of tangent at (x_1,y_1) to circle $x^2+y^2+2gx+2fy+$

$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

Note :
- For equation of tangent of circle at (x_1,y_1), substitute xx_1 for x^2, yy_1 for $y^2, \frac{x+x_1}{2}$ for $x, \frac{y+y_1}{2}$ for y and $\frac{xy_1+x_1y}{2}$ for xy and keep the constant as such.
- This method of tangent at (x_1,y_1) is applied any conics of second degree. *i.e.*, equation of tangent of

  $ax^2+2hxy+by^2+2gx+2fy+c=0$ at (x_1,y_1)

  is $axx_1+h(xy_1+x_1y)+byy_1+g(x+x_1)+f(y+y_1)+c=0$

(2) **Parametric form :** Since parametric co-ordinates of circle $x^2+y^2=a^2$ is $(a\cos\theta, a\sin\theta)$, then equation of tangent at $(a\cos\theta, a\sin\theta)$ is $x.a\cos\theta+y.a\sin\theta=a^2$ or $x\cos\theta+y\sin\theta=a$.

(3) **Slope form :** Let $y=mx+c$ is the tangent of the circle $x^2+y^2=a^2$.

$\therefore$ Length of perpendicular from centre of circle (0, 0) on line $(y=mx+c)=$ radius of circle

$\therefore \quad \frac{|c|}{\sqrt{1+m^2}}=a \Rightarrow c=\pm a\sqrt{1+m^2}$

Substituting this value of c in $y=mx+c$, we get $y=mx\pm a\sqrt{1+m^2}$. Which are the required equations of tangents.

Note :
- The reason why there are two equations $y=mx\pm a\sqrt{1+m^2}$ is that there are two tangents, both are parallel and at the ends of a diameter.
- The line $ax+by+c=0$ is a tangent to the circle $x^2+y^2=r^2$ if and only if $c^2=r^2(a^2+b^2)$.
- The condition that the line $lx+my+n=0$ touches the circle $x^2+y^2+2gx+2fy+c=0$ is $(lg+mf-n)^2=(l^2+m^2)(g^2+f^2-c)$.
- Equation of tangent to the circle $x^2+y^2+2gx+2fy+c=0$ in terms of slope is

  $y=mx+mg-f\pm\sqrt{(g^2+f^2-c)}\sqrt{(1+m^2)}$

(4) **Point of contact :** If circle be $x^2+y^2=a^2$ and tangent in terms of slope be $y=mx\pm a\sqrt{(1+m^2)}$,

Solving $x^2+y^2=a^2$ and $y=mx\pm a\sqrt{(1+m^2)}$ simultaneously, we get $x=\pm\frac{am}{\sqrt{(1+m^2)}}$ and

$y=\mp\frac{a}{\sqrt{(1+m^2)}}$

Thus, the co-ordinates of the points of contact are $\left(\pm\frac{am}{\sqrt{(1+m^2)}},\mp\frac{a}{\sqrt{(1+m^2)}}\right)$

Alternative method : Let point of contact be (x_1,y_1) then tangent at (x_1,y_1) of $x^2+y^2=a^2$ is $xx_1+yy_1=a^2$. Since $xx_1+yy_1=a^2$ and $y=mx\pm a\sqrt{(1+m^2)}$ are identical , $\therefore\ \frac{x_1}{m}=\frac{y_1}{-1}=\frac{+a^2}{\pm a\sqrt{1+m^2}}$

$\therefore\quad x_1=\pm\frac{am}{\sqrt{(1+m^2)}}$ and $y_1=\mp\frac{a}{\sqrt{(1+m^2)}}$

Thus, the co-ordinates of the points of contact are $\left(\pm\frac{am}{\sqrt{(1+m^2)}},\mp\frac{a}{\sqrt{(1+m^2)}}\right)$

Note :

- If the line $y = mx + c$ is the tangent to the circle $x^2+y^2=r^2$ then point of contact is given by $\left(-\frac{mr^2}{c},\frac{r^2}{c}\right)$
- If the line $ax+by+c = 0$ is the tangent to the circle $x^2+y^2=r^2$ then point of contact is given by $\left(-\frac{ar^2}{c},-\frac{br^2}{c}\right)$

Example : 16 The equations to the tangents to the circle $x^2+y^2-6x+4y=12$ which are parallel to the straight line 4x+3y+5=0, are **[ISM Dhanbad 1973, MP PET 1991]**

(a) $3x-4y-19=0,\ 3x-4y+31=0$ (b) $4x+3y-19=0,\ 4x+3y+31=0$

(c) $4x+3y+19=0,\ 4x+3y-31=0$ (d) $3x-4y+19=0,\ 3x-4y+31=0$

Solution : (c) Let equation of tangent be $4x+3y+k=0$, then $\sqrt{9+4+12}=\left|\frac{4(3)+3(-2)+k}{\sqrt{16+9}}\right|\Rightarrow 6+k=\pm 25\Rightarrow k=19$ and – 31

Hence the equations of tangents are $4x+3y+19=0$ and $4x+3y-31=0$

Example : 17 The equations of any tangents to the circle $x^2+y^2-2x+4y-4=0$ is

(a) $y=m(x-1)+3\sqrt{1+m^2}-2$ (b) $y=mx+3\sqrt{1+m^2}$

(c) $y=mx+3\sqrt{1+m^2}-2$ (d) None of these

Solution : (a) Equation of circle is $(x-1)^2+(y+2)^2=3^2$.

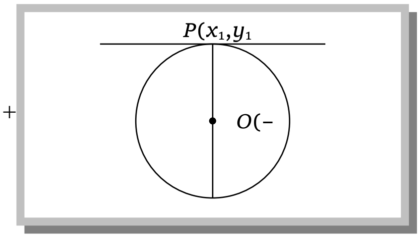
As any tangent to $x^2+y^2=3^2$ is given by $y=mx+3\sqrt{1+m^2}$

Any tangent to the given circle will be $y+2=m(x-1)+3\sqrt{1+m^2}\Rightarrow y=m(x-1)+3\sqrt{1+m^2}-2$

Example : 18 If a circle, whose centre is (–1, 1) touches the straight line $x + 2y + 12 = 0,$ then the coordinates of the point of contact are
[MP PET 1998]

(a) $\left(-\frac{7}{2}, -4\right)$ (b) $\left(-\frac{18}{5}, -\frac{21}{5}\right)$ (c) (2, –7) (d) (– 2, – 5)

Solution : (b) Let point of contact be $P(x_1, y_1)$.



This point lies on the given line , $\therefore x_1 + 2y_1 = -12$

Gradient of $OP = m_1 = \frac{y_1 - 1}{x_1 + 1}$, Gradient of $x + 2y +$

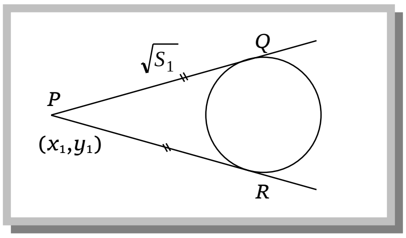
Both are perpendicular, $\therefore$ $m_1 m_2 = -1$

$\Rightarrow \left(\frac{y_1 - 1}{x_1 + 1}\right)\left(\frac{-1}{2}\right) = -1 \Rightarrow y_1 - 1 = 2x_1 + 2 \Rightarrow 2x_1 - y_1 = -3$ ..... (ii)

On solving the equation (i) and (ii), $(x_1, y_1) = \left(\frac{-18}{5}, \frac{-21}{5}\right)$

4.8 Length of Tangent

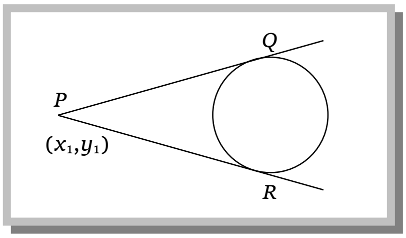
From any point, say $P(x_1, y_1)$ two tangents can be drawn to a circle which are real, coincident or imaginary according as P lies outside, on or inside the circle.



Le PQ and PR be two tangents drawn from $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. Then $PQ = PR$ is called the length of tangent drawn from point P and is given by $PQ = PR$ $= \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} = \sqrt{S_1}$

4.9 Pair of Tangents

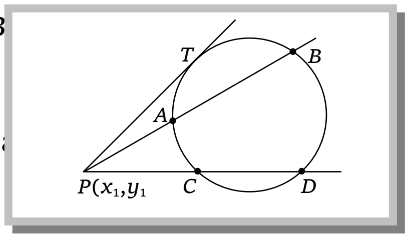
From a given point $P(x_1, y_1)$ two tangents PQ and PR can be drawn to the circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$. Their combined equation is $SS_1 = T^2$.



Where $S = 0$ is the equation of circle, $T = 0$ is the equation of tangent at (x_1, y_1) and S_1 is obtained by replacing x by x_1 and y by y_1 in S.

4.10 Power of Point with respect to a Circle

Let $P(x_1, y_1)$ be a point outside the circle and PAB and PCD drawn two secants. The power of $P(x_1, y_1)$ with respect to $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is equal to PA . PB



$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0 \Rightarrow S_1 = 0$

$\therefore$ Power remains constant for the circle *i.e.,* independent of A a

$\therefore$ $PA . PB = PC . CD = (PT)^2 = S_1 = (\sqrt{S_1})^2$

$\therefore$ $PA . PB = (\sqrt{S_1})^2 =$ square of the length of tangent.

Note : ❑ If P is outside, inside or on the circle then PA . PB is $+ve$, $-ve$ or zero respectively.

Important Tips

☞ *The length of the tangent drawn from any point on the circle* $x^2+y^2+2gx+2fy+c_1=0$ *to the circle* $x^2+y^2+2gx+2fy+c=0$ *is* $\sqrt{c-c_1}$.

☞ *If two tangents drawn from the origin to the circle* $x^2+y^2+2gx+2fy+c=0$ *are perpendicular to each other, then* $g^2+f^2=2c$.

☞ *If the tangent to the circle* $x^2+y^2=r^2$ *at the point* (a, b) *meets the coordinate axes at the points A and B and O is the origin, then the area of the triangle OAB is* $\frac{r^4}{2ab^-}$.

☞ *If* θ *is the angle subtended at* $P(x_1,y_1)$ *by the circle* $S=x^2+y^2+2gx+2fy+c=0$, *then* $\cot\frac{\theta}{2}=\frac{\sqrt{S_1}}{\sqrt{g^2+f^2-c}}$

☞ *The angle between the tangents from* (α,β) *to the circle* $x^2+y^2=a^2$ *is* $2\tan^{-1}\left(\frac{a}{\sqrt{\alpha^2+\beta^2-a^2}}\right)$.

☞ *If OA and OB are the tangents from the origin to the circle* $x^2+y^2+2gx+2fy+c=0$ *and C is the centre of the circle, then the area of the quadrilateral OACB is* $\sqrt{c(g^2+f^2-c)}$.

Example : 19 If the distances from the origin to the centres of three circles $x^2+y^2+2\lambda_i x-c^2=0$ $(i=1,2,3)$ are in *G.P.* then the lengths of the tangents drawn to them from any point on the circle $x^2+y^2=c^2$ are in

(a) *A.P.* (b) *G.P.* (c) *H.P.* (d) None of these

Solution : (b) The centres of the given circles are $(-\lambda_i, 0)$ $(i=1,2,3)$

The distances from the origin to the centres are λ_i $(i=1,2,3)$. It is given that $\lambda_2^2=\lambda_1\lambda_3$.

Let $P(h,k)$ be any point on the circle $x^2+y^2=c^2$, then, $h^2+k^2=c^2$

Now, L_i = length of the tangent from (h, k) to $x^2+y^2+2\lambda_i x-c^2=0$

$$=\sqrt{h^2+k^2+2\lambda_i h-c^2}=\sqrt{c^2+2\lambda_i h-c^2}=\sqrt{2\lambda_i h} \qquad [\because h^2+k^2=c^2 \text{ and } i=1,2,3]$$

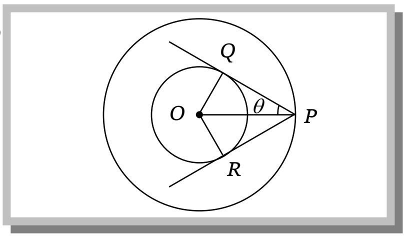
Therefore, $L_2^2=2\lambda_2 h=2h(\sqrt{\lambda_1\lambda_3})$ $\qquad [\because \lambda_2^2=\lambda_1\lambda_3]$

$=\sqrt{2h\lambda_1}\sqrt{2h\lambda_3}=L_1L_3$. Hence, L_1, L_2, L_3 are in *G.P.*

Example : 20 From a point on the circle $x^2+y^2=a^2$, two tangents are drawn to the circle $x^2+y^2=a^2\sin^2\alpha$. The angle between them is **[Rajasthan PET 2002]**

(a) α (b) $\frac{\alpha}{2}$ (c) 2α (d) None of these

Solution : (c) Let any point on the circle $x^2+y^2=a^2$ be $(a\cos t, a\sin t)$ and $\angle OPQ=\theta$



Now; PQ = length of tangent from P on the circle $x^2+y^2=a^2\sin^2\alpha$

$\therefore \quad PQ=\sqrt{a^2\cos^2 t+a^2\sin^2 t-a^2\sin^2\alpha}=a\cos\alpha$

$OQ =$ Radius of the circle $x^2 + y^2 = a^2 \sin^2 \alpha$

$OQ = a \sin \alpha$, $\therefore$ $\tan \theta = \dfrac{OQ}{PQ} = \tan \alpha \Rightarrow \theta = \alpha$; $\therefore$ Angle between tangents $\angle QPR = 2\alpha$.

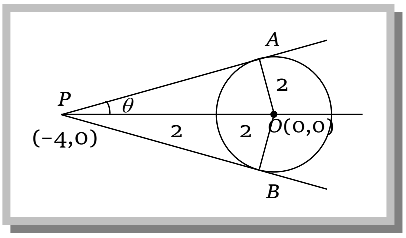
Alternative Method : We know that, angle between the tangent from (α, β) to the circle $x^2 + y^2 = a^2$ is $2\tan^{-1}\left(\dfrac{a}{\sqrt{\alpha^2 + \beta^2 - a^2}}\right)$. Let point on the circle $x^2 + y^2 = a^2$ be $(a\cos t, a\sin t)$

Angle between tangent = $2\tan^{-1}\left(\dfrac{a \sin \alpha}{\sqrt{a^2 \cos^2 t + a^2 \sin^2 t - a^2 \sin^2 \alpha}}\right) = 2\tan^{-1}\left(\dfrac{a \sin \alpha}{a \cos \alpha}\right) = 2\alpha$

Example : 21 Two tangents to the circle $x^2 + y^2 = 4$ at the points A and B meet at P (– 4, 0). The area of quadrilateral $PAOB$, where O is the origin, is

(a) 4 (b) $6\sqrt{2}$ **(c)** $4\sqrt{3}$ (d) None of these

Solution : (c) Clearly, $\sin \theta = \dfrac{2}{4} = \dfrac{1}{2}$, $\therefore \theta = 30^o$. So area $(\Delta POA) = \dfrac{1}{2}.2.4.\sin 60^o$



$\therefore$ Area (quadrilateral $PAOB$) $= 2.\dfrac{1}{2}.2.4 \sin 60^o = 8.\dfrac{\sqrt{3}}{2} = 4\sqrt{3}.$

Trick : Area of quadrilateral $= r\sqrt{S_1} = 2\sqrt{12} = 4\sqrt{3}$

Example : 22 The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2 \alpha + 13\cos^2 \alpha = 0$ is 2α. The equation of the locus of the point P is

[IIT 1996]

(a) $x^2 + y^2 + 4x - 6y + 4 = 0$ (b) $x^2 + y^2 + 4x - 6y - 9 = 0$

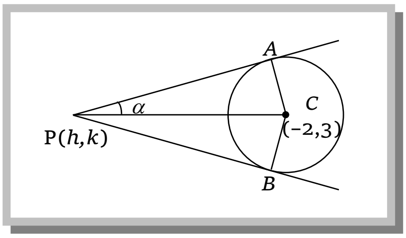
(c) $x^2 + y^2 + 4x - 6y - 4 = 0$ (d) $x^2 + y^2 + 4x - 6y + 9 = 0$

Solution : (d) The centre of the circle $x^2 + y^2 + 4x - 6y + 9\sin^2 \alpha + 13\cos^2 \alpha = 0$ is $C(-2, 3)$ and its radius is

$\sqrt{2^2 + (-3)^2 - 9\sin^2 \alpha - 13\cos^2 \alpha} = \sqrt{4 + 9 - 9\sin^2 \alpha - 13\cos^2 \alpha} = 2\sin \alpha$

Let P (h, k) be any point on the locus. The $\angle APC = \alpha$. Also $\angle PAC = \pi/2$ *i.e.* triangle APC is a right angle triangle.

Thus $\sin \alpha = \dfrac{AC}{PC} = \dfrac{2\sin \alpha}{\sqrt{(h+2)^2 + (k-3)^2}}$



$\Rightarrow \sqrt{(h+2)^2 + (k-3)^2} = 2 \Rightarrow (h+2)^2 + (k-3)^2 = 4$

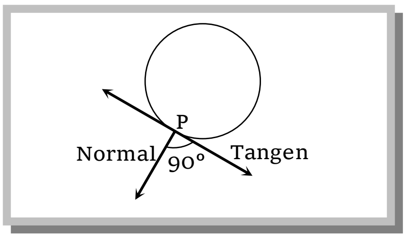
or $h^2 + k^2 + 4h - 6k + 9 = 0$

Thus the required equation of the locus is $x^2 + y^2 + 4x - 6y + 9 = 0$.

4.11 Normal to a Circle at a given Point

The normal of a circle at any point is a straight line, which is perpendicular to the tangent at the point and always passes through the centre of the circle.

(1) **Equation of normal:** The equation of normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at any point (x_1, y_1) is $y - y_1 = \dfrac{y_1 + f}{x_1 + g}(x - x_1)$ or $\dfrac{x - x_1}{x_1 + g} = \dfrac{y - y_1}{y_1 + f}$



Note : ❑ The equation of normal to the circle $x^2+y^2=a^2$ at any point (x_1,y_1) is $xy_1-x_1y=0$ or $\frac{x}{x_1}=\frac{y}{y_1}$

❑ The equation of any normal to the circle $x^2+y^2=a^2$ is $y=mx$ where m is the slope of normal.

❑ The equation of any normal to the circle $x^2+y^2+2gx+2fy+c=0$ is $y+f=m(x+g)$. where m is the slope of normal.

❑ If the line $y=mx+c$ is a normal to the circle with radius r and centre at (a, b) then $b=ma+c$.

(2) **Parametric form :** Since parametric co-ordinates of circle $x^2+y^2=a^2$ is $(a\cos\theta,\ a\sin\theta)$.

$\therefore$ Equation of normal at $(a\cos\theta,\ a\sin\theta)$ is $\frac{x}{a\cos\theta}=\frac{y}{a\sin\theta}$ or $\frac{x}{\cos\theta}=\frac{y}{\sin\theta}$

or $y=x\tan\theta$ or $y=mx$ where $m=\tan\theta$, which is slope form of normal.

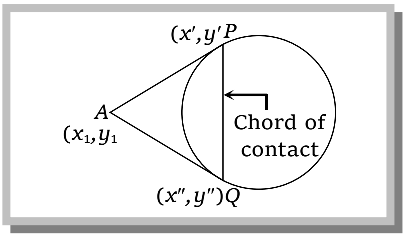
Example : 23 The line $lx+my+n=0$ is a normal to the circle $x^2+y^2+2gx+2fy+c=0$, if **[MP PET 1995]**

(a) $lg+mf-n=0$ (b) $lg+mf+n=0$ (c) $lg-mf-n=0$ (d) $lg-mf+n=0$

Solution : (a) Since normal always passes through centre of circle, therefore $(-g,-f)$ must lie on $lx+my+n=0$. Hence, $lg+mf-n=0$

4.12 Chord of Contact of Tangents

(1) **Chord of contact :** The chord joining the points of contact of the two tangents to a conic drawn from a given point, outside it, is called the chord of contact of tangents.



(2) **Equation of chord of contact :** The equation of the chord of contact of tangents drawn from a point (x_1,y_1) to the circle $x^2+y^2=a^2$ is $xx_1+yy_1=a^2$.

Equation of chord of contact at (x_1,y_1) to the circle $x^2+y^2+2gx+2fy+c=0$ is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$.

It is clear from the above that the equation to the chord of contact coincides with the equation of the tangent, if point (x_1,y_1) lies on the circle.

The length of chord of contact $=2\sqrt{r^2-p^2}$; (p being length of perpendicular from centre to the chord)

Area of ΔAPQ is given by $\frac{a(x_1^2+y_1^2-a^2)^{3/2}}{x_1^2+y_1^2}$.

(3) **Equation of the chord bisected at a given point :** The equation of the chord of the circle $S\equiv x^2+y^2+2gx+2fy+c=0$ bisected at the point (x_1,y_1) is given by $T=S'$

i.e. $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$.

Example : 24 Tangents are drawn from any point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$. If the chord of contact touches the circle $x^2 + y^2 = c^2$, $a > b$, then **[MP PET 1999, Rajasthan PET 1999]**

(a) *a, b, c* are in *A.P.* (b) *a, b, c* are in *G.P.* (c) *a, b, c* are in *H.P.* (d) *a, c, b* are in *G.P.*

Solution : (b) Chord of contact of any point $(a\cos\theta, a\sin\theta)$ on 1st circle with respect to 2nd circle is $ax\cos\theta + ay\sin\theta = b^2$

This chord touches the circle $x^2 + y^2 = c^2$,

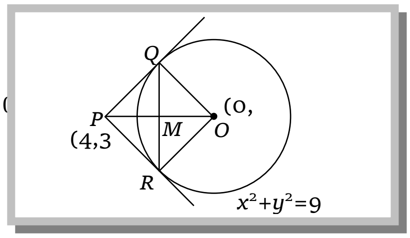
Hence, Radius = Perpendicular distance of chord from centre.

$c = \frac{b^2}{a\sqrt{\cos^2\theta + \sin^2\theta}} \Rightarrow b^2 = ac$ .Hence a,b,c are in G.P.

Example : 25 The area of the triangle formed by the tangents from the point (4, 3) to the circle $x^2 + y^2 = 9$ and the line joining their points of contact is
[IIT 1981, MP PET 1991]

(a) $\frac{25}{192}$ *sq.units* (b) $\frac{192}{25}$ *sq.units* (c) $\frac{384}{25}$ *sq.units* (d) None of these

Solution : (b) The equation of the chord of contact of tangents drawn from *P* (4, 3) to $x^2 + y^2 = 9$ is $4x + 3y = 9$. The equation of *OP* is $y = \frac{3}{4}x$.



Now, OM = (length of the perpendicular from (0, 0) on $4x + 3y - 9 = 0$

$\therefore \; QR = 2.QM = 2\sqrt{OQ^2 - OM^2} = 2\sqrt{9 - \frac{81}{25}} = \frac{24}{5}$

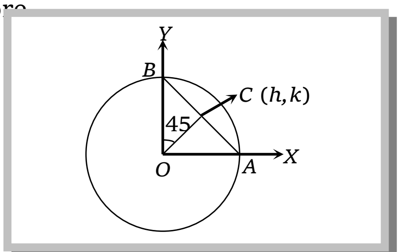
Now, $PM = OP - OM = 5 - \frac{9}{5} = \frac{16}{5}$ .So, Area of $\Delta PQR = \frac{1}{2}\left(\frac{24}{5}\right)\left(\frac{16}{5}\right) = \frac{192}{25}$ *sq. units*

Example : 26 The locus of the middle points of those chords of the circle $x^2 + y^2 = 4$ which subtend a right angle at the origin is **[MP PET 1990; IIT 1984; Rajasthan PET 1997; DCE 2000, 01]**

(a) $x^2 + y^2 - 2x - 2y = 0$ (b) $x^2 + y^2 = 4$ (c) $x^2 + y^2 = 2$ (d) $(x-1)^2 + (y-2)^2 = 5$

Solution : (c) Let the mid-point of chord is (*h, k*). Also radius of circle is 2. Therefore

$\frac{OC}{OB} = \cos 45^o \Rightarrow \frac{\sqrt{h^2 + k^2}}{2} = \frac{1}{\sqrt{2}} \Rightarrow h^2 + k^2 = 2$



Hence locus is $x^2 + y^2 = 2$

Example : 27 If two distinct chords, drawn from the point (*p, q*) on the circle $x^2 + y^2 = px + qy$ (where *p, q* ≠ 0) are bisected by the *x*-axis, then **[IIT 1999]**

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$ (c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$

Solution : (d) Let (*h*, 0) be a point on *x*-axis, then the equation of chord whose mid-point is (*h*, 0) will be

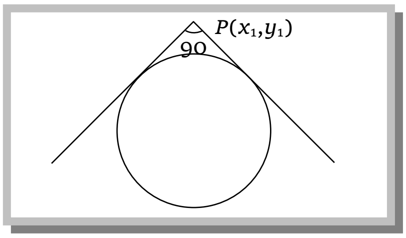
$xh - \frac{1}{2}p(x + h) - \frac{1}{2}q(y + 0) = h^2 - ph$. This passes through (*p, q*), hence $ph - \frac{1}{2}p(p + h) - \frac{1}{2}q.q = h^2 - ph$

$\Rightarrow ph - \frac{1}{2}p^2 - \frac{1}{2}ph - \frac{1}{2}q^2 = h^2 - ph \Rightarrow h^2 - \frac{3}{2}ph + \frac{1}{2}(p^2+q^2) = 0$; $\because$ h is real, hence $B^2 - 4AC > 0$

$\therefore \frac{9}{4}p^2 - 4.\frac{1}{2}(p^2+q^2) > 0 \Rightarrow 9p^2 - 8(p^2+q^2) > 0 \Rightarrow p^2 - 8q^2 > 0 \Rightarrow p^2 > 8q^2$

4.13 Director Circle

The locus of the point of intersection of two perpendicular tangents to a circle is called the Director circle.



Let the circle be $x^2+y^2=a^2$, then equation of the pair of tangents to a circle from a point is $(x^2+y^2-a^2)(x_1^2+y_1^2-a^2) = (xx_1+yy_1-a^2)^2$. If this represents a pair of perpendicular lines, coefficient of x^2 + coefficient of $y^2 = 0$ *i.e.* $(x_1^2+y_1^2-a^2-x_1^2)+(x_1^2+y_1^2-a^2-y_1^2)=0 \Rightarrow x_1^2+y_1^2 = 2a^2$

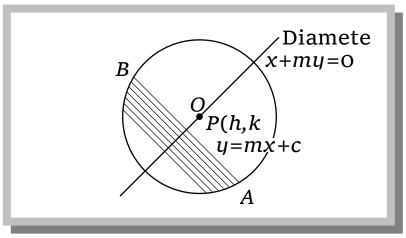
Hence the equation of director circle is $x^2+y^2=2a^2$.

Obviously director circle is a concentric circle whose radius is $\sqrt{2}$ times the radius of the given circle.

Note : ❑ Director circle of circle $x^2+y^2+2gx+2fy+c=0$ is $x^2+y^2+2gx+2fy+2c-g^2-f^2=0$.

4.14 Diameter of a Circle

The locus of the middle points of a system of parallel chords of a circle is called a diameter of the circle.



The equation of the diameter bisecting parallel chords $y = mx + c$ (c is a parameter) of the circle $x^2+y^2=a^2$ is $x+my=0$.

Note : ❑ The diameter corresponding to a system of parallel chords of a circle always passes through the centre of the circle and is perpendicular to the parallel chords.

Example : 28 A foot of the normal from the point (4, 3) to a circle is (2, 1) and a diameter of the circle has the equation $2x - y = 2$. Then the equation of the circle is

(a) $x^2+y^2+2x-1=0$ (b) $x^2+y^2-2x-1=0$ (c) $x^2+y^2-2y-1=0$ (d) None of these.

Solution : (b) The line joining (4, 3) and (2, 1) is also along a diameter. So, the centre is the intersection of the diameters $2x - y = 2$ and $y - 3 = (x-4)$. Solving these, the centre = (1, 0)

$\therefore$ Radius = Distance between (1, 0) and (2, 1) = $\sqrt{2}$.

$\therefore$ Equation of circle $(x-1)^2+y^2=(\sqrt{2})^2 \Rightarrow x^2+y^2-2x-1=0$

Example : 29 The diameter of the circle $x^2+y^2-4x+2y-11=0$ corresponding to a system of chords parallel to the line $x-2y+1=0$

(a) $x-2y+3=0$ (b) $2x-y+3=0$ (c) $2x+y-3=0$ (d) None of these

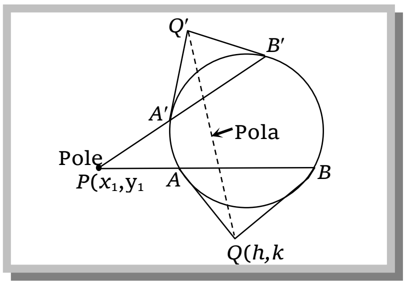
Solution : (c) The centre of the given circle is (2, –1) the equation of the line perpendicular to chord $x-2y+1=0$ is $2x+y+k=0$

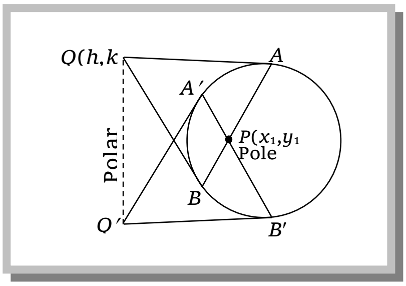
Since the line passes through the point (2, –1) therefore k = – 3. The equation of diameter is $2x + y - 3 = 0$.

4.15 Pole and Polar

Let $P(x_1, y_1)$ be any point inside or outside the circle. Draw chords AB and $A'B'$ passing through P. If tangents to the circle at A and B meet at Q (h, k), then locus of Q is called the polar of P with respect to circle and P is called the pole and if tangents to the circle at A' and B' meet at Q', then the straight line QQ' is polar with P as its pole.

If circle be $x^2 + y^2 = a^2$ then AB is the chord of contact of Q (h, k), $hx + ky = a^2$ is its equation. But $P(x_1, y_1)$ lies on AB, $\therefore$ $hx_1 + ky_1 = a^2$.





Hence, locus of Q (h, k) is $xx_1 + yy_1 = a^2$, which is polar of $P(x_1, y_1)$ with respect to the circle $x^2 + y^2 = a^2$.

(1) **Coordinates of pole of a line :** The pole of the line $lx + my + n = 0$ with respect to the circle $x^2 + y^2 = a^2$. Let pole be (x_1, y_1), then equation of polar with respect to the circle $x^2 + y^2 = a^2$ is $xx_1 + yy_1 - a^2 = 0$, which is same as $lx + my + n = 0$

Then $\frac{x_1}{l} = \frac{y_1}{m} = -\frac{a^2}{n}$, $\therefore$ $x_1 = -\frac{a^2 l}{n}$ and $y_1 = -\frac{a^2 m}{n}$. Hence, the required pole is $\left(-\frac{a^2 l}{n}, -\frac{a^2 m}{n}\right)$.

(2) **Properties of pole and polar**

(i) If the polar of $P(x_1, y_1)$ w.r.t. a circle passes through $Q(x_2, y_2)$ then the polar of Q will pass through P and such points are said to be conjugate points.

(ii) If the pole of the line $ax + by + c = 0$ w.r.t. a circle lies on another line $a_1x + b_1y + c_1 = 0$; then the pole of the second line will lie on the first and such lines are said to be conjugate lines.

(iii) The distance of any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ from the centre of a circle is proportional to the distance of each from the polar of the other.

(iv) If O be the centre of a circle and P any point, then OP is perpendicular to the polar of P.

(v) If O be the centre of a circle and P any point, then if OP (produced, if necessary) meet the polar of P in Q, then OP . OQ = (radius)^2.

Note :

- ❑ Equation of polar is like as equation of tangent *i.e.*, T = 0 (but point different)
- ❑ Equation of polar of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ with respect to (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
- ❑ If the point P is outside the circle then equation of polar and chord of contact will coincide. In this case the polar cuts the circle at two points.

- If the point P is on the circle then equation of polar, chord of contact and tangent at P will coincide. So in this case the polar touches the circle.
- If the point P is inside the circle (not its centre) then only its polar will exist. In this case the polar is outside the circle. The polar of the centre lies at infinity.
- If a triangle is like that its each vertex is a pole of opposite side with respect to a circle then it is called self conjugate triangle.

Example : 30 The polar of the point $\left(5, -\frac{1}{2}\right)$ with respect to circle $(x-2)^2 + y^2 = 4$ is **[Rajasthan PET 1996]**

(a) $5x - 10y + 2 = 0$ (b) $6x - y - 20 = 0$ (c) $10x - y - 10 = 0$ (d) $x - 10y - 2 = 0$

Solution : (b) The polar of the point $\left(5, -\frac{1}{2}\right)$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$\Rightarrow 5x - \frac{1}{2}y - 2(x+5) + 0 + 0 = 0 \Rightarrow 3x - \frac{y}{2} - 10 = 0 \Rightarrow 6x - y - 20 = 0$.

Example : 31 The pole of the straight line $9x + y - 28 = 0$ with respect to circle $2x^2 + 2y^2 - 3x + 5y - 7 = 0$ is **[Rajasthan PET 1990, 99; MNR 1984; UPSEAT 2000]**

(a) (3, 1) (b) (1, 3) (c) (3, – 1) (d) (– 3, 1)

Solution : (c) Equation of given circle is $x^2 + y^2 - \frac{3}{2}x + \frac{5}{2}y - \frac{7}{2} = 0$

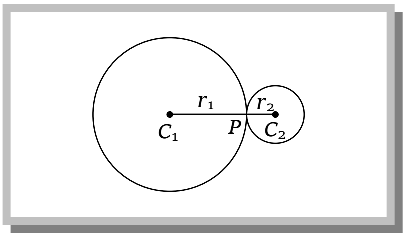
$\Rightarrow \left(x - \frac{3}{4}\right)^2 + \left(y + \frac{5}{4}\right)^2 - \frac{9}{16} - \frac{25}{16} - \frac{7}{2} = 0 \Rightarrow \left(x - \frac{3}{4}\right)^2 + \left(y + \frac{5}{4}\right)^2 - \frac{45}{8} = 0$

Put $X = x - \frac{3}{4}$ and $Y = y + \frac{5}{4}$, we get the equation of circle $X^2 + Y^2 - \frac{45}{8} = 0$ and the line $9X + Y - \frac{45}{2} = 0$

Hence pole $= \left[\frac{9 \times \frac{45}{8}}{\frac{45}{2}}, \frac{1 \times \frac{45}{8}}{\frac{45}{2}}\right] = \left(\frac{9}{4}, \frac{1}{4}\right)$. But, $x = \frac{9}{4} + \frac{3}{4} = 3$ and $y = \frac{1}{4} - \frac{5}{4} = -1$, hence the pole is (3, – 1).

4.16 Two Circles touching each other

(1) **When two circles touch each other externally :** Then distance between their centres = Sum of their radii *i.e.*, $|C_1C_2| = r_1 + r_2$

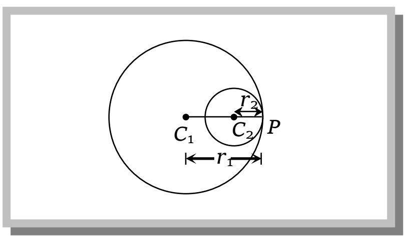


In such cases, the point of contact P divides the line joining C_1 and C_2 internally in the ratio $r_1 : r_2 \Rightarrow \frac{C_1P}{C_2P} = \frac{r_1}{r_2}$

If $C_1 \equiv (x_1, y_1)$ and $C_2 \equiv (x_2, y_2)$, then co-ordinate of P is

$$\left(\frac{r_1x_2 + r_2x_1}{r_1 + r_2}, \frac{r_1y_2 + r_2y_1}{r_1 + r_2}\right)$$

(2) **When two circles touch each other internally :** Then distance between their centres = Difference of their radii *i.e.*, $|C_1C_2| = r_1 - r_2$



In such cases, the point of contact P divides the line joining C_1 and C_2 externally in the ratio $r_1 : r_2 \Rightarrow \frac{C_1P}{C_2P} = \frac{r_1}{r_2}$

If $C_1 \equiv (x_1, y_1)$ and $C_2 \equiv (x_2, y_2)$, then co-ordinate of P is

$$\left(\frac{r_1x_2-r_2x_1}{r_1-r_2},\frac{r_1y_2-r_2y_1}{r_1-r_2}\right)$$

4.17 Common Tangents to Two circles

Different cases of intersection of two circles :

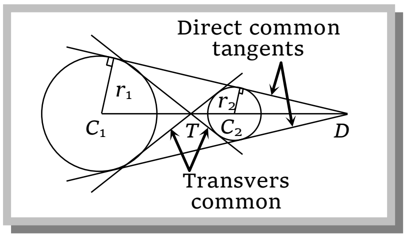
Let the two circles be $(x-x_1)^2+(y-y_1)^2=r_1^2$ .....(i)

and $(x-x_2)^2+(y-y_2)^2=r_2^2$ .....(ii)

with centres $C_1(x_1,y_1)$ and $C_2(x_2,y_2)$ and radii r_1 and r_2 respectively. Then following cases may arise :

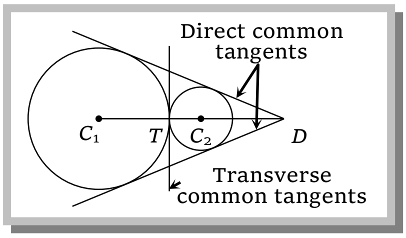
Case I : When $|C_1C_2|>r_1+r_2$ *i.e.,* the distance between the centres is greater than the sum of radii.

In this case four common tangents can be drawn to the two circles, in which two are direct common tangents and the other two are transverse common tangents.



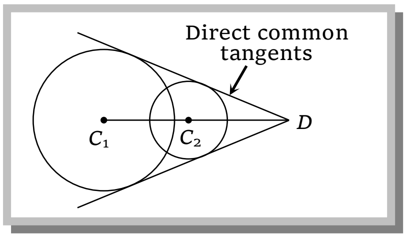
Case II : When $|C_1C_2|=r_1+r_2$ *i.e.,* the distance between the centres is equal to the sum of radii.

In this case two direct common tangents are real and distinct while the transverse tangents are coincident.



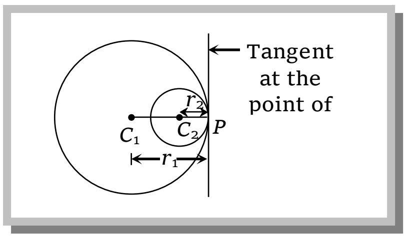
Case III : When $|C_1C_2|<r_1+r_2$ *i.e.,* the distance between the centres is less than sum of radii.

In this case two direct common tangents are real and distinct while the transverse tangents are imaginary.



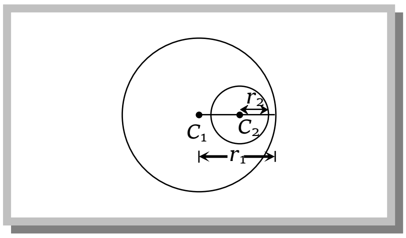
Case IV : When $|C_1C_2|=|r_1-r_2|$, *i.e.,* the distance between the centres is equal to the difference of the radii.

In this case two tangents are real and coincident while the other two tangents are imaginary.



Case V : When $|C_1C_2| < |r_1 - r_2|$, *i.e.,* the distance between the centres is less than the difference of the radii.

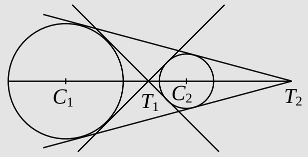
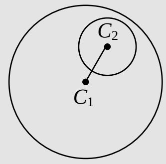
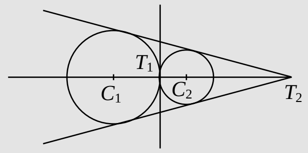
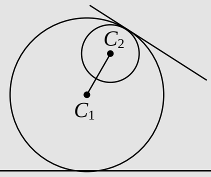
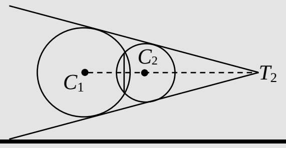
In this case, all the four common tangents are imaginary.



Note : ❑ **Points of intersection of common tangents :** The points T_1 and T_2 (points of intersection of indirect and direct common tangents) divides C_1C_2 internally and externally in the ratio $r_1 : r_2$ respectively.

❑ Equation of the common tangents at point of contact is $S_1 - S_2 = 0$.

❑ If the circle $x^2 + y^2 + 2gx + c^2 = 0$ and $x^2 + y^2 + 2fy + c^2 = 0$ touch each other, then $\frac{1}{g^2} + \frac{1}{f^2} = \frac{1}{c^2}$.

	Condition	Position	Diagram	No. of common tangents
(i)	$C_1C_2 > r_1 + r_2$	Do not intersect or one outside the other		4
(ii)	$C_1C_2 < \| r_1 - r_2\|$	One inside the other		0
(iii)	$C_1C_2 = r_1 + r_2$	External touch		3
(iv)	$C_1C_2 = \| r_1 - r_2\|$	Internal touch		1
(v)	$\| r_1 - r_2\| < C_1C_2 < r_1 + r_2$	Intersection at two real points		2

Example : 32 If circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch each other, then **[MNR 1987]**

(a) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ (c) $\frac{1}{a} + \frac{1}{b} = c^2$ (d) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$

Solution : (d) $C_1 = (-a, 0)$, $r_1 = \sqrt{a^2 - c}$; $C_2 = (0, -b)$, $r_2 = \sqrt{b^2 - c}$; $C_1C_2 = \sqrt{a^2 + b^2}$

$\because$ Circles touch each other, therefore $r_1 + r_2 = C_1C_2$

$\Rightarrow \sqrt{a^2 - c} + \sqrt{b^2 - c} = \sqrt{a^2 + b^2} \Rightarrow a^2b^2 - b^2c - a^2c = 0$

Multiplying by $\frac{1}{a^2b^2c^2}$, we get $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$.

Example : 33 If two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

[IIT 1989; Karnataka CET 2002; DCE 2000, 01; AIEEE 2003]

(a) $2 < r < 8$ (b) $r = 2$ (c) $r < 2$ (d) $r > 2$

Solution : (a) When two circles intersect each other, then difference between their radii < Distance between their centres

$\Rightarrow r - 3 < 5 \Rightarrow r < 8$ …..(i)

Sum of their radii > Distance between their centres

$\Rightarrow r + 3 > 5 \Rightarrow r > 2$ …..(ii)

Hence by (i) and (ii), $2 < r < 8$.

Example : 34 The equation of the circle having the lines $x^2+2xy+3x+6y=0$ as its normals and having size just sufficient to contain the circle $x(x-4)+y(y-3)=0$ is **[Roorkee 1990]**

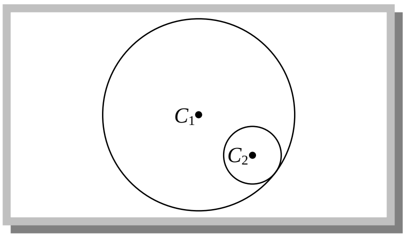
(a) $x^2+y^2+3x-6y-40=0$ (b) $x^2+y^2+6x-3y-45=0$

(c) $x^2+y^2+8x+4y-20=0$ (d) $x^2+y^2+4x+8y+20=0$

Solution : (b) Given pair of normals is $x^2+2xy+3x+6y=0$ or $(x+2y)(x+3)=0$

$\therefore$ Normals are $x+2y=0$ and $x+3=0$

The point of intersection of normals $x+2y=0$ and $x+3=0$ is the centre of required circle, we get centre $C_1=(-3,3/2)$ and other circle is



$x(x-4)+y(y-3)=0$ or $x^2+y^2-4x-3y=0$ .....(i)

Its centre $C_2=(2,\ 3/2)$ and radius $r=\sqrt{4+\frac{9}{4}}=\frac{5}{2}$

Since the required circle just contains the given circle (i), the given circle should touch the required circle internally from inside.

Therefore, radius of the required circle $=|C_1-C_2|+r=\sqrt{(-3-2)^2+\left(\frac{3}{2}-\frac{3}{2}\right)^2}+\frac{5}{2}=5+\frac{5}{2}=\frac{15}{2}$

Hence, equation of required circle is $(x+3)^2+\left(y-\frac{3}{2}\right)^2=\left(\frac{15}{2}\right)^2$ or $x^2+y^2+6x-3y-45=0$.

Example : 35 The equation of the circle which touches the circle $x^2+y^2-6x+6y+17=0$ externally and to which the lines $x^2-3xy-3x+9y=0$ are normals, is **[Roorkee 1994]**

(a) $x^2+y^2-6x-2y-1=0$ (b) $x^2+y^2+6x+2y+1=0$

(c) $x^2+y^2-6x-6y+1=0$ (d) $x^2+y^2-6x-2y+1=0$

Solution : (d) Joint equations of normals are $x^2-3xy-3x+9y=0 \Rightarrow x(x-3y)-3(x-3y)=0 \Rightarrow (x-3)(x-3y)=0$

$\therefore$ Given normals are $x-3=0$ and $x-3y=0$, which intersect at centre of circle whose coordinates are (3, 1).

The given circle is $C_1=(3,-3)$, $r_1=1$; $C_2=(3,\ 1)$, $r_2=?$

If the two circles touch externally, then $C_1C_2=r_1+r_2 \Rightarrow 4=1+r_2 \Rightarrow r_2=3$

$\therefore$ Equation of required circle is $(x-3)^2+(y-1)^2=(3)^2 \Rightarrow x^2+y^2-6x-2y+1=0$

Example : 36 The number of common tangents to the circles $x^2+y^2=4$ and $x^2+y^2-6x-8y=24$ is **[IIT 1998]**

(a) 0 (b) 1 (c) 3 (d) 4

Solution : (b) Circles $S_1\equiv x^2+y^2=(2)^2$ and $S_2\equiv(x-3)^2+(y-4)^2=(7)^2$

$\therefore$ Centres $C_1=(0,0)$, $C_2=(3,4)$ and radii $r_1=2,\ r_2=7$

$C_1C_2=\sqrt{(3)^2+(4)^2}=5$, $r_2-r_1=7-2=5$

$\therefore$ $C_1C_2=r_2-r_1$ *i.e.* circles touch internally. Hence there is only one common tangent.

Example : 37 There are two circles whose equations are $x^2+y^2=9$ and $x^2+y^2-8x-6y+n^2=0$, $n\in Z$. If the two circles have exactly two common tangents, then the number of possible values of n is

(a) 2 (b) 8 (c) 9 (d) None of these

Solution : (c) For $x^2+y^2=9$, the centre = (0, 0) and the radius = 3

For $x^2+y^2-8x-6y+n^2=0$. The centre = (4, 3) and the radius $=\sqrt{(4)^2+(3)^2-n^2}$

$\therefore$ $4^2+3^2-n^2>0$ or $n^2<5^2$ or $-5<n<5$.

Circles should cut to have exactly two common tangents.

So, $r_1+r_2>C_1C_2$, $\therefore$ $3+\sqrt{25-n^2}>\sqrt{(4)^2+(3)^2}$ or $\sqrt{25-n^2}>2$ or $25-n^2>4$

$\therefore$ $n^2<21$ or $-\sqrt{21}<n<\sqrt{21}$

Therefore, common values of n should satisfy $-\sqrt{21}<n<\sqrt{21}$.

But $n\in Z$, So, $n=-4,\ -3,........\ 3,4$. $\therefore$ Number of possible values of n = 9.

4.18 Common chord of two Circles

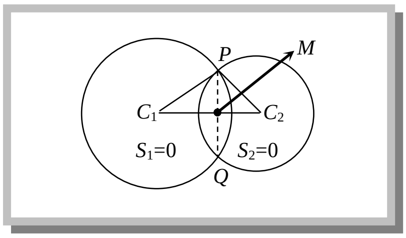
(1) **Definition :** The chord joining the points of intersection of two given circles is called their common chord.

(2) **Equation of common chord :** The equation of the common chord of two circles

$$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0 \quad \text{....(i)}$$

and $$S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0 \quad \text{....(ii)}$$

is $2x(g_1 - g_2) + 2y(f_1 - f_2) + c_1 - c_2 = 0$ *i.e.* $S_1 - S_2 = 0$.



(3) **Length of the common chord :** $PQ = 2(PM) = 2\sqrt{C_1P^2 - C_1M^2}$

Where $C_1P =$ radius of the circle $S = 0$ and $C_1M =$ length of the perpendicular from the centre C_1 to the common chord PQ.

Note : ❑ The length of the common chord is $2\sqrt{r_1^2 - p_1^2} = 2\sqrt{r_2^2 - p_2^2}$ where p_1 and p_2 are the lengths of perpendicular drawn from the centre to the chord.

❑ While using the above equation of common chord the coefficient of x^2 and y^2 in both equation should be equal.

❑ Two circle touches each other if the length of their common chord is zero.

❑ Maximum length of the common chord = diameter of the smaller circle.

Example : 38 If the common chord of the circles $x^2 + (y - \lambda)^2 = 16$ and $x^2 + y^2 = 16$ subtend a right angle at the origin, then λ is equal to

(a) 4 (b) $4\sqrt{2}$ (c) $\pm 4\sqrt{2}$ (d) 8

Solution : (c) The common chord of given circles is $S_1 - S_2 = 0$

$\Rightarrow \; x^2 + (y - \lambda)^2 - 16 - \{x^2 + y^2 - 16\} = 0$ *i.e.*, $y = \dfrac{\lambda}{2}$ $\quad (\because \lambda \neq 0)$

The pair of straight lines joining the origin to the points of intersection of $y = \dfrac{\lambda}{2}$ and $x^2 + y^2 = 16$ is $x^2 + y^2 = 16\left(\dfrac{2y}{\lambda}\right)^2$

$\Rightarrow \; \lambda^2x^2 + (\lambda^2 - 64)y^2 = 0$. These lines are at right angles if $\lambda^2 + \lambda^2 - 64 = 0$, *i.e.*, $\lambda = \pm\, 4\sqrt{2}$.

Example : 39 Which of the following is a point on the common chord of the circles $x^2 + y^2 + 2x - 3y + 6 = 0$ and $x^2 + y^2 + x - 8y - 13 = 0$ **[Karnataka CET 2003]**

(a) (1, –2) (b) (1, 4) (c) (1, 2) (d) (1, – 4)

Solution : (d) Given circles are, $S_1 \equiv x^2 + y^2 + 2x - 3y + 6 = 0$ ..... (i) and $S_2 \equiv x^2 + y^2 + x - 8y - 13 = 0$ ..... (ii)

$\therefore$ Equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow \; x + 5y + 19 = 0$, and out of the four given points only point (1, – 4) satisfies it.

Example : 40 If the circle $x^2 + y^2 = 4$ bisects the circumference of the circle $x^2 + y^2 - 2x + 6y + a = 0$, then a equals **[Rajasthan PET 1999]**

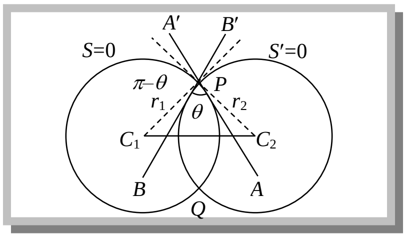
(a) 4 (b) – 4 (c) 16 (d) – 16

Solution : (c) The common chord of given circles is $S_1 - S_2 = 0 \Rightarrow \; 2x - 6y - 4 - a = 0$ .....(i)

Since, $x^2 + y^2 = 4$ bisects the circumferences of the circle $x^2 + y^2 - 2x + 6y + a = 0$, therefore (i) passes through the centre of second circle *i.e.* (1, – 3). $\therefore$ 2 + 18 – 4 – a = 0 ⇒ a = 16.

4.19 Angle of Intersection of Two Circles

The angle of intersection between two circles $S = 0$ and $S' = 0$ is defined as the angle between their tangents at their point of intersection.

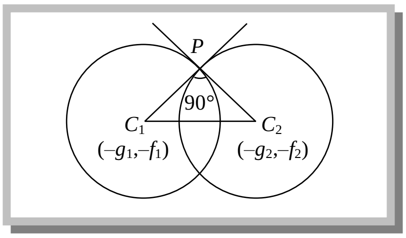


If $S \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

$S' \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

are two circles with radii r_1, r_2 and d be the distance between their centres then the angle of intersection θ between them is given by $\cos\theta = \dfrac{r_1^2 + r_1^2 - d^2}{2r_1r_2}$ or $\cos\theta = \dfrac{2(g_1g_2 + f_1f_2) - (c_1 + c_2)}{2\sqrt{g_1^2 + f_1^2 - c_1}\sqrt{g_2^2 + f_2^2 - c_2}}$

(1) **Condition of Orthogonality :** If the angle of intersection of the two circles is a right angle $(\theta = 90^o)$, then such circles are called orthogonal circles and condition for their orthogonality is $2g_1g_2 + 2f_1f_2 = c_1 + c_2$



Note : ❑ When the two circles intersect orthogonally then the length of tangent on one circle from the centre of other circle is equal to the radius of the other circle.

❑ Equation of a circle intersecting the three circles $x^2 + y^2 + 2g_ix + 2f_iy + c_i = 0$ $(i = 1, 2, 3)$ orthogonally is $\begin{vmatrix} x^2+y^2 & x & y & 1 \\ -c_1 & g_1 & f_1 & -1 \\ -c_2 & g_2 & f_2 & -1 \\ -c_3 & g_3 & f_3 & -1 \end{vmatrix} = 0$

Example : 41 A circle passes through the origin and has its centre on $y = x$. If it cuts $x^2 + y^2 - 4x - 6y + 10 = 0$ orthogonally, then the equation of the circle is **[EAMCET 1994]**

(a) $x^2 + y^2 - x - y = 0$ (b) $x^2 + y^2 - 6x - 4y = 0$ (c) $x^2 + y^2 - 2x - 2y = 0$ (d) $x^2 + y^2 + 2x + 2y = 0$

Solution : (c) Let the required circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ ....(i)

This passes through (0, 0), therefore $c = 0$

The centre $(-g, -f)$ of (i) lies on $y = x$, hence $g = f$.

Since (i) cuts the circle $x^2 + y^2 - 4x - 6y + 10 = 0$ orthogonally, therefore $2(-2g - 3f) = c + 10$

$\Rightarrow$ $-10g = 10$ $\Rightarrow$ $g = f = -1$ $(\because\ g = f$ and $c = 0)$. Hence the required circle is $x^2 + y^2 - 2x - 2y = 0$.

Example : 42 The centre of the circle, which cuts orthogonally each of the three circles $x^2 + y^2 + 2x + 17y + 4 = 0$, $x^2 + y^2 + 7x + 6y + 11 = 0$ and $x^2 + y^2 - x + 22y + 3 = 0$ is **[MP PET 2003]**

(a) (3, 2) (b) (1, 2) (c) (2, 3) (d) (0, 2)

Solution : (a) Let the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ …..(i)

Circle (i) cuts orthogonally each of the given three circles. Then according to condition $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$2g + 17f = c + 4$ …..(ii)

$7g + 6f = c + 11$ …..(iii)

$-g + 22f = c + 3$ …..(iv)

On solving (ii), (iii) and (iv), $g = -3$, $f = -2$. Therefore, the centre of the circle $(-g, -f) = (3, 2)$

Example : 43 The locus of the centre of a circle which cuts orthogonally the circle $x^2 + y^2 - 20x + 4 = 0$ and which touches $x = 2$ is **[UPSEAT 2001]**

(a) $y^2 = 16x + 4$ (b) $x^2 = 16y$ (c) $x^2 = 16y + 4$ (d) $y^2 = 16x$

Solution : (d) Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ …..(i)

It cuts the circle $x^2 + y^2 - 20x + 4 = 0$ orthogonally

$\therefore \quad 2(-10g + 0 \times f) = c + 4 \Rightarrow -20g = c + 4$ …..(ii)

Circle (i) touches the line $x = 2$; $\therefore \quad x + 0y - 2 = 0$

$\therefore \quad \left|\frac{-g + 0 - 2}{\sqrt{1}}\right| = \sqrt{g^2 + f^2 - c} \Rightarrow (g+2)^2 = g^2 + f^2 - c \Rightarrow 4g + 4 = f^2 - c$ …..(iii)

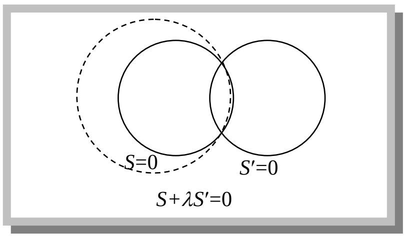
Eliminating c from (ii) and (iii), we get $-16g + 4 = f^2 + 4 \Rightarrow f^2 + 16g = 0$.

Hence the locus of $(-g, -f)$ is $y^2 - 16x = 0 \Rightarrow y^2 = 16x$.

4.20 Family of Circles

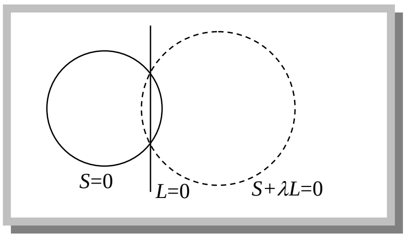
(1) The equation of the family of circles passing through the point of intersection of two given circles $S = 0$ and $S' = 0$ is given as

$$S + \lambda S' = 0 \qquad \text{(where } \lambda \text{ is a parameter, } \lambda \neq -1)$$



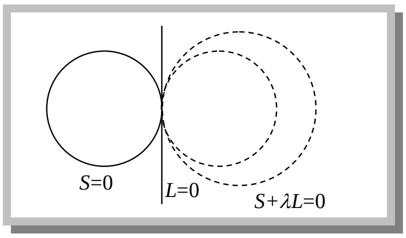
(2) The equation of the family of circles passing through the point of intersection of circle $S = 0$ and a line $L = 0$ is given as

$$S + \lambda L = 0 \qquad \text{(where } \lambda \text{ is a parameter)}$$



(3) The equation of the family of circles touching the circle $S = 0$ and the line $L = 0$ at their point of contact P is

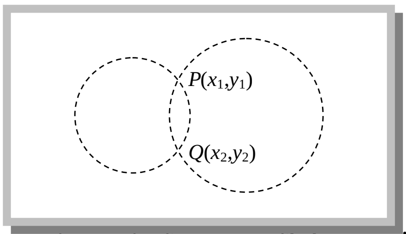
$$S + \lambda L = 0 \qquad \text{(where } \lambda \text{ is a parameter)}$$



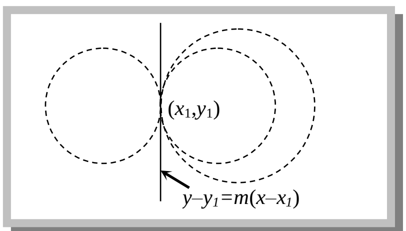
(4) The equation of a family of circles passing through two given points $P(x_1, y_1)$ and $Q(x_2, y_2)$ can be written in the form

$$\Rightarrow (x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

(where λ is a parameter)



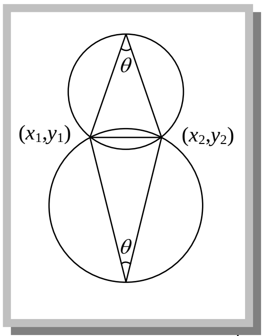
(5) The equation of family of circles, which touch $y - y_1 = m(x - x_1)$ at (x_1, y_1) for any finite m is $(x - x_1)^2 + (y - y_1)^2 + \lambda\{(y - y_1) - m(x - x_1)\} = 0$



And if m is infinite, the family of circles is

$(x-x_1)^2+(y-y_1)^2+\lambda(x-x_1)=0$ (where λ is a parameter)

(6) Equation of the circles given in diagram is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)\pm\cot\theta\{(x-x_1)(y-y_2)-(x-x_2)(y-y_1)\}=0$



Example : 44 The equation of the circle through the points of intersection of $x^2+y^2-1=0$, $x^2+y^2-2x-4y+1=0$ and touching the line $x+2y=0$ is **[Roorkee 1989]**

(a) $x^2+y^2+x+2y=0$ (b) $x^2+y^2-x+20=0$ (c) $x^2+y^2-x-2y=0$ (d) $2(x^2+y^2)-x-2y=0$

Solution : (c) Family of circles is $x^2+y^2-2x-4y+1+\lambda(x^2+y^2-1)=0$

$$x^2+y^2-\frac{2}{1+\lambda}x-\frac{4}{1+\lambda}y+\frac{1-\lambda}{1+\lambda}=0$$

Centre is $\left[\frac{1}{1+\lambda},\frac{2}{1+\lambda}\right]$ and radius $=\sqrt{\left(\frac{1}{1+\lambda}\right)^2+\left(\frac{2}{1+\lambda}\right)^2-\left(\frac{1-\lambda}{1+\lambda}\right)}=\sqrt{\frac{4+\lambda^2}{(1+\lambda)^2}}$

Since it touches the line $x+2y=0$, Hence Radius = perpendicular distance from centre to the line $=\left|\frac{\frac{1}{1+\lambda}+\frac{4}{1+\lambda}}{\sqrt{1^2+2^2}}\right|$

$$=\sqrt{\frac{4+\lambda^2}{(1+\lambda)^2}}=\frac{\sqrt{4+\lambda^2}}{1+\lambda}\Rightarrow\sqrt{5}=\sqrt{4+\lambda^2}\Rightarrow\lambda=\pm 1$$

$\lambda=-1$ cannot be possible in case of circle, so $\lambda=1$. $\therefore$ Equation of circle is $x^2+y^2-x-2y=0$

Example : 45 The equation of the circle through the points of intersection of the circles $x^2+y^2-6x+2y+4=0$, $x^2+y^2+2x-4y-6=0$ and with its centre on the line $y=x$

(a) $7x^2+7y^2+10x-10y-12=0$ (b) $7x^2+7y^2-10x-10y-12=0$

(c) $7x^2+7y^2-10x+10y-12=0$ (d) $7x^2+7y^2+10x+10y+12=0$

Solution : (b) Equation of any circle through the points of intersection of given circles is

$(x^2+y^2-6x+2y+4)+\lambda(x^2+y^2+2x-4y-6)=0\Rightarrow x^2(1+\lambda)+y^2(1+\lambda)-2x(3-\lambda)+2y(1-2\lambda)+(4-6\lambda)=0$

or, $x^2+y^2-\frac{2x(3-\lambda)}{(1+\lambda)}+\frac{2y(1-2\lambda)}{(1+\lambda)}+\frac{(4-6\lambda)}{(1+\lambda)}=0$ .....(i)

Its centre $=\left\{\frac{3-\lambda}{1+\lambda},\frac{2\lambda-1}{1+\lambda}\right\}$ lies on the line $y=x$. Then $\frac{2\lambda-1}{1+\lambda}=\frac{3-\lambda}{1+\lambda}\Rightarrow 2\lambda-1=3-\lambda\ \{\because\ \lambda\neq-1\}$

$\Rightarrow 3\lambda=4\Rightarrow\lambda=\frac{4}{3}$

Substituting the value of $\lambda=\frac{4}{3}$ in (i), we get the required equation as $7x^2+7y^2-10x-10y-12=0$.

4.21 Radical Axis

The radical axis of two circles is the locus of a point which moves such that the lengths of the tangents drawn from it to the two circles are equal.

Consider, $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ …..(i) and $S' \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ …..(ii)

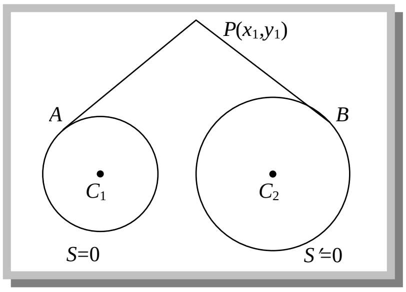
Let $P(x_1, y_1)$ be a point such that $|PA| = |PB|$

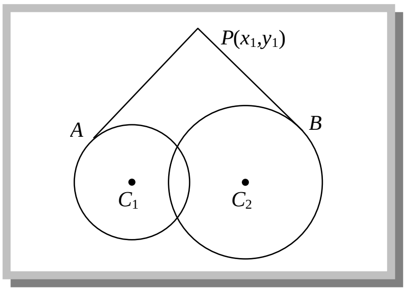
$\Rightarrow \sqrt{(x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c)} = \sqrt{(x_1^2 + y_1^2 + 2g_1x_1 + 2f_1y_1 + c_1)}$

On squaring, $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = x_1^2 + y_1^2 + 2g_1x_1 + 2f_1y_1 + c_1$

$\Rightarrow 2(g - g_1)x_1 + 2(f - f_1)y_1 + c - c_1 = 0$

$\therefore$ Locus of $P(x_1, y_1)$ is $2(g - g_1)x + 2(f - f_1)y + c - c_1 = 0$

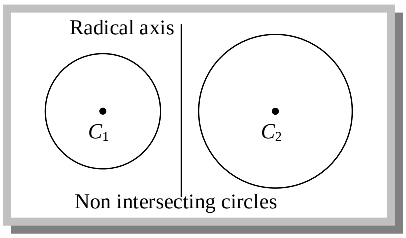


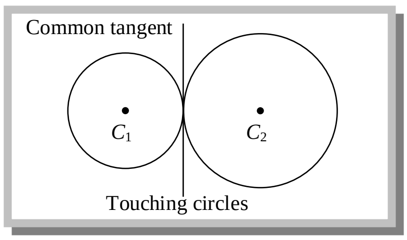


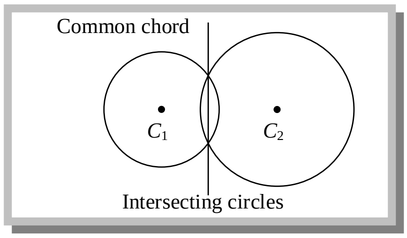
which is the required equation of radical axis of the given circles. Clearly this is a straight line.

(1) **Some properties of the radical axis**

(i) The radical axis and common chord are identical : Since the radical axis and common chord of two circles $S = 0$ and $S' = 0$ are the same straight line $S - S' = 0$, they are identical. The only difference is that the common chord exists only if the circles intersect in two real points, while the radical axis exists for all pair of circles irrespective of their position (Except when one circle is inside the other).





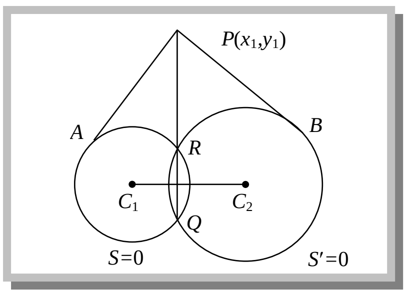


(ii) The radical axis is perpendicular to the straight line which joins the centres of the circles :

Consider, $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ …..(i)

and $S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ …..(ii)

Since $C_1 \equiv (-g, -f)$ and $C_2 \equiv (-g_1, -f_1)$ are the centres of the circles (i) and (ii), then slope of the straight line $C_1C_2 = \dfrac{-f_1 + f}{-g_1 + g} = \dfrac{f - f_1}{g - g_1} = m_1$ (say)

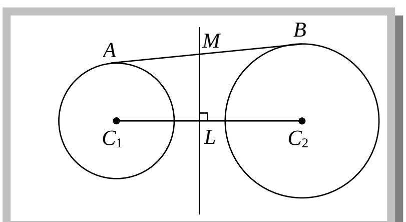


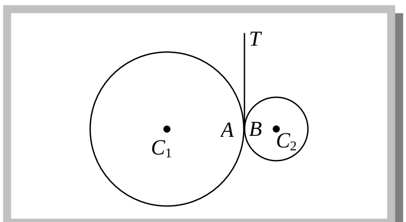
Equation of the radical axis is, $2(g - g_1)x + 2(f - f_1)y + c - c_1 = 0$

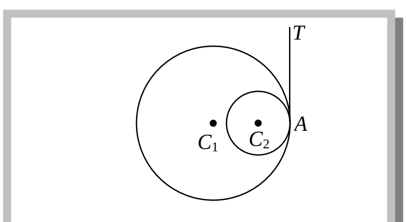
Slope of radical axis is $-\dfrac{(g - g_1)}{(f - f_1)} = m_2$ (say). $\therefore$ $m_1m_2 = -1$

Hence C_1C_2 and radical axis are perpendicular to each other.

(iii) The radical axis bisects common tangents of two circles : Let *AB* be the common tangent. If it meets the radical axis *LM* in *M* then *MA* and *MB* are two tangents to the circles. Hence *MA* = *MB*, since length of tangents are equal from any point on radical axis. Hence radical axis bisects the common tangent *AB*.







If the two circles touch each other externally or internally then *A* and *B* coincide. In this case the common tangent itself becomes the radical axis.

(iv) The radical axis of three circles taken in pairs are concurrent : Let the equations of three circles be

$$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0 \qquad .....(i)$$

$$S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0 \qquad .....(ii)$$

$$S_3 \equiv x^2 + y^2 + 2g_3x + 2f_3y + c_3 = 0 \qquad .....(iii)$$

The radical axis of the above three circles taken in pairs are given by

$$S_1 - S_2 \equiv 2x(g_1 - g_2) + 2y(f_1 - f_2) + c_1 - c_2 = 0 \qquad .....(iv)$$

$$S_2 - S_3 \equiv 2x(g_2 - g_3) + 2y(f_2 - f_3) + c_2 - c_3 = 0 \qquad .....(v)$$

$$S_3 - S_1 \equiv 2x(g_3 - g_1) + 2y(f_3 - f_1) + c_3 - c_1 = 0 \qquad .....(vi)$$

Adding (iv), (v) and (vi), we find *L.H.S.* vanished identically. Thus the three lines are concurrent.

(v) If two circles cut a third circle orthogonally, the radical axis of the two circles will pass through the centre of the third circle

or

The locus of the centre of a circle cutting two given circles orthogonally is the radical axis of the two circles.

Let $$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0 \qquad .....(i)$$

$$S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0 \qquad .....(ii)$$

$$S_3 \equiv x^2 + y^2 + 2g_3x + 2f_3y + c_3 = 0 \qquad .....(iii)$$

Since (i) and (ii) both cut (iii) orthogonally, $\therefore \; 2g_1g_3 + 2f_1f_3 = c_1 + c_3$ and $2g_2g_3 + 2f_2f_3 = c_2 + c_3$

Subtracting, we get $2g_3(g_1 - g_2) + 2f_3(f_1 - f_2) = c_1 - c_2$ .....(iv)

Now radical axis of (i) and (ii) is $S_1 - S_2 = 0$ or $2x(g_1 - g_2) + 2y(f_1 - f_2) + c_1 - c_2 = 0$

Since it will pass through the centre of circle (iii)

$\therefore \; -2g_3(g_1 - g_2) - 2f_3(f_1 - f_2) + c_1 - c_2 = 0$ or $2g_3(g_1 - g_2) + 2f_3(f_1 - f_2) = c_1 - c_2$ .....(v)

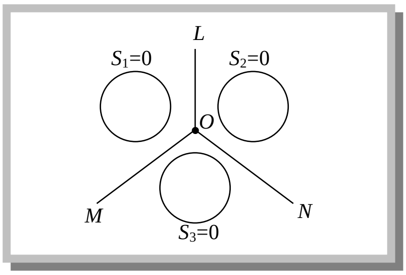
which is true by (iv).

Note : ❑ Radical axis need not always pass through the mid point of the line joining the centres of the two circles.

4.22 Radical Centre

The radical axes of three circles, taken in pairs, meet in a point, which is called their radical centre. Let the three circles be $S_1 = 0$ .....(i) , $S_2 = 0$ .....(ii) , $S_3 = 0$ .....(iii)

Let *OL*, *OM* and *ON* be radical axes of the pair sets of circles



$\{S_1 = 0, S_2 = 0\}, \{S_3 = 0, S_1 = 0\}$ and $\{S_2 = 0, S_3 = 0\}$ respectively.

Equation of *OL*, *OM* and *ON* are respectively

$S_1 - S_2 = 0$ .....(iv) , $S_3 - S_1 = 0$ .....(v), $S_2 - S_3 = 0$ .....(vi)

Let the straight lines (iv) and (v) *i.e.*, *OL* and *OM* meet in *O*. The equation of any straight line passing through *O* is $(S_1 - S_2) + \lambda(S_3 - S_1) = 0$ where λ is any constant

For $\lambda = 1$, this equation become $S_2 - S_3 = 0$, which is by (vi), equation of *ON*.

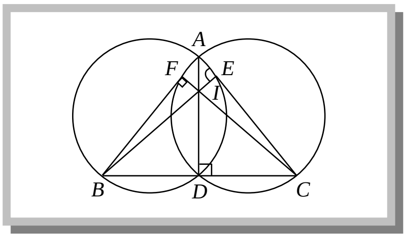
Thus the third radical axis also passes through the point where the straight lines (iv) and (v) meet.

In the above figure *O* is the radical centre.

(1) **Properties of radical centre**

(i) Co-ordinates of radical centre can be found by solving the equations

$$S_1 = S_2 = S_3 = 0$$



(ii) The radical centre of three circles described on the sides of a triangle as diameters is the orthocentre of the triangle :

Draw perpendicular from *A* on *BC*. $\therefore \quad \angle ADB = \angle ADC = \pi/2$

Therefore, the circles whose diameters are *AB* and *AC* passes through *D* and *A*. Hence *AD* is their radical axis. Similarly the radical axis of the circles on *AB* and *BC* as diameter is the perpendicular line from *B* on *CA* and radical axis of the circles on *BC* and *CA* as diameter is the perpendicular line from *C* on *AB*. Hence the radical axis of three circles meet in a point. This point *I* is radical centre but here radical centre is the point of intersection of altitudes *i.e., AD, BE* and *CF.* Hence radical centre = orthocentre.

(iii) The radical centre of three given circles will be the centre of a fourth circle which cuts all the three circles orthogonally and the radius of the fourth circle is the length of tangent drawn from radical centre of the three given circles to any of these circles.

Let the fourth circle be $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) is centre of this circle and *r* be the radius. The centre of circle is the radical centre of the given circles and *r* is the length of tangent from (h, k) to any of the given three circles.

Example : 46 The gradient of the radical axis of the circles $x^2 + y^2 - 3x - 4y + 5 = 0$ and $3x^2 + 3y^2 - 7x + 8y + 11 = 0$ is **[MP PET 2000]**

(a) $\frac{1}{3}$ (b) $-\frac{1}{10}$ (c) $-\frac{1}{2}$ (d) $-\frac{2}{3}$

Solution : (b) Equation of radical axis is $S_1 - S_2 = 0$

$S_1 \equiv x^2 + y^2 - 3x - 4y + 5 = 0$, $S_2 \equiv x^2 + y^2 - \frac{7}{3}x + \frac{8y}{3} + \frac{11}{3} = 0$

$\therefore$ Radical axis is $-2x - 20y + 4 = 0$.

Hence, gradient of radical axis = $-\frac{1}{10}$

Example : 47 The equations of three circles are $x^2 + y^2 - 12x - 16y + 64 = 0$, $3x^2 + 3y^2 - 36x + 81 = 0$ and $x^2 + y^2 - 16x + 81 = 0$. The coordinates of the point from which the length of tangents drawn to each of the three circles is equal **[Rajasthan PET 2002]**

(a) $\left(\frac{33}{4}, 2\right)$ (b) $(2, 2)$ (c) $\left(2, \frac{33}{4}\right)$ (d) None of these

Solution : (d) The required point is the radical centre of the three given circles

Now, $S_1 - S_2 = 0 \Rightarrow -16y + 37 = 0$, $S_2 - S_3 = 0 \Rightarrow 4x - 54 = 0$ and $S_3 - S_1 = 0 \Rightarrow -4x + 16y + 17 = 0$

Solving these equations, we get $x = \frac{54}{4}, \ y = \frac{37}{16} \Rightarrow x = \frac{27}{2}, \ y = \frac{37}{16}$. Hence the required point is $\left(\frac{27}{2}, \frac{37}{16}\right)$.

Example : 48 The equation of the circle, which passes through the point $(2a, 0)$ and whose radical axis is $x = \frac{a}{2}$ with respect to the circle $x^2 + y^2 = a^2$, will be **[Rajasthan PET 1999]**

(a) $x^2 + y^2 - 2ax = 0$ (b) $x^2 + y^2 + 2ax = 0$ (c) $x^2 + y^2 + 2ay = 0$ (d) $x^2 + y^2 - 2ay = 0$

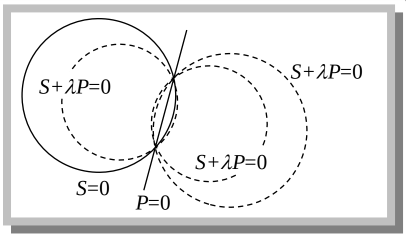
Solution : (a) Equation of radical axis is $x = \frac{a}{2} \Rightarrow 2x - a = 0$

Equation of required circle is $x^2 + y^2 - a^2 + \lambda(2x - a) = 0$

$\because$ It is passes through the point $(2a, 0)$, $\therefore$ $4a^2 - a^2 + \lambda(4a - a) = 0 \Rightarrow \lambda = -a$

$\therefore$ Equation of circle is $x^2 + y^2 - a^2 - 2ax + a^2 = 0 \Rightarrow x^2 + y^2 - 2ax = 0$

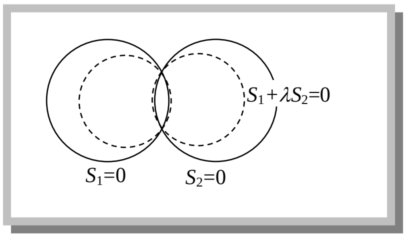
4.23 Co-Axial System of Circles

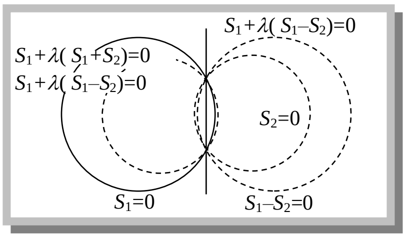


A system (or a family) of circles, every pair of which have the same radical axis, are called co-axial circles.

(1) The equation of a system of co-axial circles, when the equation of the radical axis and of one circle of the system are $P \equiv lx + my + n = 0$ and $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ respectively, is $S + \lambda P = 0$ (λ is an arbitrary constant).

(2) The equation of a co-axial system of circles, where the equation of any two circles of the system are





$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

Respectively, is $S_1 + \lambda(S_1 - S_2) = 0$, $(\lambda \neq -1)$ or $S_2 + \lambda_1(S_1 - S_2) = 0$, $(\lambda_1 \neq -1)$

Other form $S_1 + \lambda S_2 = 0$, $(\lambda \neq -1)$

(3) The equation of a system of co-axial circles in the simplest form is $x^2 + y^2 + 2gx + c = 0$, where g is variable and c is a constant.

4.24 Limiting Points

Limiting points of a system of co-axial circles are the centres of the point circles belonging to the family (Circles whose radii are zero are called **point circles**).

(1) **Limiting points of the co-axial system :** Let the circle is $x^2 + y^2 + 2gx + c = 0$ .....(i)

where g is variable and c is constant.

$\therefore$ Centre and the radius of (i) are $(-g, 0)$ and $\sqrt{(g^2 - c)}$ respectively. Let $\sqrt{g^2 - c} = 0 \Rightarrow g = \pm\sqrt{c}$

Thus we get the two limiting points of the given co-axial system as $(\sqrt{c}, 0)$ and $(-\sqrt{c}, 0)$

Clearly the above limiting points are real and distinct, real and coincident or imaginary according as $c >, =, < 0$

(2) **System of co-axial circles whose two limiting points are given :** Let (α, β) and (γ, δ) be the two given limiting points. Then the corresponding point circles with zero radii are

$(x - \alpha)^2 + (y - \beta)^2 = 0$ and $(x - \gamma)^2 + (y - \delta)^2 = 0$

or $x^2 + y^2 - 2\alpha x - 2\beta y + \alpha^2 + \beta^2 = 0$ and $x^2 + y^2 - 2\gamma x - 2\delta y + \gamma^2 + \delta^2 = 0$

The equation of co-axial system is $(x^2 + y^2 - 2\alpha x - 2\beta y + \alpha^2 + \beta^2) + \lambda(x^2 + y^2 - 2\gamma x - 2\delta y + \gamma^2 + \delta^2) = 0$

where $\lambda \neq -1$ is a variable parameter.

$\Rightarrow \quad x^2(1+\lambda) + y^2(1+\lambda) - 2x(\alpha + \gamma\lambda) - 2y(\beta + \delta\lambda) + (\alpha^2 + \beta^2) + \lambda(\gamma^2 + \delta^2) = 0$

or $\quad x^2 + y^2 - \dfrac{2(\alpha + \gamma\lambda)}{(1+\lambda)}x - 2\dfrac{(\beta + \delta\lambda)}{(1+\lambda)}y + \dfrac{(\alpha^2 + \beta^2) + \lambda(\gamma^2 + \delta^2)}{(1+\lambda)} = 0$

Centre of this circle is $\left(\dfrac{(\alpha + \gamma\lambda)}{(1+\lambda)}, \dfrac{(\beta + \delta\lambda)}{(1+\lambda)}\right)$ .....(i)

For limiting point, radius $= \sqrt{\dfrac{(\alpha + \gamma\lambda)^2}{(1+\lambda)^2} + \dfrac{(\beta + \delta\lambda)^2}{(1+\lambda)^2} - \dfrac{(\alpha^2 + \beta^2) + \lambda(\gamma^2 + \delta^2)}{(1+\lambda)}} = 0$

After solving, find λ. Substituting value of λ in (i), we get the limiting point of co-axial system.

(3) **Properties of limiting points**

(i) The limiting point of a system of co-axial circles are conjugate points with respect to any member of the system : Let the equation of any circle be $x^2 + y^2 + 2gx + c = 0$ ....(i)

Limiting points of (i) are $(\sqrt{c}, 0)$ and $(-\sqrt{c}, 0)$. The polar of the point $(\sqrt{c}, 0)$ with respect to (i) is

$x\sqrt{c} + y.0 + g(x + \sqrt{c}) + c = 0$ or $x\sqrt{c} + g(x + \sqrt{c}) + c = 0$ or $(x + \sqrt{c})(g + \sqrt{c}) = 0$ or $x + \sqrt{c} = 0$ and it clearly passes through the other limiting point $(-\sqrt{c}, 0)$. Similarly polar of the point $(-\sqrt{c}, 0)$ with respect to (i) also passes through $(\sqrt{c}, 0)$. Hence the limiting points of a system of co-axial circles are conjugate points.

(ii) Every circle through the limiting points of a co-axial system is orthogonal to all circles of the system :

Let the equation of any circle be $x^2 + y^2 + 2gx + c = 0$ .....(i)

where g is a parameter and c is constant. Limiting points of (i) are $(\sqrt{c}, 0)$ and $(-\sqrt{c}, 0)$

Now let $x^2 + y^2 + 2g'x + 2f'y + c' = 0$ .....(ii)

be the equation of any circle. If it passes through the limiting points of (i), then $c + 2g'\sqrt{c} + c' = 0$ and $c - 2g'\sqrt{c} + c' = 0$. Solving, we get $c' = -c$ and $g' = 0$

From (ii), $x^2 + y^2 + 2f'y - c = 0$ .....(iii)

where c is constant and f' is variable. Applying the condition of orthogonality on (i) and (iii) *i.e.*, $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ we find that $2 \times g \times 0 + 2 \times 0 \times f' = c - c$ *i.e.*, $0 = 0$

Hence condition is satisfied for all values of g' and f'.

Example : 49 The point (2, 3) is a limiting point of a co-axial system of circles of which $x^2 + y^2 = 9$ is a member. The coordinates of the other limiting point is given by **[MP PET 1993]**

(a) $\left(\dfrac{18}{13}, \dfrac{27}{13}\right)$ (b) $\left(\dfrac{9}{13}, \dfrac{6}{13}\right)$ (c) $\left(\dfrac{18}{13}, -\dfrac{27}{13}\right)$ (d) $\left(-\dfrac{18}{13}, -\dfrac{9}{13}\right)$

Solution : (a) Equation of circle with (2, 3) as limiting point is $(x-2)^2 + (y-3)^2 = 0$

or $(x^2 + y^2 - 9) - 4x - 6y + 22 = 0$ or $(x^2 + y^2 - 9) - \lambda(2x + 3y - 11) = 0$ represents the family of co-axial circles.

$c = \left(\lambda, \dfrac{3\lambda}{2}\right)$, $r = \sqrt{\lambda^2 + \dfrac{9\lambda^2}{4} - 11\lambda + 9}$. For limiting points $r = 0 \Rightarrow 13\lambda^2 - 44\lambda + 36 = 0 \Rightarrow \lambda = \dfrac{18}{13}, 2$

$\therefore$ The limiting points are (2, 3) and $\left[\frac{18}{13}, \frac{3}{2}\left(\frac{18}{13}\right)\right]$ or $\left(\frac{18}{13}, \frac{27}{13}\right)$.

Example : 50 In the co-axial system of circle $x^2 + y^2 + 2gx + c = 0$ where g is a parameter, if $c > 0$. Then the circles are

[Karnataka CET 1999]

(a) Orthogonal (b) Touching type (c) Intersecting type (d) Non intersecting type

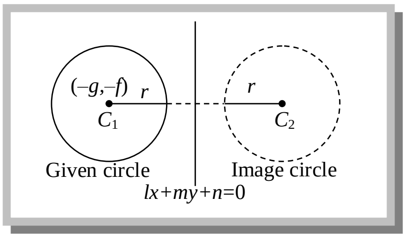
Solution : (d) The equation of a system of circle with its centre on the axis of x is $x^2 + y^2 + 2gx + c = 0$. Any point on the radical axis is $(0, y_1)$

Putting, $x = 0$, $y = \pm\sqrt{-c}$

If c is positive ($c > 0$), we have no real point on radical axis, then circles are said to be non-intersecting type.

4.25 Image of the Circle by the Line Mirror

Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ and line mirror $lx + my + n = 0$. In this condition, radius of circle remains unchanged but centre changes. Let the centre of image circle be (x_1, y_1).



Slope of $C_1C_2 \times$ slope of $lx + my + n = -1$ .....(i)

and mid point of $C_1(-g, -f)$ and $C_2(x_1, y_1)$ lie on $lx + my + n = 0$

i.e., $l\left(\frac{x_1 - g}{2}\right) + m\left(\frac{y_1 - f}{2}\right) + n = 0$ .....(ii)

Solving (i) and (ii), we get (x_1, y_1)

$\therefore$ Required image circle is $(x - x_1)^2 + (y - y_1)^2 = r^2$, where $r = \sqrt{(g^2 + f^2 - c)}$

Example : 51 The equation of the image of the circle $x^2 + y^2 + 16x - 24y + 183 = 0$ by the line mirror $4x + 7y + 13 = 0$ is

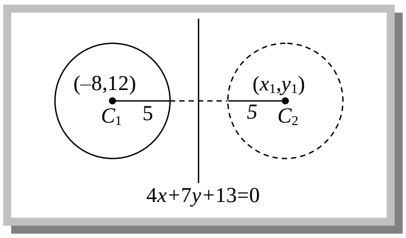
(a) $x^2 + y^2 + 32x - 4y + 235 = 0$ (b) $x^2 + y^2 + 32x + 4y - 235 = 0$

(c) $x^2 + y^2 + 32x - 4y - 235 = 0$ (d) $x^2 + y^2 + 32x + 4y + 235 = 0$

Solution : (d) The given circle and line are $x^2 + y^2 + 16x - 24y + 183 = 0$ .....(i) and $4x + 7y + 13 = 0$ .....(ii)

Centre and radius of circle (i) are (– 8, 12) and 5 respectively. Let the centre of the image circle be (x_1, y_1).

Then slope of $C_1C_2 \times$ slope of $4x + 7y + 13 = -1$



$\Rightarrow \left(\frac{y_1 - 12}{x_1 + 8}\right) \times \left(-\frac{4}{7}\right) = -1$ or $4y_1 - 48 = 7x_1 + 56$

or $7x_1 - 4y_1 + 104 = 0$ .....(iii)

and mid point of C_1C_2 *i.e.*, $\left(\frac{x_1 - 8}{2}, \frac{y_1 + 12}{2}\right)$ lie on $4x + 7y + 13 = 0$,

then $4\left(\frac{x_1 - 8}{2}\right) + 7\left(\frac{y_1 + 12}{2}\right) + 13 = 0$ or $4x_1 + 7y_1 + 78 = 0$ .....(iv)

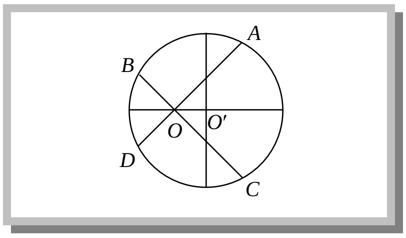
Solving (iii) and (iv), we get $(x_1, y_1) = (-16, -2)$

$\therefore$ Equation of the image circle is $(x + 16)^2 + (y + 2)^2 = 5^2$ or $x^2 + y^2 + 32x + 4y + 235 = 0$

4.26 Some Important Results

(1) **Concyclic points :** If A, B, C, D are concyclic then $OA . OD = OC . OB$, where O' be the centre of the circle.

(2) Equation of the straight line joining two points α and β on the circle

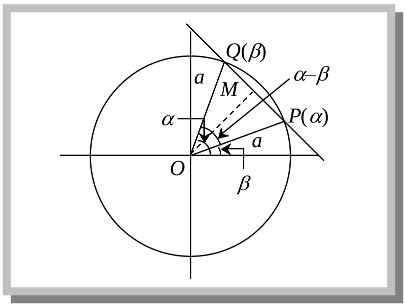


$$x^2 + y^2 = a^2$$

Required equation is $x\cos\left(\frac{\alpha + \beta}{2}\right) + y\sin\left(\frac{\alpha + \beta}{2}\right) = a\cos\left(\frac{\alpha - \beta}{2}\right)$

(3) The point of intersection of the tangents at the point $P(\alpha)$ and $Q(\beta)$ on the circle $x^2+y^2=a^2$ is

$$\left(\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \frac{a\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}\right)$$

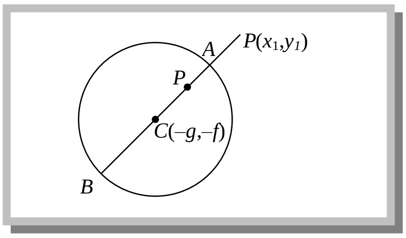


(4) Maximum and Minimum distance of a point from the circle :

Let any point $P(x_1, y_1)$ and circle $x^2+y^2+2gx+2fy+c=0$ …..(i)

The centre and radius of the circle are

$C(-g, -f)$ and $\sqrt{(g^2+f^2-c)}$ respectively.

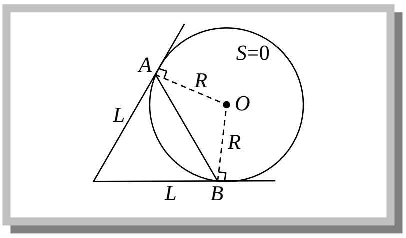


The maximum and minimum distance from $P(x_1, y_1)$ to the circle (i) are

$PB = CB + PC = r + PC$ and $PA = |CP - CA| = |PC - r|$ (P inside or outside)

where $r = \sqrt{(g^2+f^2-c)}$

(5) Length of chord of contact is $AB = \dfrac{2LR}{\sqrt{(R^2+L^2)}}$

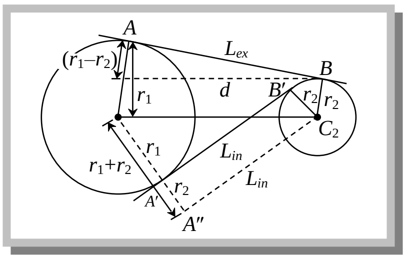


and area of the triangle formed by the pair of tangents and its chord of contact is

$$= \frac{RL^3}{R^2+L^2}$$

Where R is the radius of the circle and L is the length of tangent from $P(x_1, y_1)$ on $S=0$. Here $L=\sqrt{S_1}$.

(6) Length of an external common tangent and internal common tangent to two circles is given by



Length of external common tangent $L_{ex} = \sqrt{d^2-(r_1-r_2)^2}$

and length of internal common tangent

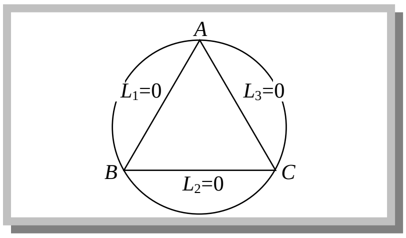
$L_{in} = \sqrt{d^2-(r_1+r_2)^2}$ [Applicable only when $d > (r_1+r_2)$]

where d is the distane between the centres of two circles *i.e.*, $|C_1C_2| = d$ and r_1 and r_2 are the radii of two circles.

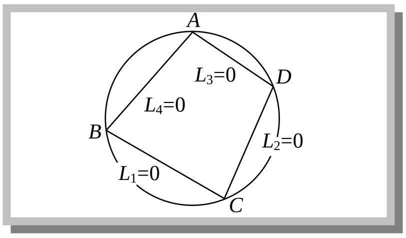
(7) Family of circles circumscribing a triangle whose sides are given by $L_1=0$; $L_2=0$ and $L_3=0$ is given by $L_1L_2+\lambda L_2L_3+\mu L_3L_1=0$ provided coefficient of $xy=0$ and coefficient of x^2 = coefficient of y^2.

Equation of the circle circumscribing the triangle formed by the lines $a_rx+b_ry+c_r=0$, where $r = 1, 2, 3$, is

$$\begin{vmatrix} \dfrac{a_1^2+b_1^2}{a_1x+b_1y+c_1} & a_1 & b_1 \\ \dfrac{a_2^2+b_2^2}{a_2x+b_2y+c_2} & a_2 & b_2 \\ \dfrac{a_3^2+b_3^2}{a_3x+b_3y+c_3} & a_3 & b_3 \end{vmatrix} = 0$$



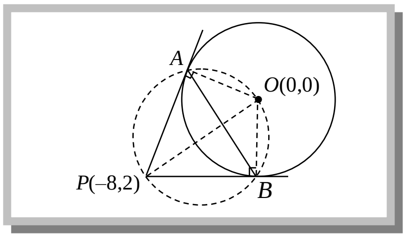
(8) Equation of circle circumscribing a quadrilateral whose sides in order are represented by the lines $L_1 = 0,\ L_2 = 0,\ L_3 = 0$ and $L_4 = 0$ is given by



$$L_1L_3 + \lambda L_2L_4 = 0$$

provided coefficient of x^2 = coefficient of y^2 and coefficient of $xy = 0$

(9) Equation of the circle circumscribing the triangle PAB is



$$(x - x_1)(x + g) + (y - y_1)(y + f) = 0$$

where $O(-g, -f)$ is the centre of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

(10) Locus of mid point of a chord of a circle $x^2 + y^2 = a^2$, which subtends an angle α at the centre is $x^2 + y^2 = (a\cos\alpha/2)^2$

(11) The locus of mid point of chords of circle $x^2 + y^2 = a^2$, which are making right angle at centre is $x^2 + y^2 = \dfrac{a^2}{2}$

(12) The locus of mid point of chords of circle $x^2 + y^2 + 2gx + 2fy + c = 0$, which are making right angle at origin is $x^2 + y^2 + gx + fy + c/2 = 0$.

(13) The area of triangle , which is formed by co-ordinate axes and the tangent at a point (x_1, y_1) of circle $x^2 + y^2 = a^2$ is $a^4/2x_1y_1$

(14) If a point is outside, on or inside the circle then number of tangents from the points is 2, 1 or none.

(15) A variable point moves in such a way that sum of square of distances from the vertices of a triangle remains constant then its locus is a circle whose centre is the centroid of the triangle.

(16) If the points where the line $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ meets the coordinate axes are concyclic then $a_1a_2 = b_1b_2$.

Example : 52 If $\left(m_i, \dfrac{1}{m_i}\right)$, $i = 1, 2, 3, 4$ are concylic points, then the value of $m_1.m_2.m_3.m_4$ is **[Karnataka CET 2002]**

(a) 1 (b) – 1 (c) 0 (d) None of these

Solution : (a) Let the equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

Since the point $\left(m_i, \dfrac{1}{m_i}\right)$ lies on this circle

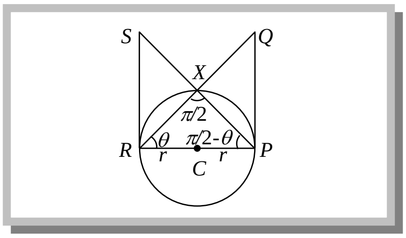
$$\therefore\ m_i^2 + \frac{1}{m_i^2} + 2gm_i + \frac{2f}{m_i} + c = 0 \Rightarrow m_i^4 + 2gm_i^3 + cm_i^2 + 2fm_i + 1 = 0$$

Clearly its roots are m_1, m_2, m_3 and m_4, $\therefore$ $m_1.m_2.m_3.m_4$ = product of roots $= \dfrac{1}{1} = 1$

Example : 53 Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then $2r$ equals **[IIT Screening 2001]**

(a) $\sqrt{PQ \,.\, RS}$ (b) $\dfrac{PQ+RS}{2}$ (c) $\dfrac{2\,PQ \,.\, RS}{PQ+RS}$ (d) $\dfrac{\sqrt{PQ^2+RS^2}}{2}$

Solution : (a) $\tan\theta = \dfrac{PQ}{PR} = \dfrac{PQ}{2r}$



Also $\tan\left(\dfrac{\pi}{2}-\theta\right) = \dfrac{RS}{2r}$

i.e. $\cot\theta = \dfrac{RS}{2r}$

$\therefore \tan\theta \,.\, \cot\theta = \dfrac{PQ \,.\, RS}{4r^2}$

$\Rightarrow 4r^2 = PQ \,.\, RS \Rightarrow 2r = \sqrt{(PQ)\,(RS)}$

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Assignment

Definition, Equation of the Circle

Basic Level

1. The two points A and B in a plane such that for all points P lies on circle satisfied $\frac{PA}{PB} = k$, then k will not be equal to **[IIT 1982]**

(a) 0 (b) 1 (c) 2 (d) None of these

2. Locus of a point which moves such that sum of the squares of its distances from the sides of a square of side unity is 9, is **[IIT 1976]**

(a) Straight line (b) Circle (c) Parabola (d) None of these

3. The equation of the circle which touches both the axes and whose radius is a, is **[MP PET 1984]**

(a) $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ (b) $x^2 + y^2 + ax + ay - a^2 = 0$

(c) $x^2 + y^2 + 2ax + 2ay - a^2 = 0$ (d) $x^2 + y^2 - ax - ay + a^2 = 0$

4. $ABCD$ is a square the length of whose side is a. Taking AB and AD as the coordinate axes, the equation of the circle passing through the vertices of the square, is **[MP PET 2003]**

(a) $x^2 + y^2 + ax + ay = 0$ (b) $x^2 + y^2 - ax - ay = 0$ (c) $x^2 + y^2 + 2ax + 2ay = 0$ (d) $x^2 + y^2 - 2ax - 2ay = 0$

5. The equation of the circle in the first quadrant touching each coordinate axis at a distance of one unit from the origin is **[Rajasthan PET 1991; MP PET 1987, 1989]**

(a) $x^2 + y^2 - 2x - 2y + 1 = 0$ (b) $x^2 + y^2 - 2x - 2y - 1 = 0$

(c) $x^2 + y^2 - 2x - 2y = 0$ (d) None of these

6. The equation of the circle which touches both axes and whose centre is (x_1, y_1), is **[MP PET 1988]**

(a) $x^2 + y^2 + 2x_1(x + y) + x_1^2 = 0$ (b) $x^2 + y^2 - 2x_1(x + y) + x_1^2 = 0$

(c) $x^2 + y^2 = x_1^2 + y_1^2$ (d) $x^2 + y^2 + 2xx_1 + 2yy_1 = 0$

7. The equation of the circle which touches x-axis and whose centre is (1, 2), is **[MP PET 1984]**

(a) $x^2 + y^2 - 2x + 4y + 1 = 0$ (b) $x^2 + y^2 - 2x - 4y + 1 = 0$

(c) $x^2 + y^2 + 2x + 4y + 1 = 0$ (d) $x^2 + y^2 + 4x + 2y + 4 = 0$

8. The equation of the circle having centre (1, – 2) and passing through the point of intersection of lines $3x + y = 14$, $2x + 5y = 18$ is **[MP PET 1990]**

(a) $x^2 + y^2 - 2x + 4y - 20 = 0$ (b) $x^2 + y^2 - 2x - 4y - 20 = 0$

(c) $x^2 + y^2 + 2x - 4y - 20 = 0$ (d) $x^2 + y^2 + 2x + 4y - 20 = 0$

9. The equation of the circle passing through (4, 5) and having the centre at (2, 2), is **[MNR 1986; MP PET 1984; UPSEAT 2000]**

(a) $x^2 + y^2 + 4x + 4y - 5 = 0$ (b) $x^2 + y^2 - 4x - 4y - 5 = 0$

(c) $x^2 + y^2 - 4x = 13$ (d) $x^2 + y^2 - 4x - 4y + 5 = 0$

10. The equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line $y - 4x + 3 = 0$, is **[Rajasthan PET 1985; MP PET 1989]**

(a) $x^2 + y^2 + 4x - 10y + 25 = 0$ (b) $x^2 + y^2 - 4x - 10y + 25 = 0$

(c) $x^2 + y^2 - 4x - 10y + 16 = 0$ (d) $x^2 + y^2 - 14y + 8 = 0$

11. The equation of the circle passing through the points (0, 0), (0, b) and (a, b) is **[AMU 1978]**

(a) $x^2+y^2+ax+by=0$ (b) $x^2+y^2-ax+by=0$ (c) $x^2+y^2-ax-by=0$ (d) $x^2+y^2+ax-by=0$

12. The equation $ax^2+by^2+2hxy+2gx+2fy+c=0$ will represent a circle, if **[MNR 1979; MP PET 1988; Rajasthan PET 1997, 2003]**

(a) $a=b=0$ and $c=0$ (b) $f=g$ and $h=0$ (c) $a=b\neq 0$ and $h=0$ (d) $f=g$ and $c=0$

13. The equation of the circle whose diameters have the end points $(a, 0)$, $(0, b)$ is given by **[MP PET 1993]**

(a) $x^2+y^2-ax-by=0$ (b) $x^2+y^2+ax-by=0$ (c) $x^2+y^2-ax+by=0$ (d) $x^2+y^2+ax+by=0$

14. The equation of the circle which touches x-axis at (3, 0) and passes through (1, 4) is given by **[MP PET 1993]**

(a) $x^2+y^2-6x-5y+9=0$ (b) $x^2+y^2+6x+5y-9=0$

(c) $x^2+y^2-6x+5y-9=0$ (d) $x^2+y^2+6x-5y+9=0$

15. From three non-collinear points we can draw **[MP PET 1984; BIT Ranchi 1990]**

(a) Only one circle (b) Three circle (c) Infinite circles (d) No circle

16. Equation of a circle whose centre is origin and radius is equal to the distance between the lines $x = 1$ and $x = -1$ is **[MP PET 1984]**

(a) $x^2+y^2=1$ (b) $x^2+y^2=\sqrt{2}$ (c) $x^2+y^2=4$ (d) $x^2+y^2=-4$

17. If the centre of a circle is (2, 3) and a tangent is $x+y=1$, then the equation of this circle is **[Rajasthan PET 1985, 1989]**

(a) $(x-2)^2+(y-3)^2=8$ (b) $(x-2)^2+(y-3)^2=3$ (c) $(x+2)^2+(y+3)^2=2\sqrt{2}$ (d) $(x-2)^2+(y-3)^2=2\sqrt{2}$

18. $ax^2+2y^2+2bxy+2x-y+c=0$ represents a circle through the origin, if **[MP PET 1984]**

(a) $a=0, b=0, c=2$ (b) $a=1, b=0, c=0$ (c) $a=2, b=2, c=0$ (d) $a=2, b=0, c=0$

19. If the equation $\frac{K\,(x+1)^2}{3}+\frac{(y+2)^2}{4}=1$ represents a circle, then $K=$ **[MP PET 1994]**

(a) 3/4 (b) 1 (c) 4/3 (d) 12

20. A circle has radius 3 units and its centre lies on the line $y=x-1$. Then the equation of this circle if it passes through point (7, 3), is **[Roorkee 1988]**

(a) $x^2+y^2-8x-6y+16=0$ (b) $x^2+y^2+8x+6y+16=0$

(c) $x^2+y^2-8x-6y-16=0$ (d) None of these

21. The equation of circle whose diameter is the line joining the points (– 4, 3) and (12, –1) is **[IIT 1971; Rajasthan PET 1984, 87, 89; MP PET 1984; Roorkee 1969; AMU 1979]**

(a) $x^2+y^2+8x+2y+51=0$ (b) $x^2+y^2+8x-2y-51=0$

(c) $x^2+y^2+8x+2y-51=0$ (d) $x^2+y^2-8x-2y-51=0$

22. The equation of the circle which passes through the points (3, – 2) and (– 2, 0) and centre lies on the line $2x-y=3$, is **[Roorkee 1971]**

(a) $x^2+y^2-3x-12y+2=0$ (b) $x^2+y^2-3x+12y+2=0$

(c) $x^2+y^2+3x+12y+2=0$ (d) None of these

23. For $ax^2+2hxy+3y^2+4x+8y-6=0$ to represent a circle, one must have

(a) $a=3, h=0$ (b) $a=1, h=0$ (c) $a=h=3$ (d) $a=h=0$

24. The equation of the circle in the first quadrant which touches each axis at a distance 5 from the origin is **[MP PET 1997]**

(a) $x^2+y^2+5x+5y+25=0$ (b) $x^2+y^2-10x-10y+25=0$

(c) $x^2+y^2-5x-5y+25=0$ (d) $x^2+y^2+10x+10y+25=0$

25. If (α, β) is the centre of a circle passing through the origin, then its equation is **[MP PET 1999]**

(a) $x^2+y^2-\alpha x-\beta y=0$ (b) $x^2+y^2+2\alpha x+2\beta y=0$ (c) $x^2+y^2-2\alpha x-2\beta y=0$ (d) $x^2+y^2+\alpha x+\beta y=0$

26. The equation of the circle whose diameter lies on $2x+3y=3$ and $16x-y=4$ and which passes through (4, 6) is **[Kurukshetra CEE 1998]**

(a) $5(x^2+y^2)-3x-8y=200$ (b) $x^2+y^2-4x-8y=200$

(c) $5(x^2+y^2)-4x=200$ (d) $x^2+y^2=40$

27. The equation of the circle of radius 5 and touching the coordinate axes in third quadrant is **[EAMCET 2002]**

(a) $(x-5)^2+(y+5)^2=25$ (b) $(x+4)^2+(y+4)^2=25$ (c) $(x+6)^2+(y+6)^2=25$ (d) $(x+5)^2+(y+5)^2=25$

28. The centre of a circle is (2, – 3) and the circumference is 10π. Then the equation of the circle is **[Kerala (Engg.) 2002]**

(a) $x^2+y^2+4x+6y+12=0$ (b) $x^2+y^2-4x+6y+12=0$

(c) $x^2+y^2-4x+6y-12=0$ (d) $x^2+y^2-4x-6y-12=0$

29. The circle described on the line joining the points (0, 1), (a, b) as diameter cuts the x-axis in points whose abscissae are roots of the equation

(a) $x^2+ax+b=0$ (b) $x^2-ax+b=0$ (c) $x^2+ax-b=0$ (d) $x^2-ax-b=0$.

30. Four distinct points $(2k, 3k)$, (1, 0), (0, 1) and (0, 0) lie on a circle for

(a) All integral values of k (b) $0<k<1$ (c) $k<0$ (d) For two values of k

31. The equations of the circles which touch both the axes and the line $x = a$ are

(a) $x^2+y^2\pm ax\pm ay+\frac{a^2}{4}=0$ (b) $x^2+y^2+ax\pm ay+\frac{a^2}{4}=0$

(c) $x^2+y^2-ax\pm ay+\frac{a^2}{4}=0$ (d) None of these.

32. The equation of the unit circle concentric with $x^2+y^2+8x+4y-8=0$ is **[EAMCET 1991]**

(a) $x^2+y^2-8x+4y-8=0$ (b) $x^2+y^2-8x+4y+8=0$

(c) $x^2+y^2-8x+4y-28=0$ (d) $x^2+y^2-8x+4y+19=0$

33. A circle of radius 2 touches the coordinate axes in the first quadrant. If the circle makes a complete rotation on the x-axis along the positive direction of the x-axis then the equation of the circle in the new position is

(a) $x^2+y^2-4(x+y)-8\pi x+(2+4\pi)^2=0$ (b) $x^2+y^2-4x-4y+(2+4\pi)^2=0$

(c) $x^2+y^2-8\pi x-4y+(2+4\pi)^2=0$ (d) None of these

34. A circle which touches the axes and whose centre is at distance $2\sqrt{2}$ from the origin, has the equation

(a) $x^2+y^2-4x+4y+4=0$ (b) $x^2+y^2+4x-4y+4=0$

(c) $x^2+y^2+4x+4y+4=0$ (d) None of these

35. If (– 1, 4) and (3, – 2) are end points of a diameter of a circle, then the equation of this circle is **[Rajasthan PET 1987, 89]**

(a) $(x-1)^2+(y-1)^2=13$ (b) $(x+1)^2+(y+1)^2=13$ (c) $(x-1)^2+(y+1)^2=13$ (d) $(x+1)^2+(y-1)^2=13$

36. The equation of the circle concentric with the circle $x^2+y^2-3x+4y-c=0$ and passing through the point (– 1, – 2) is

[Rajasthan PET 1984, 92]

(a) $x^2+y^2-3x+4y-1=0$ (b) $x^2+y^2-3x+4y=0$

(c) $x^2+y^2-3x+4y+2=0$ (d) None of these

37. If (– 3, 2) lies on the circle $x^2+y^2+2gx+2fy+c=0$ which is concentric with $x^2+y^2+6x+8y-5=0$, then c is equal to

[Rajasthan PET 1986]

(a) – 11 (b) 11 (c) – 24 (d) 24

38. Equation $x^2+y^2+4x+6y+13=0$ represents **[Roorkee 1990]**

(a) A circle (b) A pair of two different lines (c) A pair of coincident lines (d) A point

39. If the lines $2x+3y+1=0$ and $3x-y-4=0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is

[AIEEE 2004]

(a) $x^2+y^2+2x-2y-23=0$ (b) $x^2+y^2-2x-2y-23=0$

(c) $x^2+y^2+2x+2y-23=0$ (d) $x^2+y^2-2x+2y-23=0$

Advance Level

40. $y = mx$ is a chord of a circle of radius a and the diameter of the circle lies along x-axis and one end of this chord is origin. The equation of the circle described on this chord as diameter is **[MP PET 1990]**

(a) $(1+m^2)(x^2+y^2)-2ax=0$ (b) $(1+m^2)(x^2+y^2)-2a(x+my)=0$

(c) $(1+m^2)(x^2+y^2)+2a(x+my)=0$ (d) $(1+m^2)(x^2+y^2)-2a(x-my)=0$

41. If $y = 2x$ is a chord of the circle $x^2+y^2-10x=0$, then the equation of the circle of which this chord is a diameter, is

[Rajasthan PET 1988]

(a) $x^2+y^2-2x+4y=0$ (b) $x^2+y^2+2x+4y=0$ (c) $x^2+y^2+2x-4y=0$ (d) $x^2+y^2-2x-4y=0$

42. The circle on the chord $x\cos\alpha + y\sin\alpha = p$ of the circle $x^2+y^2=a^2$ as diameter has the equation **[Roorkee 1967; MP PET 1993]**

(a) $x^2+y^2-a^2-2p(x\cos\alpha+y\sin\alpha-p)=0$ (b) $x^2+y^2+a^2+2p(x\cos\alpha-y\sin\alpha+p)=0$

(c) $x^2+y^2-a^2+2p(x\cos\alpha+y\sin\alpha+p)=0$ (d) $x^2+y^2-a^2-2p(x\cos\alpha-y\sin\alpha-p)=0$

43. The equation of circle which touches the axes of coordinates and the line $\frac{x}{3}+\frac{y}{4}=1$ and whose centre lies in the first quadrant is $x^2+y^2-2cx-2cy+c^2=0$, where c is **[Ranchi BIT 1986; Kurukshetra CEE 1996]**

(a) 1 (b) 2 (c) 3 (d) 6

44. The equation of a circle which touches both axes and the line $3x-4y+8=0$ and lies in the third quadrant is **[MP PET 1986]**

(a) $x^2+y^2-4x+4y-4=0$ (b) $x^2+y^2-4x+4y+4=0$

(c) $x^2+y^2+4x+4y+4=0$ (d) $x^2+y^2-4x-4y-4=0$

45. Equation of the circle which touches the lines $x=0$, $y=0$ and $3x+4y=4$ is **[MP PET 1991]**

(a) $x^2-4x+y^2+4y+4=0$ (b) $x^2-4x+y^2-4y+4=0$

(c) $x^2+4x+y^2+4y+4=0$ (d) $x^2+4x+y^2-4y+4=0$

46. The equation of the circumcircle of the triangle formed by the lines $y+\sqrt{3}x=6$, $y-\sqrt{3}x=6$ and $y=0$, is **[EAMCET 1982]**

(a) $x^2+y^2-4y=0$ (b) $x^2+y^2+4x=0$ (c) $x^2+y^2-4y=12$ (d) $x^2+y^2+4x=12$

47. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis. The locus of the other end of the diameter through A is

[AIEEE 2004]

(a) $(y-q)^2=4px$ (b) $(x-q)^2=4py$ (c) $(y-p)^2=4qx$ (d) $(x-p)^2=4qy$

48. If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x-y+1=0$ and $x-2y+3=0$, then the value of λ is **[IIT 1991]**

(a) 1 (b) 2 (c) 3 (d) 4

49. Equation to the circles which touch the lines $3x-4y+1=0$, $4x+3y-7=0$ and pass through (2, 3) are **[EAMCET 1989]**

(a) $(x-2)^2+(y-8)^2=25$ (b) $5x^2+5y^2-12x-24y+31=0$

(c) Both (a) and (b) (d) None of these

50. The equation of the circle which passes through (1, 0) and (0, 1) and has its radius as small as possible, is

(a) $x^2+y^2-2x-2y+1=0$ (b) $x^2+y^2-x-y=0$

(c) $2x^2+2y^2-3x-3y+1=0$ (d) $x^2+y^2-3x-3y+2=0$

51. The centres of a set of circles, each of radius 3, lie on the circle $x^2+y^2=25$. The locus of any point in the set is **[AIEEE 2002]**

(a) $4\le x^2+y^2\le 64$ (b) $x^2+y^2\le 25$ (c) $x^2+y^2\ge 25$ (d) $3\le x^2+y^2\le 9$

52. The equation of the circle which touches both the axes and the straight line $4x+3y=6$ in the first quadrant and lies below it is

[Roorkee 1992]

(a) $4x^2+4y^2-4x-4y+1=0$ (b) $x^2+y^2-6x-6y+9=0$

(c) $x^2+y^2-6x-y+9=0$ (d) $4(x^2+y^2-x-6y)+1=0$

53. Three sides of a triangle have the equations $L_r \equiv y-m_r x-c_r=0;\ r=1,2,3.$ Then $\lambda L_2L_3+\mu L_3L_1+vL_1L_2=0,$ where $\lambda\neq 0,\ \mu\neq 0,\ v\neq 0,$ is the equation of the circumcircle of the triangle, if

(a) $\lambda(m_2+m_3)+\mu(m_3+m_1)+v(m_1+m_2)=0$ (b) $\lambda(m_2m_3-1)+\mu(m_3m_1-1)+v(m_1m_2-1)=0$

(c) Both (a) and (b) hold together (d) None of these

54. The equation of the circle passing through the point (1, 1) and having two diameters along the pair of lines $x^2-y^2-2x+4y-3=0$ is

(a) $x^2+y^2-2x-4y+4=0$ (b) $x^2+y^2+2x+4y-4=0$

(c) $x^2+y^2-2x+4y+4=0$ (d) None of these

55. The equation of a circle which touches x-axis and the line $4x-3y+4=0,$ its centre lying in the third quadrant and lies on the line $x-y-1=0,$ is

(a) $9(x^2+y^2)+6x+24y+1=0$ (b) $9(x^2+y^2)-6x-24y+1=0$

(c) $9(x^2+y^2)-6x+2y+1=0$ (d) None of these

56. Two vertices of an equilateral triangle are (– 1, 0) and (1, 0) and its third vertex lies above the x-axis. The equation of the circumcircle of the triangle is

(a) $x^2+y^2=1$ (b) $\sqrt{3}(x^2+y^2)+2y-\sqrt{3}=0$ (c) $\sqrt{3}(x^2+y^2)-2y-\sqrt{3}=0$ (d) None of these

57. A triangle is formed by the lines whose combined equation is given by $(x+y-4)(xy-2x-y+2)=0.$ The equation of its circumcircle is

(a) $x^2+y^2-5x-3y+8=0$ (b) $x^2+y^2-3x-5y+8=0$

(c) $x^2+y^2-3x-5y-8=0$ (d) None of these

58. If the centroid of an equilateral triangle is (1, 1) and its one vertex is (– 1, 2) then the equation of its circumcircle is

(a) $x^2+y^2-2x-2y-3=0$ (b) $x^2+y^2+2x-2y-3=0$

(c) $x^2+y^2+2x+2y-3=0$ (d) None of these

59. The equation of the circle whose one diameter is PQ, where the ordinates of P, Q are the roots of the equation $x^2+2x-3=0$ and the abscissae are the roots of the equation $y^2+4y-12=0,$ is

(a) $x^2+y^2+2x+4y-15=0$ (b) $x^2+y^2-4x-2y-15=0$

(c) $x^2+y^2+4x+2y-15=0$ (d) None of these

60. The equation of the circumcircle of an equilateral triangle is $x^2+y^2+2gx+2fy+c=0$ and one vertex of the triangle is (1, 1). The equation of incircle of the triangle is

(a) $4(x^2+y^2)=g^2+f^2$ (b) $4(x^2+y^2)+8gx+8fy=(1-g)(1+3g)+(1-f)(1+3f)$

(c) $4(x^2+y^2)+8gx+8fy=g^2+f^2$ (d) None of these

61. The equation of the circle of radius $2\sqrt{2}$ whose centre lies on the line $x-y=0$ and which touches the line $x+y=4$, and whose centre's coordinates satisfy the inequality $x+y>4$ is

(a) $x^2+y^2-8x-8y+24=0$ (b) $x^2+y^2=8$

(c) $x^2+y^2-8x+8y=24$ (d) None of these

62. The circumcircle of the quadrilateral formed by the lines $x=a,\ x=2a,\ y=-a,\ y=\sqrt{2}a$ is

(a) $x^2+y^2+3ax+a^2=0$ (b) $x^2+y^2-3ax-a^2=0$ (c) $x^2+y^2-3ax+2a^2=0$ (d) $x^2+y^2+3ax-a^2=0$

63. Equation of a circle $S(x,y)=0,\ S(2,\ 3)=16$, which touches the line $3x+4y-7=0$ at (1, 1) is given by

(a) $x^2+y^2+x+2y-5=0$ (b) $x^2+y^2+2x+2y-6=0$ (c) $x^2+y^2+4x-6y=0$ (d) None of these

Centre and Radius of a Circle

Basic Level

64. The area of the circle whose centre is at (1, 2) and which passes through the point (4, 6) is **[MNR 1982; IIT 1980; Karnataka CET 1999; MP PET 2002; DCE 2000]**

(a) 5π (b) 10π (c) 25π (d) None of these

65. The centres of the circles $x^2+y^2=1$, $x^2+y^2+6x-2y=1$ and $x^2+y^2-12x+4y=1$ are **[MP PET 1986]**

(a) Same (b) Collinear (c) Non-collinear (d) None of these

66. If a circle passes through the point (0, 0), (*a*, 0), (0, *b*), then its centre is **[MNR 1975]**

(a) (a, b) (b) (b, a) (c) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (d) $\left(\frac{b}{2}, -\frac{a}{2}\right)$

67. If the radius of the circle $x^2+y^2-18x+12y+k=0$ be 11, then $k=$ **[MP PET 1987]**

(a) 347 (b) 4 (c) – 4 (d) 49

68. The centre and radius of the circle $2x^2+2y^2-x=0$ are **[MP PET 1984, 87]**

(a) $\left(\frac{1}{4}, 0\right)$ and $\frac{1}{4}$ (b) $\left(-\frac{1}{2}, 0\right)$ and $\frac{1}{2}$ (c) $\left(\frac{1}{2}, 0\right)$ and $\frac{1}{2}$ (d) $\left(0, -\frac{1}{4}\right)$ and $\frac{1}{4}$

69. Centre of the circle $(x-3)^2+(y-4)^2=5$ is **[MP PET 1988]**

(a) (3, 4) (b) (– 3, – 4) (c) (4, 3) (d) (– 4, – 3)

70. A circle has its equation in the form $x^2+y^2+2x+4y+1=0$. Choose the correct coordinates of its centre and the right value of its radius from the following **[MP PET 1982]**

(a) Centre (– 1, –2), radius = 2 (b) Centre (2, 1), radius = 1

(c) Centre (1, 2), radius = 3 (d) Centre (– 1, 2), radius = 2

71. A circle touches the axes at the points (3, 0) and (0, – 3). The centre of the circle is **[MP PET 1992]**

(a) (3, – 3) (b) (0, 0) (c) (– 3, 0) (d) (6, – 6)

72. Radius of the circle $x^2+y^2+2x\cos\theta+2y\sin\theta-8=0$, is **[MNR 1974]**

(a) 1 (b) 3 (c) $2\sqrt{3}$ (d) $\sqrt{10}$

73. The area of a circle whose centre is (*h*, *k*) and radius *a* is **[MP PET 1994]**

(a) $\pi(h^2+k^2-a^2)$ (b) $\pi a^2 hk$ (c) πa^2 (d) None of these

74. If the coordinates of one end of the diameter of the circle $x^2+y^2-8x-4y+c=0$ are (– 3, 2), then the coordinates of other end are **[Roorkee 1995]**

(a) (5, 3) (b) (6, 2) (c) (1, – 8) (d) (11, 2)

75. The centre of the circle $x=-1+2\cos\theta$, $y=3+2\sin\theta$, is **[MP PET 1995]**

(a) (1, – 3) (b) (– 1, 3) (c) (1, 3) (d) None of these

76. If $g^2+f^2=c$, then the equation $x^2+y^2+2gx+2fy+c=0$ will represent **[MP PET 2003]**

(a) A circle of radius *g* (b) A circle of radius *f* (c) A circle of diameter $\sqrt{c}$ (d) A circle of radius 0

77. The centre of circle inscribed in square formed by the lines $x^2-8x+12=0$ and $y^2-14y+45=0$, is **[IIT Screening 2003]**

(a) (4, 7) (b) (7, 4) (c) (9, 4) (d) (4, 9)

78. The equation $x^2+y^2+2gx+2fy+c=0$ will represent a real circle if

(a) $g^2+f^2-c<0$ (b) $g^2+f^2-c\ge 0$ (c) Always (d) None of these

79. One of the diameters of the circle $x^2+y^2-12x+4y+6=0$ is given by

(a) $x+y=0$ (b) $x+3y=0$ (c) $x=y$ (d) $3x+2y=0$

80. The radius of the circle passing through the point (6, 2) two of whose diameters are $x+y=6$ and $x+2y=4$ is **[BIT Ranchi 1993]**

(a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4

81. If the equation of a circle is $ax^2+(2a-3)y^2-4x-1=0$ then its centre is

(a) (2, 0) (b) (2/3, 0) (c) (– 2/3, 0) (d) None of these

82. If $2(x^2+y^2)+4\lambda x+\lambda^2=0$ represents a circle of meaningful radius then the range of real values of λ is

(a) R (b) $(0, +\infty)$ (c) $(-\infty, 0)$ (d) None of these

83. The locus of the centres of the circles for which one end of a diameter is (1, 1) while the other end is on the line $x + y = 3$ is

(a) $x + y = 1$ (b) $2(x - y) = 5$ (c) $2x + 2y = 5$ (d) None of these

84. If A and B are two points on the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ which are farthest and nearest respectively from the point (7, 2) then

(a) $A = (2 - 2\sqrt{2},\ -3 - 2\sqrt{2})$ (b) $B = (2 + 2\sqrt{2},\ -3 + 2\sqrt{2})$

(c) $A = (2 + 2\sqrt{2},\ -3 + 2\sqrt{2})$ (d) $B = (2 - 2\sqrt{2},\ -3 + 2\sqrt{2})$

85. The radius of the circle passing through the point (5, 4) and concentric to the circle $x^2 + y^2 - 8x - 12y + 15 = 0$ is

(a) 5 (b) $\sqrt{5}$ (c) 10 (d) $\sqrt{10}$

86. The length of the radius of the circle $x^2 + y^2 + 4x - 6y = 0$ is **[Rajasthan PET 1995]**

(a) $\sqrt{11}$ (b) 12 (c) $\sqrt{13}$ (d) $\sqrt{14}$

87. $(2, y)$ is the centre of a circle. If $(x, 3)$ and (3, 5) are end points of a diameter of this circle, then **[Roorkee 1986]**

(a) $x = 1, y = 4$ (b) $x = 4, y = 1$ (c) $x = 8, y = 2$ (d) None of these

88. The greatest distance of the point P (10, 7) from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is

(a) 5 (b) 15 (c) 10 (d) None of these

89. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ be (3, 4), then the other end is **[MP PET 1986; BIT Ranchi 1991]**

(a) (0, 0) (b) (1, 1) (c) (1, 2) (d) (2, 1)

Advance Level

90. If $2x - 4y = 9$ and $6x - 12y + 7 = 0$ are the tangents of same circle, then its radius will be **[Roorkee 1995]**

(a) $\frac{\sqrt{3}}{5}$ (b) $\frac{17}{6\sqrt{5}}$ (c) $\frac{2\sqrt{5}}{3}$ (d) $\frac{17}{3\sqrt{5}}$

91. If $5x - 12y + 10 = 0$ and $12y - 5x + 16 = 0$ are two tangents to a circle, then the radius of the circle is **[EAMCET 2003]**

(a) 1 (b) 2 (c) 4 (d) 6

92. If $2x^2 + \lambda xy + 2y^2 + (\lambda - 4)x + 6y - 5 = 0$ is the equation of a circle then its radius is

(a) $3\sqrt{2}$ (b) $2\sqrt{3}$ (c) $2\sqrt{2}$ (d) None of these

93. C_1 is a circle of radius 1 touching the x-axis and the y-axis. C_2 is another circle of radius >1 and touching the axes as well as the circle C_1. Then the radius of C_2 is

(a) $3 - 2\sqrt{2}$ (b) $3 + 2\sqrt{2}$ (c) $3 + 2\sqrt{3}$ (d) None of these

94. If p and q be the longest distance and the shortest distance respectively of the point (– 7, 2) from any point (α, β) on the curve whose equation is $x^2 + y^2 - 10x - 14y - 51 = 0$ then GM of p and q is equal to

(a) $2\sqrt{11}$ (b) $5\sqrt{5}$ (c) 13 (d) None of these

95. The equation of a circle is $x^2 + y^2 = 4$. The centre of the smallest circle touching this circle and the line $x + y = 5\sqrt{2}$ has the coordinates

(a) $\left(\frac{7}{2\sqrt{2}}, \frac{7}{2\sqrt{2}}\right)$ (b) $\left(\frac{3}{2}, \frac{3}{2}\right)$ (c) $\left(-\frac{7}{2\sqrt{2}}, -\frac{7}{2\sqrt{2}}\right)$ (d) None of these

96. A circle touches the line $2x - y - 1 = 0$ at the point (3, 5). If its centre lies on the line $x + y = 5$ then the centre of that circle is **[Rajasthan PET 1992]**

(a) (3, 2) (b) (– 3, 8) (c) (4, 1) (d) (8, – 3)

97. The locus of the centre of the circle $(x\cos\theta + y\sin\theta - a)^2 + (x\sin\theta - y\cos\theta + a)^2 = a^2$ is

(a) $x^2 + y^2 = a^2$ (b) $x^2 + y^2 = 2a^2$ (c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 - 2ax - 2ay + a^2 = 0$

98. If a circle $S(x, y) = 0$ touches at the point (2, 3) of the line $x + y = 5$ and $S(1, 2) = 0$, then radius of such circle

(a) 2 units (b) 4 units (c) $\frac{1}{2}$ units (d) $\frac{1}{\sqrt{2}}$ units

Intersection of a Line and a Circle

Basic Level

99. A circle touches the y-axis at the point (0, 4) and cuts the x-axis in a chord of length 6 units. The radius of the circle is **[MP PET 1992]**

(a) 3 (b) 4 (c) 5 (d) 6

100. The radius of a circle which touches y-axis at (0, 3) and cuts intercept of 8 units with x-axis, is **[IIT 1972]**

(a) 3 (b) 2 (c) 5 (d) 8

101. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle with AB as a diameter is **[IIT 1996]**

(a) $x^2 + y^2 - x - y = 0$ (b) $x^2 + y^2 - 2x - y = 0$ (c) $x^2 + y^2 - x + y = 0$ (d) $x^2 + y^2 + x - y = 0$

102. The circle $x^2 + y^2 - 3x - 4y + 2 = 0$ cuts x-axis at **[Karnataka CET 2001]**

(a) (2, 0), (– 3, 0) (b) (3, 0), (4, 0) (c) (1, 0), (– 1, 0) (d) (1, 0), (2, 0)

103. If the line $y = x + 3$ meets the circle $x^2 + y^2 = a^2$ at A and B, then the equation of the circle having AB as a diameter will be **[Rajasthan PET 1988]**

(a) $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$ (b) $x^2 + y^2 - 3x + 3y - a^2 + 9 = 0$

(c) $x^2 + y^2 + 3x + 3y - a^2 + 9 = 0$ (d) None of these

104. If the circle $x^2 + y^2 + 2ax + 8y + 16 = 0$ touches x-axis, then the value of a is **[Rajasthan PET 1994]**

(a) ± 16 (b) ± 4 (c) ± 8 (d) ± 1

105. The length of the intercept made by the circle $x^2 + y^2 = 1$ on the line $x + y = 1$ is

(a) 2 (b) $\sqrt{2}$ (c) $1/\sqrt{2}$ (d) $2\sqrt{2}$

106. The AM of the abscissae of points of intersection of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ with x-axis is

(a) g (b) $-g$ (c) f (d) $-f$

107. The straight line $(x-2) + (y+3) = 0$ cuts the circle $(x-2)^2 + (y-3)^2 = 11$ at **[MNR 1975]**

(a) No points (b) One point (c) Two points (d) None of these

108. The equation of a circle whose centre is (3, – 1) and which cuts off a chord of length 6 on the line $2x - 5y + 18 = 0$ **[Roorkee 1977]**

(a) $(x-3)^2 + (y+1)^2 = 38$ (b) $(x+3)^2 + (y-1)^2 = 38$ (c) $(x-3)^2 + (y+1)^2 = \sqrt{38}$ (d) None of these

109. The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are **[IIT 1983]**

(a) (– 2, – 6), (4, 2) (b) (2, 6), (– 4, – 2) (c) (– 2, 6), (– 4, 2) (d) None of these

110. The line $y = mx + c$ intersects the circle $x^2 + y^2 = r^2$ at two real distinct points, if

(a) $-r\sqrt{1+m^2} < c \le 0$ (b) $0 \le c < r\sqrt{1+m^2}$ (c) (a) and (b) both (d) $-c\sqrt{1-m^2} < r$

111. A line through (0, 0) cuts the circle $x^2 + y^2 - 2ax = 0$ at A and B, then locus of the centre of the circle drawn AB as diameter is **[Rajasthan PET 2002]**

(a) $x^2 + y^2 - 2ay = 0$ (b) $x^2 + y^2 + ay = 0$ (c) $x^2 + y^2 + ax = 0$ (d) $x^2 + y^2 - ax = 0$

112. If the line $y - 1 = m(x-1)$ cuts the circle $x^2 + y^2 = 4$ at two real points then the number of possible values of m is

(a) 1 (b) 2 (c) Infinite (d) None of these

113. The *GM* of the abscissae of the points of intersection of the circle $x^2+y^2-4x-6y+7=0$ and the line $y=1$ is

(a) $\sqrt{7}$ (b) $\sqrt{2}$ (c) $\sqrt{14}$ (d) 1

114. The equation(s) of the tangent at the point (0, 0) to the circle, making intercepts of length $2a$ and $2b$ units on the coordinate axes, is (are)

(a) $ax+by=0$ (b) $ax-by=0$ (c) $x=y$ (d) None of these

Advance Level

115. A circle which passes through origin and cuts intercepts on axes a and b, the equation of circle is **[Rajasthan PET 1991]**

(a) $x^2+y^2-ax-by=0$ (b) $x^2+y^2+ax+by=0$ (c) $x^2+y^2-ax+by=0$ (d) $x^2+y^2+ax-by=0$

116. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x+y=1$. If the intercepts made by the circle $x^2+y^2-x+3y=0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1 **[IIT 1999]**

(a) $x+y=0$ (b) $x-y=0$ (c) $x+7y=0$ (d) $x-7y=0$

117. The two lines through (2, 3) from which the circle $x^2+y^2=25$ intercepts chords of length 8 units have equations

(a) $2x+3y=13,\ x+5y=17$ (b) $y=3,\ 12x+5y=39$

(c) $x=2,\ 9x-11y=51$ (d) None of these

118. Circles are drawn through the point (2, 0) to cut intercepts of length 5 units on the x-axis. If their centres lie in the first quadrant, then their equation is **[Roorkee 1992]**

(a) $x^2+y^2-9x+2ky+14=0$ (b) $3x^2+3y^2+27x-2ky+42=0$

(c) $x^2+y^2-9x-2ky+14=0$ (d) $x^2+y^2-2kx-9y+14=0$

119. A circle touches the y-axis at (0, 2) and has an intercept of 4 units on the positive side of the x-axis. Then the equation of the circle is **[IIT 1995]**

(a) $x^2+y^2-4(\sqrt{2}x+y)+4=0$ (b) $x^2+y^2-4(x+\sqrt{2}y)+4=0$

(c) $x^2+y^2-2(\sqrt{2}x+y)+4=0$ (d) None of these

120. Circles are drawn through the point (3, 0) to cut an intercept of length 6 units on the negative direction of the x-axis. The equation of the locus of their centres is

(a) The x-axis (b) $x-y=0$ (c) The y-axis (d) $x+y=0$

121. Circles $x^2+y^2=1$ and $x^2+y^2-8x+11=0$ cut off equal intercepts on a line through the point $\left(-2, \frac{1}{2}\right)$. The slope of the line is

(a) $\frac{-1+\sqrt{29}}{14}$ (b) $\frac{1+\sqrt{7}}{4}$ (c) $\frac{-1-\sqrt{29}}{14}$ (d) None of these

122. If $2l$ be the length of the intercept made by the circle $x^2+y^2=a^2$ on the line $y=mx+c$, then c^2 is equal to

(a) $(1+m^2)(a^2+l^2)$ (b) $(1+m^2)(a^2-l^2)$ (c) $(1-m^2)(a^2+l^2)$ (d) $(1-m^2)(a^2-l^2)$

123. For the circle $x^2+y^2+4x-7y+12=0$ the following statement is true

(a) The length of tangent from (1, 2) is 7 (b) Intercept on y-axis is 2

(c) Intercept on x-axis is $2-\sqrt{2}$ (d) None of these

124. The length of the chord joining the points in which the straight line $\frac{x}{3}+\frac{y}{4}=1$ cuts the circle $x^2+y^2=\frac{169}{25}$ is **[Orissa JEE 2003]**

(a) 1 (b) 2 (c) 4 (d) 8

125. A line is drawn through a fixed point $P(\alpha, \beta)$ to cut the circle $x^2+y^2=r^2$ at A and B. Then $PA \,.\, PB$ is equal to

(a) $(\alpha+\beta)^2-r^2$ (b) $\alpha^2+\beta^2-r^2$ (c) $(\alpha-\beta)^2+r^2$ (d) None of these

126. The range of values of m for which the line $y = mx + 2$ cuts the circle $x^2 + y^2 = 1$ at distinct or coincident points is

(a) $(-\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty)$ (b) $[-\sqrt{3}, \sqrt{3}]$ (c) $[\sqrt{3}, +\infty)$ (d) None of these

Position of a point w.r.t. a Circle

Basic Level

127. A point inside the circle $x^2 + y^2 + 3x - 3y + 2 = 0$ is **[MP PET 1988]**

(a) $(-1, 3)$ (b) $(-2, 1)$ (c) $(2, 1)$ (d) $(-3, 2)$

128. Position of the point (1, 1) with respect to the circle $x^2 + y^2 - x + y - 1 = 0$ is **[MP PET 1986, 1990]**

(a) Outside the circle (b) Upon the circle (c) Inside the circle (d) None of these

129. The number of tangents that can be drawn from (0, 0) to the circle $x^2 + y^2 + 2x + 6y - 15 = 0$ is **[MP PET 1992]**

(a) None (b) One (c) Two (d) Infinite

130. The number of tangents which can be drawn from the point (– 1, 2) to the circle $x^2 + y^2 + 2x - 4y + 4 = 0$ is **[BIT Ranchi 1991]**

(a) 1 (b) 2 (c) 3 (d) 0

131. The point (0.1, 3.1) with respect to the circle $x^2 + y^2 - 2x - 4y + 3 = 0$, is **[MNR 1980]**

(a) At the centre of the circle (b) Inside the circle but not at the centre

(c) On the circle (d) Outside the circle

132. The number of the tangents that can be drawn from (1, 2) to $x^2 + y^2 = 5$ is

(a) 1 (b) 2 (c) 3 (d) 0

133. The number of points on the circle $2x^2 + 2y^2 - 3x = 0$ which are at a distance 2 from the point (– 2, 1) is

(a) 2 (b) 0 (c) 1 (d) None of these

134. If $x^2 + y^2 - 6x + 8y - 11 = 0$ is a given circle and (0, 0), (1, 8) are two points, then

(a) Both the points are inside the circle (b) Both the points are outside the circle

(c) One point is on the circle another is outside the circle (d) One point is inside and another is outside the circle

Advance Level

135. A region in the x-y plane is bounded by the curve $y = \sqrt{25 - x^2}$ and the line $y = 0$. If the point $(a, a+1)$ lies in the interior of the region, then

(a) $a \in (-4, 3)$ (b) $a \in (-\infty, -1) \cup (3, +\infty)$ (c) $a \in (-1, 3)$ (d) None of these

136. If (2, 4) is a point interior to the circle $x^2 + y^2 - 6x - 10y + \lambda = 0$ and the circle does not cut the axes at any point , then λ belongs to the interval

(a) (25, 32) (b) (9, 32) (c) $(32, +\infty)$ (d) None of these

137. The range of values of $\theta \in [0, 2\pi]$ for which $(1 + \cos\theta, \sin\theta)$ is an interior point of the circle $x^2 + y^2 = 1$ is

(a) $(\pi/6, 5\pi/6)$ (b) $(2\pi/3, 5\pi/3)$ (c) $(\pi/6, 7\pi/6)$ (d) $(2\pi/3, 4\pi/3)$

138. The range of values of r for which the point $\left(-5 + \frac{r}{\sqrt{2}}, -3 + \frac{r}{\sqrt{2}}\right)$ is an interior point of the major segment of the circle $x^2 + y^2 = 16$, cut off by the line $x + y = 2$ is

(a) $(-\infty, 5\sqrt{2})$ (b) $(4\sqrt{2} - \sqrt{14}, 5\sqrt{2})$ (c) $(4\sqrt{2} - \sqrt{14}, 4\sqrt{2} + \sqrt{14})$ (d) None of these

139. If P (2, 8) is an interior point of a circle $x^2 + y^2 - 2x + 4y - p = 0$ which neither touches nor intersects the axes, then set for p is

(a) $p < -1$ (b) $p < -4$ (c) $p > 96$ (d) ϕ

Equation of Tangent, Condition for Tangency and the points of Contact

Basic Level

140. The equation of the tangent to the circle $x^2 + y^2 = r^2$ at (a, b) is $ax + by - \lambda = 0$, where λ is

(a) a^2 (b) b^2 (c) r^2 (d) None of these

141. $x = 7$ touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$, then the coordinates of the point of contact are **[MP PET 1996]**

(a) (7, 3) (b) (7, 4) (c) (7, 8) (d) (7, 2)

142. A circle with centre (a, b) passes through the origin. The equation of the tangent to the circle at the origin is **[Rajasthan PET 2000]**

(a) $ax - by = 0$ (b) $ax + by = 0$ (c) $bx - ay = 0$ (d) $bx + ay = 0$

143. If the tangent at a point $P(x, y)$ of a curve is perpendicular to the line that joins origin with the point P, then the curve is **[MP PET 1998]**

(a) Circle (b) Parabola (c) Ellipse (d) Straight line

144. The circle $x^2 + y^2 - 8x + 4y + 4 = 0$ touches **[Karnataka CET 1999]**

(a) x-axis only (b) y-axis only (c) Both x and y-axis (d) Does not touch any axis

145. The condition that the line $x\cos\alpha + y\sin\alpha = p$ may touch the circle $x^2 + y^2 = a^2$ is **[AMU 1999]**

(a) $p = a\cos\alpha$ (b) $p = a\tan\alpha$ (c) $p^2 = a^2$ (d) $p\sin\alpha = a$

146. The equation of circle with centre (1, 2) and tangent $x + y - 5 = 0$ is **[MP PET 2001]**

(a) $x^2 + y^2 + 2x - 4y + 6 = 0$ (b) $x^2 + y^2 - 2x - 4y + 3 = 0$

(c) $x^2 + y^2 - 2x + 4y + 8 = 0$ (d) $x^2 + y^2 - 2x - 4y + 8 = 0$

147. The equation of tangent to the circle $x^2 + y^2 = a^2$ parallel to $y = mx + c$ is **[Rajasthan PET 2001]**

(a) $y = mx \pm \sqrt{1 + m^2}$ (b) $y = mx \pm a\sqrt{1 + m^2}$ (c) $x = my \pm a\sqrt{1 + m^2}$ (d) None of these

148. The line $3x - 2y = k$ meets the circle $x^2 + y^2 = 4r^2$ at only one point, if $k^2 =$ **[Karnataka CET 2003]**

(a) $20r^2$ (b) $52r^2$ (c) $\frac{52}{9}r^2$ (d) $\frac{20}{9}r^2$

149. The line $lx + my + n = 0$ will be a tangent to the circle $x^2 + y^2 = a^2$ if **[MNR 1974; AMU 1981]**

(a) $n^2(l^2 + m^2) = a^2$ (b) $a^2(l^2 + m^2) = n^2$ (c) $n(l + m) = a$ (d) $a(l + m) = n$

150. The circle $x^2 + y^2 + 4x - 4y + 4 = 0$ touches **[MP PET 1988]**

(a) x-axis (b) y-axis (c) x-axis and y-axis (d) None of these

151. If the line $lx + my = 1$ be a tangent to the circle $x^2 + y^2 = a^2$, then the locus of the point (l, m) is **[MNR 1978; Rajasthan PET 1997]**

(a) A straight line (b) A circle (c) A parabola (d) An ellipse

152. The straight line $x - y - 3 = 0$ touches the circle $x^2 + y^2 - 4x + 6y + 11 = 0$ at the point whose coordinates are **[MP PET 1993]**

(a) (1, – 2) (b) (1, 2) (c) (– 1, 2) (d) (– 1, – 2)

153. If the straight line $y = mx + c$ touches the circle $x^2 + y^2 - 4y = 0$, then the value of c will be **[Rajasthan PET 1988]**

(a) $1 + \sqrt{1 + m^2}$ (b) $1 - \sqrt{m^2 + 1}$ (c) $2(1 + \sqrt{1 + m^2})$ (d) $2 + \sqrt{1 + m^2}$

154. At which point on y-axis the line $x = 0$ is a tangent to circle $x^2 + y^2 - 2x - 6y + 9 = 0$ **[Rajasthan PET 1984]**

(a) (0, 1) (b) (0, 2) (c) (0, 3) (d) (0, 4)

155. At which point the line $y = x + \sqrt{2}a$ touches to the circle $x^2 + y^2 = a^2$

or

Line $y = x + a\sqrt{2}$ is a tangent to the circle $x^2 + y^2 = a^2$ at **[Rajasthan PET 1991; MP PET 1999]**

(a) $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$ (b) $\left(-\frac{a}{\sqrt{2}}, -\frac{a}{\sqrt{2}}\right)$ (c) $\left(\frac{a}{\sqrt{2}}, -\frac{a}{\sqrt{2}}\right)$ (d) $\left(-\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$

156. If the line $3x + 4y - 1 = 0$ touches the circle $(x - 1)^2 + (y - 2)^2 = r^2$, then the value of r will be **[Rajasthan PET 1986]**

(a) 2 (b) 5 (c) $\frac{12}{5}$ (d) $\frac{2}{5}$

157. If the centre of a circle is (– 6, 8) and it passes through the origin, then equation to its tangent at the origin, is **[MNR 1976]**

(a) $2y = x$ (b) $4y = 3x$ (c) $3y = 4x$ (d) $3x + 4y = 0$

158. If the line $3x - 4y = \lambda$ touches the circle $x^2 + y^2 - 4x - 8y - 5 = 0$, then λ is equal to **[Roorkee 1972; Kurukshetra CEE 1996]**

(a) – 35, – 15 (b) – 35, 15 (c) 35, 15 (d) 35, – 15

159. The tangent to $x^2 + y^2 = 9$ which is parallel to *y*-axis and does not lie in the third quadrant touches the circle at the point

(a) (3, 0) (b) (– 3, 0) (c) (0, 3) (d) (0, – 3)

160. The points of contact of tangents to the circle $x^2 + y^2 = 25$ which are inclined at an angle of 30° to the *x*-axis are

(a) $\left(\pm\frac{5}{2}, \pm\frac{1}{2}\right)$ (b) $\left(\pm\frac{1}{2}, \pm\frac{5}{2}\right)$ (c) $\left(\mp\frac{5}{2}, \mp\frac{1}{2}\right)$ (d) None of these.

161. If the line $hx + ky = 1$ touches $x^2 + y^2 = a^2$, then the locus of the point (*h*, *k*) is a circle of radius

(a) a (b) $\frac{1}{a}$ (c) $\sqrt{a}$ (d) $\frac{1}{\sqrt{a}}$

162. The slope of the tangent at the point (*h*, *h*) of the circle $x^2 + y^2 = a^2$ is **[Roorkee 1993]**

(a) 0 (b) 1 (c) – 1 (d) Depends on *h*.

163. The line $y = mx + \sqrt{4 + 4m^2}$, $m \in R$, is a tangent to the circle

(a) $x^2 + y^2 = 2$ (b) $x^2 + y^2 = 4$ (c) $x^2 + y^2 = 1$ (d) None of these

164. The point of contact of a tangent from the point (1, 2) to the circle $x^2 + y^2 = 1$ has the coordinates

(a) $\left(\frac{1-2\sqrt{19}}{5}, \frac{2+\sqrt{19}}{5}\right)$ (b) $\left(\frac{1-2\sqrt{19}}{5}, \frac{2-\sqrt{19}}{5}\right)$ (c) $\left(\frac{1+2\sqrt{19}}{5}, \frac{2+\sqrt{19}}{5}\right)$ (d) $\left(\frac{1+2\sqrt{19}}{5}, \frac{2-\sqrt{19}}{5}\right)$

165. If the line $x + y = 1$ is a tangent to a circle with centre (2, 3), then its equation will be **[Rajasthan PET 1985, 89]**

(a) $x^2 + y^2 - 4x - 6y + 4 = 0$ (b) $x^2 + y^2 - 4x - 6y + 5 = 0$

(c) $x^2 + y^2 - 4x - 6y - 5 = 0$ (d) None of these

166. A tangent to the circle $x^2 + y^2 = a^2$ meets the axes at points *A* and *B*. The locus of the mid point of *AB* is

(a) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{a^2}$ (b) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{a^2}$ (c) $\frac{1}{x^2} + \frac{1}{y^2} = 4a^2$ (d) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{a^2}{4}$

167. If the tangent to the circle $x^2 + y^2 = 5$ at point (1, –2) touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$, then its point of contact is **[IIT 1989]**

(a) (– 1, – 3) (b) (3, – 1) (c) (– 2, 1) (d) (5, 0)

168. The equation of the tangent to the circle $x^2 + y^2 = 25$ which is inclined at 60° angle with *x*-axis, will be

(a) $y = \sqrt{3}x \pm 10$ (b) $\sqrt{3}y = x \pm 10$ (c) $y = \sqrt{3}x \pm 2$ (d) None of these

169. If $y = c$ is a tangent to the circle $x^2 + y^2 = 4$, then

(a) $|c| > 2$ (b) $|c| < 2$ (c) $|c| = 2$ (d) $|c| = 0$

Advance Level

170. If the circle $(x-h)^2 + (y-k)^2 = r^2$ is a tangent to the curve $y = x^2 + 1$ at a point (1, 2), then the possible location of the points (*h*, *k*) are given by **[AMU 2000]**

(a) $hk = 5/2$ (b) $h + 2k = 5$ (c) $h^2 - 4k^2 = 5$ (d) $k^2 = h^2 + 1$

171. If the tangent at the point *P* on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point *Q* on the *y*-axis, then the length of *PQ* is **[IIT Screening 2002]**

(a) 4 (b) $2\sqrt{5}$ (c) 5 (d) $3\sqrt{5}$

172. The tangents to $x^2+y^2=a^2$ having inclinations α and β intersect at *P*. If cot $\alpha+\cot\beta=0,$ then the locus of *P* is

(a) $x+y=0$ (b) $x-y=0$ (c) $xy=0$ (d) None of these

173. If the points *A* (1, 4) and *B* are symmetrical about the tangent to the circle $x^2+y^2-x+y=0$ at the origin then coordinates of *B* are

(a) (1, 2) (b) $(\sqrt{2}, 1)$ (c) (4, 1) (d) None of these

174. A line parallel to the line $x-3y=2$ touches the circle $x^2+y^2-4x+2y-5=0$ at the point

(a) (1, – 4) (b) (1, 2) (c) (3, – 4) (d) (3, 2)

175. The possible values of *p* for which the line $x\cos\alpha+y\sin\alpha=p$ is a tangent to the circle $x^2+y^2-2qx\cos\alpha-2qy\sin\alpha=0$ is/are **[SCRA, 1999]**

(a) 0 and *q* (b) *q* and 2*q* (c) 0 and 2*q* (d) *q*

176. A circle passes through (0, 0) and (1, 0) and touches to the circle $x^2+y^2=9,$ then the centre of circle is **[IIT 1992]**

(a) $\left(\frac{3}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},\frac{3}{2}\right)$ (c) $\left(\frac{1}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},\pm\sqrt{2}\right)$

Length of Tangent

Basic Level

177. The length of tangent from the point (5, 1) to the circle $x^2+y^2+6x-4y-3=0,$ is **[MNR 1981]**

(a) 81 (b) 29 (c) 7 (d) 21

178. Length of the tangent from (x_1, y_1) to the circle $x^2+y^2+2gx+2fy+c=0,$ is **[EAMCET 1980]**

(a) $(x_1^2+y_1^2+2gx_1+2fy_1+c)^{1/2}$ (b) $(x_1^2+y_1^2)^{1/2}$

(c) $[(x_1+g)^2+(y_1+f)^2]^{1/2}$ (d) None of these

179. The length of the tangent from the point (4, 5) to the circle $x^2+y^2+2x-6y=6$ is **[DCE 1999]**

(a) $\sqrt{13}$ (b) $\sqrt{38}$ (c) $2\sqrt{2}$ (d) $2\sqrt{13}$

180. The square of the length of the tangent from (3, – 4) on the circle $x^2+y^2-4x-6y+3=0$ is **[MP PET 2000]**

(a) 20 (b) 30 (c) 40 (d) 50

181. The length of the tangent from (0, 0) to the circle $2(x^2+y^2)+x-y+5=0$ is **[EAMCET 1994]**

(a) $\sqrt{5}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\sqrt{2}$ (d) $\sqrt{\frac{5}{2}}$

182. The length of the tangent to the circle $x^2+y^2-2x-y-7=0$ from (–1, – 3) is **[Karnataka CET 1994]**

(a) 2 (b) $2\sqrt{2}$ (c) 4 (d) 8

183. A tangent is drawn to the circle $2(x^2+y^2)-3x+4y=0$ and it touches the circle at point *A*. The tangent passes through the point *P* (2, 1). Then *PA* is equal to

(a) 4 (b) 2 (c) $2\sqrt{2}$ (d) None of these

184. Lines are drawn through the point $P(-2,-3)$ to meet the circle $x^2+y^2-2x-10y+1=0$. The length of the line segment *PA, A* being the point on the circle where the line meets the circle at coincident points, is

(a) 16 (b) $4\sqrt{3}$ (c) 48 (d) None of these

Advance Level

185. The coordinates of the point from where the tangents are drawn to the circles $x^2+y^2=1,$ $x^2+y^2+8x+15=0$ and $x^2+y^2+10y+24=0$ are of same length, are **[Roorkee 1982]**

(a) $\left(2,\frac{5}{2}\right)$ (b) $\left(-2,-\frac{5}{2}\right)$ (c) $\left(-2,\frac{5}{2}\right)$ (d) $\left(2,-\frac{5}{2}\right)$

186. Length of the tangent drawn from any point on the circle $x^2+y^2+2gx+2fy+c_1=0$ to the circle $x^2+y^2+2gx+2fy+c=0$ is

[Kerala (Engg.) 2002]

(a) $\sqrt{c_1 - c}$ (b) $\sqrt{c - c_1}$ (c) $\sqrt{c_1 + c}$ (d) None of these

187. If P is a point such that the ratio of the squares of the lengths of the tangents from P to the circles $x^2 + y^2 + 2x - 4y - 20 = 0$ and $x^2 + y^2 - 4x + 2y - 44 = 0$ is 2 : 3, then the locus of P is a circle with centre **[EAMCET 2003]**

(a) (7, – 8) (b) (– 7, 8) (c) (7, 8) (d) (– 7, – 8)

188. The lengths of the tangents from any point on the circle $15x^2 + 15y^2 - 48x + 64y = 0$ to the two circles $5x^2 + 5y^2 - 24x + 32y + 75 = 0,\ 5x^2 + 5y^2 - 48x + 64y + 300 = 0$ are in the ratio

(a) $1:2$ (b) $2:3$ (c) $3:4$ (d) None of these

189. If the squares of the lengths of the tangents from a point P to the circles $x^2 + y^2 = a^2,\ x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ are in *A. P.*, then

(a) *a, b, c* are in *G.P.* (b) *a, b, c* are in *A.P.* (c) a^2, b^2, c^2 are in *A.P.* (d) a^2, b^2, c^2 are in *G.P.*

Pair of Tangents to a Circle

Basic Level

190. A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$. The equation of the pair of tangents is

[MP PET 1990]

(a) $x^2 + y^2 + 10xy = 0$ (b) $x^2 + y^2 + 5xy = 0$ (c) $2x^2 + 2y^2 + 5xy = 0$ (d) $2x^2 + 2y^2 - 5xy = 0$

191. The equations of the tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are

[Roorkee 1989; IIT 1988; Rajasthan PET 1996]

(a) $x = 0,\ y = 0$ (b) $(h^2 - r^2)x - 2rhy = 0,\ x = 0$ (c) $y = 0,\ x = 4$ (d) $(h^2 - r^2)x + 2rhy = 0,\ x = 0$

192. The equations of the tangents drawn from the point (0, 1) to the circle $x^2 + y^2 - 2x + 4y = 0$ are **[Roorkee 1979]**

(a) $2x - y + 1 = 0,\ x + 2y - 2 = 0$ (b) $2x - y + 1 = 0,\ x + 2y + 2 = 0$

(c) $2x - y - 1 = 0,\ x + 2y - 2 = 0$ (d) $2x - y - 1 = 0,\ x + 2y + 2 = 0$

193. The two tangents to a circle from an external point are always **[MP PET 1986]**

(a) Equal (b) Perpendicular to each other (c) Parallel to each other (d) None of these

194. The equation of pair of tangents to the circle $x^2 + y^2 - 2x + 4y + 3 = 0$ from (6, – 5), is **[AMU 1980]**

(a) $7x^2 + 23y^2 + 30xy + 66x + 50y - 73 = 0$ (b) $7x^2 + 23y^2 + 30xy - 66x - 50y - 73 = 0$

(c) $7x^2 + 23y^2 - 30xy - 66x - 50y + 73 = 0$ (d) None of these

195. Tangents drawn from origin to the circle $x^2 + y^2 - 2ax - 2by + b^2 = 0$ are perpendicular to each other, if **[MP PET 1995]**

(a) $a - b = 1$ (b) $a + b = 1$ (c) $a^2 = b^2$ (d) $a^2 + b^2 = 1$

196. The equation to the tangents to the circle $x^2 + y^2 = 4$, which are parallel to $x + 2y + 3 = 0$, are **[MP PET 2003]**

(a) $x - 2y = 2$ (b) $x + 2y = \pm 2\sqrt{3}$ (c) $x + 2y = \pm 2\sqrt{5}$ (d) $x - 2y = \pm 2\sqrt{5}$

197. If $3x + y = 0$ is a tangent to the circle with centre at the point (2, – 1), then the equation of the other tangent to the circle from the origin is **[MNR 199**

(a) $x - 3y = 0$ (b) $x + 3y = 0$ (c) $3x - y = 0$ (d) $2x + y = 0$

198. The equation of a tangent to the circle $x^2 + y^2 = 25$ passing through (–2, 11) is

(a) $4x + 3y = 25$ (b) $3x + 4y = 38$ (c) $24x - 7y + 125 = 0$ (d) $7x + 24y = 230$

199. Tangents drawn from the point (4, 3) to the circle $x^2 + y^2 - 2x - 4y = 0$ are inclined at an angle

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

200. The angle between the pair of tangents from the point (1, 1/2) to the circle $x^2 + y^2 + 4x + 2y - 4 = 0$ is

(a) $\cos^{-1}\frac{4}{5}$ (b) $\sin^{-1}\frac{4}{5}$ (c) $\sin^{-1}\frac{3}{5}$ (d) None of these

201. The equation of the pair of tangents drawn from the point (0, 1) to the circle $x^2 + y^2 = 1/4$ is **[Rajasthan PET 1998]**

(a) $x^2 - 3y^2 + y + 1 = 0$ (b) $x^2 - 3y^2 - y - 1 = 0$ (c) $3x^2 - y^2 + 2y + 1 = 0$ (d) $3x^2 - y^2 + 2y - 1 = 0$

Advance Level

202. The angle between the two tangents from the origin to the circle $(x-7)^2 + (y+1)^2 = 25$ is **[MNR 1990; Rajasthan PET 1997; DCE 2000]**

(a) 0 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

203. Tangents are drawn from the point (4, 3) to the circle $x^2 + y^2 = 9$. The area of the triangle formed by them and the line joining their points of contact is **[MP PET 1991; IIT 1981, 1987]**

(a) $\frac{24}{25}$ (b) $\frac{64}{25}$ (c) $\frac{192}{25}$ (d) $\frac{192}{5}$

204. An infinite number of tangents can be drawn from (1, 2) to the circle $x^2 + y^2 - 2x - 4y + \lambda = 0$, then $\lambda =$ **[MP PET 1989]**

(a) – 20 (b) 0 (c) 5 (d) Cannot be determined

205. The area of the triangle formed by the tangents from the points (h, k) to the circle $x^2 + y^2 = a^2$ and the line joining their points of contact is **[MNR 1**

(a) $a\frac{(h^2+k^2-a^2)^{3/2}}{h^2+k^2}$ (b) $a\frac{(h^2+k^2-a^2)^{1/2}}{h^2+k^2}$ (c) $\frac{(h^2+k^2-a^2)^{3/2}}{h^2+k^2}$ (d) $\frac{(h^2+k^2-a^2)^{1/2}}{h^2+k^2}$

206. Two tangents PQ and PR drawn to the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ from point P (16, 7). If the centre of the circle is C then the area of quadrilateral $PQCR$ will be **[IIT 1981; MP PET 1994]**

(a) 75 sq. units (b) 150 sq. units (c) 15 sq. units (d) None of these

207. The tangents are drawn from the point (4, 5) to the circle $x^2 + y^2 - 4x - 2y - 11 = 0$. The area of quadrilateral formed by these tangents and radii, is **[IIT 1985]**

(a) 15 sq. units (b) 75 sq. units (c) 8 sq. units (d) 4 sq. units

208. Tangents are drawn to the circle $x^2 + y^2 = 50$ from a point 'P' lying on the x-axis. These tangents meet the y-axis at points 'P_1' and 'P_2'. Possible coordinates of 'P' so that area of triangle PP_1P_2 is minimum, is /are

(a) (10, 0) (b) $(10\sqrt{2}, 0)$ (c) (– 10, 0) (d) $(-10\sqrt{2}, 0)$

209. The angle between the tangents from α, β to the circle $x^2 + y^2 = a^2$ is, (where $S_1 = \alpha^2 + \beta^2 - a^2$)

(a) $\tan^{-1}\left(\frac{a}{\sqrt{S_1}}\right)$ (b) $2\tan^{-1}\left(\frac{a}{\sqrt{S_1}}\right)$ (c) $2\tan^{-1}\left(\frac{\sqrt{S_1}}{a}\right)$ (d) None of these

Normal and Condition of Normality

Basic Level

210. The normal to the circle $x^2 + y^2 - 3x - 6y - 10 = 0$ at the point (– 3, 4), is **[Rajasthan PET 1986, 89]**

(a) $2x + 9y - 30 = 0$ (b) $9x - 2y + 35 = 0$ (c) $2x - 9y + 30 = 0$ (d) $2x - 9y - 30 = 0$

211. The equation of normal to the circle $2x^2 + 2y^2 - 2x - 5y + 3 = 0$ at (1, 1) is **[MP PET 2001]**

(a) $2x + y = 3$ (b) $x - 2y = 3$ (c) $x + 2y = 3$ (d) None of these

212. The normal at the point (3, 4) on a circle cuts the circle at the point (–1, –2). Then the equation of the circle is **[Orissa JEE 2002]**

(a) $x^2 + y^2 + 2x - 2y - 13 = 0$ (b) $x^2 + y^2 - 2x - 2y - 11 = 0$

(c) $x^2 + y^2 - 2x + 2y + 12 = 0$ (d) $x^2 + y^2 - 2x - 2y + 14 = 0$

213. The line $\lambda x + \mu y = 1$ is a normal to the circle $2x^2 + 2y^2 - 5x + 6y - 1 = 0$ if

(a) $5\lambda - 6\mu = 2$ (b) $4 + 5\mu = 6\lambda$ (c) $4 + 6\mu = 5\lambda$ (d) None of these

214. The equation of a normal to the circle $x^2 + y^2 + 6x + 8y + 1 = 0$ passing through (0, 0) is **[Rajasthan PET 1986]**

(a) $3x + 4y = 0$ (b) $3x - 4y = 0$ (c) $4x - 3y = 0$ (d) $4x + 3y = 0$

215. The equation of the normal at the point (4, – 1) of the circle $x^2 + y^2 - 40x + 10y = 153$ is **[Rajasthan PET 1989]**

(a) $x + 4y = 0$ (b) $x - 4y = 0$ (c) $4x + y = 3$ (d) $4x - y = 0$

216. The equation of the normal to the circle $x^2 + y^2 - 4x + 6y = 0$ at (0, 0) is **[Rajasthan PET 1992]**

(a) $3x - 2y = 0$ (b) $2x - 3y = 0$ (c) $3x + 2y = 0$ (d) $2x + 3y = 0$

Advance Level

217. The area of triangle formed by the tangent, normal drawn at $(1, \sqrt{3})$ to the circle $x^2 + y^2 = 4$ and positive *x*-axis, is

[IIT 1989; Rajasthan PET 1997, 99; Kurukshetra CEE 1998]

(a) $2\sqrt{3}$ (b) $\sqrt{3}$ (c) $4\sqrt{3}$ (d) None of these

218. $y - x + 3 = 0$ is the equation of normal at $\left(3 + \frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ to which of the following circles **[Roorkee 1990]**

(a) $\left(x - 3 - \frac{3}{\sqrt{2}}\right)^2 + \left(y - \frac{\sqrt{3}}{2}\right)^2 = 9$ (b) $\left(x - 3 - \frac{3}{\sqrt{2}}\right)^2 + y^2 = 6$

(c) $(x-3)^2 + y^2 = 9$ (d) $(x-3)^2 + (y-3)^2 = 9$

219. The line $ax + by + c = 0$ is normal to the circle $x^2 + y^2 = r^2$. The portion of the line $ax + by + c = 0$ intercepted by this circle is of length

(a) r (b) r^2 (c) $2r$ (d) $\sqrt{r}$

220. If the straight line $ax + by = 2;\ a, b \neq 0$ touches the circle $x^2 + y^2 - 2x = 3$ and is normal to the circle $x^2 + y^2 - 4y = 6$ then the values of a and b are respectively **[Roorkee 2000]**

(a) 1, – 1 (b) 1, 2 (c) $-\frac{4}{3}, 1$ (d) 2, 1

221. The number of feet of normals from the point (7, – 4) to the circle $x^2 + y^2 = 5$ is

(a) 1 (b) 2 (c) 3 (d) 4

Equation of the Chord

Basic Level

222. If (a, b) is a point on the chord AB of the circle, where the ends of the chord are $A = (2, -3)$ and $B = (3, 2)$ then

(a) $a \in [-3, 2], b \in [2, 3]$ (b) $a \in [2, 3], b \in [-3, 2]$ (c) $a \in [-2, 2], b \in [-3, 3]$ (d) None of these

223. The equation of the circle with the chord $y = 2x$ of the circle $x^2 + y^2 - 10x = 0$ as its diameter is

(a) $x^2 + y^2 - 2x - 4y - 5 = 0$ (b) $x^2 + y^2 = 2x + 4y$

(c) $x^2 + y^2 = 4x + 2y$ (d) None of these

224. The radius of the circle, having centre at (2, 1), whose one of the chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$

[IIT Screening 2004]

(a) 1 (b) 2 (c) 3 (d) $\sqrt{3}$

Advance Level

225. The equation of the chord of the circle $x^2 + y^2 = 25$ of length 8 that passes through the point $(2\sqrt{3}, 2)$ and makes an acute angle with the positive direction of the x-axis is

(a) $(4\sqrt{3} - 3\sqrt{7})x + 3y = 18 - 6\sqrt{21}$ (b) $(4\sqrt{3} + 3\sqrt{7})x - 3y = 18 + 6\sqrt{21}$

(c) $(4\sqrt{3} + 3\sqrt{7})x - 3y + 18 + 6\sqrt{21} = 0$ (d) None of these

226. $P(\sqrt{2}, \sqrt{2})$ is a point on the circle $x^2 + y^2 = 4$ and Q is another point on the circle such that arc $PQ = \frac{1}{4} \times$ circumference. The coordinates of Q are

(a) $(-\sqrt{2}, -\sqrt{2})$ (b) $(\sqrt{2}, -\sqrt{2})$ (c) $(-\sqrt{2}, \sqrt{2})$ (d) None of these

227. If a line passing through the point $(-\sqrt{8}, \sqrt{8})$ and making an angle 135° with x-axis cuts the circle $x = 5\cos\theta, y = 5\sin\theta$ at points A and B, then length of the chord AB is **[Bihar CEE 1999]**

(a) 10 (b) 20 (c) 5 (d) $2\sqrt{5}$

228. Equation of chord AB of circle $x^2 + y^2 = 2$ passing through P (2, 2) such that $PB/PA = 3$, is given by

(a) $x = 3y$ (b) $x = y$ (c) $y - 2 = \sqrt{3}(x - 2)$ (d) None of these

229. If a chord of the circle $x^2 + y^2 = 8$ makes equal intercepts of length a on the coordinate axes, then

(a) $|a| < 8$ (b) $|a| < 4\sqrt{2}$ (c) $|a| < 4$ (d) $|a| > 4$

Chord of Contact

Basic Level

230. The distance between the chords of contact of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and the point (g, f) is

(a) $g^2 + f^2$ (b) $\frac{1}{2}(g^2 + f^2 + c)$ (c) $\frac{1}{2}.\frac{g^2 + f^2 + c}{\sqrt{g^2 + f^2}}$ (d) $\frac{1}{2}.\frac{g^2 + f^2 - c}{\sqrt{g^2 + f^2}}$

231. If the straight line $x - 2y + 1 = 0$ intersects the circle $x^2 + y^2 = 25$ in points P and Q, then the coordinates of the point of intersection of tangents drawn at P and Q to the circle $x^2 + y^2 = 25$ are

(a) (25, 50) (b) (– 25, – 50) (c) (– 25, 50) (d) (25, – 50)

232. If the chord of contact of tangents drawn from the point (h, k) to the circle $x^2+y^2=a^2$ subtends a right angle at the centre, then

(a) $h^2+k^2=a^2$ (b) $2(h^2+k^2)=a^2$ (c) $h^2-k^2=a^2$ (d) $h^2+k^2=2a^2$

233. The chord of contact of the pair of tangents drawn from each point on the line $2x+y=4$ to the circle $x^2+y^2=1$ pass through the point **[IIT 1997]**

(a) (1/2, – 1/4) (b) (1/2, 1/4) (c) (– 1/2, 1/4) (d) (– 1/2, – 1/4)

Advance Level

234. If the tangents are drawn to the circle $x^2+y^2=12$ at the point where it meets the circle $x^2+y^2-5x+3y-2=0$, then the point of intersection of these tangents is

(a) (6, – 6) (b) (6, 18/5) (c) (6, –18/5) (d) None of these

235. A tangent to the circle $x^2+y^2=1$ through the point (0, 5) cuts the circle $x^2+y^2=4$ at A and B. The tangents to the circle $x^2+y^2=4$ at A and B meet at C. The coordinates of C are

(a) $\left(\frac{8}{5}\sqrt{6}, \frac{4}{5}\right)$ (b) $\left(\frac{8}{5}\sqrt{6}, -\frac{4}{5}\right)$ (c) $\left(-\frac{8}{5}\sqrt{6}, -\frac{4}{5}\right)$ (d) None of these

236. Tangents drawn from (2, 0) to the circle $x^2+y^2=1$ touch the circle at A and B. Then

(a) $A=\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), B=\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ (b) $A=\left(-\frac{1}{2}, \frac{-\sqrt{3}}{2}\right), B=\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(c) $A=\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), B=\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ (d) $A=\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right), B=\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Equation of a Chord whose Middle point is given

Basic Level

237. The equation of the chord of the circle $x^2+y^2=a^2$ having (x_1, y_1) as its mid-point is **[IIT 1983; MP PET 1986]**

(a) $xy_1+yx_1=a^2$ (b) $x_1+y_1=a$ (c) $xx_1+yy_1=x_1^2+y_1^2$ (d) $xx_1+yy_1=a^2$

238. From the origin chords are drawn to the circle $(x-1)^2+y^2=1$. The equation of the locus of the middle points of these chords is

[IIT 1985; EAMCET 1991]

(a) $x^2+y^2-3x=0$ (b) $x^2+y^2-3y=0$ (c) $x^2+y^2-x=0$ (d) $x^2+y^2-y=0$

239. The equation to the chord of the circle $x^2+y^2=9$ whose middle point is (1, – 2) is **[Roorkee 1989]**

(a) $x-2y=9$ (b) $x-2y-4=0$ (c) $x-2y-5=0$ (d) $x-2y+5=0$

240. The locus of the middle point of chords of the circle $x^2+y^2=a^2$ which pass through the fixed point (h, k) is

(a) $x^2+y^2-hx-ky=0$ (b) $x^2+y^2+hx+ky=0$ (c) $x^2+y^2-2hx-2ky=0$ (d) $x^2+y^2+2hx+2ky=0$

241. Equation of the chord of the circle $x^2+y^2-4x=0$ whose mid point is (1, 0) is

(a) $y = 2$ (b) $y = 1$ (c) $x = 2$ (d) $x = 1$

242. The equation of a chord of the circle $x^2+y^2-4x=0$ which is bisected at the point (1, 1) is

(a) $x+y=2$ (b) $3x-y=2$ (c) $x-2y+1=0$ (d) $x-y=0$

243. The locus of the mid points of the chords of the circle $x^2+y^2-2y=0$ which are drawn from the origin, is **[EAMCET 199**

(a) $x^2+y^2-y=0$ (b) $x^2+y^2-x=0$ (c) $x^2+y^2-2x=0$ (d) $x^2+y^2-x-y=0$

Advance Level

244. The locus of the middle points of chords of the circle $x^2+y^2-2x-6y-10=0$ which passes through the origin, is **[Roorkee 1989]**

(a) $x^2+y^2+x+3y=0$ (b) $x^2+y^2-x+3y=0$ (c) $x^2+y^2+x-3y=0$ (d) $x^2+y^2-x-3y=0$

245. The locus of mid-point of the chords of the circle $x^2+y^2-2x-2y-2=0$ which makes an angle of 120° at the centre is **[MNR 1994]**

(a) $x^2+y^2-2x-2y+1=0$ (b) $x^2+y^2+x+y-1=0$

(c) $x^2+y^2-2x-2y-1=0$ (d) None of these

246. If the equation of a given circle is $x^2+y^2=36$, then the length of the chord which lies along the line $3x+4y-15=0$ is

(a) $3\sqrt{6}$ (b) $2\sqrt{3}$ (c) $6\sqrt{3}$ (d) None of these

247. The locus of the mid-points of a chord of the circle $x^2+y^2=4$ which subtends a right angle at the origin is

(a) $x+y=2$ (b) $x^2+y^2=1$ (c) $x^2+y^2=2$ (d) $x+y=1$

248. The equation of the locus of the middle point of a chord of the circle $x^2+y^2=2(x+y)$ such that the pair of lines joining the origin to the point of intersection of the chord and the circle are equally inclined to the x-axis is

(a) $x+y=2$ (b) $x-y=2$ (c) $2x-y=1$ (d) None of these

249. The locus of the mid-point of chords of length $2l$ of the circle $x^2+y^2=a^2$ is **[Rajasthan PET 1998]**

(a) $x^2+y^2=l^2-a^2$ (b) $x^2+y^2=l^2+a^2$ (c) $x^2+y^2=a^2-2l^2$ (d) $x^2+y^2=a^2-l^2$

Diameter of a Circle and Director Circle

Basic Level

250. The equation of the director circle of the circle $x^2+y^2=16$ is

(a) $x^2+y^2=8$ (b) $x^2+y^2=32$ (c) $x^2+y^2=64$ (d) $x^2+y^2=4$

251. If $y=2x+k$ is a diameter of the circle $2(x^2+y^2)+3x+4y-1=0$, then the value of k is

(a) 1/2 (b) – 1/2 (c) 1 (d) – 1

252. The equation of the diameter of the circle $x^2+y^2-2x+4y=0$ passing through the origin is **[Rajasthan PET 1991]**

(a) $x+2y=0$ (b) $x-2y=0$ (c) $2x+y=0$ (d) $2x-y=0$

253. The locus of the point of intersection of perpendicular tangents to the circle $x^2+y^2=a^2$ is **[MNR 1987]**

(a) A circle passing through origin (b) A circle of radius $2a$

(c) A concentric circle of radius $\sqrt{2}\,a$ (d) None of these

254. The equation of director circle of the circle $x^2+y^2=a^2$, is **[Ranchi BIT 1990]**

(a) $x^2+y^2=4a^2$ (b) $x^2+y^2=\sqrt{2}\,a^2$ (c) $x^2+y^2-2a^2=0$ (d) None of these

Advance Level

255. The equation of the diameter of the circle $3(x^2+y^2)-2x+6y-9=0$ which is perpendicular to the line $2x+3y=12$ is

(a) $3x-2y=3$ (b) $3x-2y+1=0$ (c) $3x-2y=9$ (d) None of these

256. A point on the line $x=3$ from which the tangents drawn to the circle $x^2+y^2=8$ are at right angles is

(a) $(3,-\sqrt{7})$ (b) $(3,\sqrt{23})$ (c) $(3,\sqrt{7})$ (d) $(3,-\sqrt{23})$

Pole and Polar w.r.t. a Circle

Basic Level

257. The coordinates of pole of line $lx + my + n = 0$ with respect to circle $x^2 + y^2 = 1$, is **[Rajasthan PET 1987]**

(a) $\left(\frac{l}{n}, \frac{m}{n}\right)$ (b) $\left(-\frac{l}{n}, -\frac{m}{n}\right)$ (c) $\left(\frac{l}{n}, -\frac{m}{n}\right)$ (d) $\left(-\frac{l}{n}, \frac{m}{n}\right)$

258. The equation of polar of the point (1, 2) with respect to the circle $x^2 + y^2 = 7$, is **[MNR 1973; Rajasthan PET 1983, 84]**

(a) $x - 2y - 7 = 0$ (b) $x + 2y - 7 = 0$ (c) $x - 2y = 0$ (d) $x + 2y = 0$

259. If polar of a circle $x^2 + y^2 = a^2$ with respect to (x', y') is $Ax + By + C = 0$, then its pole will be**[Rajasthan PET 1984, 95]**

(a) $\left(\frac{a^2A}{-C}, \frac{a^2B}{-C}\right)$ (b) $\left(\frac{a^2A}{C}, \frac{a^2B}{C}\right)$ (c) $\left(\frac{a^2C}{A}, \frac{a^2C}{B}\right)$ (d) $\left(\frac{a^2C}{-A}, \frac{a^2C}{-B}\right)$

260. Polar of origin (0, 0) with respect to the circle $x^2 + y^2 + 2\lambda x + 2\mu y + c = 0$ touches circle $x^2 + y^2 = r^2$ if**[Rajasthan PET 199**

(a) $c = r(\lambda^2 + \mu^2)$ (b) $r = c(\lambda^2 + \mu^2)$ (c) $c^2 = r^2(\lambda^2 + \mu^2)$ (d) $r^2 = c^2(\lambda^2 + \mu^2)$

261. The polar of the point $(5, -1/2)$ *w.r.t.* circle $(x-2)^2 + y^2 = 4$ is **[Rajasthan PET 1996]**

(a) $5x - 10y + 2 = 0$ (b) $6x - y - 20 = 0$ (c) $10x - y - 10 = 0$ (d) $x - 10y - 2 = 0$

262. The pole of the line $2x + 3y = 4$ *w.r.t.* circle $x^2 + y^2 = 64$ is **[Rajasthan PET 1996]**

(a) (32, 48) (b) (48, 32) (c) (– 32, 48) (d) (48, – 32)

263. The pole of the straight line $x + 2y = 1$ with respect to the circle $x^2 + y^2 = 5$ is **[Rajasthan PET 2000, 01]**

(a) (5, 5) (b) (5, 10) (c) (10, 5) (d) (10, 10)

264. The polars drawn from (– 1, 2) to the circles $S_1 \equiv x^2 + y^2 + 6y + 7 = 0$ and $S_2 \equiv x^2 + y^2 + 6x + 1 = 0$, are**[Rajasthan PET 20**

(a) Parallel (b) Equal (c) Perpendicular (d) Intersect at a point

265. Let the equation of a circle be $x^2 + y^2 = a^2$. If $h^2 + k^2 - a^2 < 0$ then the line $hx + ky = a^2$ is the

(a) Polar line of the point (*h, k*) with respect to the circle (b) Real chord of contact of the tangents from (*h, k*) to the circle

(c) Equation of a tangent to the circle from the point (*h, k*) (d) None of these

266. The pole of the line $4x + 3y = 50$ with respect to the circle $x^2 + y^2 = 200$ is **[Rajasthan PET 1993]**

(a) (16, 12) (b) (– 16, 12) (c) (12, 16) (d) (– 16, – 12)

267. The equation of the polar of the point (4, 4) with respect to the circle $(x-1)^2 + (y-2)^2 = 0$ is

(a) $3x + 2y = 7$ (b) $3x + 2y + 8 = 0$ (c) $3x - 2y = 8$ (d) $7x + 5y = 8$

268. The chord of contact and polar of a circle with respect to a point are coincident iff**[MP PET 1984; BIT Ranchi 1990]**

(a) The point is inside the circle (b) The point is outside the circle

(c) The point is not inside the circle (d) Never

269. The pole of the line $9x + 4y = 28$ with respect to the circle $x^2 + y^2 = 16$ is **[Rajasthan PET 1994]**

(a) (36/7, 9/7) (b) (36/7, 16/7) (c) (16/7, 36/7) (d) None of these

270. The polar of the point (– 2, 3) *w.r.t.* the circle $x^2 + y^2 - 4x - 6y + 5 = 0$ is **[Rajasthan PET 1996; EAMCET 1996]**

(a) $x = 0$ (b) $y = 0$ (c) $x = 1$ (d) $y = 1$

271. If the pole of a line *w.r.t.* the circle $x^2 + y^2 = c^2$ lies on the circle $x^2 + y^2 = 9c^2$, then that line will touch**[Rajasthan PET**

(a) $x^2 + y^2 = 4c^2$ (b) $x^2 + y^2 = c^2/9$ (c) $x^2 + y^2 = c^2/4$ (d) $x^2 + y^2 = 2c^2$

Advance Level

272. If the polar of a point (p, q) with respect to the circle $x^2 + y^2 = a^2$ touches the circle $(x-c)^2 + (y-d)^2 = b^2$, then

(a) $b^2(p^2+q^2) = (a^2 - cp - qd)^2$ (b) $b^2(p^2+q^2) = (a^2 - cq - dp)^2$

(c) $a^2(p^2+q^2) = (b^2 - cp - dq)^2$ (d) None of these

273. The equation of a circle is $x^2 + y^2 - 4x + 2y - 4 = 0$. With respect to the circle

(a) The pole of the line $x - 2y + 5 = 0$ is (1, 1)

(b) The chord of contact of real tangents from (1, 1) is the line $x - 2y + 5 = 0$

(c) The polar of the point (1, 1) is $x - 2y + 5 = 0$

(d) None of these

System of Cricles

Basic Level

274. If d is the distance between the centres of two circles, r_1, r_2 are their radii and $d = r_1 + r_2$, then **[MP PET 1986]**

(a) The circles touch each other externally (b) The circles touch each other internally

(c) The circles cut each other (d) The circles are disjoint

275. The points of intersection of the circles $x^2 + y^2 = 25$ and $x^2 + y^2 - 8x + 7 = 0$ are **[MP PET 1988]**

(a) (4, 3) and (4, – 3) (b) (4, – 3) and (– 4, – 3) (c) (– 4, 3) and (4, 3) (d) (4, 3) and (3, 4)

276. Circles $x^2 + y^2 - 2x - 4y = 0$ and $x^2 + y^2 - 8y - 4 = 0$ **[MP PET 1990]**

(a) Touch internally (b) Touch externally

(c) Intersect each other at two distinct points (d) Do not intersect each other at any point

277. For the given circles $x^2 + y^2 - 6x - 2y + 1 = 0$ and $x^2 + y^2 + 2x - 8y + 13 = 0$, which of the following is true **[MP PET 1989]**

(a) One circle lies inside the other (b) One circle lies completely outside the other

(c) Two circle intersect in two points (d) They touch each other

278. The two circles $x^2 + y^2 - 4y = 0$ and $x^2 + y^2 - 8y = 0$ **[Ranchi BIT 1985]**

(a) Touch each other internally (b) Touch each other externally

(c) Do not touch each other (d) None of these

279. Circles $x^2 + y^2 - 2x - 4y = 0$ and $x^2 + y^2 - 8y - 4 = 0$ **[IIT 1973]**

(a) Touch each other internally (b) Touch each other externally

(c) Cuts each other at two points (d) None of these

280. A tangent to the circle $x^2 + y^2 = 5$ at the point (1, – 2) ……. to the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ **[IIT 1975]**

(a) Touches (b) Cuts at real points (c) Cuts at imaginary points (d) None of these

281. If the circles $x^2 + y^2 - 9 = 0$ and $x^2 + y^2 + 2ax + 2y + 1 = 0$ touch each other, then a = **[Roorkee 1998]**

(a) – 4/3 (b) 0 (c) 1 (d) 4/3

282. The equation of the circle through the point of intersection of the circles $x^2 + y^2 - 8x - 2y + 7 = 0$, $x^2 + y^2 - 4x + 10y + 8 = 0$ and (3, – 3) is **[AI CBSE 1981]**

(a) $23x^2 + 23y^2 - 156x + 38y + 168 = 0$ (b) $23x^2 + 23y^2 + 156x + 38y + 168 = 0$

(c) $x^2 + y^2 + 156x + 38y + 168 = 0$ (d) None of these

283. The locus of the centre of a circle which touches externally the circle $x^2+y^2-6x-6y+14=0$ and also touches the y-axis is given by the equation **[IIT 1993; DCE 2000]**

(a) $x^2-6x-10y+14=0$ (b) $x^2-10x-6y+14=0$ (c) $y^2-6x-10y+14=0$ (d) $y^2-10x-6y+14=0$

284. Circles $x^2+y^2+2gx+2fy=0$ and $x^2+y^2+2g'x+2f'y=0$ touch externally, if **[MP PET 1994; Karnataka CET 2003]**

(a) $f'g=g'f$ (b) $fg=f'g'$ (c) $f'g'+fg=0$ (d) $f'g+g'f=0$

285. The circle passing through point of intersection of the circle $S=0$ and the line $P=0$ is **[Rajasthan PET 1995]**

(a) $S+\lambda P=0$ (b) $S-\lambda P=0$ (c) $\lambda S+P=0$ (d) $P-\lambda S=0$

286. The two circles $x^2+y^2-2x-3=0$ and $x^2+y^2-4x-6y-8=0$ are such that **[MNR 1995]**

(a) They touch each other (b) They intersect each other (c) One lies inside the other(d)

287. Consider the circles $x^2+(y-1)^2=9, (x-1)^2+y^2=25$. They are such that **[EAMCET 1994]**

(a) These circles touch each other (b) One of these circles lies entirely inside the other

(c) Each of these circles lies outside the other (d) They intersect in two points

288. Find the equation of the circle passing through the point (– 2, 4) and through the points of intersection of the circle $x^2+y^2-2x-6y+6=0$ and the line $3x+2y-5=0$ **[Rajasthan PET 1996]**

(a) $x^2+y^2+2x-4y-4=0$ (b) $x^2+y^2+4x-2y-4=0$

(c) $x^2+y^2-3x-4y=0$ (d) $x^2+y^2-4x-2y=0$

289. If the circles $x^2+y^2=4, x^2+y^2-10x+\lambda=0$ touch externally, then λ is equal to **[AMU 1999]**

(a) – 16 (b) 9 (c) 16 (d) 25

290. The condition that the circle $(x-3)^2+(y-4)^2=r^2$ lies entirely within the circle $x^2+y^2=R^2$, is **[AMU 1999]**

(a) $R+r\le 7$ (b) $R^2+r^2<49$ (c) $R^2-r^2<25$ (d) $R-r>5$

291. If the centre of a circle which passing through the points of intersection of the circles $x^2+y^2-6x+2y+4=0$ and $x^2+y^2+2x-4y-6=0$ is on the line $y=x$, then the equation of the circle is **[Rajasthan PET 1991; Roorkee 1989]**

(a) $7x^2+7y^2-10x+10y-11=0$ (b) $7x^2+7y^2+10x-10y-12=0$

(c) $7x^2+7y^2-10x-10y-12=0$ (d) $7x^2+7y^2-10x-12=0$

292. The equation of a circle passing through points of intersection of the circles $x^2+y^2+13x-3y=0$ and $2x^2+2y^2+4x-7y-25=0$ and point (1, 1), is **[Rajasthan PET 1988, 89; IIT 1983]**

(a) $4x^2+4y^2-30x-10y-25=0$ (b) $4x^2+4y^2+30x-13y-25=0$

(c) $4x^2+4y^2-17x-10y+25=0$ (d) None of these

293. The equation of circle passes through the points of intersection of circles $x^2+y^2-6x+8=0$ and $x^2+y^2=6$ and point (1, 1) is

[Rajasthan PET 1988; IIT 1980; MP PET 2002]

(a) $x^2+y^2-6x+4=0$ (b) $x^2+y^2-3x+1=0$ (c) $x^2+y^2-4y+2=0$ (d) None of these

294. The equation of the circle having its centre on the line $x+2y-3=0$ and passing through the points of intersection of the circles $x^2+y^2-2x-4y+1=0$ and $x^2+y^2-4x-2y+4=0$, is **[MNR 1992]**

(a) $x^2+y^2-6x+7=0$ (b) $x^2+y^2-3y+4=0$ (c) $x^2+y^2-2x-2y+1=0$ (d) $x^2+y^2+2x-4y+4=0$

295. A circle of radius 5 touches another circle $x^2+y^2-2x-4y-20=0$ at (5, 5), then its equation is **[IIT 1979]**

(a) $x^2+y^2+18x+16y+120=0$ (b) $x^2+y^2-18x-16y+120=0$

(c) $x^2+y^2-18x+16y+120=0$ (d) None of these

296. The points of intersection of circles $x^2+y^2=2ax$ and $x^2+y^2=2by$ are **[AMU 2000]**

(a) (0, 0), (a, b) (b) (0, 0), $\left(\frac{2ab^2}{a^2+b^2}, \frac{2ba^2}{a^2+b^2}\right)$ (c) (0, 0), $\left(\frac{a^2+b^2}{a^2}, \frac{a^2+b^2}{b^2}\right)$ (d) None of these

297. The equation of the circle which passes through the intersection of $x^2+y^2+13x-3y=0$ and $2x^2+2y^2+4x-7y-25=0$ and whose centre lies on $13x+30y=0$ **[DCE 2001]**

(a) $x^2+y^2+30x-13y-25=0$ (b) $4x^2+4y^2+30x-13y-25=0$

(c) $2x^2+2y^2+30x-13y-25=0$ (d) $x^2+y^2+30x-13y+25=0$

298. The two circles $x^2+y^2-2x+6y+6=0$ and $x^2+y^2-5x+6y+15=0$ **[Karnataka CET 2001]**

(a) Intersect (b) Are concentric (c) Touch internally (d) Touch externally

299. The equation of the circle passing through (1, – 3) and the points common to the two circles $x^2+y^2-6x+8y-16=0$, $x^2+y^2+4x-2y-8=0$ is

(a) $x^2+y^2-4x+6y+24=0$ (b) $2x^2+2y^2+3x+y-20=0$

(c) $3x^2+3y^2-5x-7y-19=0$ (d) None of these

300. The circles whose equations are $x^2+y^2+c^2=2ax$ and $x^2+y^2+c^2=2by$ will touch one another externally if

(a) $\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a^2}$ (b) $\frac{1}{c^2}+\frac{1}{a^2}=\frac{1}{b^2}$ (c) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$ (d) None of these

301. The equation of the circle and its chord are respectively $x^2+y^2=a^2$ and $x\cos\alpha+y\sin\alpha=p$. The equation of the circle of which this chord is a diameter is

(a) $x^2+y^2-2px\cos\alpha-2py\sin\alpha+2p^2-a^2=0$ (b) $x^2+y^2-2px\cos\alpha-2py\sin\alpha+p^2-a^2=0$

(c) $x^2+y^2+2px\cos\alpha+2py\sin\alpha+2p^2-a^2=0$ (d) None of these

302. The two circles $x^2+y^2-5=0$ and $x^2+y^2-2x-4y-15=0$

(a) Touch each other externally (b) Touch each other internally

(c) Cut each other orthogonally (d) Do not intersect

303. The circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+4x+6y+4=0$ **[EAMCET 1991]**

(a) Touch externally (b) Touch internally (c) Intersect at two points (d) Do not intersect

304. The equations of two circles are $x^2+y^2-26y+25=0$ and $x^2+y^2=25$ then

(a) They touch each other (b) They cut each other orthogonally

(c) One circle is inside the other circle (d) None of these

305. The equation of a circle C_1 is $x^2+y^2-4x-2y-11=0$. A circle C_2 of radius 1 rolls on the outside of the circle C_1. The locus of the centre of C_2 has the equation **[MP PET 2003]**

(a) $x^2+y^2-4x-2y-20=0$ (b) $x^2+y^2+4x+2y-20=0$

(c) $x^2+y^2-3x-y-11=0$ (d) None of these

306. The locus of the centres of the circles passing through the intersection of the circles $x^2+y^2=1$ and $x^2+y^2-2x+y=0$ is

(a) A line whose equation is $x+2y=0$ (b) A line whose equation is $2x-y=1$

(c) A circle (d) A pair of lines

307. If circles $x^2+y^2=9$ and $x^2+y^2+8y+c=0$ touch each other, then c is equal to **[Rajasthan PET 1994]**

(a) 15 (b) – 15 (c) 16 (d) – 16

308. The locus of the centre of the circle which touches externally the circle $x^2+y^2-6x-6y+14=0$ and also touches the y-axis, is **[IIT 1993]**

(a) $x^2-6x-10y+14=0$ (b) $x^2-10x-6y+14=0$ (c) $y^2-6x-10y+14=0$ (d) $y^2-10x-6y+14=0$

309. The circle S_1 with centre $C_1(a_1, b_1)$ and radius r_1 touches externally the circle S_2 with centre $C_2(a_2, b_2)$ and radius r_2. If the tangent at their common point passes through the origin, then

(a) $(a_1^2+a_2^2)+(b_1^2+b_2^2)=r_1^2+r_2^2$ (b) $(a_1^2-a_2^2)+(b_1^2-b_2^2)=r_1^2-r_2^2$

(c) $(a_1^2-b_2^2)+(a_2^2+b_2^2)=r_1^2+r_2^2$ (d) $(a_1^2-b_1^2)+(a_2^2+b_2^2)=r_1^2+r_2^2$

Advance Level

310. The circles $x^2+y^2-10x+16=0$ and $x^2+y^2=r^2$ intersect each other in two distinct points if **[IIT 1994]**

(a) $r<2$ (b) $r>8$ (c) $2<r<8$ (d) $2\le r\le 8$

311. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2+y^2=9$ is **[AIEEE 2002]**

(a) $\left(\frac{1}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},-\sqrt{2}\right)$ (c) $\left(\frac{3}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},\frac{3}{2}\right)$

312. The locus of the centre of the circles which touch both the circles $x^2+y^2=a^2$ and $x^2+y^2=4ax$ externally has the equation

(a) $12(x-a)^2-4y^2=3a^2$ (b) $9(x-a)^2-5y^2=2a^2$ (c) $8x^2-3(y-a)^2=9a^2$ (d) None of these

313. Tangents OP and OQ are drawn from the origin O to the circle $x^2+y^2+2gx+2fy+c=0$. Then the equation of the circumcircle of the triangle OPQ is

(a) $x^2+y^2+2gx+2fy=0$ (b) $x^2+y^2+gx+fy=0$ (c) $x^2+y^2-gx-fy=0$ (d) $x^2+y^2-2gx-2fy=0$

314. If the circle $x^2+y^2+2x+3y+1=0$ cuts $x^2+y^2+4x+3y+2=0$ in A and B, then the equation of the circle on AB as diameter is

(a) $x^2+y^2+x+3y+3=0$ (b) $2x^2+2y^2+2x+6y+1=0$ (c) $x^2+y^2+x+6y+1=0$ (d) None of these

315. The equation of the smallest circle passing through the intersection of the line $x+y=1$ and the circle $x^2+y^2=9$ is

(a) $x^2+y^2+x+y-8=0$ (b) $x^2+y^2-x-y-8=0$ (c) $x^2+y^2-x+y-8=0$ (d) None of these

316. $x^2+y^2+2(2k+3)x-2ky+(2k+3)^2+k^2-r^2=0$ represents the family of circles with centres on the line **[SCRA 1999]**

(a) $x-2y-3=0$ (b) $x+2y-3=0$ (c) $x-2y+3=0$ (d) $x+2y+3=0$

Common Tangents to Two Circles

Basic Level

317. The number of common tangents to the circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x+18y+26=0$ is **[MP PET 1995]**

(a) 1 (b) 2 (c) 3 (d) 4

318. The number of common tangents to two circles $x^2+y^2=4$ and $x^2+y^2-8x+12=0$ is **[EAMCET 1990]**

(a) 1 (b) 2 (c) 3 (d) 4

319. The number of common tangents to the circles $x^2+y^2-x=0$, $x^2+y^2+x=0$ is **[EAMCET 1994]**

(a) 2 (b) 1 (c) 4 (d) 3

320. The circles $x^2+y^2=9$ and $x^2+y^2-12y+27=0$ touch each other. The equation of their common tangent is **[MP PET 199**

(a) $4y=9$ (b) $y=3$ (c) $y=-3$ (d) $x=3$

321. The two circles $x^2+y^2-2x+6y+6=0$ and $x^2+y^2-5x+6y+15=0$ touch each other. The equation of their common tangent is **[KCET 1993; DCE 1999]**

(a) $x=3$ (b) $y=6$ (c) $7x-12y-21=0$ (d) $7x+12y+21=0$

322. The number of common tangents to the circle $x^2+y^2+2x+8y-23=0$ and $x^2+y^2-4x-10y+19=0$ is

(a) 1 (b) 2 (c) 3 (d) 4

Advance Level

323. If $a>2b>0$ then the positive value of m for which $y=mx-b\sqrt{1+m^2}$ is a common tangent to $x^2+y^2=b^2$ and $(x-a)^2+y^2=b^2$, is **[IIT Screening 2002]**

(a) $\frac{2b}{\sqrt{a^2-4b^2}}$ (b) $\frac{\sqrt{a^2-4b^2}}{2b}$ (c) $\frac{2b}{a-2b}$ (d) $\frac{b}{a-2b}$

324. Two circles, each of radius 5, have a common tangent at (1, 1) whose equation is $3x+4y-7=0$. Then their centres are

(a) (4, – 5), (– 2, 3) (b) (4, – 3), (– 2, 5) (c) (4, 5), (– 2, – 3) (d) None of these

325. The number of common tangents to the circles one of which passes through the origin and cuts off intercepts 2 from each of the axes, and the other circle has the line segment joining the origin and the point (1, 1) as a diameter, is

(a) 0 (b) 1 (c) 3 (d) 2

326. The range of values of λ for which the circles $x^2+y^2=4$ and $x^2+y^2-4\lambda x+9=0$ have two common tangents, is

(a) $\lambda \in \left[-\frac{13}{8}, \frac{13}{8}\right]$ (b) $\lambda > \frac{13}{8}$ or $\lambda < -\frac{13}{8}$ (c) $1<\lambda<\frac{13}{8}$ (d) None of these

327. Two circles with radii 'r_1' and 'r_2', $r_1>r_2\geq 2$, touch each other externally, if 'θ' be the angle between the direct common tangents, then

(a) $\theta=\sin^{-1}\left(\frac{r_1+r_2}{r_1-r_2}\right)$ (b) $\theta=2\sin^{-1}\left(\frac{r_1-r_2}{r_1+r_2}\right)$ (c) $\theta=\sin^{-1}\left(\frac{r_1-r_2}{r_1+r_2}\right)$ (d) None of these.

Common Chord of Two Circles

Basic Level

328. The common chord of the circle $x^2+y^2+4x+1=0$ and $x^2+y^2+6x+2y+3=0$ is **[MP PET 1991]**

(a) $x+y+1=0$ (b) $5x+y+2=0$ (c) $2x+2y+5=0$ (d) $3x+y+3=0$

329. The equation of line passing through the points of intersection of the circles $3x^2+3y^2-2x+12y-9=0$ and $x^2+y^2+6x+2y-15=0$, is **[IIT 1986; UPSEAT 1999]**

(a) $10x-3y-18=0$ (b) $10x+3y-18=0$ (c) $10x+3y+18=0$ (d) None of these

330. Length of the common chord of the circles $x^2+y^2+5x+7y+9=0$ and $x^2+y^2+7x+5y+9=0$ is **[Kurukshetra CEE 1996]**

(a) 9 (b) 8 (c) 7 (d) 6

331. The length of the common chord of the circles $x^2+y^2+2x+3y+1=0$ and $x^2+y^2+4x+3y+2=0$ is **[MP PET 2000]**

(a) 9/2 (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) 3/2

332. If the circle $x^2+y^2+2gx+2fy+c=0$ bisects the circumference of the circle $x^2+y^2+2g'x+2f'y+c'=0$, then

(a) $2g(g-g')+2f(f-f')=c-c'$ (b) $2g'(g-g')+2f'(f-f')=c'-c$

(c) $2g'(g-g')+2f'(f-f')=c-c'$ (d) $2g(g-g')+2f(f-f')=c'-c.$

333. If the circle $x^2+y^2+2gx+2fy+c=0$ bisects the circumference of the circle $x^2+y^2+2g'x+2f'y+c'=0$, then the length of the common chord of these two circles is

(a) $2\sqrt{g^2+f^2-c}$ (b) $2\sqrt{g'^2+f'^2-c'}$ (c) $2\sqrt{g^2+f^2+c}$ (d) $2\sqrt{g'^2+f'^2+c'}$

334. The equation of the circle described on the common chord of the circles $x^2+y^2+2x=0$ and $x^2+y^2+2y=0$ as diameter is

[EAMCET 1994]

(a) $x^2+y^2+x-y=0$ (b) $x^2+y^2-x-y=0$ (c) $x^2+y^2-x+y=0$ (d) $x^2+y^2+x+y=0$

335. The distance of the point (1, 2) from the common chord of circles $x^2+y^2-2x+3y-5=0$ and $x^2+y^2+10x+8y-1=0$ is

[EAMCET 1990]

(a) 2 units (b) 3 units (c) 4 units (d) None of these

Advance Level

336. The length of common chord of the circles $(x-a)^2+y^2=a^2$ and $x^2+(y-b)^2=b^2$ is **[MP PET 1989]**

(a) $2\sqrt{a^2+b^2}$ (b) $\frac{ab}{\sqrt{a^2+b^2}}$ (c) $\frac{2ab}{\sqrt{a^2+b^2}}$ (d) None of these

337. The length of common chord of the circles $x^2+y^2=12$ and $x^2+y^2-4x+3y-2=0$, is **[Rajasthan PET 1990, 99]**

(a) $4\sqrt{2}$ (b) $5\sqrt{2}$ (c) $2\sqrt{2}$ (d) $6\sqrt{2}$

338. The line L passes through the points of intersection of the circles $x^2+y^2=25$ and $x^2+y^2-8x+7=0$. The length of perpendicular from centre of second circle onto the line L, is **[Bihar CEE 1994]**

(a) 4 (b) 3 (c) 1 (d) 0

339. The common chord of $x^2+y^2-4x-4y=0$ and $x^2+y^2=16$ subtends at the origin an angle equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

340. The length of the common chord of the circles $(x-a)^2+(y-b)^2=c^2$ and $(x-b)^2+(y-a)^2=c^2$ is

(a) $\sqrt{c^2-(a-b)^2}$ (b) $\sqrt{4c^2-2(a-b)^2}$ (c) $\sqrt{2c^2-(a-b)^2}$ (d) $\sqrt{4c^2+(a-b)^2}$

341. If the circles $(x-a)^2+(y-b)^2=c^2$ and $(x-b)^2+(y-a)^2=c^2$ touch each other, then

(a) $a=b\pm 2c$ (b) $a=b\pm\sqrt{2}c$ (c) $a=b\pm c$ (d) None of these

342. If the circle $c_1: x^2+y^2=16$ intersects another circle c_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, the coordinates of the centre of c_2 are **[IIT 1988]**

(a) $\left(-\frac{9}{5},\frac{12}{5}\right),\left(\frac{9}{5},-\frac{12}{5}\right)$ (b) $\left(-\frac{9}{5},-\frac{12}{5}\right),\left(\frac{9}{5},\frac{12}{5}\right)$ (c) $\left(\frac{12}{5},-\frac{9}{5}\right),\left(-\frac{12}{5},\frac{9}{5}\right)$ (d) None of these

343. The common chord of the circle $x^2+y^2+6x+8y-7=0$ and a circle passing through the origin, and touching the line $y=x$, always passes through the point

(a) (– 1/2, 1/2) (b) (1, 1) (c) (1/2, 1/2) (d) None of these

344. The equation of the circle drawn on the common chord of circles $x^2+y^2+2ax+c=0$ and $x^2+y^2+2by+c=0$ as a diameter is

[Rajasthan PET 1998]

(a) $x^2+y^2+\frac{2ab^2}{a^2+b^2}x+\frac{2a^2b}{a^2+b^2}y+c=0$ (b) $x^2+y^2+\frac{ab^2}{a^2+b^2}x+\frac{a^2b}{a^2+b^2}y+c=0$

(c) $(a^2+b^2)(x^2+y^2)+2ab(bx+ay)+c=0$ (d) None of these

345. The equation of the circle drawn on the common chord of circles $(x-a)^2+y^2=a^2$ and $x^2+(y-b)^2=b^2$ as diameter is

[EAMCET 1989]

(a) $(a^2+b^2)(x^2+y^2)=2ab(bx+ay)$ (b) $(a^2+b^2)(x^2+y^2)=2ab(ax+by)$

(c) $(a^2-b^2)(x^2+y^2)=2ab(bx-ay)$ (d) $(a^2-b^2)(x^2+y^2)=2ab(ax-by)$

Angle of Intersection of Two Circles and Orthogonal System of Circles

Basic Level

346. If a circle passes through the point (1, 2) and cuts the circle $x^2+y^2=4$ orthogonally, then the equation of the locus of its centre is

[MNR 1992]

(a) $x^2+y^2-3x-8y+1=0$ (b) $x^2+y^2-2x-6y-7=0$

(c) $2x+4y-9=0$ (d) $2x+4y-1=0$

347. The locus of centre of a circle passing through (p, q) and cuts orthogonally to circle $x^2+y^2=k^2$, is **[IIT 1988]**

(a) $2px+2qy-(p^2+q^2+k^2)=0$ (b) $2px+2qy-(p^2-q^2+k^2)=0$

(c) $x^2+y^2-3px-4qy+(p^2+q^2-k^2)=0$ (d) $x^2+y^2-2px-3qy+(p^2-q^2-k^2)=0$

348. Two given circles $x^2+y^2+ax+by+c=0$ and $x^2+y^2+dx+ey+f=0$ will intersect each other orthogonally, only when

(a) $a+b+c=d+e+f$ (b) $ad+be=c+f$ (c) $ad+be=2c+2f$ (d) $2ad+2be=c+f$

349. Two circles $S_1=x^2+y^2+2g_1x+2f_1y+c_1=0$ and $S_2=x^2+y^2+2g_2x+2f_2y+c_2=0$ cut each other orthogonally, then

[Rajasthan PET 1995]

(a) $2g_1g_2+2f_1f_2=c_1+c_2$ (b) $2g_1g_2-2f_1f_2=c_1+c_2$ (c) $2g_1g_2+2f_1f_2=c_1-c_2$ (d) $2g_1g_2-2f_1f_2=c_1-c_2$

350. If the circles of same radius a and centres at (2, 3) and (5, 6) cut orthogonally, then a = **[EAMCET 1988]**

(a) 1 (b) 2 (c) 3 (d) 4

351. The circles $x^2+y^2+4x+6y+3=0$ and $2(x^2+y^2)+6x+4y+c=0$ will cut orthogonally, if c equals **[Kurukshetra CEE 1996]**

(a) 4 (b) 18 (c) 12 (d) 16

352. The equation of a circle that intersects the circle $x^2+y^2+14x+6y+2=0$ orthogonally and whose centre is (0, 2) is **[MP PET 1998]**

(a) $x^2+y^2-4y-6=0$ (b) $x^2+y^2+4y-14=0$ (c) $x^2+y^2+4y+14=0$ (d) $x^2+y^2-4y-14=0$

353. If the circles $x^2+y^2+2x+2ky+6=0$ and $x^2+y^2+2ky+k=0$ intersect orthogonally, then k is **[IIT Screening 2000]**

(a) 2 or $-\frac{3}{2}$ (b) – 2 or $\frac{3}{2}$ (c) 2 or $\frac{3}{2}$ (d) – 2 or $\frac{3}{2}$

354. The locus of the centre of circle which cuts the circles $x^2+y^2+4x-6y+9=0$ and $x^2+y^2-4x+6y+4=0$ orthogonally is

[UPSEAT 2001]

(a) $12x+8y+5=0$ (b) $8x+12y+5=0$ (c) $8x-12y+5=0$ (d) None of these

355. If the two circles $2x^2+2y^2-3x+6y+k=0$ and $x^2+y^2-4x+10y+16=0$ cut orthogonally, then the value of k is **[Kerala (Engg.) 2002]**

(a) 41 (b) 14 (c) 4 (d) 0

356. The circles $x^2+y^2+x+y=0$ and $x^2+y^2+x-y=0$ intersect at an angle of

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

357. The equation of the circle having its centre on the line $x+2y-3=0$ and passing through the point of intersection of the circles $x^2+y^2-2x-4y+1=0$ and $x^2+y^2-4x-2y+4=0$ is **[MNR 1992]**

(a) $x^2+y^2-6x+7=0$ (b) $x^2+y^2-3x+4=0$ (c) $x^2+y^2-2x-2y+1=0$ (d) $x^2+y^2+2x-4y+4=0$

358. The two circles $x^2+y^2-2x-2y-7=0$ and $3(x^2+y^2)-8x+29y=0$ **[Karnataka CET 1993]**

(a) Touch externally (b) Touch internally

(c) Cut each other orthogonally (d) Do not cut each other

359. The locus of the centres of circles passing through the origin and intersecting the fixed circle $x^2+y^2-5x+3y-1=0$ orthogonally is

(a) A straight line of the slope $\frac{3}{5}$ (b) A circle

(c) A pair of straight lines (d) None of these

360. The angle of intersection of circles $x^2+y^2+8x-2y-9=0$ and $x^2+y^2-2x+8y-7=0$ is **[EAMCET 1987]**

(a) 45^o (b) 90^o (c) 60^o (d) 30^o

361. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=4$ orthogonally, then the locus of its centre is **[AIEEE 2004]**

(a) $2ax-2by-(a^2+b^2+4)=0$ (b) $2ax+2by-(a^2+b^2+4)=0$

(c) $2ax-2by+(a^2+b^2+4)=0$ (d) $2ax+2by+(a^2+b^2+4)=0$

362. The value of λ, for which the circle $x^2+y^2+2\lambda x+6y+1=0$, intersects the circle $x^2+y^2+4x+2y=0$ orthogonally is **[MP PET 2004]**

(a) $\frac{-5}{2}$ (b) -1 (c) $\frac{-11}{8}$ (d) $\frac{-5}{4}$

Advance Level

363. The equation of a circle which cuts the three circles $x^2+y^2-3x-6y+14=0$, $x^2+y^2-x-4y+8=0$ and $x^2+y^2+2x-6y+9$ orthogonally is

(a) $x^2+y^2-2x-4y+1=0$ (b) $x^2+y^2+2x+4y+1=0$

(c) $x^2+y^2-2x+4y+1=0$ (d) $x^2+y^2-2x-4y-1=0$

364. The coordinates of the centre of the circle which intersects circles $x^2+y^2+4x+7=0$, $2x^2+2y^2+3x+5y+9=0$ and $x^2+y^2+y=0$ orthogonally are

(a) $(-2, 1)$ (b) $(-2, -1)$ (c) $(2, -1)$ (d) $(2, 1)$

365. The members of a family of circles are given by the equation $2(x^2+y^2)+\lambda x-(1+\lambda^2)y-10=0$. The number of circles belonging to the family that are cut orthogonally by the fixed circle $x^2+y^2+4x+6y+3=0$ is

(a) 2 (b) 1 (c) 0 (d) None of these

Radical Axis and Radical Centre

Basic Level

366. The equation of radical axis of the circles $x^2+y^2+x-y+2=0$ and $3x^2+3y^2-4x-12=0$, is

[Rajasthan PET 1984, 85, 86, 91, 2000]

(a) $2x^2 + 2y^2 - 5x + y - 14 = 0$ (b) $7x - 3y + 18 = 0$

(c) $5x - y + 14 = 0$ (d) None of these

367. The radical centre of three circles described on the three sides of a triangle as diameter is [EAMCET 1994]

(a) The orthocentre (b) The circumcentre (c) The incentre of the triangle (d)

368. The locus of centre of the circle which cuts the circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$
orthogonally is [Karnataka CET 1991]

(a) An ellipse (b) The radical axis of the given circles

(c) A conic (d) Another circle

369. The coordinates of the radical centre of the three circles $x^2 + y^2 - 4x - 2y + 6 = 0$, $x^2 + y^2 - 4x - 2y + 6y = 0$, $x^2 + y^2 - 12x + 2y + 30 = 0$ are

(a) (6, 30) (b) (0, 6) (c) (3, 0) (d) None of these

370. The equation of radical axis of the circles $2x^2 + 2y^2 - 7x = 0$ and $x^2 + y^2 - 4y - 7 = 0$ is [Rajasthan PET 1987, 89, 93, 96]

(a) $7x + 8y + 14 = 0$ (b) $7x - 8y + 14 = 0$ (c) $7x - 8y - 14 = 0$ (d) None of these

371. The radical centre of the circles $x^2 + y^2 - 16x + 60 = 0$, $x^2 + y^2 - 12x + 27 = 0$, $x^2 + y^2 - 12y + 8 = 0$ is [Rajasthan PET 2000]

(a) (13, 33/4) (b) (33/4, – 13) (c) (33/4, 13) (d) None of these

372. The radical axis of two circles and the line joining their centres are [Karnataka CET 2001]

(a) Parallel (b) Perpendicular

(c) Neither parallel, nor perpendicular (d) Intersecting, but not fully perpendicular

373. Radical axis of the circles $3x^2 + 3y^2 - 7x + 8y + 11 = 0$ and $x^2 + y^2 - 3x - 4y + 5 = 0$ is [Rajasthan PET 2001]

(a) $x + 10y + 2 = 0$ (b) $x + 10y - 2 = 0$ (c) $x + 10y + 8 = 0$ (d) $x + 10y - 8 = 0$

374. Two tangents are drawn from a point P on radical axis to the two circles touching at Q and R respectively then triangle formed by joining PQR is [UPSEAT 2002]

(a) Isosceles (b) Equilateral (c) Right angled (d) None of these

375. Equation of radical axis of the circles $x^2 + y^2 - 3x - 4y + 5 = 0$ and $2x^2 + 2y^2 - 10x - 12y + 12 = 0$ is [Rajasthan PET 2003]

(a) $2x + 2y - 1 = 0$ (b) $2x + 2y + 1 = 0$ (c) $x + y + 7 = 0$ (d) $x + y - 7 = 0$

376. If the circle $x^2 + y^2 + 6x - 2y + k = 0$ bisects the circumference of the circle $x^2 + y^2 + 2x - 6y - 15 = 0$, then k =[EAMCET 200

(a) 21 (b) – 21 (c) 23 (d) – 23

377. The locus of a point which moves such that the tangents from it to the two circles $x^2 + y^2 - 5x - 3 = 0$ and $3x^2 + 3y^2 + 2x + 4y - 6 = 0$ are equal, is given by

(a) $2x^2 + 2y^2 + 7x + 4y - 3 = 0$ (b) $17x + 4y + 3 = 0$

(c) $4x^2 + 4y^2 - 3x + 4y - 9 = 0$ (d) $13x - 4y + 15 = 0$

378. Two equal circles with their centres on x and y axes will possess the radical axis in the following form

(a) $ax - by - \frac{a^2 + b^2}{4} = 0$ (b) $2gx - 2fy + g^2 - f^2 = 0$ (c) $g^2x + f^2y - g^4 - f^4 = 0$ (d) $2g^2x + 2f^2y - g^4 - f^4 = 0$

379. The equations of two circles are $x^2 + y^2 + 2\lambda x + 5 = 0$ and $x^2 + y^2 + 2\lambda y + 5 = 0$. P is any point on the line $x - y = 0$. If PA and PB are the lengths of the tangents from P to the two circles and PA = 3 then PB is equal to

(a) 1.5 (b) 6 (c) 3 (d) None of these

380. The locus of a point from which the lengths of the tangents to the circles $x^2 + y^2 = 4$ and $2(x^2 + y^2) - 10x + 3y - 2 = 0$ are equal is

(a) A straight line inclined at $\pi/4$ with the line joining the centres of the circles
(b) A circle
(c) An ellipse
(d) A straight line perpendicular to the line joining the centres of the circles

381. The equation of the radical axis of circles $x^2+y^2+6x=0$ and $x^2+y^2+4x-2y+5=0$ is **[Rajasthan PET 1986]**
(a) $2x+2y=5$ (b) $4x-2y-5=0$ (c) $2x+2y+5=0$ (d) None of these

382. The equation of the radical axis of circles $(x-a)^2+(y-b)^2=c^2$ and $(x-b)^2+(y-a)^2=c^2$ is
(a) $x+y=0$ (b) $x-y=0$ (c) $x+y=c^2$ (d) None of these

Miscellaneous problems

Basic Level

383. If three circles are such that each intersects the remaining two, then their radical axes
(a) Form a triangle (b) Are coincident (c) Concurrent (d) Parallel

384. If a circle S_1 bisects the circumference of another circle S_2, then their radical axis
(a) Passes through the centre of S_1 (b) Passes through the centre of S_2
(c) Bisects the line joining their centres (d) None of these

385. If two circles intersect a third circle orthogonally, then their radical axis
(a) Touches the third circle (b) Passes through the centre of the third circle
(c) Does not intersect the third circle (d) None of these

386. The radical axis of two circles
(a) Always intersects both the circles (b) Intersects only one circle
(c) Bisects the line joining their centres (d) Bisects every common tangent to those circles

387. If the radical axis of circles $x^2+y^2-6x-8y+p=0$ and $x^2+y^2-8x-6y+14=0$ passes through the point (1, – 1), then p is equal to
(a) – 1 (b) 10 (c) – 14 (d) 14

388. The radical centre of circles $x^2+y^2+2ax+c=0$, $x^2+y^2+2by+c=0$ and $x^2+y^2+2ax+2by+c=0$ is
(a) (0, 0) (b) (a, 0) (c) (0, b) (d) (a, b)

389. The equation of the radical axis of circles $7x^2+7y^2-7x+14y+18=0$ and $4x^2+4y^2-7x+8y+20=0$ is **[Roorkee 1989]**
(a) $21x-68=0$ (b) $3y-1=0$ (c) $3x^2+3y^2+6y-6=0$ (d) None of these

Advance Level

390. If the radical axis of the circles $x^2+y^2+2gx+2fy+c=0$ and $2x^2+2y^2+3x+8y+2c=0$ touches the circle $x^2+y^2+2x+2y+1=0$, then
(a) $g=\frac{3}{4}$ and $f\neq 2$ (b) $g\neq\frac{3}{4}$ and $f=2$ (c) $g=\frac{3}{4}$ and $f=2$ (d) None of these

391. If (1, 2) is the radical centre of circle $x^2+y^2-3x-6y+d_1=0$, $x^2+y^2-x-4y+d_2=0$ and $x^2+y^2+2x-6y+d_3=0$, then
(a) $d_1+d_3=5$ (b) $d_1-d_3=5$ (c) $d_1+d_3=10$ (d) $d_1-d_3=10$

392. $x=1$ is the equation of the radical axis of two circle which intersect orthogonally. If the equation of one of these circles is $x^2+y^2=4$, then the equation of the other is **[EAMCET 1983]**
(a) $x^2+y^2-8x-4=0$ (b) $x^2+y^2-8x+4=0$ (c) $x^2+y^2+8x+4=0$ (d) None of these

Co-axial System of Circles and Limiting Points

Basic Level

393. Origin is a limiting point of a coaxial system of which $x^2+y^2-6x-8y+1=0$ is a member. The other limiting point is **[EAMCET 1994]**

(a) (– 2, – 4) (b) $\left(\frac{3}{25}, \frac{4}{25}\right)$ (c) $\left(-\frac{3}{25}, -\frac{4}{25}\right)$ (d) $\left(\frac{4}{25}, \frac{3}{25}\right)$

394. If (3, λ) and (5, 6) are conjugate points with respect to circle $x^2+y^2=3,$ then λ equals **[Rajasthan PET 1998]**

(a) 2 (b) – 2 (c) 3 (d) 4

Advance Level

395. One of the limiting point of the coaxial system of circles containing $x^2+y^2-6x-6y+4=0,\ x^2+y^2-2x-4y+3=0$ is

[EAMCET 1987]

(a) (– 1, 1) (b) (– 1, 2) (c) (– 2, 1) (d) (– 2, 2)

396. The co-axial system of circles given by $x^2+y^2+2gx+c=0$ for c < 0 represents. **[Karnataka CET 2004]**

(a) Intersecting circles (b) Non intersecting circles

(c) Touching circles (d) Touching or non-intersecting circles

Miscellaneous problems

Basic Level

397. The limit of the perimeter of the regular *n*-gons inscribed in a circle of radius *R* as $n \to \infty$ is **[MP PET 2003]**

(a) $2\pi R$ (b) πR (c) 4 *R* (d) πR^2

398. *A, B, C* and *D* are the points of intersection with the coordinate axes of the lines *ax* + by = *ab* and $bx+ay=ab,$ then

(a) *A, B, C, D* are concyclic (b) *A, B, C, D* form a parallelogram

(c) *A, B, C, D* form a rhombus (d) None of these

Advance Level

399. If the points (2, 0), (0, 1), (4, 5) and (0, c) are concyclic, then *c* is equal to **[MNR 1982]**

(a) $-1, -\frac{3}{14}$ (b) $-1, -\frac{14}{3}$ (c) $\frac{14}{3}, 1$ (d) None of these

400. Line $Ax+By+C=0$ cuts circle $x^2+y^2+ax+by+c=0$ in *P* and *Q* and the line $A'x+B'y+C'=0$ cuts the circle $x^2+y^2+a'x+b'y+c'=0$ in *R* and *S*. If the four points *P, Q, R* and *S* are concyclic, then

$$D=\begin{vmatrix} a-a' & b-b' & C-C' \\ A & B & c \\ A' & B' & c' \end{vmatrix}=$$ **[Roorkee 1986]**

(a) 1 (b) 0 (c) – 1 (d) None of these

401. A circle is inscribed in an equilateral triangle of side *a*, the area of any square inscribed in the circle is **[IIT 1994]**

(a) $\frac{a^2}{3}$ (b) $\frac{2a^2}{3}$ (c) $\frac{a^2}{6}$ (d) $\frac{a^2}{12}$

402. Any circle through the points of intersection of the lines $x+\sqrt{3}y=1$ and $\sqrt{3}x-y=2$ if intersects these lines at points P and Q, then the angle subtended by the arc PQ at its centre is **[MP PET 1998]**

(a) 180° (b) 90° (c) 120° (d) Depends on centre of radius

403. The area of the triangle formed by joining the origin to the points of intersection of the line $x\sqrt{5}+2y=3\sqrt{5}$ and circle $x^2+y^2=10$ is **[Roorkee 1998]**

(a) 3 (b) 4 (c) 5 (d) 6

404. Let AB be a chord of the circle $x^2+y^2=r^2$ subtending a right angle at the centre. Then the locus of the centroid of the ΔPAB as P moves on the circle is **[IIT Screening 2001]**

(a) A parabola (b) A circle (c) An ellipse (d) A pair of straight lines

405. A square is inscribed in the circle $x^2+y^2-2x+4y-93=0$ with its sides parallel to the coordinate axes. The coordinates of its vertices are

(a) (– 6, – 9), (– 6, 5), (8, – 9), (8, 5) (b) (– 6, 9), (– 6, – 5), (8, – 9), (8, 5)
(c) (– 6, – 9), (– 6, 5), (8, 9), (8, 5) (d) (– 6, – 9), (– 6, 5), (8, – 9), (8, – 5)

406. If the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ cut the coordinate axes in concyclic points, then

(a) $a_1a_2=b_1b_2$ (b) $a_1b_1=a_2b_2$ (c) $a_1b_2=a_2b_1$ (d) None of these

407. Let P be a point on the circle $x^2+y^2=9$, Q a point on the line $7x+y+3=0$, and the perpendicular bisector of PQ be the line $x-y+1=0$. Then the coordinates of P are

(a) (3, 0) (b) (0, 3) (c) $\left(\frac{72}{25}, -\frac{21}{25}\right)$ (d) $\left(-\frac{72}{25}, \frac{21}{25}\right)$

408. A line meets the coordinate axes in A and B. A circle is circumscribed about the triangle OAB. The distances from the end points A, B of the side AB to the tangent at O are equal to m and n respectively. Then the diameter of the circle is

(a) $m(m+n)$ (b) $n(m+n)$ (c) $m-n$ (d) None of these

409. If the circle $x^2+y^2+2gx+2fy+c=0$ is touched by $y=x$ at P such that $OP=6\sqrt{2}$, then the value of c is

(a) 36 (b) 144 (c) 72 (d) None of these

410. One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y=x+7$. If A and B are the points (–3, 4) and (5, 4) respectively, then the area of the rectangle is

(a) 16 sq. units (b) 24 sq. units (c) 32 sq. units (d) None of these

411. The maximum number of points with rational coordinates on a circle whose centre is $(\sqrt{3},0)$ is

(a) One (b) Two (c) Four (d) Infinite

412. The locus of co-ordinates of the centre of the circumcircle of the regular hexagon whose two consecutive vertices have the coordinates (–1, 0) and (1, 0) and which lies wholly above the x-axis, are

(a) $x^2+y^2-2\sqrt{3}y-1=0$ (b) $x^2+y^2-\sqrt{3}y-1=0$ (c) $x^2+y^2-2\sqrt{3}x-1=0$ (d) None of these

413. For each $k\in N$, let C_k denote the circle whose equation is $x^2+y^2=k^2$. On the circle C_k, a particle moves k units in the anticlockwise direction. After completing its motion on C_k, the particle moves to C_{k+1} in the radial direction. The motion of the particle continues in this manner. The particle starts at (1, 0). If the particle crosses the positive direction of the x-axis for the first time on the circle C_n, then n is

(a) 7 (b) 6 (c) 2 (d) None of these

414. A ray of light incident at the point (– 2, – 1) gets reflected from the tangent at (0, – 1) to the circle $x^2+y^2=1$. The reflected ray touches the circle. The equation of the line along which the incident ray moved is

(a) $4x-3y+11=0$ (b) $4x+3y+11=0$ (c) $3x+4y+11=0$ (d) None of these

415. The point P moves in the plane of a regular hexagon such that the sum of the squares of its distances from the vertices of the hexagon is $6a^2$. If the radius of the circumcircle of the hexagon is $r(<a)$ then the locus of P is

(a) A pair of straight lines (b) An ellipse
(c) A circle of radius $\sqrt{a^2-r^2}$ (d) An ellipse of major axis a and minor axis r

416. The equation of a circle is $x^2+y^2=4$. A regular hexagon is inscribed in the circle whose one vertex is (2, 0). Then a consecutive vertex has the coordinates

(a) $(\sqrt{3}, 1)$ (b) $(1, -\sqrt{3})$ (c) $(\sqrt{3}, -1)$ (d) $(1, \sqrt{3})$

417. A point $P(\sqrt{3}, 1)$ moves on the circle $x^2 + y^2 = 4$ and after covering a quarter of the circle leaves it tangentially. The equation of a line along which the point moves after leaving the circle is

(a) $y = \sqrt{3}x + 4$ (b) $\sqrt{3}y = x + 4$ (c) $\sqrt{3}y = x - 4$ (d) $y = \sqrt{3}x - 4$

418. If the curves $ax^2 + 4xy + 2y^2 + x + y + 5 = 0$ and $ax^2 + 6xy + 5y^2 + 2x + 3y + 8 = 0$ intersect at four concyclic points then the value of a is

(a) 4 (b) – 4 (c) 6 (d) – 6

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Answer Sheet

Circle and System of Circles *Assignment (Basic and Advance level)*

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
b	b	a	b	a	b	b	a	b	b	c	c	a	a	a	c	a	d	a	a
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
d	c	a	b	c	a	d	c	b	d	c	d	a	a,b,c	a	b	a	d	d	b
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
d	a	d	c	b	c	d	b	c	b	a	a	c	a	a	c	b	a	c	b
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
a	c	a	c	b	c	c	a	a	a	a	b	c	d	b	d	a	b	b	b
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
b	a	c	b	b	c	a	b	c	b	a	d	b	a	a	b	b	d	c	c
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
a	d	a	b	b	b	a	a	a	c	d	c	b	b	a	b,c	b	c	a	c
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
a,c	b	d	b	b	a	b	a	a	d	d	a	b	d	c	a	d	b	d	c
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
a	b	a	b	c	b	b	b	b	c	b	a	c	c	d	a	b	b	a	a
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
b	c	b	a,d	b	b	b	a	c	b	c	c	c	b,c	c	d	c	a	a	c
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
d	b	b	b	b	b	b	a	c	c	b	a	a	a	c	c	a	a,c	d	b
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
d	d	c	c	a	a	c	a,c	b	a	c	b	c	c	a	c	a	c	c	c
221	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**	**239**	**240**
b	b	b	c	b	b,c	a	b	c	d	c	d	b	c	a	c,d	c	c	c	a

241	**242**	**243**	**244**	**245**	**246**	**247**	**248**	**249**	**250**	**251**	**252**	**253**	**254**	**255**	**256**	**257**	**258**	**259**	**260**
d	d	a	d	a	c	c	a	d	b	a	c	c	c	a	a,c	b	b	a	c
261	**262**	**263**	**264**	**265**	**266**	**267**	**268**	**269**	**270**	**271**	**272**	**273**	**274**	**275**	**276**	**277**	**278**	**279**	**280**
b	a	b	d	a	a	a	c	b	a	b	a	a,c	a	a	a	d	a	a	a
281	**282**	**283**	**284**	**285**	**286**	**287**	**288**	**289**	**290**	**291**	**292**	**293**	**294**	**295**	**296**	**297**	**298**	**299**	**300**
a,d	a	d	a	a,b,c d	b	b	b	a	d	c	b	b	a	b	a	b	c	b	c
301	**302**	**303**	**304**	**305**	**306**	**307**	**308**	**309**	**310**	**311**	**312**	**313**	**314**	**315**	**316**	**317**	**318**	**319**	**320**
a	b	c	b	a	a	a	d	b	c	b	a	b	b	b	d	c	c	d	b
321	**322**	**323**	**324**	**325**	**326**	**327**	**328**	**329**	**330**	**331**	**332**	**333**	**334**	**335**	**336**	**337**	**338**	**339**	**340**
b	c	a	c	b	b	b	a	a	d	b	c	b	d	a	c	a	d	d	b
341	**342**	**343**	**344**	**345**	**346**	**347**	**348**	**349**	**350**	**351**	**352**	**353**	**354**	**355**	**356**	**357**	**358**	**359**	**360**
b	a	c	a	a	c	a	c	a	c	b	d	a	c	c	d	a	c	d	b
361	**362**	**363**	**364**	**365**	**366**	**367**	**368**	**369**	**370**	**371**	**372**	**373**	**374**	**375**	**376**	**377**	**378**	**379**	**380**
b	d	a	b	a	b	c	b	d	c	d	b	b	a	a	d	b	b	c	d
381	**382**	**383**	**384**	**385**	**386**	**387**	**388**	**389**	**390**	**391**	**392**	**393**	**394**	**395**	**396**	**397**	**398**	**399**	**400**
a	b	c	b	b	d	b	a	a	c	b	b	b	b	a	a	a	a	c	b
401	**402**	**403**	**404**	**405**	**406**	**407**	**408**	**409**	**410**	**411**	**412**	**413**	**414**	**415**	**416**	**417**	**418**		
c	a	c	b	a	a	a,d	d	c	c	b	a	a	b	c	b	b	b		